KEVIN OBOTE - 190696

DSA 8205: Optimization for Data Science : Assignment 1

SECTION ONE:

(i) Linear programming model for a farmer purchasing fertilizer

Minimize $W = 6x_1 + 3x_2$

Sb. to

$2x_1 + 4x_2 \geq 16$ lb of nitrogen

$4x_1 + 3x_2 \geq 24$ lb of phosphate

$x_1, x_2 \geq 0$

where x_1 = bags of super-gro fertiliser, x_2 = bags of crop-quick fertiliser and W = farmer's total cost (\$) of purchasing fertilizer.

Defining dual variables:

Let y_1 and y_2 be the dual variables associated with the constraints

y_1 be for nitrogen constraint

y_2 be for phosphate constraint

Constructing the dual Problem:

Since the problem is minimization with $\geq$ constraints, the dual will be a maximization with $\leq$ constraints

- Objective function: RHS values constraints (16 and 24) become coefficients in the objective function of the dual

$$\text{Maximize } Z = 16y_1 + 24y_2$$

Constraints: Coefficients of x_1 and x_2 in the primal constraints become the coefficients in dual constraints

For x_1 : $2y_1 + 4y_2 \leq 6$

x_2 : $4y_1 + 3y_2 \leq 3$

Non-negativity : The dual variables $y_1, y_2 \geq 0$

Dual Problem (Maximization Form)

Objective Function : Maximize $Z = 16y_1 + 24y_2$

Subject to :

$2y_1 + 4y_2 \leq 6$	$2y_1 + 4y_2 + s_1 = 6$
$4y_1 + 3y_2 \leq 3$	$4y_1 + 3y_2 + s_2 = 3$
$y_1, y_2 \geq 0$	

Initial Simplex Tableau

Basic	y_1	y_2	S_1	S_2	RHS
S_1	2	4	1	0	6
S_2	4	3	0	1	3
Z	16	24	0	0	0

Finding Pivot column : y_2 Pivot row : minimum $\left[6/4, 3/3\right]$: S_2

1st Iteration

Basic	y_1	y_2	S_1	S_2	RHS
S_1	$-10/3$	0	1	$-4/3$	2
⇒ y_2	$4/3$	1	0	$1/3$	1
Z	-16	0	0 ~~-24~~	-8	-24

∴ $Z = 24$
$S_1 = 2$
$S_2 = 0$
$y_2 = 1$
$y_1 = 0$

Interpreting : Dual variable $y_1 = 0$ suggests that the nitrogen constraint has no effect on increasing costs at the optimal solution.
$y_2 = 1$ Indicates that each additional phosphate would increase the minimum cost by \$1

SECTION ONE

ii Defining decision variables

Minimize $W = 600x_1 + 500x_2$

subject to.

$2x_1 + x_2 \geq 80$

$x_1 + 2x_2 \geq 60$

$x_1, x_2 \geq 0$

Formulating the dual problem

Let y_1 be dual variable for the first constraint: $2x_1 + x_2 \geq 80$

y_2 be dual variable for the second constraint: $x_1 + 2x_2 \geq 60$

Dual Problem:

Maximize $Z = 80y_1 + 60y_2$

Subject to: $2y_1 + y_2 \leq 600$

$y_1 + 2y_2 \leq 500$

$y_1, y_2 \geq 0$

Standardizing the constraints: $2y_1 + y_2 + S_1 = 600$

$y_1 + 2y_2 + S_2 = 500$

$y_1, y_2, S_1, S_2 \geq 0$

Initial Simplex Tableau

Basic	y_1	y_2	S_1	S_2	RHS
S_1	2	1	1	0	600
S_2	1	2	0	1	500
Z	80	60	0	0	0

Finding Pivot column: Largest positive value of Z : y_1

Pivot Row: Minimum positive ratio $\left[\frac{600}{2}, \frac{500}{1}\right]$: S_1

B

First Iteration Tableau

Basic	y_1	y_2	s_1	s_2	RHS
⇒ y_1	1	$1/2$	$1/2$	0	300
s_2	0	$3/2$	$-1/2$	1	200
Z	0	20	-40	0	-24000

Finding Pivot colum : y_2

Pivot Row : minimum $\left[\frac{300}{1/2}, \frac{200}{3/2}\right]$: s_2

$600, \frac{400}{3}$

Second Iteration Tableau

Basic	y_1	y_2	s_1	s_2	RHS
y_1	1	0	$2/3$	$-1/3$	$700/3$
⇒ y_2	0	1	$-1/3$	$2/3$	$\frac{400}{3}$
Z	0	0	$-100/3$	$-40/3$	$\frac{-80000}{3}$

$$Z = \frac{80000}{3}$$

$$y_2 = 400/3$$

$$y_1 = 700/3$$

Section Two :

(i) Maximize $Z = 3x_1 + 5x_2 + 4x_3$

S.t $\quad 2x_1 + 3x_2 \leq 8$

$2x_2 + 5x_3 \leq 10$

$3x_1 + 2x_2 + 4x_3 \leq 15 \qquad$ for $x_1, x_2, x_3 \geq 0$

Adding slack variables :

$Z = 3x_1 + 5x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3$

$2x_1 + 3x_2 + S_1 = 8$

$2x_2 + 5x_3 + S_2 = 10$

$3x_1 + 2x_2 + 4x_3 + S_3 = 15$

First Simplex Tableau

Basic variable	x_1	x_2	x_3	S_1	S_2	S_3	RHS
S_1	2	3	0	1	0	0	8
S_2	0	2	5	0	1	0	10
S_3	3	2	4	0	0	1	15
Z	3	5	4	0	0	0	0

Finding : Pivot column : Maximum positive value of the Z row

∴ x_2 is the pivot column

Pivot Row : Minimum positive ratio : $\dfrac{\text{RHS}}{\text{Pivot column value}}$

: Minimum $\left[\frac{8}{3}, \frac{10}{2}, \frac{15}{2} \right]$

∴ Minimum positive ration is $\frac{8}{3}$

S_1 Row is the pivot Row

Basic Variable	x_1	x_2	x_3	S_1	S_2	S_3	RHS
x_2	$2/3$	1	0	$1/3$	0	0	$8/3$
S_2	$-4/3$	0	5	$-2/3$	1	0	$14/3$
S_3	$5/3$	0	4	$-2/3$	0	1	$29/3$
Z	$-1/3$	0	4	$-5/3$	0	0	$-40/3$

Entering value = $\frac{\text{Row value}}{\text{Pivot column value}}$

To get other columns = Old − pivot column at that section × Corresponding New row

Pivot column : x_3 is maximum value : 4

Pivot Row :

$$\text{Min}\left[\frac{8/3}{0},\ \frac{14/3}{5},\ \frac{29/3}{4},\ \cancel{\frac{-40/3}{4}}\right]$$

∞ , 0.933 , 2.416

Min pivot ratio : $\frac{14/3}{5}$: S_2 is Pivot Row

Basic Variable	x_1	x_2	x_3	S_1	S_2	S_3	RHS
x_2	$2/3$	1	0	$1/3$	0	0	$8/3$
x_3	$-4/15$	0	1	$-2/15$	$1/5$	0	$14/15$
S_3	$41/15$	0	0	$-2/15$	$-4/5$	1	$89/15$
Z	$11/15$	0	0	$-17/15$	$-4/5$	0	$-\frac{256}{15}$

Pivot column : x_1 maximum value $11/15$

Pivot row

$$\text{min}\left[\frac{8/3}{2/3},\ \frac{14/15}{-4/15},\ \frac{89}{15} \div 41/15\right]$$

S_3 is the pivot row

Basic variable	x_1	x_2	x_3	S_1	S_2	S_3	RHS
x_2	0	1	0	$15/41$	$9/41$	$-10/41$	$50/41$
x_3	0	0	1	$-6/41$	$5/41$	$4/41$	$62/41$
x_1	1	0	0	$-2/41$	$-12/41$	$15/41$	$89/41$
Z	0	0	0	$-45/41$	$-24/41$	$-11/41$	$-\frac{765}{41}$

∴ The values of the Basic variable are:

$x_1 = 89/41$ $x_2 = 50/41$ $x_3 = 62/41$

$$Z = \frac{765}{41}$$

Section Two:

(ii) Defining the decision variables

Let x_1 be the number of belt A produced per day

x_2 be the number of belt B produced per day

Defining the objective function

The objective is to maximise profit, the profit per belt are:

4 Rs for belt A

3 Rs for belt B

∴ The objective function $Z = 4x_1 + 3x_2$

Defining the constraints

Production Time constraint

$$2x_1 + x_2 \leq 1000$$

Leather Supply constraint

$$x_1 + x_2 \leq 800$$

Backle constraint for belt A : $x_1 \leq 400$
Backle constraint for belt B : $x_2 \leq 700$
Non-negative constraints : $x_1 \geq 0$, $x_2 \geq 0$

Formulating the LP model

Maximize $Z = 4x_1 + 3x_2$

Subject to

$$2x_1 + x_2 \leq 1000$$
$$x_1 + x_2 \leq 800$$
$$x_1 \leq 400$$
$$x_2 \leq 700$$
$$x_1, x_2 \geq 0$$

Solving using Simplex method : Let us add slack variables

$$Z = 4x_1 + 3x_2 + 0S_1 + 0S_2 + 0S_3 + 0S_4$$

Subject to :

$$2x_1 + x_2 + S_1 = 1000$$
$$x_1 + x_2 + S_2 = 800$$
$$x_1 + S_3 = 400$$
$$x_2 + S_4 = 700$$

Non-negativity : $x_1, x_2, S_1, S_2, S_3, S_4 \geq 0$

Simplex Tableau

Basic Variable	x_1	x_2	S_1	S_2	S_3	S_4	RHS
S_1	2	1	1	0	0	0	1000
S_2	1	1	0	1	0	0	800
⇒ S_3	1	0	0	0	1	0	400
S_4	0	1	0	0	0	1	700
Z	4	3	0	0	0	0	0

Finding : Pivot column : Maximum positive value of the z row
: x_1 is the pivot column.

Finding pivot row : Minimum positive ratio : $\frac{RHS}{\text{Pivot column values}}$

$$: \text{Minimum} \left[\frac{1000}{2}, \frac{800}{1}, \frac{400}{1}, \frac{700}{0} \right]$$

: Pivot row is S_3

1^{st} Iteration :

Basic variable	x_1	x_2	S_1	S_2	S_3	S_4	RHS
S_1	0	1	1	0	-2	0	200
S_2	0	1	0	1	-1	0	400
⇒ x_1	1	0	0	0	1	0	400
S_4	0	1	0	0	0	1	700
Z	0	3	0	0	-4	0	-1600

Entering Value = $\frac{\text{Row value}}{\text{Pivot colum value}}$

Other columns = Old - Pivot column x Corresponding new row

2^{nd} iteration : Pivot colum : x_2 Pivot row : min $\left[\frac{200}{1}, \frac{400}{1}, \frac{400}{0}, \frac{700}{1} \right]$

: S_1

Basic v	x_1	x_2	S_1	S_2	S_3	S_4	RHS
⇒ x_2	0	1	1	0	-2	0	200
S_2	0	0	-1	1	1	0	200
x_1	1	0	0	0	1	0	400
S_4	0	0	-1	0	2	1	500
Z	-3	0	-3	0	2	0	-2200

3^{rd} Iteration Pivot row : S_2

Basic	x_1	x_2	S_1	S_2	S_3	S_4	RHS
x_2	0	1	-1	2	0	0	600
⇒ S_3	0	0	-1	1	1	0	200
x_1	1	0	-1	2	0	0	
S_4							
Z							

3rd iteration

Basic	x_1	x_2	S_1	S_2	S_3	S_4	RHS
x_2	0	1	-1	2	0	0	600
⇒ S_3	0	0	-1	1	1	0	200
x_1	1	0	1	-1	0	0	200
S_4	0	0	1	-2	0	1	100
Z	-3	0	-1	-2	0	0	-2600

∴ Maximum profit $Z = 2600$
Number of Belt A to produce $(x_1) = 200$
Number of Belt B to produce $(x_2) = 600$

Section Two:

(iii) Defining decision variables:
Let x_1 be the number of bomb P produced
x_2 be the number of bomb Q produced
x_3 be the number of bomb R produced

Defining the objective function: Maximize the total explosion yield
∴ Maximize $Z = 2x_1 + 3x_2 + 4x_3$

Defining the constraints
Explosive A constraint : $3x_1 + x_2 + 4x_3 \le 600$
limited to maximum of 600 kg of explosive A

Explosive B constraint : $2x_1 + 4x_2 + 2x_3 \ge 480$
at least 480 kg of explosive B

Explosive C constraint : $2x_1 + 3x_2 + 3x_3 = 540$
exactly 540 kg of explosive C

Non-negativity constraints
$x_1, x_2, x_3 \ge 0$

File Edit Code View Plots Session Build Debug Profile Tools Help

Go to file/function Addins

Assignment 2.3.R

Source on Save

```r
# load lpSolve package

library(lpSolve)

# Objective function coefficients (explosion yield per bomb type)
objective <- c(2, 3, 4)  # Yield from bombs P, Q, and R

# Coefficients of the constraints
constraints <- matrix(c(3, 1, 4,      # Explosive A constraint
                        2, 4, 2,      # Explosive B constraint
                        2, 3, 3),     # Explosive C constraint
                      nrow = 3, byrow = TRUE)

# Right-hand side of the constraints
rhs <- c(600,      # Maximum kg of explosive A
         480,      # Minimum kg of explosive B
         540)      # Exactly kg of explosive C

# Directions of the constraints
direction <- c("<=", ">=", "=")  # Constraints are ≤, ≥, and =

# Solve the linear programming problem
solution <- lp("max", objective, constraints, direction, rhs)

# Display results
print(solution)

# Display the optimal values of x1, x2, and x3
cat("Optimal production schedule:\n")
cat("Bomb P (x1) =", solution$solution[1], "\n")
cat("Bomb Q (x2) =", solution$solution[2], "\n")
cat("Bomb R (x3) =", solution$solution[3], "\n")
cat("Maximum explosion yield =", solution$objval, "tons\n")
```

R 4.3.1 · D:/Strathmore DSA/Module 2/TSAF/TSAR/TSA/

```
> # Objective function coefficients (explosion yield per bomb type)
> objective <- c(2, 3, 4)  # Yield from bombs P, Q, and R
>
> # Coefficients of the constraints
> constraints <- matrix(c(3, 1, 4,     # Explosive A constraint
+                         2, 4, 2,     # Explosive B constraint
+                         2, 3, 3),    # Explosive C constraint
+                       nrow = 3, byrow = TRUE)
>
> # Right-hand side of the constraints
> rhs <- c(600,      # Maximum kg of explosive A
+          480,      # Minimum kg of explosive B
+          540)      # Exactly kg of explosive C
>
> # Directions of the constraints
> direction <- c("<=", ">=", "=")  # Constraints are ≤, ≥, and =
>
> # Solve the linear programming problem
> solution <- lp("max", objective, constraints, direction, rhs)
>
> # Display results
> print(solution)
Success: the objective function is 660
>
> # Display the optimal values of x1, x2, and x3
> cat("Optimal production schedule:\n")
Optimal production schedule:
> cat("Bomb P (x1) =", solution$solution[1], "\n")
Bomb P (x1) = 0
> cat("Bomb Q (x2) =", solution$solution[2], "\n")
Bomb Q (x2) = 60
> cat("Bomb R (x3) =", solution$solution[3], "\n")
Bomb R (x3) = 120
> cat("Maximum explosion yield =", solution$objval, "tons\n")
Maximum explosion yield = 660 tons
```

Formulating LP model

Maximize $z = 2x_1 + 3x_2 + 4x_3$

Subject to

$$3x_1 + x_2 + 4x_3 + S_1 = 600$$
$$-2x_1 - 4x_2 - 2x_3 + S_2 = -480$$
$$-2x_1 - 3x_2 - 3x_3 + S_3 = -540$$
$$2x_1 + 3x_2 + 3x_3 + S_4 = 540$$

where $x_1, x_2, x_3, x_4, S_1, S_2, S_3, S_4 \geq 0$

Initial Simplex Tableau

Basic var	x_1	x_2	x_3	S_1	S_2	S_3	S_4	RHS
S_1	3	1	4	1	0	0	0	600
S_2	-2	-4	-2	0	1	0	0	-480
S_3	-2	-3	-3	0	0	1	0	-540
S_4	2	3	3	0	0	0	1	540
Z	2	3	4	0	0	0	0	0

Finding pivot column : x_3

pivot row : Minimum positive $\begin{bmatrix} \frac{600}{4}, & \frac{-480}{-2}, & \frac{-540}{-3}, & \frac{540}{3} \\ 150, & 240, & 180, & 180 \end{bmatrix}$

: S_1

First Simplex Iteration

Basic var	x_1	x_2	x_3	S_1	S_2	S_3	S_4	RHS
⇒ x_3	3/4	1/4	1	1/4	0	0	0	150
S_2	-1/2	-7/2	0	1/2	1	0	0	-180
S_3	1/4	-9/4	0	3/4	0	1	0	-90
S_4	-1/4	9/4	0	-3/4	0	0	1	90
Z	-1	2	0	-1	0	0	0	-600

Pivot column : x_2

Row : S_4

Third Simplex Tableau

Basic variable	x_1	x_2	x_3	S_1	S_2	S_3	S_4	RHS
x_3	7/9	0	1	1/3	0	0	-1/9	140
S_2	-8/9	0	0	-1/6	1	0	14/9	-40
S_3	0	0	0	0	0	1	1	0
⇒ x_2	-1/9	1	0	-1/3	0	0	4/9	40
Z	-7/9	0	0	-1/3	0	0	-2/9	680

Maximum total explosion yield $Z = 680$
The number of bomb Q produced $= 40$
The number of bomb R produced $= 140$
The number of bomb P produced $= 0$

SECTION TWO :

(iv) Defining decision variables

Let x_1 be land acre for corn
x_2 " " " for wheat
x_3 soyabean

Defining objective function : Maximize $Z = 30x_1 + 40x_2 + 20x_3$

Defining the constraints

Acreage constraint : $x_1 + x_2 + x_3 \leq 1000$
Budget constraint : $100x_1 + 120x_2 + 70x_3 \leq 100000$
Labour constraint : $7x_1 + 10x_2 + 8x_3 \leq 8000$
Non-negativity : $x_1, x_2, x_3 \geq 0$

Using Simplex Method :
Putting in standard form

$$x_1 + x_2 + x_3 + S_1 = 1000$$
$$100x_1 + 120x_2 + 70x_3 + S_2 = 100000$$
$$7x_1 + 10x_2 + 8x_3 + S_3 = 8000$$

Initial Tableau

Basic variable	x_1	x_2	x_3	S_1	S_2	S_3	RHS
S_1	1	1	1	1	0	0	1000
S_2	100	120	70	0	1	0	100 000
S_3	7	10	8	0	0	1	8000
Z	30	40	20	0	0	0	0

Finding pivot column : Largest positive value of z : x_2

row : Minimum positive ration $\frac{\text{RHS}}{\text{Pivot column values}}$: S_3

$$: \left[\frac{1000}{1}, \frac{100{,}000}{120}, \frac{8000}{10}\right]$$

$$\left[1000, \frac{2500}{3}, 800\right]$$

1st Iteration :

Basis	x_1	x_2	x_3	S_1	S_2	S_3	RHS
S_1	$3/10$	0	$1/5$	1	0	$-1/10$	200
S_2	16	0	-26	0	1	-12	4000
⇒ x_2	$7/10$	1	$8/10$	0	0	$1/10$	800
Z	2	0	-12	0	0	-4	-32000

Pivot column : x_1 Pivot row : $\left[\frac{200}{3/10}, \frac{4000}{16}, \frac{800}{7/10}\right]$: S_2

$2000/3$ 666.66, 250, $\frac{8000}{7}$

2nd Iteration

Basis	x_1	x_2	x_3	S_1	S_2	S_3	RHS
S_1	0	0	$11/16$	1	$-3/160$	$1/8$	125
⇒ x_1	1	0	$-13/8$	0	$1/16$	$-3/4$	250
x_2	0	1	$31/16$	0	$-7/160$	$5/8$	625
Z	0	0	$-35/4$	0	$-1/8$	$-5/2$	-32500

The maximum z = 32500

The land acre for corn (x_1) = 250

The land acre for wheat (x_2) = 625

The land acre for soyabean (x_3) = 0