Homework #7 - MAT 2250

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Chapter 4.4

$$y''(\theta) + 3y'(\theta) - y(\theta) = sec\theta \quad \textbf{(5)}$$

Since the given equation has $sec\theta$ we cannot use the method of undetermined coefficients to find a particular solution. This is because when we take the derivative of the trial solution, $sec\theta$, it becomes to messy to obtain a true solution.

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = xe^x \quad \textbf{(15)}$$

$$r^2 - 5r + 6 = 0$$
$$r = 3,\ r = 2$$
$$y_h = c_1e^{3t} + c_2e^{2t}$$
$$y_p = (Ax + B)e^x$$
$$y'p = (Ax + B + A)e^x$$
$$y_p'' = (Ax + B + 2A)e^x$$
$$(Ax + B + 2A)e^x - 5((Ax + B + A)e^x) + 6((Ax + B)e^x) = xe^x$$
$$Axe^x + Be^x + 2Ae^x - 5Axe^x - 5Be^x - 5Ae^x + 6Axe^x + 6Be^x = xe^x$$
$$2Axe^x + 2Be^x - 3Ae^x = xe^x$$
$$2A = 1,\ 2B - 3A = 0$$
$$A = \frac{1}{2},\ \ B = \frac{3}{4}$$
$$y_p = \frac{x}{2}e^x + \frac{3}{4}e^x$$

$$y = c_1e^{3t} + c_2e^{2t} + \frac{x}{2}e^x + \frac{3}{4}e^x$$

$$y'' - 6y' + 9y = 5t^6 e^{3t} \tag{\textbf{28}}$$

$$r^2 - 6r + 9 = 0$$
$$r = 3$$
$$y(p) = (A_6 t^6 + A_5 t^5 + A_4 t^4 + A_3 t^3 + A_2 t^2 + A^1 t + A_0)e^{3t}$$

Chapter 4.5

$$\theta'' - \theta' - 2\theta = 1 - 2t, \qquad \theta_p(t) = t - 1 \tag{5}$$

$$r^2 - r - 2 = 0$$
$$r = 2, \quad r = -1$$
$$\theta_h = c_1 e^{2r} + c_2 e^{-t}$$

$$\theta_g = c_1 e^{2r} + c_2 e^{-t} + t - 1$$

$$y'' - y = e^{2t} + te^{2t} + t^2 e^{2t} \tag{32}$$

$$r^2 - 1 = 0$$
$$r = 1, \quad r = -1$$
$$y_h = c_1 e^t + c_2 e^{-t}$$
$$y_p = t^0 (At^2 + Bt + C) e^{2t}$$
$$y_p' = (2At + B)e^{2t} + 2(Aat^2 + Bt + C)e^{2t}$$
$$y_p' = (2At^2 + (2A + 2B)t + 2C)e^{2t}$$
$$y_p'' = (4At + 2A + 2B)e^{2t} + (4At^2 + (4A + 4B)t + 4C)e^{2t}$$
$$y_p'' = (4At^2 + (8A + 4B)t + (2A + 2B + 4C)e^{2t}$$
$$(4At^2 + (8A + 4B)t + (2A + 2B + 4C)e^{2t} - (At^2 + Bt + C)e^{2t} = e^{2t} + te^{2t} + t^2 e^{2t}$$
$$(3At^2 + (8A + 3B)t + (2A + 2B + 3C))e^{2t} = e^{2t} + te^{2t} + t^2 e^{2t}$$
$$3A = 1, \quad 8A + 3B = 1, \quad 2A + 2B + 3C = 1 A = \frac{1}{3}, \quad B = -\frac{5}{9}, \quad C = \frac{13}{27}$$

$$y_p = \left(\frac{1}{3} t^2 - \frac{5}{9} t + \frac{13}{27} \right) e^{2t}$$

1 Chapter 4.9

The motion of a mass spring system with damping is governed by (**6**)

$$y''(t) + 4y'(t) + ky(t) = 0;$$
$$y(0) = 1, \quad y'(0) = 0$$

Find the equation of motion and sketch its graph for k = 2, 4, and 6.

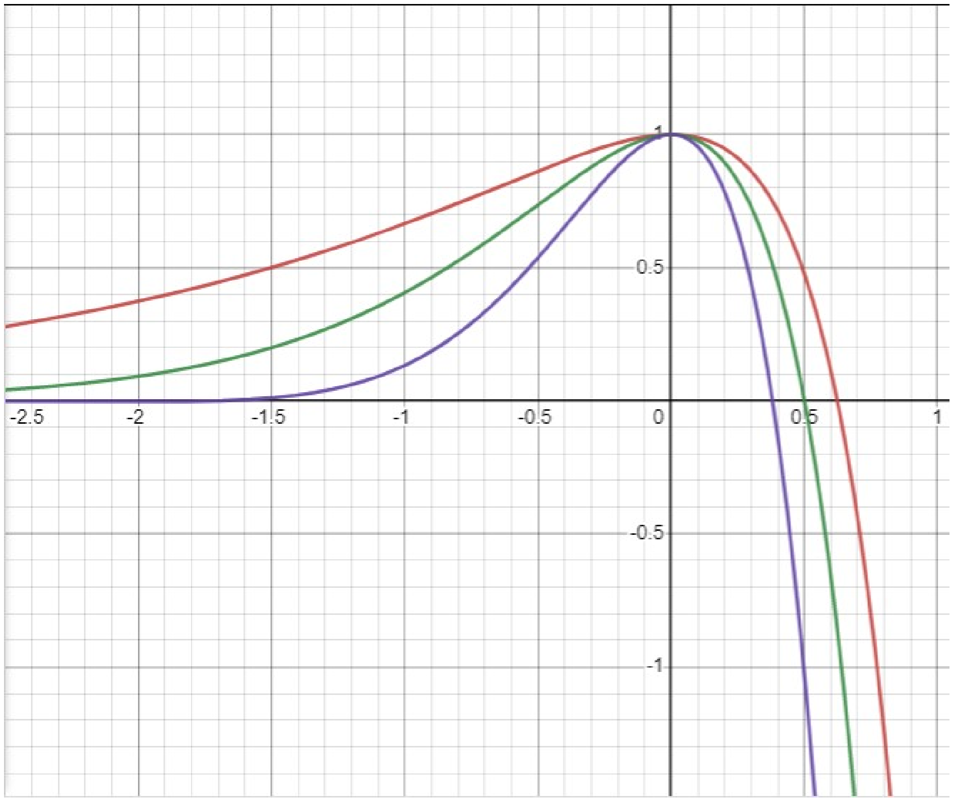
$$r^2 + 4r + k = 0$$
$$r = 2 + \sqrt{4-k}, \quad 2 - \sqrt{4-k}$$

When $k < 4$ we have two distinct roots, if $k = 4$ we have a double root,if $k > 4$ we have two complex conjugate roots

$$k = 2 \implies r = 2 + \sqrt{4-2}, \quad 2 - \sqrt{4-2}$$
$$y(t) = Ae^{(2+\sqrt{2})t} + Be^{(2-\sqrt{2})t}$$
$$1 = Ae^{(2+\sqrt{2})0} + Be^{(2-\sqrt{2})0} 1 = A + B$$
$$y'(t) = (2+\sqrt{2})Ae^{(2+\sqrt{2}t)} + (2-\sqrt{2})Be^{(2-\sqrt{2})t}$$
$$0 = (2+\sqrt{2})Ae^{(2+\sqrt{2}0)} + (2-\sqrt{2})Be^{(2-\sqrt{2})0}$$
$$(2+\sqrt{2})A + (2-\sqrt{2})B = 0$$
$$A = \frac{1-\sqrt{2}}{2}, \quad B = \frac{1+\sqrt{2}}{2}$$
$$y(t) = \frac{1-\sqrt{2}}{2}e^{(2+\sqrt{2})t} + \frac{1+\sqrt{2}}{2}e^{(2-\sqrt{2})t}$$

$$k = 4 \implies r = 2 + \sqrt{4-4}, \quad 2 - \sqrt{4-4}$$
$$y(t) = Ae^{2t} + Bte^{2t}$$
$$1 = Ae^{2(0)} + B(0)e^{2}0$$
$$1 = A$$
$$y'(t) = 2Ae^{2t} + (1+2t)Be^{2t}$$
$$0 = 2Ae^{2(0)} + (1+2(0))Be^{2t}0 = 2A + B$$
$$A = 1, \quad B = -2$$
$$y(t) = e^{2t} - 2te^{2t}$$

$$k = 6 \implies r = 2 + \sqrt{4-6}, \quad 2 - \sqrt{4-6}$$
$$y(t) = Ae^{2t}\cos(\sqrt{2}t) + Be^{2t}\sin(\sqrt{2}t)$$
$$1 = Ae^{2(0)}\cos(\sqrt{2}(0)) + Be^{2(0)}\sin(\sqrt{2}(0))$$
$$1 = A$$
$$y'(t) = Ae^{2t}(2\cos(\sqrt{2}t) - \sqrt{5}\sin(\sqrt{2}t)) + Be^{2t}(2\sin(\sqrt{2}t) + \sqrt{2}\cos(\sqrt{2}t))$$
$$0 = Ae^{2(0)}(2\cos(\sqrt{2}(0)) - \sqrt{5}\sin(\sqrt{2}(0)) + Be^{2(0)}(2\sin(\sqrt{2}(0)) + \sqrt{2}\cos(\sqrt{2}(0)))$$
$$2A + \sqrt{2}B = 0$$
$$A = 1, \quad B = -\sqrt{2}$$
$$y(t) = e^{2t}\cos(\sqrt{2}t) - \sqrt{2}e^{2t}\sin(\sqrt{2}t)$$



Chapter 6.1

$$\{x^2, x^2 - 1, 5\} \text{ on } (-\infty, \infty) \tag{8}$$

$$W(x^2, x^2 - 1, 5) = \begin{vmatrix} x^2 & x^2 - 1 & 5 \\ 2x & 2x & 0 \\ 2 & 2 & 0 \end{vmatrix}$$

$$W(x^2, x^2 - 1, 5) = 5(2(2x) - (2x)2)$$
$$W(x^2, x^2 - 1, 5) = 0$$

since the Wronskian of our functions is zero we can say our functions are linearly dependent on the interval $(-\infty, \infty)$

$$y''' + 2y'' - 11y - 12y = 0;$$
$$\{e^{3x}, e^{-x}, e^{-4x}\}$$

$$W(e^{3x}, e^{-x}, e^{-4x}) = \begin{vmatrix} e^{3x} & e^{-x} & e^{-4x} \\ 3e^{3x} & -e^{-x} & -4e^{-4x} \\ 9e^{3x} & e^{-x} & 16e^{-4x} \end{vmatrix}$$

$$\begin{gathered} W(e^{3x}, e^{-x}, e^{-4x}) = (e^{3x})(-e^{x}(16e^{-4x} - (-4e^{-4x})e^{-x}) \\ -3e^{3x}(e^{-x}(16e^{-4x}) - e^{-4x}(e^{-x}) \\ +9e^{3x}(e^{-x}(-4e^{-4x}) - e^{-4x}(-e^{-x})) \\ W(e^{3x}, e^{-x}, e^{-4x}) = -4e^{-2x}(16e^{8x} + 5) \end{gathered}$$

We can say the given functions are linearly independent for $\mathbb{R}$ and therefore form a fundamental set of solutions. We know the general solution is a linear combination of our fundamental solution set of solutions.

$$y(x) = C_1e^{3x} + C_2e^{-x} + C_3e^{-4x}$$

$$\begin{gathered} x_1' = (\cos 2t)x_1 \\ x_2' = (\sin 2t)x_2 \\ x_3' = x_1 - x_2 \\ x' = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}' = \begin{bmatrix} \cos(2t) & 0 & 0 \\ 0 & \sin(2t) & 0 \\ 1 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \end{gathered} \tag{\textbf{6}}$$

$$\begin{gathered} x'' + 3x + 2y = 0 \\ y'' - 2x = 0 \end{gathered} \tag{\textbf{11}}$$

$$x_1 = x,\ x_2 = x',\ x_3 = y,\ x_4 = y'$$
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -3 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$