FFTLasso: Large-Scale LASSO in the Fourier Domain

Adel Bibi, Hani Itani, Bernard Ghanem

King Abdullah University of Science and Technology (KAUST)

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Abstract

- The LASSO sparse-representation problem has proved to be a powerful tool for applications ranging from signal processing and information theory to computer vision and machine learning.
- Solving large scale LASSOs is often a difficulty due to the incompatibility of solvers to scale efficiently.
- This paper proposes a novel circulant reformulation of the LASSO that lifts the problem to a higher dimension where ADMM can be applied efficiently to its dual form.
- The new formulation updates all the variables with 1D FFTs as the most expensive operator followed by only elementwise operations.
- Neither system linear solvers nor matrix-vector multiplication is required.
- The proposed method can be trivially parallelized over multiple GPUs.

Solvers

- It was shown in previous works [1] that DL-ADMM (applying ADMM to the dual form) be one of the fastest solvers available.

Primal Form: $$\min_{\mathbf{c}} \; \|\mathbf{A}\mathbf{c}-\mathbf{b}\|_2^2+\lambda\|\mathbf{c}\|_1$$

Dual Form: $$\min_{\Psi} \; \frac{1}{4}\|\Psi\|_2^2+\Psi^\top\mathbf{b} \quad \text{s.t.} \quad \|\mathbf{A}^\top\Psi\|_\infty \le \lambda$$

Algorithm 1: PL-ADMM

Input : $\mathbf{b}, \mathbf{A}, \mathbf{c}=\mathbf{0}_n, \mathbf{z}=\mathbf{0}_n, \Psi=\mathbf{0}_m, \lambda, u_1, \gamma>1.$
Output: $\mathbf{c}$
while not converged **do**
- **c update:** solve $(2\mathbf{A}^\top\mathbf{A}+u_k\mathbf{I}_n)\mathbf{c}_{k+1}=2\mathbf{A}^\top\mathbf{b}-\Psi_k+u_k\mathbf{z}_k$
- **z update:** $\mathbf{z}_{k+1}=\text{soft}(\mathbf{c}_{k+1}+\Psi_k/u_k).$
- **Ψ update:** $\Psi_{k+1}=\Psi_k+u_k(\mathbf{c}_{k+1}-\mathbf{z}_{k+1})$
- $u_{k+1}=\gamma u_k$

end

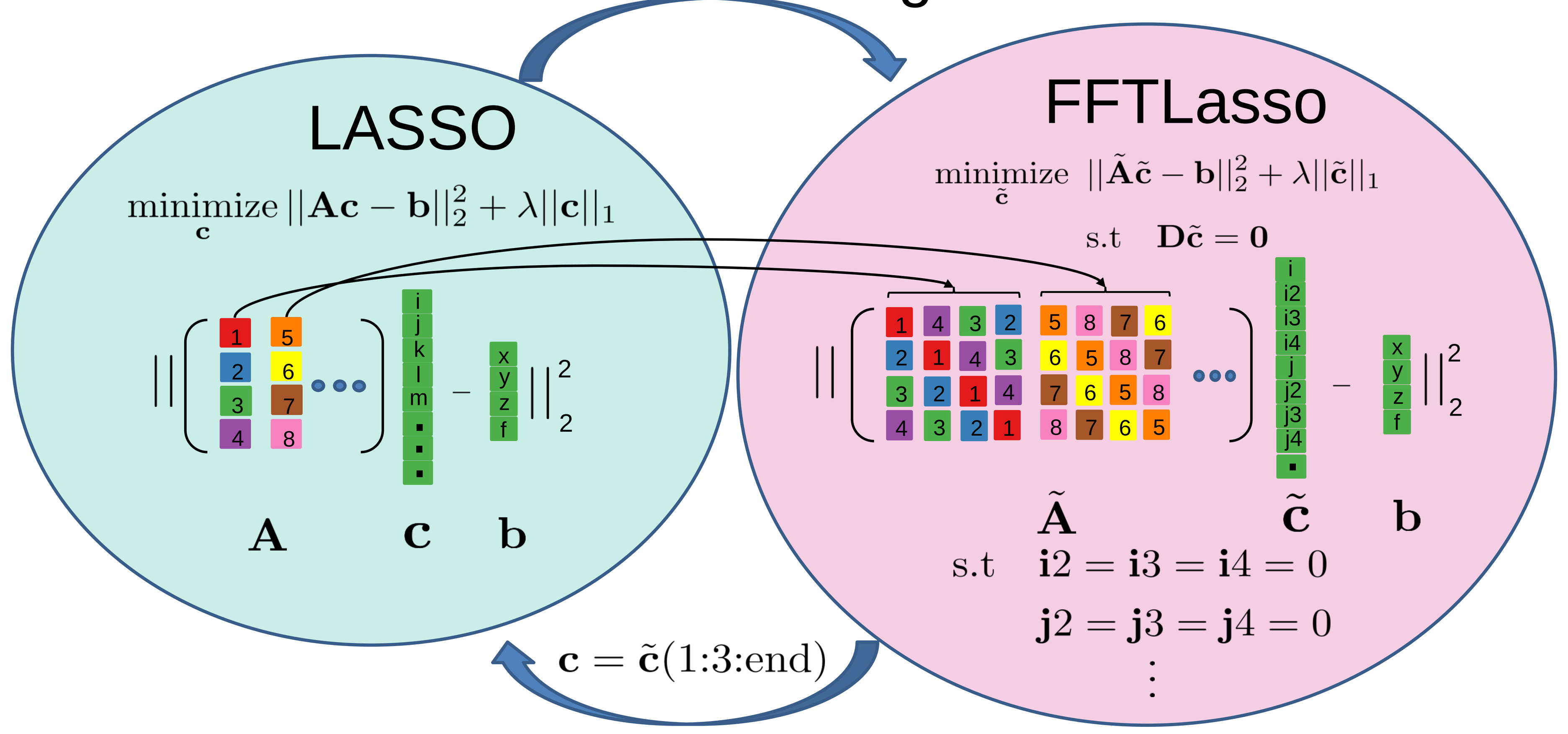
Algorithm 1: DL-ADMM

Input : $\mathbf{b}, \mathbf{A}, \mathbf{c}=\mathbf{0}_n, \zeta=\mathbf{0}_n, \Psi=\mathbf{0}_m, \lambda, \rho_1, \gamma>1$
Output: $\mathbf{c}$
while not converged **do**
- **Ψ update:** solve $(\rho_k\mathbf{A}\mathbf{A}^\top+\frac{1}{2}\mathbf{I}_m)\Psi_{k+1}=\mathbf{A}(\rho_k\zeta-\mathbf{c})-\mathbf{b}.$
- **ζ update:** $\zeta_{k+1}=\text{proj}_{\ell_\infty,\lambda}(\mathbf{A}^\top\Psi_{k+1}+\mathbf{c}_k/\rho_k).$
- **c update:** $\mathbf{c}_{k+1}=\mathbf{c}_k+\rho_k(\mathbf{A}^\top\Psi_{k+1}-\zeta_{k+1})).$
- $\rho_{k+1}=\gamma\rho_k$

end

Problem Reformulation

Problem lifting



Problem down-sampling

Primal Form: $$\min_{\tilde{\mathbf{c}}} \; \|\tilde{\mathbf{A}}\tilde{\mathbf{c}}-\mathbf{b}\|_2^2+\lambda\|\tilde{\mathbf{c}}\|_1 \quad \text{s.t} \quad \mathbf{D}\tilde{\mathbf{c}}=\mathbf{0}$$

Dual Form: $$\min_{\Psi,\theta} \; \frac{1}{4}\|\Psi\|_2^2+\Psi^{\mathbf{H}}\mathbf{b} \quad \text{s.t} \quad \|\tilde{\mathbf{A}}^{\mathbf{H}}\Psi+\mathbf{D}^{\mathbf{H}}\theta\|_\infty \le \lambda$$

- In FFTLasso, the most expensive operation is m-FFTs (n times).
- Nesterov's accelerated gradient is applied to speed up convergence.

Algorithm 1: FFTLasso

Input : $\mathbf{b}, \mathbf{A}, \tilde{\mathbf{c}}_1=\tilde{\mathbf{y}}_1=\tilde{\mathbf{r}}_1=\mathbf{e}_1=\mathbf{t}_1=\zeta_1=\mathbf{D}^{\mathbf{H}}\theta_1=\mathbf{0}_{mn}, \Psi=\mathbf{0}_m, \lambda, \rho_1, \gamma>1, q.$
Output: $\mathbf{c}$
while not converged **do**
- **compute:** $\mathbf{e}_{k+1}=\rho_k\zeta_k-\rho_k\mathbf{D}^{\mathbf{H}}\theta_k-\tilde{\mathbf{c}}_k$
- **$\hat{\Psi}^*$ update:** $\hat{\Psi}^*_{k+1}=\frac{\sum_i^N \hat{\mathbf{a}}_i^*\odot\hat{\mathbf{e}}^*_{ik+1}-\hat{\mathbf{b}}^*}{\rho_k\sum_i^N \hat{\mathbf{a}}_i\odot\hat{\mathbf{a}}_i^*+\frac{1}{2}}$
- **compute:** $\tilde{\mathbf{A}}^{\mathbf{H}}\Psi_{k+1},$
- **$\mathbf{D}^{\mathbf{H}}\theta$ update:** $(\mathbf{D}^{\mathbf{H}}\theta_{k+1})=(\zeta_k-\frac{1}{\rho_k}\tilde{\mathbf{c}}_k-\tilde{\mathbf{A}}^{\mathbf{H}}\Psi_{k+1})$
- $(\mathbf{D}^{\mathbf{H}}\theta_{k+1})_{1:\text{m}:\text{end}}=\mathbf{0}_n$
- **compute:** $\mathbf{t}_{k+1}=\tilde{\mathbf{A}}^{\mathbf{H}}\Psi_{k+1}+\mathbf{D}^{\mathbf{H}}\theta_{k+1}+\tilde{\mathbf{c}}_k/\rho_k$
- **ζ update:** $\zeta_{k+1}=\text{sign}(\mathbf{t}_{k+1})\odot\min(|\mathbf{t}_{k+1}|,\lambda)$
- **$\tilde{\mathbf{c}}$ update:** $\tilde{\mathbf{c}}_{k+1}=\tilde{\mathbf{y}}_k+\rho_k(\tilde{\mathbf{A}}^{\mathbf{H}}\Psi_{k+1}+\mathbf{D}^{\mathbf{H}}\theta_{k+1}-\zeta_{k+1})$
- **compute:** $\tilde{\mathbf{y}}_{k+1}=(1+q)\tilde{\mathbf{c}}_{k+1}-q\tilde{\mathbf{c}}_k$
- $\rho_{k+1}=\gamma\rho_k$

end
$\mathbf{c}\leftarrow\tilde{\mathbf{c}}(1{:}m{:}\text{end})$

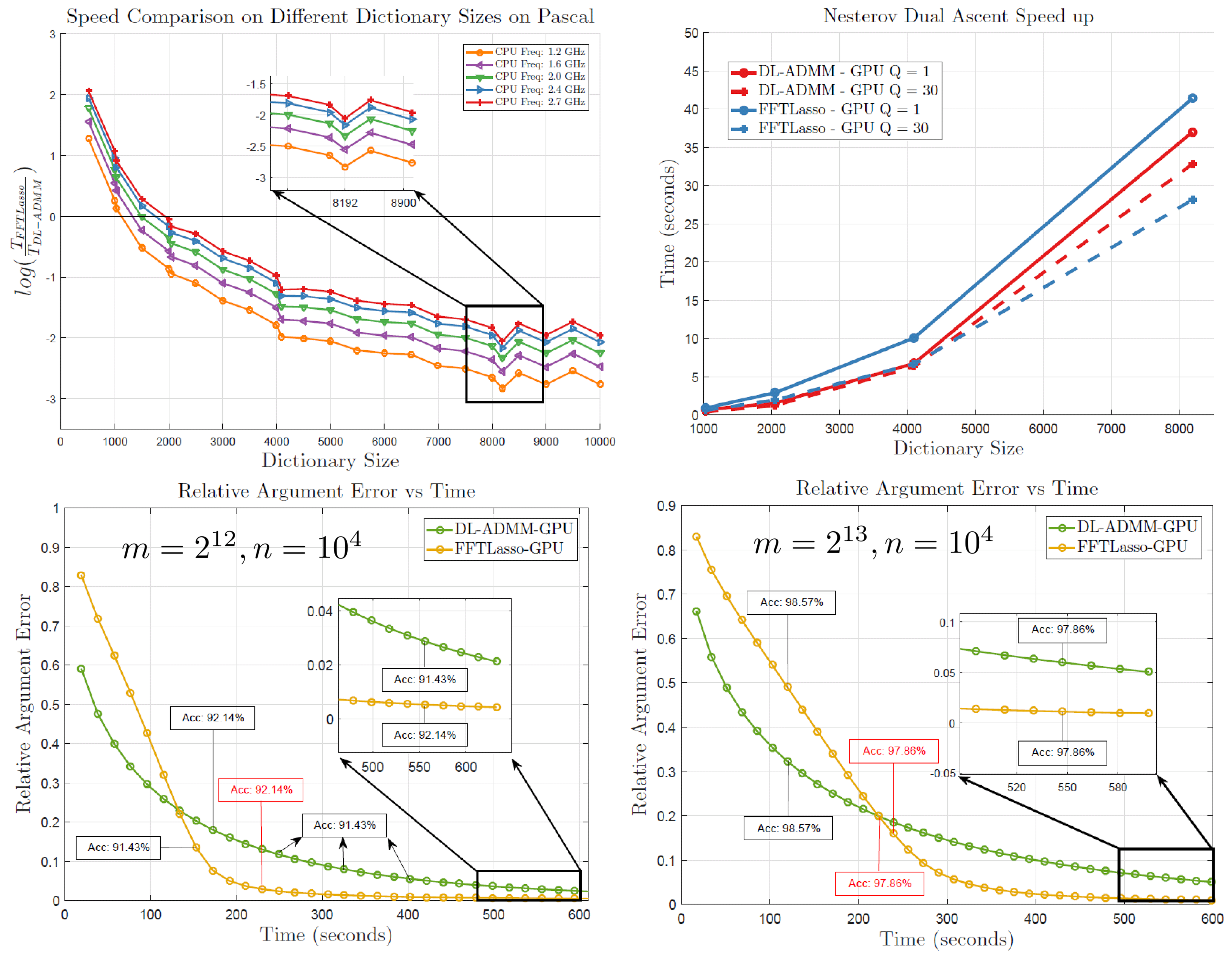
Experiments

- Memory Efficiency. Table [1] demonstrates that FFTLasso is much more memory efficient that DL-ADMM.

Table 1: Memory efficiency comparison between DL-ADMM and FFTLasso on a TitanX GPU with 12GB memory. Dimensions marked with ✓(*) are reached using simple dictionary splitting that is only possible for FFTLasso. Refer to the text for details.

Dictionary Size	2^{12}	6500	2^{13}	8500	9000	9500
DL-ADMM	✓	✓	✗	✗	✗	✗
FFTLasso	✓	✓	✓	✓	✓(*)	✓(*)

- Speed Comparison. We compare FFTLasso against DL-ADMM on both synthetic and Face Recognition datasets.
- Synthetic experiments for DL-ADMM are conducted on both multi core CPUs and on a GPU too.
- Face Recognition experiments are conducted on 2 size-varying problems.



References: [1] A. Yang et al. "A Review for fast l1-Minimization Algorithms for Robust Face Recognition"

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