HW3

Tuesday, February 25, 2020 8:38 PM

ATANAS DELEVSKI ECE 407 HW#3

1.) a)

$$E(z) = 1\cdot\tfrac{1}{8} + 2\cdot\tfrac{1}{8} + 3\cdot\tfrac{1}{8} + 4\cdot\tfrac{1}{8} + 5\cdot\tfrac{1}{4} + 6\cdot\tfrac{1}{4}$$

$$= \tfrac{10}{8} + \tfrac{11}{4} = \tfrac{5}{4} + \tfrac{11}{4} = \tfrac{16}{4} = 4$$

$\boxed{E(z) = 4}$

$Var(Z) = E(x^2) - (E(x))^2 \Rightarrow$ $E(x^2) = \frac{1}{8}(1^2+2^2+3^2+4^2) + \frac{1}{4}(5^2+6^2) = 19$

$(E(x))^2 = 4^2 = 16$

$\Rightarrow 19 - 16 = 3$

$\therefore \boxed{Var(z) = 3}$

b) $E(X) = E(z_1) + E(z_2) + \ldots + E(z_{10}) \Rightarrow 4\cdot 10 = 40$

$\Rightarrow \boxed{E(X) = 40}$

Variance works the same way for independent events.

$\therefore Var(X) = Var(z_1) + Var(z_2) + \ldots + Var(z_{10}) = 3\cdot 10 = 30$

$\boxed{Var(X) = 30}$

2) a) $\dfrac{2+4+0+7+1+2+0+3+2+1+5+4+3}{13} = \dfrac{34}{13}$

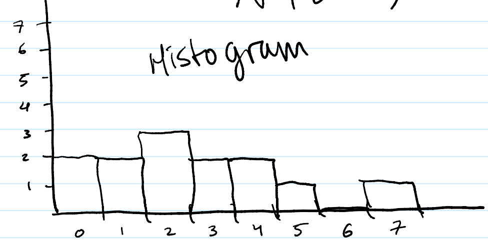
$\boxed{\text{Mean} = \tfrac{34}{13} \text{ or } 2.615}$

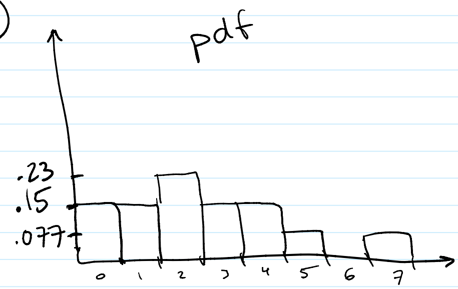
b) $Var(s) = E(s^2) - (E(s))^2 \Rightarrow \dfrac{2^2+4^2+0+7^2+1^2+2^2+3^2+2^2+1^2+5^2+4^2+3^2}{13}$

$\Rightarrow \dfrac{138}{13} - \left(\dfrac{34}{13}\right)^2 = \dfrac{638}{169}$

$\boxed{\text{Variance} = \tfrac{638}{169} \text{ or } 3.775}$

3.) $N(2.615, 3.775)$





4.)

a) $\begin{matrix} x \\ y \end{matrix}$ $\begin{bmatrix}1\\2\end{bmatrix} + \begin{bmatrix}2\\3\end{bmatrix} + \begin{bmatrix}2\\4\end{bmatrix} + \begin{bmatrix}3\\4\end{bmatrix} + \begin{bmatrix}1.5\\5\end{bmatrix} + \begin{bmatrix}2\\5\end{bmatrix} + \begin{bmatrix}1\\5\end{bmatrix} + \begin{bmatrix}1\\4\end{bmatrix} + \begin{bmatrix}2\\2\end{bmatrix} = \begin{bmatrix}15.5\\34\end{bmatrix}$

$\Rightarrow \frac{1}{9}\begin{bmatrix}15.5\\34\end{bmatrix} \Rightarrow$ mean vector $= \begin{bmatrix}1.722\\3.777\end{bmatrix}$

$$\Rightarrow \frac{1}{9}\begin{bmatrix}15.5\\34\end{bmatrix} \Rightarrow \boxed{\text{mean vector} = \begin{bmatrix}1.722\\3.777\end{bmatrix}}$$

Covariance matrix:

$$\Sigma = \begin{bmatrix}\sigma_1^2 & \sigma_{xy}\\ \sigma_{xy} & \sigma_2^2\end{bmatrix} \Rightarrow$$

$$\sigma_1^2 = E[x^2] - [E[x]]^2 \Rightarrow 3.361 - 1.722^2$$

$$\therefore \sigma_1^2 = .3958 \approx .396$$

$$\sigma_2^2 = \text{'' \quad ''} \Rightarrow \frac{140}{9} - 3.777^2$$

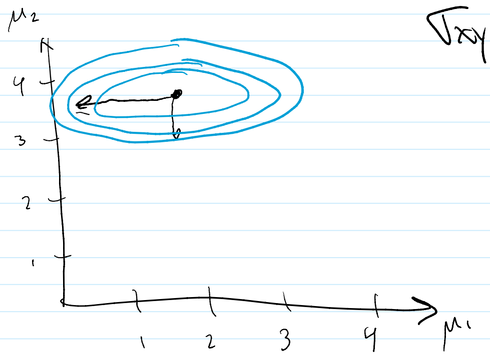
$$\therefore \sigma_2^2 = 1.2898 \approx 1.29$$

$$\frac{\sum (X-\bar{x})(Y-\bar{y})}{N-1} = \frac{(1-1.722)(2-3.77)+(2-1.722)(3-3.77)\ldots}{8}$$

$$\sigma_{xy} = -0.0069$$

$$\Rightarrow \boxed{\text{Cov Matrix} = \begin{bmatrix}.396 & -.0069\\ -.0069 & 1.29\end{bmatrix}}$$

b)



5)

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\ell(\lambda) = \sum_{i=1}^{n} (x_i \log\lambda - \lambda - \log x_i!)$$

$$= \log\lambda \sum_{i=1}^{n} x_i - n\lambda - \sum_{i=1}^{n} \log x_i!$$

$$\Rightarrow \ell'(\lambda) = \frac{1}{\lambda}\sum_{i=1}^{n} x_i - n = 0$$

$$\therefore \hat{\lambda} = \bar{X}$$