R Program for Calculating and Plotting a Parametric Curve

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Task

Write an R program to calculate the coordinates of points on a parametric curve and display the curve length.

Description

A parametric curve is given by the following equations:

$$x(t) = \sin(2t)$$
$$y(t) = \cos(t)$$

The parameter range t is from 0 to $\frac{\pi}{2}$. The program should:

1. Calculate and display the coordinates of the curve points for various values of t, dividing the interval into 50 equal segments.
2. Calculate the arc length of the curve using the following formula:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Sample Expected Output

1. The coordinates of the curve points should be displayed as follows:

```
t = 0.00000000 : (x, y) = (0.000000e+00, 1.000000e+00)
t = 0.03205707 : (x, y) = (6.407022e-02, 9.994862e-01)
t = 0.06411414 : (x, y) = (1.278772e-01, 9.979454e-01)
...
...
t = 1.57079633 : (x, y) = (1.224647e-16, 6.123234e-17)
Length of the curve: 2.357358
```

Hints

1. Use the derivatives for $\frac{dx}{dt}$ and $\frac{dy}{dt}$ as follows:

$$\frac{dx}{dt} = 2\cos(2t)$$

$$\frac{dy}{dt} = -\sin(t)$$

2. You may use the `plot` command or the `ggplot2` package to plot the curve.
3. Use the `integrate` function to perform the integration.
4. Commenting (adding brief explanations of the code) will be part of the evaluation.