MEG 401

DESIGN II

BY

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•Course Content (Expected)

Design of various joints (riveted, brazed, welded, key, pins, splines). Design and production matching (Limits and fits). Design of gear systems (spur, helical, bevel, worm gear) including strength of cast, forged and welded housing and structures. Joints, fasteners, shaft and bearing mountings. Design project (to be carried out in groups of 3 to 4 students per group).

Learning outcomes:

On completion of this course, students will be able to:

- Demonstrate knowledge on basic machine elements used in machine design
- Design machine elements to withstand the loads and deformations.
- To apply mechanical engineering design theory to identify and quantify machine elements in the design of commonly used mechanical systems.

Apply the concepts of stress analysis, theories of failure and material science to analyze, design and/or select commonly used machine components.

• Course Content (Aspired)

1], 4] Design of keys, splines and couplings

[2, 4] Design of gear systems (spur, helical, bevel, worm gear)

[3, 4] Joints, fasteners, shaft and bearing mountings.

[4] Design group projects

• Chapter Objectives

1. Design the section of the most commonly used rectangular key on the basis of torque and allowable shear and bearing stresses in key.
2. Design the thickness and radius of semicircular portion of Woodruff key on the basis of torque and allowable shear and bearing stresses in key.
3. Design the section of a spline in a splined shaft on the basis of torque and allowable normal stress on spline.
4. Design the section of Kennedy's key for heavy torque transmission on the basis of allowable shear and bearing stresses in key.
5. Design a rigid coupling for two shafts on the basis of torque, allowable shear stress in flange, in bolts connecting two flanges and in hub of the coupling..

• Chapter 1: Design of keys, splines and couplings

1.1 Design of keys

1.2 Design of splines

1.3 Design of couplings

• 1.1 Design of various keys: Introduction

IC engines, turbines, rotating elements like wheels, gears, pulleys and sprocket are connected with the help of a key between the shaft of a prime mover and the hub of a rotating element.

There are several types of keys, such as rectangular key, woodruff key, round key, saddle key, etc., depending upon the magnitude of the torque to be transmitted.

The keyways create *a region of stress concentration* both in shaft and hub.

• 1.1 Design of various keys: Introduction

A "key" is a demountable machinery part, which, when assembled into key seats, provides positive means for transmitting torque between the shaft and the hub. Keys are classified on the basis of their size and shape, as shown in Fig. 1-1 and 1-2.

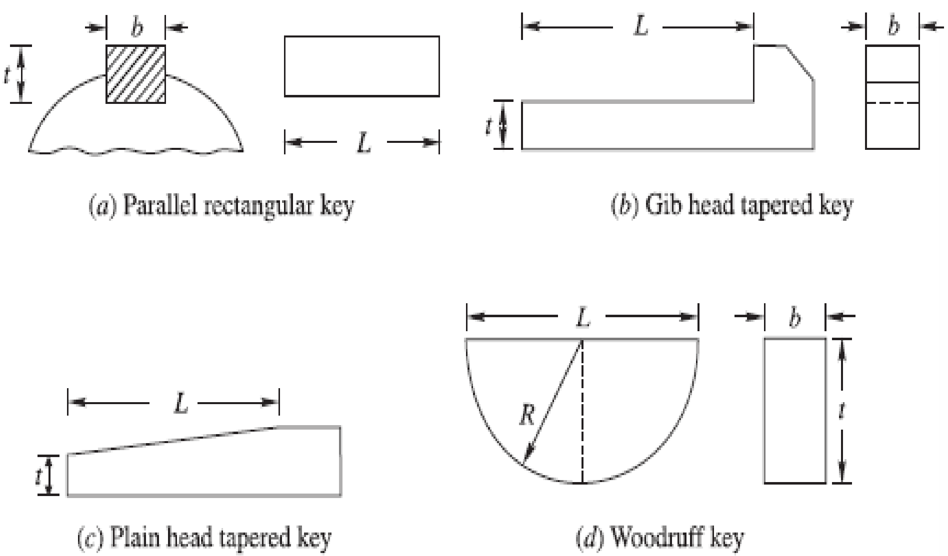


Fig. 1-1.

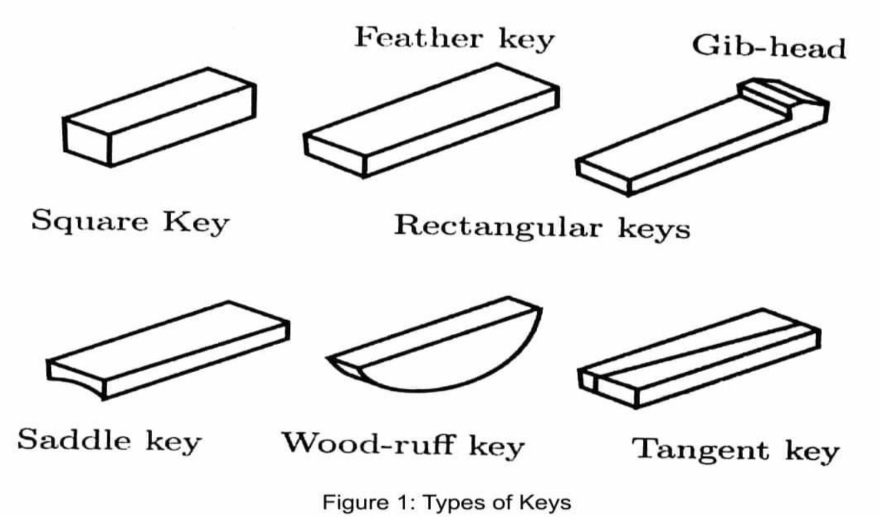


Fig. 1-2.

• 1.1 Design of keys

Shear stress developed in key leading to shear failure:

$$\tau = \frac{\text{Tangential force on key due to torque transmitted}}{\text{Area of key under shear}}$$

Average bearing stress developed in key leading to bearing failure:

$$\sigma = \frac{\text{Tangential force on key due to torque transmitted}}{\text{Area of key under bearing}}$$

Based on experimental results, H. F. Moore gave the relation for the weakening effect of a keyway in the shaft in the following manner:

$$K_w = \text{weakening factor}$$
$$= 1 - 0.2\left(\frac{b}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

• 1.1 Design of keys

$$h = \frac{t}{2} = \text{half the key thickness}$$

where, b = breadth of the key, and h = depth of key in the shaft.

In general practice, K_w = 0.75

If the key way is too long, and key is of the sliding type, then the angle of twist is increased by factor K_θ in the shaft.

$$K_\theta = 1 + 0.4\left(\frac{b}{d}\right) + 0.7\left(\frac{h}{d}\right)$$

• 1.1 Design of keys

Example 1-1 The dimensions of a woodruff key for a 40 mm shaft, are shown in Fig. 1-3.One third of the depth of the key is in the hub portion. The shaft transmits 6 kW at 350 rpm. The key is made of steel and $\sigma_{yt} = \sigma_{yc} = 380$ N/mm^2. Calculate the factor of safety in the design of the key.

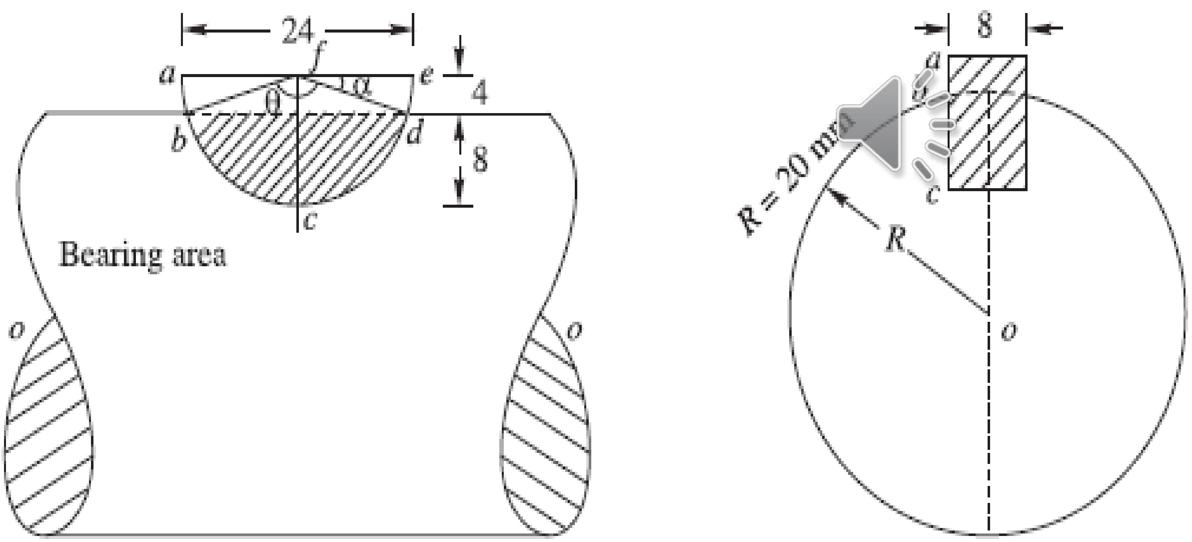


Fig. 1-3 a

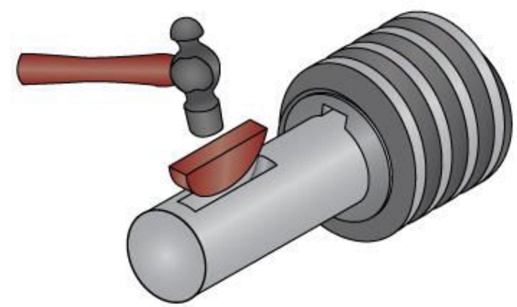


Fig. 1-3 b

• 1.1 Design of keys

Total area of key $A = \dfrac{\pi R^2}{2} = \dfrac{\pi \times 12^2}{2} = 226.2 \text{ mm}^2$

Angle $\theta = \pi - 2\tan^{-1}\dfrac{4}{12} = 180^\circ - 2 \times 19.5$

$= 141^\circ$, angle $\alpha = (180 - 141)/2$

Length $bd = 2 \times 12 \times \cos\alpha$

$= 2 \times 12 \times \cos 19.5^\circ = 22.62 \text{ mm}$

Area of $\Delta f\ bd = \dfrac{1}{2} \times bd \times 4$

$= \dfrac{1}{2} \times 22.62 \times 4 = 45.24 \text{ mm}^2$

Tan α = 4/12 ; α = $\tan^{-1}$ 4/12

Θ = 180 – 2 x α

cos α = od/12 ; od =12 x cos α

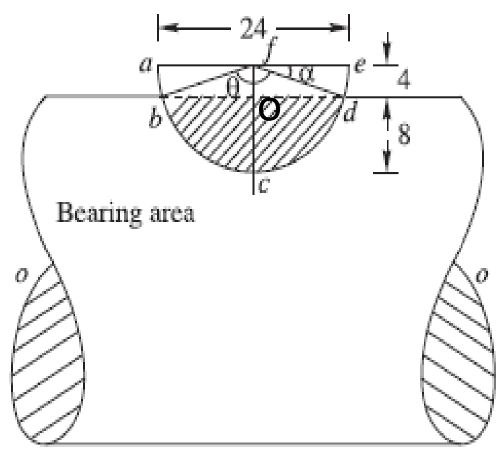
bd = 2 x od

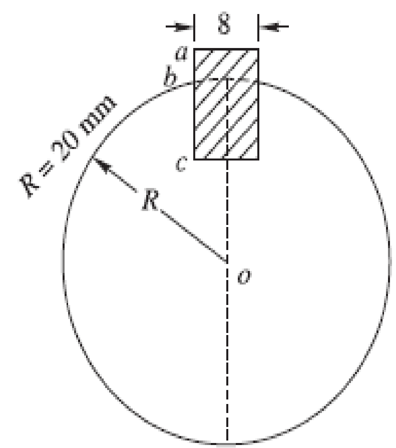
Area of key under bearing pressure = area *bcd*

$=$ area of section $f\ bcd - \Delta\ f\ bd$

$= 226.2 \times \dfrac{141}{180} - 45.24 = 177.9 - 45.24$

$= 131.95 \text{ mm}^2$





• 1.1 Design of keys

Area of key under bearing above line *bd* =
226.2 − 131.95 = 94.25 mm²

Area of woodruff key under shear = *bd* × 8 =
22.62 × 8 = 180.96 mm²

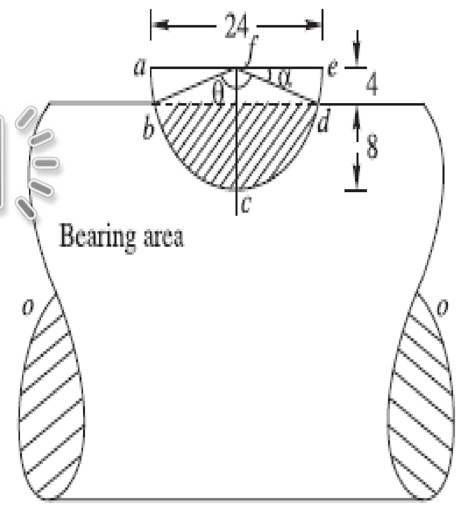
Power = 6 kW

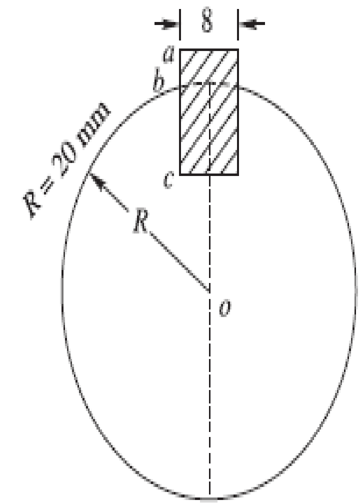
$$\text{Angular speed } \omega = \frac{2\pi \times 350}{60}\ 36.6 \text{ rad/sec}$$

$$\text{Torque} = \frac{6000 \text{ Nm}}{36.65} = 163.71 \text{ Nm}$$

Shaft radius $r = 20$ mm

$$\text{Tangential force } P_t = \frac{163710}{20} = 8185.5 \text{ N}$$

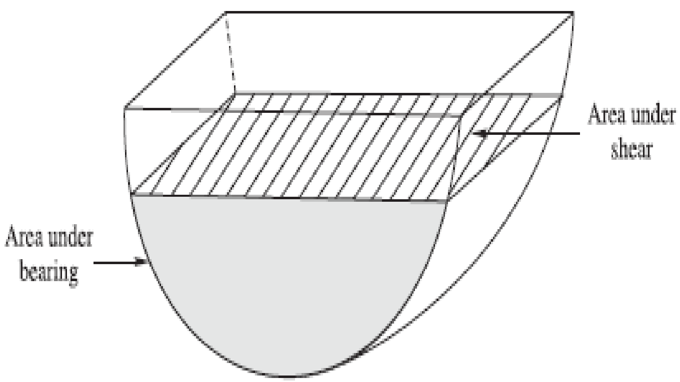




• 1.1 Design of keys

Bearing stress developed in key $$\sigma_b = \frac{8185.5}{94.25} = 86.85$$

Shearing stress developed in key $$= \frac{8185.5}{180.96} = 45.23 \text{ N/mm}^2$$



Material $\sigma_{yt} = \sigma_{yc}$ = 380 N/mm^2

σ_a = allowable stress in bearing

$$= \frac{380}{\text{FOS}} = 86.85$$

τ_a = allowable stress in shearing

$$= \frac{0.577 \times 380}{\text{FOS}} = \frac{219.26}{\text{FOS}} = 45.23$$

$$\text{FOS in bearing} = \frac{380}{86.85} = 4.375$$

$$\text{FOS in shear} = \frac{219.26}{45.23} = 4.85$$

• 1.1 Design of keys

SPECIAL-PURPOSE KEYS

The following keys are used for undertaking very heavy or very light torques:

1. Kennedy's keys
2. Saddle key
3. Tangent key

For the transmission of heavy torques, two square keys, rather than one, at right angles to each other, as shown in Fig. 1-4 are used. The arrangement is known as Kennedy's keys.

The torque is equally divided between two keys.

Tangential force on each key

$$P_t = \frac{T}{2r} = \frac{T}{d}$$

where, *d* is diameter of the shaft.
The key section is square, $a \times a$. Say, length of key is *l*.
Area of the key under shear $= \sqrt{2}al$

• 1.1 Design of keys

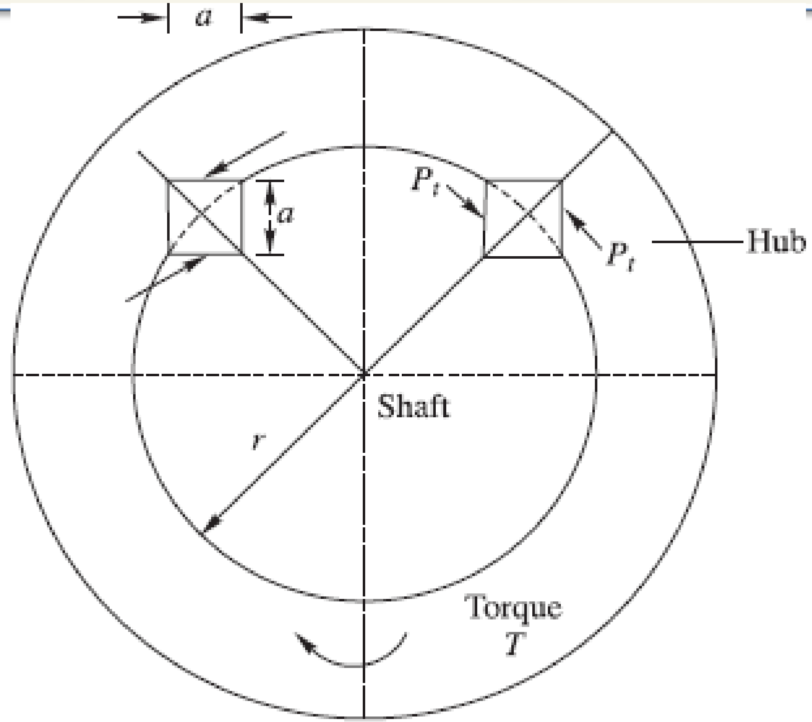


Figure 1-4 Transmission of heavy torque by two square keys

$$\text{Area of the key under bearing pressure} = \frac{\sqrt{2}a}{2} \times l = \frac{al}{\sqrt{2}}$$

$$\text{Bearing stress developed in key } \sigma_b = \frac{P_t \times \sqrt{2}}{al} = \frac{T}{d} \times \frac{\sqrt{2}}{al}$$

$$\text{Shearing stress developed in key } \tau = \frac{P_t}{\sqrt{2}al} = \frac{T}{d} \times \frac{1}{\sqrt{2}al}$$

• 1.1 Design of keys

If σ_a and τ_a are allowable bearing stress and allowable shearing stress, respectively, for the material, then:

$$\text{Factor of safety against shear failure} = \frac{\tau_a}{\tau}$$

$$\text{Factor of safety against crushing failure} = \frac{\sigma_a}{\sigma_b}$$

Example 1-2 Kennedy keys of 12 mm × 12 mm are used to connect a shaft of 50 mm diameter, transmitting 40 kW at 360 rpm. The keys are made of 40 C 8 steel with $\sigma_{yt} = \sigma_{yc} = 380$ N/mm^2. Taking a factor of safety of 3, determine the required length of the keys.

Solution:

$$N = 360 \text{ rpm}$$

$$\omega = \frac{2\pi \times 360}{60} = 37.7 \text{ rad/sec}$$

$$\text{Torque transmitted } T = \frac{\text{Power}}{\omega} = \frac{40 \times 1000}{37.7}$$

$$= 1061 \text{ Nm}$$

$$= 1061 \times 10^3 \text{ Nmm}$$

• 1.1 Design of keys

Shaft radius r = 25 25mm

Tangential force per key $P_r = \frac{T}{2r} = \frac{1061 \times 10^3}{2 \times 25} = 21,220\text{ N}$

Section of the key = 12 × 12 mm

Length of the key = l mm

$$\sigma_{yt} = \sigma_{yc} = 380\text{ N/mm}^2$$

$$\text{FOS} = 3$$

$$\sigma_{\text{allowable}} = \frac{380}{3}\text{ MPa} = 126.66\text{ N/mm}^2$$

$$\tau_{\text{allowable}} = 0.577 \times \frac{380}{3} = 73.1\text{ N/mm}^2$$

$$\text{Section of key under shear} = \sqrt{2} \times 12 \times l\text{ mm}^2$$

$$= \sqrt{2} \times 12 \times l \times 73.1 = P_t = 21220\text{ N}$$

$$l = \frac{21220}{\sqrt{2} \times 12 \times 73.1} = 17\text{ mm}$$

$$\text{Taking allowable bearing stress } \sigma_a = 126.66\text{ N/mm}^2$$

$$P_t = \frac{\sigma_a \times 12 \times l}{\sqrt{2}} = 126.66 \times 12 \times \frac{l}{\sqrt{2}}$$

$$\text{Length of key } l = \frac{21220 \times \sqrt{2}}{126.66 \times 12} = 19.74\text{ mm}$$

• 1.1 Design of Splines

For more torque transmission, splines are used in place of keys. These are built-in tooth-like keys formed on the outside of the shaft, and inside the hub. Earlier, splines were of trapezoidal section, with the depth of the section at the shaft less than the depth of the section at the hub. Therefore, a shaft section was weaker than a hub section. Nowadays involute splines are used (see Fig. 1-5a). In this form, *the depth of the spline section at the shaft is more than the depth of the spline section at the hub*. The standard, involute spline-tooth forms have a 20° pressure angle. Standard splines can have from 6 to 50 teeth.

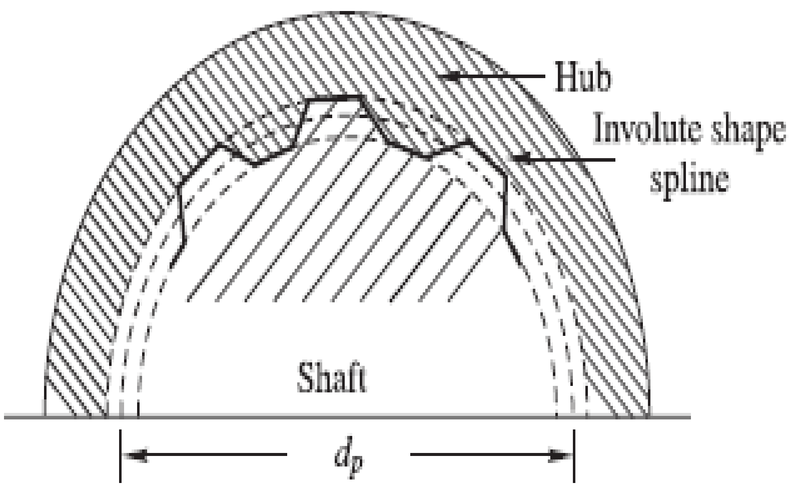


Fig. 1-5a

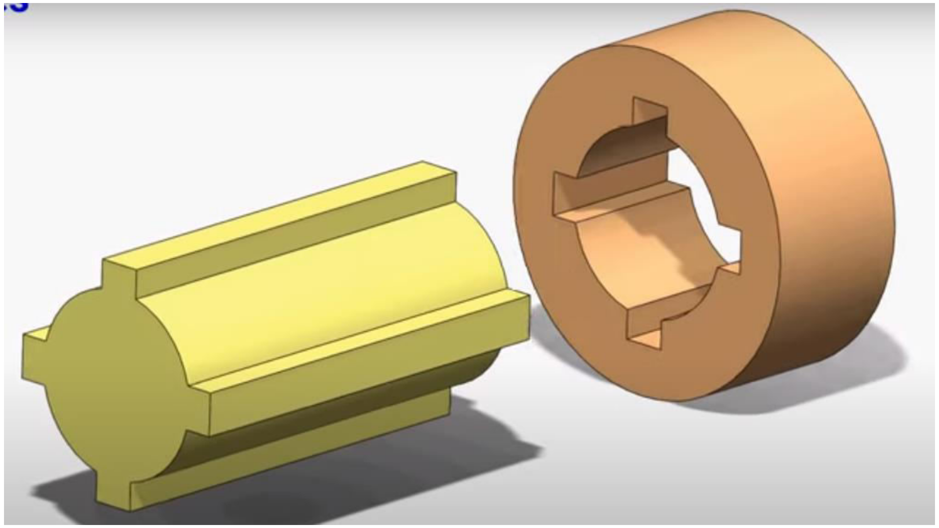


Fig. 1-5b

• 1.1 Design of Splines

Splines provide maximum strength at the root of tooth, accuracy of tooth form, and superior machined surface finish. Splines can accommodate large axial movements between the shaft and the hub, and at the same time transmit torque. Engine torque is usually passed into the transmission through a spline which connects the engine clutch to the transmission input shaft, and allows the axial motion, which is necessary for disengaging the clutch from the flywheel. Splines are loaded under pure torsion. If the splines are made perfectly, with no variation in tooth thickness or spacing, all the teeth would share the load equally.

Length of the spline = l

Pitch circle diameter of splines = d_p

Area under shear $= \frac{\pi d_p l}{2}$

The Society of Automobile Engineers (SAE) assumes that only 25% of the teeth are actually sharing the load at any one time. Shear stress $\tau = \frac{16T}{\pi d_p^2 l}$

where T is the orque transmitted.

• 1.1 Design of Splines

Example 4-3 A standard splined connection, 12 × 45 × 50 mm, is used for a gear and shaft assembly, rotating at 400 rpm. The length of the gear hub is 60 mm, and the normal pressure on the splines is limited to 6.5 MPa. Calculate the power which can be transmitted from the gear to the shaft. What is the shear stress developed in the splined shaft and in the splined hub?

Solution

Number of splines n = 12

Angle $\frac{\phi}{2} = \frac{360}{12 \times 2} = 15^{\circ}$

Minor radius $R_1 = \frac{45}{2} = 22.5$ mm;

Major radius $R_2 \frac{50}{2} = 25$ mm (see Fig. 1-6).

Length of the spline l = 60 mm

Mean radius $R_m = \frac{22.5 + 25.0}{2} = 23.75\,\text{mm}$

Normal pressure on spline p = 6.5 N/mm^2

Normal force per spline $P_n = (R_2 - R_1) \times l \times p = 2.5 \times 60 \times 6.5 = 975$ N

Torque per spline $T' = P_n \times R_m = 975 \times 23.75 = 23156.25 = 23.156$ N

Total torque for spline shaft $T = T' \times n = 23.15625 \times 12 = 277.875$ Nm

Angular speed $\omega = \frac{2\pi \times 400}{60} =$ 41.89 rad/sec

Power Transmission capacity $P = 41.89 \times 277.875 = 11{,}640$ Nm/s = 11.64 kW

• 1.1 Design of Splines

Shearing area per spline on shaft $= \dfrac{2\pi R_1}{2 \times n} \times l = \dfrac{\pi \times 22.5}{12} \times 60$

$= 353.43 \text{ mm}^2$

$$\text{Shear stress in splined shaft} = \frac{P_n}{353.43} = \frac{975}{353.43} = 2.76 \text{ N/mm}^2$$

$$\text{Shearing area per spline on hub} = \frac{2\pi R_2}{2 \times n} \times l = \frac{2 \times \pi \times 25 \times 60}{2 \times 12} = 392.7 \text{ mm}^2$$

$$\text{Shear stress in splined hub} = \frac{975}{392.7} = 2.48 \text{ N/mm}^2$$

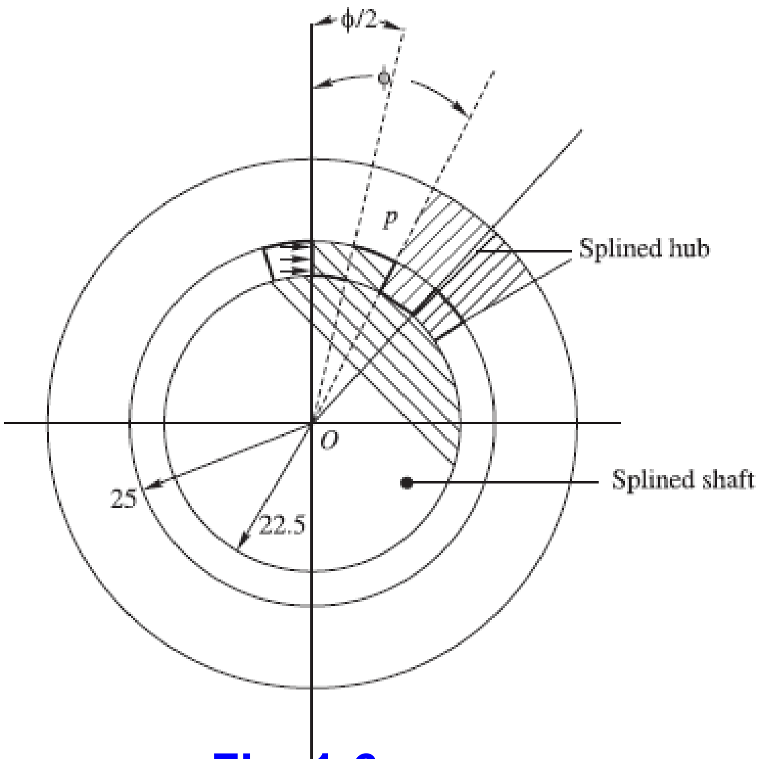


Fig. 1-6

• 1.3 Design of couplings

A coupling is referred to as a type of device which is usually found having its application in order to connect the two shafts together for the purpose of power transmission.

A wide variety of couplings are commercially available, ranging from a simple rigid coupling to elaborate flexible couplings using gears, elastomers and fluids for transmission of torque from one shaft to another shaft or from a shaft to a device. Couplings can be broadly classified into rigid and flexible couplings. Flexible couplings can absorb same misalignment between the two shafts, but rigid couplings do not permit misalignment between the two shafts. A rigid coupling locks the two shafts and allows no relative moment between them. There are three types of rigid couplings:

(1) set-screw coupling
(2) rigid coupling
(3) clamp coupling

• 1.3 Design of couplings

Fig. 1-7

• 1.3 Design of couplings: Selection Guide

Coupling Type

Rigid Flange Coupling

Rigid Sleeve Coupling

Rigid Compression Sleeve Coupling

Elastomeric Couplings

Characteristics And Uses

- Angular And Parallel Misalignment Between Coupled Shaft Is Negligible
- Frequent Uncoupling Is Required
- Keyed Or Splined To Each Shaft
- No Vibration Isolation Provided
- Easier To Remove And Install
- Requires High-Strength Bolts To Join
- A Small In-Line Misalignment Can Be Tolerated
- The Friction Between Shaft And Coupling Is The Clamping Force
- An Angular Misalignment Of 2°, Or A Parallel Misalignment 7 Mm, May Be Tolerated
- Easily Replaceable
- Electrical And Partial Vibration Isolation Provided
- Reduced Torsional Capacity In Comparison To Rigid Coupling

• 1.3 Design of couplings: Selection Guide

Flexible Metal Couplings	• Angular, In-Line And Parallel Misalignment May Be Accommodated At Higher Torques
Gear Couplings	• Used For Large Torque • In-Line And Angular Misalignment Is Present
Spring Couplings	• Suitable For Low Torque And Large Angular Misalignment, Up To 60°
Schmidt Couplings	• Designed For Parallel Misalignment With Adequate Space Between Shafts To Accommodate Coupling
	• Low Torque Application Used With Fractional HP Motors • Designed To Provide Vibration Isolation
Fluid Couplings	• Can Be Used For A Wide Range Of Torque; From 1 To 373 KW, At 1,200 Rpm.

• 1.3 Design of Sleeve couplings

A sleeve or muff is inserted over the two shafts to be coupled, and a gib-head key is inserted between the sleeve and the shafts to provide a connection for torque transmission.

Outer diameter of sleeve $d_1 = 2d + 13$ mm

where, *d* is the diameter of th shaft.

Internal diameter of sleeve $= d$

Length of the sleeve, $l = 3.5d$ (to provide axial stability)

A gib-head key is fitted in the keyways cut in the seeve and in the shaft. A gib head is provided for easy assembly and removal of key.

Length of the key $L > l$ length of sleeve

Section of the Key

Half of the key is inserted into the keyways of the shafts

• 1.3 Design of Sleeve couplings

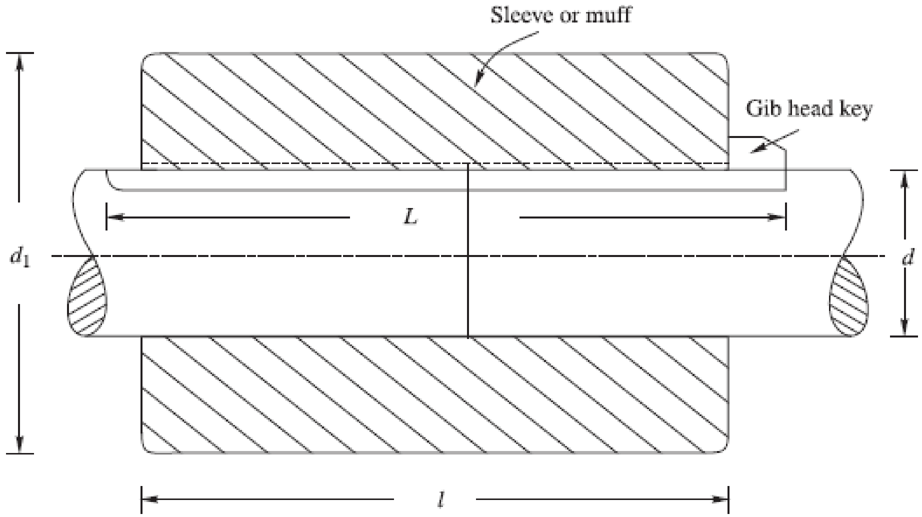


Figure 1.8 Sleeve or muff coupling

$$\text{Tangential force on key } F_t = \frac{\text{Torque}}{\text{Shaft radius}}$$

$$\text{Breadth of the key } b = \frac{F_t}{\tau_{ak} \times l'}$$

where, τ_{ak} = allowable shear stress in key with thickness (t).

$$\text{Thickness of key } t = \frac{2F_t}{\sigma_{ack} \times l'}$$

where, σ_{ack} = allowable crushing stress in key

.

$l' = \frac{l}{2}$ (Length of the key in each shaft)

• 1.3 Design of couplings

Example 1-4 Design a sleeve coupling for the transmission of 12 kW at 300 rpm by two connected steel shafts. Take service factor $K_s = 1.25$. The sleeve is made of CI. The key and the shaft are made of the same material.

Allowable stress:

Shear stresses in key and shaft = 50 MPa

Crushing stress in key = 100 MPa

Shear stress in CI sleeve = 10 MPa

Solution:

Power = 12 Kw

Service factor $K_s = 1.25$

Design power $P_d = 1.25 \times 12 = 15\text{kN}$

Speed $N = 300$ rpm

• 1.3 Design of couplings

$$\text{Angular speed } \omega = \frac{2\pi \times N}{60}$$

$$= \frac{2\pi \times 300}{60} = 31.416 \text{ rad/sec}$$

$$\text{Torque } T = \frac{P_d}{\omega} = \frac{15 \times 1000}{31.416} = 481.386 \text{ Nm} = 481386 \text{ Nmm}$$

$$= \frac{\pi}{16} d^3 \times \tau_{as}$$

where, τ_{as} = allowable shear stress in shaft.

$$\text{Shaft diameter } d^3 = \frac{16T}{\pi\tau_{as}} = \frac{16 \times 481386}{\pi \times 50} = 49.03 \times 10^3$$

$$d = 36.55 \text{ mm} \simeq 40 \text{ mm}$$

Sleeve

Allowable shear stress in sleeve = 10 MPa

Outer diameter of sleeve = d_1 mm = 2d + 13 = 93 mm

Shear stress developed in sleeve of CI:

• 1.3 Design of couplings

$$T = \frac{\pi}{16}\left(\frac{d_1^4 - d^4}{d_1}\right) \times \tau$$

$$481386 = \frac{\pi}{16}\left(\frac{93^4 - 40^4}{93}\right)\tau = 152530\tau$$

$$\tau = \frac{481386}{152530} = 3.16 \text{ MPa} << 10 \text{ MPa (allowed)}$$

Length of the sleeve l = 3.5 × 40 = 140 mm

Key

$$\text{Tangential force } F_t = \frac{T}{d/2} = \frac{481386}{20} = 24069 \text{ N}$$

$$l' = \text{length of key in each shaft}$$

$$\tau_{ak} = 50 \text{ MPa}$$

$$\text{Breadth } b = \frac{F_t}{l' \times \sigma_{ak}} = \frac{24069}{70 \times 50} = 6.88 \simeq 7 \text{ mm}$$

$$\text{Thickness of key } t = \frac{2F_t}{\sigma_{ack} \times l'} = \frac{2 \times 24069}{100 \times 70} = 6.88 \simeq 7 \text{ mm}$$

- **1.3 Design of couplings**