UNCERTAIN HEALTH DYNAMICS AND WORKING DECISIONS OF OLDER ADULTS*

Angela Denis†

October 22, 2020

PENDING: add link to most recent version

Abstract

As the population ages, there is a general concern and effort on trying to lengthen the labor-force participation of older adults, for whom health is an important determinant of working decisions. In this paper, I introduce heterogeneity in health dynamics with age and argue that uncertainty about this heterogeneity affects the working decisions of older adults. Using the Health and Retirement Study, I first show evidence of heterogeneity in the rate health changes with age, and use subjective survival expectations to infer health beliefs in a Bayesian-learning framework. I then estimate how working decisions depend on those beliefs flexibly, using a neural-network approach that does not require additional structure. The results show beliefs have substantial negative bias, that is, on average, individuals incorrectly believe their health will deteriorate too fast. Furthermore, eliminating that bias would increase labor-force participation by up to 2 percentage points. At the same time, changes in health translate into only small changes in beliefs. Hence, I look at one possible policy that could affect beliefs: the provision of information on blood glucose and cholesterol levels. The results show that this information has only small effects on beliefs and working decisions, and, consequently, policies with larger effects on beliefs are needed to delay retirement.

Keywords: health dynamics, older adults, retirement, uncertainty, beliefs

JEL Classification: D83, I14, J14, J26

*I would like to thank Stéphane Bonhomme, Alessandra Voena, Derek Neal, and David Meltzer for serving on my thesis committee and providing invaluable guidance and support. I am also grateful for the discussions and feedback of many people, including Siying Cao, Kai-Wei Hsu, Cristhian Seminario-Amez, the Labor Party student group, and the participants of the Econometrics working group and the public-labor seminar at the University of Chicago. I gratefully acknowledge financial support from the University of Chicago Social Sciences Division and the University of Chicago Kenneth C. Griffin Department of Economics

†University of Chicago. E-mail: adenis@uchicago.edu

1 Introduction

- Due to rapid population aging, the number of older people out of the labor force who will need to be supported by each worker is projected to rise by around 40% between 2018 and 2050 (OECD, 2019). From 42 per 100 workers in 2018 to 58 per 100 workers in 2050. In 2015, the OECD adopted an agenda to promote employment at older ages, to protect living standards and public finances
- There is extensive evidence on the effect of health and health shocks on older adults' working choices
  - Negative health shocks lead to early retirement (Kerkhofs and Lindeboom, 1997; Bound et al, 1999; Disney et al, 2006; French, 2005)
  - Changes in health affect retirement expectations of workers (McGarry, 2004)
- However, most of the literature allows only for limited heterogeneity, and assumes that health changes with age at an homogeneous rate across individuals, i.e. homogeneous dynamics.
- In this paper, I show evidence of heterogeneity in the dynamics of health, in particular in the rate health changes with age
- The effect this heterogeneity has on working decisions depends on how much individuals know about it. Hence, a key question is how uncertain individuals are about their future health.
- I argue there is uncertainty an that it plays a role in the working decisions of older adults. Thus, the goal of this paper is to document this heterogeneity and to study its effects on working decisions of older adults.
- In particular, I want to quantify how uncertain individuals are about their heterogeneous profile, and how accurate are their beliefs.
- Furthermore, I want to measure the effects that beliefs about future health play a role in the working decisions of older adults.
- Finally, I ask whether we can change working behavior of older adults by providing additional information and changing their beliefs.
- Use longitudinal data from the Health and Retirement Study (HRS), and answer these questions in steps.
- First, I estimate a dynamic model for health with heterogeneity in levels and change rates with age, showing evidence of heterogeneity in health dynamics
- Second, I study individuals beliefs about that heterogeneity, and propose a Bayesian learning model, where individuals update beliefs as their health changes over time.
- To identify the key parameter of beliefs, I use data on *subjective survival expectations*. The level of this variables speak to bias in beliefs, while its changes with health speak to uncertainty in beliefs. This is because the more uncertain an individual is the more he update his beliefs with a same signal, in this case, a change in health.

- This intuition is proved for an ideal variable of survival expectations (expectations rates instead of probabilities), and show in simulations the intuition also holds for the observed data on survival.
- I find evidence of uncertainty about health profiles and systematic negative bias in beliefs about health changes, according to which individuals expect their health to deteriorate faster on average than what actually happens.
- Third, I turn into the working decisions of older adults. In a life cycle model, working decisions = f(health,assets,income, and beliefs about future health), i.e. beliefs are part of the state variables. Under Bayes' assumptions, we only need to keep track of mean and variance of beliefs
- I estimate working decisions as function of these beliefs. First, I do it assuming a probit error with linear index and find a positive effect of beliefs. However, this is a strong assumption that I do not derive from primitives of the model.
- Instead, I estimate the policy rule flexibly, using neural networks. The flexibility of neural networks inform these models without imposing additional structure. The use of survival expectations to infer beliefs means that no assumptions are required for the relation between the outcome and beliefs, which can be fully flexible.
- As some of the inputs are unobserved to the econometrician, including beliefs, I use an interative approach in the same spirit as EM algorithms.
- I find, first, better expected health is associated with larger probability of work (positive marginal effects). Second, that eliminating initial bias in beliefs would increase participation by more than to 2pp. Third, the effects of health shocks on participation are mostly due to persistence of the health process, with negligible information effects by affecting beliefs
- Preliminary evidence also shows that beliefs matter for other outcomes: health insurance and assets
- Consequently, an important question relates to whether we can shift beliefs and affect working decisions of older adults. I study that question using an information experiment available in the HRS, where additional health information (in the form of blood-based biomarkers) is collected and provided to a random half of the sample
- The objective of the HRS was not to run an experiment but to save cost, and hence this is not a proper experiment, and it does not have the right control group (individuals for whom the blood-base biomarkers are collected but not informed). Still, it provides us with exogenous variation that I can use to analyze the effects of additional information, both through the lens of the data and through the lens of the model
- I find the information experiment has only small effect on expectations and negligible effect on working decisions. Reduced form evidence show larger effects for college educated individuals (model results by education pending)
- Overall, the positive effect of beliefs on working decisions implies that policies targeting those beliefs have the potential of affecting outcomes that are the focus of many public policy debates, like

retirement age and health insurance coverage. However, I conclude that larger signal are required to shift beliefs enough to affect working decisions of older adults.

- This paper relates to three strands of the literature: a literature on health dynamics, where my contribution is allowing heterogeneous dynamics and uncertainty; a literature on micro/empirical learning, where my contribution is to combine expectations and outcome data and to allow for systematic bias; and a literature on outcomes of older individuals and effects of health, where my contribution is adding a role for health beliefs.

2 Framework

This paper introduces two elements into a standard model of labor-participation decisions in late life: heterogeneity in health dynamics and incomplete information about that heterogeneity. This section formalizes this idea and describes a framework in which older adults choose labor participation based on their health and on their beliefs about how their health will change with age. Let i denote an individual and let t denote his age. I focus on individuals fifty years and older and define t as zero for age fifty.

2.1 Health process with heterogeneous dynamics

Health is a dynamic process that, as people get older, naturally deteriorates in a heterogeneous way across individuals. In particular, I assume,

$$h_{it} = \rho h_{it-1} + \alpha_i + \delta_i \cdot t + \epsilon_{it}, \tag{1}$$

where the parameter $\rho \in (0, 1)$ captures persistence in health, α_i captures heterogeneous levels in health, δ_i captures heterogeneous changes of health with age, and ϵ_{it} represents health shocks. The first novel element in this paper is to allow for heterogeneous slopes of health with age, δ_i. Larger values of h_{it} represent better health, and health decreases with age. Therefore, $\delta_i < 0$. Give antecedents for this equation

2.2 Uncertain health dynamics and beliefs

The second novel element is to allow for individuals to be uncertain about their own health dynamics. Given that health deteriorates in old age, I assume 50-years old individuals do not know δ_i, which has not affected them before. For simplicity, I assume they know their heterogeneous level α_i,[1] as they have observed their health for several decades. I assume further that health shocks are unobserved, and individuals observe their overall health.

Under uncertainty, rational individuals form beliefs about their health slopes δ_i (henceforth, slope beliefs) and update those beliefs as they see their health changing with age. In particular, I assume

[1] This assumption can be generalized. In studying income profiles, Guvenen (2007) proposes a similar process with heterogeneous intercepts and slopes, both unknown. He finds the learning process for intercepts is much faster than the learning process for slopes.

individuals are Bayesian learners, with initial beliefs (at age 50) about δ_i equal to $N(\hat{\delta}_{i0}, \hat{\sigma}_0^2)$.[2] By further assuming that health shocks ϵ_{it} are i.i.d normally distributed, posterior beliefs in period t after observing health h_{it} are also normally distributed, $N(\hat{\delta}_{it}, \hat{\sigma}_t^2)$, with mean and variance defined recursively by

$$\frac{\hat{\delta}_{it}}{\hat{\sigma}_t^2} = \frac{\hat{\delta}_{it-1}}{\hat{\sigma}_{t-1}^2} + \frac{(h_{it} - \rho h_{it-1} - \alpha_i)t}{\sigma_\epsilon^2} \tag{2}$$

$$\frac{1}{\hat{\sigma}_t^2} = \frac{1}{\hat{\sigma}_{t-1}^2} + \frac{t^2}{\sigma_\epsilon^2}. \tag{3}$$

Equation (2) shows the posterior mean is a weighted average of the prior mean and the signal, with weights that depend on precision. The more certain an individual is to begin with (lower $\hat{\sigma}_{t-1}^2$), the more weight he gives to what he already knows, namely, the prior. The more precise is the signal (lower σ_ϵ^2) more weight is given to its information. Equation (3) shows precision increases over time, and increases more when the signals are more precise.

Conditional on health history, the key parameters determining beliefs are the parameters governing initial beliefs,

$$b = \mathbb{E}(\hat{\delta}_{i0} - \delta_i) \tag{4}$$

$$\lambda^2 = \frac{\hat{\sigma}_0^2}{Var(\delta_i)}. \tag{5}$$

The parameter b measures the bias in initial beliefs at the population level. If $b = 0$, individuals are overall unbiased, in the sense that $\mathbb{E}(\hat{\delta}_{i0}) = \mathbb{E}(\delta_i)$. If b is positive (negative), individuals are upward (downward) biased and, hence, they believe health deteriorates on average more slowly (faster) than the average rate. The parameter λ measures the degree of initial uncertainty individuals face with respect to δ_i, which affects the amount of learning they do over time. Setting $\lambda = 0$ implies no uncertainty and therefore no learning. The larger the value of λ the more uncertain individuals are and the more weight they give to new information. The Bayesian learning assumption allows me to reduce the dimensionality of the problem, giving structure to time-varying beliefs that are unobserved by the econometrician.

2.3 Embedding health uncertainty in a model of labor supply

In a life-cycle model, forward-looking individuals attempt to predict variables that will affect their future utility or future set of options in order to choose their best current action. The need for those predictions is given by the inherent uncertainty on many key variables. In this paper, I focus on working decisions of older adults and argue that a key source of uncertainty for this group is related to their future health. In particular, I focus on the heterogeneous way health changes with age δ_i and how beliefs about them, given by $N(\hat{\delta}_{it}, \hat{\sigma}_t^2)$, relate to their working decisions.

Consider a model where individual i must choose consumption c_{it} and labor participation p_{it} every

[2] The assumption of common-prior variance across individuals is usual in the learning literature. See for example, add examples. It is, however, an important assumption for the identification results provided later.

period. I focus on the extensive margin of labor participation and assume p_{it} is a binary decision.[3] Individual i's health is given by h_{it}, which follows equation (1). The main components of this life-cycle model are the following.

Preferences. Individual i's flow utility is given by a function U that depends on his participation and consumption decisions, p_{it} and c_{it}, as well as on his health h_{it}. Furthermore, preferences depend on past labor participation, for example, to reflect psychological costs of going back to work after retirement and adjusting to a new work environment. I summarize this dependence by allowing p_{it-1} to enter the utility function. Hence, flow utility is given by $U(p_{it}, c_{it}, h_{it}, p_{it-1})$. The individual discounts the future, and when he dies his remaining assets a are left as a bequest.

Budget constraint. Let a_{it-1} denote individual i's assets at the end of period $t-1$. If the individual chooses to work, he receives labor income, which depends on his past labor income w_{it-1}, his health h_{it} through the effects of health on productivity, and his past participation p_{it-1}, through wage penalties of reentering the labor market after retirement. His assets at the end of the period depend also on his consumption choice, his other sources of income, including pension and social security, and other health-related costs.

Uncertainty. Individuals are uncertain about their future health, in part because due to unpredictable health shocks ϵ_{it}, and in part because because they don't know their health slopes δ_i. They form beliefs about their slopes δ_i and update those beliefs as they see their health changing over time according to equations (2) and (3). Future wages are also uncertain, and assume random conditional on past information.

Timing. At the beginning of period t, an individual must choose participation p_{it} and consumption c_{it} before health shocks are realized and health h_{it} is observed. Then, beliefs are updated. At the end period t, individual i may or may not die.

Information set. The information set of individual i at the beginning of period t is given by his history up to $t-1$ in terms of labor participation p_i^{t-1} (where superscripts denote histories), consumption c_i^{t-1}, and health h_i^{t-1}, as well as labor income w_i^{t-1}. It also includes his known value α_i and his prior-beliefs parameters $\hat{\delta}_{i0}$ and $\hat{\sigma}_0^2$. The relevant information from this set can be summarized in his state variables, given by

$$\Omega_{it-1} \quad = \quad \left\{p_{it-1}, a_{it-1}, w_{it-1}, h_{it-1}, \hat{\delta}_{it-1}, \hat{\sigma}_{t-1}^2, \alpha_i\right\}.$$

Slope uncertainty implies δ_i does not belong to Ω_{it-1}, but beliefs about δ_i do, with those beliefs summarized by $\hat{\delta}_{it-1}$ and $\hat{\sigma}_t^2$. Note that I am assuming only heterogeneity in health; thus, no other individual-level heterogeneity is stated in Ω_{it-1}. A natural extension of this research is to allow for more flexible dimensions of heterogeneity.

[3] Add references in the literature about this, whether explaining the median number of hours, or the lack of flexibility of older individuals in finding part time jobs, or whatever holds.

The solution to this problem is a policy rule for labor participation, which is a function of the state variables and the parameters of the model θ (parameters defining flow utility, health, parameters entering the budget constraint, discount factor and so on), which I omit for ease of notation:

$$\mathbb{P}(p_{it} = 1|\Omega_{it-1}) \quad = \quad \mathbb{P}(p_{it} = 1|p_{it-1}, a_{it-1}, w_{it-1}, h_{it-1}, \hat{\delta}_{it-1}, \hat{\sigma}^2_{it-1}, \alpha_i) \tag{6}$$

Similarly, policy rules for other decisions, including consumption and assets, can be written as functions of the state variables Ω_{it-1}.

With these elements, the model is a standard model of labor participation in late life and includes several channels through which health can play a role. First, health directly affects utility by changing the marginal utility of consumption and the disutility of work. Second, it enters the budget constraint via health-related costs and via effects on labor income due to changes in productivity. Third, health affects the probabilities of survival. The overall effect of health on individuals' participation decisions depends on all of these channels. The novel element in this paper is that beliefs about future health also play a role. They could have a positive or negative effect depending on the relative importance of these channels in the individual's problem.

2.4 Objective of this paper

The objective of this paper is, first, to document the heterogeneity in health dynamics among older adults; second, to study their beliefs about their health dynamics; and third, to examine whether these beliefs have an effect on their working decisions, by studying the effect of marginal changes in beliefs on those decisions,

$$\frac{\partial \mathbb{P}(p_{it} = 1|\Omega_{it-1})}{\partial \hat{\delta}_{it-1}} \tag{7}$$

I estimate this relation flexibly, without imposing any additional structure on the model of labor supply. However, this framework could be applied under a structural approach under additional assumptions. To get robust results, I study these questions while avoiding making those extra assumptions. Nevertheless, those additional assumptions would allow me study some interesting counterfactuals, which left for future research.

In this context of uncertain health dynamics, an interesting question is related to the dual role of health shocks ϵ_{it-1} in working decisions. On the one hand, a health shock ϵ_{it-1} moves h_{it-1}, which in turn affects h_{it} through persistence of the health process. This persistence effect disappears if $\rho = 0$. On the other hand, an uncertain individual can not perfectly distinguish ϵ_{it-1} within h_{it-1}. Hence, the shocks' effect on health is partly interpreted as new information regarding δ_i, changing beliefs $\hat{\delta}_{it-1}$. This information channel disappears if $\lambda = 0$. Using Bayes' rule, we can write

$$\frac{d\mathbb{P}(p_{it}=1|\Omega_{it-1})}{d\epsilon_{it-1}} = \underbrace{\frac{\partial\mathbb{P}(p_{it}=1\Omega_{it-1})}{\partial h_{it-1}}}_{\text{persistence channel}} + \underbrace{\frac{\partial\mathbb{P}(p_{it}=1|\Omega_{it-1})}{\partial\hat{\delta}_{it-1}} \overbrace{\frac{(t-1)\hat{\sigma}^2_{t-1}}{\sigma^2_\epsilon}}^{factor}}_{\text{information channel}}, \tag{8}$$

where the change in the posterior mean $\hat{\delta}_{it-1}$ depends on prior uncertainty, as well as on the signal' strength.

3 Data and some statistics

For this study, I use data from waves 4 to 12 of the Health and Retirement Study (HRS), a longitudinal survey representative of the population 50 years and older in the US. This survey interviews individuals and their spouses every two years and includes several measures of health, questions about expectations, information about labor participation and retirement, as well as income and wealth variables.[4]. For most of the analysis, I use the RAND version of these data. In this section, I briefly describe the variables more pertinent to this work: health measures and survival expectations.

Add table of summary statistics of the main variables (health, survival expectations, working decisions, demographics, assets, pension

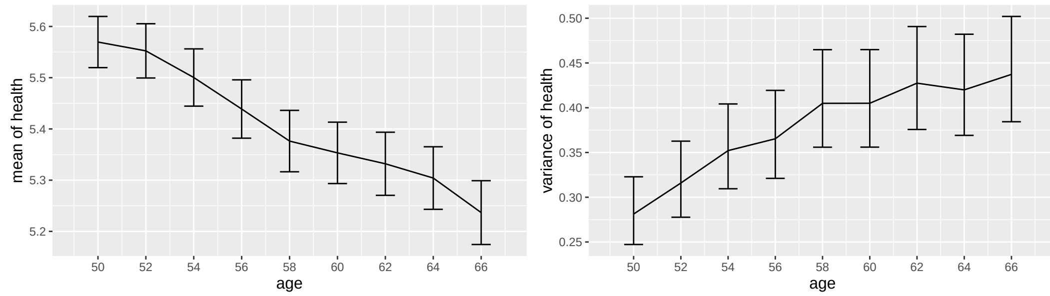
3.1 Data on health

The most common measure of health used in the literature is *self-assessed health*, an ordinal variable taking five values from very poor to excellent. It has been shown to correlate with several variables, including education, income, savings, retirement and health insurance. Still, its limited range and its subjective nature make it not ideal in studying health changes with age. The HRS, however, provides a larger battery of health-related questions, which I exploit to construct a summary measure of health via factor analysis that I use throughout the paper. The appendix lists the health measures used and provides details on the factor analysis estimation of the summary measure h_{it}. The scale of the summary measure h_{it} is set to be the inverse scale of the number of chronic conditions, which range from 0 to 7. Therefore, larger values of h_{it} represent better health, and an increase of one unit in h_{it} corresponds to one less chronic condition.

Figure 1 shows the mean and variance of health by age. Given the two-years time between waves, throughout this paper I consider age as measured in two-years bins. These plots are the starting point for thinking about health in old age: they show that with age, the average health in the population decays while the variance of health in the population increases. This pattern of decreasing mean and increasing variance is robust to sample composition and also holds for most of the individual measures, as shown in the appendix. Similarly, Figure 2 shows percentiles of health with age, which also reflect an increasing

[4] I use only in-person interviews and exclude proxy interviews, because questions on survival expectations are not asked in proxy interviews.

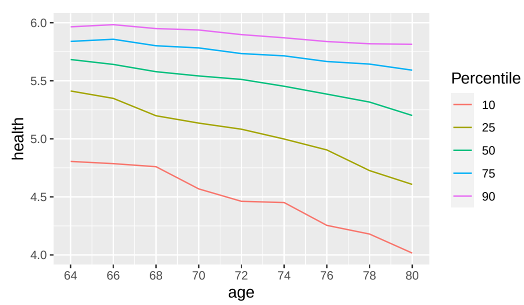
Figure 1: Mean and variance of health with age



(a) Mean of h_{it} with age (b) Variance of h_{it} with age

Note: Results from a balanced sample of 433 individuals observed at 50 years with at least 9 consecutive waves. The bars represent the 95% confidence intervals. Standard errors need to be adjusted to account for the estimation of the health summary measure.

Figure 2: Health percentiles with age



(a) Mean of h_{it} with age

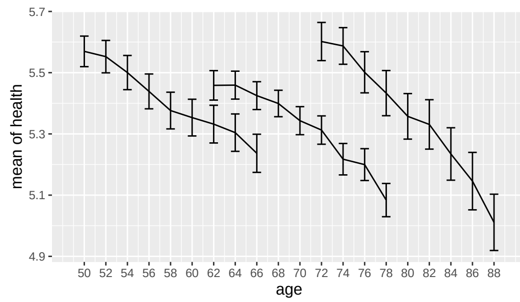
Note: Results from a balanced sample of 414 individuals observed at age 64 with at least 9 consecutive waves.

variance over time. The pattern in these plots suggests a process with heterogeneous slopes with age, which I empirically investigate next. Furthermore, Figure 3 shows the mean of health for different cohorts of individuals. It suggest survival bias, because cohorts of individuals surviving to older ages have better health than cohorts that may not survive that long. Add plot relating health and SAH

3.2 Data on subjective survival expectations

The HRS includes a battery of questions relating subjective expectations, including subjective survival expectations, which is relevant in this paper. The question asks *What is the percentage chance you will live to be (80, 85, 90, 95 or 100) or more?*, where the reference age is a function on the individual's age and the wave of the survey. Mention what is know from the literature. Add a histogram plot

Figure 3: Mean of health with age for different cohort of individuals



(a) Mean of h_{it} with age

Note: Results from three balanced samples of individuals with at least 9 consecutive waves: 433 individuals observed from age 50, 509 individuals observed from age 62, and 153 individuals observed from 72. The bars represent the 95% confidence intervals. Standard errors need to be adjusted to account for the estimation of the health summary measure.

4 Health process with heterogeneous dynamics

This section estimates a health process with heterogeneous intercepts and slopes. As Figure 3 suggests, for a population of older adults, we need to control for survival bias, which I address by jointly modeling the two processes, given the lack of a suitable instrument affecting survival chances but not health.

4.1 Empirical strategy

Let S_{it} be a binary variable for surviving up to the beginning of period t with $S_{i0} = 1$ and let the health and survival processes be given by

$$\begin{aligned} h_{it} &= \rho h_{it-1} + \alpha_i + \delta_i \cdot t + \tau \cdot t^2 + \epsilon_{it}, & \epsilon_{it}\ it\text{-iid } N(0, \sigma^2_\epsilon) \quad (9) \\ S_{it} &= \mathbb{1}\{\gamma h_{it-1} + \theta_0 + \theta_1 \cdot t + \theta_2' x_i + \eta_{it}\} S_{it-1}, & \eta_{it}\ it\text{-iid } N(0, 1) \quad (10) \end{aligned}$$

with individual-level heterogeneity (α_i, δ_i),

$$\begin{pmatrix} \alpha_i \\ \delta_i \end{pmatrix} \Bigg| x_i, h_{i0} \sim N\left(\begin{pmatrix} \mu_\alpha + \nu_\alpha' x_i + \omega_\alpha h_{i0} \\ \mu_\delta + \nu_\delta' x_i + \omega_\delta h_{i0} \end{pmatrix}, \begin{bmatrix} \sigma^2_\alpha & \phi \sigma_\alpha \sigma_\delta \\ \phi \sigma_\alpha \sigma_\delta & \sigma^2_\delta \end{bmatrix} \right), \quad (11)$$

The health process has the same features described before, with heterogeneous levels α_i, heterogeneous sloes δ_i and persistence parameter ρ. The survival process depends on health, through the parameter γ_1, and it depends on age, through θ_1. η_{it} denotes survival shocks, assumed independent of health shocks. The variables in x_i are time-invariant binary variables for female, white, Hispanic, and an education level below high school graduation. These variables potentially affect health (through the individual-level heterogeneity) and survival. I also allow for the unobserved heterogeneity to depend on health h_{i0} (health at age 50) in order to address initial-conditions concerns.

The panel structure of the data provides identification for the distribution of α_i and δ_i under these

Table 1: MLE results on health and survival

	Symbol	Coefficient	Pvalue
Persistence	ρ	0.222	0.000
Mean* of α_i	μ_α	0.967	0.000
Mean* of δ_i	μ_δ	-0.059	0.006
SD of α_i	σ_α	0.236	0.000
SD of δ_i	σ_δ	0.043	0.000
$Corr(\alpha_i, \delta_i)$	ϕ	-0.034	0.605
SD of health shocks	σ_ϵ	0.266	0.000
Survival dependence on health	γ_1	0.656	0.002
Controls		Yes	
N alive observations		8,901	
N dead observations		112	
N individuals		1,671	
-Log likelihood		3,021.1	

Note: Main results of estimating equations (9), (10) and (11). Full set of results are shown in the appendix. The standard errors need to be adjusted to account for the estimation of the health measure.

assumptions.

Let Θ be the set of parameters of this random-coefficients model,[5] which I estimate by maximum likelihood:

$$\max_{\Theta} \sum_{i=1}^{N} log\Big(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \prod_{t=1}^{T_i} \mathbb{P}\big(h_{it}, S_{it} | h_{it-1}, S_{it-1} = 1, x_i, \alpha, \delta\big) \cdot \phi(\alpha, \delta | x_i, h_{i0}) d\alpha d\delta \Big).$$

4.2 Results

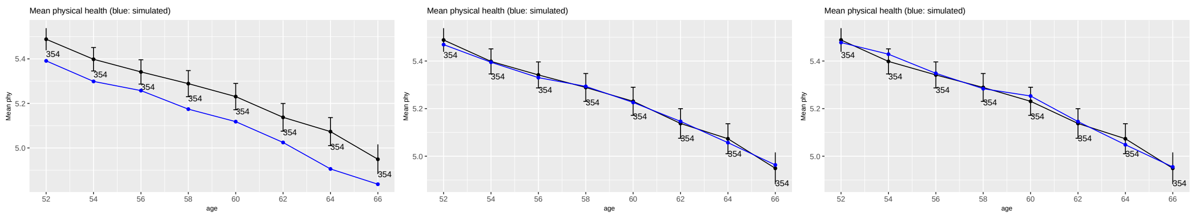
I use a sample of 8,901 correlative observations from 1,671 individuals observed since they were 50 years old ($t = 0$).[6] Over the span of the following eight waves, 112 of these individuals died. The main results are shown in Table 1 and full results are shown in the appendix. The table shows, first, heterogeneity in both the intercepts and the slopes of the health process, with positive and significant σ_α^2 and σ_δ^2. Second, these two sources of heterogeneity are uncorrelated, which implies knowing α_i does not provide additional information on δ_i. Health decreases with age and the persistence of the health process is relatively low, with $\rho = 0.22$. The results in the appendix further show low levels of education are associated with worse health every period, health decreases faster for white individuals, and women and Hispanic individuals have higher probabilities of survival on average. Those results also show h_{i0} is correlated with α_i, but it does not provide information on δ_i.

I want to emphasize two aspects of this model: the inclusion of heterogeneous slopes with age and the

[5] $\Theta = \{\rho, \tau, \sigma_\epsilon^2, \gamma, \theta_0, \theta_1, \theta_2, \mu_\alpha, \mu_\delta, \nu_\alpha, \nu_\delta, \omega_\alpha, \omega_\delta, \sigma_\alpha^2, \sigma_\delta^2, \phi\}$

[6] I approximate the double integral by using one thousand draws from a bivariate normal distribution.

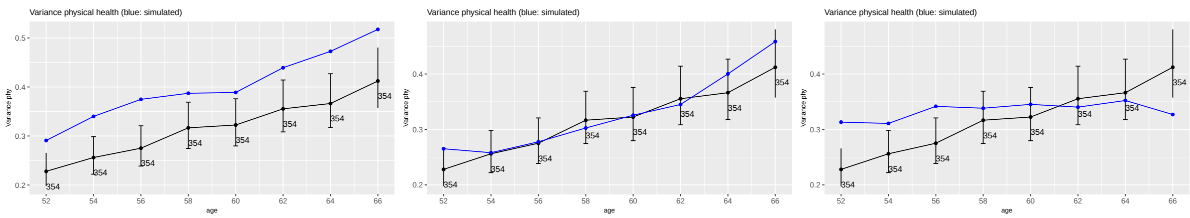
Figure 4: Mean and variance of predicted health in three different versions of the model



(a) Heterogeneous slopes without survival equation

(b) Heterogeneous slopes and survival equation

(c) Homogeneous slopes and survival equation



(d) Heterogeneous slopes without survival equation

(e) Heterogeneous slopes and survival equation

(f) Homogeneous slopes and survival equation

Note: The sample consists of 26,950 correlative observations from 7,301 individuals observed since they were 66 years old. Over the span of the following eight waves, 996 of them died. The black lines plot the health data and the blue lines plot the predicted values of health in each model. The standard errors need to be adjusted to account for the estimation of the health measure. pending: improve labels

joint estimation with survival. To understand how these two aspects influence my results, I estimate two additional versions of the model: (i) one excluding the equation for survival but allowing for heterogeneous slopes with age, and (ii) another one assuming homogenous slopes with age but including an equation for survival. The results are in the appendix and show qualitatively similar results for the coefficients that are common across specifications. Their main difference is that ignoring slope heterogeneity increases the point estimate of the persistence parameter ρ by over 50% (from 0.22 to 0.37). However, a key takeaway is that these models achieve very different fits of health over time. This takeaway is more clearly seen in Figure 4, which repeats the exercise for a sample of individuals observed from 66 years old and plots the predicted mean and variance of health with age. The figure shows that ignoring survival leads to a downward bias of average health and an upward bias of its variance, consistent with a model that includes the lower tail of the health distribution, which is dropped from the data as people die. The figure also shows that when ignoring slope heterogeneity, we predict a rather constant variance of health, contrary to what the data show. In that sense, these plots support a model with slope heterogeneity, though they don't discard alternative explanations. As a robustness check, I estimate a version with heteroskedastic error ϵ_{it} allowing for its variance to depends on age. The results shows increasing variance of health shocks do not explain away the heterogeneity in slopes δ_i.

Finally, I add two robustness checks included in the appendix. First, I estimate a similar model using *self-assessed health* instead of the constructed summary measure of health. The results show the presence of heterogeneous slopes with age is robust to the use of this measure alone. Second, I estimate a version of the model adding the unobserved heterogeneity (α_i, δ_i) directly to the survival equation. The results show (α_i, δ_i) is not (jointly) significant, which implies being alive does not provide an additional signal on the heterogeneous slopes.

Table 1 shows evidence of that slope heterogeneity, but it does not reveal anything about how much individuals know or don't know about their own slope δ_i, which I address next.

5 Uncertain health dynamics and beliefs

To study the effect of beliefs on labor-participation decisions of older adults, the main difficulty is that those beliefs are unobserved by the econometrician. The Bayesian learning model implis beliefs are updated over time using health, starting from initial beliefs. Hence, a key issue is the identification of those initial beliefs, that is, how much individuals know about their slopes at age 50 and how biased those beliefs are to begin with. Because the health process does not reveal slopes beliefs, this section proposes the use of *survival expectations*, available in the data. Equation (10) implies survival is a health-related process given. Therefore, expectations about future survival are related to expectations about future health; thus, they are related to slope beliefs.

5.1 Empirical strategy

The exact wording of the HRS questions follows:

> $[plive10_{it}]$ *What is the percentage chance you will live to be (80, 85, 90, 95 or 100) or more?*,

where the reference age depends on the individual's age t at the time of the survey (and wave), and it is approximately 10 years in the future. Let s denote this reference age. Then, this question corresponds to

$$plive10_{it} \quad = \quad \mathbb{P}(S_{is} = 1|\Omega_{it}) = \prod_{l=t+1}^{s} \mathbb{P}(S_{il} = 1|S_{il-1} = 1, \Omega_{it}) = \prod_{l=t+1}^{s} \mathbb{P}(\gamma_1 h_{il} + \eta_{il+1} \geq 0|\Omega_{it}).$$

Applying the equation for health (9) recursively, we can write:

$$h_{il} \quad = \quad \underbrace{\rho^{l-t} h_{it} + \alpha_i \sum_{k=0}^{l-t-1} \rho^k}_{\text{known under } \Omega_{it}} + \underbrace{\delta_i \sum_{k=0}^{l-t-1} (l-k)\rho^k + \sum_{k=0}^{l-t-1} \rho^k \epsilon_{i(l-k)}}_{\text{unknown under } \Omega_{it}}.$$

From the view point of Ω_{it}, the second term is random, with a normal distribution that depends on $(\hat{\delta}_{it}, \hat{\sigma}_t^2)$ (and the parameters of the model). Because age-t beliefs depend on health history h_i^t and initial beliefs $N(\hat{\delta}_{i0}, \hat{\sigma}_0^2)$, this second term is a function of λ and b. Therefore,

$$plive10_{it} \quad = \quad plive10_{it}(\alpha_i, h_{it}, \hat{\delta}_{it}, \hat{\sigma}_t^2) = plive10_{it}(\alpha_i, h_i^t, \hat{\delta}_{i0}, \hat{\sigma}_0^2).$$

Exact formulas are given in the appendix. They shows $plive10_{it}$ are complex non-linear functions of slope beliefs.

Each period, individuals observe their health and update their beliefs regarding their unknown δ_i. This new information allows them to also update their expectations about their future health, and hence, their expectations about future survival. Hence the slopes bias, unobserved by the econometrician, are closely linked to survival beliefs, which are observed by the econometrician. Intuitively, the bias parameter b affects expected health and, hence, the average survival expectation. Thus, levels of *survival expectation* identify bias b. In what follows, I discuss identification of the uncertainty parameter λ.

Identification using subjective expectations about survival rates (ideal data)

I start by discussing identification using ideal data, which I do not actually observe. Let Ω_{it} be the information set of individual i after observing his health up to period t. Thus, $\alpha_i, \hat{\delta}_{it}, \hat{\sigma}_t^2 \in \Omega_{it}$.

Proposition 5.1 (Identification of λ) *Let health and survival process by given by equations (9) and (10), and assume individuals are Bayesian learners with prior beliefs about δ_i following $N(\hat{\delta}_{i0}, \hat{\sigma}_0^2)$. Consider the subjective expectations about survival rates:*

$$\begin{aligned} bsr_{it} &\equiv \mathbb{P}(S_{it+3} = 1 | S_{it+2} = 1, \Omega_{it}) \\ bsr_{it+1} &\equiv \mathbb{P}(S_{it+3} = 1 | S_{it+2} = 1, \Omega_{it+1}). \end{aligned}$$

Then,

$$Cov(\Delta_w \Phi^{-1}(bsr_{it+1}), \Delta h_{it+1}) \quad = \quad \mathbb{C}_t Var(\Delta h_{it+1}),$$

where the time-varying constant $\mathbb{C}_t$ is increasing in λ.

The proof is in the appendix. The proposition says we can identify λ with longitudinal data on subjective expectations about survival rates and health. The key equation behind this result,

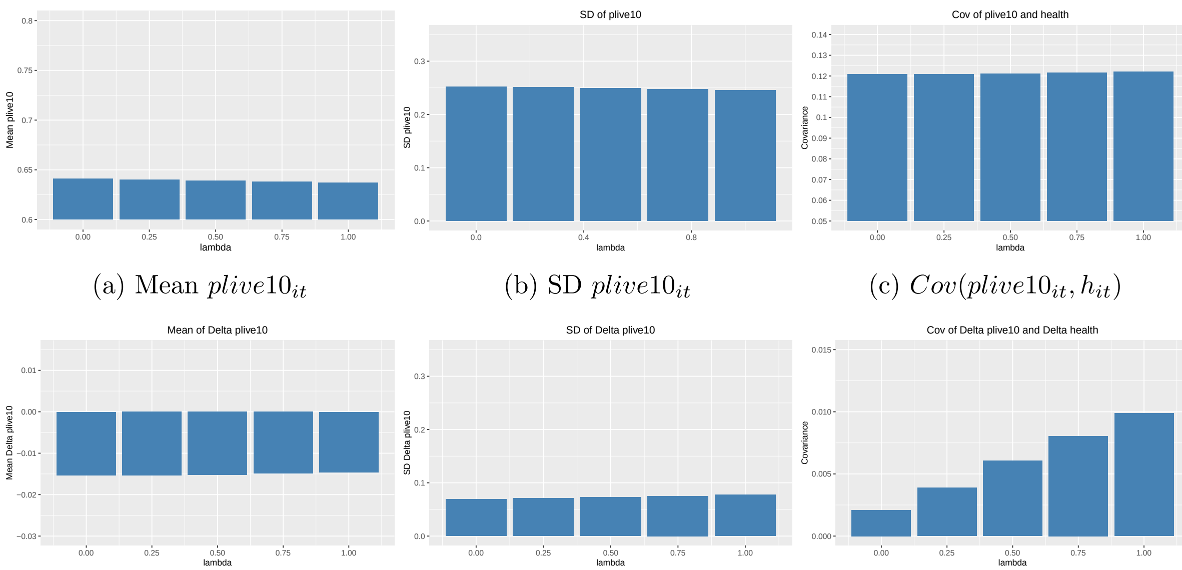
$$\Delta_w \Phi^{-1}(bsr_{it+1}) = \underbrace{\rho(h_{it+1} - \rho h_{it} - \alpha_i - \hat{\delta}_{it}(t+1) - \beta x_{it+1})}_{\text{due to persistency}} + \underbrace{(t+2)(\hat{\delta}_{it+1} - \hat{\delta}_{it})}_{\text{due to learning}}, \tag{12}$$

shows individuals update their survival expectations for two reasons. The first reason is that health is a persistent process; thus, any change in health will have future repercussions on health and therefore on survival. Note that if $\rho = 0$, this channel disappears. The second reason is that learning implies a change in future predictions of health and therefore of survival. Note that if $\lambda = 0$, $\hat{\delta}_{it+1} = \hat{\delta}_{it}$ and this channel disappears.

Identification using subjective expectations about survival probabilities (available data)

We can not use the previous result directly, because the HRS does not exactly measure subjective expectations about survival rates. However, Figure 5 shows the intuition of proposition 5.1 extends to the available data. It shows the results of a simulation exercise, where I simulated data on h_{it} and $plive10_{it}$ for different values of the uncertainty parameter λ, plotted in the x-axis of each figure. The six plots correspond to the six moments used later for estimation. The top row considers moments in levels, and the bottom row considers moments in differences. The figure clearly shows that, as before, the covariance between changes in health and changes in survival expectations depends on the underlying uncertainty.

Figure 5: Simulated moments of $plive10_{it}$ by uncertainty λ in data-generating process



(a) Mean $plive10_{it}$ (b) SD $plive10_{it}$ (c) $Cov(plive10_{it}, h_{it})$

(d) Mean $\Delta plive10_{it}$ (e) SD $\Delta plive10_{it}$ (f) $Cov(\Delta plive10_{it}, \Delta h_{it})$

Note: pending: explain briefly. Improve labels

This model has two simplifying assumptions. First, the model assumes the health process is exogenous, with no choice variable that affects the evolution of health; that is, no investment is purposefully made in the form of health behaviors (like exercising or smoking) and working decisions do not affect health. This assumption is not uncommon in the literature on labor market decisions among older individuals, and it emphasizes changes in health due to aging. Second, the model assumes health is the only signal available to individuals and with no endogenous acquisition of health information (e.g., demand for preventive care). This assumption is more restrictive in this context, and I address it in the last section of the paper by looking at another source of information that may shift beliefs, hence reducing reliance on the model.

Under these assumptions, $plive10_{it}$ is a function of initial beliefs $N(\hat{\delta}_{i0}, \hat{\sigma}_0^2)$, heterogeneous intercept α_i, and health history up to t, $(h_{i0}, \ldots h_{it})$. Hence, for any value of b and λ, I can use the estimated health process to simulate draws α_i and $\hat{\delta}_{i0}$, and then use those variables to simulate $plive10_{it}$.[7] I estimate the parameters governing initial beliefs, b and λ, by simulated method of moments. I use six moments, three in levels and three in differences, corresponding to the mean of $plive10_{it}$, its variance, and its covariance with h_{it}.[8] Details of the implementation are given in the appendix.

[7] The distribution of $\hat{\delta}_{i0}$ depends on b and λ. Hence, I first simulate α_i and δ_i conditional on health history $h_{i0}, \ldots h_{iT_i}$, and then for a given value of b and λ, I draw $\hat{\delta}_{i0}$ conditional on α_i, δ_i, and h_{i0}.

[8] As described in the appendix, most individuals are first observed in sample at age t_0 older than 50, and I modify the simulation process for them accordingly. Overall, I target these six moments averaged across time for different subgroups

Table 2: Estimated parameters of prior beliefs

	Symbol	Coefficient	Lower bound	Upper bound
Uncertainty	λ	0.338	0.336	0.340
Bias	b	-0.061	-0.061	-0.060
Mean of measurement error	μ_{merror}	0.121	0.118	0.123
SD of measurement error	σ_{merror}	0.177	0.176	0.177

Note: Prior beliefs about slopes are unobserved $N(\delta_i^k+b, \lambda^2\sigma_\delta^2)$, whereas subjective survival expectations $plive10_{it}$ are observed but measured with error. The estimation uses a subsample of 2,000 individuals with eight periods, chosen randomly for computational reasons. Moments are simulated using 20 draws of measurement error and 20 draws of unobserved heterogeneity. The bounds correspond to a 95% confidence interval. The standard errors need to be adjusted to account for the estimation of the health measure.

Subjective survival expectations are measured with substantive error, which is well established in the literature (add examples). Similar to Kleinjans and Van Soest (2014), I allow for non-classical i.i.d. measurement error $\nu_{it} \sim N(\mu_{merror}, \sigma^2_{merror})$, such that the observed survival expectations are given by

$$\widetilde{plive}10_{it} \quad = \quad \max\{\min\{plive10_{it} + \nu_{it}, 1\}, 0\}.$$

Note the measurement error shifts *observed survival expectations* by μ_{merror} on average. Similarly, the bias in initial beliefs b also shifts *observed survival expectations*. However, these two biases have different effects over time: the average shift due to measurement error is constant on age, given the i.i.d. assumption, while the average shift due to initial bias in beliefs is decreasing with age as individuals observe their health and update their beliefs. Thus, we can separately identify both effects.

5.2 Results

The estimation results presented in Table 2 show individuals face a sizable amount of uncertainty and a large amount of negative initial bias; that is, though uncertain, individuals believe their health will decay with age at a faster rate than what is actually true on average. Add a more clear interpretation of these two magnitudes. In line with previous literature, subjective survival expectations are subject to large amounts of measurement error. Following Manski and Molinari (2010), I also estimate a version including rounding and find similar results (not shown). Panel a of Table 3 shows the fit of the targeted moments using $plive10_{it}$, while panel b shows the fit of similar un-targeted moments using survival expectations to age 75.[9]

With these estimated parameters, I can simulate slope beliefs, which I use in the next section to study their effect on working decisions of older adults.

of individuals, depending on the age t_0 I first observe them, for a total of 78 moments.

[9] The HRS includes two questions on survival expectations every wave: $plive10_{it}$ asks for a reference age approximately 10 years ahead, and $plive75_{it}$ asks for a reference age equal to 75 years. However, this last question is only asked to individuals 65 or younger, limiting the sample.

Table 3: Estimation results of initial beliefs by simulated method of moments: Moments' fit

a. Targeted moments			
	Data moment	SE	Simulated moment
$\mathbb{E}(plive10)$	0.531	(0.00011)	0.538
$\mathbb{E}(plive10^2)$	0.371	(0.00012)	0.357
$\mathbb{E}(plive10 \cdot h)$	2.890	(0.00065)	2.957
$\mathbb{E}(\Delta plive10)$	-0.013	(0.00002)	-0.014
$\mathbb{E}((\Delta plive10)^2)$	0.070	(0.00003)	0.066
$\mathbb{E}(\Delta plive10 \Delta h)$	0.007	(0.00002)	0.007
b. Other moments (not targeted)			
	Data moment	SE	Simulated moment
$\mathbb{E}(plive75)$	0.702	(0.00017)	0.806
$\mathbb{E}(plive75^2)$	0.556	(0.00021)	0.687
$\mathbb{E}(plive75 \cdot h)$	3.886	(0.00101)	4.469
$\mathbb{E}(\Delta plive75)$	-0.001	(0.00010)	0.018
$\mathbb{E}((\Delta plive75)^2)$	0.054	(0.00008)	0.042
$\mathbb{E}(\Delta plive75 \Delta h)$	0.006	(0.00005)	0.003

Note: Panel a. uses the same sample used for estimation. Panel b. uses a subsample of 1,247 individuals up to 65 years old for whom $plive75_{it}$ (*the percentage chance you will live to be 75*) is asked. The standard errors need to be adjusted to account for the estimation of the health measure.

6 Working decisions as functions of beliefs about health

The dynamic problem outlined in section 2 shows the policy rule for labor participation is a function of the state variables in the model. The novelty in this paper is that those state variables include individuals' beliefs about their future health. These beliefs are the result of two key elements: heterogeneity in health dynamics and uncertainty about that heterogeneity. These elements imply beliefs about that heterogeneity -instead of just a common parameter- enter individuals' choices.

In this section, I estimate the probability of work as a function of those state variables,

$$\mathbb{P}(p_{it} = 1|\Omega_{it-1}) \;\; = \;\; \mathbb{P}(p_{it} = 1|p_{it-1}, a_{it-1}, w_{it-1}, h_{it-1}, \hat{\delta}_{it-1}, \hat{\sigma}^2_{it-1}, \alpha_i). \tag{13}$$

By using the results from the previous section, we can simulate all of the state variables, and hence identify their effect on working decision. Furthermore, by using *survival expectations* to identify and simulate beliefs, no additional assumption on the relation between belieds and working decisions has been made. In particular, there is no restriction on the sign of the effect of $\hat{\delta}_{it-1}$ on working decisions.[10] Note also that, conditional on states variables in Ω_{it-1}, survival expectations $plive10_{it}$ do not play an additional role in working decisions p_{it}.

6.1 Probit results on working decisions

I first estimate equation (13) using a probit approach, i.e. assuming that $\mathbb{P}(p_{it} = 1|\Omega_{it-1}) = \Phi\big(\beta'\Omega_{it-1}\big)$ and integrating out the unobserved underlying heterogeneity α_i, δ_{i0}.[11]

Table 4 presents these results. It shows that beliefs do matter for working decisions, with a positive and significant coefficient for $\hat{\delta}_{it-1}$. This positive sign implies that expecting better health, that is, expecting health to deteriorate more slowly with age, is associated with larger probabilities of work.[12] On the other hand, *survival expectations* $plive10_{it}$ are also significant predictors in the probability of work, but that significance holds only while slope beliefs are not accounted for. Though interesting, these results assume a linear index for the probability of work, which is a very strong assumption, that is not justified by assumptions on the fundamentals of the model. Thus, in what follows, I estimate the probability of work flexibly, using instead a neural network approach.

[10] If individuals expecting better future health want to work longer, the sign would be positive. This could happen if the dominant effect were the desire to save more given the longer life expectancy implied by better health. If individuals expecting worse future health want to work longer, the sign would be negative. This could happen if the dominant effect were the desire to save more given the higher cost of future healthcare implied by worse health.

[11] Some of the input variables are unobserved by the econometrician; namely, heterogeneity in health level α_i and beliefs about slope heterogeneity, $\hat{\delta}_{it}$ and $\hat{\sigma}^2_t$. Conditional on health history, these unobserved variables depend on individual-level heterogeneity, which is integrated out. See the appendix for details on the likelihood specification.

[12] The assumptions of the learning model imply the posterior variance $\hat{\sigma}^2_t$ is constant across individuals of the same age t. Given that age is a also a relevant determinant of working decisions, I don't have enough variation to disentangle these two effects separately; any results would be based on functional form assumptions alone. Therefore, I focus instead on interpreting the effects of the posterior mean.

Table 4: Probit results on probability of work

		(1)		(2)		(3)	
		Coefficient	SE	Coefficient	SE	Coefficient	SE
Age	t	-0.196	(0.016)	-0.082	(0.003)	-0.192	(0.016)
Past work	p_{it-1}	2.032	(0.018)	2.031	(0.019)	2.034	(0.019)
Past health	h_{it-1}	0.169	(0.024)	0.261	(0.033)	0.175	(0.046)
Health intercept	α_i	0.244	(0.036)	0.074	(0.046)	0.243	(0.075)
Slope beliefs: mean	$\hat{\delta}_{it-1}$	1.933	(0.249)			1.903	(0.499)
Slope beliefs: variance*	$\hat{\sigma}_t^2/\sigma_\delta^2$	-13.854	(2.048)			-13.335	(2.102)
Survival expectations	$plive10_{it}$			0.114	(0.031)	0.007	(0.043)
Controls	other vars Ω_{it-1}	Yes		Yes		Yes	
N individuals		14,969		14,718		14,718	
N observations		58,040		55,592		55,592	

Note: Results of estimating equation (13) using a probit approach. pending: change format

6.2 Neural-network approach

A neural network provides a very flexible tool for estimation. In the case of a binary outcome, and under some particular specifications, a neural network corresponds to a maximum likelihood estimation with logistic errors, where the probability of success is a complex non-linear index of the inputs. As mentioned by Farrell et al. (2020), we can think of neural networks as a type of non-parametric or sieve estimation whereby the basis functions are learned from the data, hence allowing for greater flexibility.

In this case, I also need to account for the fact that some of the input variables are unobserved by the econometrician. These unobserved variables are slopes beliefs $(\hat{\delta}_{it-1}, \hat{\sigma}_t^2)$ and heterogeneous health levels α_i. Though they are time-varying variables, they can be written as functions of time-invariant unobserved variables (δ_{i0}, α_i) and the observed health path $(h_{i1}, \dots h_{iT_i})$ of each individual.[13] Thus, following a standard likelihood approach, I want to maximize the log of the likelihood integrating out this time-invariant unobserved heterogeneity. To do so, I follow the insight of EM-type algorithms.

Let θ be the parameters governing an outcome variable, in this case, working decisions. When there is underlying heterogeneity, we estimate θ by maximizing a likelihood that integrates out that heterogeneity. In this context, EM-type algorithms provide us with two key insights. First, the parameter θ that maximizes the *log of the integral* also maximizes the *integral of the log* if instead of using the prior distribution of the heterogeneity, we use its posterior distribution given the outcome variable. However, because this posterior distribution depends on θ, it is unknown. Thus, the second insight of EM-type algorithms is to solve the problem for θ iteratively: in iteration k, the E step uses θ_{k-1} to update the posterior distribution of the heterogeneity, and the M step estimates θ_k by maximizing the integral of the

[13] This relationship depends also on the parameters of the health process (ρ, σ_ϵ^2) and the parameters of beliefs (b and λ), but it does not depend on the parameters defining the relation between working decisions and state variables.

log using that posterior.

I use this same iterative logic as a convenient implementation for maximizing the integrated likelihood under a neural network approach. In this case, the E step is done by Markov chain Monte Carlo (MCMC) and provides draws from the posterior distribution of $(\alpha_i, \hat{\delta}_{i0})$ given working decisions p_i.[14]. Then, the M step uses those draws to expand the data, simulate the inputs $(\hat{\delta}_{it}, \hat{\sigma}_t^2, \alpha_i)$ using health, and estimates θ by using a neural network on the expanded data.[15] I start this iterative process at an M step using an incomplete posterior: the distribution of $(\alpha_i, \hat{\delta}_{i0})$ conditional on the health history $(h_{i1}, \dots h_{iT_i})$ and the history of survival expectations $(plive10_{i1}, \dots plive10_{iT_i})$. This distribution is incomplete because it does not condition on the working decisions, but it does already include the heterogeneity information contained in the health and expectations variables.

6.3 Neural-network results on working decisions

Following this strategy, I estimate the probability of work conditional on the state variables. I use a sample of individuals who are attached to the labor market, defined as individuals with at least 20 years of working experience. The loss and fit of the model is given in the appendix. pending: update results after iteration

> ***(1)*** *Beliefs play a role in the participation decisions of older adults, with positive average marginal effects that are similar in orders of magnitude to the average marginal effects of health and assets.*

Table 5 presents the effects on the probability of work of a marginal change in expected beliefs $\hat{\delta}_{it-1}$, health h_{it-1}, and assets a_{it-1}, respectively, conditional on age and past participation p_{it-1}, averaged across individuals. Even though the effects are of different magnitudes and signs, they are similar in orders of magnitude. The same result holds in Figures 6 and 7, which show the marginal effects of beliefs $\hat{\delta}_{it-1}$ by deciles of health and beliefs for adults aged 52-59 and 66-75, respectively.

> ***(2)*** *The total effect of a health shock ϵ_{it-1} on working decisions goes mostly through the persistence channel, with negligible effects through the information channel.*

This result is shown in Table 5, which includes the decomposition of the effects of a health shock into a persistence channel and an information channel, according to equation (8). Note the small values on the column Factor, which imply that a health shock has only a small effect on beliefs $\hat{\delta}_{it-1}$ and, therefore, only a small effect through the information channel This result highlights that, even though individuals are uncertain and biased, to significantly affect their decisions, we need large enough signals. Section 8 looks at one possible such variable: health information in the form of blood glucose and cholesterol levels.

[14] MCMC uses the likelihood of p_i given $(\alpha_i, \hat{\delta}_{i0})$ from the previous-iteration M step and the prior distribution of $(\alpha_i, \hat{\delta}_{i0})$.

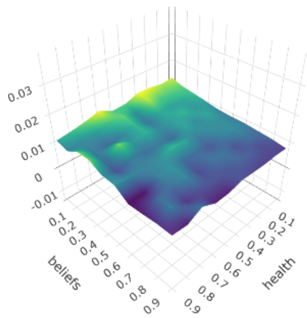
[15] The standard EM algorithm is known to converge, as the likelihood increases in each step of the sequence. This convergence does not hold in this case, given the lack of uniqueness of the estimation in each step. First, the E step does not return a closed-form solution for the posterior, but it returns draws from MCMC. Second, the non-convexity of the neural network's optimization problem implies non-uniqueness of the solution of the M step. Therefore, the approach is not aimed at getting at the unique solution, which does not necessarily exists, but as a convenient implementation.

Table 5: Average marginal effects on the probability of work and decomposition of the effects of a health shock

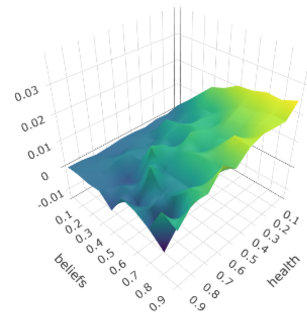
Age	Average marginal effects			Decomposition of a health shock		
	Health h_{it-1}	Beliefs $\hat{\delta}_{it-1}$	Assets a_{it-1}	Factor	Persistence channel	Information channel
$p_{it-1} = 0$						
52	0.055	0.011	-0.043	0.003	1.00	0.00
54	0.046	0.011	-0.037	0.006	1.00	0.00
56	0.040	0.012	-0.033	0.008	1.00	0.00
58	0.034	0.012	-0.028	0.011	1.00	0.00
60	0.027	0.011	-0.023	0.012	0.99	0.01
62	0.021	0.010	-0.018	0.014	0.99	0.01
64	0.016	0.010	-0.013	0.014	0.99	0.01
66	0.014	0.009	-0.010	0.015	0.99	0.01
68	0.013	0.006	-0.010	0.014	0.99	0.01
70	0.011	0.004	-0.008	0.014	1.00	0.00
72	0.009	0.002	-0.007	0.013	1.00	0.00
74	0.007	0.001	-0.005	0.012	1.00	0.00
$p_{it-1} = 1$						
52	0.015	0.007	-0.012	0.003	1.00	0.00
54	0.018	0.009	-0.014	0.006	1.00	0.00
56	0.021	0.010	-0.016	0.008	1.00	0.00
58	0.024	0.013	-0.017	0.011	0.99	0.01
60	0.026	0.015	-0.018	0.012	0.99	0.01
62	0.028	0.018	-0.016	0.014	0.99	0.01
64	0.027	0.021	-0.010	0.014	0.99	0.01
66	0.024	0.023	-0.004	0.015	0.99	0.01
68	0.023	0.024	-0.001	0.014	0.98	0.02
70	0.021	0.025	0.002	0.014	0.98	0.02
72	0.020	0.026	0.004	0.013	0.98	0.02
74	0.019	0.027	0.007	0.012	0.98	0.02

Note: The columns on persistence and information channels correspond to the terms in equation (8), expressed as a proportion of the total partial effect.

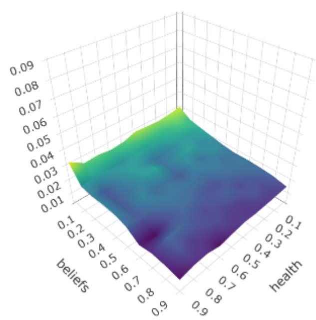
Figure 6: Average marginal effect of expected beliefs $\hat{\delta}_{it-1}$, health h_{it-1}, and assets a_{it-1} on the probability of work p_{it} for adults in their 50s



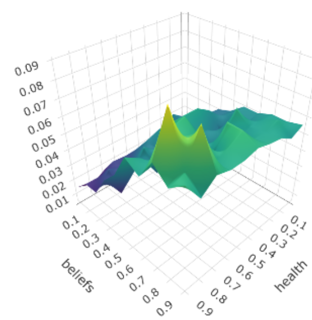
(a) Marginal change in $\hat{\delta}_{it-1}$ conditional on $p_{it-1} = 1$



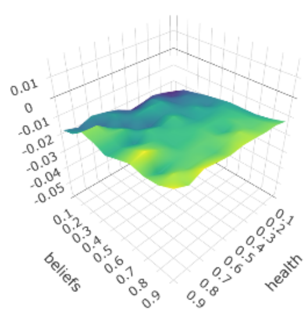
(b) Marginal change in $\hat{\delta}_{it-1}$ conditional on $p_{it-1} = 0$



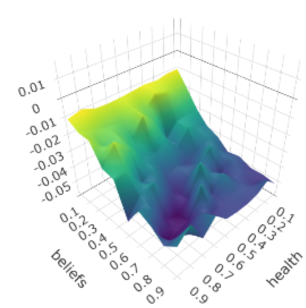
(c) Marginal change in h_{it-1} conditional on $p_{it-1} = 1$



(d) Marginal change in h_{it-1} conditional on $p_{it-1} = 0$



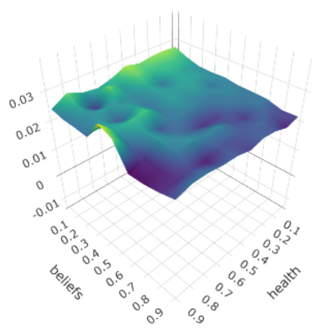
(e) Marginal change in a_{it-1} conditional on $p_{it-1} = 1$



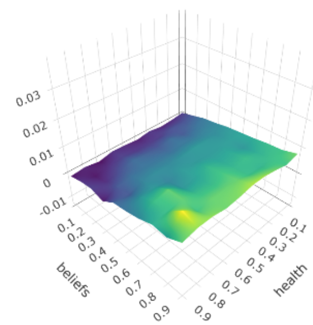
(f) Marginal change in a_{it-1} conditional on $p_{it-1} = 0$

Note: Each row corresponds to the average marginal effects with respect to $\hat{\delta}_{it-1}$, h_{it-1}, and a_{it-1}, respectively. The left column conditions on individuals who were working, $p_{it-1} = 1$, and the right column conditions on individuals who were not working, p_{it-1}=0, in the previous period. In each plot, the x- and y-axis correspond to deciles of health h_{it-1} and expected beliefs $\hat{\delta}_{it-1}$ for the corresponding subsample of the plot. Note the z-axis changes in each row.

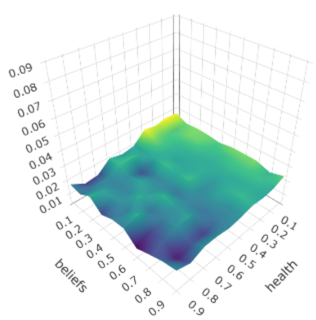
Figure 7: Average marginal effect of expected beliefs $\hat{\delta}_{it-1}$, health h_{it-1}, and assets a_{it-1} on the probability of work p_{it} for adults between 66 and 75 years old



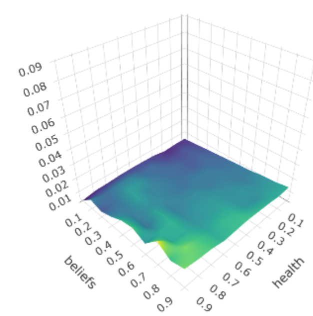
(a) Marginal change in $\hat{\delta}_{it-1}$ conditional on $p_{it-1} = 1$



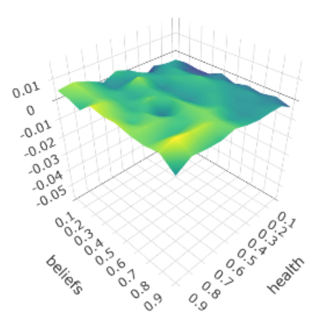
(b) Marginal change in $\hat{\delta}_{it-1}$ conditional on $p_{it-1} = 0$



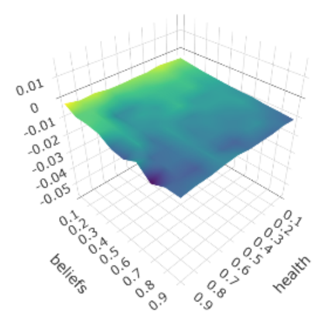
(c) Marginal change in h_{it-1} conditional on $p_{it-1} = 1$



(d) Marginal change in h_{it-1} conditional on $p_{it-1} = 0$



(e) Marginal change in a_{it-1} conditional on $p_{it-1} = 1$



(f) Marginal change in a_{it-1} conditional on $p_{it-1} = 0$

Note: Each row corresponds to the average marginal effects with respect to $\hat{\delta}_{it-1}$, h_{it-1}, and a_{it-1}, respectively. The left column conditions on individuals who were working, $p_{it-1} = 1$, and the right column conditions on individuals who were not not working, p_{it-1}=0, in the previous period. In each plot, the x- and y-axis correspond to deciles of health h_{it-1} and expected beliefs $\hat{\delta}_{it-1}$ for the corresponding subsample of the plot. Note the z-axis changes in each row.

7 Counterfactuals

In this section, I study how labor participation would change if we could eliminate bias in initial beliefs. In particular, I look at two questions:

1. How much would labor participation change if initial beliefs were unbiased at the population level, that is, $\mathbb{E}(\hat{\delta}_{i0}) = \mathbb{E}(\delta_i)$?

2. How much would labor participation change if we could reduce each individual's bias in half, by closing the distance between $\hat{\delta}_{i0}$ and δ_i?

To look at these questions, I use an impulse-response function approach, using the results of the previous section. That is, I simulate working decisions under a sample's baseline scenario, and compare those predictions against the predictions simulated under each of these two potential changes in initial beliefs.[16]

> ***(1)*** *Eliminating the bias in prior beliefs* b *would increase participation by more than 2 percentage points around the formal retirement age (66-67).*

Figure 8 shows the average change in the probability of work after eliminating the initial bias in prior beliefs, b. Note this effect has an inverted-U shape. In the early 50s, the effect is small given that individuals are still mostly working. But as people start to retire, the new beliefs imply larger probabilities of work that do not vanish completely over time and remain above 2 percentage points for individuals in their early seventies. Note that, in this sample, the average probability of work prior the change in beliefs is 34% at age 66 and 17% at age 78; hence, the increment in the figure is not trivial. Furthermore, as this effect results from eliminating a misconception at the population level, it is an easier target policy that could be addressed by information campaigns, without the need of providing individual-specific information.

> ***(2)*** *Reducing the initial bias of each individual in half has a heterogeneous effect, with larger gains in the probability of work for individuals who are initially more biased.*

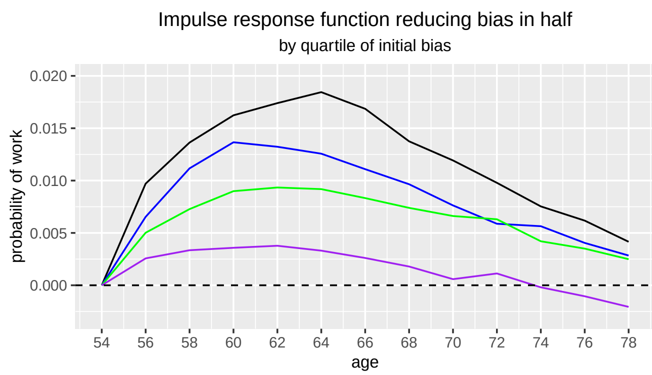
Figure 9 shows this results, distinguishing by quartile of initial bias, $\hat{\delta}_{i0} - \delta_i$. Given that the overall initial bias $b < 0$, most individuals are initially downward biased. Thus, reducing bias in half per each individual means increasing initial beliefs for most of them, which translates into the effects being positive, as shown in the figure.

[16] The Figures 8 9 present the response in terms of labor-participation decisions by age, given a change in initial beliefs. Over time, this change in initial beliefs translates into changes in posterior beliefs, labor-participation decisions, as well as decisions regarding assets and health-insurance. The effects on these last two variables were also predicted using a neural-network approach.

Figure 8: Impulse response function to a shift in prior beliefs eliminating overall bias b

Note: Impulse response function using the subsample of individuals used in estimation that are observed at 52 years old, corresponding to 1,184 individuals.

Figure 9: Impulse response function to reducing individuals' initial bias by half



Note: pending: improve labels

8 An information experiment: Blood-based biomarkers as signals of health

The results of the previous section show beliefs matter for the working decisions of older adults. Furthermore, because those beliefs are initially biased, eliminating bias has non-trivial effects on working decisions. Bias can be eliminated through signals that provide better information. However, the results also show the size of the signal matters: a health shock, for example, translates into only a small change in beliefs, and hence has a negligible information effect on working decisions (its effects go through changing the stock of health, which persists in time).

In this section, I look at one additional signal of health slopes with age δ_i: blood-based biomarkers. By providing information about health, blood-based biomarkers can signal individuals' slopes. In 2006, the HRS introduced the collection of a blood sample for measuring biomarkers.[17] With the blood sam-

[17] The collection introduced more detailed measures of health, including physical measures, a saliva sample for DNA analysis, and a blood sample for measuring biomarkers. The physical measures include blood pressure and pulse, lung function, hand grip strength, balance test, timed walk test, height, weight, and waist circumference. These variables are valuable measures

ple, three biomarkers are measured and individuals are informed of their levels: HDL cholesterol (known as the *good cholesterol*), total cholesterol, and blood glucose hbA1c. The results are provided around a month after the survey has ended[18] (see Edwards (2018) for more details). In this section, I study whether receiving this information provides individuals with an additional signal about their health, in particular, about their slopes δ_i, and study whether they change their survival expectations and working behavior accordingly.

To achieve that goal, I make use of a key aspect in the introduction of these measures: to control costs associated with their collection, the HRS randomly split the sample into two halves, and in each wave, the HRS collects these biomarkers in only one of those halves. Hence, this collection scheme provides us with an information experiment, that is, with exogenous variation in who receives this additional information. Note, however, that this was not the intended goal of the HRS, and as such this is not an ideal experiment. An ideal experiment would include a control arm of individuals who get their blood taken but are not informed on their results. Still, the HRS collection scheme of biomarkers does provide us with exogenous variation that I use in this section. Another advantage of looking at this additional source of information is that it allows me to relax the assumption of health as the only (or sufficient) signal[19] and to use additional sources of variation when estimating the effects of beliefs on the working decisions of older adults.[20]

For these biomarkers to be a valid slope signal, being correlated with health is not enough; they must be correlated with δ_i. The appendix shows this last correlation does in fact hold. It presents the results of estimating an equation for health, similar to that of section 4, allowing for the distribution of the heterogeneity to depend on these biomarkers. The results show both the heterogeneous intercepts α_i and heterogeneous slopes δ_i are correlated with these particular biomarkers.

8.1 Reduced-form approach

I then turn to estimate the overall effect of receiving this information on individuals' survival expectations and working decisions. To that end, I use the experiment introduced with the biomarker collection in 2006 (wave 8), when the sample was randomly divided into two. To be able to generate this information, the experiment also introduces a difference in interview mode between the two groups, because an in-person interview is required to collect the blood sample.[21] The interview mode could have an effect on

of health, but I do not include them in this paper, given that they are measured only every two waves. Furthermore, their value as signals of health is limited given that, on one hand, they reflect aspects of health already experienced by individuals in their everyday life, and on the other, the results of the measures are immediately communicated to individuals before asking them about their survival expectations.

[18] Two other biomarkers are measured: C-reactive protein (CRP), a general marker of systemic inflammation, and Cystatin C, an indicator of kidney functioning. However, individuals are not informed of their levels on these two biomarkers; hence, these results do not play a role as health signals.

[19] The signal analyzed here is provided exogenously to individuals. Hence, this paper does not address endogenous acquisition of information, which is left for future work.

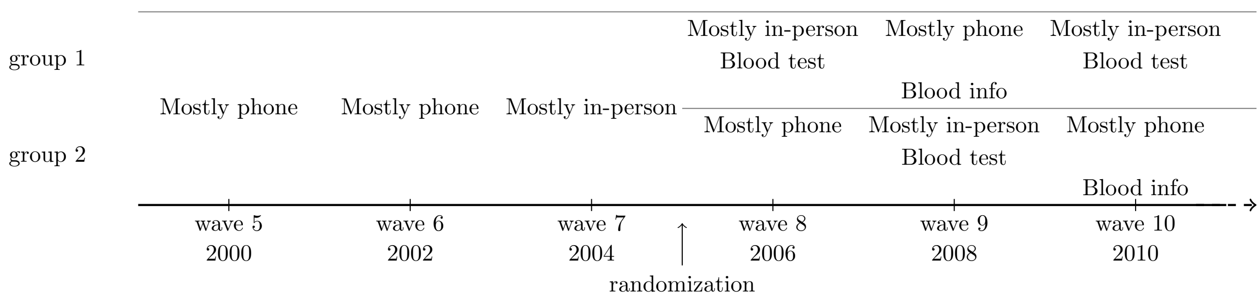
[20] In inferring slope beliefs and using them to study their effect on labor-participation decisions in the previous section, I only use cross-sectional variation given by differences in initial beliefs, conditional on health and survival-expectations histories.

[21] The HRS survey is usually conducted by phone, except for first interviews of new cohorts, people who request in-person interviews, and individuals residing in nursing homes. A shift to in-person interviews in 2004 also occurred in an attempt by the HRS to increase individuals' consent to link their survey responses with administrative data. These differences in

individuals' answers, in particular, to questions regarding opinions and expectations. Though potentially problematic, the timing of the information provision allows me to separately identify the interview-mode effect from the information effect of the biomarker results, because that information is only provided to individuals after the fieldwork has ended. Hence, individuals do not have the information in the wave when the blood is collected, but in the following wave.

Figure 10 presents the structure of the biomarker collection and the information experiment, and it helps us visualize the identification strategy. On one hand, a difference-in-differences analysis using waves 7 and 8 returns the interview-mode effect. On the other hand, a difference-in-differences analysis using waves 7 and 9 returns the interview-mode effect (with the opposite sign) plus the information effect of receiving the additional signal. Hence, we can identify the information effect by adding these two terms. Under the parallel-trends assumption, the same idea holds if we construct these terms using wave 5 instead of wave 7.

Figure 10: Timing of the biomarker collection and information experiment



Therefore, I estimate the following equation:

$$y_{iw} \quad = \quad \beta_0 + \beta_1 d_{g_i} + \beta_{2w} d_w + \beta_{3w} d_{g_i} \cdot d_w + \epsilon_{iw}, \tag{14}$$

where i denotes an individual and w denotes a wave. I use w instead of t, because in this paper, t denotes age. I consider two dependent variables separately, survival expectations $plive10_{iw}$ and a binary of work p_{iw}. I estimate these equations using a balanced sample of individuals observed from waves 5 to 9.[22] d_{g_i} is a dummy for the group of individuals set for blood collection in wave 8 (group 1 in Figure 10, with group 2 as the reference category), and d_w are dummies for waves 6 to 9 (wave 5 is the reference category). Hence, the interview-mode effect is given by β_{3w_8}, and the information effect of the signal is given by $\beta_{3w_8} + \beta_{3w_9}$, where the interview-mode effects in each group cancel each other out. Trends are parallel before the randomization happens if $\beta_{3w_6} = \beta_{3w_7} = 0$, and randomization in the selection of the two groups implies $\beta_1 = 0$.

Table 6 presents the estimation results of equation (14) for both $plive10_{iw}$ and p_{iw}. When looking

interview mode are unimportant for the analysis as long as they are applied in the same way across the two groups.

[22] I use only up to wave 9, because from wave 10 onward, the groups are no longer comparable, given that they have been provided information with different timing.

Table 6: Interview mode and information effects of biomarker experiment

		Survival expectation ($plive10_{iw}$)			Work decision (p_{iw})		
		All	Less college	College	All	Less college	College
Group 1	β_1	-0.47	-0.24	-1.38	0.00	0.01	-0.01
Wave 6	β_{2w_6}	-1.42***	-1.21**	-2.09**	-0.07***	-0.07***	-0.09***
Wave 7	β_{2w_7}	-1.50***	-1.44***	-1.72**	-0.12***	-0.12***	-0.12***
Wave 8	β_{2w_8}	-6.41***	-6.12***	-7.37***	-0.16***	-0.16***	-0.19***
Wave 9	β_{2w_9}	-3.57***	-3.22***	-4.70***	-0.20***	-0.20***	-0.22***
Group 1, wave 6	β_{3w_6}	0.28	-0.06	1.37	0.01	0.00	0.02
Group 1, wave 7	β_{3w_7}	-0.27	-0.24	-0.33	0.01	0.01	0.01
Group 1, wave 8	β_{3w_8} (a)	1.77**	1.29	3.31***	0.01	0.00	0.03
Group 1, wave 9	β_{3w_9} (b)	-0.42	-1.12	1.82	0.01	0.01	0.00
Constant	β_0	53.97***	52.42***	58.96***	0.49***	0.45***	0.61***
Observations		41,930	31,815	10,115	41,923	31,810	10,113
R-squared		0.004	0.004	0.005	0.021	0.021	0.022
Interview mode effect (a)		1.77**	1.29	3.31**	0.01	0.00	0.03
Information effect (a)+(b)		1.36	1.65	5.12**	0.02	0.01	0.04

Note: Estimation results are from equation (14). The sample consists of $N = 8,386$ individuals with non-proxy interviews who are at least 50 years old in wave 8 and who give a valid answer to $plive10_{iw}$ every wave between waves 5 and 9. Seven of these observations do not have information on p_{iw}. Standard errors are clustered at the household level. *** p<0.01, ** p<0.05, * p<0.1

at the results for survival expectation, $plive10_{iw}$, the table shows the two groups are similar and that pre-trends are parallel. The table also shows a positive and significant interview-mode effect of 1.77 percentage points and a similar but insignificant information effect of 1.36 percentage points. Though insignificant, this positive sign is aligned with what we already know about beliefs; on average, individuals' beliefs about health and survival are downward biased. Therefore, providing more information moves those expectations up. When looking at the results for working decisions, p_{iw}, the two groups are similar to begin with and have parallel pre-trends, but we find no significant effect of interview mode[23] or information. Overall, these results suggest the signal is not large enough to have a significant effect on expectations and decisions.

The table also presents the results separately by education level. It shows that for adults with a college degree, both the interview-mode and information effects are larger and significant when looking at survival expectations. For adults with less than a college degree, only the interview-mode effect is significant. When looking at working decisions, no significant effects -interview-mode or information effects- for either group are seen. These differences by education level suggest the ability to process the information matters, with more educated adults internalizing the provided information better. The effect on their working decisions is also larger though still not significant.[24]

[23] The lack of an interview-mode effect on working decisions is expected, given the more objective nature of working outcomes versus survival expectations.

[24] I run a similar regression with the number of doctor visits since the last interview as a dependent variable and find no effects

The appendix further decomposes group 1 into adults who receive a bad biomarker result versus those who do not. However, because we cannot make the same distinction in group 2,[25] we cannot identify information effects by the type of signal received (good or bad biomarker results). Still, this analysis is interesting because it shows older adults who receive bad results have lower survival expectations to begin with, suggesting they already know at least some of this information. Consistently, by wave 7, people who later receive bad biomarker results also work less on average than those who receive good results.

8.2 Model-based approach

These reduced-form results are also consistent with the predictions of a learning model. For illustration, I estimate survival expectations allowing for the biomarker information to be a second signal for group 1 in wave 9.[26] Bayes' rule implies that in this case, the posterior mean of δ_i is a weighted average of the prior at that period, the signal provided by health and the signal provided by the biomarker information. The weights depend on how uncertain individuals are to begin with, and on how precise is the information provided by each signal. Thus, to predict beliefs $\hat{\delta}_{it}$, a key issue is to predict the precision of the additional signal. To measure that precision, I use future biomarker results from group 2. These individuals were tested in wave 9 and received the information before wave 10. Hence, I use their biomarker information and their survival expectations in waves 9 and 10 to estimate the parameters of the additional signal using simulated method of moments.[27] The randomness in the selection of the groups implies the parameters recovered by looking at group 2 must also represent the parameters governing the biomarker signal for group 1.

Hence, using those parameters, I go back to group 1 and predict their survival expectations at wave 9. As shown in Table 7, the learning model suggests that by having the additional signal on health, group 1 increases their survival expectations between waves 8 and 9 by 0.4 percentages points more than the control group. This change in survival expectations is positive but negligible, consistent with the results in Table 6. In that sense, the reduced-form evidence supports the predictions of the learning model.[28]

Pending: replicate model results with differences by education level. When estimating b and λ parameters by education level, I find individuals with less education have more measurement error (more

(results not shown), neither interview-mode nor information effects, for either group. This result suggests the difference in survival expectations between these two groups is not explained by a different number of doctor visits. However, more educated individuals may still be better able to incorporate the new information with the help of their physicians, even if the number of doctor visits remains the same.

[25] One possibility would be to use the biomarker results in wave 9 to attempt the same distinction for group 2. However, an analysis using repeated biomarker results from future waves shows these results change over time, introducing noise when using results from wave 9 to assign wave 8 status for the second group.

[26] I consider in this analysis only blood glucose because it is the biomarker more consistently related to slopes δ_i.

[27] In an alternative version, I use a maximum likelihood approach to jointly estimate health and lab results as a function of slope heterogeneity. I then use those parameters to predict slope beliefs and survival expectations. Under this alternative approach, I get qualitatively the same results as the ones from using SMM.

[28] I do not look at a model-based effect on working decisions, because this paper has attempted to study the effect of beliefs on working decisions without making any assumption about their relation. Hence, I do not have a model-based equivalent to test. However, the information experiment does provide an interesting way to test a fully structural model, which I plan to use in future research.

Table 7: Predicted survival expectations in a model with health and blood glucose as signals

	Number of observations	Predicted survival expectations		
		wave 8	wave 9	wave 9 - wave 8
Control (group 2)	4,852	45.8	45.4	-0.3
Treated (group 1)	5,357	44.8	44.9	0.1
Treated with bad blood glucose result	552	39.1	38.5	-0.5
Treated with good blood glucose result	3,649	46.0	46.3	0.3
Treated no blood glucose result	1,156	43.8	43.7	-0.2

Note: The sample consists of $N = 10,209$ individuals with non-proxy interviews who are at least 50 years old in wave 8 and who give a valid answer to $plive10_{iw}$ in waves 8 and 9. Survival expectations are predicted from a model with one signal for the control group (health) and two signals for the treated group (health and blood glucose results). These two signals are assumed to be independent conditional on individual heterogeneity. The parameters determining the strength of blood glucose as a signal of δ_i come from an estimation using future values of the control group (waves 9 and 10)

biased and larger standard error), and they are more initially biased in their beliefs b. Results on uncertainty depend on gender, with more educated women more uncertain the less educatd women, while more educated men are less uncertain than less educated men. I need to predict survival expectations, adding biomarkers result to see if I can replicate the larger effects for the more educated older adults.

9 Conclusion

References

Edwards, R. (2018). If my blood pressure is high, do i take it to heart? behavioral effects of biomarker collection in the health and retirement study. *Demography*, 55:403–434.

Farrell, M., Liang, T., and Misra, S. (2020+). Deep neural networks for estimation and inference. *Econometrica (forthcoming)*.

Guvenen, F. (2007). Learning your earning: Are labor income shocks really very persistent? *The American Economic Review*, 97(3):687–712.

Kleinjans, K. and Van Soest, A. (2014). Rounding, focal point answers and nonresponse to subjective probability questions. *Journal of Applied Econometrics*, 29:567–585.

Manski, C. and Molinari, F. (2010). Rounding probabilistic expectations in surveys. *Journal of Business & Economic Statistics*, 28(2):219–231.