Chapter 1: Systems of Linear Equations Worksheet

Note: Since this may be your first discussion of the semester, this worksheet is slightly longer than usual.

1. Are each of the following statements true or false?

(a) The system $\{2x + 4y = 0, 3x + 4y = 1\}$ has the same solution set as the system $\{3x + 4y = 1, 2x + 4y = 0\}$.

(b) A linear system with two equations and three unknowns will always have infinitely many solutions.

(c) The system $\{x - 2y = -2, 2x + y = 3\}$ has the same solution set as $\{x - 2y = -2, (2x + y)c = 3c\}$ for any choice of number c.

2. Consider the following system of linear equations.

$$\begin{cases} 2x - 5y + z &= 1 \\ 3x - 4y + z &= 0 \end{cases}$$

(a) Verify that setting $x = -2$, $y = 1$, and $z = 10$ gives a solution to this system of equations.

(b) More generally, show that for any number $y = r$, setting $x = -r - 1$, $y = r$, and $z = 7r + 3$ gives a solution to this system of equations.

3. A system of equation is ***consistent*** if there exists at least one solution to all equations in the system, and ***inconsistent*** if there exists no solution with this property.

Given the linear system

$$\begin{cases} 5x - 3y &= s \\ -10x + 6y &= t, \end{cases}$$

(a) Determine particular values of s and t so that the system is consistent.

(b) Determine particular values of s and t so that the system is inconsistent.

(c) What relationship between the values of s and t will guarantee that the system is consistent?

4. Identify which systems of equations are ***consistent***, and which are ***inconsistent***:

(a) $x - y = 0$.

(b) $\{x + y = 2, x - z = 3\}$.

(c) $\{x + y = 1, 2x + 2y = 3\}$.

5. Graph the following three equations on the xy-plane, and use these graphs to find the unique solution to the system of these equations:

$$\begin{cases} 2x + y &= 4 \\ x - y &= 2 \\ x + 2y &= 2 \end{cases}$$

6. Recall that a system of equations

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + & \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + & \cdots + a_{2n}x_n = b_2 \\ & \vdots \\ a_{m1}x_1 + a_{m2}x_2 + & \cdots + a_{mn}x_n = b_m \end{cases}$$

is ***homogeneous*** if $b_1 = b_2 = \cdots = b_m = 0$.

(a) Consider the following homogeneous system of linear equations in 2 variables.

$$\begin{cases} 4x + 5y &= 0 \\ 2x + 3y &= 0 \end{cases}$$

Is the system consistent or inconsistent?

(b) Consider the following homogeneous system of linear equations in 3 variables.

$$\begin{cases} 2x - 3y + 5z &= 0 \\ 3x - 5y + 2z &= 0 \\ -7x + y - 3z &= 0 \end{cases}$$

Is the system consistent or inconsistent?

(c) Is every homogeneous linear system always consistent? Explain.