# A Method for Efficiency Measurement

## Python Implementation using pyDEA

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# What is Efficiency Measurement?

Refers to quantifying how well an entity (like a company, department, or individual) uses its resources to achieve its goals, aiming for maximum output with minimal input.
Data Envelopment Analysis (DEA) is a non-parametric mathematical technique used to measure the relative efficiency of a set of decision-making units (DMUs) by comparing their inputs and outputs.

- Data envelopment analysis (DEA) is a popular mathematical tool for analyzing the relative efficiency of homogenous decision-making units (DMUs).
- With multiple inputs and outputs, DEA can measure the relative efficiency of DMUs by using a ratio of the weighted sum of outputs to the weighted sum of inputs.

# Real World Applications of DEA

- Healthcare: Compare the efficiency of hospitals using inputs i.e. no. of doctors, nurses, and beds against outputs i.e. no. of patients treated or survival rates. Goal: Identify underperforming facilities and improve resource utilization.
- Banking: Evaluate branches based on inputs (staff size, operational costs) and outputs (number of accounts opened, loans processed). Goal: Optimize operations and close underperforming branches.
- Education: Compare schools using inputs like teachers, classroom space, and funding with outputs like test scores and graduation rates. Goal: Improve educational outcomes and funding efficiency.
- Energy Sector (Power Plants): Measure the efficiency of power plants by comparing fuel and labor inputs to energy output and environmental impact. Goal: Reduce waste and improve sustainability.
- Public Sector (Municipal Services): Assess performance of local governments or public services (e.g., waste management, policing). Goal: Ensure public funds are being used efficiently and equitably.

# Types of DEA Models

- CCR Model (Constant Returns to Scale): The CCR model, assumes constant returns to scale. This means that if inputs are increased proportionally, outputs will also increase proportionally. It is often used as a benchmark for efficiency evaluation.
- BCC Model (Variable Returns to Scale): The BCC model, allows for variable returns to scale. This means that if inputs are increased proportionally, outputs may increase, decrease, or remain the same.
- Orientation of DEA Models: Input-oriented-These models focus on minimizing the inputs required to produce a given level of outputs. Output-oriented- These models focus on maximizing the outputs produced from a given level of inputs. Non-oriented: These models do not have a specific orientation and are used when both input and output reductions are of interest.
- Other DEA Model Types: Two-stage DEA, Slack-Based Model (SBM),Network DEA, Super-efficiency DEA, Fuzzy DEA, Stochastic DEA.

# Basic DEA Model (CCR Input-Oriented)

- The CCR input-oriented model focuses on minimizing the amount of inputs used while maintaining at least the same level of outputs.
- This model assumes constant returns to scale, meaning doubling the inputs should double the outputs.
- The objective is to determine how much each DMU (Decision-Making Unit) can proportionally reduce its inputs without reducing output levels.
- The model includes constraints that ensure no other DMU performs better (i.e., achieves more output with less input).
- The result of the model is an efficiency score between 0 and 1. A score of 1.0 indicates full efficiency, while scores below 1 indicate potential for improvement.

# Graphical Illustration

- To visually understand DEA, we can plot DMUs on a graph with input on one axis and output on the other.
- The efficient frontier is formed by connecting the DMUs that are the most efficient—those producing the most output for the least input.
- DMUs that fall below the efficient frontier are considered inefficient because they use more input than necessary to produce a given level of output.
- This visual tool helps stakeholders easily identify which units are best performing and which are lagging behind.

# Introducing pyDEA

- pyDEA is an open-source Python package designed for conducting DEA.
- It supports common DEA models, including CCR and BCC, and both input- and output-oriented approaches.
- Data is provided in a straightforward CSV format, making it accessible to users with basic data science skills.
- After running the analysis, pyDEA returns a set of results including efficiency scores, reference sets (peer DMUs), and slack values, which help explain inefficiencies.

# Why use pyDEA?

- pyDEA is both powerful and user-friendly, making it a great choice for researchers and analysts.
- Being open-source, it can be freely used and modified for custom projects.
- It allows seamless integration with other Python tools, enabling automation, data cleaning, and reporting within the same pipeline.
- It is ideal for batch processing large datasets, making it suitable for simulation studies and real-time evaluation dashboards.

# Choosing Model Type in pyDEA

Before running DEA, you must choose the appropriate model configuration. Options include:

- CCR model (assumes constant returns to scale)
- BCC model (allows for variable returns to scale)
- Additionally, you must decide whether to take an:
- Input-oriented approach (minimize inputs for given outputs)
- Output-oriented approach (maximize outputs for given inputs)
- Also, consider the returns to scale assumption, which affects how DEA constructs the efficiency frontier.