The Mandelbrot Set
Hybrid MPI/OpenMP Implementation

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Introduction

The goal is to implement and analyze a **hybrid MPI/OpenMP** implementation of the computation of the Mandelbrot set, which is defined as:

$$\mathcal{M} = \{c \in \mathbb{C} : \lim_{n \to \infty} z_n < \infty\}$$

where $z_{n+1} = z_n^2 + c$ and $z_0 = 0$.

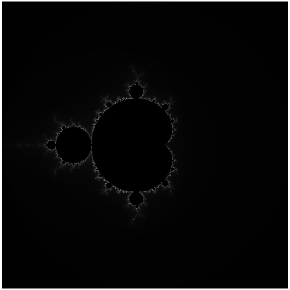

Figure: Rendering of the Mandelbrot set.

Encoding:

- each pixel represents a complex number c.
- the color of the pixel depends on the number of iterations before z_n diverges.

Computational architecture

ORFEO cluster
EPYC paritition

- 8 nodes
- 2x AMD EPYC 7742 64-Core Processor on each node
- 512 GB RAM

For our purposes we will use at most 2 nodes of the EPYC partition.

Parallelization strategy

We adopt a sequential fashion:

1. using MPI, initialize P processes
2. each process computes a portion of the image using T OpenMP threads (loop scheduling policy is set to `dynamic`)
3. the master process uses `MPI_Gatherv` to collect the results

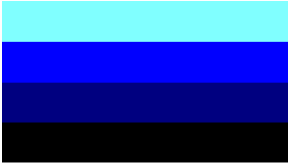

Figure: Suppose to have 4 processes. Each process will compute a portion of the image.

Let $N = n_x \times n_y$ be the total number of pixels. Each process will compute approximately N/P pixels.

Experimental setup

- **MPI strong scaling**: we fix $T = 1$,
$$n_x = n_y = 4096$$
and $P = 1, 2, 4, 8, 16, 32, 64, 80, 96, 112, 128$.
- **MPI weak scaling**: we fix $T = 1$,
$$n_x = n_y = 1024 \times \texttt{round}\{\sqrt{P}\}$$
for $P = 1, 2, 4, 8, 16, 32, 64, 80, 96, 112, 128$;
- **OpenMP strong scaling**: we fix $P = 1$,
$$n_x = n_y = 4096$$
for $T = 2, 4, 6, 8, ...62, 64$;
- **OpenMP weak scaling**: we fix $P = 1$
$$n_x = n_y = 1024 \times \texttt{round}\{\sqrt{T}\}$$
for $T = 2, 4, 6, 8, ...62, 64$.

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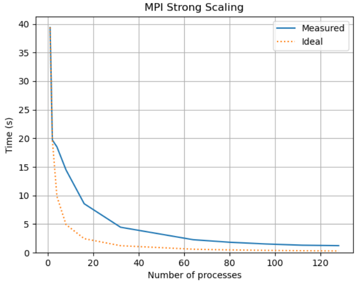
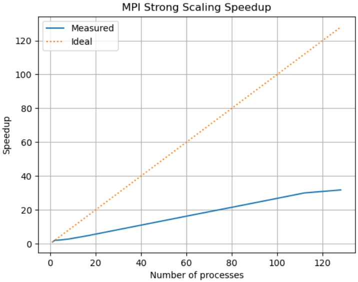
Other parameters

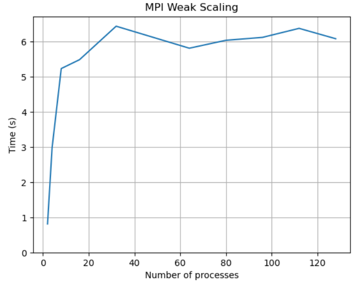
- For MPI, we set `--map-by core`
- For OpenMP, we set `OMP_PLACES=cores`

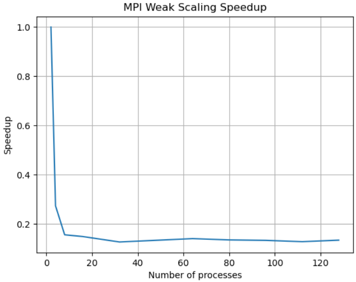
MPI strong scaling results



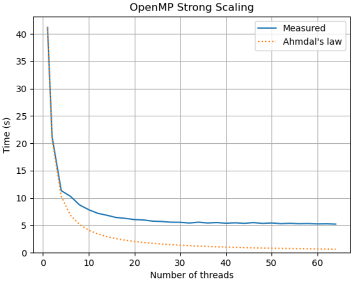


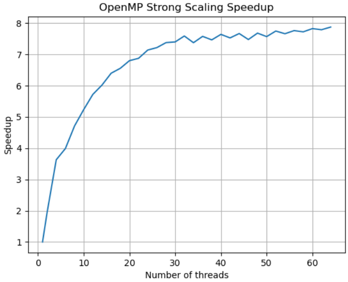
MPI weak scaling results



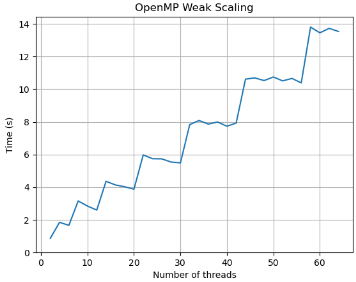


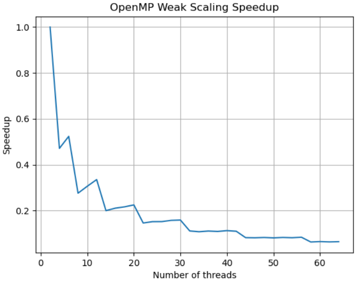
OpenMP strong scaling results





OpenMP weak scaling results





Conclusions

- MPI strong scaling: the speedup is almost linear up to 64 processes.
- MPI weak scaling: the speedup is almost linear up to 64 processes.
- OpenMP strong scaling: the speedup is almost linear up to 32 threads.
- OpenMP weak scaling: the speedup is almost linear up to 32 threads.