Atanas Delevski
ECE 407 HW #8
4/26/2020

Question 1: (I did by hand on paper)

ATANAS DELEVSKI ECE 407 HW #8

4/24/20

Q1) $\begin{bmatrix}5\\6\\0\end{bmatrix}, \begin{bmatrix}6\\-5\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$ orthonormal basis of $\mathbb{R}^3$?

⇒ these vectors are indeed orthogonal to each other, but they are certainly not normalized, since their lengths are not 1.

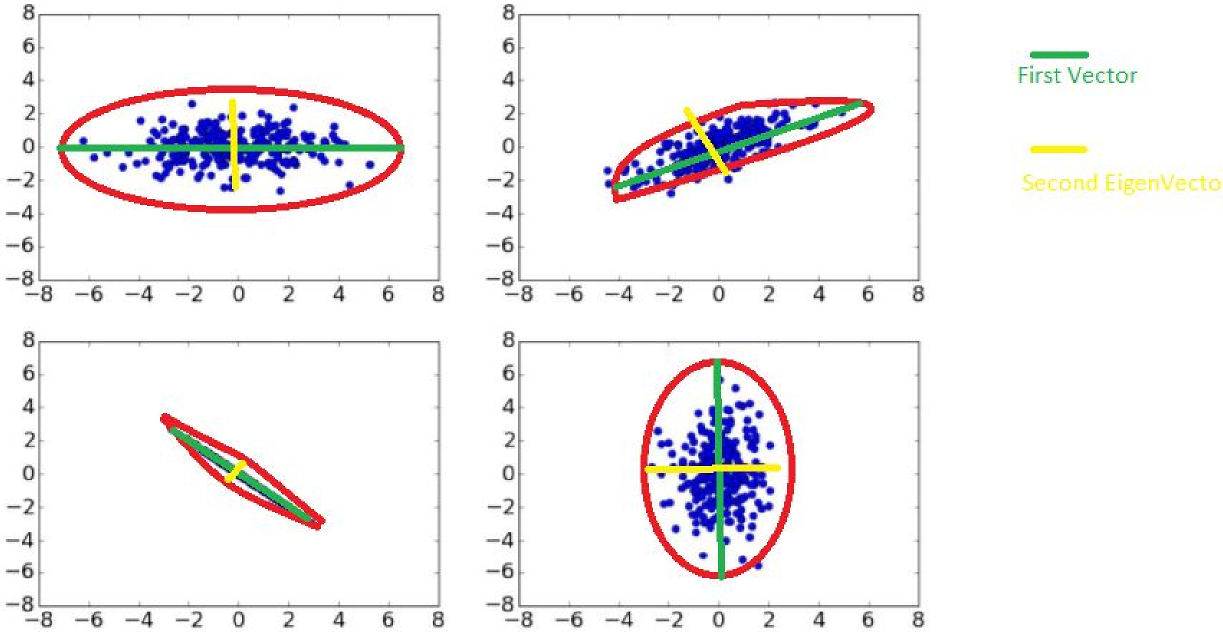
⇒ Therefore they are <u>not</u> <u>orthonormal</u>

$\|v_1\|^2 = v_1 \cdot v_1 = 25+36 = 61$

$\|v_1\| = \sqrt{61}$,

(only the third vector is normal)

Question 2:



The first eigenvector is the axis of most variation (green). The second eigenvector is the axis with the second most variation (yellow).

Note: in the bottom left data-set, the line of data points were so perfectly straight that one could say they were 1-D, and therefore it is not possible to fit the second eigenvector (yellow) since there is no second axis of variation.

Question 3: (I did electronically in One Note)

Q3) u_1, u_2 are two orthonormal vectors part of U.

$U = \begin{pmatrix} \uparrow & \uparrow \\ u_1 & u_2 \\ \downarrow & \downarrow \end{pmatrix} p$

a)

$\boxed{U = p \times 2}$

$\boxed{U^T = 2 \times p}$

$\boxed{UU^T = p \times p}$ (where p is the # of dimensions that $u_{1,2}$ span)

$$\underbrace{\begin{bmatrix} \uparrow & & \uparrow \\ u_1 & \cdot & u_2 \\ \downarrow & & \downarrow \end{bmatrix}}_{p \times 2} \cdot \underbrace{\begin{bmatrix} \leftarrow u_1 \rightarrow \\ \leftarrow u_2 \rightarrow \end{bmatrix}}_{2 \times p} = \underbrace{\overbrace{\begin{bmatrix} & \\ & \end{bmatrix}}^{p}}_{p \times p} p$$

$\boxed{u_1 \cdot u_1^T = p \times p}$

$$\begin{bmatrix} \uparrow \\ u_1 \\ \downarrow \end{bmatrix} \begin{bmatrix} \leftarrow u_1 \rightarrow \end{bmatrix} = \overbrace{\begin{bmatrix} & \\ & \end{bmatrix}}^{p \times p} p \times p$$

b)

$x \to (u_1 \cdot x, u_2 \cdot x) = U^T x$

$x \to (u_1 \cdot x)u_1 + (u_2 \cdot x)u_2 = UU^T x$

one case

$x \to U^T x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

another case

$x \to UU^T x = \begin{pmatrix} \uparrow \\ x_1 \\ x_2 \\ \downarrow \end{pmatrix}$

One projects the vector's non-zero values, and the other projects everything

Question 4:

I used a variety of tools to solve this question.
First, I used python to get my covariance matrix.

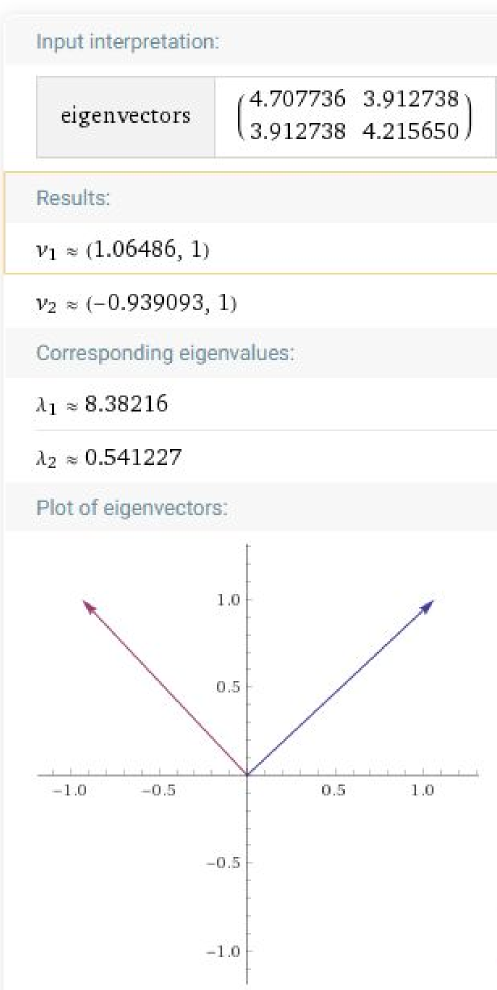
```
import pandas as pd

data = pd.read_excel('D:\Python\School\ECE407\HW8\data2.xlsx')
df = pd.DataFrame(data)

x = df.cov()
print(x)
```

	X	**Y**
X	**4.707736**	**3.912738**
Y	**3.912738**	**4.215650**

Then, I used wolfram alpha to calculate my eigenvectors and eigenvalues from the covariance matrix.



Therefore, the eigenvalues are: **8.38**, **0.54** and the Eigenvectors are **[1.06, 1]^T**, **[-0.939, 1]^T**

This is showing the top two directions of variance within the dataset. One is along the positive correlation axis between x and y, and the other is along the negative correlation axis between x and y.