

**Heat and Mass Transfer: Fundamentals & Applications**

**Fourth Edition**

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# **Chapter 11**

# **HEAT EXCHANGERS**

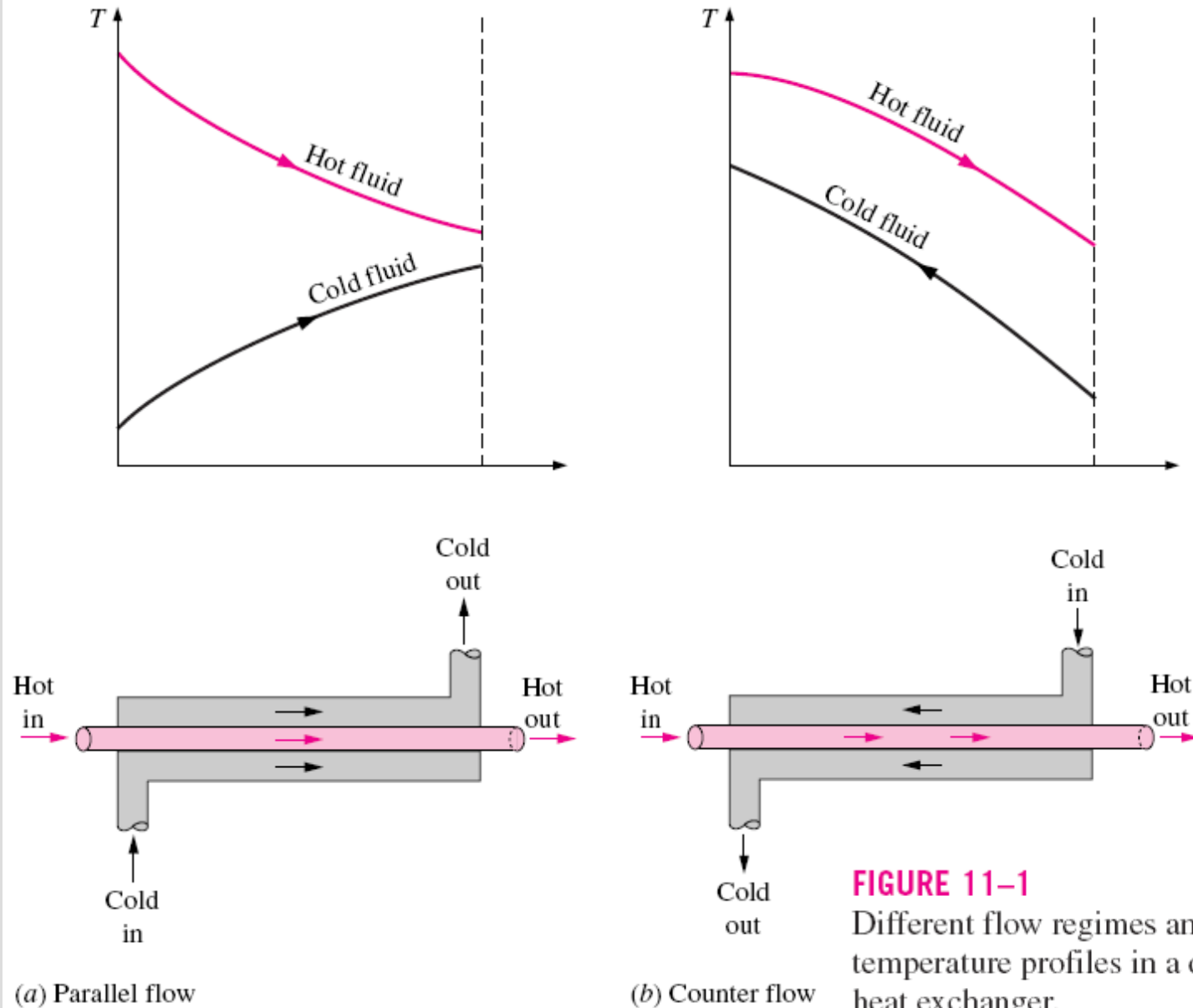
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# Objectives

- Recognize numerous types of heat exchangers, and classify them
- Develop an awareness of fouling on surfaces, and determine the overall heat transfer coefficient for a heat exchanger
- Perform a general energy analysis on heat exchangers
- Obtain a relation for the logarithmic mean temperature difference for use in the LMTD method, and modify it for different types of heat exchangers using the correction factor
- Develop relations for effectiveness, and analyze heat exchangers when outlet temperatures are not known using the effectiveness-NTU method
- Know the primary considerations in the selection of heat exchangers.

# TYPES OF HEAT EXCHANGERS

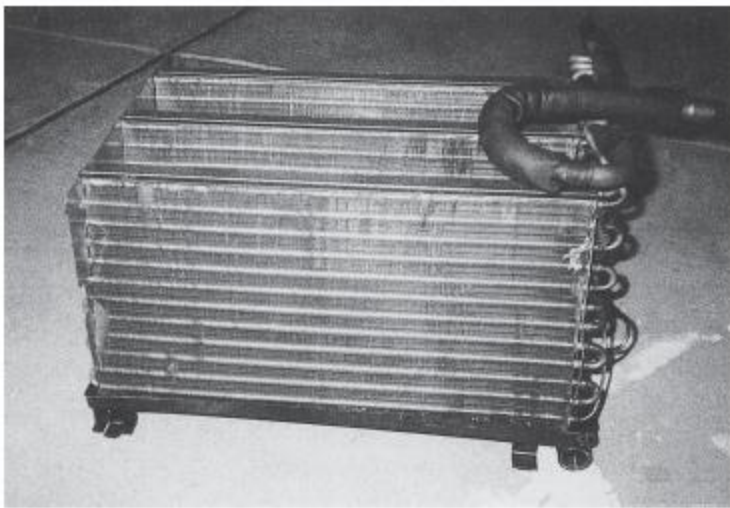


**FIGURE 11-1**

Different flow regimes and associated temperature profiles in a double-pipe heat exchanger.

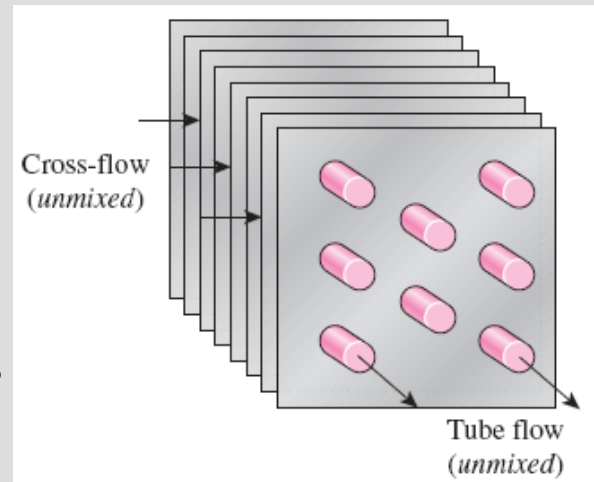
**Compact heat exchanger:** It has a large heat transfer surface area per unit volume (e.g., car radiator, human lung). A heat exchanger with the *area density*  $\beta > 700 \text{ m}^2/\text{m}^3$  is classified as being compact.

**Cross-flow:** In compact heat exchangers, the two fluids usually move *perpendicular* to each other. The cross-flow is further classified as *unmixed* and *mixed flow*.

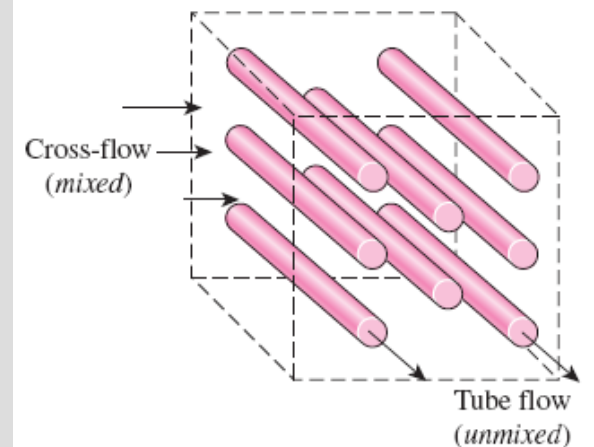


**FIGURE 11-2**

A gas-to-liquid compact heat exchanger for a residential air-conditioning system.



(a) Both fluids unmixed



(b) One fluid mixed, one fluid unmixed

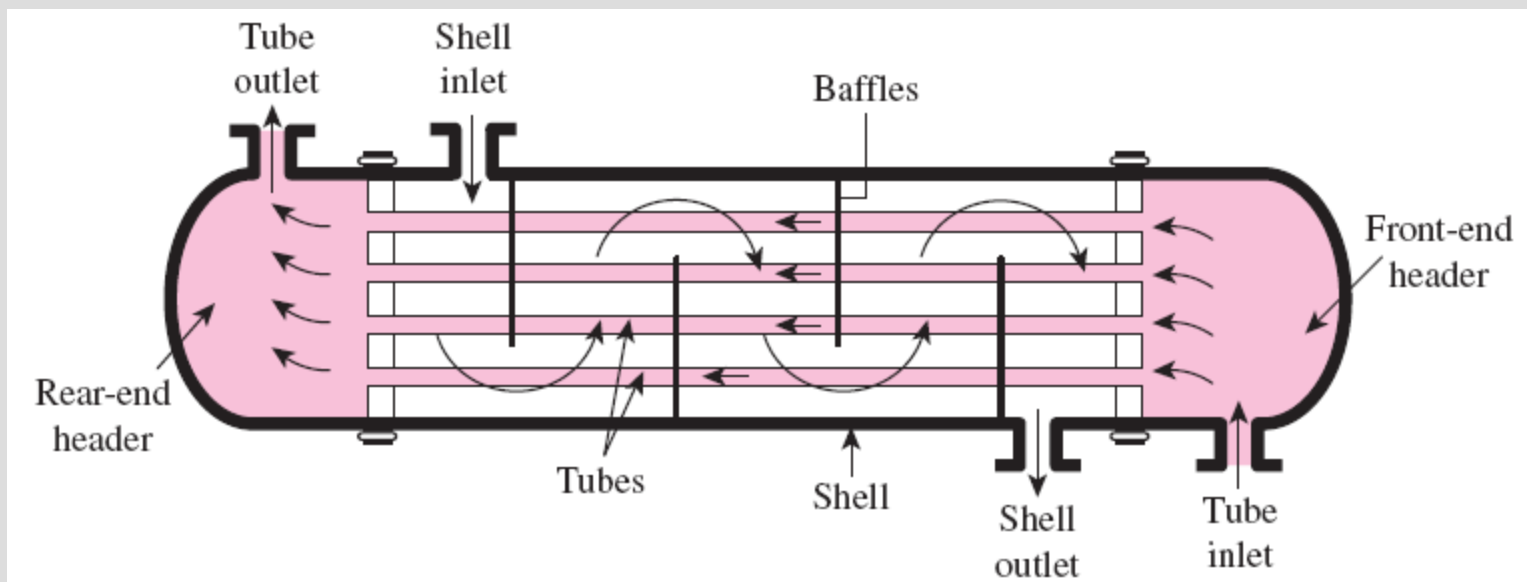
**FIGURE 11-3**

Different flow configurations in cross-flow heat exchangers.

**Shell-and-tube heat exchanger:** The most common type of heat exchanger in industrial applications.

They contain a large number of tubes (sometimes several hundred) packed in a shell with their axes parallel to that of the shell. Heat transfer takes place as one fluid flows inside the tubes while the other fluid flows outside the tubes through the shell.

Shell-and-tube heat exchangers are further classified according to the number of shell and tube passes involved.



**FIGURE 11-4**

The schematic of a shell-and-tube heat exchanger (one-shell pass and one-tube pass).

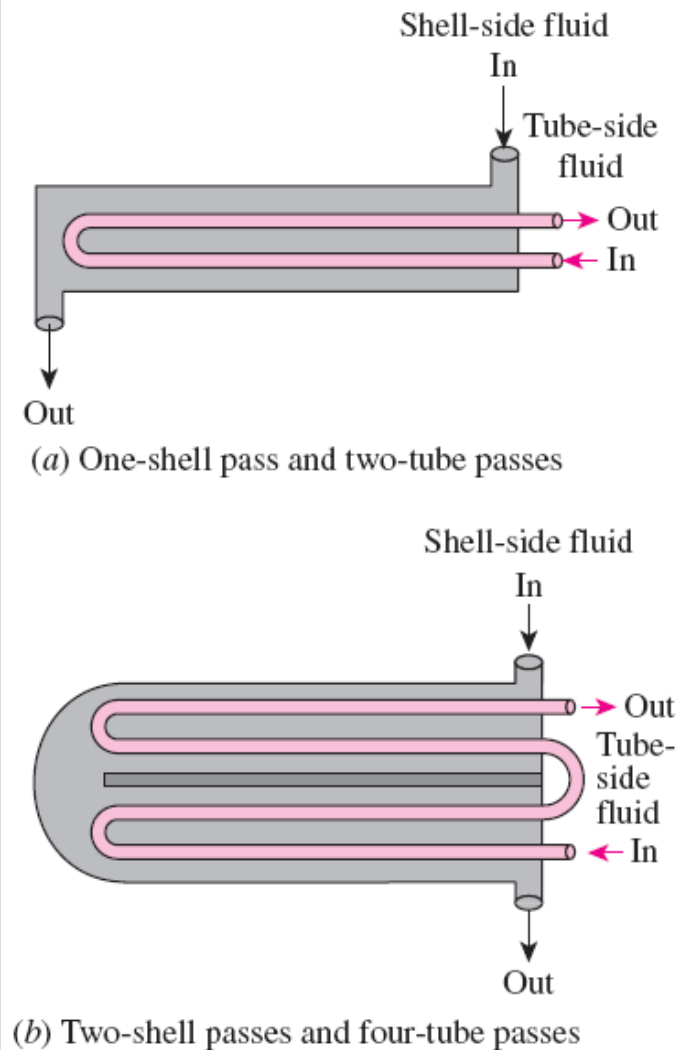
### Regenerative heat exchanger:

Involves the alternate passage of the hot and cold fluid streams through the same flow area.

**Dynamic-type regenerator:** Involves a rotating drum and continuous flow of the hot and cold fluid through different portions of the drum so that any portion of the drum passes periodically through the hot stream, storing heat, and then through the cold stream, rejecting this stored heat.

**Condenser:** One of the fluids is cooled and condenses as it flows through the heat exchanger.

**Boiler:** One of the fluids absorbs heat and vaporizes.

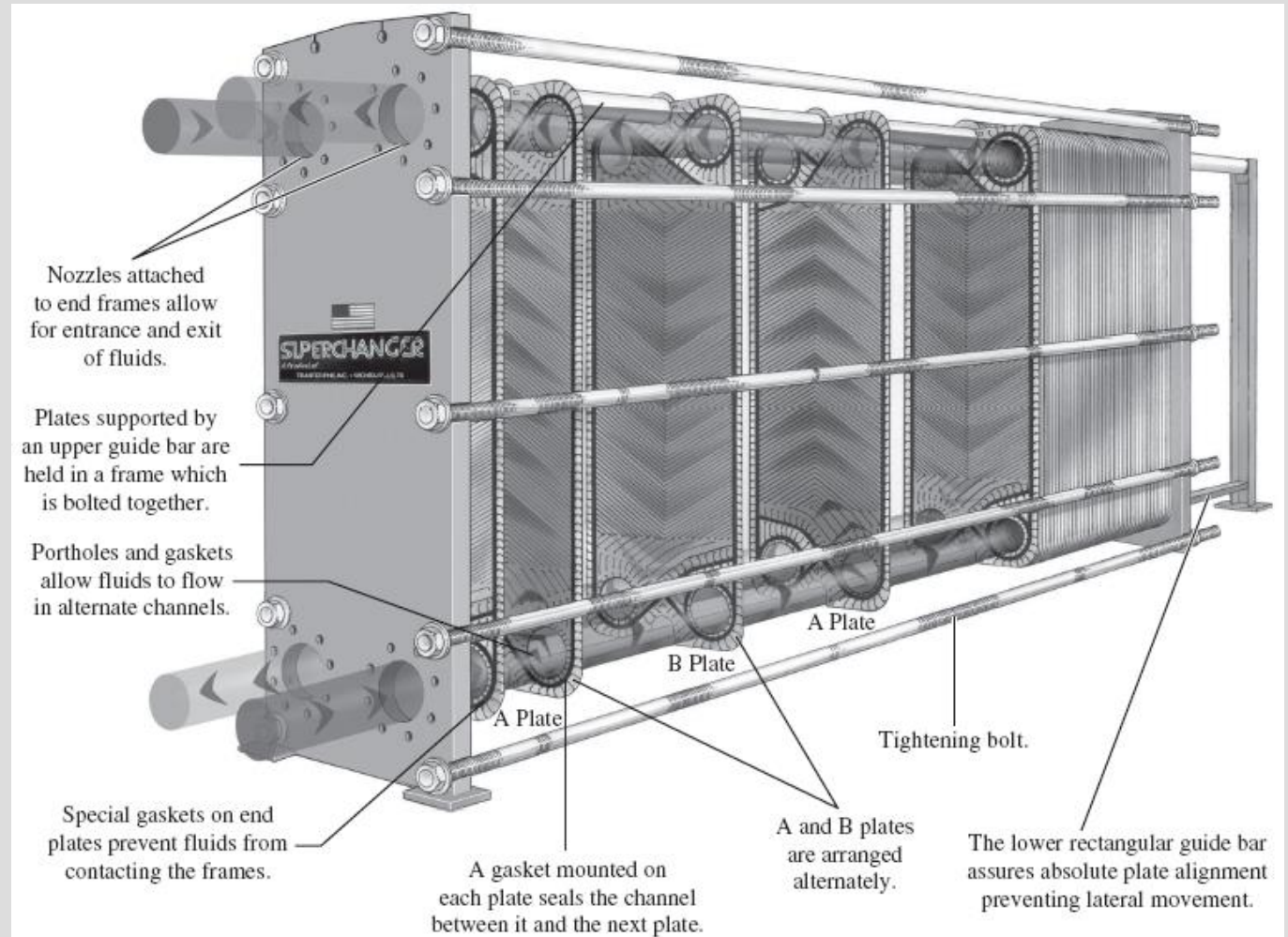


**FIGURE 11-5**

Multipass flow arrangements in shell-and-tube heat exchangers.

**Plate and frame (or just plate) heat exchanger:** Consists of a series of plates with corrugated flat flow passages. The hot and cold fluids flow in alternate passages, and thus each cold fluid stream is surrounded by two hot fluid streams, resulting in very effective heat transfer. Well suited for liquid-to-liquid applications.

A plate-and-frame liquid-to-liquid heat exchanger.



# THE OVERALL HEAT TRANSFER COEFFICIENT

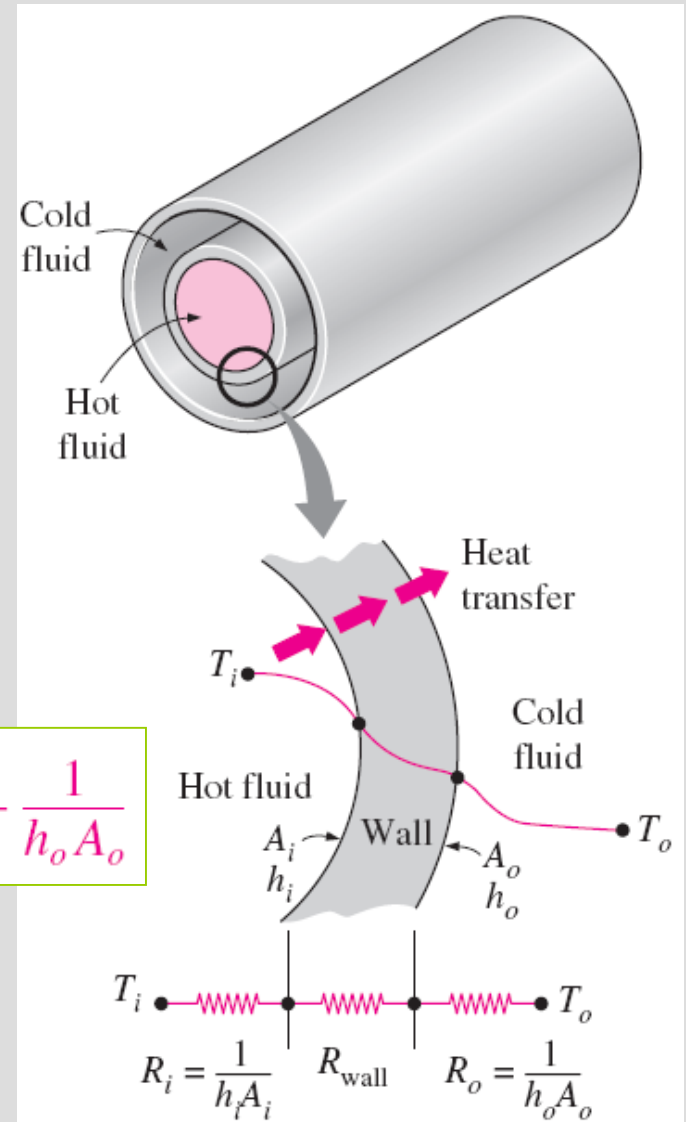
- A heat exchanger typically involves two flowing fluids separated by a solid wall.
- Heat is first transferred from the hot fluid to the wall by *convection*, through the wall by *conduction*, and from the wall to the cold fluid again by *convection*.
- Any radiation effects are usually included in the convection heat transfer coefficients.

$$R_{\text{wall}} = \frac{\ln(D_o/D_i)}{2\pi kL}$$

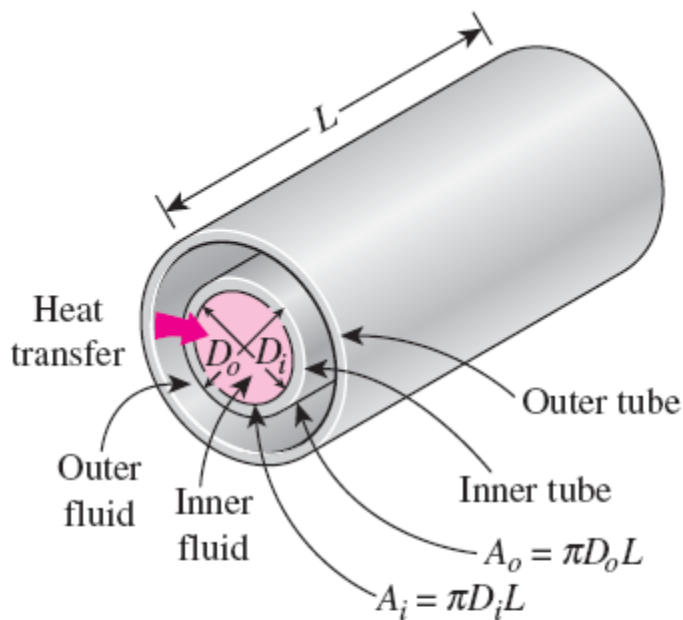
$$R = R_{\text{total}} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{1}{h_o A_o}$$

$$A_i = \pi D_i L \text{ and } A_o = \pi D_o L$$

Thermal resistance network associated with heat transfer in a double-pipe heat exchanger.







**FIGURE 11-8**

The two heat transfer surface areas associated with a double-pipe heat exchanger (for thin tubes,  $D_i \approx D_o$  and thus  $A_i \approx A_o$ ).

$$\dot{Q} = \frac{\Delta T}{R} = UA\Delta T = U_i A_i \Delta T = U_o A_o \Delta T$$

$U$  the overall heat transfer coefficient,  $\text{W/m}^2 \cdot ^\circ\text{C}$

$$\frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + R_{\text{wall}} + \frac{1}{h_o A_o}$$

$$U_i A_i = U_o A_o, \text{ but } U_i \neq U_o \text{ unless } A_i = A_o$$

**When**  $R_{\text{wall}} \approx 0$   $A_i \approx A_o \approx A_s$

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o} \quad U \approx U_i \approx U_o$$

The overall heat transfer coefficient  $U$  is dominated by the *smaller* convection coefficient. When one of the convection coefficients is *much smaller* than the other (say,  $h_i \ll h_o$ ), we have  $1/h_i \gg 1/h_o$ , and thus  $U \approx h_i$ . This situation arises frequently when one of the fluids is a gas and the other is a liquid. In such cases, fins are commonly used on the gas side to enhance the product  $UA$  and thus the heat transfer on that side.

The overall heat transfer coefficient ranges from about 10 W/m<sup>2</sup>.°C for gas-to-gas heat exchangers to about 10,000 W/m<sup>2</sup>.°C for heat exchangers that involve phase changes.

When the tube is *finned* on one side to enhance heat transfer, the total heat transfer surface area on the finned side is

$$A_s = A_{\text{total}} = A_{\text{fin}} + A_{\text{unfinned}}$$

For short fins of high thermal conductivity, we can use this total area in the convection resistance relation

$$R_{\text{conv}} = 1/hA_s$$

$$A_s = A_{\text{unfinned}} + \eta_{\text{fin}} A_{\text{fin}}$$

To account for fin efficiency

**TABLE 11–1**

Representative values of the overall heat transfer coefficients in heat exchangers

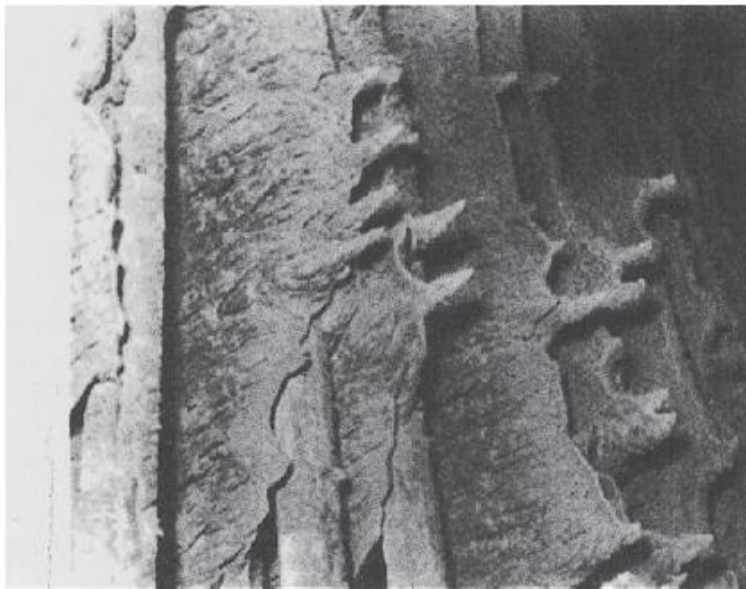
Type of heat exchanger	$U$ , W/m <sup>2</sup> .K*
Water-to-water	850–1700
Water-to-oil	100–350
Water-to-gasoline or kerosene	300–1000
Feedwater heaters	1000–8500
Steam-to-light fuel oil	200–400
Steam-to-heavy fuel oil	50–200
Steam condenser	1000–6000
Freon condenser (water cooled)	300–1000
Ammonia condenser (water cooled)	800–1400
Alcohol condensers (water cooled)	250–700
Gas-to-gas	10–40
Water-to-air in finned tubes (water in tubes)	30–60 <sup>†</sup>
	400–850 <sup>†</sup>
Steam-to-air in finned tubes (steam in tubes)	30–300 <sup>†</sup>
	400–4000 <sup>‡</sup>

# Fouling Factor

The performance of heat exchangers usually deteriorates with time as a result of accumulation of *deposits* on heat transfer surfaces. The layer of deposits represents *additional resistance* to heat transfer. This is represented by a **fouling factor  $R_f$** .

$$\frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

The fouling factor increases with the *operating temperature* and the *length of service* and decreases with the *velocity* of the fluids.



**FIGURE 11–9**

Precipitation fouling of ash particles on superheater tubes.

**TABLE 11–2**

Representative fouling factors  
(thermal resistance due to fouling for  
a unit surface area)

Fluid	$R_f$ , m <sup>2</sup> ·K/W
Distilled water, sea-water, river water, boiler feedwater:	
Below 50°C	0.0001
Above 50°C	0.0002
Fuel oil	0.0009
Steam (oil-free)	0.0001
Refrigerants (liquid)	0.0002
Refrigerants (vapor)	0.0004
Alcohol vapors	0.0001
Air	0.0004

# ANALYSIS OF HEAT EXCHANGERS

An engineer often finds himself or herself in a position

1. to select a heat exchanger that will achieve a specified temperature change in a fluid stream of known mass flow rate - **the log mean temperature difference (or LMTD) method**.
2. to predict the outlet temperatures of the hot and cold fluid streams in a specified heat exchanger - **the effectiveness–NTU method**.

The rate of heat transfer in heat exchanger (HE is insulated):

$$\dot{Q} = \dot{m}_c c_{pc} (T_{c, \text{out}} - T_{c, \text{in}})$$

$$\dot{Q} = \dot{m}_h c_{ph} (T_{h, \text{in}} - T_{h, \text{out}})$$

$\dot{m}_c, \dot{m}_h$  = mass flow rates

$c_{pc}, c_{ph}$  = specific heats

$T_{c, \text{out}}, T_{h, \text{out}}$  = outlet temperatures

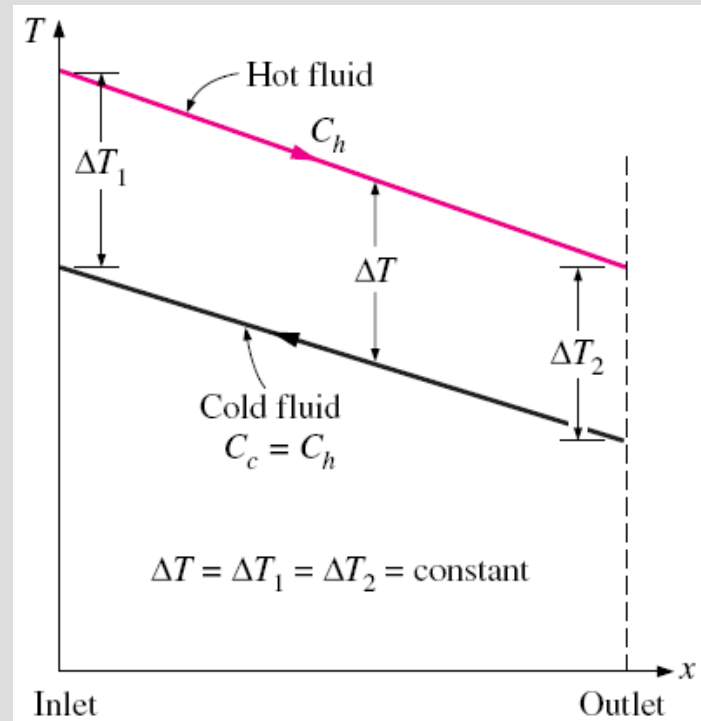
$T_{c, \text{in}}, T_{h, \text{in}}$  = inlet temperatures

$$C_h = \dot{m}_h c_{ph} \quad \text{and} \quad C_c = \dot{m}_c c_{pc}$$

heat capacity rate

$$\dot{Q} = C_c (T_{c, \text{out}} - T_{c, \text{in}}) \quad \dot{Q} = C_h (T_{h, \text{in}} - T_{h, \text{out}})$$

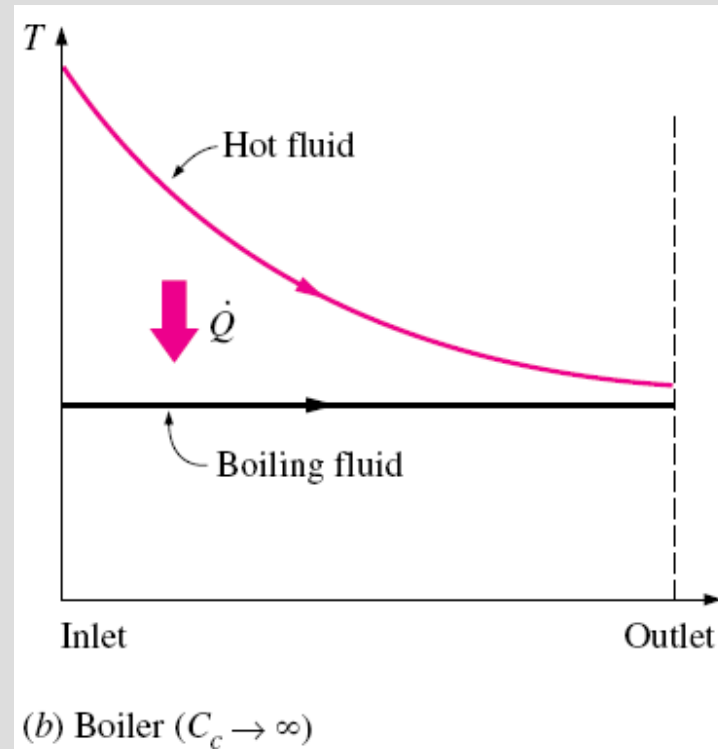
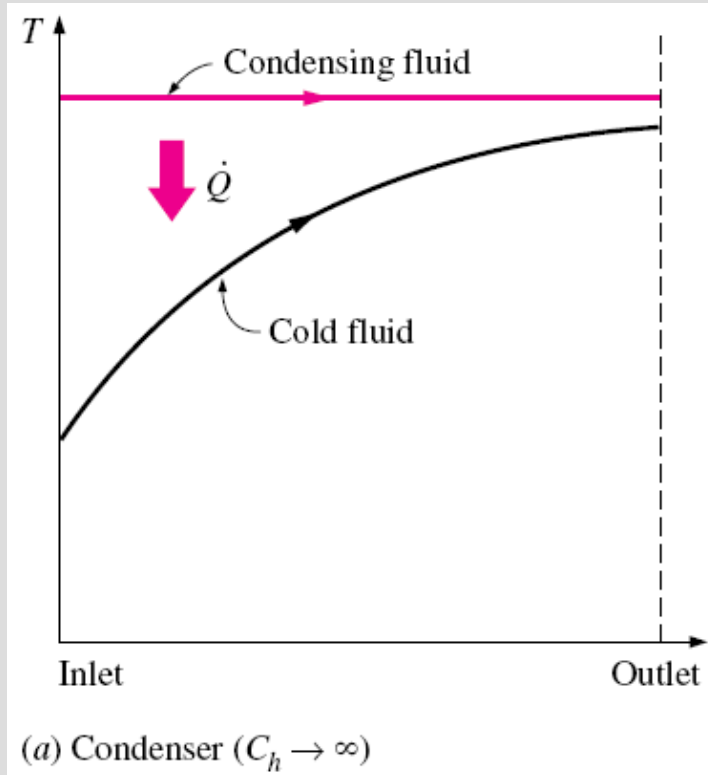
Two fluid streams that have the same capacity rates experience the same temperature change in a well-insulated heat exchanger.



$\dot{m}$  is the rate of evaporation or condensation of the fluid  $\dot{Q} = \dot{m}h_{fg}$

$h_{fg}$  is the enthalpy of vaporization of the fluid at the specified temperature or pressure.

The heat capacity rate of a fluid during a phase-change process must approach infinity since the temperature change is practically zero.



Variation of fluid temperatures in a heat exchanger when one of the fluids condenses or boils.

$$\dot{Q} = UA_s \Delta T_m$$

$\Delta T_m$  an appropriate mean (average) temperature difference between the two fluids

# THE LOG MEAN TEMPERATURE DIFFERENCE METHOD

$$\delta\dot{Q} = -\dot{m}_h c_{ph} dT_h \quad \delta\dot{Q} = \dot{m}_c c_{pc} dT_c$$

$$dT_h = -\frac{\delta\dot{Q}}{\dot{m}_h c_{ph}} \quad dT_c = \frac{\delta\dot{Q}}{\dot{m}_c c_{pc}}$$

$$dT_h - dT_c = d(T_h - T_c) = -\delta\dot{Q} \left( \frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}} \right)$$

$$\delta\dot{Q} = U(T_h - T_c) dA_s$$

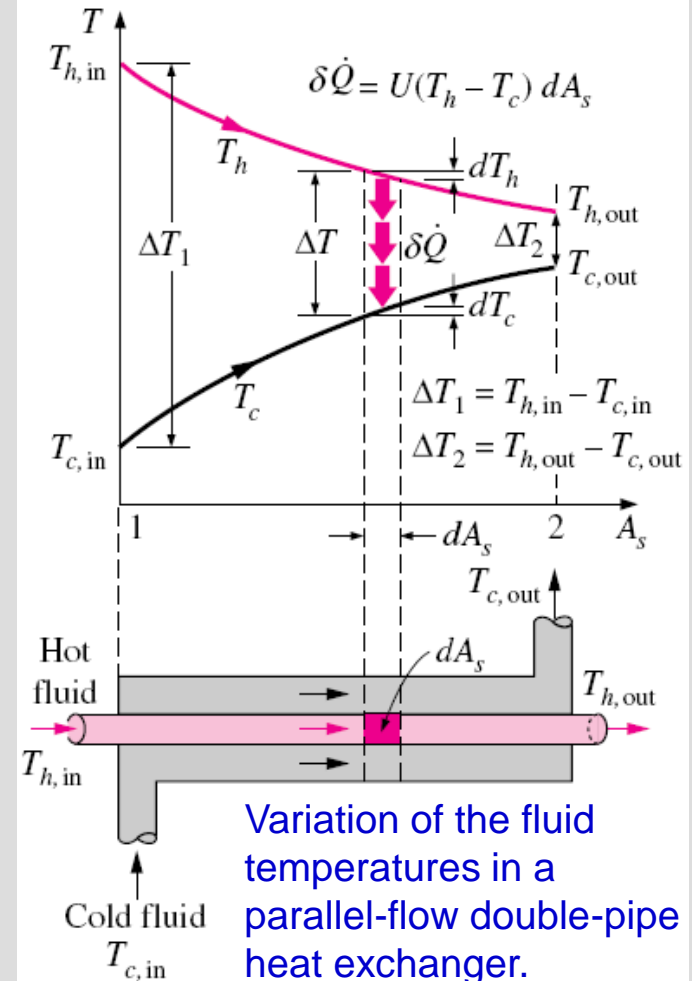
$$\frac{d(T_h - T_c)}{T_h - T_c} = -U dA_s \left( \frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}} \right)$$

$$\ln \frac{T_{h, out} - T_{c, out}}{T_{h, in} - T_{c, in}} = -UA_s \left( \frac{1}{\dot{m}_h c_{ph}} + \frac{1}{\dot{m}_c c_{pc}} \right)$$

$$\dot{Q} = UA_s \Delta T_{lm}$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

log mean temperature difference



## The arithmetic mean temperature difference

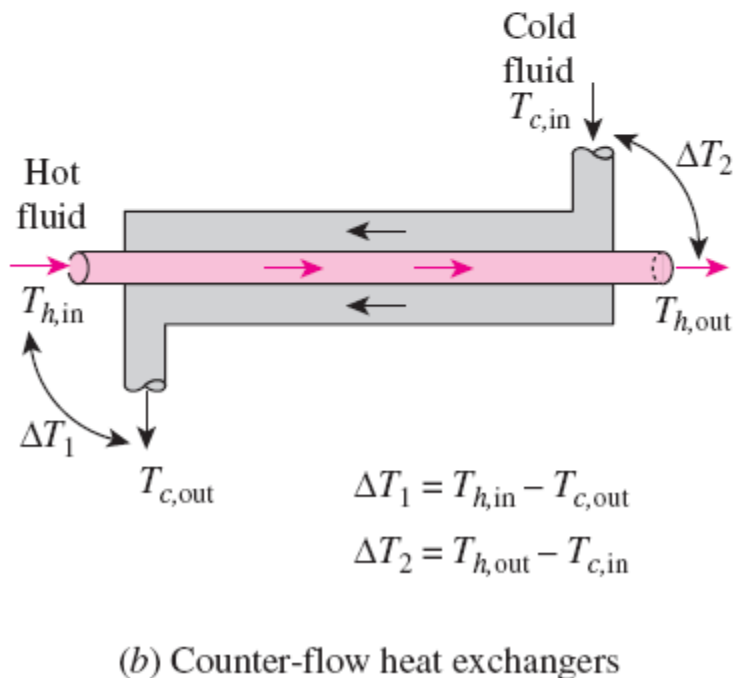
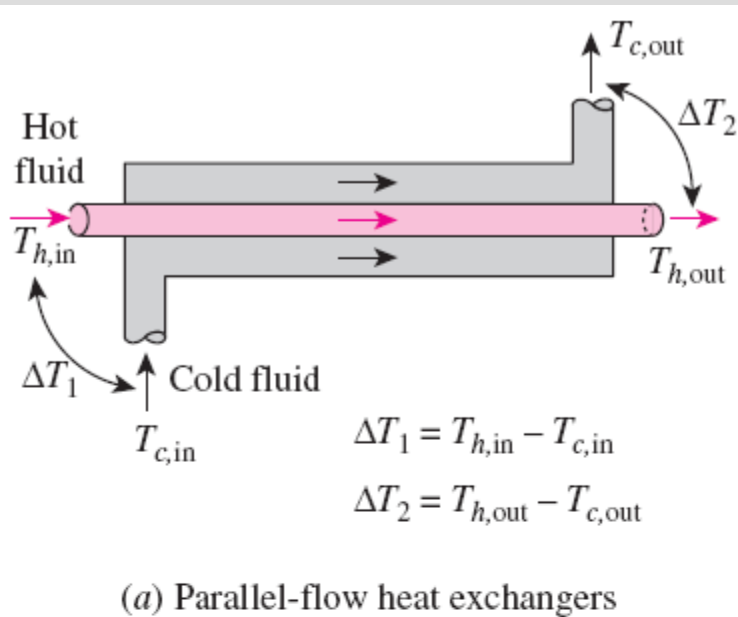
$$\Delta T_{\text{am}} = \frac{1}{2}(\Delta T_1 + \Delta T_2)$$

The logarithmic mean temperature difference  $\Delta T_{\text{lm}}$  is an *exact* representation of the *average temperature difference* between the hot and cold fluids.

Note that  $\Delta T_{\text{lm}}$  is always less than  $\Delta T_{\text{am}}$ . Therefore, using  $\Delta T_{\text{am}}$  in calculations instead of  $\Delta T_{\text{lm}}$  will overestimate the rate of heat transfer in a heat exchanger between the two fluids.

When  $\Delta T_1$  differs from  $\Delta T_2$  by no more than 40 percent, the error in using the arithmetic mean temperature difference is less than 1 percent. But the error increases to undesirable levels when  $\Delta T_1$  differs from  $\Delta T_2$  by greater amounts.

**FIGURE 11-15**  
The  $\Delta T_1$  and  $\Delta T_2$  expressions in parallel-flow and counter-flow heat exchangers.





# Counter-Flow Heat Exchangers

In the limiting case, the cold fluid will be heated to the inlet temperature of the hot fluid.

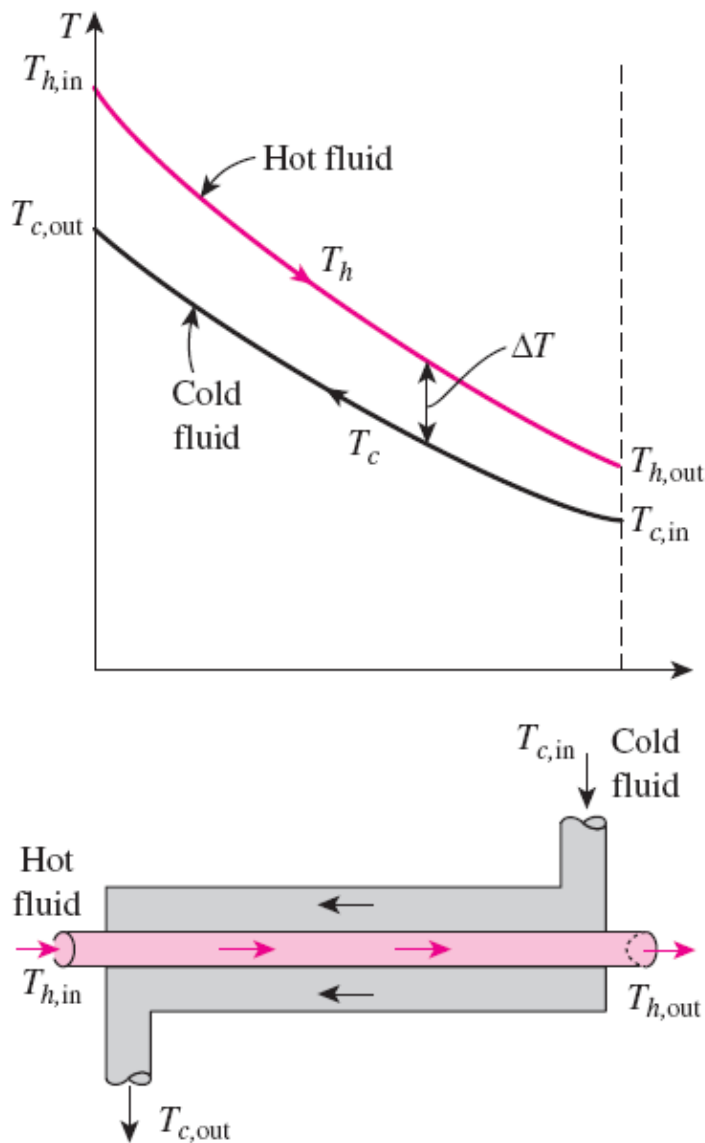
However, the outlet temperature of the cold fluid can *never* exceed the inlet temperature of the hot fluid.

For specified inlet and outlet temperatures,  $\Delta T_{lm}$  a counter-flow heat exchanger is always greater than that for a parallel-flow heat exchanger.

That is,  $\Delta T_{lm, CF} > \Delta T_{lm, PF}$ , and thus a smaller surface area (and thus a smaller heat exchanger) is needed to achieve a specified heat transfer rate in a counter-flow heat exchanger.

When the *heat capacity rates* of the two fluids are *equal*

$$\Delta T_{lm} = \Delta T_1 = \Delta T_2$$

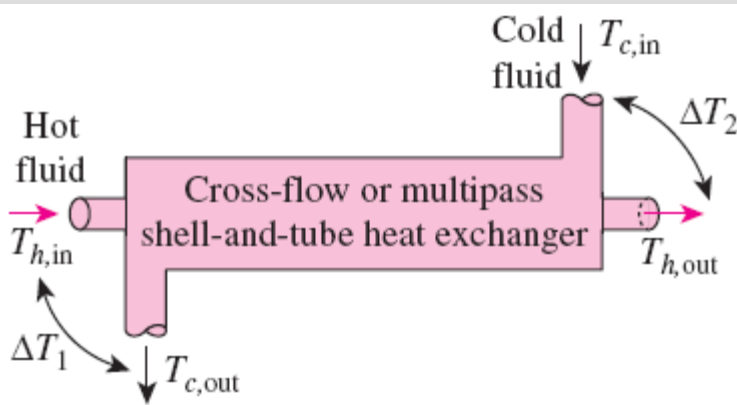


**FIGURE 11-16**

The variation of the fluid temperatures in a counter-flow double-pipe heat exchanger.



# Multipass and Cross-Flow Heat Exchangers: Use of a Correction Factor



Heat transfer rate:

$$\dot{Q} = UA_s F \Delta T_{lm,CF}$$

where

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

$$\Delta T_1 = T_{h,in} - T_{c,out}$$

$$\Delta T_2 = T_{h,out} - T_{c,in}$$

and  $F = \dots$  (Fig. 11-18)

## FIGURE 11-17

The determination of the heat transfer rate for cross-flow and multipass shell-and-tube heat exchangers using the correction factor.

$$\Delta T_{lm} = F \Delta T_{lm,CF}$$

***F* correction factor** depends on the *geometry* of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams.

*F* for common cross-flow and shell-and-tube heat exchanger configurations is given in the figure versus two temperature ratios *P* and *R* defined as

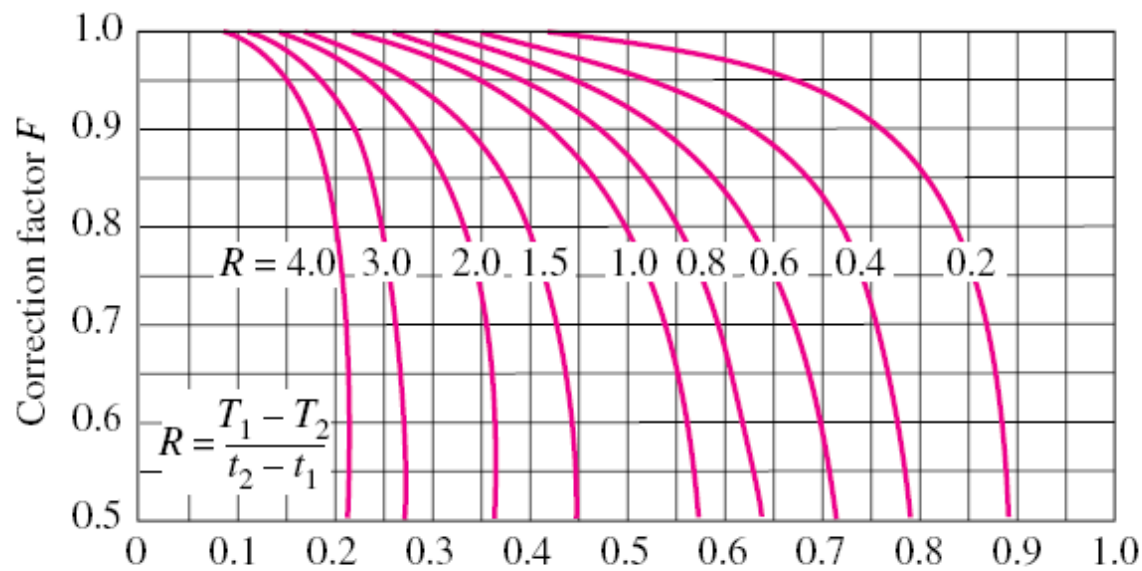
$$P = \frac{t_2 - t_1}{T_1 - t_1}$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}c_p)_{\text{tube side}}}{(\dot{m}c_p)_{\text{shell side}}}$$

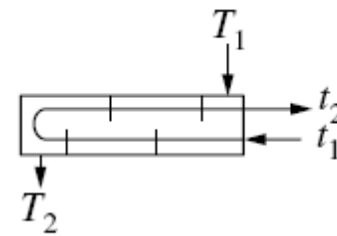
**1 and 2** *inlet* and *outlet*

***T* and *t*** *shell-* and *tube-side* temperatures

***F* = 1 for a condenser or boiler**

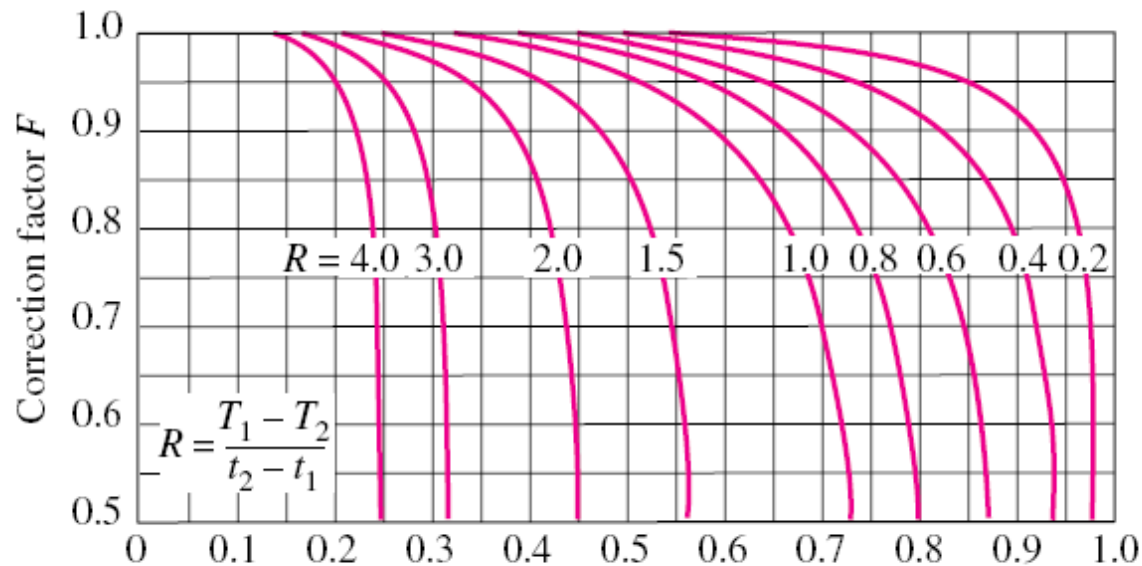


(a) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes

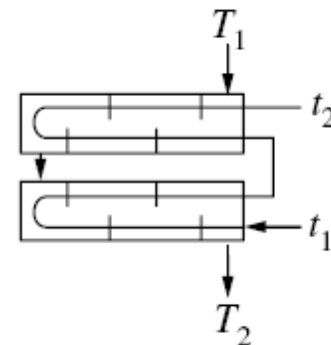


Correction factor  $F$  charts for common shell-and-tube heat exchangers.

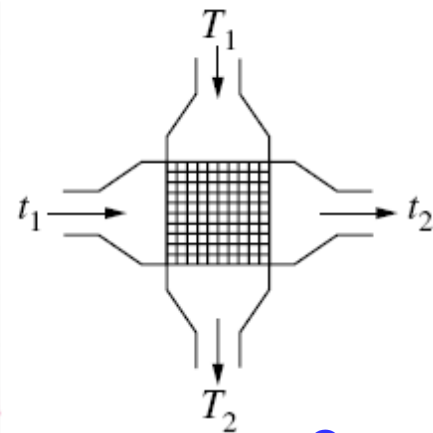
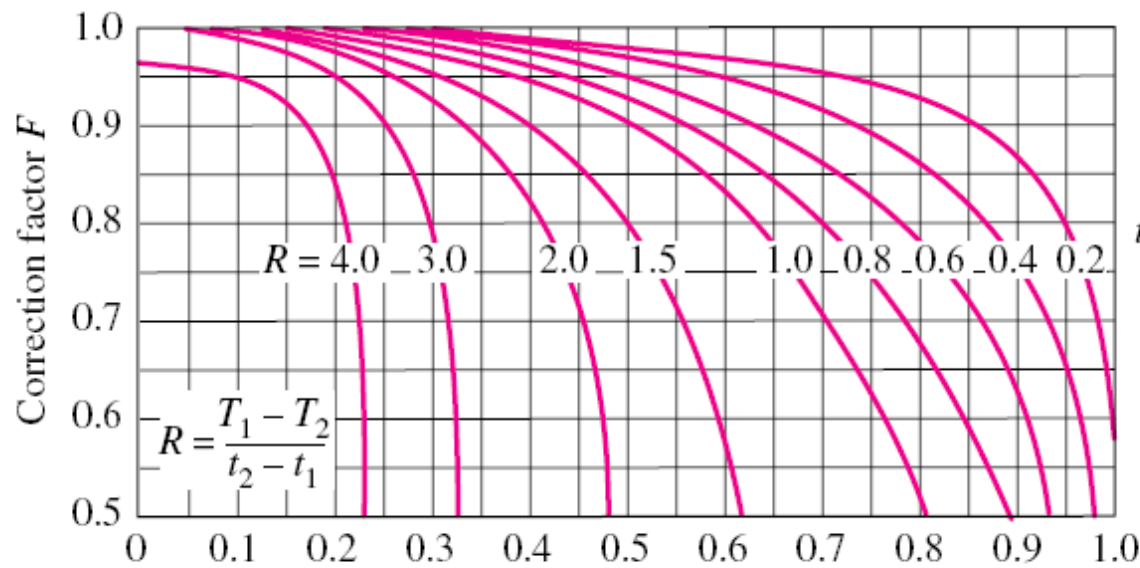
$$P = \frac{t_2 - t_1}{T_1 - t_1}$$



(b) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes

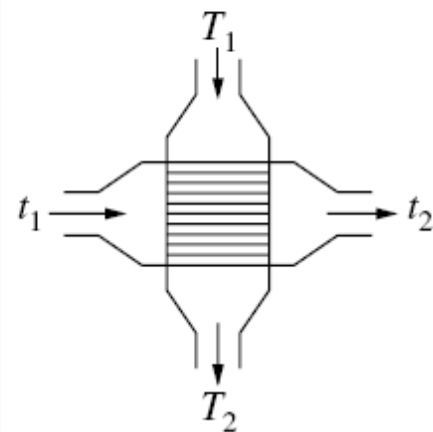
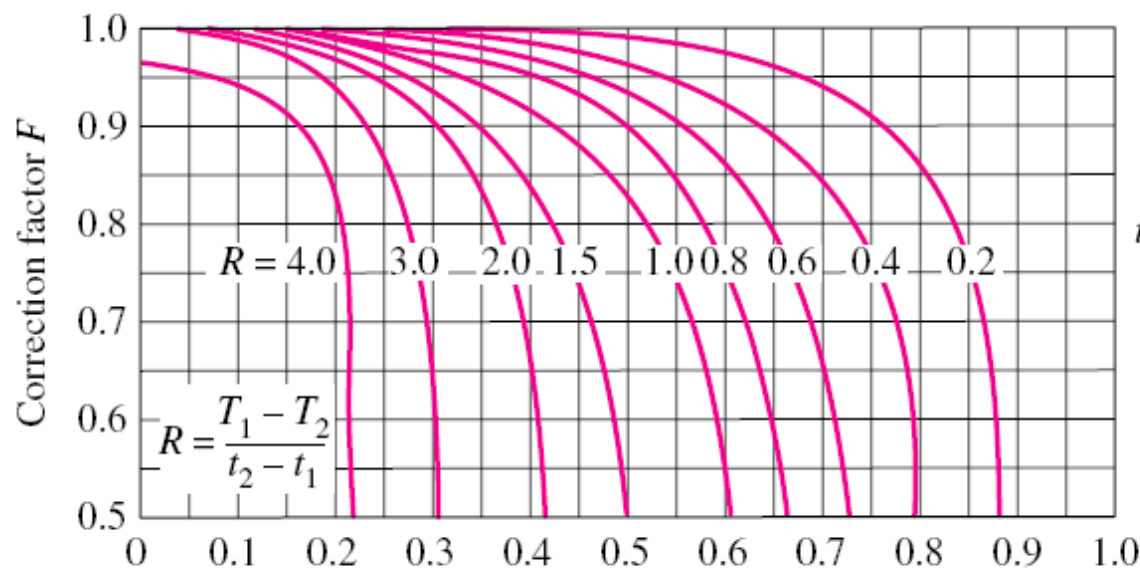


$$P = \frac{t_2 - t_1}{T_1 - t_1}$$



$$P = \frac{t_2 - t_1}{T_1 - t_1}$$

Correction factor  $F$  charts for common cross-flow heat exchangers.



$$P = \frac{t_2 - t_1}{T_1 - t_1}$$

(d) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

The LMTD method is very suitable for determining the *size* of a heat exchanger to realize prescribed outlet temperatures when the mass flow rates and the inlet and outlet temperatures of the hot and cold fluids are specified.

With the LMTD method, the task is to **select** a heat exchanger that will meet the prescribed heat transfer requirements. The procedure to be followed by the selection process is:

1. Select the type of heat exchanger suitable for the application.
2. Determine any unknown inlet or outlet temperature and the heat transfer rate using an energy balance.
3. Calculate the log mean temperature difference  $\Delta T_{lm}$  and the correction factor  $F$ , if necessary.
4. Obtain (select or calculate) the value of the overall heat transfer coefficient  $U$ .
5. Calculate the heat transfer surface area  $A_s$ .

The task is completed by selecting a heat exchanger that has a heat transfer surface area equal to or larger than  $A_s$ .

# THE EFFECTIVENESS-NTU METHOD

A second kind of problem encountered in heat exchanger analysis is the determination of the *heat transfer rate* and the *outlet temperatures* of the hot and cold fluids for prescribed fluid mass flow rates and inlet temperatures when the *type* and *size* of the heat exchanger are specified.

## Heat transfer effectiveness

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$$

$$\dot{Q} = C_c(T_{c, \text{out}} - T_{c, \text{in}}) = C_h(T_{h, \text{in}} - T_{h, \text{out}})$$

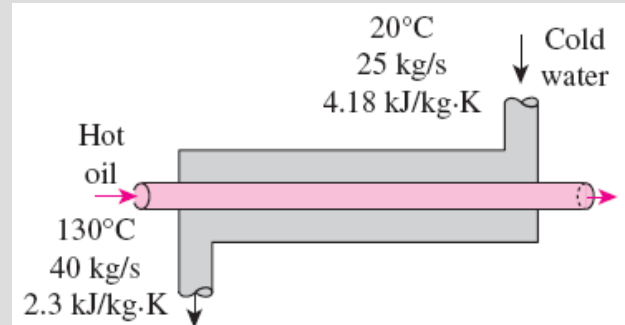
$$C_c = \dot{m}_c c_{pc} \text{ and } C_h = \dot{m}_h c_{ph}$$

$$\Delta T_{\max} = T_{h, \text{in}} - T_{c, \text{in}}$$

$$\dot{Q}_{\max} = C_{\min}(T_{h, \text{in}} - T_{c, \text{in}})$$

the maximum possible heat transfer rate

$C_{\min}$  is the smaller of  $C_h$  and  $C_c$



$$C_c = \dot{m}_c c_{pc} = 104.5 \text{ kW/K}$$

$$C_h = \dot{m}_h c_{ph} = 92 \text{ kW/K}$$

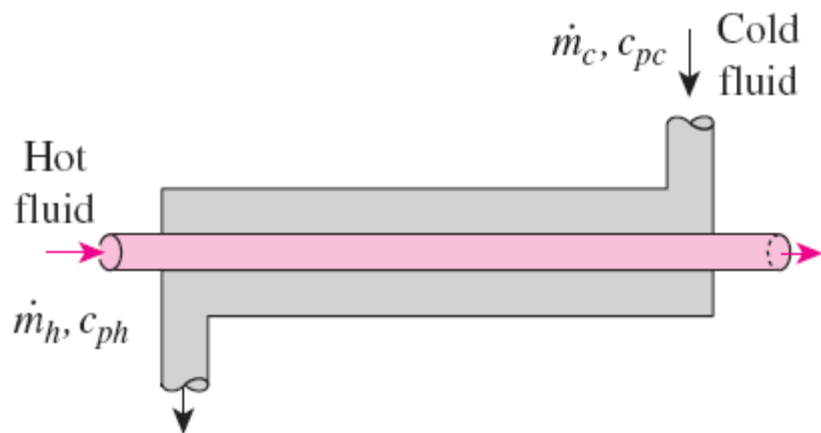
$$C_{\min} = 92 \text{ kW/K}$$

$$\Delta T_{\max} = T_{h, \text{in}} - T_{c, \text{in}} = 110^\circ\text{C}$$

$$\dot{Q}_{\max} = C_{\min} \Delta T_{\max} = 10,120 \text{ kW}$$

**FIGURE 11–23**

The determination of the maximum rate of heat transfer in a heat exchanger.



$$\begin{aligned}\dot{Q} &= \dot{m}_h c_{ph} \Delta T_h \\ &= \dot{m}_c c_{pc} \Delta T_c\end{aligned}$$

If  $\dot{m}_c c_{pc} = \dot{m}_h c_{ph}$

then  $\Delta T_h = \Delta T_c$

## FIGURE 11-25

The temperature rise of the cold fluid in a heat exchanger will be equal to the temperature drop of the hot fluid when the heat capacity rates of the hot and cold fluids are identical.

## Actual heat transfer rate

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon C_{\min} (T_{h,\text{in}} - T_{c,\text{in}})$$

if  $C_c = C_{\min}$ :

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c (T_{c,\text{out}} - T_{c,\text{in}})}{C_c (T_{h,\text{in}} - T_{c,\text{in}})} = \frac{T_{c,\text{out}} - T_{c,\text{in}}}{T_{h,\text{in}} - T_{c,\text{in}}}$$

if  $C_h = C_{\min}$ :

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_h (T_{h,\text{in}} - T_{h,\text{out}})}{C_h (T_{h,\text{in}} - T_{c,\text{in}})} = \frac{T_{h,\text{in}} - T_{h,\text{out}}}{T_{h,\text{in}} - T_{c,\text{in}}}$$

The effectiveness of a heat exchanger depends on the *geometry* of the heat exchanger as well as the *flow arrangement*.

Therefore, different types of heat exchangers have different effectiveness relations.

We illustrate the development of the effectiveness  $\epsilon$  relation for the double-pipe *parallel-flow* heat exchanger.

$$\ln \frac{T_{h, \text{out}} - T_{c, \text{out}}}{T_{h, \text{in}} - T_{c, \text{in}}} = -\frac{UA_s}{C_c} \left(1 + \frac{C_c}{C_h}\right)$$

$$T_{h, \text{out}} = T_{h, \text{in}} - \frac{C_c}{C_h} (T_{c, \text{out}} - T_{c, \text{in}})$$

$$\ln \frac{T_{h, \text{in}} - T_{c, \text{in}} + T_{c, \text{in}} - T_{c, \text{out}} - \frac{C_c}{C_h} (T_{c, \text{out}} - T_{c, \text{in}})}{T_{h, \text{in}} - T_{c, \text{in}}} = -\frac{UA_s}{C_c} \left(1 + \frac{C_c}{C_h}\right)$$

$$\ln \left[ 1 - \left(1 + \frac{C_c}{C_h}\right) \frac{T_{c, \text{out}} - T_{c, \text{in}}}{T_{h, \text{in}} - T_{c, \text{in}}} \right] = -\frac{UA_s}{C_c} \left(1 + \frac{C_c}{C_h}\right)$$

$$\epsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c(T_{c, \text{out}} - T_{c, \text{in}})}{C_{\min}(T_{h, \text{in}} - T_{c, \text{in}})} \longrightarrow \frac{T_{c, \text{out}} - T_{c, \text{in}}}{T_{h, \text{in}} - T_{c, \text{in}}} = \epsilon \frac{C_{\min}}{C_c}$$

$$\epsilon_{\text{parallel flow}} = \frac{1 - \exp \left[ -\frac{UA_s}{C_c} \left(1 + \frac{C_c}{C_h}\right) \right]}{\left(1 + \frac{C_c}{C_h}\right) \frac{C_{\min}}{C_c}}$$

$$\epsilon_{\text{parallel flow}} = \frac{1 - \exp \left[ -\frac{UA_s}{C_{\min}} \left(1 + \frac{C_{\min}}{C_{\max}}\right) \right]}{1 + \frac{C_{\min}}{C_{\max}}}$$

Effectiveness relations of the heat exchangers typically involve the *dimensionless* group  $UA_s/C_{\min}$ .

This quantity is called the **number of transfer units NTU**.

$$NTU = \frac{UA_s}{C_{\min}} = \frac{UA_s}{(\dot{m}c_p)_{\min}}$$

For specified values of  $U$  and  $C_{\min}$ , the value of NTU *is a measure of the surface area  $A_s$* . Thus, the larger the NTU, the larger the heat exchanger.

$$c = \frac{C_{\min}}{C_{\max}} \quad \begin{array}{l} \text{capacity} \\ \text{ratio} \end{array}$$

The effectiveness of a heat exchanger is a function of the number of transfer units NTU and the capacity ratio  $c$ .

$$\varepsilon = \text{function} (UA_s/C_{\min}, C_{\min}/C_{\max}) = \text{function} (NTU, c)$$

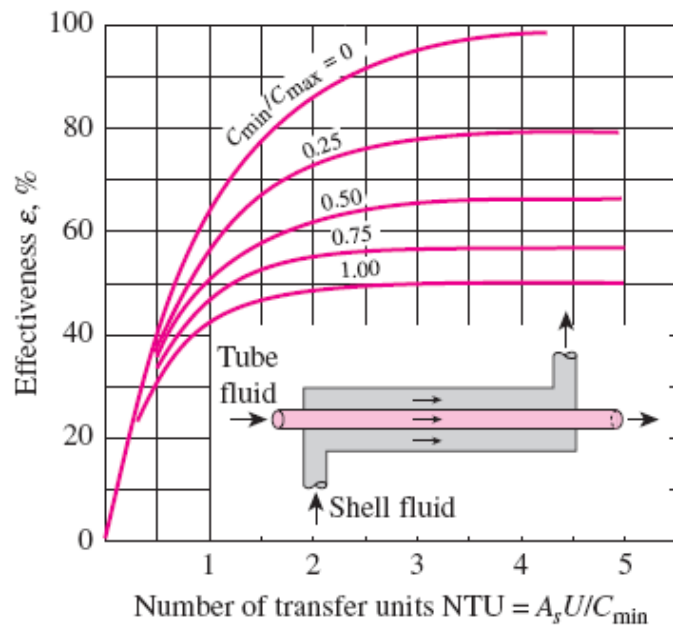


**TABLE 11-4**

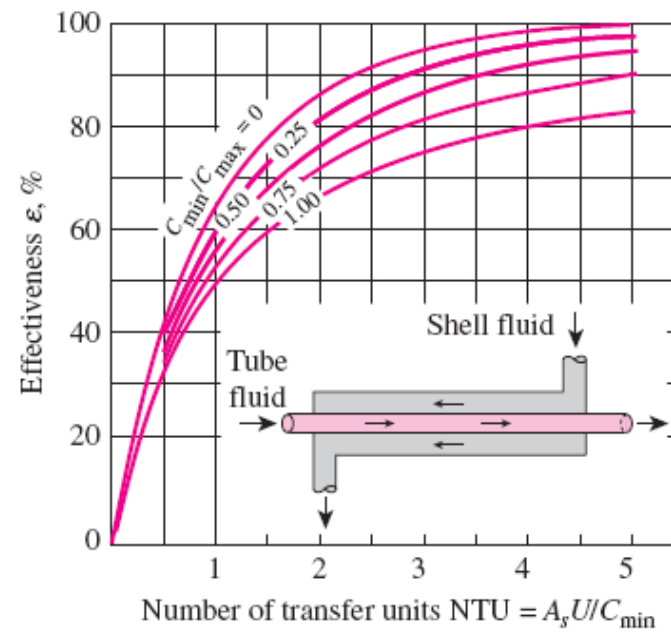
Effectiveness relations for heat exchangers:  $NTU = UA_s/C_{\min}$  and  $c = C_{\min}/C_{\max} = (\dot{m}c_p)_{\min}/(\dot{m}c_p)_{\max}$

Heat exchanger type	Effectiveness relation
1 <i>Double pipe:</i>	
Parallel-flow	$\varepsilon = \frac{1 - \exp[-NTU(1 + c)]}{1 + c}$
Counter-flow	$\varepsilon = \frac{1 - \exp[-NTU(1 - c)]}{1 - c \exp[-NTU(1 - c)]}$
2 <i>Shell-and-tube:</i>	
One-shell pass 2, 4,...tube passes	$\varepsilon = 2 \left\{ 1 + c + \sqrt{1 + c^2} \frac{1 + \exp[-NTU\sqrt{1 + c^2}]}{1 - \exp[-NTU\sqrt{1 + c^2}]} \right\}^{-1}$
3 <i>Cross-flow</i> (single-pass)	
Both fluids unmixed	$\varepsilon = 1 - \exp \left\{ \frac{NTU^{0.22}}{c} [\exp(-c NTU^{0.78}) - 1] \right\}$
$C_{\max}$ mixed, $C_{\min}$ unmixed	$\varepsilon = \frac{1}{c} (1 - \exp \{-c[1 - \exp(-NTU)]\})$
$C_{\min}$ mixed, $C_{\max}$ unmixed	$\varepsilon = 1 - \exp \left\{ -\frac{1}{c} [1 - \exp(-c NTU)] \right\}$
4 <i>All heat exchangers</i> with $c = 0$	$\varepsilon = 1 - \exp(-NTU)$

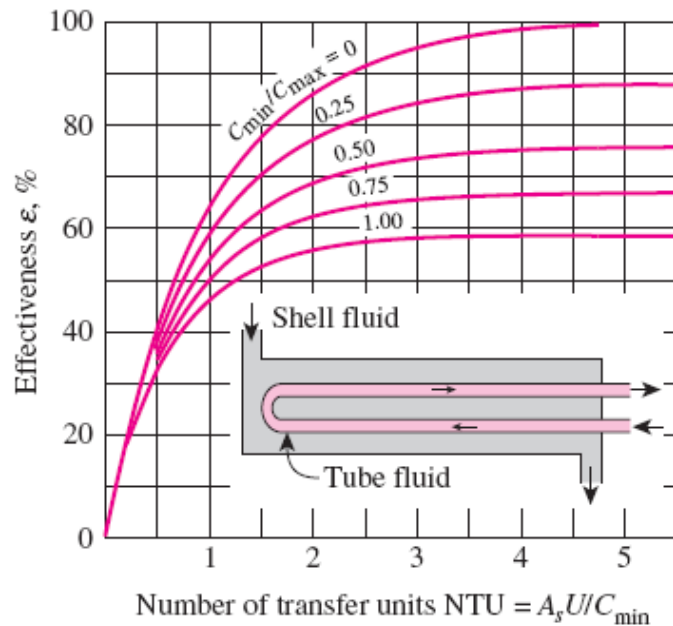
# Effectiveness for heat exchangers.



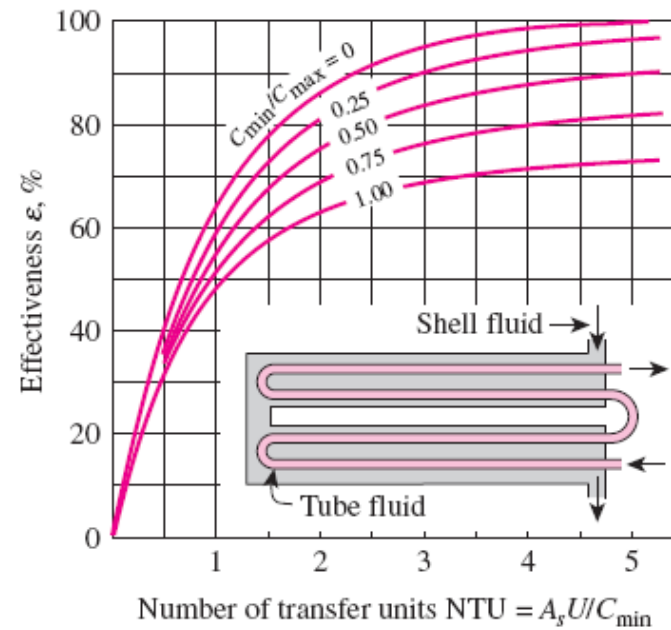
(a) Parallel-flow



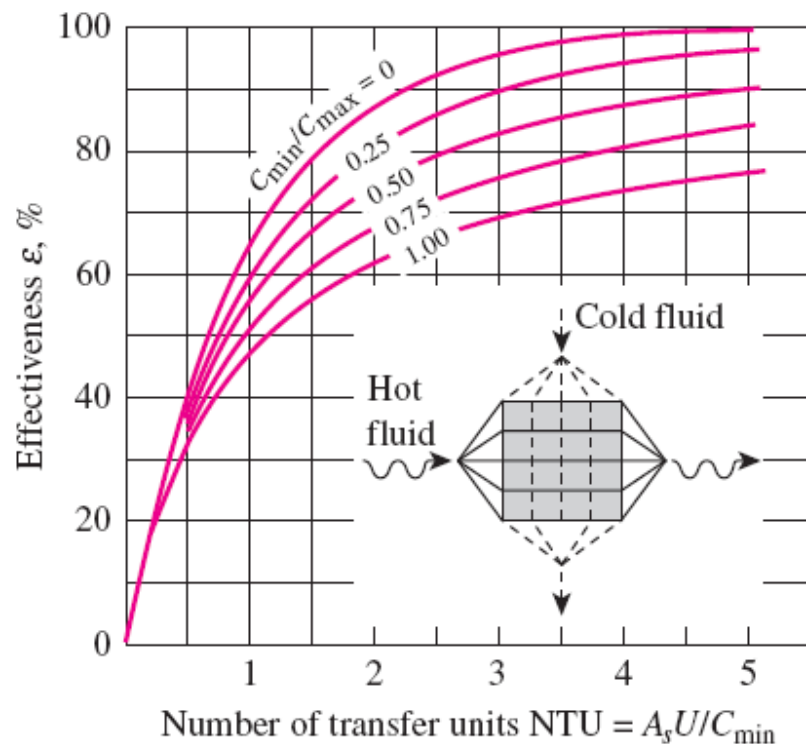
(b) Counter-flow



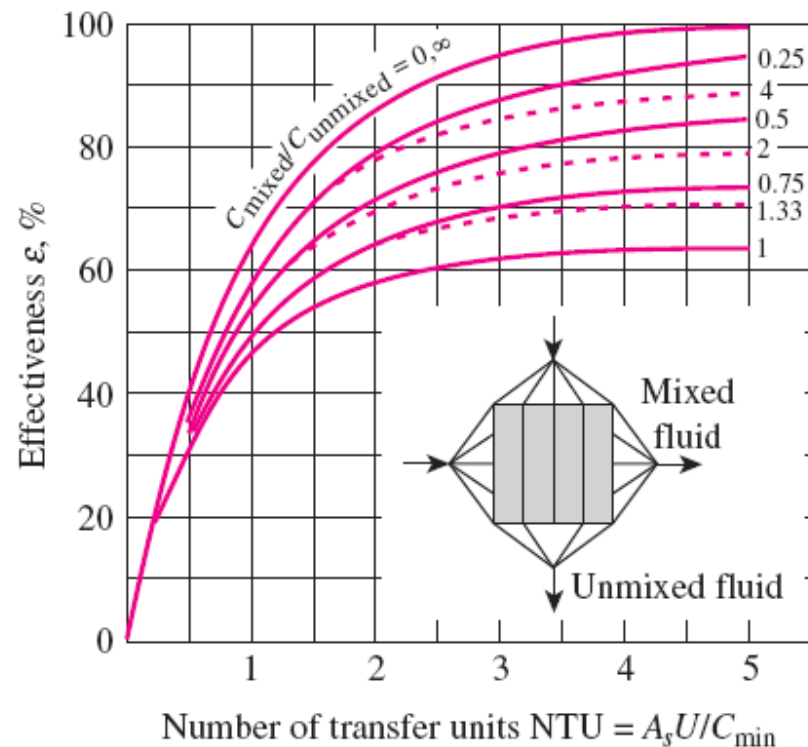
(c) One-shell pass and 2, 4, 6, ... tube passes



(d) Two-shell passes and 4, 8, 12, ... tube passes



(e) Cross-flow with both fluids unmixed



(f) Cross-flow with one fluid mixed and the other unmixed

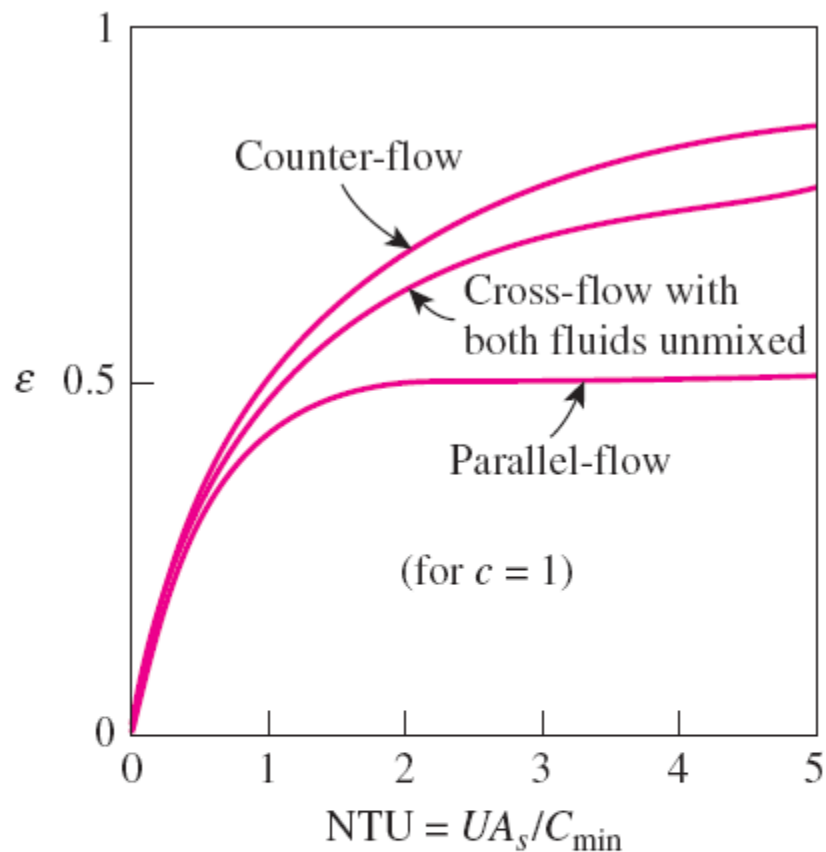
**FIGURE 11–26**  
Effectiveness for heat exchangers.

**TABLE 11–5**

NTU relations for heat exchangers:  $NTU = UA_s/C_{\min}$  and  $c = C_{\min}/C_{\max} = (\dot{m}c_p)_{\min}/(\dot{m}c_p)_{\max}$

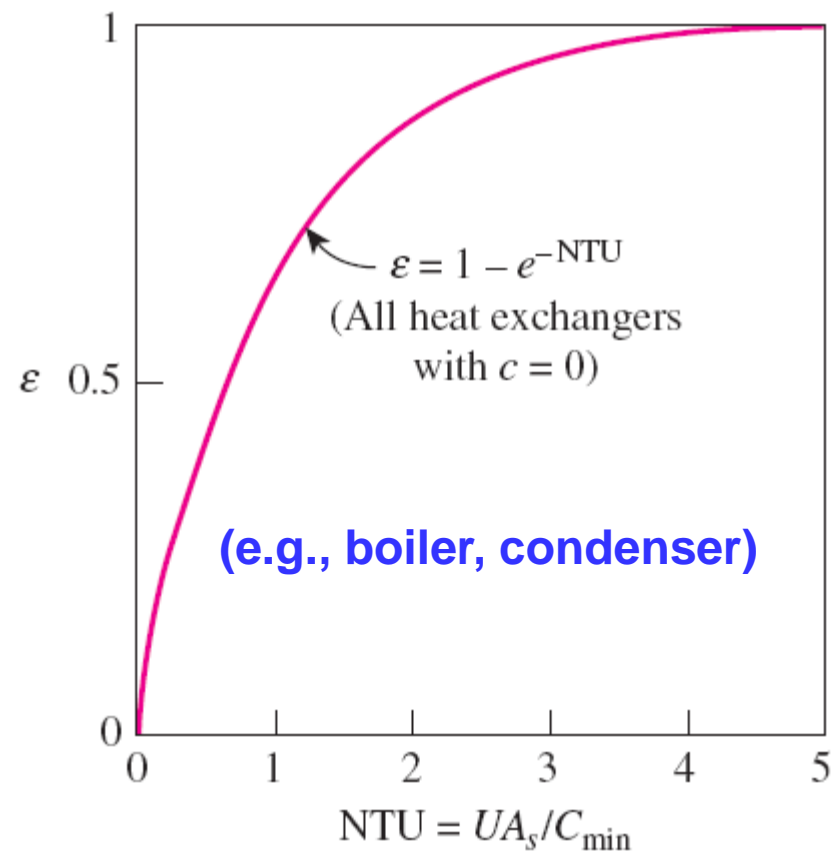
Heat exchanger type	NTU relation
1 <i>Double-pipe:</i>	
Parallel-flow	$NTU = -\frac{\ln [1 - \varepsilon(1 + c)]}{1 + c}$
Counter-flow	$NTU = \frac{1}{c - 1} \ln \left( \frac{\varepsilon - 1}{\varepsilon c - 1} \right)$
2 <i>Shell and tube:</i>	
One-shell pass 2, 4,...tube passes	$NTU = -\frac{1}{\sqrt{1 + c^2}} \ln \left( \frac{2/\varepsilon - 1 - c - \sqrt{1 + c^2}}{2/\varepsilon - 1 - c + \sqrt{1 + c^2}} \right)$
3 <i>Cross-flow (single-pass):</i>	
$C_{\max}$ mixed, $C_{\min}$ unmixed	$NTU = -\ln \left[ 1 + \frac{\ln (1 - \varepsilon c)}{c} \right]$
$C_{\min}$ mixed, $C_{\max}$ unmixed	$NTU = -\frac{\ln [c \ln (1 - \varepsilon) + 1]}{c}$
4 <i>All heat exchangers     with <math>c = 0</math></i>	$NTU = -\ln(1 - \varepsilon)$

When all the inlet and outlet temperatures are specified, the size of the heat exchanger can easily be determined using the LMTD method. Alternatively, it can be determined from the effectiveness–NTU method by first evaluating the effectiveness from its definition and then the NTU from the appropriate NTU relation.



**FIGURE 11-27**

For a specified NTU and capacity ratio  $c$ , the counter-flow heat exchanger has the highest effectiveness and the parallel-flow the lowest.



**FIGURE 11-28**

The effectiveness relation reduces to  $\varepsilon = \varepsilon_{\max} = 1 - \exp(-NTU)$  for all heat exchangers when the capacity ratio  $c = 0$ .

## Observations from the effectiveness relations and charts

- The value of the effectiveness ranges from 0 to 1. It increases rapidly with NTU for small values (up to about  $NTU = 1.5$ ) but rather slowly for larger values. Therefore, the use of a heat exchanger with a large NTU (usually larger than 3) and thus a large size cannot be justified economically, since a large increase in NTU in this case corresponds to a small increase in effectiveness.
- For a given NTU and capacity ratio  $c = C_{\min} / C_{\max}$ , the *counter-flow* heat exchanger has the *highest* effectiveness, followed closely by the cross-flow heat exchangers with both fluids unmixed. The lowest effectiveness values are encountered in parallel-flow heat exchangers.
- The effectiveness of a heat exchanger is independent of the capacity ratio  $c$  for NTU values of less than about 0.3.
- The value of the capacity ratio  $c$  ranges between 0 and 1. For a given NTU, the effectiveness becomes a *maximum* for  $c = 0$  (e.g., boiler, condenser) and a *minimum* for  $c = 1$  (when the heat capacity rates of the two fluids are equal).

# SELECTION OF HEAT EXCHANGERS

The uncertainty in the predicted value of  $U$  can exceed 30 percent. Thus, it is natural to tend to overdesign the heat exchangers.

Heat transfer enhancement in heat exchangers is usually accompanied by *increased pressure drop*, and thus *higher pumping power*.

Therefore, any gain from the enhancement in heat transfer should be weighed against the cost of the accompanying pressure drop.

Usually, the *more viscous fluid is more suitable for the shell side* (larger passage area and thus lower pressure drop) and *the fluid with the higher pressure for the tube side*.

**The proper selection of a heat exchanger depends on several factors:**

- **Heat Transfer Rate**
- **Cost**
- **Pumping Power**
- **Size and Weight**
- **Type**
- **Materials**

The *rate of heat transfer* in the prospective heat exchanger

$$\dot{Q} = \dot{m}c_p(T_{\text{in}} - T_{\text{out}})$$

The annual cost of electricity associated with the operation of the pumps and fans

$$\text{Operating cost} = (\text{Pumping power, kW}) \times (\text{Hours of operation, h}) \\ \times (\text{Unit cost of electricity, \$/kWh})$$

# Summary

- Types of Heat Exchangers
- The Overall Heat Transfer Coefficient
  - ✓ Fouling factor
- Analysis of Heat Exchangers
- The Log Mean Temperature Difference Method
  - ✓ Counter-Flow Heat Exchangers
  - ✓ Multipass and Cross-Flow Heat Exchangers:  
Use of a Correction Factor
- The Effectiveness–NTU Method
- Selection of Heat Exchangers