Problem 3

a) T quantile (T* a, int left, int right, int k){

1. if (right ≤ left + 10){ → $\Theta(1)$
   InsertionSort (a, left, right); → $\boxed{\Theta(n^2)}$ → only for small arrays
   return a[k+left]; → $\Theta(1)$
   }

   else{

   int smallsize = (right - left)/5; → $\Theta(1)$
   T* b = new T[smallsize]; → $\Theta(1)$
   for(int i=0; i<smallsize; i++)
   b[i] = quantile (a, left+5*i, left+5(i+1)-1, 2); recursive call
   T pivot = quantile (b, 0, smallsize-1, smallsize/2); recursive call
   int p = linearsearch (a, left, right, pivot); → $\boxed{\Theta(n)}$
   a.swap (p, right); → $\Theta(1)$
   int m = partition(a, left, right); → $\boxed{\Theta(n)}$

   if (left + k == m) return a[m]; → $\Theta(1)$
   else if (left + k < m) return quantile(a, left, m-1, k); recursive call
   else return quantile(a, m+1, right, k-(m+1-left)); recursive call
   }

}

2. input size (right → left) is always 5, so the recursive call here always takes $\Theta(5^2)$ or $\Theta(25)$. Smallsize amount of recursive calls are made. Total amount of time for this is smallsize × 25 = $\frac{\text{right} - \text{left}}{5} \cdot 25 = 5n$

3. in the recursive call assigning pivot, the array is size smallsize-1, or ((right-left)/5) - 1 ⇒ $(n/5) - 1$

4. at least $3n/10$ elements in a are $\leq$ the pivot and at least $3n/10$ elements in a are $\geq$ the pivot, because the pivot is chosen as the median of the medians of the subgroups of a (median of b). Because of this, you know that 3/5 of the groups less than the pivot and group with the pivot are $\leq$ the pivot (b/c pivot is chosen as median over median of other groups, these groups have 2 other values also less than the pivot). There are approx $n/2$ of these groups containing values guaranteed less than the pivot (half b/c median), $\Rightarrow \frac{n}{2} \cdot \frac{3}{5} = \frac{3n}{10}$ atleast $\frac{3n}{10}$ elements in $a \leq$ pivot. Same reasoning for $\frac{3n}{10}$ elements $\geq$ pivot.

5. at most, the array for the recursive call in the last 2 lines is $n - \frac{3n}{10}$ size, because of the pivot above.
$\Rightarrow$ at most size $7n/10$

6.

recursion in last 2 lines

$$T(n) = T(n/5) + T(7n/10) + \Theta(n) \quad , \quad n \geq 10$$

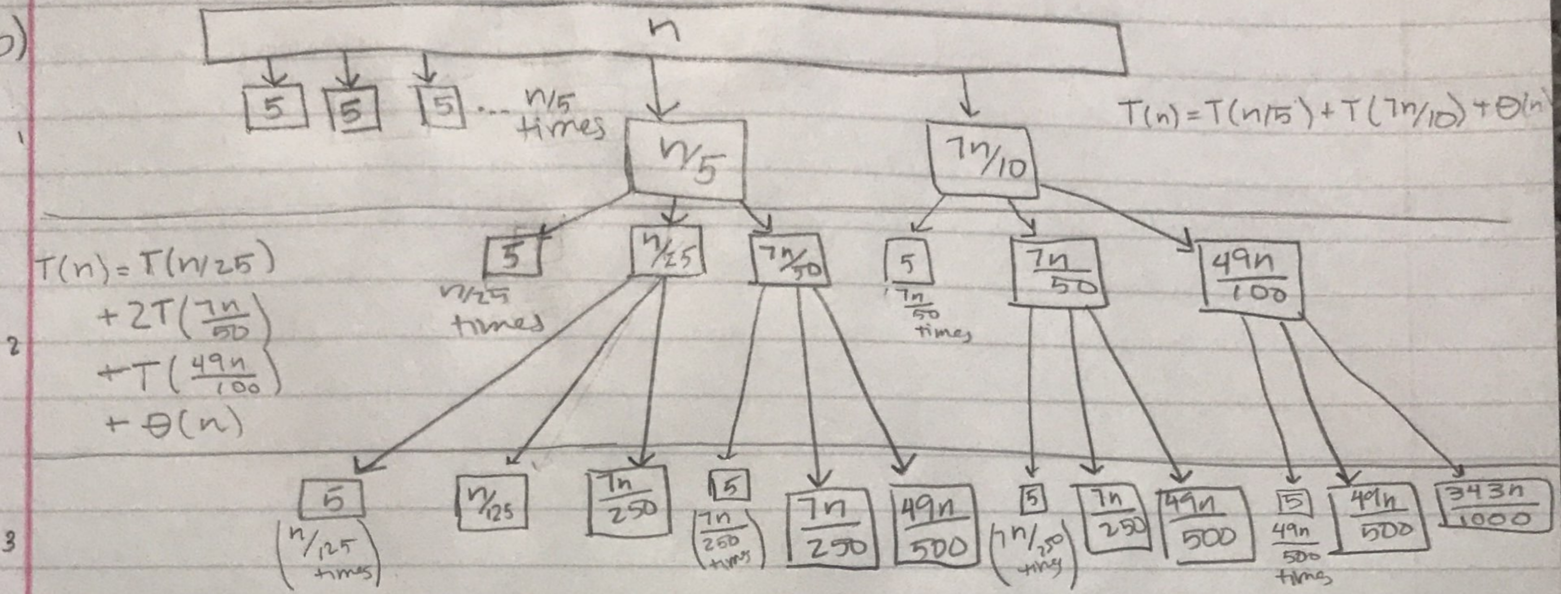
assigning pivot

loop
assigning values in b array

basecase: $T(10) \Rightarrow \Theta(n)$

Problem 3

b)



$$T(n) = T(n/125) + 3T\left(\frac{7n}{250}\right) + 3T\left(\frac{49n}{500}\right) + T\left(\frac{343n}{1000}\right) + \Theta(n)$$

4. $T(n) = T\left(\frac{n}{5^i}\right) + iT\left(\frac{7n}{10(5)^i}\right) \ldots + \Theta(n)$

5. $\log_{10/7} n$ layers

6. $\sum_{0}^{\log_{10/7} n} T\left(\frac{n}{5^i}\right) + iT\left(\frac{7n}{10(5)^i}\right) \ldots + \Theta(n)$

$$= \frac{n}{\underset{\underset{n}{\downarrow}}{5^{\log n}}} + \log n\left(\frac{7n}{\underset{\underset{n}{\downarrow}}{10(5)^{\log n}}}\right) + \Theta(n) \Rightarrow 1 + \log n + \Theta(n) \Rightarrow \Theta(n)$$