Analyzing the network structure of public procurement in Germany: Implications for corruption risk using single bidder rates

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1. Introduction

Background and motivation

Public procurement contracts are an essential part of the public sector. They are a critical instrument for governments to acquire goods and services, and are essential for ensuring that public services are delivered effectively. However, these contracts can be susceptible to corruption, fraud, and other forms of misconduct, which can result in significant financial losses for the government and undermine public trust.

In recent years, there has been a growing interest in understanding and preventing corruption in public procurement (see Wachs et al., 2021). The availability of administrative datasets from European Union member states has enabled researchers to analyze large volumes of data and identify patterns and trends related to corruption risk.

This paper focuses on the use of a competitive measure called single bidding rate as a proxy of corruption risk. This measure is quantified as the rate at which a contract attracted only a single bidder. While this measure is used as an effective proxy for corruption, it is essential to note that the presence of single bidding does not necessarily indicate corruption, as there may be valid reasons why only one bidder participated in a tender.

The results of this paper have many implications. For instance, under the competition lens, new approaches using the network data can be developed and utilized for detecting signals of bid-rigging activities to aid the enforcement arm of antitrust authorities.

Research objectives

This paper builds on the study of Wachs et al. (2021), which analyzed the relationship between the degree of centralization of a market and its corruption risk, and investigated whether centralization induces corruption. The goal of this paper is to primarily visualize and describe the network of public procurement contracts in Germany from 2008 to 2016. Specifically, it focuses on understanding the structure of the German market in relation to corruption risk across years.

To achieve these objectives, data science tools are applied to conduct network-based approaches such as R-A clustering, degree distributions, k-core decomposition, modularity maximization, and measurement of nodal centrality.

In the following sections, a detailed description of the data, methodology, and analysis of results, are provided. A discussion of the policy implications of the findings as well as recommendations for future work is also included to highlight the need for further research to combat corruption and foster market competition in public procurement.

2. Data and framework

2.1. Data description and preparation

EU public procurement contracts

For this paper, a bipartite network of public procurement contracts awarded by the European Union from 2008 to 2016 was utilized. The data was collected from Tenders Electronic Daily (TED) and is available on Netzschleuder[1]. The dataset has been further processed by Wachs et al. (2021) to include country estimates of corruption risk. The current analysis used the same two datasets: contract-level tidy data (*contracts*) and aggregated country-year corruption indicators data (*aggregated_country_year*). Both datasets along with their data dictionary are available at https://zenodo.org/record/3537986#.Xis4mC2ZNGV.

The *contracts* dataset comprises information on 4,098,711 contracts awarded from 2008 to 2016 across 26 EU member states. It includes issuer-winner pairs, where the issuers or buyers are public institutions like ministries or city halls, while the winners or suppliers are firms providing goods and services. Each row represents a contract identified by a unique *tender_id*. The dataset also provides the location and country of origin of both issuers and winners.

Other useful information contained in the dataset includes the estimated value of the contract (*est_value*), the type of contract (*contract_type*), whether or not the contract was supported by EU funding (*contract_eu_funded*), and the type of procedure used to award the contract (*procedure*). Additional variables include the number of bids received (*n_bids*), whether or not there was a call for tenders found (*nocft*), and a single-bidder measure (*single_bidder*). This paper focuses on the use of the *single_bidder* variable which is the single bidder rate (*sbr*).

On the other hand, the *aggregated_country_year* dataset consists of survery-based corruption risk measures including the Transparency International's Corruption Perception Index (TI CPI), World Bank's Control of Corruption Index (WB CoC), Varieties of Democracy's Corruption Index (V-Dem Corruption Index), and the Quality of Government (QoG) Institute's European Quality of Governance Index (European Qual. Gov. Index), which can be used for further analysis later using the full dataset of countries and years.

Creating the graph object with NetworkX

The python library `networkx` was used to create a graph object representing the bipartite network of public procurement contracts awarded by the European Union from 2008 to 2016. The network consists of two sets of nodes, issuers and winners, with edges connecting contracts to the firms that won them.

[1] Netzschleuder: the network catalogue, repository and centrifuge (skewed.de)

The bipartite graph was created using the `from_pandas_edgelist` function of the `networkx` library, which simplifies the process of creating a bipartite graph as it allows the user to directly load the edge list into the graph object without needing to iterate through the dataframe. Edges are annotated with single bidder rate and are weighted by the number of contracts between the issuers and winners. Each set of nodes are annotated with location information, including country of origin using the `set_node_attributes` function.

Github repository

The GitHub repository used for this paper can be found at https://github.com/adellegia/network-analysis-EU-procurements. This repository contains the code written in Python by the author and uses various packages, including *networkx*, *pandas*, *numpy*, and *matplotlib*, for network analysis, statistical calculations, and data visualization. The repository includes scripts for data cleaning, network construction, and analysis, making it possible for others to replicate and extend the study. Additionally, the repository provides visualizations and tables that summarize the main findings of the analysis.

2.2. Scope of the analysis

The full dataset includes 26 EU member states from 2008 to 2016, resulting in 234 bipartite networks, one for each country-year combination. For instance, Figure 1 displays the public procurement markets in Austria, Belgium, Finland, Germany, Hungary, and the Netherlands in 2014, arranged from top-left to bottom-right, respectively. The figure shows that German public institutions awarded more contracts than their counterparts in the other five countries in 2014. Notably, each subgraph appears to have a distinct core and periphery as plotted using the *spring_layout* (more on this in subsection 3.2).

Due to limited computational resources including computing time for handling a very large network, this analysis focuses on the German public procurement market over the nine-year period, which consists of 291,056 contracts. The exploratory analysis builds heavily on the study by Wachs et al. (2021), which analyzed public procurement markets and corruption risk in EU member states including Germany using network-based approaches. This paper attempts to replicate their methods including core-periphery analysis and community detection, and inspect market structure measures to provide a more comprehensive view of the German public procurement market.

This paper primarily discusses and interprets the observed descriptive measures of the bipartite networks across time. Further experiments including the relationship between the degree of centralization of a market and its corruption risk will be conducted in the continuation of this paper. Furthermore, the question on whether centralization induces corruption by fostering corruption among core issuers and winners will not be answered yet.

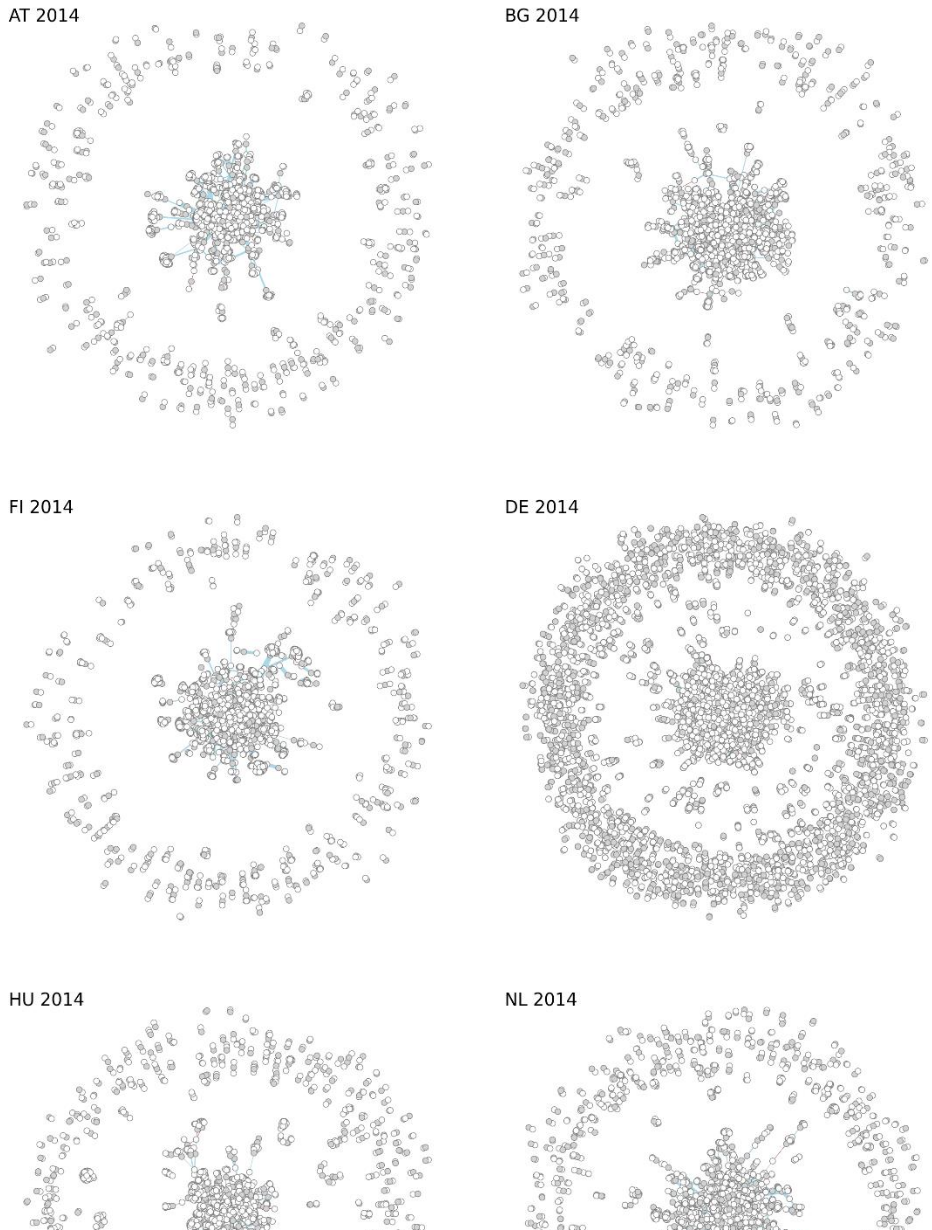

Figure 1. Austria, Belgium, Finland, Germany, Hungary, and the Netherlands in 2014

3. Network analysis

3.1. Descriptive analysis

To start the network analysis, Table 1 presents the summary statistics of the German network between 2008 and 2016. The bipartite network for Germany comprises a total of 291,056 contracts, with the number of contracts increasing steadily over the years except for a small dip in 2011. On average, there are 4,049 public institutions and 15,395 providers of goods and services. The last four columns present the averages and standard deviations of the winner and issuer degrees, weighted by the number of contracts.

A node's weighted degree represents the total number of contracts that the node (either issuer or winner) is involved in. This gives an indication of the node's overall level of participation and involvement in the public procurement market, and can be used to identify nodes that are particularly central or influential in the network. The weighted degree is a useful measure in this case because the weights on the edges (i.e., the number of contracts) represent the strength of the connection between issuers and winners.

Figure 2 visually inspects the degree heterogeneity or the variation in the number of contracts awarded (in gray) and won (in white) across years by presenting the log-log plots of degree distributions for both issuers and winners. The alpha parameter of a power law distribution fitted to both distributions is also shown in the plot describing the steepness of the curve. A larger alpha value indicates a more heterogeneous distribution with a larger number of nodes having a small degree and a smaller number of nodes having a large degree. Conversely, a smaller alpha value indicates a more homogenous distribution with a larger number of nodes having a large degree and a smaller number of nodes having a small degree.

The results reveal that the degree distributions for issuers and winners are highly heterogeneous across years, with some rare issuers and winners involved in hundreds, and even thousands, of contracts, while majority are involved in only a few contracts throughout the analyzed period.

Table 1 also displays the Robins-Alexander clustering coefficient (R-A clust.) which measures the local correlation of connectivity in bipartite networks, similar to the clustering coefficient in monopartite networks. Robins–Alexander clustering is defined as “four times the number of four cycles divided by the number of three paths in a bipartite graph.”[2] The interpretation of the coefficient by Wachs et al. (2021) is followed as: “Given that a firm wins contracts from two issuers, and that another firm wins a contract from one of these two issuers, R-A clust. can be interpreted as the probability that this second firm also wins a contract from the other issuer. The expected Robins–Alexander clustering of random bipartite networks approaches their density as they get large.” Similar to their findings, the observed R-A clustering coefficient is an order of

[2] robins_alexander_clustering — NetworkX 3.1 documentation

magnitude higher than the observed density, indicating significant local correlations in the market across years.

The R-A clust. and descriptive statistics indicate that the German public procurement networks have rich structure, which can be tested later on if it significantly deviates from random behavior. In the next subsections, two key measures of market structure will be examined: its centralization, which reflects the degree of concentration of ties around a few central actors, and its clustering, which captures the extent to which actors tend to form tightly connected groups.

Table 1. Summary statistics of German procurement market from 2008 to 2016

Year	Contracts	Nodes	Winners	Issuers	Density	R-A clust.	$\mu(Deg_W)$	$\sigma(Deg_W)$	$\mu(Deg_I)$	$\sigma(Deg_I)$
2008	19,627	14,863	11,430	3,433	0.00018	0.01	1.72	2.47	5.72	13.57
2009	23,789	16,578	12,715	3,863	0.00017	0.01	1.87	4.93	6.16	16.67
2010	30,204	17,858	14,150	3,708	0.00019	0.02	2.13	11.03	8.15	39.99
2011	28,710	18,253	14,489	3,764	0.00017	0.02	1.98	3.38	7.63	18.09
2012	31,762	19,438	15,534	3,904	0.00017	0.02	2.04	4.04	8.14	21.77
2013	34,127	20,040	15,885	4,155	0.00017	0.02	2.15	4.62	8.21	22.19
2014	37,258	20,589	16,506	4,083	0.00018	0.04	2.26	5.89	9.13	28.26
2015	40,425	22,324	17,898	4,426	0.00016	0.05	2.26	5.37	9.13	26.09
2016	45,154	25,059	19,954	5,105	0.00014	0.05	2.26	5.36	8.85	24.88
Average	32,339	19,444	15,395	4,049	0.00017	0.03	2.08	5.23	7.90	23.50
Total	291,056	175,002	138,561	36,441						

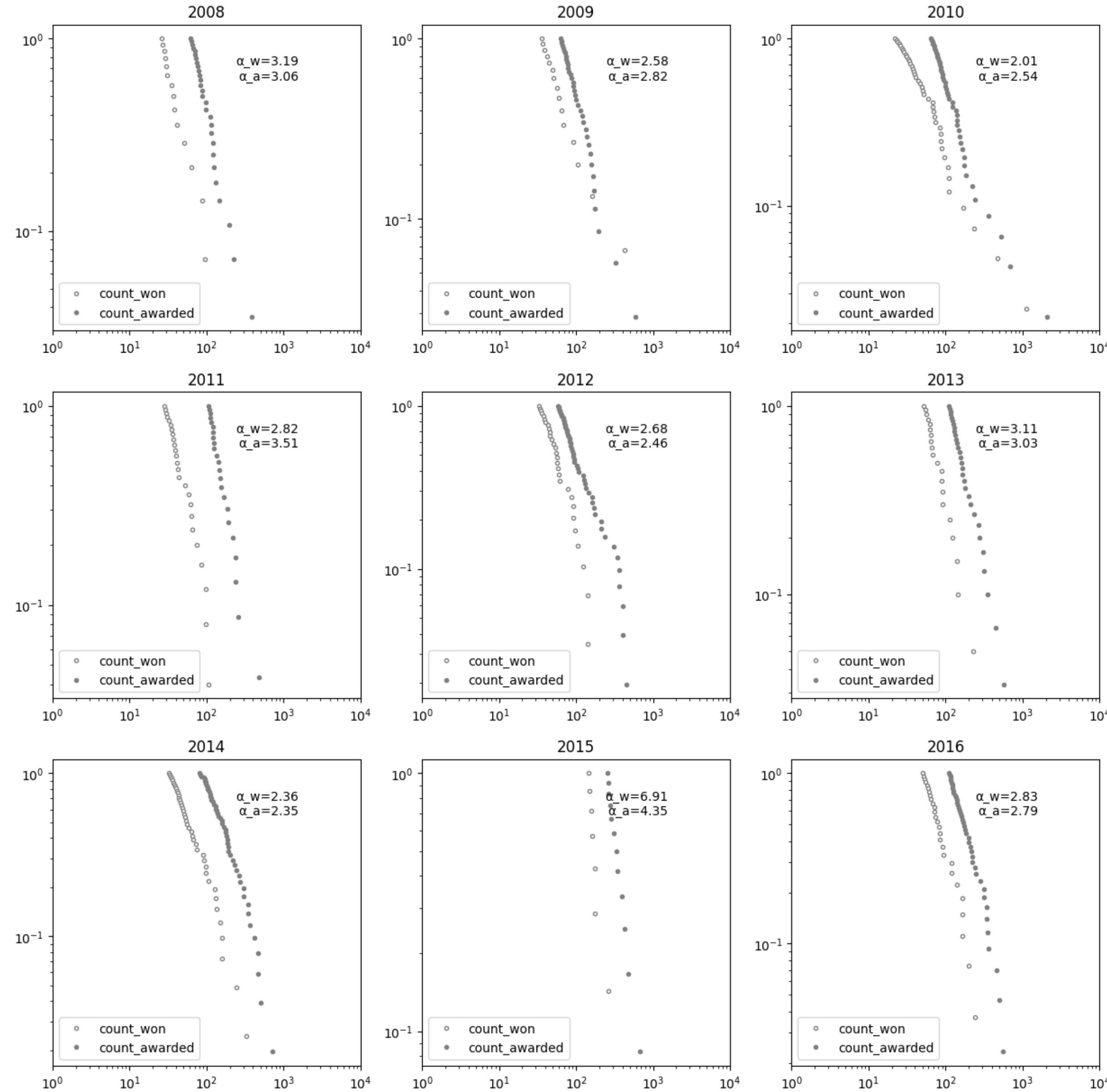

Figure 2. Distribution of the number of contracts awarded and won in the German market from 2008 to 2016

3.2. k-core analysis

Given that the networks have a unique structure with significant local correlations as discussed above and can be seen in Figure 1 (DE 2014), it is important to highlight the most central and active nodes. These nodes are located at the network's center and form a group of densely connected actors, called cores.

It is worth mentioning that in this paper, the k-core decomposition method is utilized rather than the k-shell decomposition method employed by Wachs et al. (2021). The k-core decomposition method identifies the largest subgraph in which all nodes have at least degree k. This is achieved by iteratively removing nodes with degree less than k until no such nodes remain. In this paper, the k-core is preferred due to its simplicity, robustness, and efficiency (i.e., identifies a single maximal core rather than multiple shells, is robust to the presence of hubs, and has fewer recursive steps).

The subgraph per year in Figure 3 can be compared to how the corresponding k-core graph where k=3 in Figure 4. The choice of k=3 for the k-core graph is based on manual checking wherein, setting k<3 retains nodes that appear to be in the periphery and setting k>3 leaves a smaller core. However, it must be noted that the value of k could be based on various considerations, such as the desired level of network simplification or the need to highlight the core structure of the network.

In Figure 4, gray nodes depict the issuers of contracts, while white nodes denote winners. Nodes are considered to be part of the core if they participate in numerous contracts with other highly active nodes through an iterative process. The edges between the issuer and winner of contracts are marked as red if the rate of single bidding exceeds 50%. The edges' width is weighted by the number of contracts between an issuer-winner pair.

It is observed that the 3-core has a consistent size (i.e., density less than 1%) over time and remains densely connected in all years. The weighted 3-core subgraph also has a much higher density as seen in Table 2 with an average annual core density of 0.0048 as compared to 0.00017 in Table 1. The higher density implies that a stronger level of connectivity in the network's core, which could be interpreted as a higher level of interaction or cooperation between the issuer and winners.

As a robustness check, the results when k=*None* (the default settings) is found in the Appendix A Figure A.1 along with the summary statistics in Table A.1. When k=*None*, the method will recursively remove nodes and edges from the graph until no more nodes can be removed. This means that the resulting k-core will have the maximum possible value of k for the given graph. In other words, it will find the largest possible k-core. Therefore, the density of this subgraph is expected to be higher than that of a 3-core. This is confirmed by the results suggesting an even larger average density of 0.376, which implies a more connected network than that of a 3-core.

This analysis reveals that the core plays a critical role in the overall structure and function of the German public procurement network.

Table 2. Summary statistics of the 3-core German market from 2008 to 2016

Year	Contracts	Nodes	Winners	Issuers	Density	R-A clust.	$\mu(Deg_W)$	$\sigma(Deg_W)$	$\mu(Deg_I)$	$\sigma(Deg_I)$
2008	1,179	516	289	227	0.00887	0.07	4.08	2.33	5.19	3.32
2009	2,221	866	477	389	0.00593	0.05	4.66	3.00	5.71	3.90
2010	3,121	1,127	661	466	0.00492	0.11	4.72	3.51	6.70	5.68
2011	3,717	1,306	768	538	0.00436	0.06	4.84	3.72	6.91	6.45
2012	3,769	1,326	778	548	0.00429	0.10	4.84	3.62	6.88	6.27
2013	4,339	1,543	879	664	0.00365	0.11	4.94	3.71	6.53	6.60
2014	4,936	1,650	965	685	0.00363	0.19	5.12	4.17	7.21	8.06
2015	6,612	2,130	1,256	874	0.00292	0.17	5.26	4.82	7.57	8.02
2016	7,144	2,359	1,355	1,004	0.00257	0.17	5.27	5.02	7.12	7.59
Average	4,115	1,424	825	599	0.00457	0.11	4.86	3.77	6.65	6.21
Total	37,038	12,823	7,428	5,395						

Further analysis can be done to compute the average single bidding rate in the k-core across years. With additional data points from other countries, corruption indices can be used to correlate and measure the relationship of single bidding rate and perception-based measures of corruption.

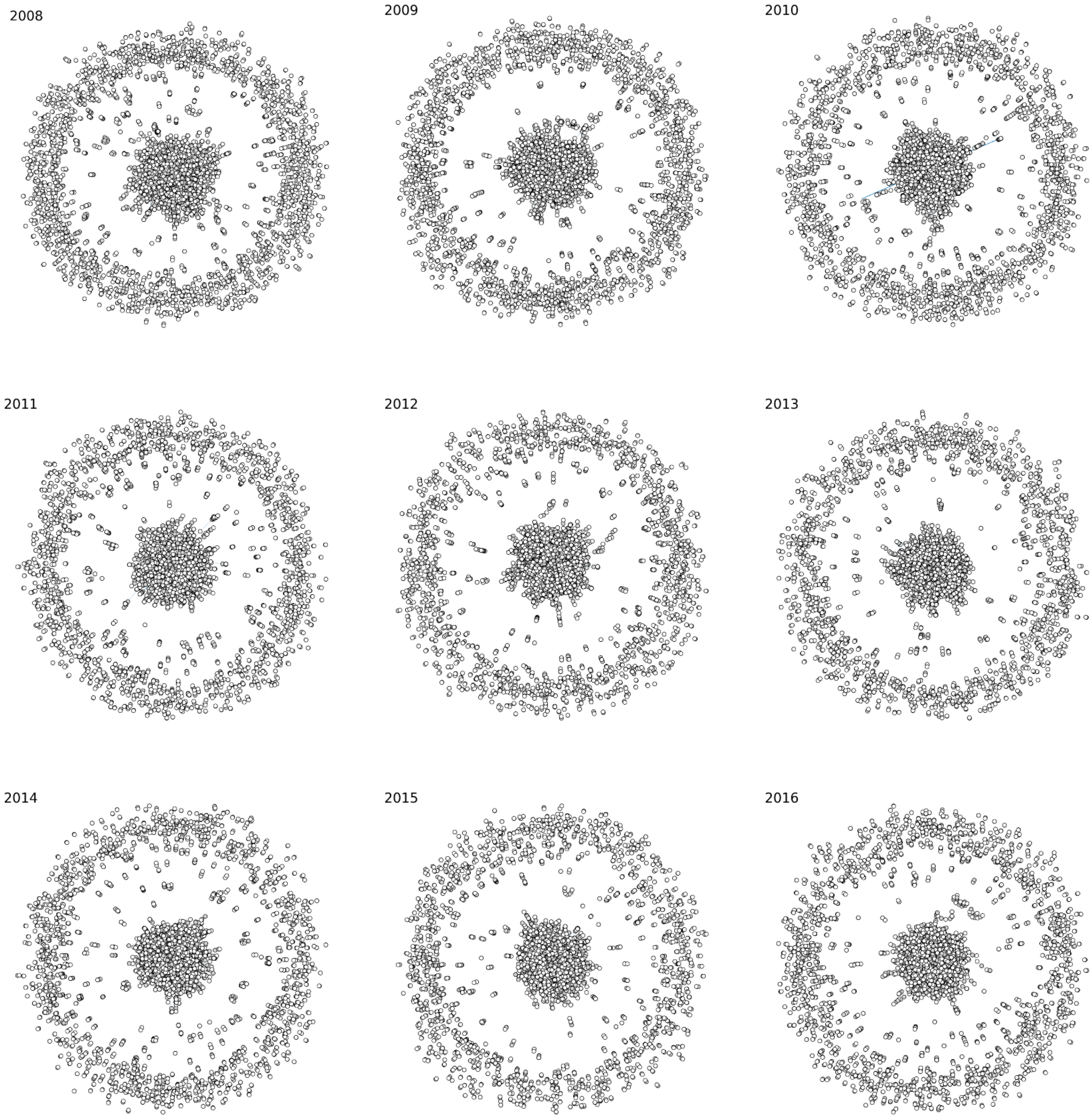

Figure 3. German public procurement market from 2008 to 2016

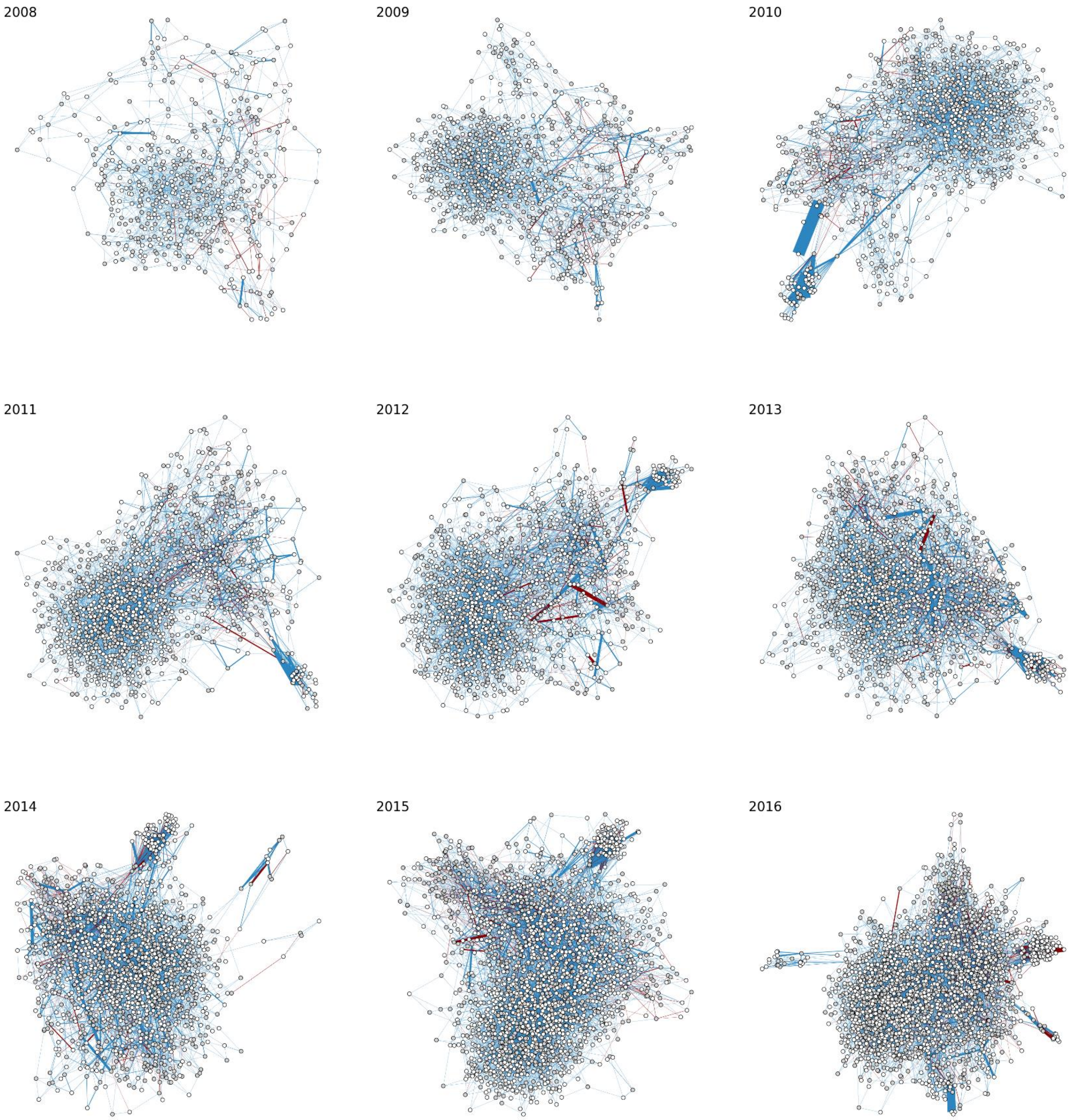

Figure 4. Weighted 3-core of the German market from 2008 to 2016

3.3. Community detection and centrality

In addition to using k-core decomposition, the Clauset-Newman-Moore greedy modularity maximization algorithm, which is readily available in the `networkx` library through its `greedy_modularity_communities`[3] function, is utilized to aid in community detection. While k-core decomposition iteratively removes lowest-degree nodes until all remaining nodes have a minimum degree of k, the greedy algorithm iteratively merges nodes or communities to increase the modularity of the network partition.

Modularity ranges from -1 to 1, where a value of 1 indicates a perfect community structure, meaning all edges within a community and no edges between different communities, while a value of -1 means the opposite, that is all edges between communities and none within a community. A value close to 0 indicates that the community structure is not significantly different from a random partition of the same network.

Before performing the community detection algorithm, the largest connected component of each network is first obtained by getting the largest subgraph of the results of `networkx` `connected_components` function. For example, Figure 5 shows the largest connected component of the German market in 2016 (left) and the results of the greedy algorithm (right) with a modularity of 0.84, indicating that edges are much more likely to be between nodes of the same community rather than across communities.

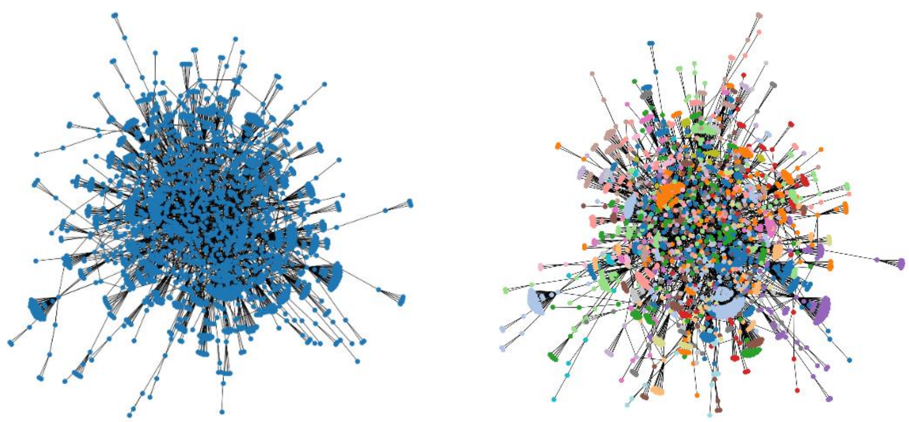

Figure 5. The largest connected component (left) and community detection using greedy algorithm (left) of the German market in 2016

As seen in Figure 5, the greedy algorithm detects multiple communities varying from 82 to 136 clusters per year with modularity between 0.83 to 0.88, suggesting significant topological clustering across years (see Table 3). One possible explanation for this is that the nodes in the

[3] greedy_modularity_communities — NetworkX 3.1 documentation

network are highly heterogeneous or diverse, which naturally grouped into distinct clusters based on certain attributes.

Table 3 also reports the coefficient of variation of single bidding rate across clusters, calculated as the ratio of the standard deviation of single bidder rates across the clusters to the mean. For example, in 2012, the standard deviation of the mean single bidding rate of 113 clusters divided by its average across clusters is equal to 1.26. A CV greater than 1 indicates high variability, while a CV less than 1 indicates low variability. Results suggest that the variability of the *sbr* is relatively high, with some years showing higher variability than others.

The last column presents the number of clusters with an average single bidding rate greater than or equal 50%. For instance, in year 2012, six clusters were found to have 50% average *sbr* or above. Note that the single bidding rate is equal to 1 when there was no competition for the contract.

To understand better these "risky" communities which are prone to corruption, Table 4 displays the mean and standard deviation of the community with the highest average single bidding rate. The mean varies between 0.41 to 0.94, with the highest mean of a community detected in 2013 followed by 2012.

Table 3. Results of greedy modularity maximization algorithm from 2008 to 2016

Year	# Communities	Modularity	CV(*sbr*)	# Communities >= 50% *sbr*
2008	82	0.88	1.01	0
2009	107	0.86	1.21	2
2010	124	0.86	1.32	3
2011	95	0.84	0.98	1
2012	113	0.84	1.26	6
2013	95	0.85	1.15	3
2014	86	0.84	0.84	2
2015	136	0.83	0.99	4
2016	126	0.84	0.92	6
Average	107.11	0.85	1.08	3.00

Table 4. Summary statistics of the community with the highest average single bidder rate

Year	μ(sbr)	σ(sbr)
2008	0.41	0.42
2009	0.66	0.47
2010	0.61	0.49
2011	0.50	0.50
2012	0.83	0.37
2013	0.94	0.24
2014	0.71	0.45
2015	0.81	0.36
2016	0.73	0.45

Note: CV=coefficient of variation, SD=standard deviation

Table 5 presents summary statistics for the subgraph of communities with average single bidding rate of 50% or higher from 2009 to 2016. For instance, in 2010, the subgraph of three detected communities has 96 nodes with 2,157 edges/contracts awarded by 7 German public institutions to 89 private firms. The fact that there are over 2,000 connections between the sets of nodes suggests that the same issuer-winner pairs may frequently be involved in multiple contracts, which may indicate relatively low competition.

To investigate which nodes in the communities with average single bidding rate of 50% or higher tend to have more influence or importance on other nodes, three measures of centrality including the degree, closeness, betweenness centrality are computed and the average values are reported in Table 6. Since the public procurement market is a bipartite network, the `network.algorithms.bipartite`[4] is used to calculate centrality measures separately for each set of nodes, issuers and winners. The degree centrality of a node measures the number of connections it has to other nodes, while the closeness centrality measures how quickly a node can reach other nodes in the network. The betweenness centrality measures how often a node acts as a bridge between other nodes in the network (i.e., extent to which a node lies on the shortest path between other nodes).

Figure 6 displays these three centrality measures for issuers (red nodes) and winners (blue nodes) in the subgraph of communities for 2010 (year with the highest number of contracts) and 2012 (year with six communities having above 50% *sbr*). (See Appendix B for all plots.)

For example, for the 2010 degree centrality plot, the blue node with a value of about 140 represents a node with a high number of connections to red nodes suggesting its importance. The higher closeness centrality of the blue nodes at the bottom left cluster as well as the bottom sub-cluster of the top one suggests that they are more central to their immediate neighbors and have shorter average path lengths to other nodes in their respective clusters compared to the ones on top. The blue node on the top cluster having the highest betweenness centrality implies that it is located in a critical position, in which it is an important firm providing a particular service between pairs of nodes in the set of issuers. The same interpretation follows for the two darker shaded red nodes or issuers.

These centrality measures provide insights into the structure and importance of nodes in the network. For example, nodes with high degree centrality have many connections and may be important hubs in the network. This information can be used to identify influential public procurement entities in Germany and their role in the network. Further analysis can be done by investigating the type of contract, whether or not the contract was supported by EU funding, and the type of procedure used to award the contract.

[4] Bipartite — NetworkX 3.1 documentation

Table 5. Summary statistics of communities with average single bidding rate of 50% or above from 2009 to 2016

Year	# Communities >= 50% *sbr*	Contracts	Nodes	Winners	Issuers	μ(Deg_W)	σ(Deg_W)	μ(Deg_I)	σ(Deg_I)
2009	2	604	27	17	10	35.53	98.55	60.40	177.20
2010	3	2157	96	89	7	24.24	126.48	308.14	710.62
2011	1	8	8	7	1	1.14	0.35	8.00	0.00
2012	6	130	73	45	28	2.89	6.89	4.64	6.18
2013	3	52	31	28	3	1.86	2.98	17.33	3.68
2014	2	73	18	15	3	4.87	6.70	24.33	29.47
2015	4	45	31	22	9	2.05	2.88	5.00	6.27
2016	6	75	60	53	7	1.42	1.00	10.71	5.70
Average	3	393	43	34	8.5	9.25	30.73	54.82	117.39
Total	30	3144	344	276	68				

Table 6. Average centrality measures of communities with average single bidding rate of 50% or above from 2009 to 2016[5]

Year	# Communities >= 50% *sbr*	μ($CDeg_W$)	μ($CDeg_I$)	μ(CC_W)	μ(CC_I)	μ(CB_W)	μ(CB_I)
2009	2	3.553	3.553	0.767	0.689	0.014	0.048
2010	3	3.462	3.462	0.770	0.576	0.003	0.092
2011	1	1.143	1.143				
2012	6	0.103	0.103	0.625	0.567	0.004	0.008
2013	3	0.619	0.619	1.019	1.067	0.000	0.132
2014	2	1.622	1.622	0.895	0.779	0.006	0.157
2015	4	0.227	0.227	0.881	0.754	0.004	0.021
2016	6	0.202	0.202	0.991	0.989	0.000	0.025

Note: CDeg=Degree centrality, CB=Betweenness, CC=Closeness

[5] After manual inspection, it is indeed that the average degree centrality of issuer nodes is equal to that of winner nodes since it is bipartite network. This is because each edge in the graph connects a node from one set to a node from the other set, and so the number of edges connected to nodes in one set is always equal to the number of edges connected to nodes in the other set.

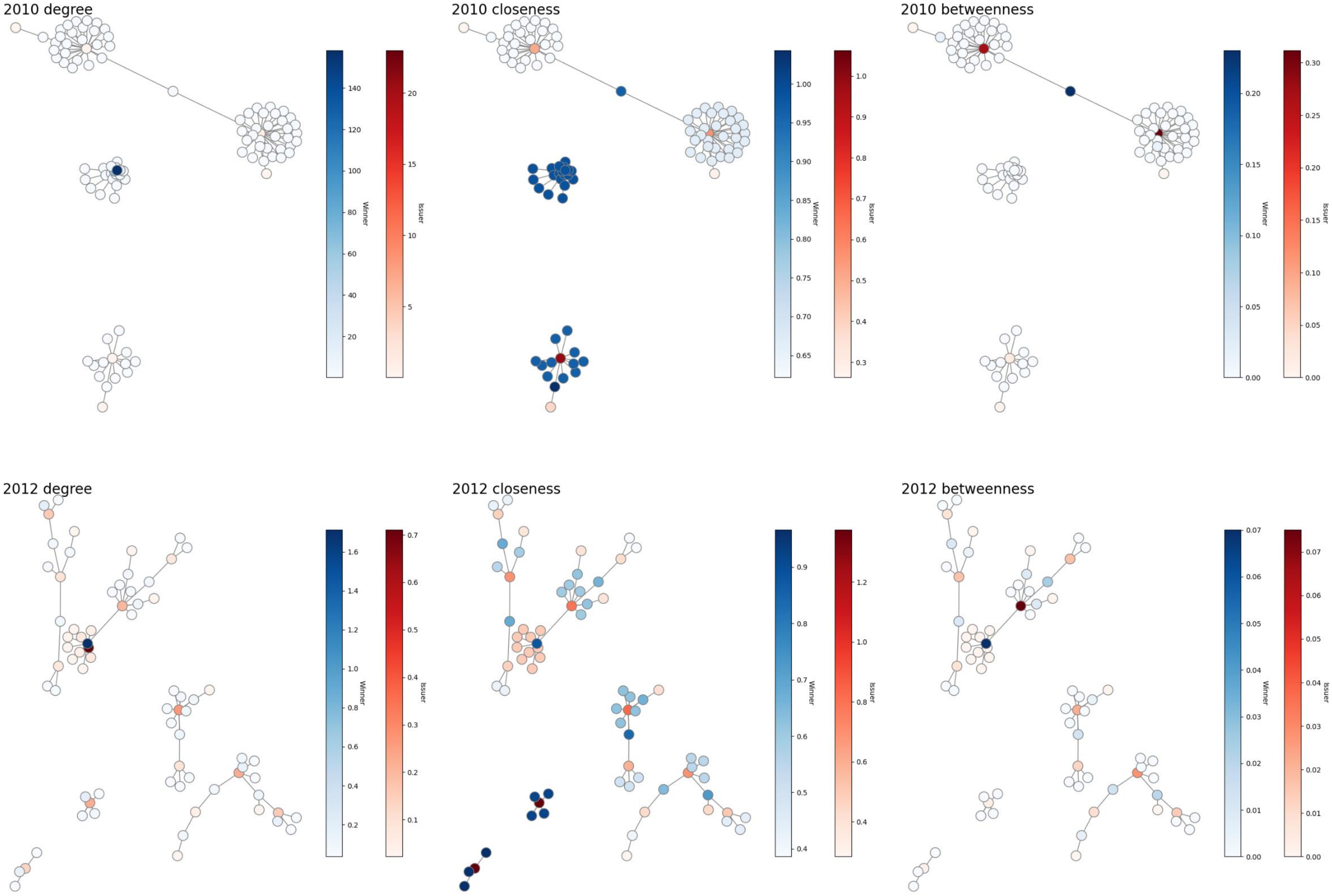

Figure 6. Nodal centrality of communities with average single bidder rate of 50% or above in 2010 and 2012

4. Conclusion and future work

One of the main findings of this paper is that the full network of German public procurement contracts across years has a rich structure as captured by the average R-A clustering coefficient of 0.03, implying significant local correlations. After plotting the degree distributions for issuers and winners on a log-log scale, it is also discovered that there exist rare issuers and winners involved in hundreds, and even thousands, of contracts, while majority are involved in a few contracts. This is measured by the alpha greater than 2, indicating a high heterogeneity of the degree distributions.

By applying the k-core decomposition method, the study was able to identify the most central and densely connected nodes of the network, also known as cores. The choice of k=3 results to a core having a higher density (0.0048) than that of the full network (0.00017) implying a stronger level of connectivity in the network's core. This could be interpreted as a higher level of interaction or cooperation between issuers and winners and suggests that the core plays a critical role in the overall structure and function of the network.

This type of analysis can be extended to compare the characteristics of the nodes and edges between the core and periphery, which could provide additional insights into the network's structure and dynamics including the measure of corruption risk. By examining the differences between the core and periphery, it is be possible to identify key features contributing to the overall connectivity and behavior of the network.

Furthermore, the Clauset-Newman-Moore greedy modularity maximization algorithm identified an average of 107 communities and 0.85 modularity within the largest connected component per year. Analyzing the clusters with high single bidding rates of 50% or more confirmed the previous findings regarding the heterogeneity of degree distributions, indicating that rare issuers and winners are involved in hundreds of contracts. Additionally, the nodal centrality of these communities highlights the presence of certain influential public procurement entities in Germany. As a result, further investigation into the corruption risk associated with these entities is crucial.

Overall, this paper demonstrates the value of network analysis in understanding complex systems such as public procurement networks and highlights the importance of ongoing research and analysis in this area to address corruption and promote fair and efficient procurement practices.

5. References

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Appendix A

Table A.1. Summary statistics of the k-core German market from 2008 to 2016

Year	**Contracts**	Nodes	Winners	Issuers	**Density**	R-A clust.	$\mu(Deg_W)$	$\sigma(Deg_W)$	$\mu(Deg_I)$	$\sigma(Deg_I)$
2008	1179	516	289	227	0.00887	0.07	4.08	2.33	5.19	3.32
2009	2221	866	477	389	0.00593	0.05	4.66	3.00	5.71	3.90
2010	68	17	9	8	0.50000	0.94	7.56	0.50	8.50	0.50
2011	56	15	8	7	0.53333	1.00	7.00	0.00	8.00	0.00
2012	104	22	13	9	0.45022	0.88	8.00	0.78	11.56	1.17
2013	78	18	9	9	0.50980	0.96	8.67	0.47	8.67	0.47
2014	188	29	17	12	0.46305	0.93	11.06	0.87	15.67	1.65
2015	182	28	15	13	0.48148	0.93	12.13	0.72	14.00	1.18
2016	204	31	17	14	0.43871	0.87	12.00	1.61	14.57	2.03
Average	475	171	94	76	0.37682	0.74	8.35	1.14	10.21	1.58
Total	4280	1542	854	688						

Notes: k=maximum possible value of k. When k=*None* in the k-core decomposition method, the largest k value for which a k-core can be extracted is used, which corresponds to the entire graph.

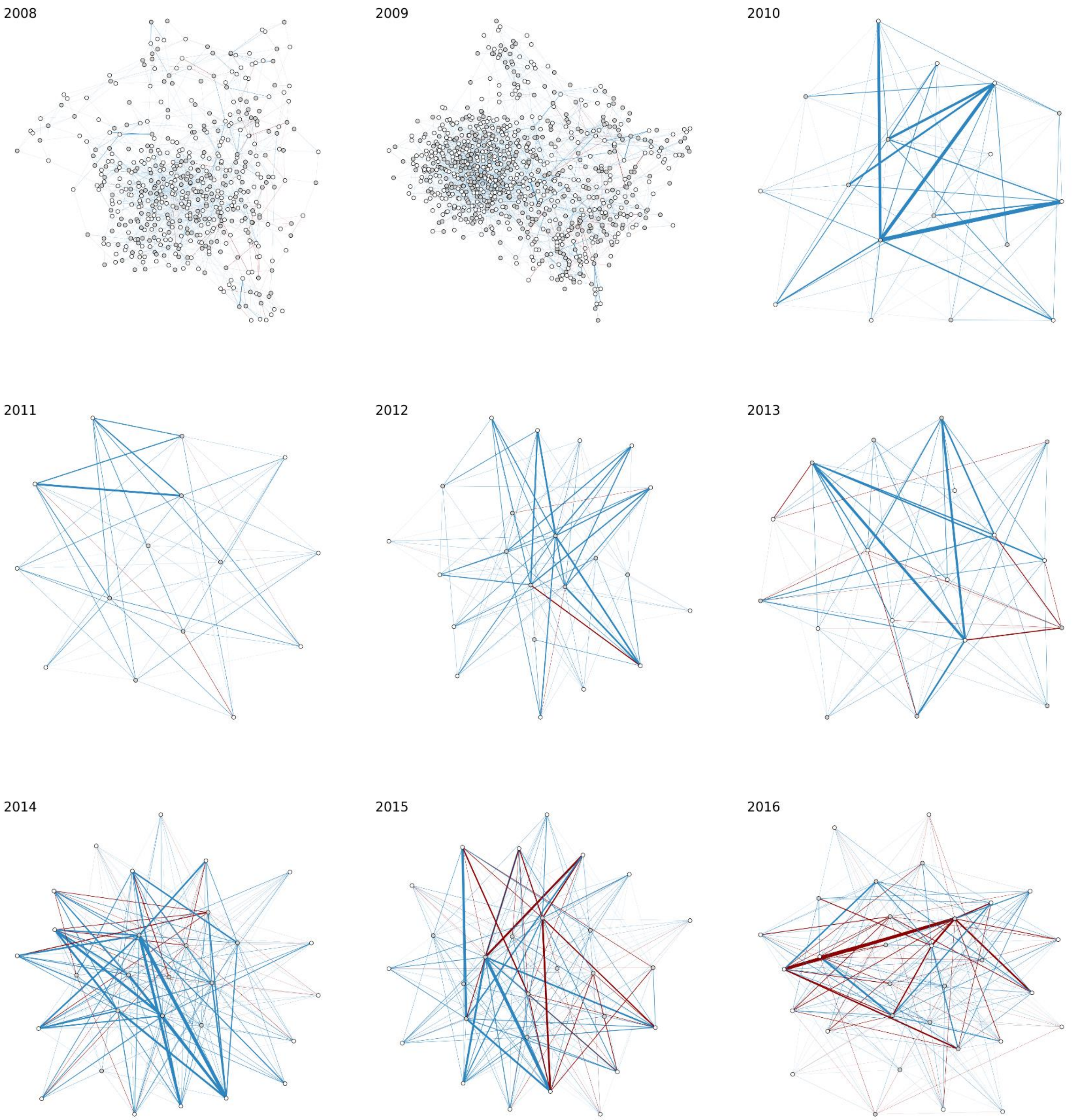


Figure A.1. Weighted k-core of the German market from 2008 to 2016 (k=maximum possible value of k)

Appendix B

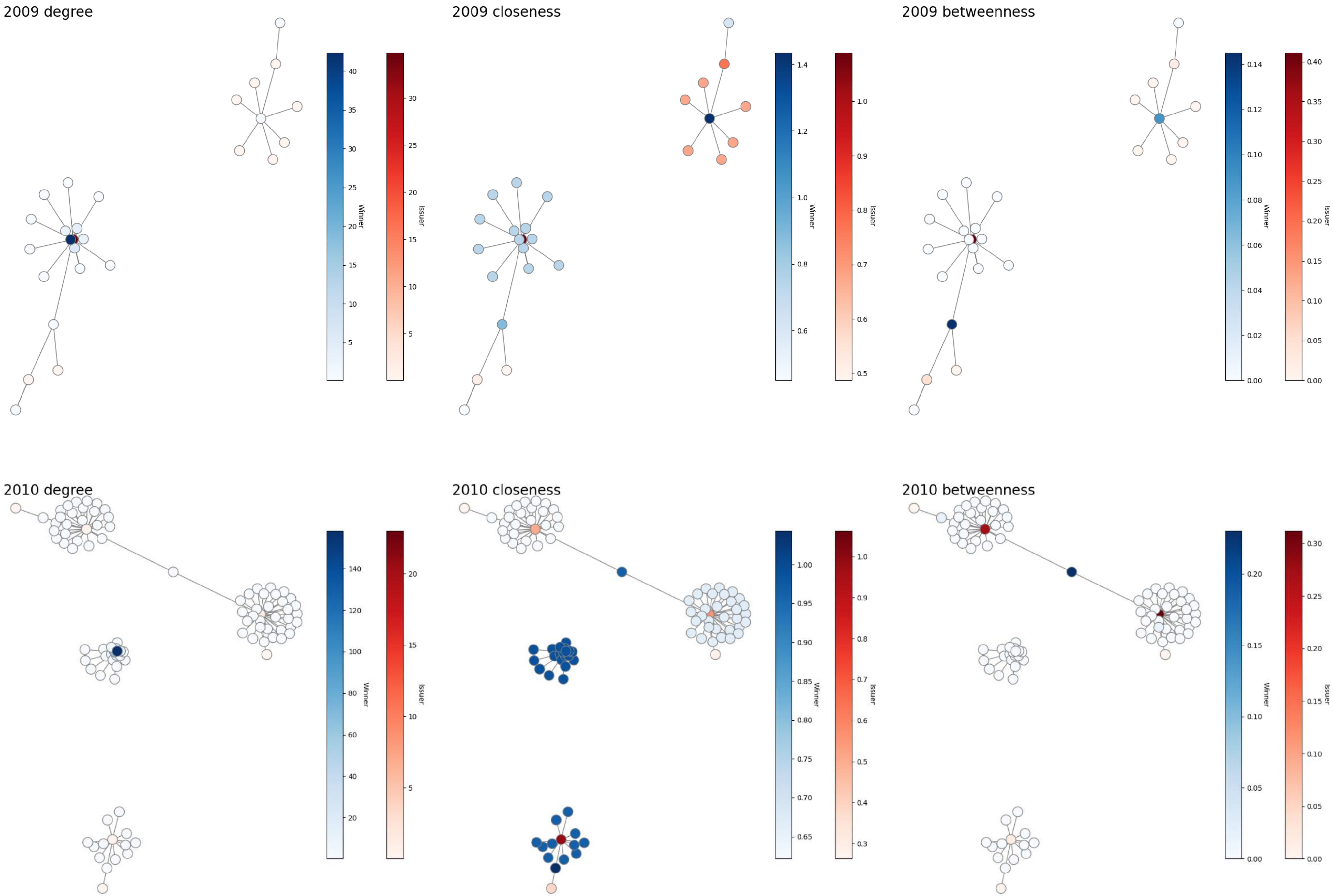


Figure B.1. Nodal centrality of communities with average single bidder rate of 50% or above from 2009 to 2016

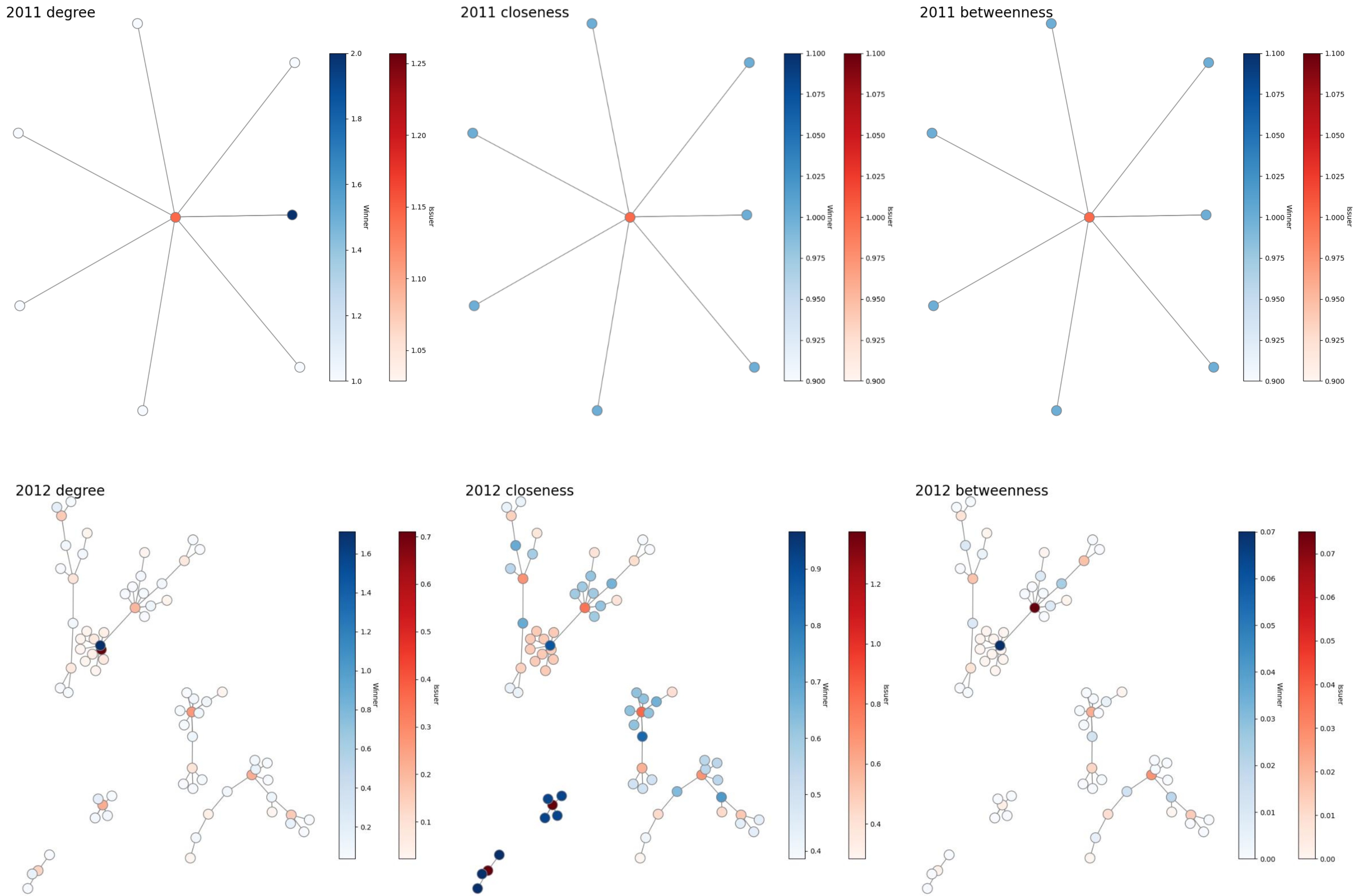
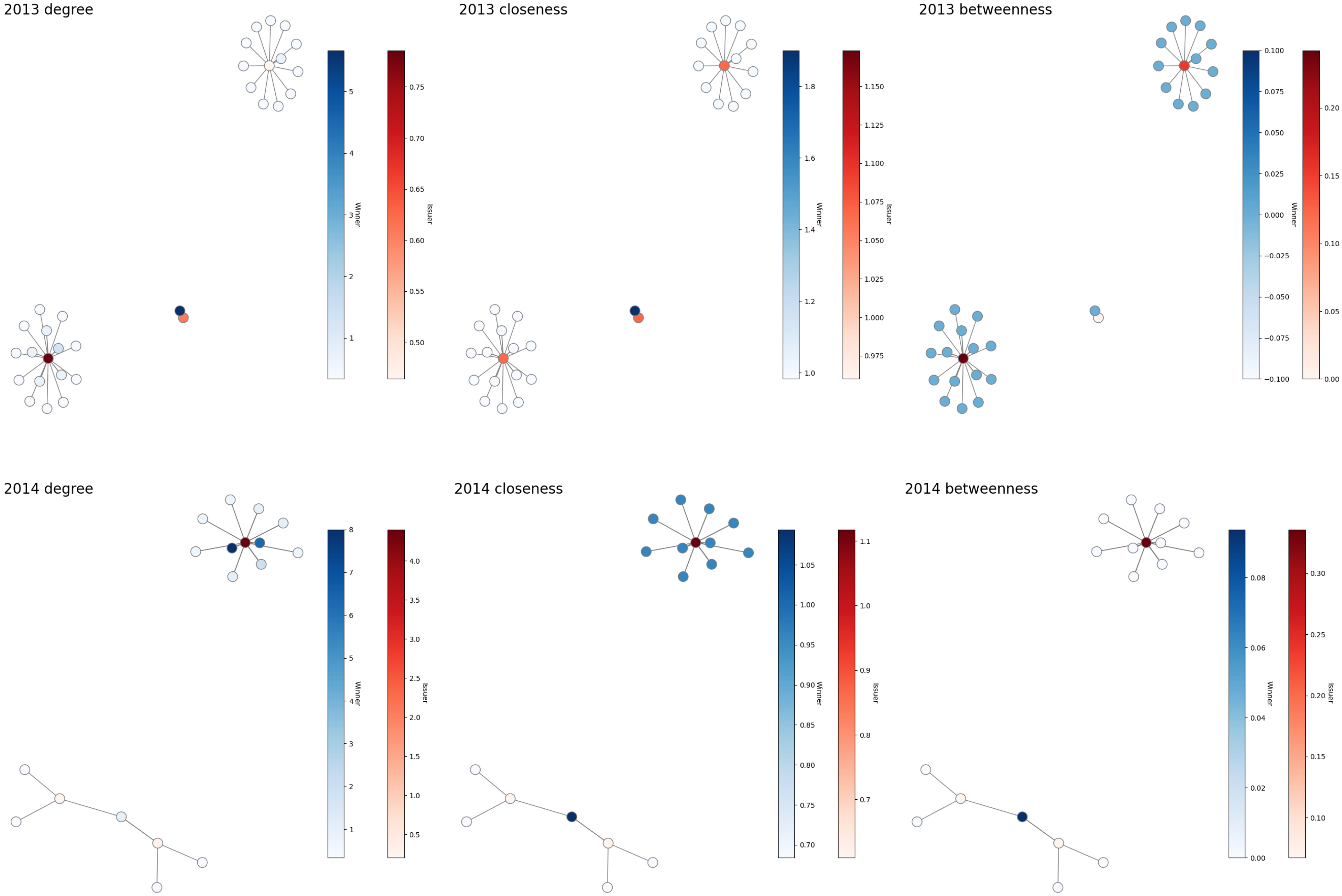


Figure B.1 Nodal centrality of communities with average single bidder rate of 50% or above from 2009 to 2016



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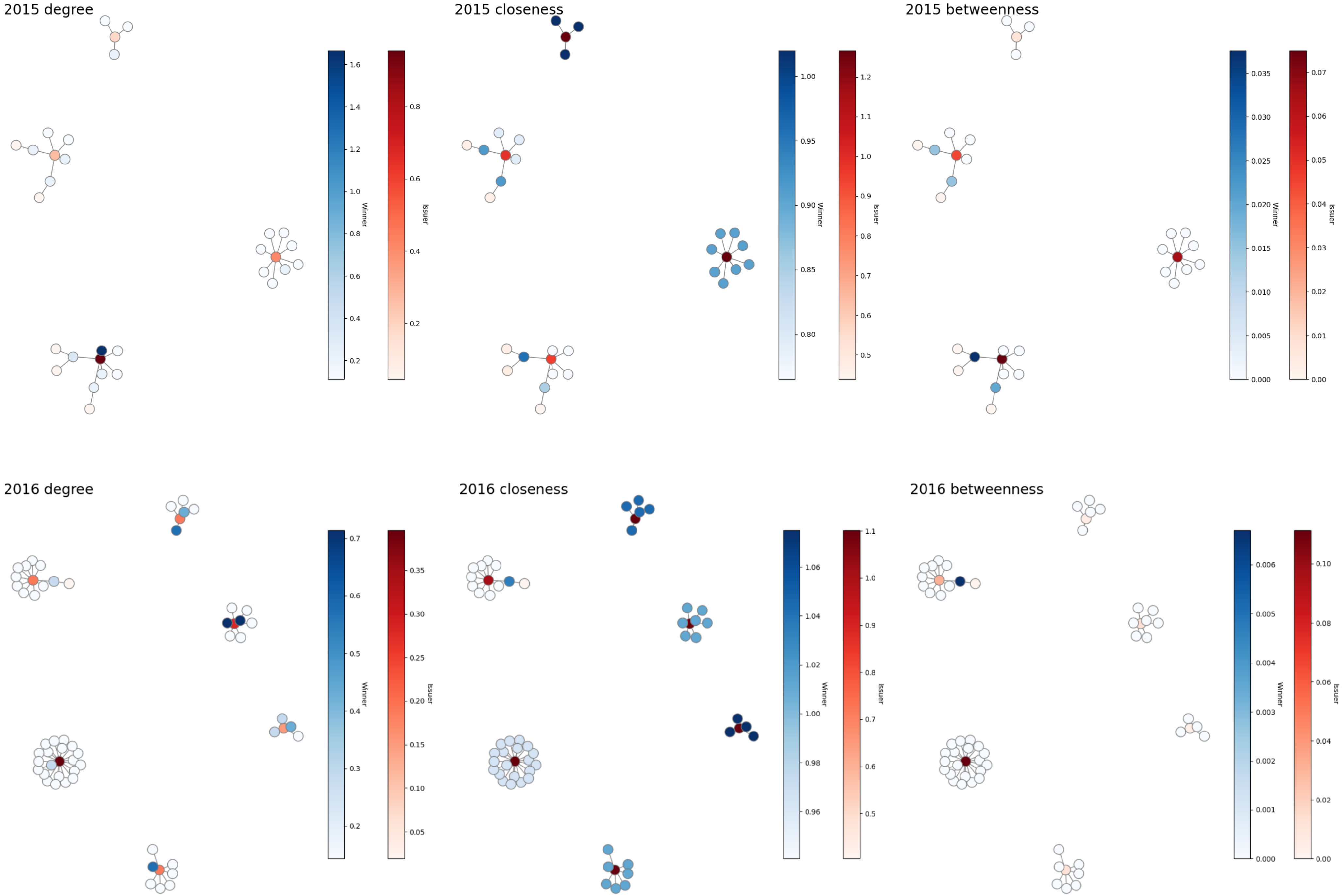

Figure B.1 Nodal centrality of communities with average single bidder rate of 50% or above from 2009 to 2016