

MATH 421

Announcements

September 5, 2025

- Syllabus reading quiz was due at 9:00 am today
- Introduce yourself! (Before class, after class, during drop-in hours)
- Logic basics reading quiz due Monday – relates to content we will cover today.
- Other assignments will be posted later today (reading quizzes for next week's content and first homework).

Definitions

Definition: A **definition** is an agreement about the meaning of a particular (mathematical) word.

Q: What agreements would we have to make in order to formulate a definition for a sandwich?

- what is "bread"?
- same material or different?

Statements

Definition: A **statement** is a declarative sentence which is either true or false but not both.

Example: Our section of Math 421 is held in B135.

Today is Friday. $1+1=2$. Grass is blue.

Non-example:

This statement is false. Purple is the best

color. $1+1$. $2x^2$. $x+y=z$. $2x^2=8$

"If $x=2$, then $2x^2=8$ "

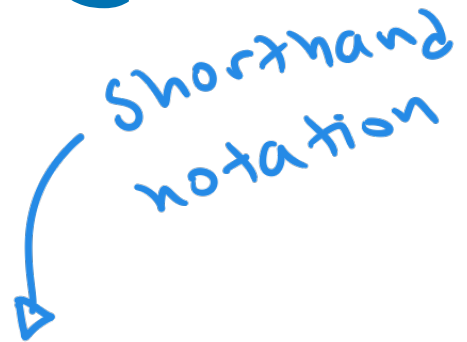
Group Activity: True or False?

Form groups of 4-6 (ideally with different people from Wednesday!). Introduce yourselves and then as a group, determine the truth value of the following statements:

- [Insert your name] is a math major.
- Every person in your group drank coffee today.
- There exists a person in your group who was not born in Wisconsin.
- There exists a unique person in your group who [complete this statement so that it is true].
- There does not exist a person in your group who [complete this statement so that it is true].

Quantifiers

Shorthand
notation



\forall For all integers x , $2x^2 = 8$. F

\exists There exists an integer x , such that $2x^2 = 8$. T

$\exists!$ There exists a unique integer x , such that $2x^2 = 8$. F

$$2x^2 = 8$$

If $2x^2 = 8$ is true, then ...

If $2x^2 = 8$ is false, then ...

Negation

2, 3,
5, 7

* the negation of a "for all" statement involves a "there exists" statement.

$\neg P$
 $\sim P$

Truth table

P	not P
T	F
F	T

If P is a statement, then the statement **not P** is

- True when P is false
- False when P is true

Examples:

- P : 6 is an even number.

Not P : 6 is not an even number OR 6 is an odd number.

- Q : Every number is an even number. F

Not Q : There exists an odd number. T

Every is an odd number F
not the negation.

Conjunction

$$P \wedge Q$$

If P and Q are statements, then the statement **P and Q** is

- True when P and Q are both true
- False when at least one of P and Q is false

P	Q	$P \text{ and } Q$
T	T	T
T	F	F
F	T	F
F	F	F

True or False?

1. For all integers x , x is even and x is prime.

2. There exists an integer x such that x is even and x is prime.

$T \wedge F$ a unique

Disjunction

$P \vee Q$

If P and Q are statements, then the statement **P or Q** is

- True when at least one of P and Q is true
- False when P and Q are both false

P	Q	$P \text{ or } Q$
T	T	T
T	F	T
F	T	T
F	F	F

True or False?

1. For all integers x , x is even or x is prime.

F

2. There exists ^{a unique} an integer x such that x is even or x is prime.

T. F.

Activity

Complete the following truth table to show that “not(P or Q) is logically equivalent to “not P and not Q ”

P	Q	not P	not Q	P or Q	not(P or Q)	not P and not Q
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T