How Incentive Mechanisms Reduce the Price of Anarchy in Networks

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Abstract

Inefficiencies caused by selfish decision-making are a problem in many different forms of networks. This paper examines incentive-based strategies as tools to make the behavior of individual agents align with what makes the system as a whole more efficient. Theoretical approaches to this often involve modeling the system with optimization techniques to develop schemes that decrease the inefficiency caused by selfish routing. Experimental work attempts to test these techniques out and develop new ones by studying how they interact with real human agents, rather than perfectly rational ones. By comparing theory, experimental work, and machine learning, the literature suggests that hybrid approaches using both tolls and subsidies, commonly referred to as "carrot-and-stick" strategies, often outperform methods solely using one or the other. More broadly, this research highlights how these schemes can address real-world problems, like traffic management or data flow.

1. Introduction

Selfish routing is a common issue in network systems, occurring whenever individuals make routing decisions to minimize their own costs rather than considering the system's overall efficiency. These routing choices can be viewed from the perspective of game theory as a Nash equilibrium, where no single user can improve their outcome by changing their route. But, while such equilibria are stable, they aren't always efficient for the system as a whole. This decentralized behavior can often result in significant inefficiencies, especially in large-scale networks such as road systems or data communication networks. These inefficiencies are measured by the Price of Anarchy (PoA), which measures how much worse the outcome of self-interested behavior is when compared to a centrally coordinated system; a higher PoA represents a greater difference between selfish and optimal outcomes.

Researchers have approached the study of selfish routing and how to decrease it from various angles. Theoretical works use optimization and game theory in order to decrease the inefficiencies caused by selfish routing [Roughgarden and Tardos [2002]]. Experimental studies, on the other hand, test these concepts in environments to observe how humans respond to them, often revealing changes in results from completely rational agents [Hartman [2012]]. Additionally, more recent studies using machine learning have also produced models which relate to both theoretical and experimental findings [Güemes-Palau et al. [2025]].

These problems are not purely theoretical, however, and are also visible in real-world systems such as the previously mentioned traffic systems, where drivers independently choosing routes often results in congestion. Instead, approaches that use incentives can achieve much more efficient outcomes. One example of such an organization is Singapore's Electric Road Pricing system, described in Chicago Metropolitan Agency for Planning [2021], which dynamically adjusts toll prices based on traffic levels. This nudges individual drivers into behavior that benefits the system as a whole.

This paper examines how these interventions, typically categorized as carrot-and-stick strategies (i.e., rewards and penalties), can be employed to reduce the PoA in network routing settings. Additionally, rather than relying solely on the use of tolls or subsidies, this paper also explores the potential of a mixed strategy that combines both tolls and subsidies. By analyzing how different incentive schemes affect outcomes, both in theoretical models and practical applications, this paper seeks to answer the question: How can the PoA be minimized in network routing through carrot/stick strategies?

2. Practical Robustness

This section examines two approaches to studying the robustness of Toll and Subsidy Schemes (TSS). Some researchers use mathematical modeling to explore how these systems ideally behave. This approach yields general insights into the potential outcome of implementing the modeled system. However, other researchers utilize experimental evaluations, either in simulations or in real-world studies, to test how systems perform when agent behavior is considered. Together, these perspectives account for the majority of work in this area, offering complementary views on the strengths and limitations of different strategies.

2.1. Theoretical Perspective

Much of the research in this field involves theoretical analysis of how various systems of tolls and subsidies would perform on a network. Examples of this theoretical analysis involve the use of linear programming, graph theory, and sometimes even machine learning. These methods generally provide evidence from mathematical models to prove or explain an idea, such as defining an upper bound for the PoA caused by selfish routing as in Roughgarden and Tardos [2002] or generating a toll system to decrease this PoA as in Fleischer et al. [2004]. Mathematical optimization (discussed in more detail in Section 3.2) is prevalent in this area of research. Roughgarden used convex programs to determine the maximum possible latency of a system caused by selfish routing. Similarly, Fleischer used linear programming to generate a system of tolls that attempted to minimize the total cost of selfish routing on a system.

One trade-off across these studies is between mathematical depth and real-world applicability. While very simple models can make guarantees like tight bounds on the effects of toll systems, they often overlook dynamic conditions and behavioral factors.

2.2. Experimental/Practical Perspective

In contrast to the theoretical form of analysis, experimental work typically focuses on using toll and subsidy schemes in real-life or simulated environments to assess their performance. Instead of mathematical theory – such as upper bounds on the PoA or mathematical proofs that a certain system will lower PoA – some researchers use simulations to observe how real agents actually respond to them, while others choose to use experimental studies, and some even implement both. However, both the theoretical and experimental approaches are mathematical in nature and have a strong quantitative footing. For example, in Tian et al. [2022], participants in the simulation had to choose which time to leave an origin each day to arrive at a destination with the lowest cost; the original cost is determined by the travel time (which was determined by the congestion of the route at the time), and if they arrived early or late, the cost increased. This process was iterated 40 times, resulting in data to determine how carrot and stick penalties affected the overall cost of the system.

This figure presents what the participants actually saw during the experiment. Each round gave them information about their previous travel time and costs, as well as the

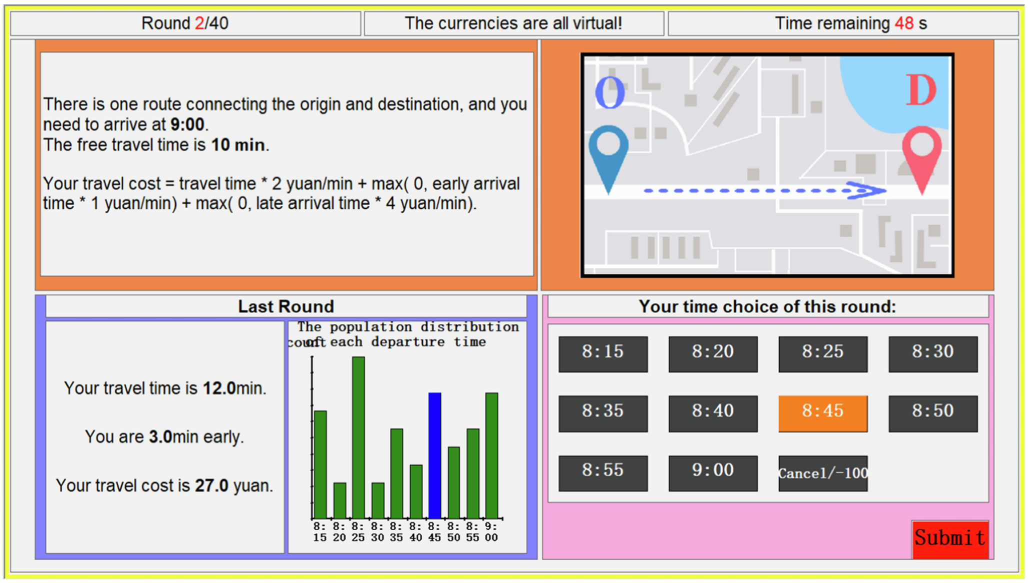

Figure 1: Simulation of traffic routing from Tian et al. [2022].

congestion at the various times people left. Then, they were able to choose when to leave the next round, making decisions possibly based on data from earlier rounds. Over repeated rounds, these choices revealed patterns that the researchers could use to analyze the effects of incentive structures.

3. Methods

3.1. Modeling Experiment Design

Before running experiments in this domain, researchers must translate theoretical network models into simplified models which they can feasibly engage with without too high of a computational cost. All three of the techniques presented in this section: linear programming, experimental economics, and machine learning, attempt to form solutions to the underlying problem under these simplified models. Additionally, they often work together, where experimental methods often rely on models developed in theoretical work while machine learning often uses data from both experimental and theoretical research. Understanding the construction of these experimental setups is key in contextualizing the results and whether they align with what theory suggests.

3.2. Linear Programming And Algorithmic Models

A linear program (LP) is an optimization in the form of:

$$\begin{aligned}
\min \quad & c^\top x && (1)\\
\text{s.t.} \quad & Ax = b, && (2)\\
& x \geq 0, && (3)
\end{aligned}$$

Where x is the decision vector, c is the cost vector, and A, b encode the linear constraints Stein [2009]. LPs and their solutions are key to the analysis of selfish routing in network systems; these models typically assume perfectly rational agents who all attempt to minimize their cost. A Nash equilibrium is a strategy profile where no player can reduce their cost by deviating, as described in Sethi and Weibull [2016]

One work in this space is Roughgarden and Tardos [2002], which quantifies how much selfish routing degrades the overall efficiency of a network; they first defined networks G to be composed of $(V,\ E)$ with vertex set V and edge set E. They then model routing as a convex program, defining the cost $C(f)$ of a flow f in G.

$$C(f) = \sum_{e \in E} \ell_e(f_e)\, f_e \tag{4}$$

where $\ell_e(x)$ is the latency on edge e and f_e is the total flow on edge e. Here, latency is a function which gives the travel time/delay experienced on edge e when the amount of traffic on the edge is x.

A Nash flow (selfish routing) is when drivers chose the route with lowest latency for themselves. It satisfies

$$\ell_{P_1}(f) \leq \ell_{P_2}(f) \quad \text{for all used paths } P_1, P_2. \tag{5}$$

This paper then defines marginal-cost latencies on an edge e as

$$\ell_e^*(x) = \ell_e(x) + x\, \ell_e'(x) \tag{6}$$

which is the effective latency, accounting for both the agent's travel time and the extra delay its presence causes for others. Then, an optimal (minimum latency) flow can be defined as a Nash equilibrium with respect to the marginal-cost latencies ℓ^*.

For linear latency functions, $\ell_e(x) = a_e x + b_e$, the marginal-latency function $\ell_e^*(x)$ simplifies to $2a_e x + b_e$. In this case, Roughgarden and Tardos prove the bound

$$\frac{C(f)}{C(f^*)} \leq \frac{4}{3}. \tag{7}$$

Both Christodoulou et al. [2014] and Fleischer et al. [2004] study selfish routing with linear latency functions $\ell_e(x) = a_e x + b_e$, however they cover different concepts.

Like Roughgarden, Christodoulou et al. [2014] proves the worst-case PoA is $\frac{4}{3}$. To improve this, however, they introduced coordination mechanisms which modified edge latencies. In the case of Pigou's network, which consists of two parallel links, their optimal mechanism enforces:

$$\hat{\ell}_1(x) = \begin{cases} \ell_1(x) & \text{if } x \le x_1, \\ \ell_1(x_2) & \text{if } x_1 < x \le x_2, \\ \ell_1(x) & \text{if } x > x_2, \end{cases} \tag{8}$$

Where $\hat{\ell}_1(x)$ is the latency of an edge and x_1, x_2 are thresholds, which act in a way where when flow exceeds x_1, the latency artificially spikes to avoid overuse, causing selfish users to distribute traffic in a manner that is closer to the system-optimal flow. This tweak increases efficiency, lowering the PoA from $1.\overline{333}$ to 1.191.

Fleischer et al. [2004] examines heterogeneous users, meaning different agents value time and money differently. This is measured by a sensitivity parameter α_i; A high α_i means a user is very averse to congestion, and a low α_i means a user is less sensitive. Their key result states that a congestion (description of the traffic on each edge of a network) $\mathbf{g}$ is enforceable by tolls, which can most directly be modeled through this dual problem:

$$\text{maximize} \quad \sum_i d_i z_i - \sum_e g_e \tau_e \tag{9}$$

$$\text{subject to} \quad z_i \le \alpha_i \ell_p(g) + \sum_{e \in p} \tau_e \quad \forall i, \forall p \in \mathcal{P}_i, \tag{10}$$

$$\tau_e \ge 0 \quad \forall e \in E. \tag{11}$$

Here, d_i is the demand of user i, P_i is the set of all available paths for the user, and $\ell_p(g)$ is the latency of path p under congestion $\mathbf{g}$. The dual variables are z_i and τ_e, where z_i is the effective cost faced by user i and τ_e is the toll on edge e. The constraint states that z_i can't exceed the cost of any path $p \in P_i$, where cost is comprised of the congestion-caused latency multiplied by the sensitivity parameter a_i, plus the toll on the path.

The tolls τ_e are optimal solutions to the dual problem, which enforce the congestion system $\mathbf{g}$. They effectively cause users to follow the intended pattern of congestion. This holds even when a_i differs. Additionally, this program has a corresponding primal program, which effectively minimizes the total latency across all users. Then, by solving an LP pair formed by the primal and dual equations, it is proven that a congestion $\mathbf{g}$ is truly enforceable through tolls.

Like Christodoulou et al. [2014] and Fleischer et al. [2004], Xiao and Zhang [2014] proposed an incentive scheme; However, this scheme uniquely combined the use of tolls and subsidies. First, they proved that via linear equations, a beneficial (pareto-improving, system-optimal, and revenue-neutral) TSS can always be found for a one-origin or one-destination network. Here, system-optimal refers to a traffic flow arrangement that minimizes the total cost across the network, not just for one user. They then determined that for multi-origin networks, the TSS might require external subsidies, as it may not always be self-sustainable depending on the network's structure.

For one-origin networks, the TSS makes sure the overall system benefits by solving the equation:

$$\tilde{t}ij + \rho ij + \pi_i - \pi_j = 0 \quad \forall \tilde{x}, i, j > 0, \tag{12}$$

Where π_i are node potentials and p_{ij} are toll and subsidy rates.

For multi-origin networks, the problem becomes a balanced transportation problem, which is solvable through LP. Pareto improvement, which occurs when a change in a system is beneficial or neutral for all users, is found by minimizing:

$$\ell_{P_1}(f) \leq \ell_{P_2}(f) \quad \text{for all used paths } P_1, P_2 \tag{13}$$

In their numerical examples, Xiao and Zhang emphasize the scheme's effectiveness in lowering travel disutilities across all Origin-Destination pairs. However, they do note limitations in multi-origin networks: if the network structure prevents the toll and subsidy flows from balancing, the scheme ends up with negative revenue, resulting in external subsidies being necessary.

3.2.1 Pigou-Style Worked Example

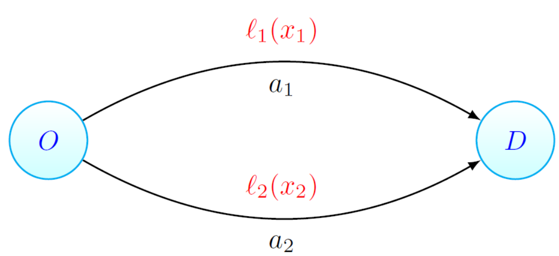

Figure 2: Visual of Pigou's network from Guzmán and Kleer [2016]

We illustrate the mechanics of a revenue-neutral TSS on Pigou´s network, shown in Fig. 2. The network has one origin-destination pair connected by two parallel edges: Edge one has the latency $\ell_1(x) = x$, where the more it's being used, the slower it becomes. Edge two has latency $\ell_2(x) = 1$, so no matter the level of usage, there is a consistent level of latency.

3.3. Experimental Economics

In contrast to theoretical approaches like linear programming, experimental methods observe how real humans behave when subject to various incentive schemes. In these types of experiments, participants typically play simplified congestion games in a simulation.

For example, Tian et al. [2022] studies various ways that the human aspect of this form of research causes results to differ from what would be expected solely from theory. Firstly, they discovered that due to the phenomenon of loss aversion, penalties are more impactful at first. But, as time passes, the effect of loss aversion weakens. Additionally, a traditional penalty-based congestion pricing system was contrasted with a new Incentive-Based Traffic Demand Management (IBTDM) model. Another result of the study was that while a theoretically optimal penalty scheme performs better in alleviating congestion, a reward scheme that considers psychological factors can reduce the system's total travel time in the same way.

Similarly, Hartman [2012] examined how participants reacted to a toll system that minimizes PoA. He conducted a series of experiments where 18 subjects repeatedly chose between a congested "bridge" and an uncongested "highway," first without tolls, and then with a toll that attempted to induce the system-optimal outcome. The results showed that, as expected, inefficient congestion occurred in the no-toll setting, while the use of tolls resulted in a solution closer to the theoretical prediction.

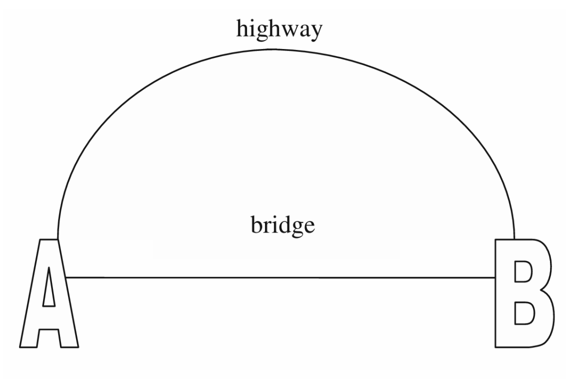

Figure 3: A visual of the scenario that subjects see for their travel situation from Hartman [2012].

3.4. Machine Learning

Recent advances in machine learning have made it possible to generate models for these situations beyond traditional hand-coded models. These new models can often make predictions faster than older approaches, especially in networks that are large-scale or changing in shape.

One example of a use of machine learning in this field is M4, a learned flow-level network simulator developed by Li et al. [2025]. M4 is trained on ground-truth data, which is verified real-world data used to train models like M4. This training allows M4 to simulate how network traffic behaves, including measuring metrics like the network's overall latency and throughput. The authors also show that it aligns with the results given by packet-level simulators like ns-3 while achieving a significant speedup. As depicted in Fig. 4, m4 converts a snapshot of a network in time into a bipartite graph and then uses a graph neural network(GNN) to predict the changes over time.

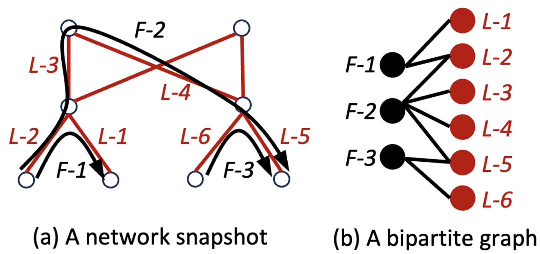

Figure 4: Basic process of m4.

Another approach is RouteNet-Gauss (Güemes-Palau et al. [2025]), which also implements a GNN to model network performance. Unlike previous models which relied on synthetic data, RouteNet-Gauss is trained with a dataset from real hardware, which allows it to adapt better to real-world network systems. This model has been shown to be almost 500x as fast as traditional discrete-event simulators while being able to generalize to networks much larger than those used during training, all while keeping it's error rate at less than 3

Both Li et al. [2025] and Güemes-Palau et al. [2025] use GNNs, which are machine learning models made for graph-structured data, to predict network performance. As shown in Fig. 5, a GNN updates nodes and edges over time through the use of multiple GNN blocks. The transformed graph is then used for predictions, allowing models like m4 and RouteNet-Gauss to accurately predict network performance.

A third tool called MimicNet(Zhang et al. [2021]) focuses on performance prediction in data center networks. Instead of simulating every packet across an entire network, MimicNet

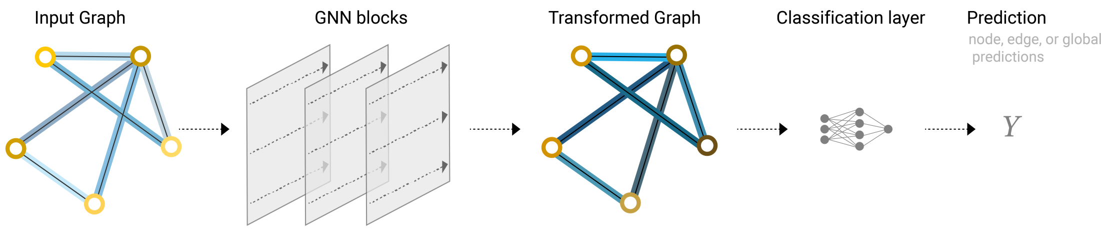

Figure 5: An overview of a GNN showing its general process from Sanchez-Lengeling et al. [2021].

trains on small-scale simulations, and uses the results of those simulations to build simplified models of unobserved clusters in the network. Then, by creating a system combining one real cluster with several simplified models, the system is able to achieve speedups of up to 950x traditional simulators.

Together, these models show how machine learning offers high levels of speed and accuracy in simulating complex networks. While traditional simulators are important for detailed analysis, these learned models are key for rapid testing and optimization when attempting to develop incentive schemes.

4. Models And Metrics

In many cases, the Price of Anarchy is measured as a ratio of the total latency of the system caused by selfish routing to the total latency of the system with optimal routing, as defined in Tian et al. [2022]. This is typically used in networks, like the one shown in Fig. 6.

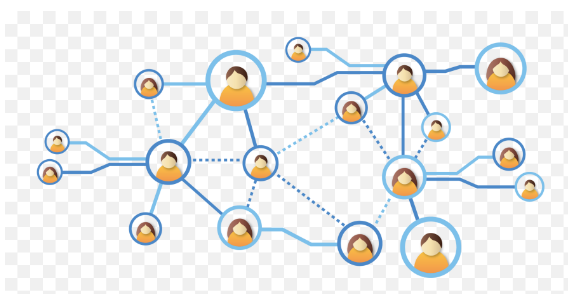

Figure 6: Network Diagram.

Other papers use different metrics to measure the overall cost. For example, in Fleischer et al. [2004], for an agent of commodity i, they use Eq. (14) to measure the cost. Many papers have similar equations for the cost in this form; however, they vary slightly between papers.

$$\text{Cost} = \alpha_i \cdot l_p(f) + \tau_p \tag{14}$$

- α_i: sensitivity of that commodity's agents to the latency
- $l_p(f)$: total latency on path p for flow f
- τ_p: total toll on path p

These various formulations of the cost equation reveal how researchers adapt it to capture something specific with their model. Some focus solely on latency, while others include tolls, subsidies, and behavioral parameters to better reflect user decision-making. This flexibility allows for studies to look into different forms of inefficiency: minimizing total travel time, ensuring revenue neutrality, or balancing fairness so no individual agent is detrimented. While the specific equation may vary, the common goal across these papers is to evaluate and compare incentive schemes.

5. Pure Vs. Mixture Strategies

Many papers have compared the use of taxes and subsidies, while others have also considered the use of both together. Additionally, because of the difference in methods that were used to analyze and compare the incentive systems, researchers have made different conclusions.

5.1. Taxes Vs Subsidies

When comparing the use of taxes with the use of subsidies, the results are mixed. In one study based on graph theory (Ferguson et al. [2021]), researchers determined that in situations where users are completely selfish, smaller subsidies are actually more effective than tolls. However, with unknown user heterogeneity, tolls are generally more effective.

However, when examined from the perspective of experimental economics in Tian et al. [2022], the concepts of loss and risk aversion have a significant impact. Although penalties and rewards can have positive influences, the results show that penalty-based strategies are more impactful early on, as people tend to avoid the losses inflicted by penalties. However, over time, the penalties and subsidies grew to have roughly the same effect.

5.2. Taxes And Subsidies Together

In Xiao and Zhang [2014], they used linear programming to study Toll and Subsidy Systems(TSS), where tolls and subsidies are used together in a network. They determined that TSSs have several advantages over traditional tolling schemes, including a lower travel disutility than traditional schemes on average, alongside other social benefits.

Additionally, Bagloee and Sarvi [2017] develops an algorithm for making a TSS with any network. The approach first transforms the problem into a Constrained Traffic Assignment Problem(CTAP), where tolls and subsidies are then interpreted as Lagrangian multipliers. By continuously updating the delay function with these values, the algorithm guides traffic flow, ensuring efficiency throughout the network.

6. Conclusion

This paper has examined the impact of TSS schemes in reducing inefficiencies caused by selfish routing in networks. From theoretical work, experimental work, and machine learning, researchers have developed complementary perspectives on how to design these schemes. The consistent takeaway with these methods is that hybrid approaches(combining tolls and subsidies) have the potential to outperform approaches solely using one or the other. These results are not abstract; however, they can be applied directly to real-world issues like traffic congestion or data bottlenecks in communication networks.

So, the next step in this field is implementation. While researchers have shown that these strategies can work successfully in models and experiments, it's also imperative to see these methods tested in real life. Policymakers should attempt to test hybrid TSS schemes in real cities rather than simple practices like toll booths. Similarly, network engineers should consider implementing factors such as loss aversion into their algorithms, as these have been shown to impact outcomes in practice.

Additionally, several open questions still remain. Some subsidy-based approaches have only been tested on small-scale networks, so it's unknown how well they will scale to larger, more complex systems. Experimental economics also provides evidence that loss aversion and other psychological factors are relevant; however, translating this into optimization is still a challenge.

Addressing these questions requires collaboration between economists, computer scientists, and policymakers - each of these members has work to do in order to finally result in more efficient networks. Ultimately, the challenge now isn't proving that these systems can work, but implementing them properly. Hybrid TSS schemes offer a path towards more efficient systems in the future, resulting in transportation and communication systems that are faster and more sustainable.

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