

TOP 20 PROBLEM PATTERNS

Probability Theory Final Exam — December 16, 2025 — With Section References

1. CLT Game/Total

“n games”, “total winnings”

Steps:

$$1. \text{ Find } E[X_i], \text{Var}(X_i)$$

$$2. S_n \approx N(n\mu, n\sigma^2)$$

3. Standardize

Ref: §6.1, Template B

2. Gaussian Vector

“Gaussian vector”, “indep. components”

Key: $\rho = 0 \Leftrightarrow$ independent (MVN)

Solve: Cov = 0

Ref: §4.5, Template A, G

3. Lognormal Stock

$S = S_0 e^Z$ ”, “stock price”

Key: $E[e^X] = e^{\mu + \sigma^2/2}$

Ref: §7.3, Template D

4. Bayesian Discrete

“prior”, “posterior”, “defective”

Steps:

1. List θ_i , priors $P(\theta_i)$

2. Likelihoods $P(\text{data}|\theta_i)$

3. Bayes: posterior \propto prior \times likelihood

4. Normalize

Ref: §7.2, Template E

5. BVN Conditional

“ $Y|X = x$ ”, “bivariate normal”

$$\mu_{Y|X} = \mu_Y + \rho \frac{\sigma_Y}{\sigma_X} (x - \mu_X)$$

$$\sigma_{Y|X}^2 = \sigma_Y^2 (1 - \rho^2)$$

Ref: §4.5, Template F

10. Predictive Dist.

“predict next”, “posterior predictive”

$$P(X_{n+1}|data) = \sum_{\theta} P(X|\theta)P(\theta|data)$$

Ref: Template H

$$z_{0.025} = 1.96$$

Ref: §6.4

16. Conjugate Prior

“Beta-Binomial”

$\text{Beta}(\alpha, \beta) + x$ successes in n

$\Rightarrow \text{Beta}(\alpha + x, \beta + n - x)$

Ref: §7.2

6. Exponential + CLT

“i.i.d. Exp”, “mean θ ”

Mean $\theta \Rightarrow \lambda = 1/\theta!$

$$\bar{X} \approx N(1/\lambda, 1/(n\lambda^2))$$

Ref: §3.4, §6.1, Template C

11. Product Lognormal

“ XY where lognormal”

$$\ln(XY) = \ln X + \ln Y$$

Still lognormal!

Ref: Template I

7. Monty Hall

“Monty Hall”, “contestant picks”

Sober: Switch wins 2/3

Dizzy: No advantage

Ref: §8.1, Template J

8. Max/Min i.i.d.

“maximum of n”, “minimum”

$$P(\max \leq a) = [F(a)]^n$$

$$P(\min > a) = [1 - F(a)]^n$$

Ref: §4.7, Template L

9. Linear Combo BVN

“ $P(X + Y > c)$ ”, “sum of jointly normal”

$$X + Y \sim N(\mu_X + \mu_Y, \sigma_{X+Y}^2)$$

$$\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y$$

Ref: §4.5, §8.3

12. BVN Parameters

“find μ_Y, σ_Y, ρ ”

Match $E[Y|X]$, $\text{Var}(Y|X)$ formulas

Ref: Template N

13. Ratio + CLT

“ $\bar{X}/(\bar{X} + c) < p$ ”

Key: Transform inequality first!

Then apply CLT

Ref: Template O

14. Find n for CLT

“smallest n such that”

$$n \geq (z^* \sigma / (c - \mu))^2$$

Ref: Template K

15. Confidence Int.

“95% CI”, “confidence interval”

$$\bar{X} \pm z_{\alpha/2} \cdot \sigma / \sqrt{n}$$

17. Total Expectation

“ $E[X] = ?$ ”, condition on Y

$$E[X] = E[E[X|Y]]$$

Ref: §7.1, §8.9

18. Total Variance

“ $\text{Var}(X)$ ”, condition on Y

$$\text{Var}(X) = E[\text{Var}(X|Y)] + \text{Var}(E[X|Y])$$

Ref: §7.1

19. Sum of Poisson

“sum of Poisson”

$$X + Y \sim \text{Poisson}(\lambda_1 + \lambda_2)$$

Ref: Template P

20. Transformation

“find dist. of $Y = g(X)$ ”

$$\text{CDF: } F_Y(y) = P(g(X) \leq y)$$

Jacobian: $f_Y = f_X | dx/dy|$

Ref: §4.6

CLT: $Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Bayes: $P(H|E) = \frac{P(E|H)P(H)}{P(E)}$

Lognormal: $E[e^X] = e^{\mu + \sigma^2/2}$

BVN Cond: $\mu_{Y|X} = \mu_Y + \rho \frac{\sigma_Y}{\sigma_X} (x - \mu_X)$

Cov: $E[XY] - E[X]E[Y]$

Var Sum: $\sigma_X^2 + \sigma_Y^2 + 2\text{Cov}$

Total Exp: $E[X] = E[E[X|Y]]$

Max: $P(\max \leq a) = [F(a)]^n$

CRITICAL TRAP: “Mean $\theta = 3$ ” (Exponential) $\Rightarrow \lambda = 1/3$ NOT 3! — “Gaussian” = Normal — “Independent components” = $\rho = 0$ for MVN