Mission to the Epsilon Planetary System

Introduction

In the year 2403, deep within the Milky Way galaxy, a planetary system named **Epsilon** has been discovered. This system consists of three planets with fascinating and unique orbits around a shining star called **Io**. Astronomers are eager to study the behavior of these planets and their movements in their respective orbits. Your mission in this project is to model the motion of these planets, mathematically analyze the related parameters, and perform numerical computations. You are required to complete this mission using the **R programming language**.

Project Objectives

1. Analyze and model the motion of planets in the Epsilon system using **polar coordinates**.
2. Use **numerical differentiation** to analyze the velocity and acceleration of the planets.
3. Employ **numerical integration** to calculate the arc length of the orbits.
4. Visualize and plot the trajectories of the planets.

Mission Details

Level 1: Defining the Orbits of the Planets

Three planets named **Compass**, **Navigator**, and **Gravity Center** orbit their star as follows:

- **Compass:**
$$r(t) = 3 + 2 \cdot \sin(t), \quad \theta(t) = 2t$$
- **Navigator:**
$$r(t) = 4 + \cos(3t), \quad \theta(t) = t^2$$
- **Gravity Center:**
$$r(t) = 5 - \sin(2t), \quad \theta(t) = 3t$$

Your Tasks:

1. Convert the polar equations for each planet into Cartesian coordinates (x, y) using the formulas:
$$x(t) = r(t) \cdot \cos(\theta(t)), \quad y(t) = r(t) \cdot \sin(\theta(t))$$
2. Compute the (x, y) coordinates for each planet over the time interval $[0, 2\pi]$ divided into 50 points.
3. Plot the trajectory of each planet in Cartesian coordinates.

Level 2: Calculating Arc Length of Orbits

Calculate the arc length of each orbit using the formula:

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Your Tasks:

1. Use numerical integration methods to compute the arc length of each orbit over the interval $[0, 2\pi]$.

Level 3: 3D Modeling of the Epsilon System

To enhance visualization, simulate the motion of the planets in a 3D space. Assume the $z(t)$ coordinates of each planet are as follows:

- **Compass:**
$$z(t) = \cos(t)$$
- **Navigator:**
$$z(t) = \sin(2t)$$
- **Gravity Center:**
$$z(t) = \sin(t) + \cos(t)$$

Your Tasks:

1. Calculate the 3D coordinates (x, y, z) for each planet.
2. Plot the 3D motion of the planets using the **plotly** library.

Level 4: Analyzing Velocity and Acceleration (Bonus Points)

Use numerical differentiation to calculate the velocity and acceleration of the planets:

- **Velocity Formula:**
$$v(t) = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$
- **Acceleration Formula:**
$$a(t) = \frac{dv(t)}{dt}$$

Your Tasks:

1. Calculate the instantaneous velocity $v(t)$ and acceleration $a(t)$ of each planet over the interval $[0, 2\pi]$ using numerical methods.
2. Plot the velocity and acceleration for each planet separately.

Important Notes

1. The project can be completed individually or in teams of two. In the case of teamwork, only the team leader should submit the project.
2. An online presentation of the project is mandatory. Failure to present will result in no grade being awarded.
3. The online presentation schedule will be announced at the end of the semester.
4. Submitting a project explanation document is mandatory.
5. A portion of the grade will be based on commenting and documentation in the R code.
6. The final submission must include the R code files and a PDF containing project explanations.