Math 421

September 8, 2025

Announcements

- Homework 1 due Friday
 - LaTeX is optional, but encouraged for proofs!
- MLC opens today
 - Proof table
 - Mon-Thurs: 3pm-7:30pm VV B227
 - Course Assistant
 - Thurs: 4-7 in MLC
- Errors on Set Basics & 1.1
 Reading Quizzes

Conditional

If P and Q are statements, then the statement if P, then Q is

- True when either
 - lacktriangle P and Q are both true
 - P is false
- \circ False when P is true and Q is false

We sometimes write " $P \Rightarrow Q$ " or "P implies Q".

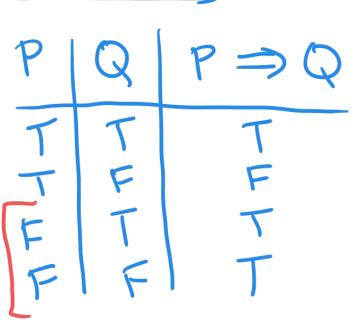
True or False?

1. If f is differentiable, then f is continuous.

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2. If x is prime, then x is odd.





Biconditional/Equivalence

If P and Q are statements, then the statement P if and only if Q is

- \circ True when both P and Q are either both true or both false
- False otherwise

We sometimes write " $P \Leftrightarrow Q$ " or "P iff Q".

P	Q)	$P \iff Q$		
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True or False?

1. A function f is differentiable if and only if it is continuous.



2. A number is divisible by 6 if and only if it is divisible by 2 and 3.



Converse & Contrapositive **Definition:** The converse of "if P, then Q" is "if Q, then P".

Definition: The contrapositive of "if P, then Q" is "if not Q, then not *P*". P >> Q not Q >> not P

Example: If f is differentiable, then f is continuous, \top Converse:

If f is continuous, then f is differentiable F

Contrapositive:

If f is not continuous, then f is not differentiable.

Q: Is a statement equivalent to its converse? Its contrapositive? Zint tes.

1f x is divisible by 6, Activity Activity by 2 & 3.

Use the following truth table to show that " $P \Rightarrow Q$ " is logically equivalent to the contrapositive "not $Q \Rightarrow$ not P"

P	Q	not P	$\operatorname{not} Q$	$P \Rightarrow Q$	$notQ \Rightarrow notP$
Т	Т	F	F	T	T
Т	F	F	T	F	F
F	Т	T	F	T	T
F	F	7	7	T	T

Pairs Activity

Person A: You only know the following information –

- The definition of the integers, i.e. that it is the set $\{..., -2, -1, 0, 1, 2, ...\}$
- The operations of addition, +, and multiplication, ·, and their basic properties, i.e. commutativity, associativity, distributive law, etc.

Person B: Convince Person A that the following theorem is true **Theorem:** If x is an even integer, then x^2 is an even integer.

Definitions, Theorems & Proofs

Much of class will have the following structure

- 1. We will introduce a mathematical object using a definition.
 - For yourself: Think of an example and a non-example.
- 2. We will state a property of it using a theorem.
 - For yourself: What are the hypotheses and why are they needed? What is the <u>conclusion</u>?
- 3. We will explain why the property is true using a proof.
 - **For yourself:** What proof technique is being used? What is the big idea? What common strategies are being employed?

Even & Odd – Definition & Examples

Definition:

- An integer x is **even** if there exists an integer k such that x = 2k.
- An integer x is **odd** if there exists an integer k such that x=2k+1.

Examples:

$$7 \text{ is odd, because}$$
 $2.3+1=7.$
 $k=3.$

· integers are closed under addition and multiplication

Even & Odd - Theorem

Theorem: If x is an even integer, then x^2 is an even integer.

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Proof: Let x be an even integer. There fore there exists an integer k such that x = 2k.

Thus x^2 = x \cdot x = (2k)(2k) = 4k^2 = 2(2k^2).

So, x^2 = 2m where m = 2k^2 is an integer and there fore x^2 is an even integer.
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Activity

Theorem: Suppose x is an integer. If x is odd, then $x^2 + 1$ is even. *Discussion:*

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Proof: let x be an odd integer. Then there exists an integer k such that x = 2k+1.

Thus x^2+1 = (2k+1)(2k+1)+1 = 4k^2+4k+1+1
= 2(2k^2+2k+1).

Therefore, x^2+1=2m where m=2k^2+2k+1 is an integer. Thus x^2+1 is an even integer.
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