171.107 Quiz 4 Solutions

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October 7, 2016

Figure 1: Bird's-eye view of the spring system.

Quiz Question

A block with mass $m = 10$ kg rests on a frictionless table and is accelerated by a spring with spring constant $k = 2500$ N/m after being compressed a distance $x_1 = 0.5$ m from the spring's unstretched length. The floor is frictionless except for a rough patch a distance $d = 2$ m long. For this rough path, the coefficient of friction is $\mu_k = 0.5$.

1. How much work is done by the spring as it accelerates the block?
2. What is the speed of the block right after it leaves the spring?
3. What is the speed of the block after it passes the rough spot?

Quiz Solution

The system in question is shown in Figure 1. As usual, no calculators were allowed and we let $g = 10$ m/s^2.

1. The work done by the spring is equivalent to the change in its potential energy, that is,

$$W = \Delta U = \frac{1}{2} k x_1^2 \qquad \boxed{\text{2 pts for eqn}}$$
$$= \frac{1}{2} \times 2500 \times 0.5^2$$
$$= \boxed{312.5 \text{ J.}} \qquad \boxed{\text{1 pt for answer + } \tfrac{1}{3} \text{ pts for units}}$$

2. To find the speed of the block, we set the change in potential energy above equal to the kinetic energy of the block and solve for the speed:

$$\frac{1}{2}kx_1^2 = \frac{1}{2}mv^2$$
$$\implies v = \sqrt{\frac{kx^2}{m}} \qquad \boxed{\text{2 pts for eqn}}$$
$$= \sqrt{\frac{2500 \times 0.25}{10}}$$
$$= \boxed{\sqrt{62.5}\ \text{m/s.}} \qquad \boxed{\text{1 pt for answer + } \tfrac{1}{3} \text{ pts for units}}$$

3. To find the speed after the block has passed the rough spot, we find the change in energy due to the work done by the rough spot on the block and subtract that from the energy of the block before reaching the rough spot. We find for the final speed v_f:

$$\Delta E = \frac{1}{2}kx_1^2 - \mu_k mgd = \frac{1}{2}mv_f^2$$
$$\implies v_f = \sqrt{\frac{kx_1^2 - 2\mu_k mgd}{m}} \qquad \boxed{\text{2 pts for eqn}}$$
$$= \sqrt{\frac{2500 \times 0.25 - 1 \times 10 \times 10 \times 2}{10}}$$
$$= \boxed{\sqrt{42.5}\ \text{m/s.}} \qquad \boxed{\text{1 pt for answer + } \tfrac{1}{3} \text{ pts for units}}$$