Chapter 2: Solving systems using matrices Worksheet

1. Are each of the following statements true or false?

(a) $\left[\begin{array}{cc|c} 2 & 8 & 6 \\ 2 & 3 & 2 \end{array}\right]$ The systems of linear equations represented by the augmented matrices $\left[\begin{array}{cc|c} 2 & 8 & 6 \\ 2 & 3 & 2 \end{array}\right]$ and $\left[\begin{array}{cc|c} 1 & 4 & 3 \\ 0 & 1 & \frac{4}{5} \end{array}\right]$ have the same set of solutions.

(b) If A is a 3×2 matrix, it's possible for the rank of A to be 3.

(c) A homogeneous system of 2 equations in 3 unknowns always has nontrivial solutions.

(d) If A is a 3×3 matrix with rank 2, then the homogeneous system with coefficient matrix given by A will have only the unique trivial solution.

2. Perform row operations on each matrix to reduce it to a row echelon form. Afterwards, find the reduced row echelon form for the matrices. What is the rank of each?

(a) $A = \begin{bmatrix} 0 & 1 \\ 1 & -2 \\ 2 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & 2 & 2 & -2 \\ 4 & -5 & -5 & 5 \\ 0 & -4 & -4 & 4 \end{bmatrix}$

3. Solve each system of equations by 1.) putting it into an augmented matrix and 2.) Using Gaussian Elimination to get the system into row-echelon form and 3.) Extracting the solutions. If a system has no solutions, explain why. If a system has multiple solutions, describe the family of solutions in terms of a free variable/parameter.

(a)
$$\begin{cases} 4x + y + 3z & = 5 \\ 2x - 2z & = 4 \\ 8x + y - z & = 9 \end{cases}$$

(b)
$$\begin{cases} 4x + y + 3z & = 5 \\ 2x - 2z & = 4 \\ 8x + y - z & = 13 \end{cases}$$

4. Find a nontrivial solution of the homogeneous system represented by the augmented matrices. If there is no nontrivial solution, explain why.

(a) $\left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 5 & 2 & -1 & 0 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 2 & 5 & 0 & 0 \\ -3 & 3 & 1 & 0 \end{array}\right]$

5. Find the general solution and basic solutions to the homogeneous system represented by the augmented matrix $\left[\begin{array}{cccc|c} 1 & 1 & -1 & 5 & 0 \\ 2 & -1 & -14 & 1 & 0 \\ -3 & 1 & 19 & -3 & 0 \end{array}\right]$

6. Given the matrix $M = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$, find two 3×3 (coefficient) matrices A_1 and A_2, each with the above reduced row echelon form M, but $\left[\begin{array}{c|c} \quad A_1 \quad & \begin{matrix} 1 \\ -2 \\ 3 \end{matrix} \end{array}\right]$ has no solution, and $\left[\begin{array}{c|c} \quad A_2 \quad & \begin{matrix} 1 \\ -2 \\ 3 \end{matrix} \end{array}\right]$ has infinitely many solutions.