Math 421

Monday, September 15

Announcements

- Homework 2 due Friday, September 19 at 11:59 pm
- Drop-in hours
  - Wednesday 12-1 pm in VV 311
  - Thursday 9-11 in VV B205
- MLC
  - Proof Table (M-Th 3:30-7 VV B227)
  - Course Assistant (Th 4-7 Table 4)

Example 2 – Discussion

Theorem: If $n \in \mathbb{N}$, then $n! \leq n^n$.

- What proof technique should we use? *Why*?

- Do you believe the statement? Verify for $n = 2, 3$.

- What is the base case?

- What is the inductive assumption?

Note: In simple cases we don't need to explicitly state P_n.

Example 2 – Proof

Proof.

Activity – Proof

Theorem: For all $n \in \mathbb{N}$,

$$\sum_{j=1}^{n}(2j-1) = n^2.$$

Example 3 - Discussion

Theorem: For all $n \in \mathbb{N}$ and $n \geq 4$, we have $2^n < n!$.

Note: We can use any number as the “base case”.

- What proof technique should we use? *Why*?

- Do you believe the statement? *Why does it start at 4*?

- What is the base case?

- What is the inductive assumption?

Example 3 – Proof

Proof.

Definition – Power Set

Def: The power set, $\mathcal{P}(S)$, of a set S is a set of all subsets of S.

Examples

1. $S = \{a\}, \mathcal{P}(S) =$

2. $T = \{1,2\}, \mathcal{P}(T) =$

Group Activity

Theorem: Let $n \in \mathbb{N}$. The power set of a set S with n elements has 2^n elements.

1. Convince yourselves this statement is true.
2. This statement can be proven using a proof by induction.
   A. What is the base case?
   B. Assume P_2 is true, how would you prove P_3 is true?
   C. Can you generalize this to the case where you assume P_k is true and prove that P_{k+1} is true?