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ECE 407 4/21/2020
Midterm #2

Q1) 2D data: $\begin{bmatrix}1\\2\end{bmatrix}$ $\begin{bmatrix}1\\3\end{bmatrix}$ $\begin{bmatrix}4\\3\end{bmatrix}$ $\begin{bmatrix}4\\4\end{bmatrix}$

a) Setting initial centers to: $\begin{bmatrix}1\\1\end{bmatrix}$ $\begin{bmatrix}4\\4\end{bmatrix}$

First iteration:

	Cluster 1	Cluster 2
$x=\begin{bmatrix}1\\2\end{bmatrix}$:	✓ $\begin{bmatrix}1\\1\end{bmatrix}-\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}0\\-1\end{bmatrix}$	$\begin{bmatrix}4\\4\end{bmatrix}-\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}3\\2\end{bmatrix}$
$x=\begin{bmatrix}1\\3\end{bmatrix}$:	✓ $\begin{bmatrix}1\\1\end{bmatrix}-\begin{bmatrix}1\\3\end{bmatrix}=\begin{bmatrix}0\\-2\end{bmatrix}$	$\begin{bmatrix}4\\4\end{bmatrix}-\begin{bmatrix}1\\3\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix}$
$x=\begin{bmatrix}4\\3\end{bmatrix}$:	$\begin{bmatrix}1\\1\end{bmatrix}-\begin{bmatrix}4\\3\end{bmatrix}=\begin{bmatrix}-3\\-2\end{bmatrix}$	$\begin{bmatrix}4\\4\end{bmatrix}-\begin{bmatrix}4\\3\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix}$ ✓
$x=\begin{bmatrix}4\\4\end{bmatrix}$:	$\begin{bmatrix}1\\1\end{bmatrix}-\begin{bmatrix}4\\4\end{bmatrix}=\begin{bmatrix}-3\\-3\end{bmatrix}$	$\begin{bmatrix}4\\4\end{bmatrix}-\begin{bmatrix}4\\4\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$ ✓

Cluster 1: $\dfrac{\begin{bmatrix}1\\2\end{bmatrix}+\begin{bmatrix}1\\3\end{bmatrix}}{2}=\begin{bmatrix}1\\2.5\end{bmatrix}$ = new center for cluster 1

Cluster 2: $\dfrac{\begin{bmatrix}4\\3\end{bmatrix}+\begin{bmatrix}4\\4\end{bmatrix}}{2}=\begin{bmatrix}4\\3.5\end{bmatrix}$ = new center for cluster 2

After doing 1 iteration, it is clear that the centers will not move if we do more iterations.

b) Given $\begin{bmatrix}5\\5\end{bmatrix}$, we will now classify it.

Cluster 1: $\begin{bmatrix}1\\2.5\end{bmatrix}-\begin{bmatrix}5\\5\end{bmatrix}=\begin{bmatrix}-4\\-2.5\end{bmatrix}$

Cluster 2: $\begin{bmatrix}4\\3.5\end{bmatrix}-\begin{bmatrix}5\\5\end{bmatrix}=\begin{bmatrix}-1\\-1.5\end{bmatrix}\rightarrow$ smaller

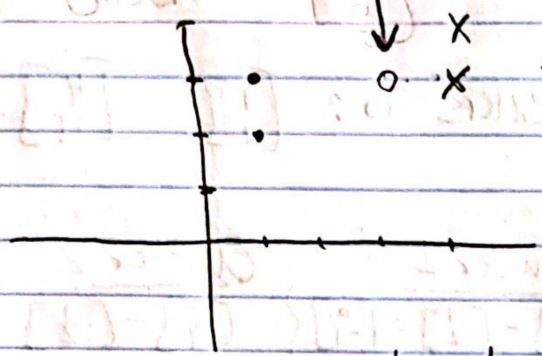
$\Rightarrow$ $\boxed{\begin{bmatrix}5\\5\end{bmatrix} \text{ is classified in cluster 2!}}$

EXAM Continued

Q2)

$$\text{Data} = \begin{array}{cccc} 1 & 1 & -1 & -1 \\ \begin{bmatrix}1\\2\end{bmatrix} & \begin{bmatrix}1\\3\end{bmatrix} & \begin{bmatrix}4\\3\end{bmatrix} & \begin{bmatrix}4\\4\end{bmatrix} \end{array}$$

a) Given $\begin{bmatrix}3\\3\end{bmatrix}$:



closer to X's

$\Rightarrow$ Using $K=1$, $\begin{bmatrix}3\\3\end{bmatrix}$ is closest to the data point $\begin{bmatrix}4\\3\end{bmatrix}$,

$\Rightarrow$ therefore $\begin{bmatrix}3\\3\end{bmatrix}$ is classified as -1 class using $k=1$

b) Using $K=3$,

so

$$\begin{bmatrix}3\\3\end{bmatrix} \rightarrow \begin{bmatrix}4\\3\end{bmatrix} = \sqrt{1^2+0^2} = \sqrt{1} = 1$$

$$\begin{bmatrix}3\\3\end{bmatrix} \rightarrow \begin{bmatrix}4\\4\end{bmatrix} = \sqrt{1^2+1^2} = \sqrt{2}$$

$$\begin{bmatrix}3\\3\end{bmatrix} \rightarrow \begin{bmatrix}1\\3\end{bmatrix} = \sqrt{2^2+0^2} = \sqrt{4} = 2$$

$$\begin{bmatrix}3\\3\end{bmatrix} \rightarrow \begin{bmatrix}1\\2\end{bmatrix} = \sqrt{2^2+1^2} = \sqrt{5}$$

3 shortest distances

two of three nearest neighbors are class -1

$\Rightarrow$ Using $K=3$, $\begin{bmatrix}3\\3\end{bmatrix}$ is still classified as class -1

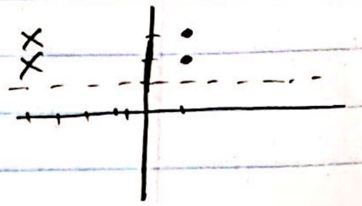
Scanned with CamScanner

1 1 -1 -1

$x_0 = \begin{bmatrix}1\\2\end{bmatrix}$ $x_1 = \begin{bmatrix}1\\3\end{bmatrix}$ $x_2 = \begin{bmatrix}-4\\3\end{bmatrix}$ $x_3 = \begin{bmatrix}-4\\2\end{bmatrix}$

x_0 x_1 x_2

Q3)



a) Initial $w = \begin{bmatrix}0\\0\end{bmatrix}$

Step 0) $\hat{y}_i = \text{sign}(w^T x_0) = [0\ 0]\begin{bmatrix}1\\2\end{bmatrix} = \text{sgn}(0) \neq y_0 = 1$

$\Rightarrow W_{new} = W_{old} + y_0 \cdot x_0 = \begin{bmatrix}0\\0\end{bmatrix} + 1 \cdot \begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix}$

Step 1) $\hat{y}_i = \text{sign}(w^T x_1) = [1\ 2]\begin{bmatrix}1\\3\end{bmatrix} = 1+6=7 \Rightarrow \text{sgn}(7) = y_1 = 1$

$\Rightarrow$ no update

Step 2) $\hat{y}_i = \text{sign}(w^T x_2) = [1\ 2]\begin{bmatrix}-4\\3\end{bmatrix} = -4+6 = 2 \neq$

$\text{sgn}(2) \neq y_2 = -1$

$W_{new} = W_{old} + y_2 x_2 = \begin{bmatrix}1\\2\end{bmatrix} + (-1)\begin{bmatrix}-4\\3\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix} + \begin{bmatrix}4\\-3\end{bmatrix} = \begin{bmatrix}5\\-1\end{bmatrix}$

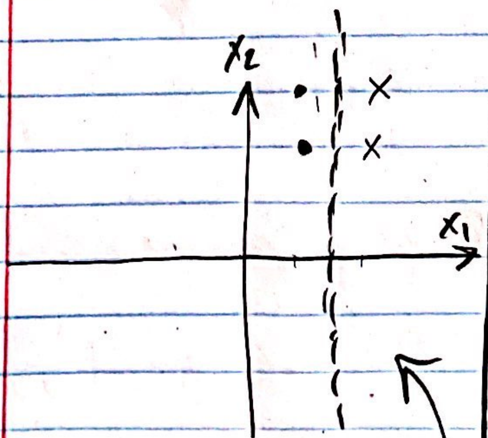
Step 3) $\hat{y}_i = \text{sgn}(w^T x_3) = [5\ -1]\begin{bmatrix}-4\\2\end{bmatrix} = -20-2 = -22$

$\text{sgn}(-22) = y_3 = -1$

$\Rightarrow$ no update

- Final weight vector $\Rightarrow$ $\boxed{w = \begin{bmatrix}5\\-1\end{bmatrix}}$

b)



Yes you can classify this with a perceptron since you can split the classes linearly like it is shown in the diagram