

# Simplicity in Composition

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# Composition

- $A$ : type (set) of values



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- $\oplus: A \times A \rightarrow A$



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- $1_A$ : identity for  $\oplus$

$$x \oplus (y \oplus z) = (x \oplus y) \oplus z$$

$$x = x \oplus 1_A = 1_A \oplus x$$



$$(\mathbb{Z}, +, 0)$$



$([a], ++, [])$



$$(A \rightarrow A, \circ, a \mapsto a)$$





```
trait Monoid[A] {  
  def combine(x: A, y: A): A  
  def empty: A  
}
```











36





3

7

11

15





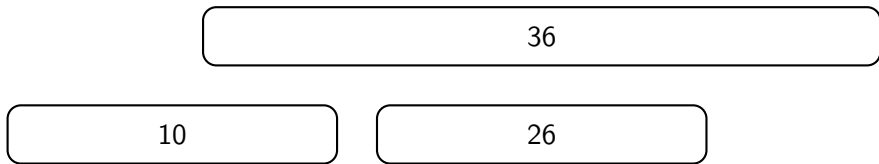
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36





**Associative composition** allows for **modular**  
**decomposition** and **reasoning**.



# Composing programs

A



# Composing programs

$F[A]$



# Composing programs

$$(F[A], F[A]) \Rightarrow F[A]$$



# Composing programs

$$(F[A], F[B]) \Rightarrow F[?]$$





# Composing programs

$$(F[A], F[B]) \Rightarrow F[(A, B)]$$



# Composing programs

```
def zipOption[A, B]  
  (oa: Option[A], ob: Option[B]): Option[(A, B)] =  
  
  (oa, ob) match {  
    case (Some(a), Some(b)) => Some((a, b))  
    case _                  => None  
  }
```



# Composing programs

```
def zipList[A, B]  
  (la: List[A], lb: List[B]): List[(A, B)] =  
  
  la match {  
    case Nil      => Nil  
    case h :: t   => lb.map((h, _)) ++ zipList(t, lb)  
  }
```



# Composing programs

```
def zipFunction[A, B, X]  
  (f: X => A, g: X => B): X => (A, B) =  
  
  (x: X) => (f(x), g(x))
```



# Composing programs

- $F(-)$ : type of program



# Composing programs

- $F(-)$ : type of program
- $\otimes : F(A) \times F(B) \rightarrow F((A, B))$



# Composing programs

- $F(\_)$ : type of program
- $\otimes : F(A) \times F(B) \rightarrow F((A, B))$
- $\eta : A \rightarrow F(A)$



# Composing programs

- $F(-)$ : type of program
- $\otimes : F(A) \times F(B) \rightarrow F((A, B))$
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$$fa \otimes (fb \oplus fc) \cong (fa \oplus fb) \oplus fc$$

$$fa \cong fa \otimes \eta_{Unit} \cong \eta_{Unit} \otimes fa$$





```
trait Monoidal[F[_]] {  
  def zip[A, B](fa: F[A], fb: F[B]): F[(A, B)]  
  def pure[A](a: A): F[A]  
  
  /*  
  def map[A, B](fa: F[A])(f: A => B): F[B]  
  */  
}
```



# Composing programs

$$(F[A], F[B]) \Rightarrow F[(A, B)]$$



# Composing programs

$F[A]$



# Composing dependent programs

$$(F[A], A \Rightarrow F[B]) \Rightarrow F[B]$$



# Composing dependent programs

```
def flatMapOption[A, B]  
  (oa: Option[A], f: A => Option[B]): Option[B] =  
  
  oa match {  
    case Some(a) => f(a)  
    case None    => None  
  }
```



# Composing dependent programs

```
def flatMapList[A, B]  
  (la: List[A], f: A => List[B]): List[B] =  
  
  la match {  
    case Nil => Nil  
    case h :: t => f(h) ++ flatMapList(t, f)  
  }
```



# Composing dependent programs

```
def flatMapFunction[A, B, X]  
  (fa: X => A, f: A => (X => B)): X => B =  
  
  (x: X) => f(fa(x))(x)
```



# Composing dependent programs

- $F(-)$ : type of program





# Composing dependent programs

- $F(\_)$ : type of program
- $(\gg= : F(A) \times A \rightarrow F(B)) \rightarrow F(B)$



# Composing dependent programs

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- $(\gg= : F(A) \times A \rightarrow F(B)) \rightarrow F(B)$
- $\eta : A \rightarrow F(A)$



# Composing dependent programs

- $F(\_)$ : type of program
- $(\gg= : F(A) \times A \rightarrow F(B)) \rightarrow F(B)$
- $\eta : A \rightarrow F(A)$

$$(fa \gg= f) \gg= g = fa \gg= (f \gg= g)$$

$$fx = \eta(x) \gg= f$$

$$fa = fa \gg= \eta$$



```
trait Monad[F[_]] {  
  def flatMap[A, B](fa: F[A], f: A => F[B]): F[B]  
  def pure[A](a: A): F[A]  
}
```



## A nicer monad

$$(fa \gg= f) \gg= g = fa \gg= (f \gg= g)$$

$$fx = \eta(x) \gg= f$$

$$fa = fa \gg= \eta$$



## A nicer monad

$$f : A \rightarrow F(B)$$

$$g : B \rightarrow F(C)$$

$$h : C \rightarrow F(D)$$



## A nicer monad

$$f : A \rightarrow F(B)$$

$$g : B \rightarrow F(C)$$

$$h : C \rightarrow F(D)$$

$$(f \rhd\!\rhd g) \rhd\!\rhd h = f \rhd\!\rhd (g \rhd\!\rhd h)$$

$$f = f \rhd\!\rhd \eta = \eta \rhd\!\rhd f$$



**Category theory** studies the algebra of composition.

