

Documentation

Bellman-Ford Algorithm

The algorithm keeps two mappings:

- $dist[x]$ = the cost of the minimum cost walk from s to x known so far
- $prev[x]$ = the vertex just before x on the walk above.

Initially, $dist[s]=0$ and $dist[x]=\infty$ for $x \neq s$; this reflects the fact that we only know a zero-length walk from s to itself.

Then, we repeatedly performs a *relaxation* operation defined as follows: if (x,y) is an edge such that $dist[y] > dist[x] + c(x,y)$, then we set:

- $dist[y] = dist[x] + c(x,y)$
- $prev[y] = x$

Algorithm:

Initialization

```
# step 1
distance = [inf for _ in range(self._no_of_vertices)]
distance[source] = 0
predecessor = [False for _ in range(self._no_of_vertices)]
```

Relaxation

```
# step 2
for _ in range(self._no_of_vertices - 1):
    flag = False
    for item in self._cost_dict.items():
        if distance[item[0][1]] > distance[item[0][0]] + item[1]:
            distance[item[0][1]] = distance[item[0][0]] + item[1]
            predecessor[item[0][1]] = item[0][0]
            flag = True
    # if there are no modifications in the distance then the minimum distance was found
    if not flag:
        break
```

Search for negative cycles

```
# step 3
for item in self._cost_dict.items():
    if distance[item[0][1]] > distance[item[0][0]] + item[1]:
        raise ValueError("Graph contains a negative cost cycle!\n")
```

Path construction

```
# construct the path
print("A lowest cost walk from " + str(source) + " to " + str(target) + " is " +
      str(distance[target]))
path = ""
current_vertex = target
while current_vertex != source:
    path += str(current_vertex) + " <- "
    current_vertex = predecessor[current_vertex]
path += str(source)
print(path)
```