MEG 401

DESIGN II

BY

DR SIMOLOWO O. E.

MECHANICAL ENGINEERING

OLABISI ONABANJO UNIVERSITY

- Chapter 2: Design of gear systems (spur, helical, bevel, worm gear)

Sub-Topics

Types of Gears

Nomenclature

Conjugate Action

Involute Properties

Fundamentals

Contact Ratio

Interference

The Forming of Gear Teeth

Straight Bevel Gears

Parallel Helical Gears

Worm Gears

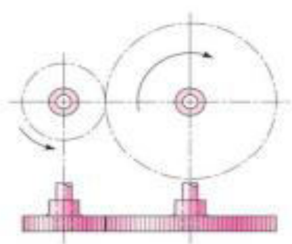
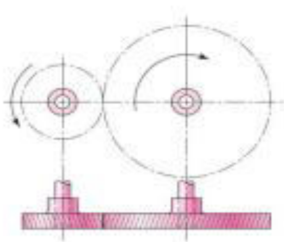
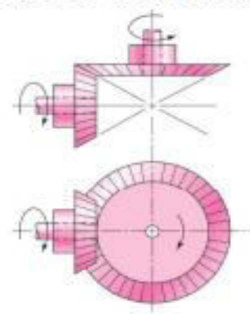
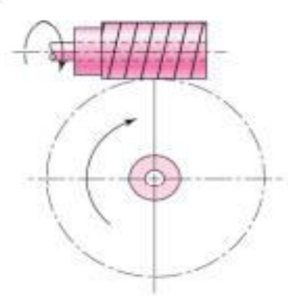
Tooth Systems

Gear Trains

• Chapter 2

Types of Gears

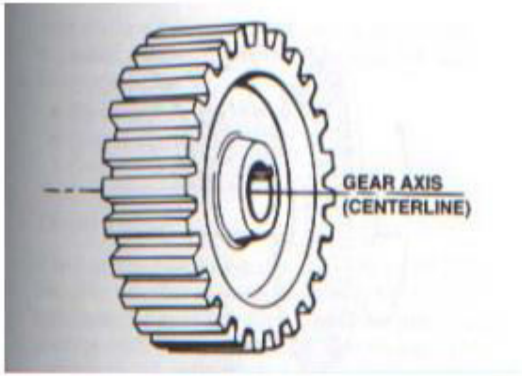
- **Spur gears** have teeth parallel to the axis of rotation and are used to transmit motion from one shaft to another, parallel, shaft.
- **Helical gears** have teeth inclined to the axis of rotation. Helical gears are not as noisy, because of the more gradual engagement of the teeth during meshing.
- **Bevel gears** have teeth formed on conical surfaces and are used mostly for transmitting motion between intersecting shafts.
- **Worms and worm gears ,**The worm resembles a screw. The direction of rotation of the worm gear, also called the worm wheel, depends upon the direction of rotation of the worm and upon whether the worm teeth are cut right-hand or left-hand.

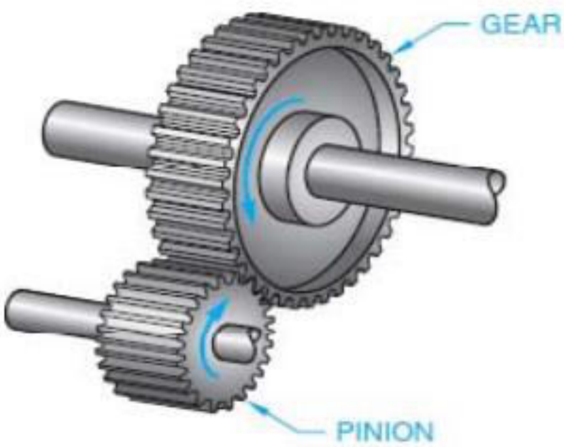





• Chapter 2

SPUR GEAR

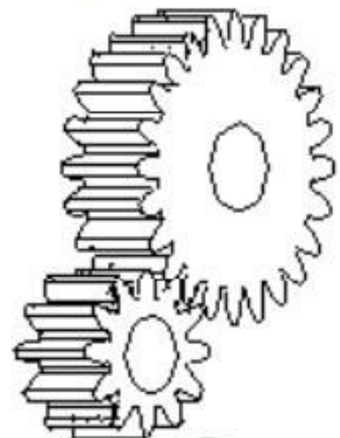
- Teeth is parallel to axis of rotation
- Transmit power from one shaft to another parallel shaft
- Used in Electric screwdriver, oscillating sprinkler, windup alarm clock, washing machine and clothes dryer





- Chapter 2

External and Internal spur Gear…



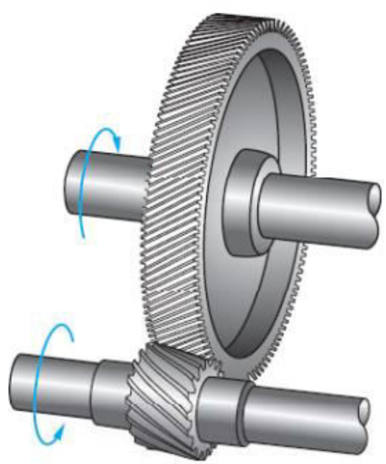
- Advantages:
  - Economical
  - Simple design
  - Ease of maintenance
- Disadvantages:
  - Less load capacity
  - Higher noise levels

Chapter 2

Helical Gear

- The teeth on helical gears are cut at an angle to the face of the gear
- This gradual engagement makes helical gears operate much more smoothly and quietly than spur gears
- Carry more load than equivalent-sized spur gears

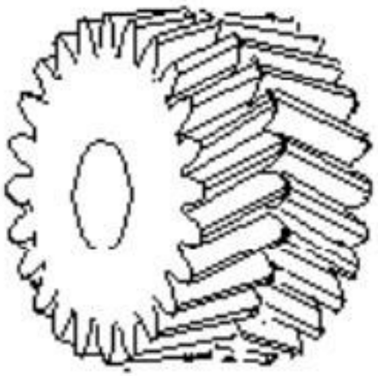
Helical Gear…



• Chapter 2

Herringbone gears

- To avoid axial thrust, two helical gears of opposite hand can be mounted side by side, to cancel resulting thrust forces



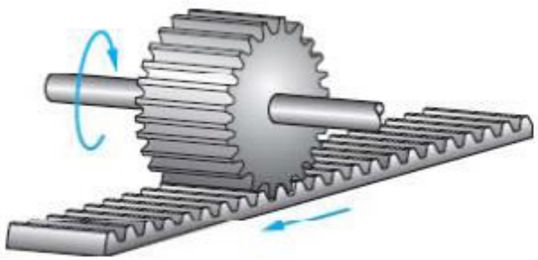
- Herringbone gears are mostly used on heavy machinery.

- Chapter 2

Rack and pinion

- **Rack and pinion gears** are used to convert rotation (From the pinion) into linear motion (of the rack)

- A perfect example of this is the steering system on many cars



• Chapter 2

Bevel gears

- **Bevel gears** are useful when the direction of a shaft's rotation needs to be changed
- They are usually mounted on shafts that are 90 degrees apart, but can be designed to work at other angles as well
- The teeth on bevel gears can be **straight**, **spiral** or **hypoid**
- locomotives, marine applications, automobiles, printing presses, cooling towers, power plants, steel plants, railway track inspection machines, etc.

• Chapter 2

Straight and Spiral Bevel Gears

- Chapter 2

WORM AND WORM GEAR

- **Worm gears** are used when large gear reductions are needed. It is common for worm gears to have reductions of 20:1, and even up to 300:1 or greater
- Many worm gears have an interesting property that no other gear set has: the worm can easily turn the gear, but the gear cannot turn the worm
- Worm gears are used widely in material handling and transportation machinery, machine tools, automobiles etc

- Chapter 2

WORM AND WORM GEAR

• Chapter 2

Nomenclature

- ✓ The **pitch circle** is a theoretical circle upon which all calculations are usually based; its diameter is the **pitch diameter**.
- ✓ A **pinion** is the smaller of two mating gears. The larger is often called the **gear**.
- ✓ The **circular pitch** p is the distance, measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth. It is equalto the sum of the tooth thickness and width of space.
- ✓ The **module** m is the ratio of the pitch diameter to the number of teeth.
- ✓ The **diametral pitch** P is the ratio of the number of teeth on the gear to the pitch diameter.

$$P = \frac{N}{d}$$

$$m = \frac{d}{N}$$

$$p = \frac{\pi d}{N} = \pi m$$

$$pP = \pi$$

where

P = diametral pitch, teeth per inch

N = number of teeth

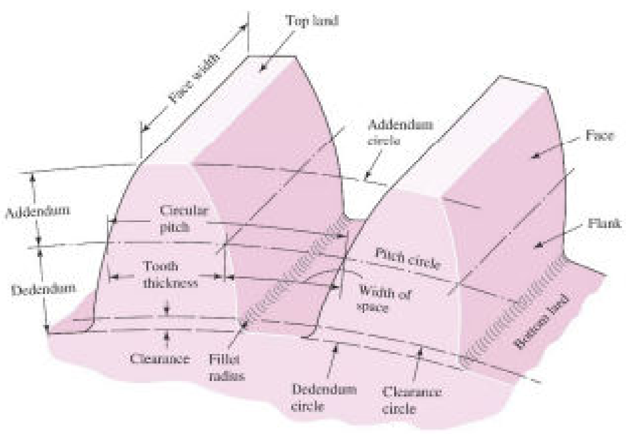
d = pitch diameter, in

m = module, mm

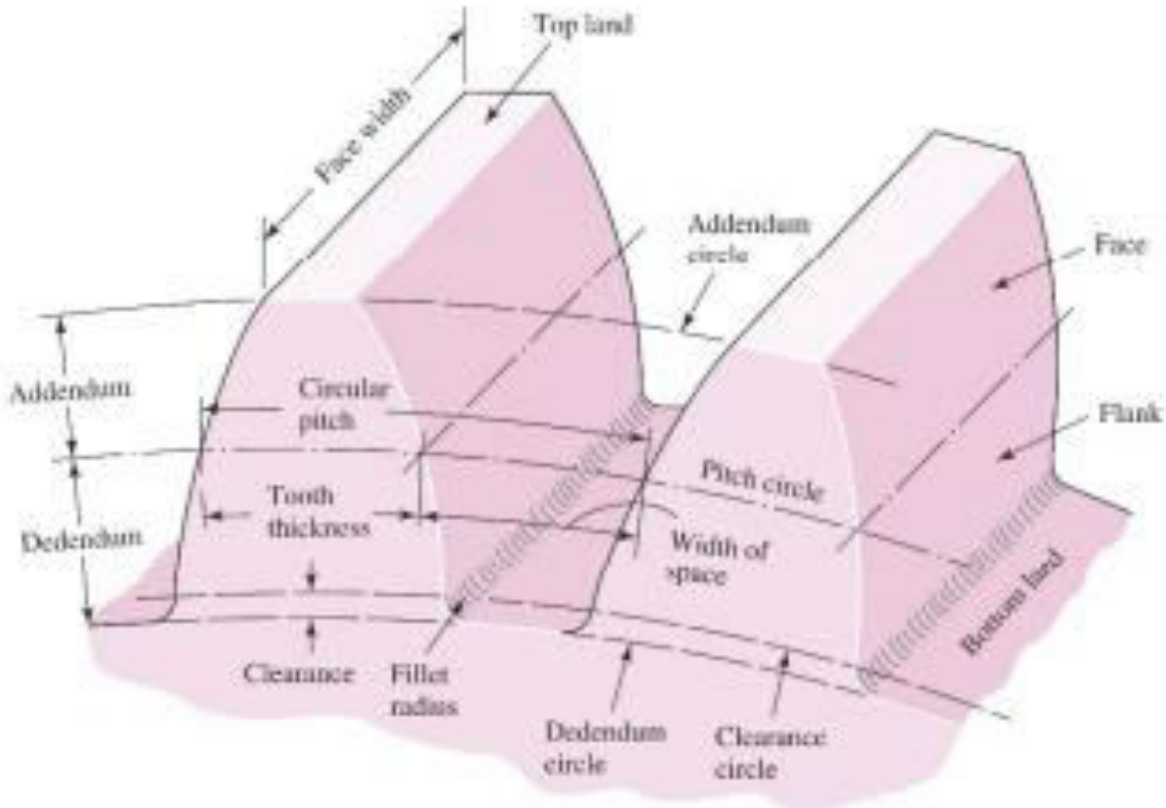
d = pitch diameter, mm

p = circular pitch

Figure 13–5
Nomenclature of spur-gear teeth.



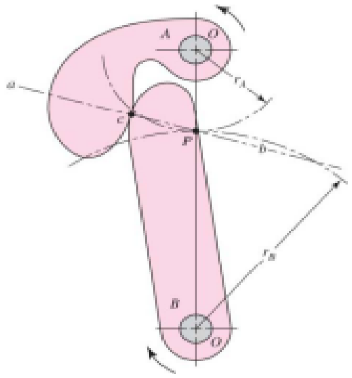
- Chapter 2



Top land
Face width
Addendum circle
Face
Addendum
Circular pitch
Flank
Pitch circle
Tooth thickness
Dedendum
Width of space
Bottom land
Clearance
Fillet radius
Dedendum circle
Clearance circle

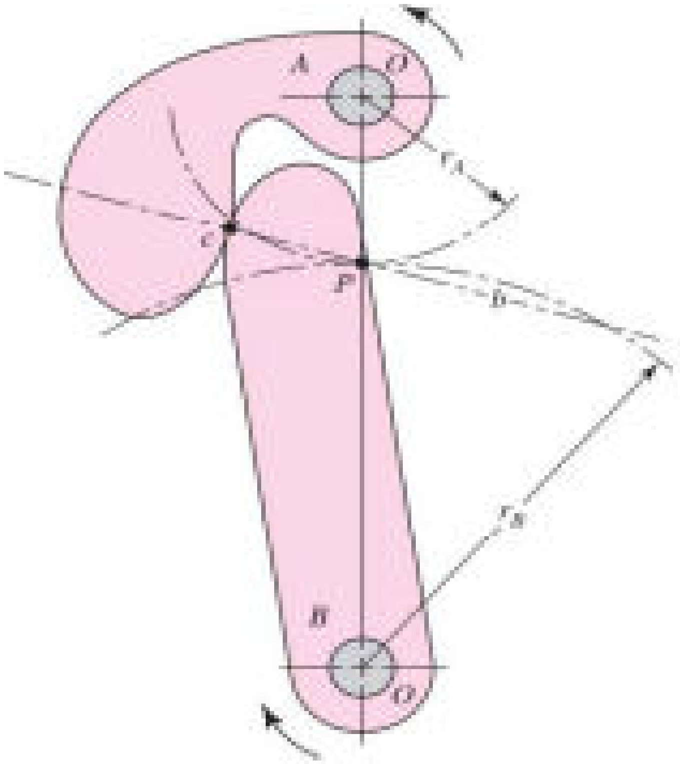
Conjugate Action

- Tooth profiles are designed so as to produce a *constant angular velocity ratio during meshing*, **conjugate action**.
- When one curved surface pushes against another ,the point of contact occurs where the two surfaces are tangent to each other (point *c*), and the forces at any instant are directed along the common normal *ab (line of action)* to the two curves.
- The angular-velocity ratio between the two arms is inversely proportional to their radii to the point *P*.
- Circles drawn through point P are called pitch circles, and point P is called the pitch point.
- To transmit motion at a constant angular-velocity ratio, the pitch point must remain fixed; that is, all the lines of action for every instantaneous point of contact must pass through the same point *P*.



Mating gear teeth produce rotary motion similar to cams

- Chapter 2



• Chapter 2

VELOCITY RATIO OF GEAR DRIVE

- In the case of involute profiles, all points of contact occur on the same straight line *ab*. All normal to the tooth profiles at the point of contact coincide with the line ab, thus these profiles transmit uniform rotary motion.
- *When two gears are in mesh their pitch circles roll on one another without slippage. Then the pitch line velocity is* $V = r_1\omega_1 = r_2\,\omega_2$

d = Diameter of the wheel

N =Speed of the wheel

ω = Angular speed

$$\text{velocity ratio (n)} = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

A
G

- Chapter 2

Fundamentals

- When two gears are in mesh, their pitch circles roll on one another without slipping. The pitch-line velocity $V = |r_1\omega_1| = |r_2\omega_2|$
- Thus the relation between the radii on the angular velocities is

$$\left|\frac{\omega_1}{\omega_2}\right| = \frac{r_2}{r_1}$$

- The addendum and dedendum distances for standard interchangeable teeth are, 1/P and 1.25/P, respectively.
- The pressure line (line of action)

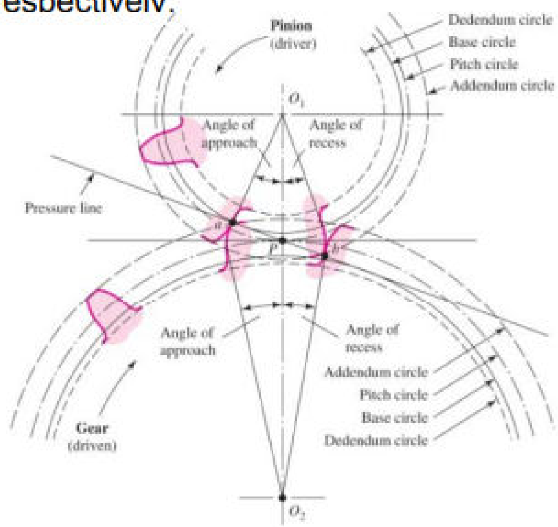
represent the direction in which the

resultant force acts between the gears.

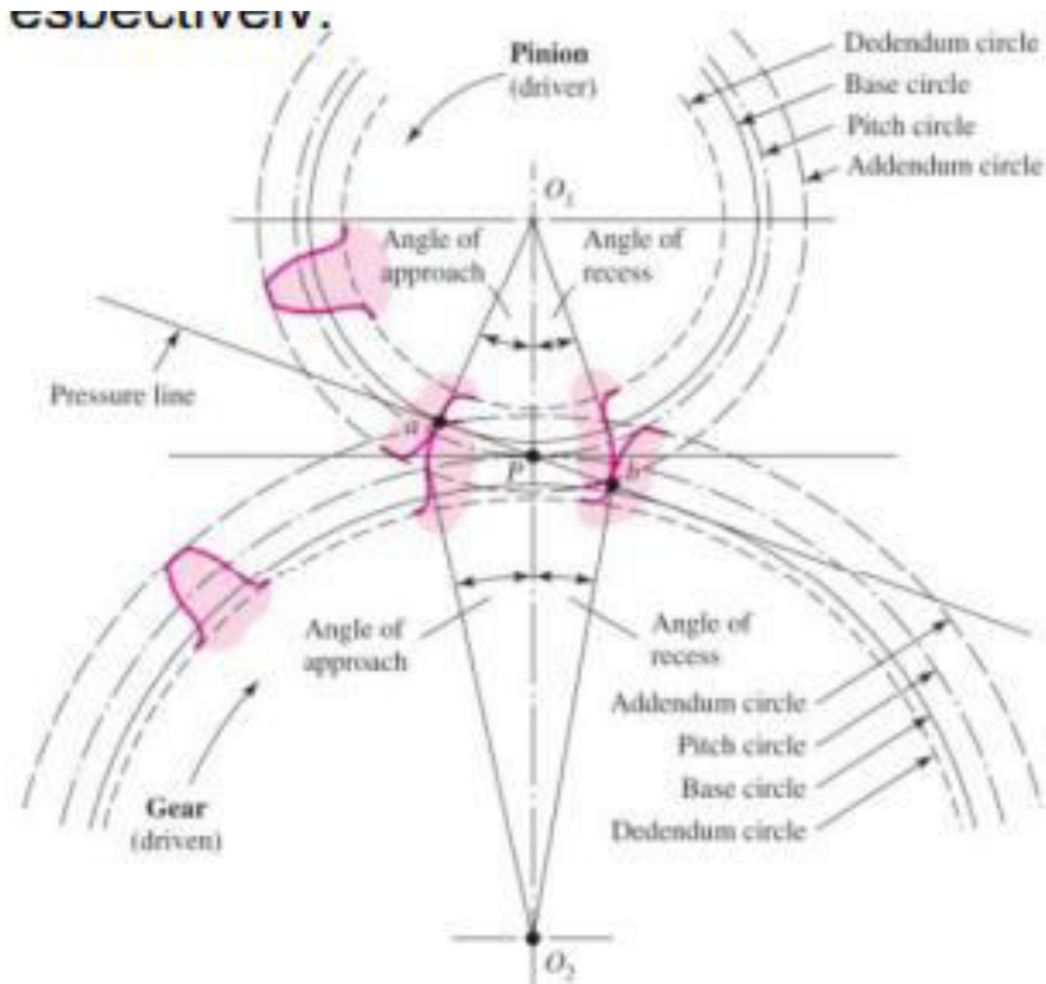
- The angle ϕ is called the pressure angle

and it usually has values of 20° or 25°

- **The involute begins at the base circle and is undefined below this circle.**



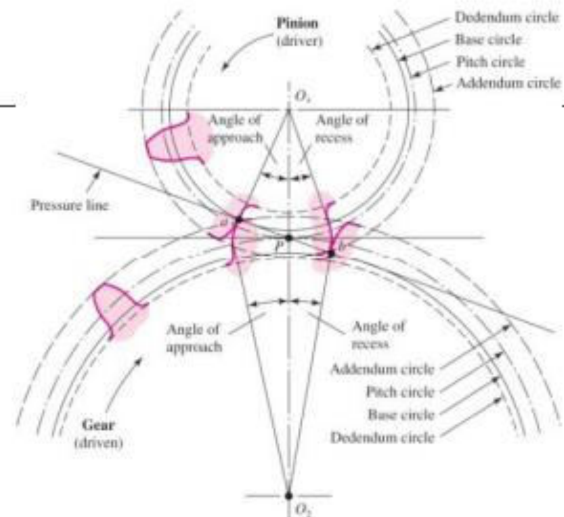
- Chapter 2



Pinion
(driver)
Dedendum circle
Base circle
Pitch circle
Addendum circle
O_1
Angle of
approach
Angle of
recess
Pressure line
a
P
b
Angle of
approach
Angle of
recess
Addendum circle
Pitch circle
Base circle
Dedendum circle
Gear
(driven)
O_2

• Chapter 2

Fundamentals

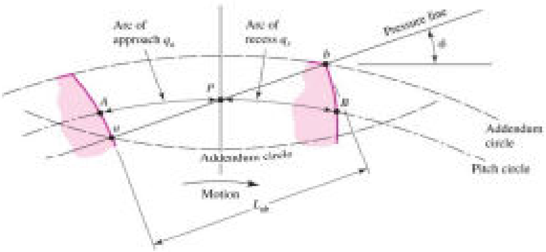


- If we construct tooth profiles through point *a* and draw radial lines from the intersections of these profiles with the pitch circles to the gear centers, we obtain the **angle of approach** for each gear.
- The final point of contact will be where the addendum circle of the driver crosses the pressure line. The **angle of recess** for each gear is obtained in a manner similar to that of finding the angles of approach.
- We may imagine a **rack** as a spur gear having an infinitely large pitch diameter. Therefore, the rack has an infinite number of teeth and a base circle which is an infinite distance from the pitch point.

• Chapter 2

Contact Ratio

- The zone of action of meshing gear teeth is shown with the distance *AP* being the **arc of approach** q_a , and the distance *P B* being the **arc of recess** q_r.
- Tooth contact begins and ends at the intersection of the two addendum circles with the pressure line.
- When a tooth is just beginning contact at *a*, the previous tooth is simultaneously ending its contact at *b* for cases when one tooth and its space occupying the entire arc *AB*.
- Because of the nature of this tooth action, either one or two pairs of teeth in contact, it is convenient to define the term contact ratio m_c as

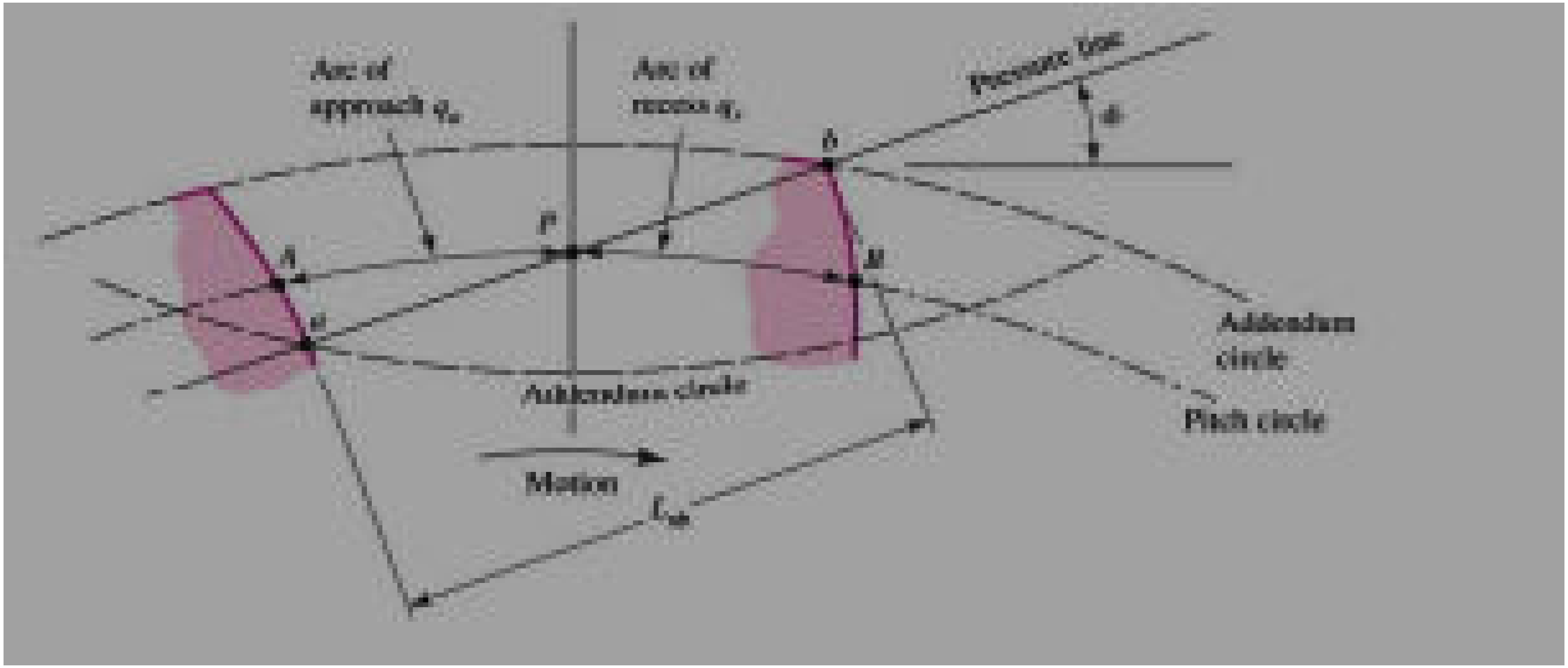


$$m_c = \frac{q_t}{p} = \frac{L_{ab}}{p \cos \phi}$$

a number that indicates the average number of pairs of teeth in contact.

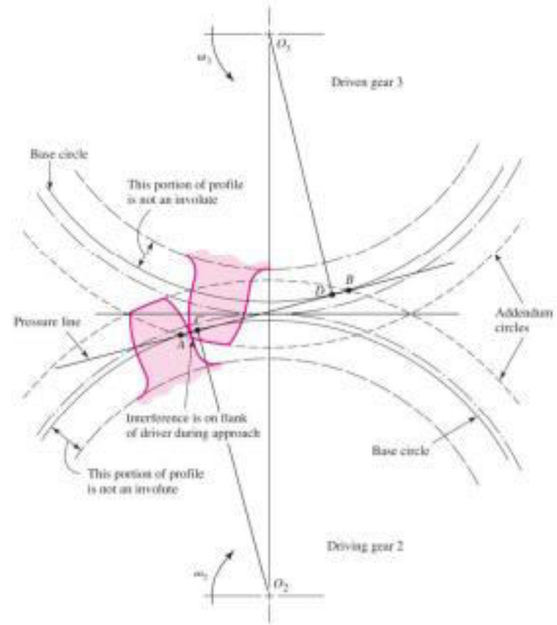
•Gears should not generally be designed having contact ratios less than about 1.20, because inaccuracies in mounting might reduce the contact ratio even more, increasing the possibility of impact between the teeth as well as an increase in the noise level.

- Chapter 2

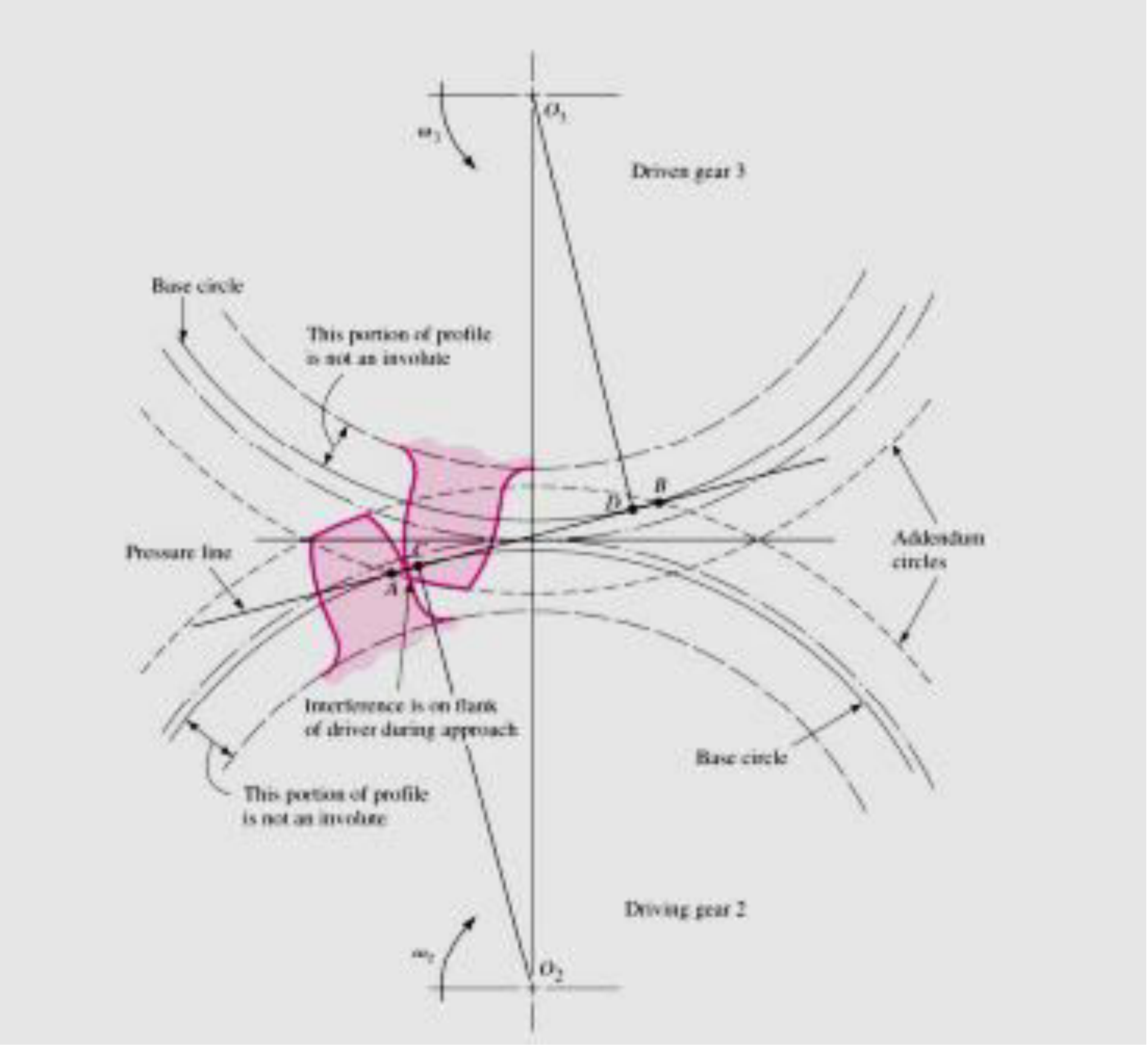

Arc of approach q_a
Arc of recess q_r
Pressure line
ϕ
b
P
A
B
a
Addendum circle
Pitch circle
Addendum circle
Motion
L_{ab}

Interference

- The contact of portions of tooth profiles that are not conjugate is called **interference**.
- When the points of tangency of the pressure line with the base circles *C* and *D* are located **inside** of points *A* and *B* (initial and final points of contact), interference is present.
- The actual effect of interference is that the involute tip or face of the driven gear tends to dig out the noninvolute flank of the driver.
- When gear teeth are produced by a generation process, interference is automatically eliminated because the cutting tool removes the interfering portion of the flank. This effect is called **undercutting.**



• Chapter 2



O_3
ω_3
Driven gear 3
Base circle
This portion of profile is not an involute
D
B
Pressure line
C
A
Addendum circles
Interference is on flank of driver during approach
Base circle
This portion of profile is not an involute
Driving gear 2
ω_2
O_2

• Chapter 2

Interference Analysis

- The smallest number of teeth on a spur pinion and gear, one-to-one gear ratio, which can exist without interference is N_P .
- The number of teeth for spur gears is given by

$$N_P = \frac{2k}{3\sin^2\phi}\left(1 + \sqrt{1 + 3\sin^2\phi}\right)$$

where k = 1 for full-depth teeth, 0.8 for stub teeth and φ = pressure angle.

- If the mating gear has more teeth than the pinion, that is, $m_G = N_G/N_P = m$ is more than one, then the smallest number of teeth on the pinion without interference is given by

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

- The largest gear with a specified pinion that is interference-free is

$$N_G = \frac{N_P^2\sin^2\phi - 4k^2}{4k - 2N_P\sin^2\phi}$$

- The smallest spur pinion that will operate with a rack without interference is

$$N_P = \frac{2(k)}{\sin^2\phi}$$

• Chapter 2

$$N_P = \frac{2k}{3\sin^2\phi}\left(1 + \sqrt{1 + 3\sin^2\phi}\right) \tag{13–10}$$

where $k = 1$ for full-depth teeth, 0.8 for stub teeth and ϕ = pressure angle.

For a 20° pressure angle, with $k = 1$,

$$N_P = \frac{2(1)}{3\sin^2 20^\circ}\left(1 + \sqrt{1 + 3\sin^2 20^\circ}\right) = 12.3 = 13 \text{ teeth}$$

Thus 13 teeth on pinion and gear are interference-free. Realize that 12.3 teeth is possible in meshing arcs, but for fully rotating gears, 13 teeth represents the least number. For a $14\frac{1}{2}^\circ$ pressure angle, $N_P = 23$ teeth, so one can appreciate why few $14\frac{1}{2}^\circ$-tooth systems are used, as the higher pressure angles can produce a smaller pinion with accompanying smaller center-to-center distances.

If the mating gear has more teeth than the pinion, that is, $m_G = N_G/N_P = m$ is more than one, then the smallest number of teeth on the pinion without interference is given by

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right) \tag{13–11}$$

• Chapter 2

For example, if $m = 4, \phi = 20^\circ$,

$$N_P = \frac{2(1)}{[1+2(4)]\sin^2 20^\circ}\left[4+\sqrt{4^2+[1+2(4)]\sin^2 20^\circ}\right] = 15.4 = 16 \text{ teeth}$$

Thus a 16-tooth pinion will mesh with a 64-tooth gear without interference.

The largest gear with a specified pinion that is interference-free is

$$N_G = \frac{N_P^2 \sin^2\phi - 4k^2}{4k - 2N_P \sin^2\phi} \tag{13–12}$$

For example, for a 13-tooth pinion with a pressure angle ϕ of 20°,

$$N_G = \frac{13^2 \sin^2 20^\circ - 4(1)^2}{4(1) - 2(13)\sin^2 20^\circ} = 16.45 = 16 \text{ teeth}$$

For a 13-tooth spur pinion, the maximum number of gear teeth possible without interference is 16.

The smallest spur pinion that will operate with a rack without interference is

$$N_P = \frac{2(k)}{\sin^2\phi} \tag{13–13}$$

For a 20° pressure angle full-depth tooth the smallest number of pinion teeth to mesh with a rack is

$$N_P = \frac{2(1)}{\sin^2 20^\circ} = 17.1 = 18 \text{ teeth}$$

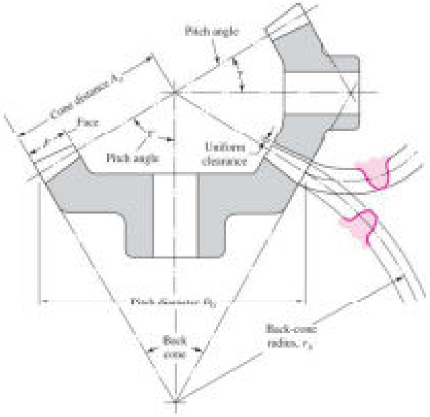
• Chapter 2

Straight Bevel Gears (read)

- When gears are used to transmit motion between intersecting shafts, some form of bevel gear is required.
- The terminology of bevel gears is illustrated.
- The pitch angles are defined by the pitch cones meeting at the apex, as shown in the figure. They are related to the tooth numbers as follows:

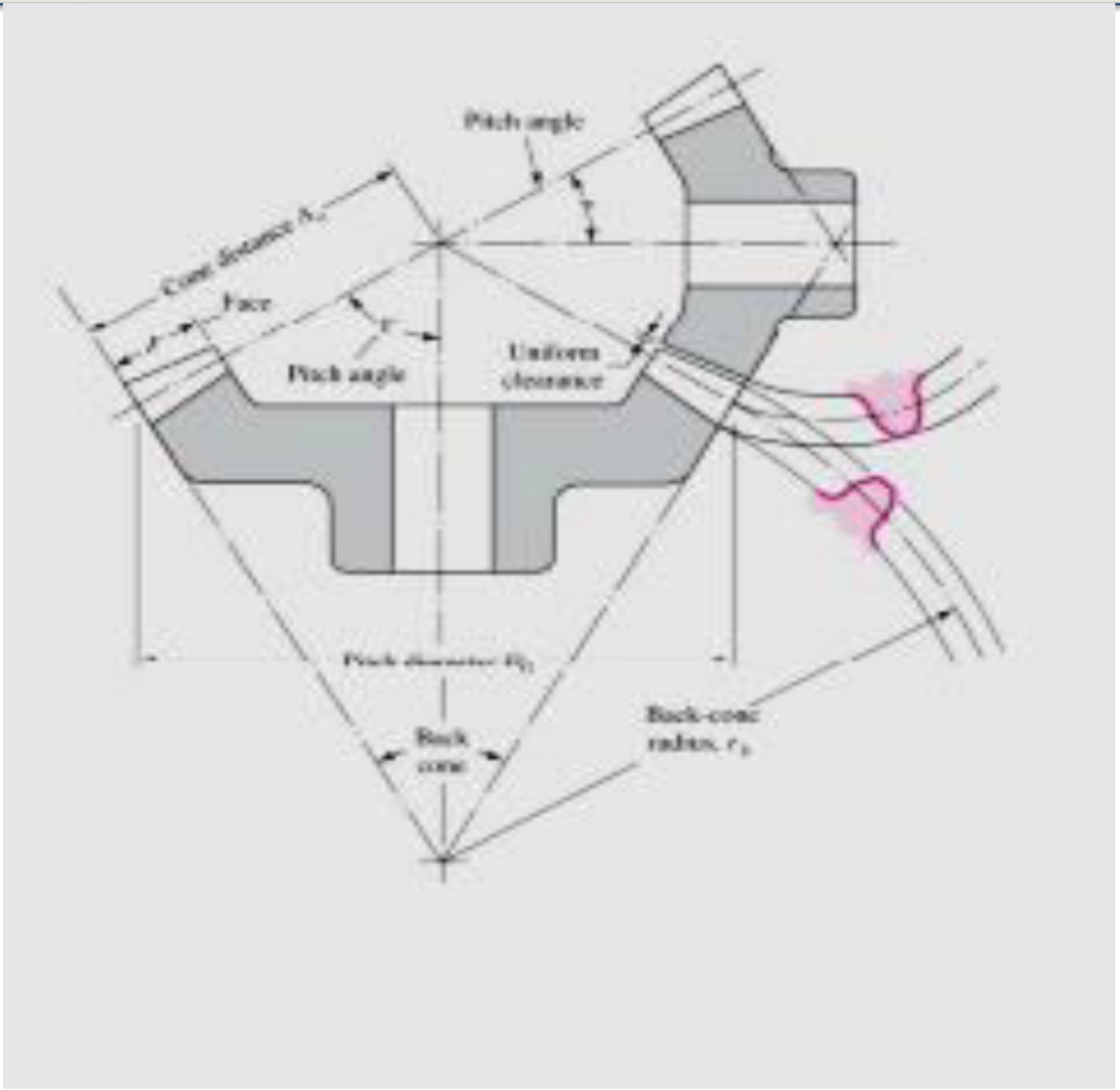
$$\tan \gamma = \frac{N_P}{N_G} \qquad \tan \Gamma = \frac{N_G}{N_P}$$

where the subscripts P and G refer to the pinion and gear, respectively, and where γ and Γ are, respectively, the pitch angles of the pinion and gear.



Standard straight tooth bevel gears are cut by using a 20° pressure angle and full depth teeth. This increases contact ratio, avoids undercut, and increases the strength of the pinion.

• Chapter 2



Pitch angle
Cone distance
Face
Pitch angle
Uniform clearance
Back-cone radius, r_b
Back cone

- Chapter 2

Parallel Helical Gears

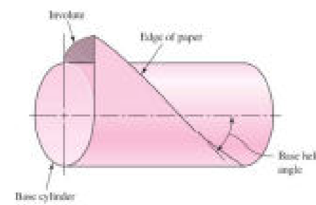
- Helical gears subject the shaft bearings to both radial and thrust loads. When the thrust load become high it maybe desirable to use double helical gears (herringbone) which is equivalent to helical gears of opposite hand, mounted side by side on the same shaft. They develop opposite thrust reactions and thus cancel out.

- When two or more single helical gears are mounted on the same shaft. The hand of the gears should be selected to minimize the thrust load.

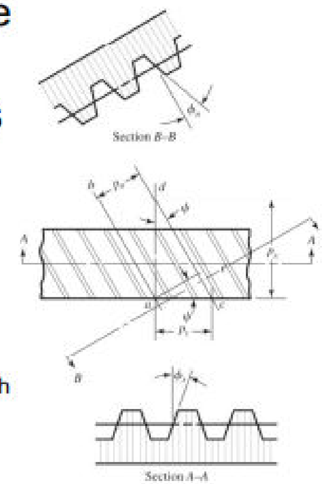
• Chapter 2

Parallel Helical Gears

- The shape of the tooth of Helical gears is an involute helicoid.



- The initial contact of helical-gear teeth is a point that extends into a line as the teeth come into more engagement. In spur gears the line of contact is parallel to the axis of rotation; in helical gears the line is diagonal across the face of the tooth.



- The distance *ae* is the **normal circular pitch** p_n and is related to the transverse circular pitch as follows:

$$p_n = p_t \cos \psi$$

- The distance *ad* is called the **axial pitch** p_x and is related by the expression

$$p_x = \frac{p_t}{\tan \psi}$$

- The normal diametral pitch $P_n = \frac{P_t}{\cos \psi}$ Transverse diametral pitch
- Normal circular pitch x normal diametral pitch ($p_n x P_n = \pi$)
- The pressure angle φ_n in the normal direction is different from the pressure angle φ_t in the direction of rotation. These angles are related by the equation

$$\cos \psi = \frac{\tan \phi_n}{\tan \phi_t}$$

Parallel Helical Gears (Cont.)

- The pressure angle φ_t in the tangential (rotation) direction is

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right)$$

- The smallest tooth number N_P of a helical-spur pinion that will run without interference with a gear with the same number of teeth is

$$N_P = \frac{2k\cos\psi}{3\sin^2\phi_t}\left(1+\sqrt{1+3\sin^2\phi_t}\right)$$

- The largest gear with a specified pinion is given by

$$N_G = \frac{N_P^2\sin^2\phi_t - 4k^2\cos^2\psi}{4k\cos\psi - 2N_P\sin^2\phi_t}$$

- The smallest pinion that can be run with a rack is

$$N_P = \frac{2k\cos\psi}{\sin^2\phi_t}$$

• Chapter 2

The smallest tooth number N_P of a helical-spur pinion that will run without interference[2] with a gear with the same number of teeth is

$$N_P = \frac{2k\cos\psi}{3\sin^2\phi_t}\left(1+\sqrt{1+3\sin^2\phi_t}\right) \tag{13–21}$$

For example, if the normal pressure angle ϕ_n is 20°, the helix angle ψ is 30°, then ϕ_t is

$$\phi_t = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 30^\circ}\right) = 22.80^\circ$$

$$N_P = \frac{2(1)\cos 30^\circ}{3\sin^2 22.80^\circ}\left(1+\sqrt{1+3\sin^2 22.80^\circ}\right) = 8.48 = 9 \text{ teeth}$$

For a given gear ratio $m_G = N_G/N_P = m$, the smallest pinion tooth count is

$$N_P = \frac{2k\cos\psi}{(1+2m)\sin^2\phi_t}\left[m+\sqrt{m^2+(1+2m)\sin^2\phi_t}\right] \tag{13–22}$$

The largest gear with a specified pinion is given by

$$N_G = \frac{N_P^2\sin^2\phi_t - 4k^2\cos^2\psi}{4k\cos\psi - 2N_P\sin^2\phi_t} \tag{13–23}$$

For example, for a nine-tooth pinion with a pressure angle ϕ_n of 20°, a helix angle ψ of 30°, and recalling that the tangential pressure angle ϕ_t is 22.80°,

$$N_G = \frac{9^2\sin^2 22.80^\circ - 4(1)^2\cos^2 30^\circ}{4(1)\cos 30^\circ - 2(9)\sin^2 22.80^\circ} = 12.02 = 12$$

The smallest pinion that can be run with a rack is

$$N_P = \frac{2k\cos\psi}{\sin^2\phi_t} \tag{13–24}$$

For a normal pressure angle ϕ_n of 20° and a helix angle ψ of 30°, and $\phi_t = 22.80^\circ$,

$$N_P = \frac{2(1)\cos 30^\circ}{\sin^2 22.80^\circ} = 11.5 = 12 \text{ teeth}$$

Chapter 2

EXAMPLE 13-2 A stock helical gear has a normal pressure angle of 20°, a helix angle of 25°, and a transverse diametral pitch of 6 teeth/in, and has 18 teeth. Find:

(*a*) The pitch diameter
(*b*) The transverse, the normal, and the axial pitches
(*c*) The normal diametral pitch
(*d*) The transverse pressure angle

Solution

Answer (*a*)

$$d = \frac{N}{P_t} = \frac{18}{6} = 3 \text{ in}$$

Answer (*b*)

$$p_t = \frac{\pi}{P_t} = \frac{\pi}{6} = 0.5236 \text{ in}$$

Answer

$$p_n = p_t \cos\psi = 0.5236 \cos 25^\circ = 0.4745 \text{ in}$$

Answer

$$p_x = \frac{p_t}{\tan\psi} = \frac{0.5236}{\tan 45^\circ} = 1.123 \text{ in}$$

Answer (*c*)

$$P_n = \frac{P_t}{\cos\psi} = \frac{6}{\cos 25^\circ} = 6.620 \text{ teeth/in}$$

Answer (*d*)

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 25^\circ}\right) = 21.88^\circ$$

Worm Gears (read)

- The worm and worm gear of a set have the same hand of helix as for crossed helical gears.
- It is usual to specify the lead angle λ on the worm and helix angle ψ_G on the gear; the two angles are equal for a 90° shaft angle.
- Since it is not related to the number of teeth, the worm may have any pitch diameter; this diameter should, however, be the same as the pitch diameter of the hob used to cut the worm-gear teeth. Generally,

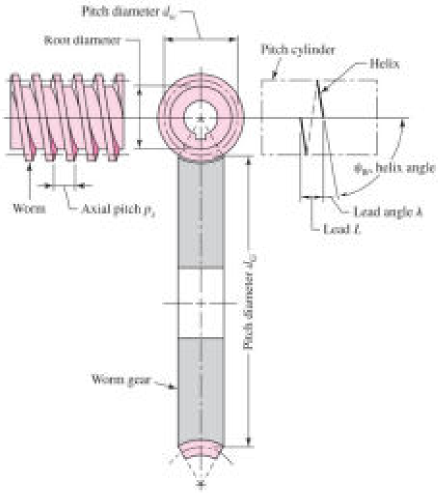
$$\frac{C^{0.875}}{3.0} \leq d_W \leq \frac{C^{0.875}}{1.7}$$

where C is the center distance.

•The lead L and the lead angle λ of the worm have the following relations:

$$L = p_x N_W$$

$$\tan \lambda = \frac{L}{\pi d_W}$$



• Chapter 2

Tooth Systems

Spur gears

- A **tooth system** is a standard that specifies the relationships involving addendum, dedendum, working depth, tooth thickness, and pressure angle.
- Tooth forms for worm gearing have not been highly standardized, perhaps because there has been less need for it.
- The **face width** F_G of the worm gear should be made equal to the length of a tangent to the worm pitch circle between its points of intersection with the addendum circle.

Tooth System	Pressure Angle ϕ, deg	Addendum a	Dedendum b
Full depth	20	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	$22\frac{1}{2}$	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	25	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
Stub	20	$0.8/P_d$ or $0.8m$	$1/P_d$ or $1m$

Diametral Pitch	
Coarse	2, $2\frac{1}{4}$, $2\frac{1}{2}$, 3, 4, 6, 8, 10, 12, 16
Fine	20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200

Modules	
Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

Worm gears

Lead Angle λ, deg	Pressure Angle ϕ_n, deg	Addendum a	Dedendum b_G
0–15	$14\frac{1}{2}$	$0.3683p_x$	$0.3683p_x$
15–30	20	$0.3683p_x$	$0.3683p_x$
30–35	25	$0.2865p_x$	$0.3314p_x$
35–40	25	$0.2546p_x$	$0.2947p_x$
40–45	30	$0.2228p_x$	$0.2578p_x$

• Chapter 2

Standard Tooth Properties

Quantity*	Formula	Quantity*	Formula
Addendum	$\frac{1.00}{P_n}$ [$1.0\, m_n$]	*External gears:*	
Dedendum	$\frac{1.25}{P_n}$ [$1.25\, m_n$]	*Standard center distance*	$\frac{D+d}{2}$
Pinion pitch diameter	$\frac{N_P}{P_n \cos\psi}$ $\left[\frac{N_P m_n}{\cos\psi}\right]$	*Gear outside diameter*	$D+2a$
Gear pitch diameter	$\frac{N_G}{P_n \cos\psi}$ $\left[\frac{N_P m_n}{\cos\psi}\right]$	*Pinion outside diameter*	$d+2a$
Normal arc tooth thickness[†]	$\frac{\pi}{P_n} - \frac{B_n}{2}$ $\left[\pi m_n - \frac{B_n}{2}\right]$	*Gear root diameter*	$D-2b$
Pinion base diameter	$d \cos\phi_t$	*Pinion root diameter*	$d-2b$
		Internal gears:	
Gear base diameter	$D \cos\phi_t$	*Center distance*	$\frac{D-d}{2}$
Base helix angle	$\tan^{-1}(\tan\psi \cos\phi_t)$	*Inside diameter*	$D-2a$
		Root diameter	$D+2b$

*All dimensions are in inches, and angles are in degrees.
[†]B_n is the normal backlash.
Corresponding SI units formula in square brackets.

Helical gears

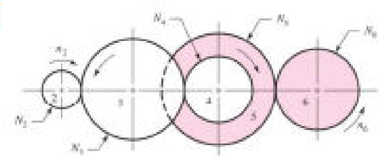
Bevel gears

Item	Formula
Working depth	$h_k = 2.0/P$ [$= 2.0$ m]
Clearance	$c = (0.188/P) + 0.002$ in [$= 0.188$ m $+ 0.05$ mm]
Addendum of gear	$a_G = \frac{0.54}{P} + \frac{0.460}{P(m_{90})^2}$ $\left[= 0.54\,\text{m} + \frac{0.46\,\text{m}}{(m_{90})^2}\right]$
Gear ratio	$m_G = N_G/N_P$
Equivalent 90° ratio	$m_{90} = m_G$ when $\Gamma = 90°$
	$m_{90} = \sqrt{m_G \frac{\cos\gamma}{\cos\Gamma}}$ when $\Gamma \neq 90°$
Face width	$F = 0.3A_0$ or $F = \frac{10}{P}$, whichever is smaller $\left[F = \frac{A_0}{3} \text{ or } F = 10\,\text{m}\right]$
Minimum number of teeth	Pinion: 16, 15, 14, 13; Gear: 16, 17, 20, 30

Corresponding SI units formula in square brackets.

Gear Trains

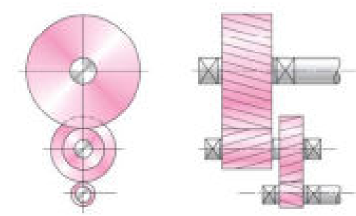
- Consider a pinion 2 driving a gear 3. The speed of the driven gear is



$$n_3 = \left|\frac{N_2}{N_3} n_2\right| = \left|\frac{d_2}{d_3} n_2\right|$$

where n = revolutions or rev/min N = number of teeth
d = pitch diameter

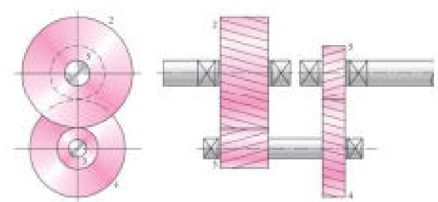
- Gear 3 is an idler that affects only the direction of rotation of gear 6.



- Gears 2, 3, and 5 are drivers, while 3, 4, and 6 are driven members. We define the **train value** e as

$$e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}} \qquad n_6 = -\frac{N_2}{N_3}\frac{N_3}{N_4}\frac{N_5}{N_6} n_2$$

- As a rough guideline, a train value of up to 10 to 1 can be obtained with one pair of gears. A two-stage compound gear train can obtain a train value of up to 100 to 1.

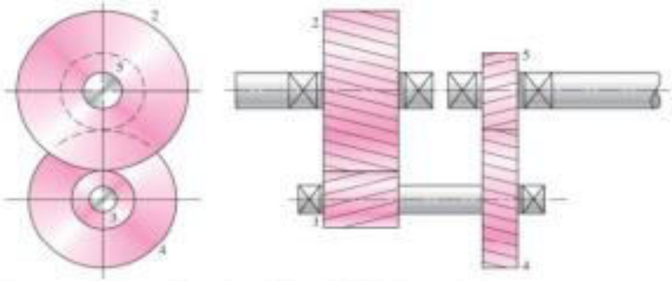


$$d_2/2 + d_3/2 = d_4/2 + d_5/2$$

- It is sometimes desirable for the input shaft and the output shaft of a two-stage compound gear train to be in-line.

- Chapter 2

$$d_2/2 + d_3/2 = d_4/2 + d_5/2$$



The diametral pitch relates the diameters and the numbers of teeth, $P = N/d$. Replacing all the diameters gives

$$N_2/(2P) + N_3/(2P) - N_4/(2P) + N_5/(2P)$$

Assuming a constant diametral pitch in both stages, we have the geometry condition stated in terms of numbers of teeth:

$$N_2 + N_3 = N_4 + N_5$$

This condition must be exactly satisfied, in addition to the previous ratio equations, to provide for the in-line condition on the input and output shafts.

- Chapter 2

$$e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}} \tag{13-30}$$

Note that pitch diameters can be used in Eq. (13–30) as well. When Eq. (13–30) is used for spur gears, e is positive if the last gear rotates in the same sense as the first, and negative if the last rotates in the opposite sense.

Now we can write

$$n_L = en_F \tag{13-31}$$

where n_L is the speed of the last gear in the train and n_F is the speed of the first.

Planetary Gear Train

- Planetary trains always include a **sun gear**, a **planet carrier** or **arm**, and one or more **planet gears**.
- The figure shows a planetary train composed of a sun gear 2, an arm or carrier 3, and planet gears 4 and 5.
- The angular velocity of gear 2 relative to the arm in rev/min is

$$n_{23} = n_2 - n_3$$

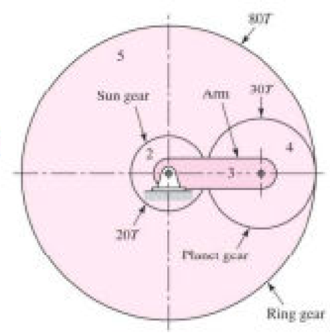
- The ratio of gear 5 to that of gear 2 is the same and is proportional to the tooth numbers, whether the arm is rotating or not. It is the train value.

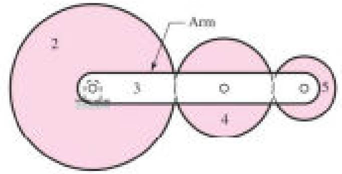
$$e = \frac{n_5 - n_3}{n_2 - n_3} \quad \text{or} \quad e = \frac{n_L - n_A}{n_F - n_A}$$

where n_F = rev/min of first gear in planetary train

n_L = rev/min of last gear in planetary train

n_A = rev/min of arm





• Chapter 2:

EXAMPLE 13–3 A gearbox is needed to provide a 30:1 ($\pm$ 1 percent) increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

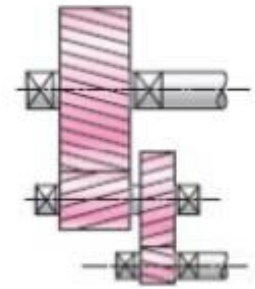
Solution Since the ratio is greater than 10:1, but less than 100:1, a two-stage compound gear train, such as in Figure 13–28, is needed. The portion to be accomplished in each stage is $\sqrt{30} = 5.4772$. For this ratio, assuming a typical 20° pressure angle, the minimum number of teeth to avoid interference is 16, according to Eq. (13–11). The number of teeth necessary for the mating gears is

Answer
$$16\sqrt{30} = 87.64 \doteq 88$$

From Eq. (13–30), the overall train value is

$$e = (88/16)(88/16) = 30.25$$

This is within the 1 percent tolerance. If a closer tolerance is desired, then increase the pinion size to the next integer and try again.



- Chapter 2

EXAMPLE 13–4 A gearbox is needed to provide an *exact* 30:1 increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

Solution The previous example demonstrated the difficulty with finding integer numbers of teeth to provide an exact ratio. In order to obtain integers, factor the overall ratio into two integer stages.

$$e = 30 = (6)(5)$$

$$N_2/N_3 = 6 \quad \text{and} \quad N_4/N_5 = 5$$

With two equations and four unknown numbers of teeth, two free choices are available. Choose N_3 and N_5 to be as small as possible without interference. Assuming a 20° pressure angle, Eq. (13–11) gives the minimum as 16.

Then

$$N_2 = 6\,N_3 = 6(16) = 96$$

$$N_4 = 5\,N_5 = 5(16) = 80$$

The overall train value is then exact.

$$e = (96/16)(80/16) = (6)(5) = 30$$

• Chapter 2

EXAMPLE 13–5 A gearbox is needed to provide an exact 30:1 increase in speed, while minimizing the overall gearbox size. The input and output shafts should be in-line. Specify appropriate teeth numbers.

Solution The governing equations are

$$N_2/N_3 = 6$$

$$N_4/N_5 = 5$$

$$N_2 + N_3 = N_4 + N_5$$

With three equations and four unknown numbers of teeth, only one free choice is available. Of the two smaller gears, N_3 and N_5, the free choice should be used to minimize N_3 since a greater gear ratio is to be achieved in this stage. To avoid interference, the minimum for N_3 is 16.

Applying the governing equations yields

$$N_2 = 6N_3 = 6(16) = 96$$

$$N_2 + N_3 = 96 + 16 = 112 = N_4 + N_5$$

Substituting $N_4 = 5N_5$ gives

$$112 = 5N_5 + N_5 = 6N_5$$

$$N_5 = 112/6 = 18.67$$

• Chapter 2

If the train value need only be approximated, then this can be rounded to the nearest integer. But for an exact solution, it is necessary to choose the initial free choice for N_3 such that solution of the rest of the teeth numbers results exactly in integers. This can be done by trial and error, letting $N_3 = 17$, then 18, etc., until it works. Or, the problem can be normalized to quickly determine the minimum free choice. Beginning again, let the free choice be $N_3 = 1$. Applying the governing equations gives

$$N_2 = 6N_3 = 6(1) = 6$$

$$N_2 + N_3 = 6 + 1 = 7 = N_4 + N_5$$

Substituting $N_4 = 5N_5$, we find

$$7 = 5N_5 + N_5 = 6N_5$$

$$N_5 = 7/6$$

This fraction could be eliminated if it were multiplied by a multiple of 6. The free choice for the smallest gear N_3 should be selected as a multiple of 6 that is greater than the minimum allowed to avoid interference. This would indicate that $N_3 = 18$. Repeating the application of the governing equations for the final time yields

$$N_2 = 6N_3 = 6(18) = 108$$

$$N_2 + N_3 = 108 + 18 = 126 = N_4 + N_5$$

$$126 = 5N_5 + N_5 = 6N_5$$

$$N_5 = 126/6 = 21$$

$$N_4 = 5N_5 = 5(21) = 105$$

Thus,

Answer

$$N_2 = 108$$

$$N_3 = 18$$

$$N_4 = 105$$

$$N_5 = 21$$

Checking, we calculate $e = (108/18)(105/21) = (6)(5) = 30$.

And checking the geometry constraint for the in-line requirement, we calculate

$$N_2 + N_3 = N_4 + N_5$$

$$108 + 18 = 105 + 21$$

$$126 = 126$$

• Chapter 2

EXAMPLE 13–6 In Fig. 13–30 the sun gear is the input, and it is driven clockwise at 100 rev/min. The ring gear is held stationary by being fastened to the frame. Find the rev/min and direction of rotation of the arm and gear 4.

Solution Designate $n_F = n_2 = -100$ rev/min, and $n_L = n_5 = 0$. Unlocking gear 5 and holding the arm stationary, in our imagination, we find

$$e = -\left(\frac{20}{30}\right)\left(\frac{30}{80}\right) = -0.25$$

Substituting this value in Eq. (13–32) gives

$$-0.25 = \frac{0 - n_A}{(-100) - n_A}$$

or

Answer
$$n_A = -20 \text{ rev/min}$$

To obtain the speed of gear 4, we follow the procedure outlined by Eqs. (*b*), (*c*), and (*d*). Thus

$$n_{43} = n_4 - n_3 \qquad n_{23} = n_2 - n_3$$

and so

$$\frac{n_{43}}{n_{23}} = \frac{n_4 - n_3}{n_2 - n_3} \tag{1}$$

But

$$\frac{n_{43}}{n_{23}} = -\frac{20}{30} = -\frac{2}{3} \tag{2}$$

Substituting the known values in Eq. (1) gives

$$-\frac{2}{3} = \frac{n_4 - (-20)}{(-100) - (-20)}$$

Solving gives

Answer
$$n_4 = 33\tfrac{1}{3} \text{ rev/min}$$