Links

Google Slides:
https://docs.google.com/presentation/d/1zMYlFBUiE7uI4MfQvC_pb7pvGcYhIIMjsdc6bDp24Yc/edit?usp=sharing

Github:
https://github.com/ademiltonnunes/Machine-Learning/tree/main/ANN/Design%20XOR%20Gate

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Project: Design XOR Gate

Week 7 - Homework 1
CS550 - Machine Learning and Business Intelligence

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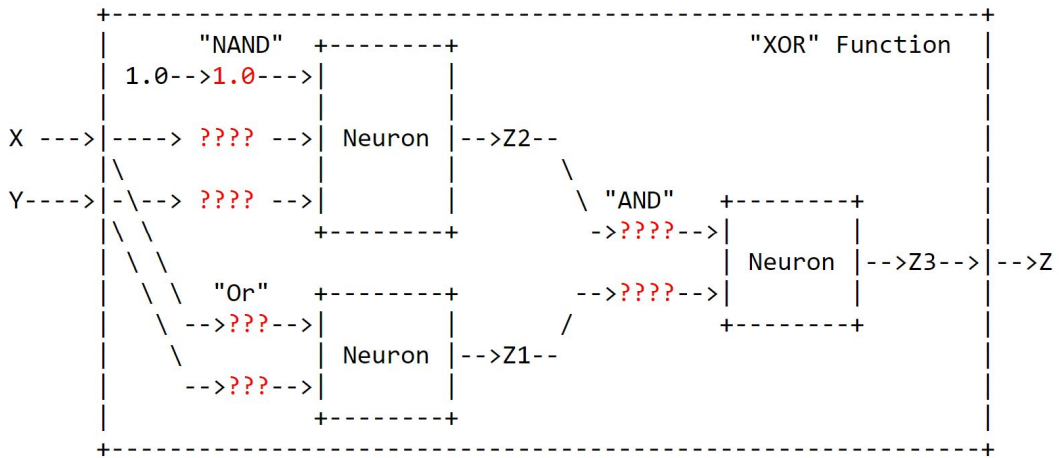
Introduction

This project aims to design an Artificial Neural Network (ANN) for the XOR gate. To design that it is necessary to use three neurons, first neuron OR gate, second NAND gate, and third AND gate, generating the designer below:



Introduction

Each neuron has an input X and Y and an output Z under the function

Z= (W0 * C + W1 * X + W2 * Y >= 1.0)

Where:

- W0, W1, and W2 = weight
- C is a constant number
- W0 * C = Bias. Using bias is optional, for this project bias will be 1.0*1.0
- X, Y = Inputs
- Z = Output

Introduction

Backward process: For this project, if Z has an output lower or higher than the expected value we are going to increase or decrease weight by 0.5.

Activation function:

- if Z result is smaller or equal 1, Z =0
- If Z result is greater or equal 1, Z = 1

OR GATE

OR GATE has the following desired result:

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1

This gate doesn’t use Bias, so the function formula will be:

Z= (W1 * X + W2 * Y >= 1.0)

OR GATE

Loop 1

W1/W2 = 0

X	Y	Z	Activation Function
0	0	0	0
0	1	0	0
1	0	0	0
1	1	0	0

Loop 2

W1/W2 = 0.5

X	Y	Z	Activation Function
0	0	0	0
0	1	0.5	0
1	0	0.5	0
1	1	1	1

OR GATE

Loop 3

W1/W2 = 1

X	Y	Z	Activation Function
0	0	0	0
0	1	1	1
1	0	1	1
1	1	2	1

Desired output found. So, OR GATE is under the function formula:

Z= (1 * X + 1 * Y >= 1.0)

NAND GATE

NAND GATE is similar to NOT AND GATE. So, NAND GATE output is:

AND Desired

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

NAND Desired

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

NAND GATE

Loop 1

W0 = 1

W1/W2 = 0

C	X	Y	Z	Activation Function
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

The value is to high compared to the desired result, so we have to decrease weight.

NAND GATE

Loop 2

W0 = 1

W1/W2 = -0.5

The value is now too low, so for this we have to increase weight. Since the step before we decrease W1/W2, I will increase only W0.

C	X	Y	Z	Activation Function
1	0	0	1	1
1	0	1	0.5	0
1	1	0	0.5	0
1	1	1	0	0

NAND GATE

Loop 3

W0 = 1.5

W1/W2 = -0.5

Desired output found. So, NAND GATE is under the function formula:

Z= (C*1.5 -0.5 * X -0.5 * Y >= 1.0)

C	X	Y	Z	Activation Function
1	0	0	1.5	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0.5	0

XOR GATE

Since XOR GATE:

Z = (X OR Y) AND (X NAND Y)

We have to figure out AND GATE as well.

AND GATE

AND Desired

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

Loop 1

The weights are too low, let’s increase it.

W1/W2 = 0

X	Y	Z	Activation Function
0	0	0	0
0	1	0	0
1	0	0	0
1	1	0	0

AND GATE

Loop

W1/W2 = 0.5

X	Y	Z	Activation Function
0	0	0	0
0	1	0.5	0
1	0	0.5	0
1	1	1	1

Desired output found. So, AND GATE is under the function formula:

Z= (0.5 * X + 0.5 * Y >= 1.0)

XOR GATE - Continuation

Since XOR GATE:

Z = (X OR Y) AND (X NAND Y) and we already figure out AND GATE. OR GATE output will be X and NAND GATE output will be Y. Applying the formula of AND GATE:

X = OR GATE

Y = AND GATE

Z = X AND Y

Z = (X OR Y) AND (X NAND Y)

Z = (1 * X + 1 * Y >= 1.0) AND (C*1.5 -0.5 * X -0.5 * Y >= 1.0)

Z= (0.5 * **(1 * X + 1 * Y >= 1.0)** + 0.5 ***(C*1.5 -0.5 * X -0.5 * Y >= 1.0)** >= 1.0)

XOR GATE - Continuation

Let’s test XOR GATE Formula:

Desired:

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

XOR GATE - Continuation

OR				NAND					XOR			
				W0	1.5							
W1/W2	1			W1/W2	-0.5				W1/W2	0.5		
X	Y	Z	Z1	C	X	Y	Z	Z2	X = Z1	Y = Z2	Z	Activation Function
0	0	0	0	1	0	0	1.5	1	0	1	0.5	0
0	1	1	1	1	0	1	1	1	1	1	1	1
1	0	1	1	1	1	0	1	1	1	1	1	1
1	1	2	1	1	1	1	0.5	0	1	0	0.5	0

Conclusion

Because the XOR GATE output is similar than was expected for XOR gate, the XOR GATE formula was proved as correct. So, XOR GATE is:

Z= (0.5 * **(1 * X + 1 * Y >= 1.0)** + 0.5 ***(C*1.5 -0.5 * X -0.5 * Y >= 1.0)** >= 1.0)