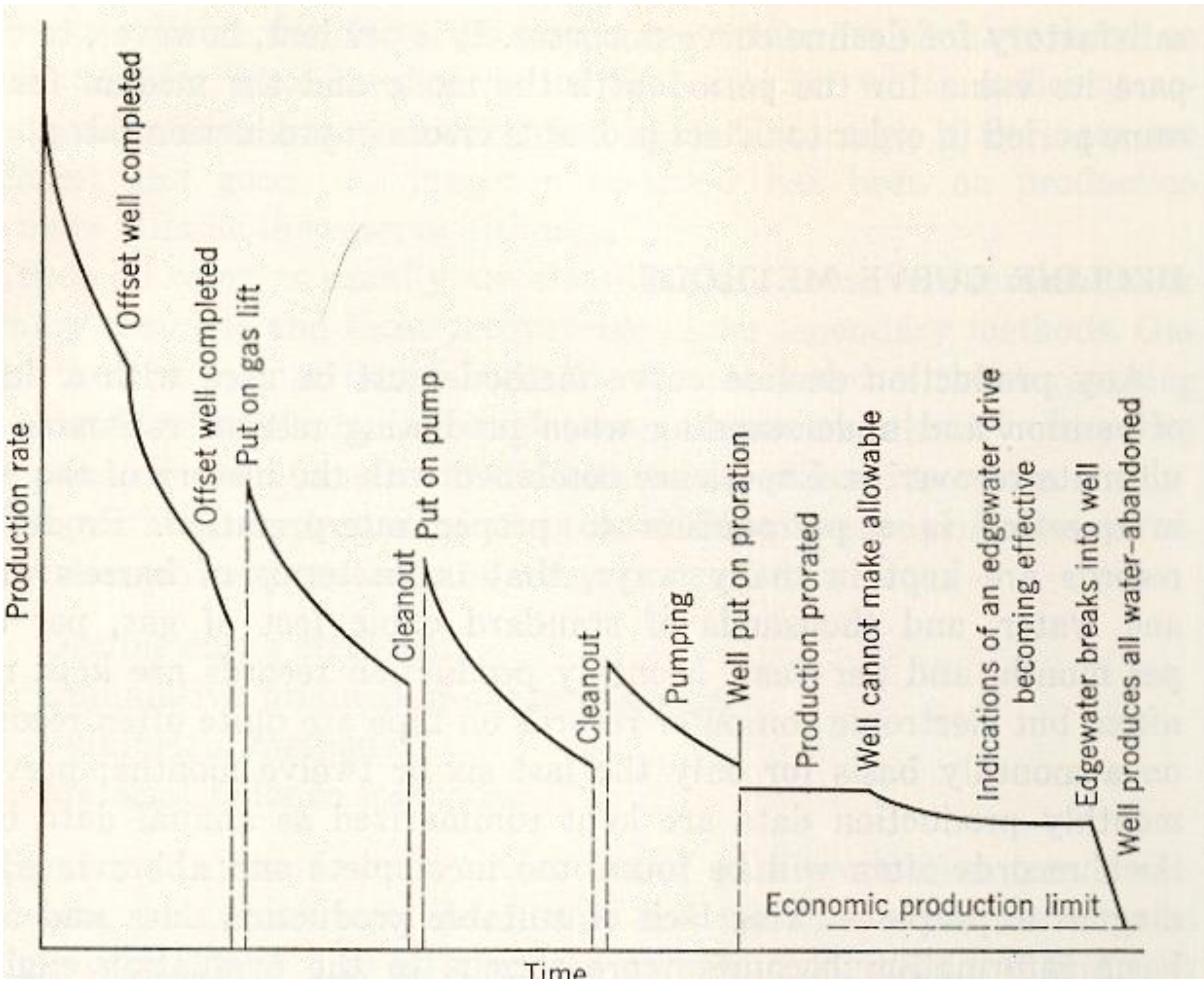
Decline Curves



Decline Curves that plot flow rate vs. time are the most common tools for forecasting production and monitoring well performance in the field. These curves quickly show by graphic means which wells or fields are producing as expected or under producing. Mainly used because they are easy to set up and to use in the field. They are not based on any of the physics of the flow of oil and gas through the rock formations, empirical in nature. The most common forms are daily flow rates vs. the month. Water and gas rates are commonly plotted along with the oil rate, or GOR and WOR. Cumulative production vs. the months is also very common, both oil and water can be plotted.

These plots are plotted both on linear plots and semi-log plots with the q on the log scale.

Arps Empirical Equations

Express the rate of change in terms of decline rate D.

$$D = -\frac{1}{q}\frac{dq}{dt}$$

Differential form

$$D = -\frac{d(\ln q)}{dt}$$

Define the *b*-exponent term as the time-rate-change of the reciprocal of the decline rate

$$b = \frac{d\left(1/D\right)}{dt} = constant$$

First derivative

$$b = -\frac{d}{dt}\left[\frac{q}{dq/dt}\right]$$

The *b*-exponent term should remain constant as the producing rate declines from the initial value to a later value. Operating conditions can cause changes during the life of the well. Divide the production history into like segments to better understand the future production.

Three types of decline curves

Exponential $b=0$

Harmonic $b=1$

Hyperbolic $0<b<1$

Equations can be derived from the previous equations

Integrating over the range 0 to t, and defining the initial decline rate at $t=0$ as D_i

$$D = \frac{D_i}{1+bD_i t}$$

Gives us

$$-\frac{d(\ln q)}{dt} = \frac{D_i}{1+bD_i t}$$

There is two cases to consider for the exponent term, $b = 0$ and $b \neq 0$.

For the case of b = 0 then $D = D_i$, the decline rate is not a function of time, it remains constant throughout the life of the well.

$$D = -\frac{d(\ln q)}{dt}$$

Integrating over 0 to t develops a producing rate

$$q_2 = q_1 exp(-Dt)$$

For $b \neq 0$ the rate equation

$$q_2 = \frac{q_1}{(1-bD_i t)^{1/b}}$$

Combine equations

$$\boldsymbol{D = \frac{1}{q}\frac{dq}{dt} = D_i\left[\frac{q}{q_i}\right]^b}$$

Exponential Equations

Expressing the $b =$ 0 equation in log terms and in the form of a straight line with ln q as the y axis and t on the x axis

$$\boldsymbol{ln\ q_2 = -Dt + ln\ q_2}$$

The intercept is q_1 and the slope on the line is -D.

So $\boldsymbol{D = \frac{ln\left[\frac{q_1}{q_2}\right]}{t}}$

Rearranging for time

$$\boldsymbol{t = \frac{ln\left[\frac{q_1}{q_2}\right]}{D}}$$

Generalized equation for cummlative production

$$\boldsymbol{N_p = \int_0^t q dt}$$

For the exponential case, integrating over t > 0 range

$$\boldsymbol{N_p = \frac{q_i}{D}[1 - exp(-Dt)]}$$

Using the rate equation to simplify

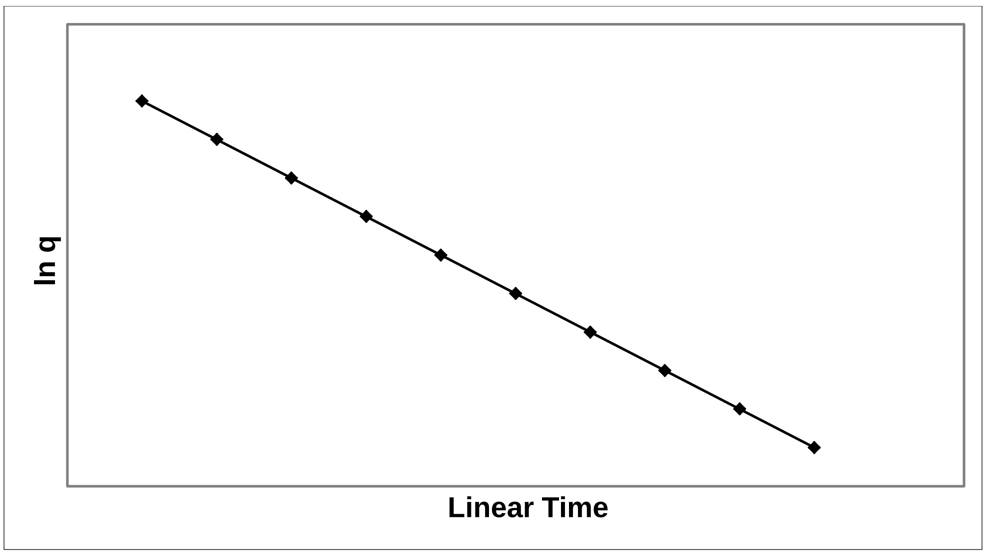
$$\boldsymbol{N_p = \frac{q_i - q_2}{D}}$$

Rearrange for the line equation

$$q_2 = -DN_p + q_i$$

Assume $q_i \rightarrow 0$ a theoretical maximum cumm can be calculated

$$Q_{p\,max} = \frac{q_i}{D}$$



Exponential Decline Plot.

$$q = q_i e^{-Dt}$$

q = production rate at time t, bbls/time unit

q_i = initial production rate, bbls/time unit

D = nominal exponential decline rate, $^1/_{\text{unit time}}$

Effective decline is

$$D_e = \frac{q_i - q}{q_i}$$

Nominal decline is

$$D = -\ln(1 - D_e)$$

or

$$D_e = 1 - e^{-D}$$

<u>Example 1:</u> Given that a well has declined from 100 BOPD to 96 BOPD during a one month period

a) Predict the rate after 11 more months using nominal exponential decline.
b) Same as part a), using effective decline.

a) Nominal decline

$q_i = 100$ BOPD

$q = 96$ BOPD

$t = 1$ month

$q = q_i e^{-Dt}$

$\frac{q}{q_i} = e^{-Dt}$

$\ln \frac{q}{q_i} = -Dt$

$$D = \left[\ln\left(\frac{q_i}{q} \right) \right] \left(\frac{1}{t} \right)$$

$$D = \left[\ln\left(\frac{100}{96} \right) \right] \left(\frac{1}{1} \right) = 0.04082 / month$$

Rate at end of one year

$q = q_i e^{-Dt}$

$q = 100 e^{-0.04082(12)}$

$q = 61.27$ BOPD

b) Effective decline

$$D_e = \frac{q_i - q}{q_i} = \frac{100-96}{100} = 0.04 / month$$

convert to yearly

$$1 - D_{ey} = (1 - D_{em})^{12}$$

$$1 - D_{ey} = (1 - 0.04)^{12}$$

$$D_{ey} = 0.3875 / year$$

Rate at end of one year

$$q = q_i (1 - D_e)$$

$$q = 100(1 - 0.3875)$$

$$q = 61.27 \text{ BOPD}$$

Cumulative production

$$N_p = \int_0^t q dt$$

$$N_p = \int_0^t q_i e^{-Dt} dt$$

$$N_p = \frac{q_i}{-D} e^{-Dt} \Big|_0^t$$

$$N_p = \frac{q_i}{-D}\left[e^{-Dt} - e^{o}\right]$$

$$N_p = \frac{q_i}{D}\left(1 - e^{-Dt}\right)$$

$$N_p = \frac{q_i - q}{D}$$

<u>Example 2:</u> Calculate the amount of oil produced during the first year for the well above

$$q_i = 100 \text{ BOPD}$$

$$q = 61.27 \text{ BOPD after one year}$$

$$D = 0.04082(12) = 0.48986 / year$$

$$N_p = \left(\frac{100 - 61.27}{0.48986}\right)365 = 28{,}858 \text{ STB}$$

or using nominal decline

$$q_i = 100 \text{ BOPD}$$

$$q = 96 \text{ BOPD } (30.42 \text{ days})$$

$$D = \left[\ln\left(\frac{100}{96}\right)\right]\left(\frac{1}{30.42}\right) = 0.001342 \frac{1}{day}$$

$$N_p = \frac{100}{0.001342}\left(1 - e^{-0.001342(365)}\right)$$

$$N_p = 28{,}858 \text{ STB}$$

Better to stay consistent with time.

<u>Example 3:</u> Project the yearly production for the well in the above examples for the next 5 years.

$$N_p = \frac{61.27}{0.001342}\left(1 - e^{-0.001342(365)}\right)$$

$$N_p = 17{,}681$$

$$N_p = \frac{37.54}{0.001342}\left(1 - e^{-0.001342(365)}\right)$$

$$N_p = 10{,}834$$

Year	Rate End of Year (BOPD)	Yearly Production (STB)	
0	100.00	-	
1	61.27	28,858	
2	37.54	17,681	
3	23.00	10,834	
4	14.09	6,639	
5	8.64	4,061	
		68,073	Cumulative STB after 5