Math 421

September 8, 2025

Announcements

- Homework 1 due Friday
  - LaTeX is optional, but encouraged for proofs!
- MLC opens today
  - Proof table
    - Mon-Thurs: 3pm-7:30pm VV B227
  - Course Assistant
    - Thurs: 4-7 in MLC
- Errors on Set Basics & 1.1 Reading Quizzes

Conditional

hypothese

conclusion

If P and Q are statements, then the statement ***if P, then Q*** is

- True when either
  - P and Q are both true
  - P is false
- False when P is true and Q is false

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

We sometimes write "$P \Rightarrow Q$" or "P implies Q".

True or False?

1. If f is differentiable, then f is continuous.

   T

2. If x is prime, then x is odd.

   F

Biconditional/Equivalence

If P and Q are statements, then the statement ***P if and only if Q*** is

- True when both P and Q are either both true or both false
- False otherwise

We sometimes write "$P \Leftrightarrow Q$" or "P iff Q".

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

True or False?

1. A function f is differentiable if and only if it is continuous.

   F

2. A number is divisible by 6 if and only if it is divisible by 2 and 3.

   T

Converse & Contrapositive

$P \Rightarrow Q$

$Q \Rightarrow P$

Definition: The converse of "if P, then Q" is "if Q, then P".

Definition: The contrapositive of "if P, then Q" is "if not Q, then not P".

$P \Rightarrow Q$ not $Q \Rightarrow$ not P

Example: If $\underbrace{f \text{ is differentiable}}_{P}$, then $\underbrace{f \text{ is continuous}}_{Q}$. T

Converse:

If f is continuous, then f is differentiable F

Contrapositive:

If f is not continuous, then f is not differentiable. T

Q: Is ~~a~~ this statement equivalent to its converse? Its contrapositive?

No. Yes.

Activity

If x is divisible by 6, then x is divisible by 2 & 3.

Use the following truth table to show that "$P \Rightarrow Q$" is logically equivalent to the contrapositive "not $Q \Rightarrow$ not P"

P	Q	not P	not Q	$P \Rightarrow Q$	not $Q \Rightarrow$ not P
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Pairs Activity

Person A: You **only** know the following information –

- The definition of the integers, i.e. that it is the set $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$
- The operations of addition, $+$, and multiplication, $\cdot$, and their basic properties, i.e. commutativity, associativity, distributive law, etc.

Person B: Convince Person A that the following theorem is true

Theorem: If x is an even integer, then x^2 is an even integer.

- No division :(
- Define even integer
- define exponentiation

Definitions, Theorems & Proofs

Much of class will have the following structure

1. We will introduce a mathematical object using a **definition**.
   - **For yourself:** Think of an example and a non-example.
2. We will state a property of it using a **theorem**.
   - **For yourself:** What are the hypotheses and why are they needed? What is the conclusion?
3. We will explain why the property is true using a proof.
   - **For yourself:** What proof technique is being used? What is the big idea? What common strategies are being employed?

Even & Odd – Definition & Examples

Definition:

- An integer x is ***even*** if there exists an integer k such that $x = 2k$.
- An integer x is ***odd*** if there exists an integer k such that $x = 2k + 1$.

Examples:

- Even: 2, -4, 364, 0
- Odd: -17, 1, 7

9 → k=4

7 is odd, because
2·3+1 = 7.
k=3.

• integers are closed under addition and multiplication

Even & Odd – Theorem

Theorem: If x is an even integer, then x^2 is an even integer.

Proof: Let x be an even integer. Therefore there exists an integer k such that $x = 2k$.

Thus $x^2 = x \cdot x = (2k)(2k) = 4k^2 = 2(2k^2)$.

So, $x^2 = 2m$ where $m = 2k^2$ is an integer and therefore x^2 is an even integer.

Activity

Theorem: Suppose x is an integer. If x is odd, then $x^2 + 1$ is even.

Discussion:

Proof: Let x be an odd integer. Then there exists an integer k such that $x = 2k+1$.

Thus $x^2+1 = (2k+1)(2k+1)+1 = 4k^2+4k+1+1$

$= 2(2k^2+2k+1)$.

Therefore, $x^2+1 = 2m$ where $m = 2k^2+2k+1$ is an integer. Thus x^2+1 is an even integer.