(i) Describe bisection method and use it to solve the following problem.

$$\max f(x) = x^3 + 4x^2 - 10$$

(ii) Describe Newton's method and use it to solve the following optimization problem.

$$f(x) = e^{2x} - x - 6$$

(iii) Clearly describe Gradient search procedure for a multivariate unconstrained maximization problem and give a solved example.

(iv) Describe Karush-Kuhn-Tucker (KKT) conditions for constrained optimization.

(v) Describe how to solve the KKT conditions.

(vi) Use KKT to solve the following

$$\begin{aligned}
\text{Max } f(x_1, x_2) \quad &= 15x_1 + 30x_2 + 4x_1 x_2 - 2x_1^2 - 4x_2^2 \\
\text{Subject to:} \qquad 2x_1 + 2x_2 \quad &\leq \quad 30 \\
x_1, x_2 \quad &\geq \quad 0
\end{aligned}$$

(vii) Consider the following linearly constrained convex programming problem.

$$\begin{aligned}
\text{Min } f(x_1, x_2) \quad &= x_1^2 - 6x_1 + x_2^3 - 3x_2 \\
\text{Subject to:} \qquad x_1 + x_2 \quad &\leq \quad 1 \\
x_1, x_2 \quad &\geq \quad 0
\end{aligned}$$

(a) Obtain the KKT conditions for this problem.

(b) Use the KKT conditions to check whether $(x_1, x_2) = \left(\frac{1}{2}, \frac{1}{2}\right)$

(c) Use KKT conditions to derive an optimal solution.

END