Modes of Heat transfer

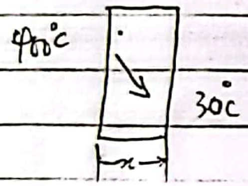
There are 3 modes of heat transfer.

* Conduction
* Convention
* Radiation.

8.t5

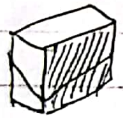
CONDUCTION

8/+



$Q \propto \Delta T$

$Q \propto \frac{1}{x}$



$Q \propto A$

$Q \propto K$

$$\boxed{Q = -KA\frac{\Delta T}{\Delta x}} \rightarrow \text{Thermal Conductivity}$$

Fourier Heat Conduction equation

$\frac{\Delta T}{\Delta x}$ = Temp gradient.

Fourier law state that the negative gradient of temp and the time rate of heat transfer is proportional to the area at right angle of that gradient through which the heat flows.

$$q = -K\nabla T$$

q = Local heat flux density in Wm^2

K = is the Conductivity of the material in $W \cdot m^{-1} \cdot K^{-1}$

ΔT = is the temp gradient in $K \cdot m^{-1}$

CONVECTION

Newton's law of cooling

$$Q = hA\Delta T = hA(T_s - T_\infty)$$

where h = convective Heat range Coefficient

Note: h is not a material property.

RADIATION

Stefan-Boltzman Radiation equation (state)

$$Q_{Rad} \propto T^4$$

$$Q = \sigma T^4$$

where σ (sigma) = stefan bolzman Coefficient

$$\sigma = 5.67 \times 10^{-8}$$

$$Q_{rad} = A\sigma T^4$$

$$Q_{rad} = \sigma A \varepsilon T^4$$

$$Q_{rad} = \sigma A_1 \varepsilon_1 (T_1^4 - T_2^4)$$

$$Q_{rad_{1-2}} = \sigma A_1 \varepsilon_1 f_{1-2}(T_1^4 - T_2^4)$$

where f_{1-2} = view factor.

$$Q = KA\frac{\Delta T}{L} \quad \text{or} \quad Q = -KA\frac{\Delta T}{\Delta x}$$

$$\frac{Q}{A} = q = \frac{K\Delta T}{L} = \text{Heat flux } (w/m^2)$$

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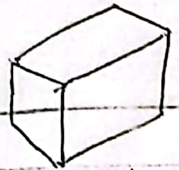
...rmed question)

Insurance

inside

$T_1 = 1400°C$



outside

$T_2 = 30°C$

$K_{brick} = 1.31 W/m^{-1}K^{-1}$ $\quad L = 500mm = 0.5m$

$T_1 = 1400 + 273 = 1673K$ $\quad T_2 = 30 + 273 = 303$

from $Q = -KA\frac{\Delta T}{\Delta x}$ since A is not given we use

$$\frac{Q}{A} = q = K\frac{\Delta T}{L}$$

$$\frac{Q}{A} = 1.31\left(\frac{1673 - 303}{0.5}\right) = 3589.4\ w/m^2$$

Reducing the heat loss by 50%

50% of 3589 = $1794\ w/m^2$

$3589.4 + 1794\ w/m^2 = 5383.4$

$$5383.4 = \frac{1.31(1370)}{L}$$

$$5383.4 = \frac{1794.7}{L}$$

$$L = \frac{1794.7}{5383.4} = 0.33m$$

If there is no space, consider changing the material

25/10/2022

<u>Revision</u>

$$\boxed{Q = -KA\frac{dT}{dx}}$$

<u>factor affecting heat flow</u>

$Q \propto dT$

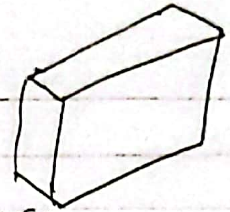
$Q \propto \frac{1}{dx}$ or $\frac{1}{L}$

$Q \propto A$

$Q \propto K$

600°C

873K



0.5m

30°C

303K

Low thermal conductivity material are insulators

Negative sign is used for correcting the thickness difference, when using (L) there's no need for (−) negative sign.

Convection happen btw hot surface and a fluid.

Newton's law of cooling

$$\boxed{Q = hA\Delta T}$$

cold fluid goes down

Hot fluid goes up

Convection.

h = heat transfer coefficient

<u>Radiation</u>

A cold body radiate

stefan Boltzmann

$$\boxed{Q \propto T^4}$$

$$\boxed{\sigma = 5.67 \times 10^{-8}}$$

S-B constant.

$$Q = \sigma T^4$$

Because all bodies are not black-bodies.

ε = Emissivity

$\varepsilon = 1$ = black body

$\varepsilon = < 1$ = Gray body.

Electrical Analogy of Thermal System.

$V = IR$

$I = V/R$

$$Q = \frac{\text{Temperature difference}}{R_{th}}$$

R_{th} = Thermal Resistance

$$Q_{conduct} = KA\frac{\Delta T}{L} = \frac{\Delta T}{L} \times \frac{1}{\frac{1}{KA}}$$

$$= \frac{\Delta T}{\frac{L}{KA}}$$

$$= R_{conduction} = \frac{L}{KA}$$

$$\frac{\Delta T}{L} \times \frac{1}{\frac{1}{KA}} = \frac{\Delta T}{\frac{L}{KA}}$$

$$Q = \frac{\Delta T}{R_{th}} = \frac{\Delta T}{\frac{L}{KA}} = R_{th} = \frac{L}{KA}$$

$$Q = \frac{\Delta T}{R_{th}}$$

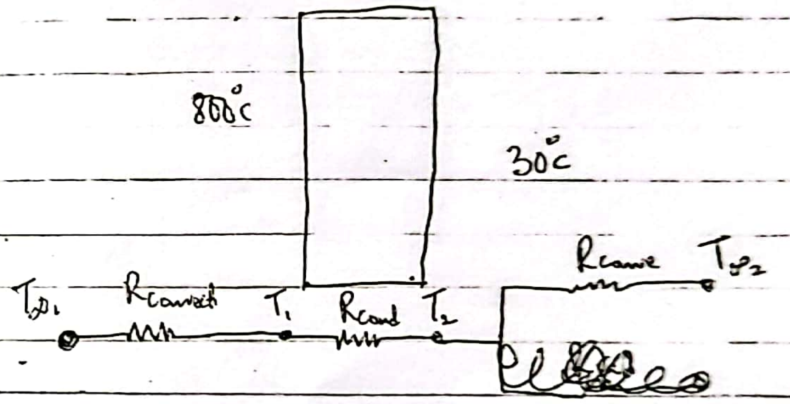
$$Q = KA\frac{\Delta T}{L} = \frac{\Delta T}{L} \times \frac{1}{\frac{1}{KA}}$$

Convection

$$Q = hA\Delta T = \frac{\Delta T}{\frac{1}{hA}}$$

$$\boxed{R_{convection} = \frac{1}{hA}}$$

Example



$$Q = h_{\infty 1}A(T_{\infty 1} - T_1) = KA\frac{T_1 - T_2}{L} = h_{\infty 2}A(T_2 - T_{\infty 2})$$

We can use any two

using the extremes

$$Q = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

$$R_{total} = R_{convection} + R_{conduction} + R_{convection}$$

Note

The rate of heat conduction through a medium in a specific direction (say in the x-direction) is proportional to the temperature difference across the medium and the area normal to the direction of heat transfer, but inversely proportional to the distance in that direction. This was expressed in the differential form by Fourier's law of heat conduction for one dimensional heat conduction as

$$\dot{Q}_{cond} = -KA\frac{dT}{dx}$$

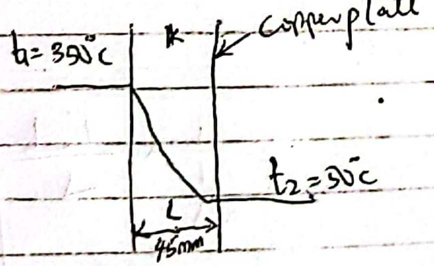
Example under Conduction

[illegible]e rate of heat transfer per unit area through a copper [illegible] thick, whose one face is maintained at 350°c and the [illegible] 50°c. Take thermal conductivity of copper as 370 W/m°c

$= (50-350)$ $L = 45mm = 0.045m$ $K = 370 W/m°c$

Fourier's law

$$-KA\frac{dt}{dx} = -KA\left(\frac{t_2-t_1}{L}\right)$$



$$= -k\frac{dt}{dx}$$

$$[illegible]0 \times \frac{(50-350)}{0.045}$$

$2.466 \times 10^6 W/m^2$

$= 2.466 MW/m^2$

Exp 2: A plane wall is 150mm thick and its wall area is $4.5m^2$. If its conductivity is 9.35 W/m°c and surface temp are steady at 150°c and 45°c determine (i) Heat flow across the plane wall (ii) Temp gradient in the flow direction

Solution

$L = 150mm = 0.15m$ $A = 4.5m^2$ $dt = t_2 - t_1 = 45 - 150 = -105°c$

$K = 9.35 W/m°c$

(i) Heat flow across the plane wall (Q)

As per Fourier's law

$$Q = -KA\frac{dt}{dt} = \frac{-KA(t_2 - t_1)}{L}$$

$$= -9.35 \times 4.5 \times \frac{(-105)}{0.15}$$

$$= 29452.5W$$

(ii) Temp gradient dt/dx:

From Fourier's law we have

$$\frac{dt}{dx} = -\frac{Q}{KA} = \frac{29452.5}{9.35 \times 4.5} = -700°c/m$$

Example under Convection

Exp 3: A hot plate is 1m x 1.5m is maintained at 300°c. Air at 20°c blows over the plate. If the convective heat transfer coeff is $20W/m^2°c$. Cal the rate of heat transfer.

Sol $A = 1 \times 1.5 = 1.5m^2$ $t_s = 300°c$ $t_f = 20°c$

$h = 20W/m^2°c$

Newton's law of cooling

$$Q = hA(t_s - t_f)$$
$$= 20 \times 1.5 \times (300-20) = 8400 = 8.4kw$$

A wire of 1.5mm in diameter and 150mm long is submerged ... atmospheric pressure. An electric current is passed through ... is increased until the water boils at 100°C under ... If convective heat transfer coefficient is 4500W/m²°C ... electric power must be supplied to the wire to ... the wire surface at 120°C?

$d = 1.5mm = 0.0015m$ $L = 150mm = 0.15m$

Surface area of the wire (exposed to heat transfer) =

$A = \pi d L = \pi \times 0.0015 \times 0.15 = 7.068 \times 10^{-4} m^2$

$t_s = 120°C$ $t_f = 100°C$ $h = 4500 W/m^2 °C$

Power to be supplied

Electric power which must be supplied = Total convection loss (Q)

$Q = hA(t_s - t_f) = 4500 \times 7.068 \times 10^{-4}(120-100)$

$= 63.6w$

Example under Radiation.

A surface having an area of 1.5m² and maintained at 300°C ... by radiation with another surface at 40°C. The value ... to the geometric location and emissivity is 0.52 determine ... by radiation (ii) the value of thermal resistance ... value of equivalent convection coefficient.

Sol Given $A = 1.5m^2$ $T_1 = t_1 + 273 = 300 + 273 = 573K$

$T_2 = t_2 + 273 = 40 + 273 = 313K$ $F = 0.52$

(i) $Q = F\sigma A(T_1^4 - T_2^4)$ where $\sigma = 5.67 \times 10^{-8} W/m^2K^4$

$Q = 0.52 \times 5.67 \times 10^{-8} \times 1.5 (573^4 - 313^4)$

$= 0.52 \times 5.67 \times 1.5 \left[\left(\frac{573}{100}\right)^4 - \left(\frac{313}{100}\right)^4\right]$

$Q = 4343 W$

(ii) $Q = \frac{T_1 - T_2}{(R_{th})_{rad}} \Rightarrow \frac{573-313}{4343} = R_{th}(rad)$

$= 0.0598 °C/w$

(iii) $Q = hA(t_1 - t_2)$

$h_r = \frac{Q}{A(t_1-t_2)} = \frac{4343}{1.5(300-40)}$

$= 11.13 W/m^2 °C$

Exp 6 A carbon steel plate (thermal conductivity = 45W/m°C) 600mm x 900mm x 25mm is maintained at 310°C. Air at 15°C blows over the hot plate. If convection heat transfer coefficient is 22W/m²°C and 250W is lost from the plate surface by radiation. Calculate the inside plate temperature

Soln

$A = 600mm \times 900mm = 0.6m \times 0.9 = 0.54m^2$ $L = 25mm = 0.025m$

$t_s = 310°C$ $t_f = 15°C$ $h = 22W/m^2°C$ $Q_{rad} = 250W$ $k = 45W/m°C$

Inside plate temp (t_i)

Heat conducted through plate = Convection heat losses + radiation heat losses

$$Q_{cond} = Q_{conv} + Q_{rad}$$

$$KA\frac{dt}{dx} = hA(t_s - t_f) + F\sigma A(T_s^4 - T_f^4)$$

$$45 \times 0.54 \times \frac{(t_s - t_i)}{L} = 22 \times 0.54(310-15) + 250$$

$$-45 \times 0.5 \times \frac{(310 - t_i)}{0.025} = 22 \times 0.54 \times 295 + 250$$

$$972(t_i - 310) = 3754.6$$

$$t_1 = \frac{3754.6}{972} + 310$$

$$t_1 = 313.86^{\circ}c$$

7/11/2022

MEG407

- Steady state $\rightarrow$ no change with time.
- Transient $\rightarrow$ Change with time
- Lumped system $\rightarrow$ uniform Heat change

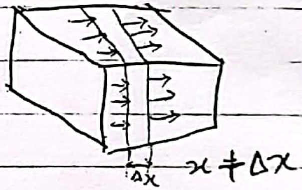
Heat Generation.

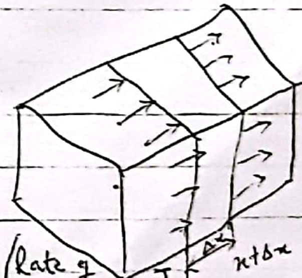
$$E_{gen} = W/m^3$$

$$E_{gen} = \int_v \dot{e}_{gen}\, dV$$

$$E_{gen} = \dot{e}_{gen} V$$

One-dimensional Heat Conduction Equation





$$\left(\begin{array}{c}\text{Heat Conduction}\\ \text{at } x\end{array}\right) - \left(\begin{array}{c}\text{Heat conduction}\\ \text{out } (x+\Delta x)\end{array}\right) + \left(\begin{array}{c}\text{Heat}\\ \text{Generation}\end{array}\right) = \left(\begin{array}{c}\text{Rate of}\\ \text{change of Energy}\\ \text{Content of the element}\end{array}\right)$$

$$\dot{Q} - \dot{Q}_{x+\Delta x} + \dot{E}_{gen} = \frac{\Delta E_{element}}{\Delta t}$$

$$\Delta E_{element} = \rho(A\Delta x(T_{t+\Delta t} - T_t)$$

$$\dot{E} = \dot{e}_{gen} A\Delta x$$

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{e}_{gen}A\Delta x = \rho(A\Delta x(T_{t+\Delta t} - T_t)$$

$$\boxed{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\dot{e}_{gen}=\rho c\frac{\partial T}{\partial t}} \rightarrow \text{Variable Conductivity.}$$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\dot{e}_{gen}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \alpha=\frac{k}{\rho c}$$

Steady state.

$$\frac{\partial}{\partial t}=0 \qquad \frac{\partial^2 T}{\partial x^2}+\dot{e}_{gen}=0 \quad \text{Poission equation}$$

Transient no heat generation

$$\dot{e}_{gen}=0 \qquad \frac{d^2T}{dx^2}=\frac{1}{\alpha}\frac{dT}{dx} \text{ — diffusion equation}$$

Steady state no heat generation.

$$\dot{e}_{gen}=0 \qquad \frac{d^2T}{dx^2}=\frac{1}{\alpha}\frac{dT}{dt} \rightarrow \text{Laplace Equation } \cancel{\text{diffusion eqn}}$$

Heat Conduction Equation Cylindrical Coordinate.

Conduction

$$Q=2\pi rLk\frac{T_1-T_2}{\ln\left(\frac{r_2}{r_1}\right)}$$

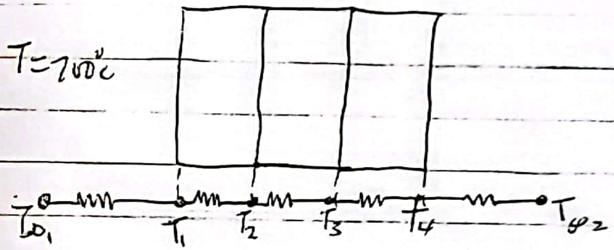
Convention

$$Q=h2\pi rL\Delta T$$

Radiation -

$$Q_{rad}=h_{rad}A\Delta T$$

Composite walls



$$R_{conv}=\frac{1}{h_1A},\ R_{cond}\ \frac{L_1}{K_1A},\ \frac{L_2}{K_2A},\ \frac{L_3}{K_3A},\ \frac{1}{h_2A}$$

$$Q=\frac{T_{\infty_1}-T_{\infty_2}}{R_{total}}$$

$$R_{total}=R_1+R_2+R_3+R_4+R_5$$
$$=\frac{1}{h_1A}+\frac{L_1}{K_1A}+\frac{L_2}{K_2A}+\frac{L_3}{K_3A}+\frac{1}{h_2A}$$

8/11/2022

Spherical

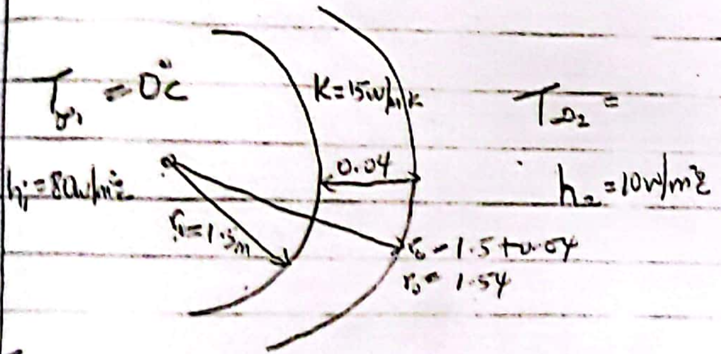
Conduction

$$Q=\frac{4\pi K(T_1-T_0)}{\left(\frac{1}{r_1}-\frac{1}{r_0}\right)} \qquad \text{when } T(r) \text{ negligible}$$

$$=\frac{r_0-r_1}{4\pi K r_0 r_1}$$

$$Q = 4\pi r^2 h \Delta T$$

Solution to the class quiz



$$= \frac{1}{h_1(4\pi r^2)} + \frac{r_o - r_1}{4\pi k r_o r_1} + \frac{1}{h_2(4\pi r^2)}$$

$$= \frac{1}{80\{(4\times\pi\times(1.5)^2)\}} + \frac{1.54 - 1.5}{4\times\pi\times 15\times 1.54\times 1.5} + \frac{1}{10\times\{(4\times\pi\times(1.54)^2)\}}$$