



STRATHMORE INSTITUTE OF MATHEMATICAL SCIENCES
MASTER OF SCIENCE IN DATA SCIENCE & ANALYTICS
CAT 1- Open Book
DSA 8505: Bayesian Statistics

DATE: 23rd Jan 2026

Instruction

- (a) Answer All Question
 - (b) Scan and submit your answer sheet through the Google Classroom by 23h59, 23rd Jan 2026.
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1. A researcher is studying the probability θ that a patient responds to a new treatment. Before collecting data, the researcher specifies the prior distribution:

$$\theta \sim \text{Beta}(\alpha, \beta), \quad \alpha = 3, \beta = 7.$$

A clinical trial is conducted on $n = 20$ patients and $k = 14$ respond successfully.

Required:

- a. Write the likelihood function $p(D | \theta)$ under the Binomial model.
- b. Using Bayes' theorem, derive the posterior distribution $p(\theta | D)$ showing all steps.
- c. State the posterior distribution clearly in the form:

$$\theta | D \sim \text{Beta}(\alpha, \beta).$$

2. In a hospital, the number of emergency cases arriving per night is assumed:

$$Y | \lambda \sim \text{Poisson}(\lambda).$$

Assume a Gamma prior:

$$\lambda \sim \Gamma(\alpha, \beta), \quad \alpha = 4, \beta = 2.$$

In one night, $y = 7$ emergency cases are observed.

Required:

- a. Derive the posterior distribution for λ .
 - b. Find the posterior mean $E(\lambda \mid y)$.
3. Suppose a single observation $x = 42$ is recorded from the model:

$$x \mid \theta \sim \mathcal{N}(\theta, \sigma^2), \quad \sigma^2 = 9.$$

Assume the prior distribution:

$$\theta \sim \text{Uniform}(0, 100).$$

Required:

- a. Write Bayes' theorem for $p(\theta \mid x)$.
- b. Show that the posterior has a Normal kernel.
- c. State the posterior distribution form and explain why it is a **truncated Normal distribution**.