MEG 403

Selected Problems and Solutions In Vibration

Free Vibration

PROBLEMS

2.1. A 0.453-kg mass attached to a light spring elongates it 7.87 mm. Determine the natural frequency of the system.

2.2. A spring-mass system, k_1 and m, has a natural frequency of f_1. If a second spring k_2 is added in series with the first spring, the natural frequency is lowered to $\frac{1}{2}f_1$. Determine k_2 in terms of k_1.

2.3. A 4.53-kg mass attached to the lower end of a spring whose upper end is fixed vibrates with a natural period of 0.45 s. Determine the natural period when a 2.26-kg mass is attached to the midpoint of the same spring with the upper and lower ends fixed.

2.4. An unknown mass of m kg attached to the end of an unknown spring k has a natural frequency of 94 cpm. When a 0.453-kg mass is added to m, the natural frequency is lowered to 76.7 cpm. Determine the unknown mass m and the spring constant k N/m.

2.5. A mass m_1 hangs from a spring k N/m and is in static equilibrium. A second mass m_2 drops through a height h and sticks to m_1 without rebound, as shown in Fig. P2.5. Determine the subsequent motion.

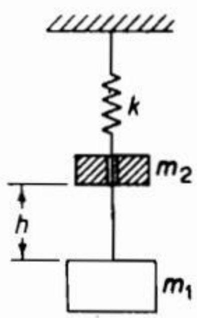

FIGURE P2.5.

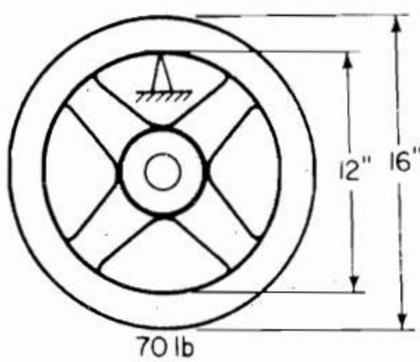

FIGURE P2.7.

2.6. The ratio k/m of a spring-mass system is given as 4.0. If the mass is deflected 2 cm down, measured from its equilibrium position, and given an upward velocity of 8 cm/s, determine its amplitude and maximum acceleration.

2.34. A mass of 0.907 kg is attached to the end of a spring with a stiffness of 7.0 N/cm. Determine the critical damping coefficient.

2.35. To calibrate a dashpot, the velocity of the plunger was measured when a given force was applied to it. If a $\frac{1}{2}$-lb weight produced a constant velocity of 1.20 in./s, determine the damping factor ζ when used with the system of Prob. 2.34.

2.36. A vibrating system is started under the following initial conditions: $x = 0$ and $\dot{x} = v_0$. Determine the equation of motion when (a) $\zeta = 2.0$, (b) $\zeta = 0.50$, and (c) $\zeta = 1.0$. Plot nondimensional curves for the three cases with $\omega_n t$ as abscissa and $x\omega_n/v_0$ as ordinate.

2.60. If two springs are connected in series, as shown in the first figure in the table of spring stiffness, derive the resulting spring stiffness and the natural frequency of the motion.

2.61. If two springs are connected in parallel, as shown in the second figure in the table of spring stiffness, derive the resulting spring stiffness and the natural frequency of the motion.

2.62. Write down the equations of motion and find the effective spring constant for the system shown in Fig. P2.62.

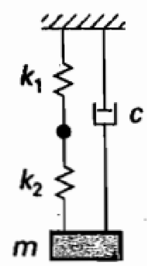

FIGURE P2.62.

Forced Vibration

3.1. A machine part of mass 1.95 kg vibrates in a viscous medium. Determine the damping coefficient when a harmonic exciting force of 24.46 N results in a resonant amplitude of 1.27 cm with a period of 0.20 s.

3.2. If the system of Prob. 3.1 is excited by a harmonic force of frequency 4 cps, what will be the percentage increase in the amplitude of forced vibration when the dashpot is removed?

3.3. A weight attached to a spring of stiffness 525 N/m has a viscous damping device. When the weight is displaced and released, the period of vibration is 1.80 s, and the ratio of consecutive amplitudes is 4.2 to 1.0. Determine the amplitude and phase when a force $F = 2\cos 3t$ acts on the system.

3.4. Show that for the dampled spring-mass system, the peak amplitude occurs at a frequency ratio given by the expression

$$\left(\frac{\omega}{\omega_n}\right)_p = \sqrt{1 - 2\zeta^2}$$

3.5. A spring-mass is excited by a force $F_0 \sin \omega t$. At resonance, the amplitude is measured to be 0.58 cm. At 0.80 resonant frequency, the amplitude is measured to be 0.46 cm. Determine the damping factor ζ of the system.

3.13. A counterrotating eccentric mass exciter shown in Fig. P3.13 is used to determine the vibrational characteristics of a structure of mass 181.4 kg. At a speed of 900 rpm, a stroboscope shows the eccentric masses to be at the top at the instant the structure is moving upward through its static equilibrium position, and the corresponding amplitude is 21.6 mm. If the unbalance of each wheel of the exciter is 0.0921 kg · m, determine (a) the natural frequency of the structure, (b) the damping factor of the structure, (c) the amplitude at 1200 rpm, and (d) the angular position of the eccentrics at the instant the structure is moving upward through its equilibrium position.

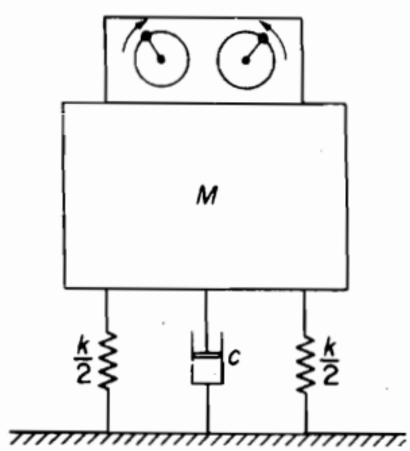

FIGURE P3.13.

3.19. For turbines operating above the critical speed, stops are provided to limit the amplitude as they run through the critical speed. In the turbine of Prob. 3.18, if the clearance between the 2.54-cm shaft and the stops is 0.0508 cm, and if the eccentricity is 0.0212 cm, determine the time required for the shaft to hit the stops. Assume that the critical speed is reached with zero amplitude.

3.21. The springs of an automobile trailer are compressed 10.16 cm under its weight. Find the critical speed when the trailer is traveling over a road with a profile approximated by a sine wave of amplitude 7.62 cm and wavelength of 14.63 m. What will be the amplitude of vibration at 64.4 km/h? (Neglect damping.)

3.26. An industrial machine of mass 453.4 kg is supported on springs with a static deflection of 0.508 cm. If the machine has a rotating unbalance of 0.2303 kg · m, determine (a) the force transmitted to the floor at 1200 rpm and (b) the dynamic amplitude at this speed. (Assume damping to be negligible.)

3.27. If the machine of Prob. 3.26 is mounted on a large concrete block of mass 1136 kg and the stiffness of the springs or pads under the block is increased so that the statical deflection is still 0.508 cm, what will be the dynamic amplitude?

3.28. An electric motor of mass 68 kg is mounted on an isolator block of mass 1200 kg and the natural frequency of the total assembly is 160 cpm with a damping factor of $\zeta = 0.10$ (see Fig. P3.28). If there is an unbalance in the motor that results in a harmonic force of $F = 100 \sin 31.4t$, determine the amplitude of vibration of the block and the force transmitted to the floor.

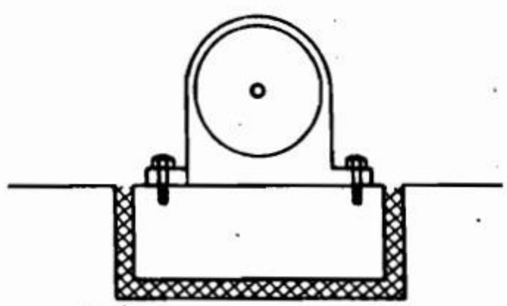

FIGURE P3.28.

5.2. Determine the normal modes and frequencies of the system shown in Fig. P5.2 when $n = 1$.

5.3. For the system of Prob. 5.2, determine the natural frequencies as a function of n.

5.4. Determine the natural frequencies and mode shapes of the system shown in Fig. P5.4.

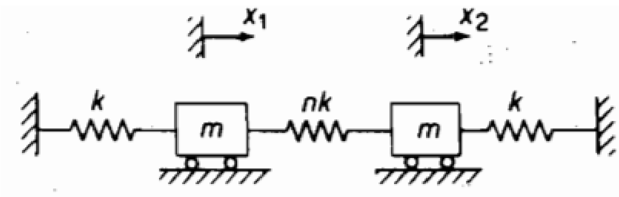

FIGURE P5.2.

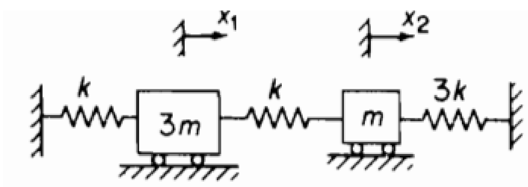

FIGURE P5.4.

1. The 8kg body is moved 0.2m to the right of the equilibrium position and released from rest at t = 0.
    a. Determine its displacement at time, t = 2s, the viscous damping coefficient, c is 20 Ns/m and the spring stiffness, k is 32 N/m.
    b. Determine the logarithmic decrement, δ
    c. Measure two successive amplitudes x_1 and x_2 of a full cycle.

2. Show that for the damped spring-mass system, the peak amplitude occurs at a frequency ratio given by the expression:

$$\frac{1}{1} = \overline{\rho \quad \varsigma -}$$

3. a. What is *Whirling?*(In addition, state the general equation for *Whirling*)
   b. A body of mass *m* is supported by three springs of stiffness k_1, k_2 and k_3. The arrangement of the springs is shown in the figure 2 below. Determine the equivalent spring stiffness of the three (3) springs, hence find the natural frequency of the system if the mass m = 20 kg, K_1 = 40 N/m, $K_2 = K_3$ = 32 N/m

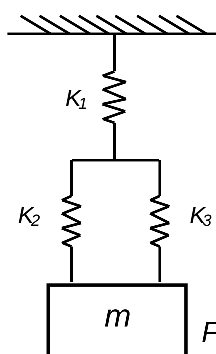

Figure2.0 [15 Marks]

4. a. Briefly describe what *Support* motion vibration with the aid of a well-labeled diagram. [10 Marks]
   b. A machine of 100kg mass is supported on springs of total stiffness 700 kN/m and has an unbalanced rotating element, which results in a disturbing force of 350 N at a speed of 3000 rpm. Assuming a damping factor of ζ = 0.20, determine:
   (i.) Its amplitude of motion due to the unbalance,
   (ii.) The transmissibility, and
   (iii.) The transmitted force.

5. An electric motor of mass 50kg is supported by elastic mountings providing a total stiffness of 26.3 kN/m, and a damping coefficient of 575 Ns/m. The motor has a rotating unbalance equivalent to a mass of 8kg at a radius of 12.5mm from the axis of rotation of the rotor. If the motor is running at a speed of 1200 revolutions per minute and constrained to move vertically,

determine the amplitude of the steady-state vibration of the motor. (Provide the simple schematic and free-body diagram).

6. The Figure 3.0 below shows an undamped *two-degreeof-freedom*(*DOF*) system with specific parameters where m = 10 kg and k = 35 N/m. For the normal mode of oscillation,
    a. Write the equations of motion for the system,
    b. Determine the characteristic equation and hence the Eigen values
    c. Determine its natural frequencies (e.g. •$_1$, •$_2$), and
    d. Determine its mode shapes.

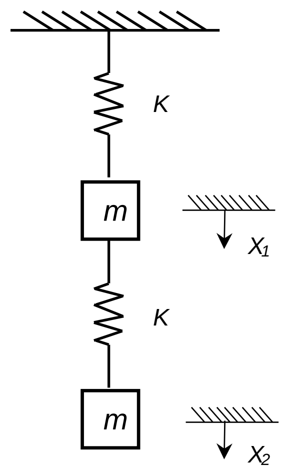

Figure 30