Generalizing from individuals to populations: Hierarchical inference supports convention formation on networks

Anonymous CogSci submission

Abstract

Linguistic conventions allow us to communicate efficiently even with novel members of our community. At the same time, much of our intended meaning is partner-specific. Exactly how do agents make the inferential leap to community-wide expectations from their experiences with specific partners? We propose a hierarchical Bayesian model that explains how partner-specific pacts may gradually spread through a network. We collected experimental data showing how people conventionalize referring expressions in a series of interactive reference games with different partners in a small community. Results were used to evaluate a hierarchical Bayesian cognitive model formalizing a theory of the generalization mechanisms underlying convention formation.

Keywords: convention; generalization;

To communicate successfully, speakers and listeners must share a common system of semantic meaning in the language they are using. These meanings are *conventional* in the sense that they are sustained by the expectations each person has about others (Bicchieri, 2006; Lewis, 1969). A key property of linguistic conventions is that they hold over entire communities, allowing us to communicate efficiently even with people we've never met before. But exactly how do we make the inferential leap to community-wide expectations from our experiences with specific partners? Grounding collective convention formation in individual cognition requires an explicit *theory of generalization* capturing how people transfer what they have learned from one partner to the next.

One influential theory is that speakers simply ignore the identity of different partners and update a single monolithic representation after every interaction (Baronchelli, 2018; Barr, 2004; Steels, 1995; Young, 2015). We call this a *complete-pooling* theory (Gelman & Hill, 2006) because data from each partner is collapsed into an undifferentiated pool of evidence. Complete-pooling models have been remarkably successful at predicting collective behavior on networks, but have typically been evaluated only in settings where anonymity is enforced. For example, Centola & Baronchelli (2015) asked how large networks of participants coordinated on conventional names for novel faces. On each trial, participants were paired with a random neighbor but were not informed of that neighbor's identity, or even the total number of different possible neighbors.

While complete-pooling may be appropriate for some everyday social interactions, such as coordinating with anonymous drivers on the highway, it is less tenable for everyday communicative settings. Knowledge about a partner's identity is both available and relevant for conversation (Eckert, 2012). Extensive evidence from psycholinguistics has demonstrated the *partner-specificity* of our language use (Brennan & Hanna, 2009; Clark, 1996). Because meaning is grounded in the evolving 'common ground' shared with each partner, meanings established over a history of interaction with one partner are not necessary transfered to other partners (Metzing & Brennan, 2003; Wilkes-Gibbs & Clark, 1992). Partner-specificity thus poses clear problems for complete-pooling theories but can be easily explained by another simple model, where agents maintain separate expectations about meaning for each partner. We call this a *no-pooling* model. The problem with no-pooling, of course, is that agents are forced to start from scratch with each partner. Community-level expectations never get off the ground.

What theory of generalization, then, can explain partner-specific meaning but also allow conventions to spread through communities? We propose a *partial-pooling* account that offers a compromise between these extremes. Unlike complete-pooling and no-pooling models, we propose that beliefs about meaning have hierarchical structure. That is, the meanings used by different partners are expected to be drawn from a shared community-wide distribution but are also allowed to differ from one another in systematic, partner-specific ways. This structure provides an inductive pathway for abstract population-level expectations to be distilled from partner-specific experience (see also Kleinschmidt & Jaeger, 2015; Tenenbaum, Kemp, Griffiths, & Goodman, 2011). In other words, we suggest that conventional meanings result from agents solving a meta-learning problem, adapting to each partner along the way.

We begin by formalizing this account in a probabilistic model of communication and presenting several simulations of listener and speaker behavior within and across partners. Next, we test the qualitative predictions of this model in a behavioral experiment. Participants were paired for a series of extended reference games with each neighbor in small networks. Our results showed signatures of *ad hoc* convention formation within dyads, but also gradual generalization of these local pacts across subsequent partners as the network converged. Taken together, this work suggests that local partner-specific learning is not only compatible with global convention formation but may facilitate it when coupled with a powerful hierachical inductive mechanism.

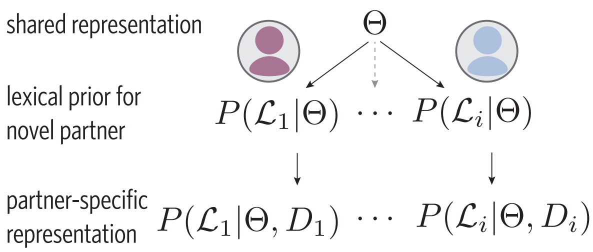

Figure 1: Schematic of hierachical Bayesian model.

A hierarchical Bayesian model of convention

In this section, we provide an explicit computational account of the cognitive mechanisms supporting the balance between community-level stability and partner-specific flexibility. Specifically, we show how the dyadic convention formation model of Hawkins, Frank, & Goodman (2017) can be extended with a principled mechanism for generalization across multiple partners. This model begins with the idea that knowledge about meanings can be represented probabilistically: agents have uncertainty about what lexical meaning their current partner is using (Bergen, Levy, & Goodman, 2016). In our hierarchical model, this lexical uncertainty is represented by a multi-level prior.

At the highest level of the hierarchy is a *community-level* variable Θ parameterizing the agent's *partner-specific* expectations $P(\theta_i|\Theta)$, where θ_i represents the latent system of meanings used by partner i (see Fig. 1). Given observations D_i from repeated communicative interactions with a specific partner i, the agent update their beliefs about the latent system of meaning using Bayes rule:

$$\begin{aligned} P(\theta_i, \Theta|D_i) &\propto P(D_i|\theta_i, \Theta)P(\theta_i, \Theta) \\ &= P(D_i|\theta_i)P(\theta_i|\Theta)P(\Theta) \end{aligned}$$

This inference decomposes the problem of partner-specific learning into two terms, a prior term $P(\theta_i|\Theta)P(\Theta)$ and a likelihood term $P(D_i|\theta_i)$. The prior captures the idea that different partners will share some aspects of meaning in common. In the absence of strong information about language use departing from this common structure, the agent ought to be regularized toward generalizable knowledge of their community's conventions. The likelihood represents predictions about how a partner using a particular system of meaning will use language, which we specify in more detail below.

Critically, in addition to allowing agents to update partner-specific expectations and adapt to their partner as an interaction continues, agents may also update their community-level expectations by marginalizing over data accumulated from different partners:

$$P(\Theta|D) = P(\Theta)\int_{\theta_i} P(D_i|\theta_i)P(\theta_i|\Theta)$$

where $D = \bigcup_{i=1}^{k} D_i$. This inductive pathway allows new data from individual partners to systematically inform beliefs about community-wide conventions. Given weak expectations about Θ, a partner's behavior may at first be more parsimoniously explained as a partner-specific meaning. After multiple partners are inferred to have a similar system of meaning, however, it becomes more likely that a novel partner will share it as well: beliefs about Θ will shift to represent this commonality. This transfer is sometimes referred to as a "sharing of statistical strength" across partners.

Model simulations

We investigate the qualitative predictions of this model under three increasingly complex reference games scenarios. In all of these scenarios, speaker and listener agents are shown a set of two objects $\{o_1, o_2\}$. One of these objects is designated for the speaker as the *target*, and they must select an utterance u_j to convey the identity of the target to the listener. Upon hearing this utterance, the listener must select which of the objects they believe to be the target. Thus, the available data D_i from an interaction with partner i consists of utterance-object pairs $\{(u, o)_t\}$ for each trial of the task.

Given this concrete setting, we can now explicitly specify forms of the likelihood and prior terms. We consider a likelihood $P(D_i|\theta_i)$ given by the Rational Speech Act (RSA) framework, which formalizes the Gricean assumption of cooperativity (Franke & Jäger, 2016; Goodman & Frank, 2016). A pragmatic speaker S_1 attempts to trade off informativity against the cost of producing an utterance, while a pragmatic listener L_1 inverts their model of the speaker to infer the intended target. The chain of recursive social reasoning grounds out in a *literal listener* L_0, which looks up the meaning in a parameterized lexicon $\mathcal{L}_\theta$. This model can be formally specified as follows:

$$\begin{aligned} L_0(o_i|u_j, \theta) &\propto \exp\{\mathcal{L}_\theta(u_j, o_i)\} \\ S_1(u_j|o_i, \theta) &\propto \exp\{w_I \cdot \log L_0(o_i|u_j, \theta) - w_C \cdot \text{cost}(u_j)\} \\ L_1(o_i|u_j, \theta) &\propto S_1(w|o_i, \theta)P(o_i) \end{aligned}$$

where w_I and w_C are free parameters controlling the relative weights on the informativity and cost term, respectively.

Finally, to derive the model's behavior, we must also specify the form of the lexical prior and a method to perform inference. We assume Θ is an $N \times M$ matrix with an entry for each utterance-object pair, and use independent Gaussian distributions for each $\Theta_{ij} \in \Theta$ as a hyper-prior. We then centered our partner-specific prior $\theta_{ij} \in \theta$ at the shared value for a particular partner:

$$\begin{aligned} P(\Theta_{ij}) &\sim \mathcal{N}(0, 1) \\ P(\theta_{ij}) &\sim \mathcal{N}(\Theta_{ij}, 1) \end{aligned}$$

The variances chosen in these priors represent assumptions about how strongly adaptation is regularized.

For all simulations, we used variational inference (VI; Ranganath, Gerrish, & Blei, 2013) to perform inference. Variational methods transform probabilistic inference problems into optimization problems by approximating the true posterior with a parameterized family. Specifically, we make a

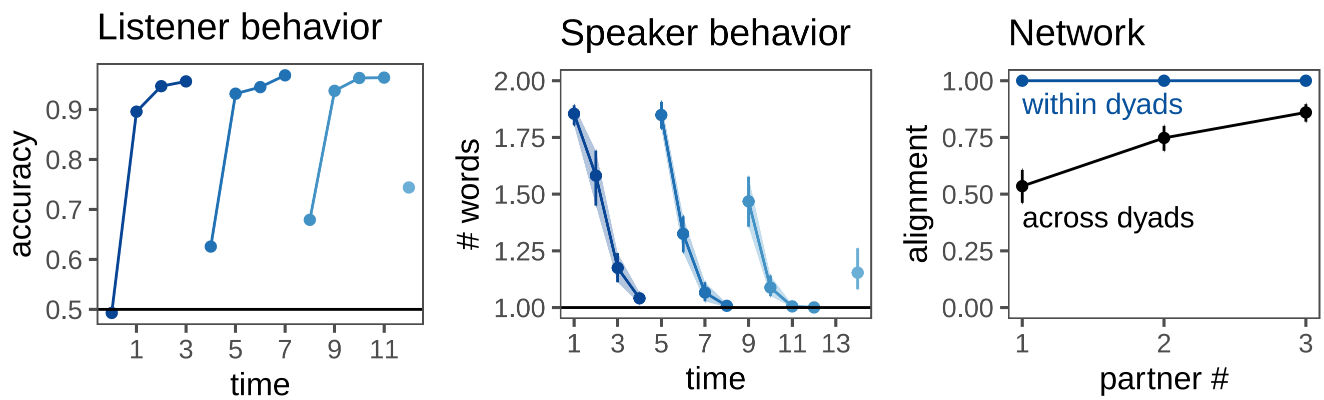

Figure 2: Model predictions across a series of different partners.

mean-field approximation and assume that the full posterior can be factored into independent Gaussians for each random variable in the lexicon. We then optimize the parameters of these posterior Gaussians by minimizing the evidence lower bound (ELBO) objective (see Murphy, 2012 for more details). Variational inference allows us to amortize inference as additional data is observed. We run 50,000 steps of gradient descent on the first observation to obtain a posterior, compute the agent's marginal prediction for the next observation by taking the expectation over 50,000 samples from this posterior, then continue running gradient descent on the same parameters after adding the new observation in the data.

Simulation 1: Listener accuracy across partners The key predictions of our model concern the pattern of generalization across partners. In our first simulation, we consider the partner-specificity of a listener's expectations about which object is being referred to. To observe the model's behavior in the simplest case, we assume the speaker can choose from two utterances $\{u_1, u_2\}$ and feed the model the same utterance-object pair ($\{o_1, u_1\}$) on every trial. Instead of presenting this stream of data from a single partner, or randomly choosing a different partner on every trial, we swap in a new partner every block of 4 trials.

Our simulation results are shown in Fig. 2A. We find that the listener begins at chance due to its uninformative prior, but after several trials with the same partner, it updates its beliefs to be highly accurate at guessing the target. When a new partner is introduced, we find that it reverts nearly to its original state. Because of the hierarchical structure of its lexical expectations, it was ambiguous whether the evidence from the first partner was idiosyncratic or due to shared structure. After adapting to multiple partners, however, we find that it brings stronger expectations into its interaction with a fourth partner. Thus, expectations about what an individual partner means have gradually been incorporated into community-level expectations.

Simulation 2: Speaker utterance length across partners Next, we show that our model accounts for the *partner-specificity* of the speaker's referring expressions (Wilkes-Gibbs & Clark, 1992). We supplement the utterance space with multi-word utterances built from a set of four primitives: $\{u_1, u_2, u_3, u_4\}$. Under our model, speakers revert back to a longer description with a novel partner because evidence from a single listener is relatively uninformative about the community-level prior. After interacting with enough partners in a tight-knit community, speakers should become increasingly confident that labels are not simply idiosyncratic features of a particular partner's lexicon but are shared across the entire community. In other words, the partner-specific expectations agents form within an interaction to solve novel communication problems gradually generalize to community-wide expectations as they gain additional evidence of the latent population-level distribution from which different partners are sampled. These expectations manifest in an increasing willingness to use shorter labels with novel partners (Fig. 2B).

Simulation 3: Network convergence The first two simulations presented a single agent with a fixed sequence of data to understand its gradient of generalization within and across partners. Here, we test the consequences of the proposed hierarchical inference scheme for a network of interacting agents. How does the network as a whole coordinate? Do agents come to share a similar Θ, suggesting that community-wide conventions have formed? We used a round-robin scheme to schedule four agents into three blocks of interaction, with agents taking turns in the speaker and listener roles. From each individual agent's perspective, this experiment is identical to the earlier simulation (i.e. a series of 3 partners). Because all agents are not learning from the others, however, the network as a whole faces a coordination problem. For example, in the first block, agent 1 and 2 may coordinate on using u_1 while agent 3 and 4 coordinate no using u_2. Once they swap partners, they must re-negotiate this potential miscoordination. We find that the network as a whole gradually aligns (Fig. 2C).

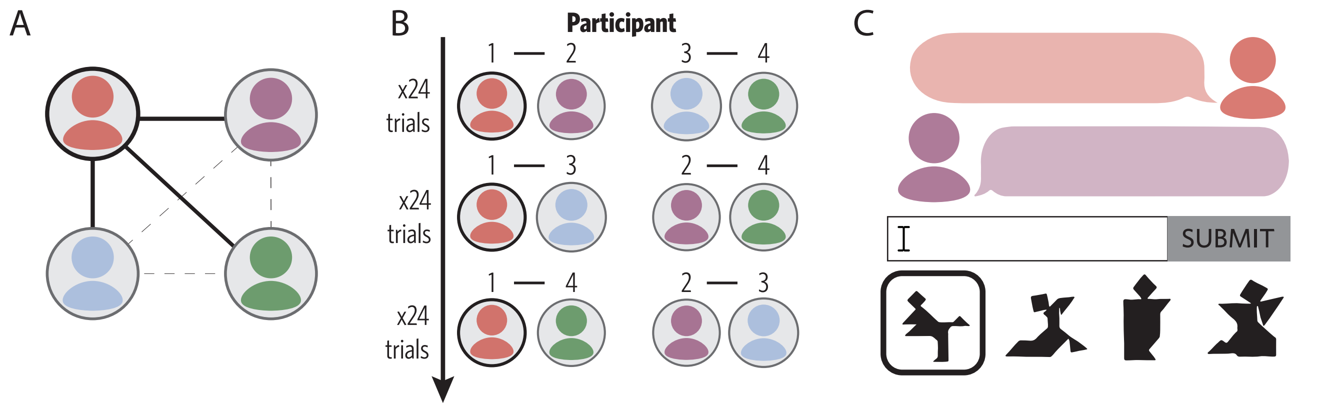

Figure 3: Experimental design. (A) Participants were placed in fully-connected networks of 4 and (B) played repeated reference games with each partner.

Behavioral experiment: Convention formation on a network

To evaluate these qualitative predictions, we designed a communication experiment on a small network. Rather than anonymizing partners, we divided the experiment into blocks of contiguous interaction with stable partners (see Fay, Garrod, Roberts, & Swoboda, 2010; Garrod & Doherty, 1994 for similar designs). Each block was a full repeated reference game, where participants had to coordinate on an *ad hoc* convention, or *pact*, for how to refer to reoccuring target objects with their partner. While it has been frequently observed that messages reduce in length across repetitions as common ground is built a single partner (Krauss & Weinheimer, 1964), and sharply jump revert to their initial length when a new partner is introduced (Wilkes-Gibbs & Clark, 1992), we were interested in the effect at subsequent partner boundaries. Complete-pooling accounts predict no change in the number of words when a new partner is introduced and are thus inconsistent even with the results of Wilkes-Gibbs & Clark (1992). No-pooling accounts predict that roughly the same initial description length will re-occur with every subsequent interlocutor. Contrary to either of these extremes, our hierarchical Bayesian model predicts that description length will increase at partner boundaries but that the initial length will decrease incrementally over successive interactions: after each partner, agents should be more willing to transfer expectations from one partner to another in their community.

Methods

Participants We recruited participants from Amazon Mechanical Turk to play an interactive, natural-language reference game implemented with the Dallinger platform[1]. Participants were randomly assigned to one of 23 fully-connected four-person communities.

Stimuli and procedure Participants were paired with each of their three neighbors for a series of dyadic interactions. In each interaction, they played a real-time, natural-language reference game where they repeatedly referred to a set of four abstract tangram shapes taken from [Clark & Wilkes-Gibbs (1986); see Fig. 3]. These stimuli have been used extensively in the literature on coordination and common ground. They were designed such that participants will not already have strong pre-existing lexical conventions for how to refer to them (unlike photographs of common objects), but are structured enough to support many possible descriptions (unlike images of white noise).

On each trial of a reference game, one of these four shapes was highlighted as the *target object* for the "speaker" who was instructed to use a chatbox to communicate the identity of this object to their partner, the "listener". The listener could reply through the chatbox but must ultimately make a selection from the array. The trial sequence for a given partner was constructed so that each of four targets appear six times each, spread evenly across the session, for a total of 24 trials.

After completing 24 trials with one partner, they were introduced to their next partner and asked to play the repeated reference game again with the same four objects. Each participant in a network was assigned a distinct avatar so that participants were clear they were speaking to distinct partners. This process repeated until each participant had partnered with all three neighbors. Players were given full feedback on each about their partner's choice and received bonus payment for each correct response. Because some pairs within the network took longer than others to complete the trial sequence, we sent participants to a temporary waiting room if their next partner was not yet ready.

Results

Listener accuracy TODO.

Speaker utterance length The key empirical predictions distinguishing our model from alternatives concern behavior across partner boundaries. We operationalized the degree of conventionalization as the mean number of words used per description, a standard measure of coding efficiency in ref-

[1] http://docs.dallinger.io/

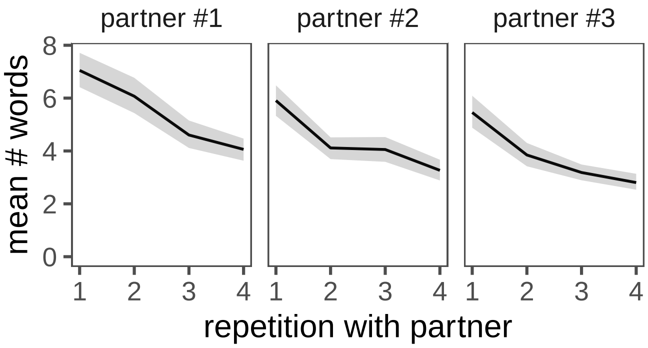

Figure 4: Reduction in number of words within and across partner boundaries.

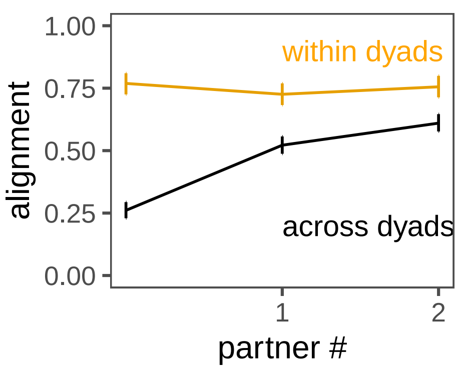

Figure 5: Reduction in number of words within and across partner boundaries.

erence games . We tested these predictions using mixed-effects regressions of partner number and repetition number on the number of words in a speaker's description, with random-effect structure including item-effects at the object and speaker level. We find a positive jump in description length across partner-boundaries overall, $t(91) = 3.7$, $p < 0.001$, indicating sensitivity to different partners, but a successive incremental decrease in the lengths of these initial descriptions, $t(79.2) = -6.8$, $p < 0.001$, consistent with our proposal.

Network convergence TODO.

Discussion

1. Other advantages of hierarchical model. e.g. it's more robust to deviations than complete-pooling; if we have a lot of interactions with idiosyncratic speakers (e.g. children), we don't replace our conventional community-level expectations. But agent-based models with a memory window or single representation predict this. Also there are other kinds of partner-specific information that may need to be tracked (e.g. visual access/knowledge, e.g. if you know someone is an expert in an area).

2. Possible connection to memory mechanisms: partners as contexts that get reinstated (**???**; Horton & Gerrig, 2016), and

3. Suggest ideas about different communities and code-switching as targets of future work.

4. Summary: More broadly, hierarchical generalization may be a foundational cognitive building block for establishing conventionality at the group level while maintaining flexibility within interactions. # Acknowledgements

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