Complete Problem Index

with Solution Paths & Variations

Probability Theory Final Exam Preparation

December 2025

Total Problems:	57 main problems, 131 sub-parts
Sources:	HW1-HW6, Practice Midterms 1-2, Practice Final
Reference:	FINAL_EXAM_CHEATSHEET.tex sections

Contents

1 How to Use This Document

1.1 Color Coding

- **Blue** = Homework problems (HW1-HW6)
- **Red** = Practice exam problems
- `Section Green` = Study guide section references
- `Formula: Purple` = Specific formula references
- `Template Orange` = Multi-step problem templates

1.2 Finding What You Need

Method 1: By Problem Source

1. Find the homework/exam in the Table of Contents
2. Look up the specific problem number
3. Follow the PRIMARY SOLUTION PATH

Method 2: By Problem Type

1. Go to Section 4 (Problems by Type)
2. Find your problem type
3. See all related problems and their solutions

Method 3: By Keyword

1. Go to Section 6 (Terminology Dictionary)
2. Look up the keyword from your problem
3. Get directed to the correct section

1.3 CRITICAL Terminology Reminders

If You See	It Means	Go To
"Gaussian"	Normal distribution	`Section 3.3`
"Gaussian vector"	Multivariate Normal (MVN)	`Section 4.5`
"Independent components"	$\rho = 0$ (for MVN: independent!)	`Section 4.5`
"Mean $\theta = 3$" (Exponential)	$\lambda = 1/3$ NOT $\lambda = 3$!	`Section 3.4`
"Large n" or "approximate"	Use CLT	`Section 6.1`

2 Homework Problems

2.1 Homework 1

2.1.1 HW1-Q1: Six-Card Poker Hands

Problem: Calculate probability of two-pair and three-of-a-kind in 6-card hands.

Primary Solution Path:

- `Section 1.5` - Counting Methods
- `Formula:` $C(n,k) = \binom{n}{k}$

Key Steps:

1. Total hands: $C(52, 6)$
2. Count each hand type using combinations
3. Divide for probability

Variations:

- Different hand types → Different counting
- 5-card instead of 6-card → Same method
- With/without wild cards → Adjust counts

2.1.2 HW1-Q2: Non-Transitive Dice

Problem: Compare irregular dice with faces Blue: 333336, Orange: 144444, White: 222555.

Primary Solution Path:

- `Section 2.1` - PMF and CDF
- Method: Enumerate all 36 outcomes for each pair

Variations:

- Different dice faces → Same enumeration
- Sum of dice → Convolution
- Three-way comparison → Check transitivity

2.1.3 HW1-Q3: Birthday Problem

Problem: Find probability of shared birthdays in group of n.

Primary Solution Path:

- `Section 1.5` - Counting Methods
- `Formula:` $P(\texttt{shared}) = 1 - \frac{365!/(365-n)!}{365^n}$

Key Insight: Use complement! $P(\text{at least one shared}) = 1 - P(\text{all different})$

Variations:

- Different number of days → Replace 365
- Three people share → More complex counting
- Simulation approach → Monte Carlo

2.1.4 HW1-Q4 to HW1-Q8: Basic Probability

Problem	Type	Section
Q4: Dice difference	Enumeration	Section 2.1
Q5: School grades	Weighted probability	Section 1.1
Q6: Three coins	Simple counting	Section 1.5
Q7: Elevator	Permutations	Section 1.5
Q8: Ball drawing	Symmetry	Section 1.1

2.2 Homework 2

2.2.1 HW2-Q1: Polya Urn Model

Problem: Box with r red, b blue balls. Draw, return with k additional of same color. Find P(first 3 red, 4th blue).

Primary Solution Path:

- Section 1.2 - Conditional Probability
- Formula: Chain rule: $P(A_1 \cap \ldots \cap A_n) = P(A_1)P(A_2|A_1)\cdots$

Solution:

$$P = \frac{r}{r+b} \cdot \frac{r+k}{r+b+k} \cdot \frac{r+2k}{r+b+2k} \cdot \frac{b}{r+b+3k}$$

2.2.2 HW2-Q7: Five Coins (Bayesian)

Problem: Five coins with $p_i = 0, 1/4, 1/2, 3/4, 1$. Select coin randomly, observe head. Find posterior probabilities.

Primary Solution Path:

- Section 7.2 - Bayesian Statistics
- Section 1.3 - Bayes' Theorem
- Formula: $P(\texttt{coin}_i|H) = \frac{P(H|\texttt{coin}_i)P(\texttt{coin}_i)}{\sum_j P(H|\texttt{coin}_j)P(\texttt{coin}_j)}$

Key Steps:

1. Prior: $P(\text{coin}_i) = 1/5$
2. Likelihood: $P(H|\text{coin}_i) = p_i$
3. Posterior: Apply Bayes formula

2.2.3 HW2-Q9: Blue Taxi (Base Rate Fallacy)

Problem: 1 blue taxi, 99 green. Witness 99% accurate on blue, 2% false positive on green.

Primary Solution Path:

- Section 7.2 - Bayesian Statistics
- Template E (Bayesian Discrete Prior)

Key Insight: $P(\text{blue}|\text{says blue}) \approx 0.33$ despite 99% witness accuracy!

Variations:

- Different base rates → Posterior changes dramatically
- Different test accuracy → Sensitivity analysis
- Multiple tests → Sequential updating

2.3 Homework 3

2.3.1 HW3-Q1: Joint PMF

Problem: $p_{X,Y}(x, y) = c(x^2 + y^2)$ for $x \in \{1, 2, 4\}$, $y \in \{1, 3\}$.

Primary Solution Path:

- Section 4.1 - Joint Distributions
- Section 4.2 - Marginals
- Section 4.4 - Covariance

Solution Steps:

1. Find c: $\sum\sum p(x, y) = 1$
2. Marginals: $p_X(x) = \sum_y p(x, y)$
3. Expectations: $E[X] = \sum x \cdot p_X(x)$
4. Variance: $\mathrm{Var}(X) = E[X^2] - E[X]^2$

2.3.2 HW3-Q5: Reward for HT Patterns

Problem: Coin tosses with $P(H) = p$. Reward 1 for each HT pattern. Find $E[R]$, $\mathrm{Var}(R)$.

Primary Solution Path:

- Section 7.4 - Indicator Random Variables
- Formula: $R = \sum I_i$ where $I_i = 1$ if HT at position i

Key Formulas:

$$E[R] = (n - 1) \cdot p(1 - p)$$
$$\mathrm{Var}(R) = \sum \mathrm{Var}(I_i) + 2\sum \mathrm{Cov}(I_i, I_j)$$

2.3.3 HW3-Q6: Exponential Distribution

Problem: $X \sim \mathrm{Exp}(\lambda)$. Find survival function, moments, distribution of $\min(X_1, X_2)$.

Primary Solution Path:

- Section 3.4 - Exponential Distribution
- Formula: $P(X > x) = e^{-\lambda x}$

! If "mean $\theta = 3$" then $\lambda = 1/3$ NOT $\lambda = 3$!

Key Results:

- $E[X] = 1/\lambda$, $\mathrm{Var}(X) = 1/\lambda^2$
- $\min(X_1, X_2) \sim \mathrm{Exp}(\lambda_1 + \lambda_2)$
- Memoryless: $P(X > s + t|X > s) = P(X > t)$

2.4 Homework 4

2.4.1 HW4-Q1: Joint PDF Analysis

Problem: $f(x,y) = c(x^2 + xy)$ on $[0,1]^2$. Find c, marginals, covariance, check independence.

Primary Solution Path:

- Section 4.1 - Joint Distributions
- Section 4.2 - Marginal/Conditional
- Section 4.4 - Covariance/Correlation
- Section 4.3 - Independence Check

Solution:

1. $c = 12/7$ (integrate to 1)
2. $f_X(x) = \frac{12}{7}(x^2 + x/2)$
3. $E[X] = 5/7$, $\text{Var}(X) \approx 0.0469$
4. $\text{Cov}(X,Y) \approx -0.0034$
5. NOT independent (joint $\neq$ product of marginals)

2.4.2 HW4-Q4: Meeting Problem

Problem: $A, B \sim \text{Uniform}(0,60)$ independent. Find $P(|A-B| < 15)$.

Primary Solution Path:

- Section 4.1 - Joint Distributions
- Section 3.2 - Uniform Distribution
- Template U17 (Meeting Problem)

Key Method: Draw square $[0,60]^2$, shade region where condition holds, compute area fraction.
Solution: $P(|A-B| < 15) = 1 - (45/60)^2 = 1 - 9/16 = 7/16$

2.4.3 HW4-Q5: Overlapping Sums

Problem: $X_i \sim \text{Exp}(2)$ i.i.d. $X = \sum_{i=1}^{n} X_i$, $Y = \sum_{i=n-7}^{2n-8} X_i$. Find $\text{Cov}(X,Y)$.

Primary Solution Path:

- Section 4.4 - Covariance
- Formula: $\texttt{Cov}(\sum X_i, \sum X_j) = \sum_{\texttt{overlap}} \texttt{Var}(X_i)$

Solution: Overlap has 8 terms. $\text{Cov}(X,Y) = 8 \cdot \text{Var}(X_i) = 8 \cdot (1/4) = 2$.

2.5 Homework 5

2.5.1 HW5-Q1: Bivariate Normal

Problem: (X, Y) jointly normal with $\mu_X = 1$, $\sigma_X^2 = 2$, $\mu_Y = -2$, $\sigma_Y^2 = 3$, $\rho = -2/3$.

Primary Solution Path:

- Section 4.5 - Bivariate Normal
- Template A (Gaussian Vector Problems)
- Template F (BVN Conditional)
- Template G (Linear Combination Independence)

Part (a): $P(X + Y > 0)$

1. $X + Y$ is normal
2. $E[X + Y] = 1 + (-2) = -1$
3. $\text{Var}(X + Y) = 2 + 3 + 2(-2/3)\sqrt{6} \approx 1.73$
4. Standardize and use Φ table

Part (b): Find a for independence

1. Set $\text{Cov}(aX + Y, X + 2Y) = 0$
2. For MVN: $\text{Cov} = 0 \Leftrightarrow$ independent

2.5.2 HW5-Q2: Lognormal Distribution

Problem: X lognormal with parameters $\mu = 3$, $\sigma^2 = 1.44$. Find $P(X \leq 6.05)$.

Primary Solution Path:

- Section 7.3 - Lognormal Distribution
- Formula: $P(X \leq x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$

Solution:

$$P(X \leq 6.05) = \Phi\left(\frac{\ln(6.05) - 3}{1.2}\right) = \Phi\left(\frac{1.8 - 3}{1.2}\right) = \Phi(-1) \approx 0.159$$

2.5.3 HW5-Q4: Conditional BVN

Problem: Test scores bivariate normal. If score on A is 80, find $P(B > 90)$.

Primary Solution Path:

- Section 4.5 - Bivariate Normal
- Template F (BVN Conditional)
- Formula: $Y|X = x \sim N\left(\mu_Y + \rho\frac{\sigma_Y}{\sigma_X}(x - \mu_X), (1 - \rho^2)\sigma_Y^2\right)$

2.6 Homework 6

2.6.1 HW6-Q1: Monty Hall (Sober vs Dizzy)

Problem: Three doors, one car. Contestant picks A, Monty opens B (goat). Compare sober vs dizzy Monty.

Primary Solution Path:

- Section 7.2 - Bayesian Statistics
- Section 1.3 - Bayes' Theorem
- Template J (Monty Hall Variants)

Sober Monty (knows locations):

Hypothesis	Prior	Likelihood	Posterior
H_A (car at A)	1/3	1/2	1/3
H_B (car at B)	1/3	0	0
H_C (car at C)	1/3	1	2/3

Strategy: SWITCH! (probability 2/3)

Dizzy Monty (random):

Hypothesis	Prior	Likelihood	Posterior
H_A (car at A)	1/3	1/2	1/2
H_C (car at C)	1/3	1/2	1/2

Strategy: Doesn't matter (both 1/2)

2.6.2 HW6-Q3: CLT Sample Size

Problem: $\sigma = 3$. Find minimum n for $P(|\bar{X} - \mu| < 0.3) \geq 0.95$.

Primary Solution Path:

- Section 6.1 - CLT
- Template K (Finding n for CLT)

Solution:

$$\begin{aligned} P\left(|Z| < \frac{0.3\sqrt{n}}{3}\right) &\geq 0.95 \\ \frac{0.3\sqrt{n}}{3} &\geq 1.96 \\ \sqrt{n} &\geq 19.6 \\ n &\geq 385 \end{aligned}$$

2.6.3 HW6-Q5: Beta-Binomial

Problem: Prior: $\theta \sim \text{Uniform}(0, 1)$. Data: 3 defective in 8. Find posterior.

Primary Solution Path:

- Section 7.2 - Bayesian Statistics
- Formula: Beta$(\alpha, \beta) + k$ in $n \to$ Beta$(\alpha + k, \beta + n - k)$

Solution:

- Prior: $\theta \sim \text{Beta}(1, 1)$
- Posterior: $\theta|data \sim \text{Beta}(1 + 3, 1 + 5) = \text{Beta}(4, 6)$
- $E[\theta|data] = 4/10 = 0.4$

3 Practice Exam Problems

3.1 Practice Midterm 1

Problem	Description	Section	Template
PM1-Q1	Dice rolled 11 times, multinomial	Section 1.5, Section 2.2	-
PM1-Q2	Set theory true/false	Section 1.2, Section 1.4	-
PM1-Q3	Married couples seating	Section 1.5	-
PM1-Q4	Dice and cards game + Bayes	Section 2.6, Section 1.3	Template E

3.2 Practice Midterm 2

Problem	Description	Section	Template
PM2-Q1	Joint PDF $f(x,y) = 1/y$ on triangle	Section 4.1, Section 4.4	-
PM2-Q2	Joint PDF with exponential	Section 4.1, Section 7.1	-
PM2-Q3	Uniform[-1,1], max, correlation	Section 3.2, Section 4.7, Section 4.4	Template L
PM2-Q4	Coin game with MGF, random N	Section 5.1, Section 7.1	Template B

3.3 Practice Final

3.3.1 PF-Q1: Coin Game with CLT

Problem: Win \$3 if HH, lose \$1 if TT, \$0 otherwise. Play 400 times.

Primary Solution Path:

- Section 6.1 - CLT
- Template B (CLT Game Problems)

Solution:

1. Single game: $E[X] = 3(1/4) - 1(1/4) = 0.5$, $\text{Var}(X) = 2.25$
2. Total: $S_{400} \sim N(200, 900)$
3. $P(S > 240) = P(Z > 4/3) = 1 - \Phi(1.33) \approx 0.092$

3.3.2 PF-Q2: Gaussian Vector Independence

Problem: X_1, X_2 i.i.d. $N(0,1)$. $Y_1 = aX_1 + X_2$, $Y_2 = X_1 + bX_2$. Find a, b for independence.

Primary Solution Path:

- Section 4.5 - Bivariate Normal
- Template A (Gaussian Vector)
- Template G (Linear Combination Independence)

Key Insight: For MVN, independence $\Leftrightarrow$ Cov $= 0$.
Solution: $\text{Cov}(Y_1, Y_2) = a + b = 0 \Rightarrow b = -a$.

3.3.3 PF-Q3: Exponential CLT

Problem: $X_1, \ldots, X_{100}$ i.i.d. Exp with mean 3. Find $P\left(\frac{\bar{X}}{\bar{X}+3} < 0.5\right)$.

Primary Solution Path:

- Section 6.1 - CLT
- Section 3.4 - Exponential
- Template C (Exponential + CLT)
- Template O (Ratio with Sample Mean)

! Mean $\theta = 3$ means $\lambda = 1/3$!

Solution:

1. Transform: $\frac{\bar{X}}{\bar{X}+3} < 0.5 \Leftrightarrow \bar{X} < 3$
2. By CLT: $\bar{X} \approx N(3, 9/100)$
3. $P(\bar{X} < 3) = \Phi(0) = 0.5$

3.3.4 PF-Q4: Lognormal Stock Price

Problem: $S = S_0 e^Z$ where $Z \sim N(r - \sigma^2/2, \sigma^2)$. Find $E[e^{-r}S]$ and $P(S > 100)$.

Primary Solution Path:

- Section 7.3 - Lognormal
- Template D (Lognormal Stock Price)

Key Formula: $E[e^X] = e^{\mu+\sigma^2/2}$ for $X \sim N(\mu, \sigma^2)$.
Solution (a): $E[e^{-r}S] = S_0 = 100$ (risk-neutral pricing result)
Solution (b): $P(S > 100) = P(Z > 0) = 1 - \Phi\left(\frac{-(r-\sigma^2/2)}{\sigma}\right)$

3.3.5 PF-Q5: Discrete Bayesian

Problem: Prior: $P(\theta = 1/2) = P(\theta = 3/4) = 1/2$. Data: 0 defective in 10.

Primary Solution Path:

- Section 7.2 - Bayesian Statistics
- Template E (Bayesian Discrete Prior)

Solution:

- $P(\theta = 1/2|data) \approx 0.999$
- $P(\theta = 3/4|data) \approx 0.001$

4 Problems by Type

4.1 Counting & Combinatorics

- HW1-Q1: Poker hands → Section 1.5
- HW1-Q3: Birthday problem → Section 1.5
- HW1-Q7: Elevator → Section 1.5
- PM1-Q1: Multinomial dice → Section 1.5
- PM1-Q3: Circular seating → Section 1.5

4.2 Conditional Probability

- HW2-Q1: Polya urn → Section 1.2
- HW2-Q2: Craps game → Section 1.2
- HW2-Q8: Binary paradox → Section 1.2
- HW3-Q2: Three-sided die → Section 1.2

4.3 Bayes' Theorem

- HW2-Q7: Five coins → Section 7.2
- HW2-Q9: Blue taxi → Section 7.2
- HW2-Q10: Four dice → Section 7.2
- HW6-Q1: Monty Hall → Section 7.2, Template J
- HW6-Q2: Dice prediction → Section 7.2, Template H
- HW6-Q5, Q6: Beta-Binomial → Section 7.2
- PF-Q5: Discrete prior → Section 7.2, Template E

4.4 Joint Distributions

- HW3-Q1: Joint PMF → Section 4.1
- HW4-Q1: Joint PDF → Section 4.1, Section 4.4
- HW4-Q2: Independence check (PMF) → Section 4.3
- HW4-Q4: Meeting problem → Section 4.1, Template U17
- PM2-Q1, Q2: Joint PDF with different regions → Section 4.1

4.5 Bivariate Normal

- HW5-Q1: BVN with all parts → Section 4.5, Template A,F,G
- HW5-Q4: Conditional BVN → Section 4.5, Template F
- HW5-Q5: Parameter recovery → Section 4.5, Template N
- PF-Q2: Gaussian vector independence → Section 4.5, Template A,G

4.6 CLT / Limit Theorems

- HW6-Q3: Sample size determination → `Section 6.1`, `Template K`
- HW6-Q4: Digit average → `Section 6.1`
- PF-Q1: Coin game 400 times → `Section 6.1`, `Template B`
- PF-Q3: Exponential average → `Section 6.1`, `Template C,O`

4.7 Lognormal / Finance

- HW5-Q2: Lognormal CDF → `Section 7.3`
- HW5-Q3: Product of lognormals → `Section 7.3`, `Template I`
- HW5-Q6: Stock returns data → `Section 7.3`
- PF-Q4: Stock price model → `Section 7.3`, `Template D`

4.8 Exponential Distribution

- HW3-Q6: Exp basics, min of Exp → `Section 3.4`
- HW4-Q5: Overlapping sums of Exp → `Section 3.4`, `Section 4.4`
- PF-Q3: Exp with CLT → `Section 3.4`, `Section 6.1`, `Template C`

5 Problems by Study Guide Section

Section	Problems
Section 1.1-1.2	HW1-Q4,Q5,Q6,Q8; HW2-Q1,Q2,Q8; PM1-Q2
Section 1.3 (Bayes)	HW2-Q7,Q9,Q10; HW6-Q1,Q2; PM1-Q4; PF-Q5
Section 1.5 (Counting)	HW1-Q1,Q3,Q7; PM1-Q1,Q3
Section 2.2 (Binomial)	HW2-Q4,Q5,Q6; HW3-Q2,Q4
Section 2.6 (Hypergeometric)	PM1-Q4
Section 3.2 (Uniform)	HW4-Q4; PM2-Q3
Section 3.3 (Normal)	HW5-Q1,Q4,Q5; PF-Q2
Section 3.4 (Exponential)	HW3-Q6; HW4-Q5; PF-Q3
Section 4.1 (Joint)	HW3-Q1; HW4-Q1,Q4; PM2-Q1,Q2
Section 4.4 (Cov/Cor)	HW3-Q1; HW4-Q1,Q3,Q5; PM2-Q1,Q3
Section 4.5 (BVN)	HW5-Q1,Q4,Q5; PF-Q2
Section 4.6 (Transform)	HW3-Q7,Q8
Section 4.7 (Order Stats)	PM2-Q3
Section 5.1 (MGF)	PM2-Q4
Section 6.1 (CLT)	HW6-Q3,Q4; PF-Q1,Q3
Section 7.1 (Cond Exp)	PM2-Q2,Q4
Section 7.2 (Bayesian)	HW2-Q7,Q9,Q10; HW6-Q1,Q2,Q5,Q6; PF-Q5
Section 7.3 (Lognormal)	HW5-Q2,Q3; PF-Q4

6 Terminology Dictionary

Term in Problem	**Meaning**	**Go To**	**Example**
"Gaussian"	Normal distribution	Section 3.3 or Section 4.5	HW5-Q1, PF-Q2
"Gaussian vector"	Multivariate Normal	Section 4.5	PF-Q2
"Independent components"	$\rho = 0$ (for MVN: independent!)	Section 4.5	PF-Q2
"Mean θ" (Exp)	$\lambda = 1/\theta$ NOT θ!	Section 3.4	PF-Q3
"i.i.d."	Independent, identical	Often Section 6.1	PF-Q1,Q3
"Large n"	Use CLT	Section 6.1	HW6-Q3,Q4
"Approximate"	Use CLT	Section 6.1	PF-Q1,Q3
"Prior" / "Posterior"	Bayesian	Section 7.2	HW6-Q1,Q5
"Conjugate"	Beta-Binomial etc.	Section 7.2	HW6-Q5,Q6
"Stock price" / "S_0e^Z"	Lognormal	Section 7.3	PF-Q4
"Memoryless"	Exponential (continuous)	Section 3.4	HW3-Q6
"Max" / "Min"	Order statistics	Section 4.7	PM2-Q3
"$\psi(t)$"	MGF	Section 5.1	PM2-Q4
"E[X—Y]"	Conditional expectation	Section 7.1	PM2-Q2,Q4
"Monty Hall"	Bayesian	Section 7.2, Template J	HW6-Q1
"Without replacement"	Hypergeometric	Section 2.6	PM1-Q4
"n games/trials"	CLT	Section 6.1, Template B	PF-Q1

7 Template Quick Reference

Template	**When to Use**	**Key Formula**
Template A	Gaussian vector, MVN	Linear combos are normal; $\rho = 0 \Leftrightarrow$ indep
Template B	CLT game/coin problems	$S_n \approx N(n\mu, n\sigma^2)$
Template C	Exponential + CLT	Watch λ vs mean; $\bar{X} \approx N(1/\lambda, 1/(n\lambda^2))$
Template D	Lognormal stock	$E[e^X] = e^{\mu+\sigma^2/2}$
Template E	Bayesian discrete prior	Bayes table
Template F	BVN conditional	$Y\|X = x \sim N(\mu_Y + \rho\frac{\sigma_Y}{\sigma_X}(x - \mu_X), (1 - \rho^2)\sigma_Y^2)$
Template G	Linear combo independence	Set Cov = 0, solve
Template H	Predictive distribution	$P(x_{n+1}\|data) = \int P(x\|\theta)\pi(\theta\|data)d\theta$
Template I	Product of lognormals	$\ln(XY) = \ln X + \ln Y$ is normal
Template J	Monty Hall variants	Different likelihoods!
Template K	Find n for CLT	$n \geq (z\sigma/\epsilon)^2$
Template L	Order statistics	$F_{\max}(x) = [F(x)]^n$; $F_{\min}(x) = 1 - [1 - F(x)]^n$
Template N	BVN from conditions	Match conditional formulas
Template O	Ratio with sample mean	Transform inequality first!

A Emergency Problem-Solving Flowchart

1. **Read problem completely**
2. **Identify keywords** (see Terminology Dictionary)
3. **Map to section** using keyword table
4. **Check for template** match
5. **Write down relevant formulas**
6. **Execute step by step**
7. **Verify answer** (probability in [0,1]? Units correct?)

If Still Stuck:

- Is it asking for probability? → CDF or integration
- Is it asking for expectation? → $E[X] = \int xf(x)dx$
- Is it "given" something? → Conditional probability/distribution
- Is it "update belief"? → Bayes' theorem
- Is it large n? → CLT
- Does it say "Gaussian"? → It's Normal!

B Formula Quick Reference

Probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad P(A) = \sum P(A|B_i)P(B_i)$$

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)} \qquad \text{(Bayes)}$$

Expectation/Variance:

$$E[aX + b] = aE[X] + b \qquad \text{Var}(aX + b) = a^2\text{Var}(X)$$

$$\text{Var}(X) = E[X^2] - E[X]^2 \qquad \text{Cov}(X, Y) = E[XY] - E[X]E[Y]$$

CLT:

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0, 1)$$

Bivariate Normal Conditional:

$$Y|X = x \sim N\left(\mu_Y + \rho\frac{\sigma_Y}{\sigma_X}(x - \mu_X), (1 - \rho^2)\sigma_Y^2\right)$$

Lognormal:

$$E[e^X] = e^{\mu+\sigma^2/2} \quad \text{for } X \sim N(\mu, \sigma^2)$$

Beta-Binomial Conjugate:

$$\text{Beta}(\alpha, \beta) + k \text{ successes in } n \to \text{Beta}(\alpha + k, \beta + n - k)$$

C Problem Count Summary

Source	Problems	Sub-parts
Homework 1	8	15
Homework 2	11	22
Homework 3	8	24
Homework 4	5	13
Homework 5	6	11
Homework 6	6	15
Practice Midterm 1	4	6
Practice Midterm 2	4	12
Practice Final	5	13
TOTAL	**57**	**131**