Math 421

Wednesday, September 17

Announcements

- Homework 2 Due Friday
- Course assistant office hours had been moved to Tuesday this week – so none tomorrow.
- Grace’s drop-in hours
    - Today, 12-1 VV 311
    - Tomorrow, 9-11 VV B205

Chapter 1: Introduction

Section 2: The Set $\mathbb{R}$ of Real Numbers

Real Numbers: $\mathbb{R}$

Axioms:

- Associative laws (**A1.** & **M1.**)
- Commutative laws (**A2.** & **M2.**)
- Existence of an identity element (**A3.** & **M3.**)
- Existence of inverse elements (**A4.** & **M4.**)
- Distributive law (**DL**)
- Closure under addition and multiplication
- Order structure (**O1.**-**O5.**)

Example Proof: $\sqrt{2} \notin \mathbb{Q}$

Thm: $\sqrt{2}$ is irrational.

Proof.

Discussion Questions

Answer the following questions, if the answer is yes or the statement is true, you do not need to provide a proof. If the answer is no or the statement is false, you should provide a counterexample.

- Are the rational numbers closed under multiplication? Closed under addition?
- Are the irrational numbers closed under multiplication? Closed under addition?
- **True or False:** If x is rational and y is irrational, then $x + y$ is irrational.
- **True or False:** if x is rational and y is irrational, then xy is irrational.

Absolute Value & Distance

Def: The **absolute value function** is defined by

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Notice that for all $x \in \mathbb{R}$

- $|-x| = |x|$
- $-|x| \leq x \leq |x|$

Example Proof: Triangle Inequality

Thm. (The Triangle Inequality) If $x, y \in \mathbb{R}$, then $|x + y| \leq |x| + |y|$.

Proof.

Example: Proof (cont.)

Group Activity – Proof Practice

Thm. If $x, y \in \mathbb{R}$, then $|xy| = |x||y|$.

Form groups of 4.

- What proof technique should you use? *Why?*

- Divide the cases up among the group – prove your case individually and then share back out with the group to compare and contrast.

Distances

If $a, b \in \mathbb{R}$, then $|a - b|$ is the distance between a and b.

Notice that if $c \in \mathbb{R}$, then

$$|a - b| = \qquad \leq |a - c| + |c - b|$$

Proof Practice

Thm. Suppose $a, b \in \mathbb{R}$. Then $a = b$ if and only if for every positive real number $\varepsilon > 0$ we have $|a - b| < \varepsilon$.

Discussion.

- What is this theorem saying in your own words? Do you believe it?
- What are the two conditional statements we will have to prove?

Proof Practice

Proof.