TOP 20 PROBLEM PATTERNS

Probability Theory Final Exam — December 16, 2025 — With Section References

1. CLT Game/Total

"n games", "total winnings"

Steps:

1. Find $E[X_i]$, $\text{Var}(X_i)$
2. $S_n \approx N(n\mu, n\sigma^2)$
3. Standardize

Ref: §6.1, Template B

2. Gaussian Vector

"Gaussian vector", "indep. components"

Key: $\rho = 0 \Leftrightarrow$ independent (MVN)
Solve: Cov $= 0$
Ref: §4.5, Template A, G

3. Lognormal Stock

"$S = S_0 e^Z$", "stock price"

Key: $E[e^X] = e^{\mu+\sigma^2/2}$
Ref: §7.3, Template D

4. Bayesian Discrete

"prior", "posterior", "defective"

Steps:

1. List θ_i, priors $P(\theta_i)$
2. Likelihoods $P(\text{data}|\theta_i)$
3. Bayes: posterior $\propto$ prior $\times$ likelihood
4. Normalize

Ref: §7.2, Template E

5. BVN Conditional

"$Y|X = x$", "bivariate normal"

$\mu_{Y|X} = \mu_Y + \rho\frac{\sigma_Y}{\sigma_X}(x - \mu_X)$
$\sigma^2_{Y|X} = \sigma^2_Y(1 - \rho^2)$
Ref: §4.5, Template F

6. Exponential + CLT

"i.i.d. Exp", "mean θ"

TRAP: Mean $\theta \Rightarrow \lambda = 1/\theta$!
$\bar{X} \approx N(1/\lambda, 1/(n\lambda^2))$
Ref: §3.4, §6.1, Template C

7. Monty Hall

"Monty Hall", "contestant picks"

Sober: Switch wins 2/3
Dizzy: No advantage
Ref: §8.1, Template J

8. Max/Min i.i.d.

"maximum of n", "minimum"

$P(\max \le a) = [F(a)]^n$
$P(\min > a) = [1 - F(a)]^n$
Ref: §4.7, Template L

9. Linear Combo BVN

"$P(X+Y > c)$", "sum of jointly normal"

$X + Y \sim N(\mu_X + \mu_Y, \sigma^2_{X+Y})$
$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y + 2\rho\sigma_X\sigma_Y$
Ref: §4.5, §8.3

10. Predictive Dist.

"predict next", "posterior predictive"

$P(X_{n+1}|data) = \sum_\theta P(X|\theta)P(\theta|data)$
Ref: Template H

11. Product Lognormal

"XY where lognormal"

$\ln(XY) = \ln X + \ln Y$
Still lognormal!
Ref: Template I

12. BVN Parameters

"find μ_Y, σ_Y, ρ"

Match $E[Y|X]$, $\text{Var}(Y|X)$ formulas
Ref: Template N

13. Ratio + CLT

"$\bar{X}/(\bar{X} + c) < p$"

Key: Transform inequality first!
Then apply CLT
Ref: Template O

14. Find n for CLT

"smallest n such that"

$n \ge (z^*\sigma/(c - \mu))^2$
Ref: Template K

15. Confidence Int.

"95% CI", "confidence interval"

$\bar{X} \pm z_{\alpha/2} \cdot \sigma/\sqrt{n}$
$z_{0.025} = 1.96$
Ref: §6.4

16. Conjugate Prior

"Beta-Binomial"

$\text{Beta}(\alpha, \beta)$ + x successes in n
$\Rightarrow \text{Beta}(\alpha + x, \beta + n - x)$
Ref: §7.2

17. Total Expectation

"$E[X]$ =?", condition on Y

$E[X] = E[E[X|Y]]$
Ref: §7.1, §8.9

18. Total Variance

"Var(X)", condition on Y

$\text{Var}(X) = E[\text{Var}(X|Y)] + \text{Var}(E[X|Y])$
Ref: §7.1

19. Sum of Poisson

"sum of Poisson"

$X + Y \sim \text{Poisson}(\lambda_1 + \lambda_2)$
Ref: Template P

20. Transformation

"find dist. of $Y = g(X)$"

CDF: $F_Y(y) = P(g(X) \le y)$
Jacobian: $f_Y = f_X|dx/dy|$
Ref: §4.6

QUICK FORMULA REFERENCE

CLT: $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$ | **Lognormal:** $E[e^X] = e^{\mu+\sigma^2/2}$ | **Cov:** $E[XY] - E[X]E[Y]$ | **Total Exp:** $E[X] = E[E[X|Y]]$

Bayes: $P(H|E) = \frac{P(E|H)P(H)}{P(E)}$ | **BVN Cond:** $\mu_{Y|X} = \mu_Y + \rho\frac{\sigma_Y}{\sigma_X}(x - \mu_X)$ | **Var Sum:** $\sigma_X^2 + \sigma_Y^2 + 2\text{Cov}$ | **Max:** $P(\max \leq a) = [F(a)]^n$

CRITICAL TRAP: "Mean $\theta = 3$" (Exponential) $\Rightarrow \lambda = 1/3$ NOT 3! — "Gaussian" = Normal — "Independent components" = $\rho = 0$ for MVN