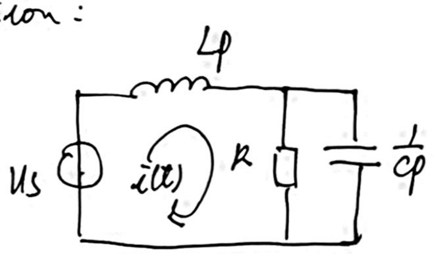
Solution:

1.
2分



—www— 是 $R=Lp$ 的元器件

—||— 是 $R=\frac{1}{cp}$ 的元器件.

[1pt] (1) KCL: $I_R(t)+I_c(t)=i(t)$.

$$i(t)=\frac{U_s(t)}{Lp+\frac{1}{\frac{1}{R}+cp}},\qquad I_R(t)=\frac{U_R(t)}{R},\qquad I_c(t)=\frac{U_c(t)}{\frac{1}{cp}}.$$

$U_R(t)=U_c(t)$. 则

$$\frac{U_c(t)}{R}+\frac{U_c(t)}{\frac{1}{cp}}=\frac{1}{Lp+\frac{1}{\frac{1}{R}+cp}}U_s(t)$$

$$\left(\frac{1}{R}+\frac{1}{\frac{1}{cp}}\right)U_c(t)=\left(\frac{1}{Lp+\frac{1}{\frac{1}{R}+\frac{1}{cp}}}\right)U_s(t).$$

分母为cp

$$\Rightarrow\left(\frac{1}{R}+\frac{1}{\frac{1}{cp}}\right)\left(Lp+\frac{1}{\frac{1}{R}+\frac{1}{cp}}\right)U_c(t)=U_s(t)$$

分母为cp

最终结果没有问题

$$\left(\frac{Lp}{R}+\frac{cp\cdot Lp}{1}+1\right)U_c(t)=U_s(t)$$

$\frac{Lp}{R\cdot C}$

$$\left(\frac{Lp}{R}+cLp^2+1\right)U_c(t)=U_s(t).$$ 整理, 得

$$\left(p^2+\frac{1}{RC}p+\frac{1}{LC}\right)U_c(t)=\frac{1}{LC}U_s(t).$$

[1pt] (2) 上式中, $i_L(t)=i(t)=\frac{1}{Lp+\frac{1}{\frac{1}{R}+cp}}\cdot U_s(t)$

$$\Rightarrow\left(Lp+\frac{1}{\frac{1}{R}+cp}\right)i_L(t)=U_s(t)$$

$$\left(Lp+\frac{R}{1+Rcp}\right)\cdot(1+Rcp)\,i_L(t)=(1+Rcp)U_s(t)$$

$$\Rightarrow\left(Lp+RCLp^2+R\right)i_L(t)=(1+Rcp)U_s(t)$$

$$\Rightarrow\left(p^2+\frac{1}{RC}p+\frac{1}{CL}\right)i_L(t)=\left(\frac{1}{L}p+\frac{1}{RCL}\right)U_s(t)$$

用正常方法算完后改写成p算子形式的也可以给分。

4分 2. $y'(t)+y(t)=f(t)$

$\boxed{1pt}$ ① 特征根为1，故 $y_{zi}=C_1e^{-t}$

因为 y(0+) = y(0-)=3 故 $y_{zi}=3e^{-t}u(t)$， 则 $y_{zs}=2e^{-3t}u(t)$

$\boxed{1pt}$ ② 零输入 $\Rightarrow y'(0_-)=0 \Rightarrow y'(0_-)+y(0_-)=10$

$\Rightarrow y_{zi}=C_1e^{-t}=10\big|_{t=0} \Rightarrow C_1=10 \Rightarrow y_{zi}=10e^{-t}u(t)$

（注意：$y'(0)=10$，$y(0_+)=y(0_-)$.）.

$\boxed{1pt}$ ③ $y_{zs}=2e^{-3(t-2)}u(t-2)$ 时移特性

$\boxed{1pt}$ ④ $y_{zs}=\left(2e^{-3t}u(t)\right)'+3\left(2e^{-3t}u(t)\right)$

$$=2\delta(t)+\cancel{(-3)\cdot 2e^{-3t}u(t)+6e^{-3t}u(t)}=2\delta(t)$$

$\boxed{3pts}$

3. (1) $\boxed{1pt}$

$$F_n=\frac{1}{T}\int_{-\tau}^{\tau}e^{-jn\Omega t}dt=\frac{1}{T}\cdot\frac{1}{-jn\Omega}e^{-jn\Omega t}\Big|_{-\tau}^{\tau}$$

$$=\frac{1}{T}\cdot\frac{1}{-jn\Omega}\left(e^{-jn\Omega\tau}-e^{-jn\Omega(-\tau)}\right)$$

$$=(\text{令 }\theta=n\Omega\tau)\ \frac{1}{T}\cdot\frac{2\tau}{2j\theta}\left(e^{j\theta}-e^{-j\theta}\right)$$

$$=\frac{2\tau}{T}\cdot\frac{\sin\theta}{\theta}$$

$$=\frac{2\tau}{T}\cdot\frac{\sin(n\Omega\tau)}{n\Omega\tau}=\boxed{\frac{2\tau}{T}\cdot Sa(n\Omega\tau)\xlongequal{T=4\tau}\frac{1}{2}Sa(n\Omega\tau)}$$

欧拉公式

$\sin\theta=\frac{1}{2j}\left(e^{j\theta}-e^{-j\theta}\right)$

$n\Omega=\frac{\theta}{\tau}$

$n\Omega\tau=\pi$　$\Omega=\frac{2\pi}{T}$

$\omega\tau=\pi$　$\omega=\frac{\pi}{\tau}$

$\omega\tau=\pi$

$n\Omega=\frac{\pi}{\tau}$

$n\cdot\frac{2\pi}{T}=\frac{\pi}{\tau}$

$n=\frac{T}{\tau}\cdot\frac{1}{2}=4\cdot\frac{1}{2}=2$

(2) $\boxed{1pt}$

给分点：（可酌情给分）

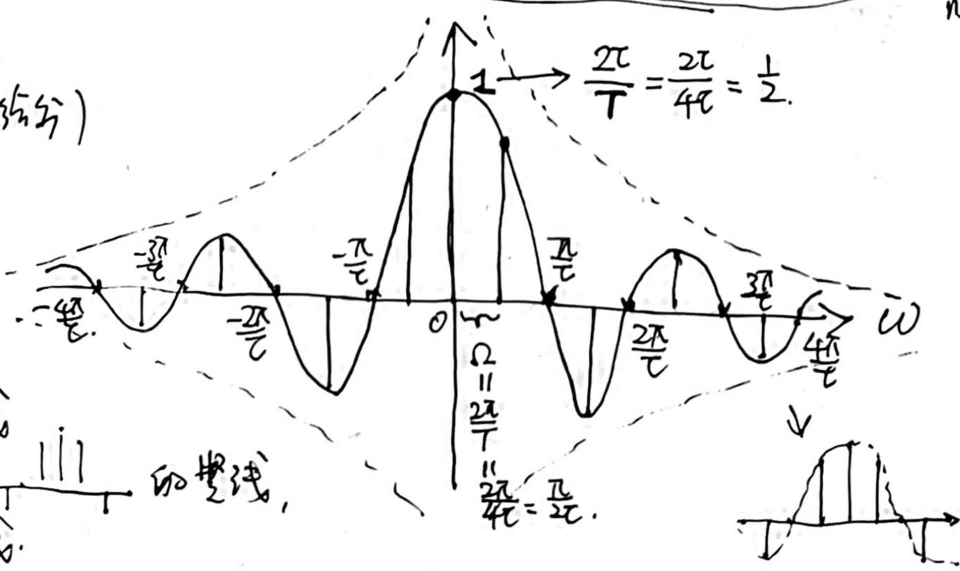
① Sinc 包络

② 幅度 $\frac{1}{2}$

③ 第一个0点 $\frac{\pi}{\tau}$

④ $0\to\frac{\pi}{\tau}$ 有两条线

⑤ 最终答案是 的谱线，少线画出谱线.



(3) $\boxed{1pt}$. $T\to 2T$，包络线不变，采样的线密一倍。

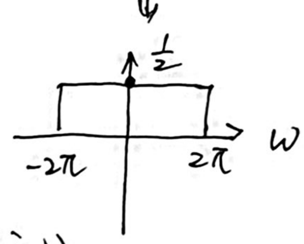
4. Solution

(1) 2pts

设 $x(t) = \dfrac{\sin Wt}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow} X(j\omega) = \begin{cases} 1 & |\omega| \le W \\ 0 & |\omega| > W. \end{cases}$

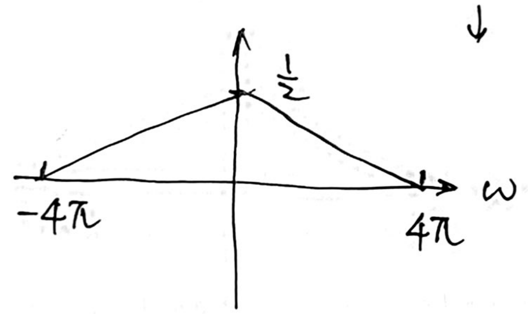
$g(t) = \dfrac{\sin 2\pi t}{2\pi t} = \dfrac{1}{2}\dfrac{\sin 2\pi t}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow} G(j\omega) = \begin{cases} \frac{1}{2} & |\omega| \le 2\pi \\ 0 & |\omega| > 2\pi \end{cases}$

("$W = 2\pi$").



[时域卷积定理]

$f(t) = (g(t))^2 \longleftrightarrow \mathcal{F}\{j\omega\} = \dfrac{1}{2\pi} G(j\omega) * G(j\omega)$



注意：$\displaystyle\int_{-2\pi}^{2\pi} \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2\pi} = 4\pi \cdot \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2\pi} = \frac{1}{2}$. 所以幅度为$\frac{1}{2}$.

写成：$\mathcal{F}\{j\omega\} = \begin{cases} \frac{1}{2}\left(1 - \frac{t}{4\pi}\right) & |t| \le 4\pi \\ 0 & |t| \ge 4\pi \end{cases}$ 也可得分，画图和公式均可。

(2) $\displaystyle\int_{-\infty}^{+\infty} f(t)\,dt = \mathcal{F}\{j\omega\}\Big|_{\omega=0} = \mathcal{F}\{j0\} = \frac{1}{2}$ (上图)

2pts

$-p-4+2p+6$

5. Solution: (1) $\boxed{2pts}$

4分 $(p^2+7p+12)y(t) = (p+2)f(t)$

$$H(p) = \frac{p+2}{p^2+7p+12} = \frac{p+2}{(p+3)(p+4)} = \frac{-1}{p+3} + \frac{2}{p+4}$$

$$\Rightarrow h(t) = -e^{-3t}u(t) + 2e^{-4t}u(t).$$

(2) $\boxed{2pts}$

$$H(j\omega) = \frac{-1}{j\omega+3} + \frac{2}{j\omega+4}. \qquad F(j\omega) = \frac{6}{j\omega+1}$$

$$Y_{zs}(j\omega) = H(j\omega)F(j\omega) = \frac{6}{j\omega+1}\left(\frac{-1}{j\omega+3} + \frac{2}{j\omega+4}\right)$$

$$= \frac{1}{j\omega+1} + \frac{3}{j\omega+3} + \frac{-4}{j\omega+4}.$$

$$\Rightarrow y_{zs}(t) = [e^{-t}+3e^{-3t}+e^{-4t}]u(t). = (e^{-t}+3e^{-3t}-4e^{-4t})u(t)$$

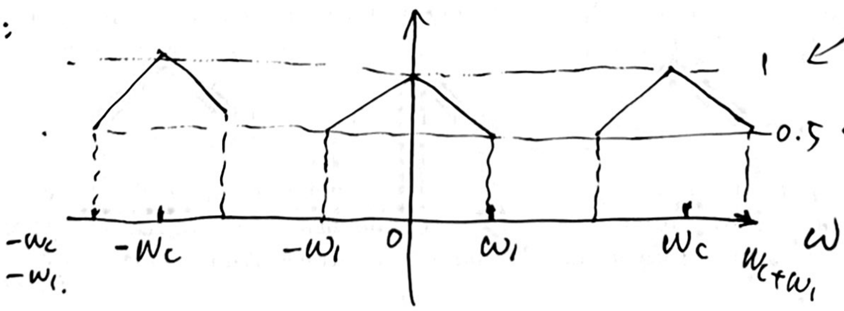
-4

（如果用时域卷积做出来也可得分。）

6. (1) $\boxed{1pt}$ $f_s \geq 2\max f \Rightarrow f_s \geq 2\frac{\omega_1}{2\pi} = \frac{\omega_1}{\pi}.$

3分 (2) $\boxed{1pt}$ 一个域里面的抽样是另一个域里面的重复。

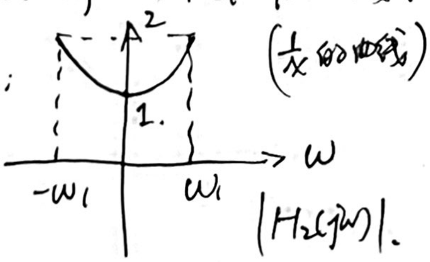
故：



需要标出来，有没有Ts关系不大，主要体现相对变化

(3) $\boxed{1pt}$ 理想无失真传输：恒定幅度，线性相位。故：

~~相位~~ 幅度谱：



（反的曲线）

相位谱是一个线性的