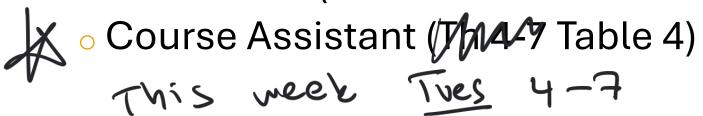
Math 421

Monday, September 15

Announcements

- Homework 2 due Friday,
 September 19 at 11:59 pm
- Drop-in hours
 - Wednesday 12-1 pm in VV 311
 - Thursday 9-11 in VV B205
- MLC
 - Proof Table (M-Th 3:30-7 VV B227)



Example 2 – Discussion

Theorem: If $n \in \mathbb{N}$, then $n! \le n^n$. P_n : $n! \le n^n$

- What proof technique should we use? Why?
- ightharpoonup Do you believe the statement? Verify for n=2,3./

2 4 4 / 6 = 27 /

What is the base case?

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* What is the inductive assumption? What is the conditional statement you are trying to prove.

17 n! < n + then
(n+1)! < (n+1).

Note: In simple cases we don't need to explicitly state P_n .

N=11 IF K > K+11

Pn: "n! = n""

Example 2 – Proof

Proof. We argue by induction.

Base Case: If N=1, then 1!=1=1. So true for N=1.

If $N! \leq N^n$ then $(N+1)! \leq (N+1)!$ Induction Step Suppose $k! \leq k$ for some $k \in \mathbb{N}$. Then

LHS rewrite to use induction hypothesis $(k+1)! \leq k! (k+1) \leq k' (k+1) \leq (k+1)! \leq (k+1) = (k+1)!$ LHS rewrite to use induction hypothesis $(k+1)! \leq k! (k+1) \leq k' (k+1)! \leq (k+1)! \leq (k+1)! \leq (k+1)!$ The properties of the properties of

Therefore $n! = n^n$ for all $n \in \mathbb{N}$ by the principle of mathematical induction.

Activity — Proof then $1+3+5+\cdots+(2k-1)=k^2$ Theorem: For all $n \in \mathbb{N}$, algebra.

 $(2(k+1)-1) = (k+1)^2$

$$\sum_{j=1}^{n} (2j-1) = n^2.$$

 $17 \stackrel{k}{\lesssim} (2j-i) = k^2 + hen \stackrel{k+1}{\lesssim} (2(j)-i) = (k+i)^2$

$$\sum_{j=1}^{k+1} (z_{j} - 1) = (2(k+1)-1) + \sum_{j=1}^{k} (z_{j} - 1) = (2(k+1)-1) + k^{2}$$

$$= 2k+2-1+k^{2} = |k^{2}+2k+1| = (k+1)^{2}. \Box$$

Example 3 - Discussion

Theorem: For all $n \in \mathbb{N}$ and $n \ge 4$, we have $2^n < n!$.

Note: We can use any number as the "base case".

What proof technique should we use? Why?

- Do you believe the statement? Why does it start at 4?
- What is the base case?
- What is the inductive assumption?

Example 3 – Proof

Proof. We argue by induction.

Base Case: Let n=4 $2^4=16 < 24=4!$ Induction Step: Suppose $2^k \ge k!$ for some $k \in \mathbb{N}$,

with $(k \ge 4)$ Then, $2^{k+1} = 2 \cdot 2^k \ge 2 \cdot k! < (k+)k! = (k+1)!$

Therefore, Z" < N! is true for all N = 4.

Definition - Power Set

Def: The power set, $\mathcal{P}(S)$, of a set S is a set of all subsets of S.

Examples

1.
$$S = \{a\}, \mathcal{P}(S) =$$

2.
$$T = \{1,2\}, \mathcal{P}(T) =$$

Group Activity

Theorem: Let $n \in \mathbb{N}$. The power set of a set S with n elements has 2^n elements.

- 1. Convince yourselves this statement is true.
- 2. This statement can be proven using a proof by induction.
 - A. What is the base case?
 - B. Assume P_2 is true, how would you prove P_3 is true?
 - C. Can you generalize this to the case where you assume P_k is true and prove that P_{k+1} is true?