Financial derivatives

Lecture 5: Option trading strategies

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Trading of options

Option trading

Similar to forwards and futures, **option trading** can occur **over-the-counter** or through organized **exchanges**. There are a few things we should consider when trading options:

- **Option styles**: European, American, or exotics.
- **Strike price**.
- **Open interest and volume**. market liquidity.
- **Time to maturity**.
- **Implied volatility**. → higher vol → higher insurance value.

Option trading strategies

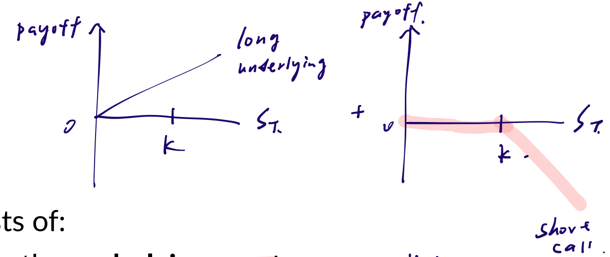
We can **combine** different option contracts to **customize** payoffs of our trading strategies. Typical examples include:

- **Covered positions**.
- **Vertical spreads** (same time to maturity).
- **Combinations**.
- **Horizontal (Calendar) spreads** (different time to maturity).

For the following cash flow tables, we assume that options are not **early exercised**.

Covered positions

Covered calls

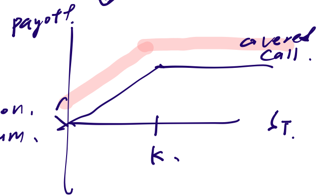


A **covered call** consists of:

1. a **long position** in the **underlying asset**;
2. a **short position** in the **call option**.

Use this strategy when you:

- **expect** the market to **remain flat**;
- **expect** the **implied volatility** to be **low**.



option premium

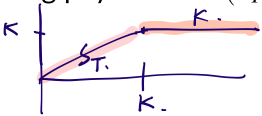
earned from forgo of upward growing potential

Covered calls

Table: Covered call

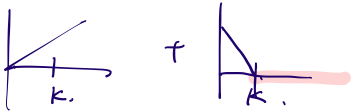
Investment	**Cash flows at**		
	t	$T : S_T < K$	$T : S_T \geq K$
Long underlying	$-S_t$	S_T	S_T
Short call	$C_{K,t}$	0	$K - S_T$ $-(S_T - K)$
Net cash flows	$C_{K,t} - S_t$	S_T	K

The resulting payoff is $\min(S_T, K)$. ⇐. get S_T. when $S_T < K$. + get K. when $S_T \geq K$.



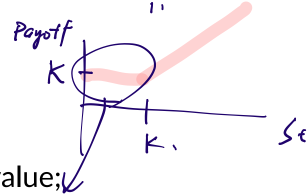
Protective puts

(Buy insurance.)



A **protective put** consists of:

1. a **long position** in the **underlying asset**;
2. a **long position** in the **put option**.



Use this strategy when you:

- **expect** the **underlying asset** to drop in value;
- want to **buy insurance** to **protect against downside risk**.

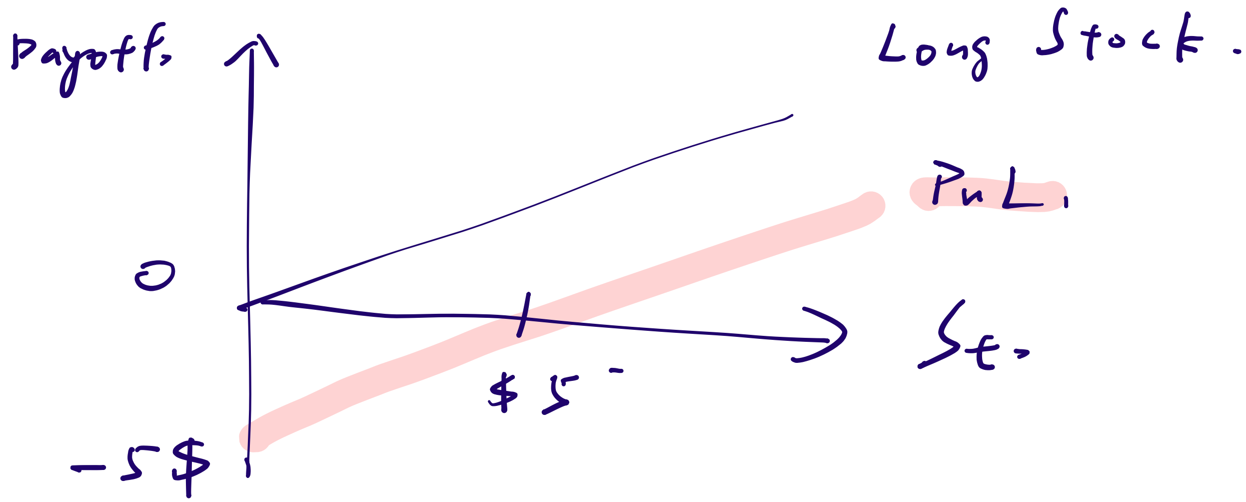
Protective puts

Table: Protective put

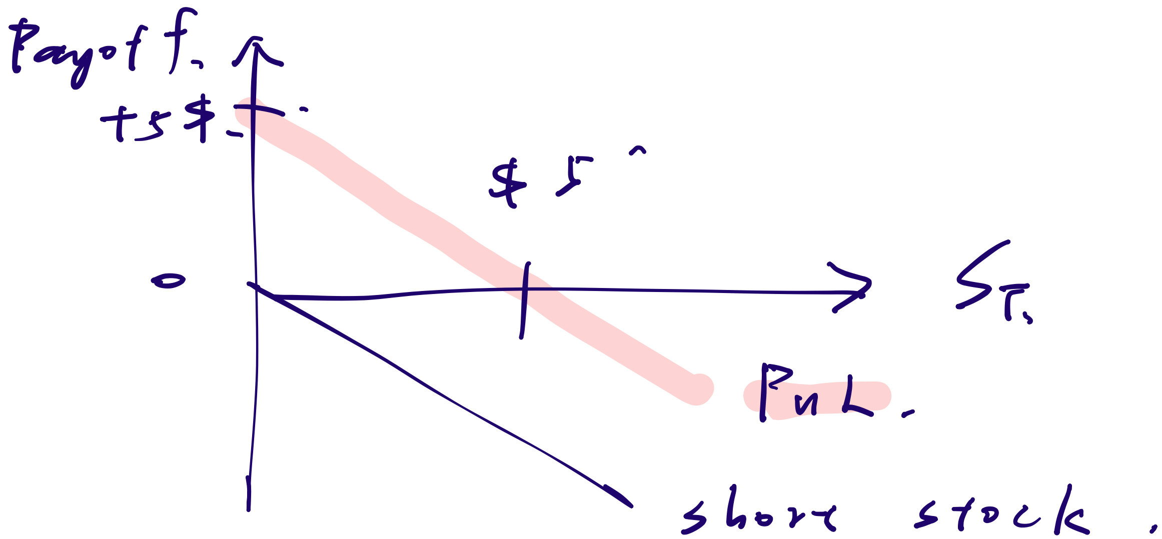
Investment	**Cash flows at**		
	t	$T: S_T < K$	$T: S_T \geq K$
Long underlying	$-S_t$	S_T	S_T
Long put	$-P_{K,t}$	$K - S_T$ (ITM)	0
Net cash flows	$-S_t - P_{K,t}$	K	S_T

The resulting payoff is $\max(S_T, K)$.

<Long>

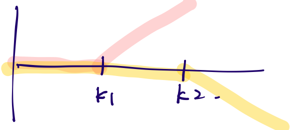


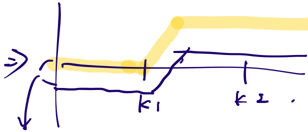
<Short>.



Vertical spreads

Bull spreads





A **bull spread with calls** consists of:

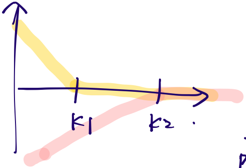
1. a **long position** in the **call option** at K_1;   ($C_1 > C_2$) more ITM.
2. a **short position** in the **call option** at K_2 with $K_2 > K_1$.

A **bull spread with puts** consists of:

1. a **long position** in the **put option** at K_1;   ($P_2 > P_1$)
2. a **short position** in the **put option** at K_2 with $K_2 > K_1$.

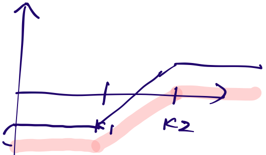
Use this strategy when you:

- anticipate a **limited upside** for the **underlying**;
- want to **limit the maximum loss** of the position.



=>

earn premium



Bull spreads

Table: Bull spread using calls

$(K_2 > K_1) \Rightarrow (C_{K_1} > C_{K_2})$

Investment	Cash flows at			
	t	$S_T < K_1$	$K_1 \leq S_T < K_2$	$S_T \geq K_2$
Long K_1 call	$-C_{K_1,t}$	0 OTM	$S_T - K_1$ ITM	$S_T - K_1$
Short K_2 call	$C_{K_2,t}$	0 OTM	0 OTM	$K_2 - S_T$
Net cash flows	$C_{K_2,t} - C_{K_1,t}$	0	$S_T - K_1$	$K_2 - K_1$
	$-$	0	$+$	$+$

pay (cost) to get future $+$ payoff

Bull spreads

Table: Bull spread using puts

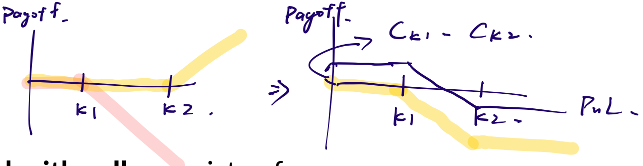
$K_2 > K_1 \Rightarrow P_{K_2} > P_{K_1}$.

Investment	Cash flows at			
	t	$S_T < K_1$ (K2 -)	$K_1 \leq S_T < K_2$	$S_T \geq K_2$
Long K_1 put	$-P_{K_1,t}$	$K_1 - S_T$ ITM	0 OTM	0 OTM
Short K_2 put	$P_{K_2,t}$	$S_T - K_2$ ITM,	$S_T - K_2$ ITM,	0 OTM.
Net cash flows	$P_{K_2,t} - P_{K_1,t}$	$K_1 - K_2$	$S_T - K_2$	0
	+	(−	−	0.)

get + now, get − in the future

Payoff ≤ 0.

Bear spreads



A **bear spread with calls** consists of:

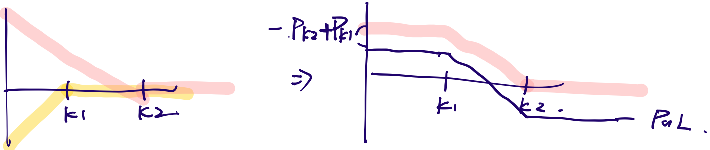
1. a **short position** in the **call option** at K_1; $C_{K1} > C_{K2}$.
2. a **long position** in the **call option** at K_2 with $K_2 > K_1$.

A **bear spread with puts** consists of:

1. a **short position** in the **put option** at K_1; $P_{K2} > P_{K1}$.
2. a **long position** in the **put option** at K_2 with $K_2 > K_1$.

Use this strategy when you:

- anticipate a **limited downside** for the **underlying**;
- want to **limit the maximum loss** of the position.



Bear spreads

Table: Bear spread using calls

$K_2 > K_1$, $C_{K1} > C_{K2}$.

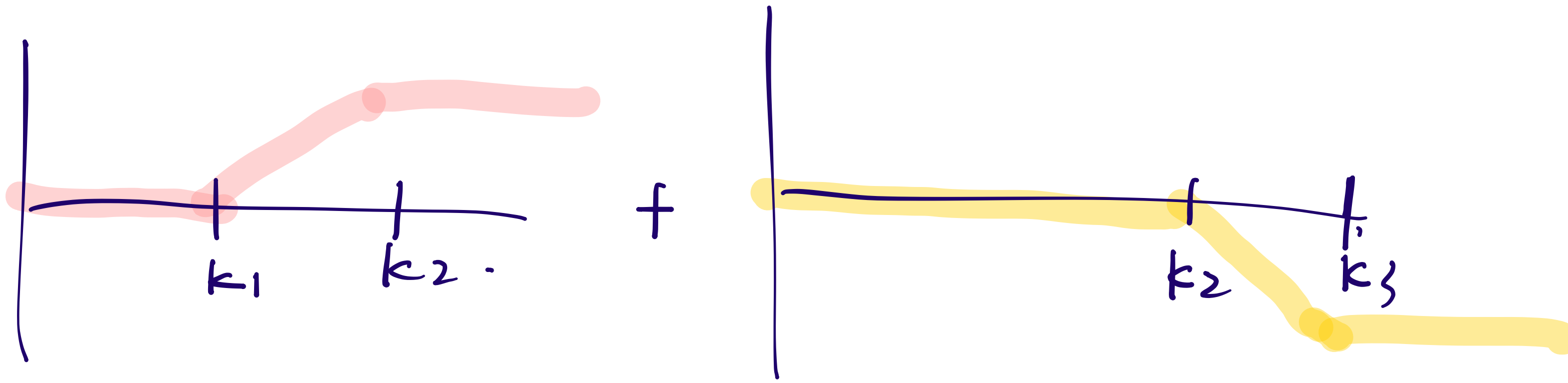
Investment	Cash flows at			
	t	$S_T < K_1$	$K_1 \leq S_T < K_2$	$S_T \geq K_2$
Short K_1 call	$C_{K_1,t}$	0	$K_1 - S_T$ ITM.	$K_1 - S_T$
Long K_2 call	$-C_{K_2,t}$	0	0	$S_T - K_2$
Net cash flows	$C_{K_1,t} - C_{K_2,t}$	0	$K_1 - S_T$	$K_1 - K_2$
	+	0	−	−

Bear spreads

Table: Bear spread using puts

$K_2 > K_1$, $P_{K_2} > P_{K_1}$.

Investment	Cash flows at			
	t	$S_T < K_1$	$K_1 \leq S_T < K_2$	$S_T \geq K_2$
Short K_1 put	$P_{K_1,t}$	$S_T - K_1$	0	0
Long K_2 put	-$P_{K_2,t}$	$K_2 - S_T$	$K_2 - S_T$	0
Net cash flows	$P_{K_1,t} - P_{K_2,t}$	$K_2 - K_1$	$K_2 - S_T$	0
	$-$	($+$	$+$	0).



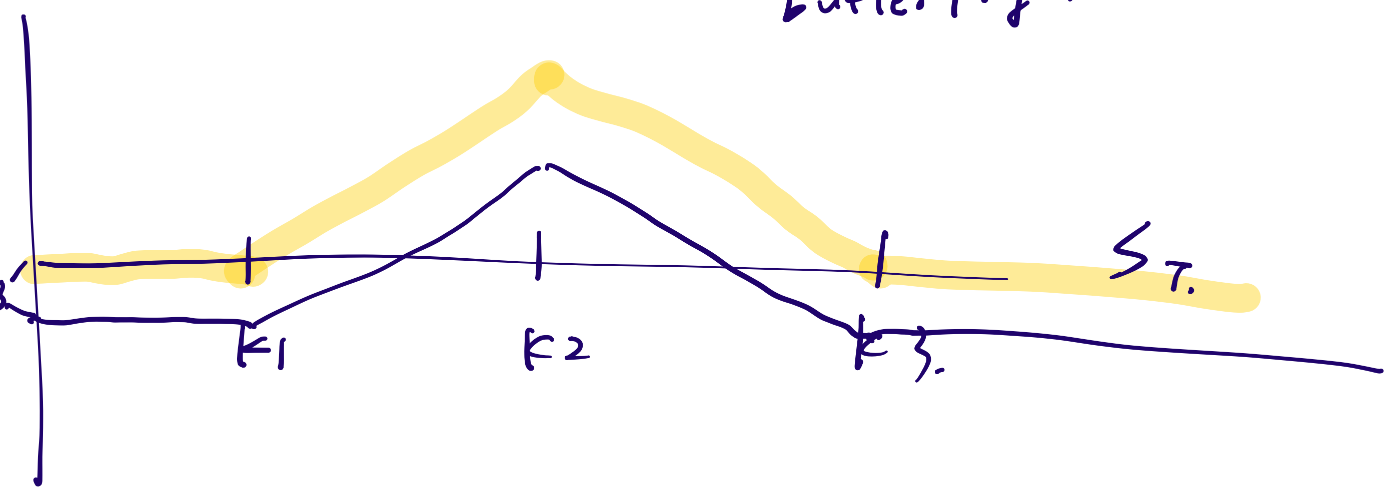
?

<calls>.

Payoff.

butterfly.

$2C_{K_2} - C_{K_1} - C_{K_3}$



Butterfly spreads

A **butterfly spread with calls** consists of:

1. a **long position** in the **call option** at K_1; $-C_{K1}$
2. **two short positions** in the **call option** at K_2; $+C_{K2}$
3. a **long position** in the **call option** at K_3 with $K_3 > K_2 > K_1$ and $K_1 + K_3 = 2K_2$. (symmetric butterfly spread) $-C_{K3}$.

A **butterfly spread with puts** consists of:

1. a **long position** in the **put option** at K_1;
2. **two short positions** in the **put option** at K_2;
3. a **long position** in the **put option** at K_3 with $K_3 > K_2 > K_1$ and $K_1 + K_3 = 2K_2$.

Butterfly spreads are simply joining a **bull spread** and a **bear spread** at a common middle strike price. Use this strategy when you:

- predict **limited fluctuations** in the **underlying**;
- want to **limit the maximum loss** of the position.

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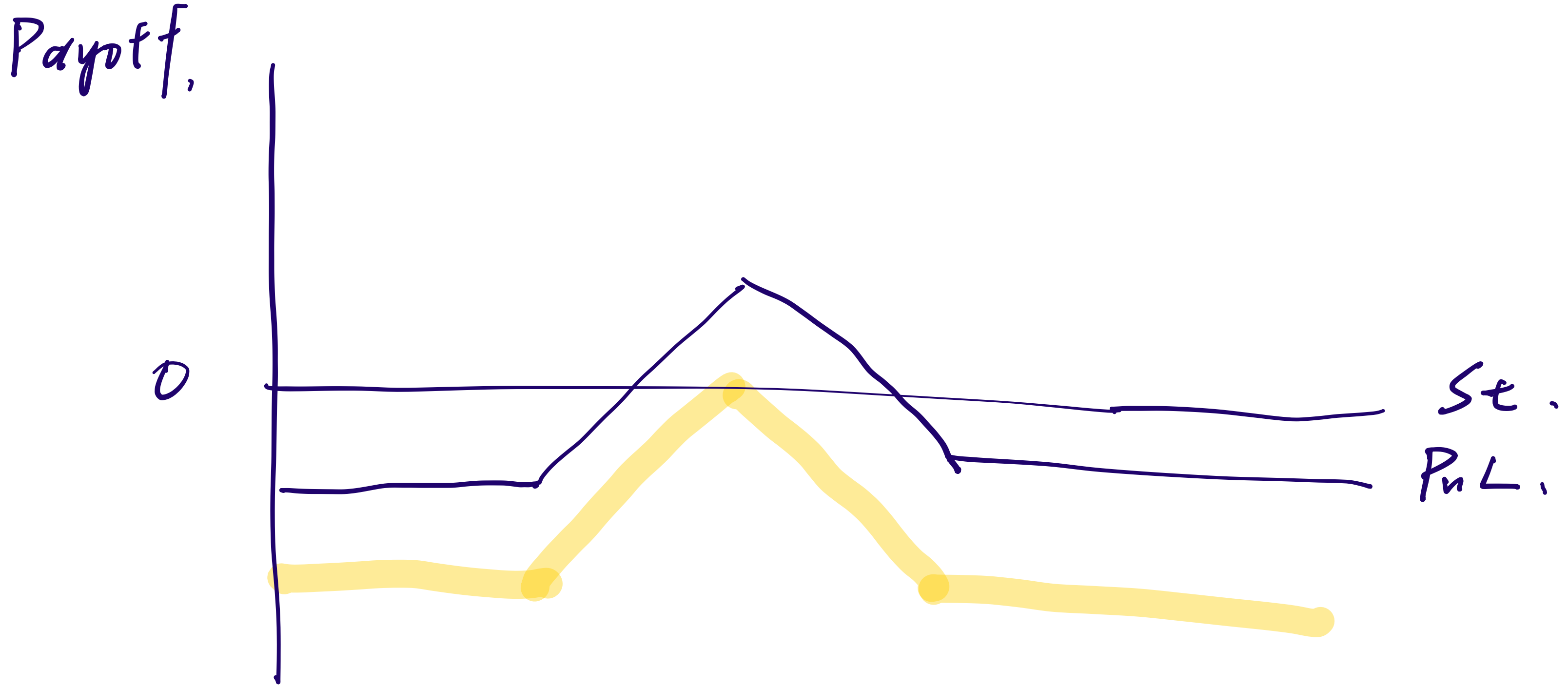

< put >.

$$wk_1 + (1-w)k_3 = k_2$$

$$S_T - wk_1 - (1-w)k_3.$$

$$= S_T - (wk_1 + (1-w)k_3)$$

$$= S_T - k_2.$$

$$wS_T - w\cancel{k_1} + w\cancel{k_1} + (1-w)k_3 - S_T.$$

$$= (w-1)S_T + (1-w)k_3.$$

Butterfly spreads

Table: Butterfly spread using calls

$K_3 > K_2 > K_1$. ($C_{K_1} > C_{K_2} > C_{K_3}$)

Investment	**Cash flows at**				
	t	$S_T < K_1$	$K_1 \le S_T < K_2$	$K_2 \le S_T < K_3$	$S_T \ge K_3$
Long 1 K_1 call	$-C_{K_1,t}$	0	$S_T - K_1$	$S_T - K_1$ $+$	$S_T - K_1$
Short 2 K_2 call	$2C_{K_2,t}$	0	0	$2K_2 - 2S_T$ $-$	$2K_2 - 2S_T$
Long 1 K_3 call	$-C_{K_3,t}$	0	0	0	$S_T - K_3$
Net cash flows	$2C_{K_2,t} - C_{K_1,t} - C_{K_3,t}$	0	$S_T - K_1$	$2K_2 - S_T - K_1$	$2K_2 - K_1 - K_3$
	$-$	0	$+$	$K_1 + K_3 - S_T - \cancel{K_1}$	0

For a **symmetric** butterfly spread, we have $K_1 + K_3 = 2K_2$:

$\Rightarrow$ **Net cash flows**	$2C_{K_2,t} - C_{K_1,t} - C_{K_3,t}$	0	$S_T - K_1$	$K_3 - S_T$	0

Butterfly spreads and call option value bounds

Since the **symmetric** butterfly spread produces **non-negative pay-offs** in the future, the price of **buying** the spread must be **positive**:

$$C_{K_1,t} + C_{K_3,t} - 2C_{K_2,t} \geq 0$$
$$\frac{C_{K_1,t} + C_{K_3,t}}{2} \geq C_{K_2,t}$$

To eliminate arbitrage opportunity, we must have **convexity** in call option prices:

$$\text{If } K_1 < K_2 < K_3, \text{ then } wC_{K_1,t} + (1-w)C_{K_3,t} \geq C_{K_2,t}$$

where:

$$w = \frac{K_3 - K_2}{K_3 - K_1}$$

Butterfly spreads

Table: Butterfly spread using puts

Investment	Cash flows at				
	t	$S_T < K_1$	$K_1 \leq S_T < K_2$	$K_2 \leq S_T < K_3$	$S_T \geq K_3$
Long 1 K_1 put	$-P_{K_1,t}$	$K_1 - S_T$	0	0	0
Short 2 K_2 put	$2P_{K_2,t}$	$2S_T - 2K_2$	$2S_T - 2K_2$	0	0
Long 1 K_3 put	$-P_{K_3,t}$	$K_3 - S_T$	$K_3 - S_T$	$K_3 - S_T$	0
Net cash flows	$2P_{K_2,t} - P_{K_1,t} - P_{K_3,t}$	$K_1 + K_3 - 2K_2$	$S_T \underbrace{-2K_2}_{= -(K_1+\cancel{K_3})} + \cancel{K_3}$	$K_3 - S_T$	0

For a **symmetric** butterfly spread, we have $K_1 + K_3 = 2K_2$:

Net cash flows	$2P_{K_2,t} - P_{K_1,t} - P_{K_3,t}$	0	$S_T - K_1$	$K_3 - S_T$	0

Butterfly spreads and put option value bounds

Since the **symmetric** butterfly spread produces **non-negative pay-offs** in the future, the price of **buying** the spread must be **positive**:

$$P_{K_1,t} + P_{K_3,t} - 2P_{K_2,t} \geq 0$$
$$\frac{P_{K_1,t} + P_{K_3,t}}{2} \geq P_{K_2,t}$$

To eliminate arbitrage opportunity, we must have **convexity** in put option prices:

$$\text{If } K_1 < K_2 < K_3, \text{ then } wP_{K_1,t} + (1-w)P_{K_3,t} \geq P_{K_2,t}$$

where:

$$w = \frac{K_3 - K_2}{K_3 - K_1}$$

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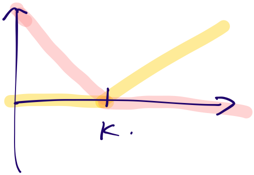
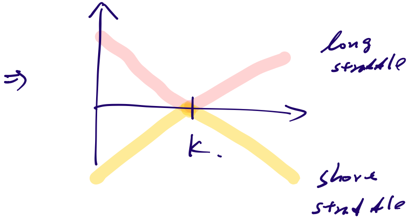
Straddles





A **straddle** consists of:

1. a **long position** in the **call option** at K;
2. a **long position** in the **put option** at K.

Use this strategy when you:

- predict **high volatility** in the **underlying**.

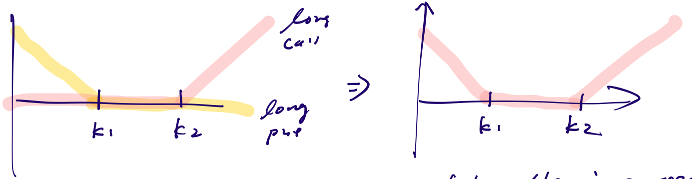
Taking a **short position** in straddle is essentially **selling insurance** to other investors. (SR > 2, However, if financial crisis. you're in trouble !)

Straddles

Table: Straddles

Investment	Cash flows at		
	t	$T: S_T < K$	$T: S_T \geq K$
Long call	$-C_{K,t}$	0	$S_T - K$
Long put	$-P_{K,t}$	$K - S_T$	0
Net cash flows	$-C_{K,t} - P_{K,t}$	$K - S_T$	$S_T - K$
	$-$	$+$	$+$

Strangles



(straddle is a special case of strangles with $k_1 = k_2$)

A **strangles** consists of:

1. a **long position** in the **put option** at K_1;
2. a **long position** in the **call option** at K_2 with $K_2 > K_1$.

Use this strategy when you:

- predict **high volatility** in the **underlying**;
- want to pay **less option premiums** than straddle.

(ATM options are expensive :)

k_1, k_2 are OTM, ⇒ cheap

Strangles

Table: Strangles

Investment	**Cash flows at**			
	t	$S_T < K_1$	$K_1 \leq S_T < K_2$	$S_T \geq K_2$
Long K_1 put	$-P_{K_1,t}$	$K_1 - S_T$	0	0
Long K_2 call	$-C_{K_2,t}$	0	0	$S_T - K_2$
Net cash flows	$-P_{K_1,t} - C_{K_2,t}$	$K_1 - S_T$	0	$S_T - K_2$
	−	+	0.	+

Strips

A **strip** consists of:

1. a **long position** in the **call option** at K;
2. two **long position** in **put options** at K.

Use this strategy when you:

- predict **high volatility** in the **underlying**;
- expect **price declines are more likely** than price increases.

Strips

Table: Strips

Investment	Cash flows at		
	t	$T: S_T < K$	$T: S_T \geq K$
Long 1 call	$-C_{K,t}$	0	$S_T - K$
Long 2 put	$-2P_{K,t}$	$2K - 2S_T$	0
Net cash flows	$-C_{K,t} - 2P_{K,t}$	$2(K - S_T)$	$S_T - K$

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Straps

A **strap** consists of:

1. two **long position** in **call options** at K;
2. a **long position** in the **put option** at K.

Use this strategy when you:

- predict **high volatility** in the **underlying**;
- expect **price increases are more likely** than price declines.

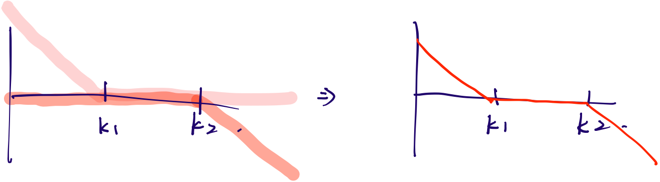
Straps

Table: Straps

Investment	Cash flows at		
	t	$T: S_T < K$	$T: S_T \geq K$
Long 2 call	$-2C_{K,t}$	0	$2S_T - 2K$
Long 1 put	$-P_{K,t}$	$K - S_T$	0
Net cash flows	$-2C_{K,t} - P_{K,t}$	$K - S_T$	$2(S_T - K)$

Other strategies

Collars



A **collar** consists of:

1. a **long position** in the **put option** at K_1;
2. a **short position** in the **call option** at K_2 with $K_2 > K_1$.

Use this strategy when you:

- expect **price decrease are more likely** than price increases.

By combining with a **long position** in the **underlying**, collar **protects** the value of the underlying from **market fluctuations**.

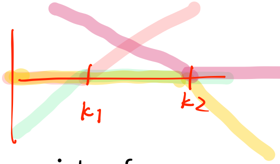
Collars

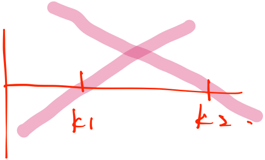
Table: Collars

Investment	Cash flows at			
	t	$S_T < K_1$	$K_1 \leq S_T < K_2$	$S_T \geq K_2$
Long K_1 put	$-P_{K_1,t}$	$K_1 - S_T$	0	0
Short K_2 call	$C_{K_2,t}$	0	0	$K_2 - S_T$
Net cash flows	$C_{K_2,t} - P_{K_1,t}$	$K_1 - S_T$	0	$K_2 - S_T$

+ −

Box spreads





A **box spread** consists of:

1. a **long position** in the **call option** at K_1;
2. a **short position** in the **put option** at K_1;
3. a **short position** in the **call option** at K_2 with $K_2 > K_1$;
4. a **long position** in the **put option** at K_2 with $K_2 > K_1$.

It is equivalent to a **synthetic zero-coupon bond** by taking **long-short** positions in **synthetic forwards**. All the options must be **European style** to avoid early exercises.

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Box spreads

Table: Box spreads

Investment	**Cash flows at**			
	t	$S_T < K_1$	$K_1 \leq S_T < K_2$	$S_T \geq K_2$
Long K_1 call	$-C_{K_1,t}$	0	$S_T - K_1$	$S_T - K_1$
Short K_1 put	$P_{K_1,t}$	$S_T - K_1$	0	0
Long K_2 put	$-P_{K_2,t}$	$K_2 - S_T$	$K_2 - S_T$	0
Short K_2 call	$C_{K_2,t}$	0	0	$K_2 - S_T$
Net cash flows	$C_{K_2,t} + P_{K_1,t} - C_{K_1,t} - P_{K_2,t}$	$K_2 - K_1$	$K_2 - K_1$	$K_2 - K_1$

$-$ $+$ $+$ $+$

Ratio spreads

A **ratio spread** consists of:

1. a **long position** in the **call option** at K_1;
2. two **short positions** in **call options** at K_2 with $K_2 > K_1$.

Use this strategy when you:

- expect **price will rise above** K_1 **but not more than** K_2.

It can be modified to **speculate** in a **bearish** market by taking:

1. a **long position** in the **put option** at K_2;
2. two **short positions** in **put options** at K_1 with $K_2 > K_1$.

Ratio spreads

Table: Ratio spread using calls

$C_{K1} > C_{K2}$.

Investment	Cash flows at			
	t	$S_T < K_1$	$K_1 \leq S_T < K_2$	$S_T \geq K_2$
Long 1 K_1 call	$-C_{K_1,t}$	0	$S_T - K_1$	$S_T - K_1$
Short 2 K_2 call	$2C_{K_2,t}$	0	0	$2K_2 - 2S_T$
Net cash flows	$2C_{K_2,t} - C_{K_1,t}$	0	$S_T - K_1$	$2K_2 - K_1 - S_T$

$+$ $(+, -)$

Condors

A **condor with calls** consists of:

1. a **long position** in the **call option** at K_1;
2. a **short position** in the **call option** at K_2;
3. a **short position** in the **call option** at K_3;
4. a **long position** in the **call option** at K_4 with $K_4 > K_3 > K_2 > K_1$ and $K_1 + K_4 = K_2 + K_3$.

Similar to butterfly spreads, condors are joining a **bull spread** and a **bear spread** but at different strike prices. Use this strategy when you:

- predict **limited fluctuations** in the **underlying**;
- want to **pay less option premiums** than a butterfly spread;
- want to **limit the maximum loss** of the position.

Condors

Table: Condor using calls

Investment	Cash flows at					
	t	$S_T < K_1$	$K_1 \leq S_T < K_2$	$K_2 \leq S_T < K_3$	$K_3 \leq S_T < K_4$	$S_T \geq K_4$
Long 1 K_1 call	$-C_{K_1,t}$	0	$S_T - K_1$	$S_T - K_1$	$S_T - K_1$	$S_T - K_1$
Short 1 K_2 call	$C_{K_2,t}$	0	0	$K_2 - S_T$	$K_2 - S_T$	$K_2 - S_T$
Short 1 K_3 call	$C_{K_3,t}$	0	0	0	$K_3 - S_T$	$K_3 - S_T$
Long 1 K_4 call	$-C_{K_4,t}$	0	0	0	0	$S_T - K_4$
Net cash flows	$C_{K_2,t} + C_{K_3,t} - C_{K_1,t} - C_{K_4,t}$	0	$S_T - K_1$	$K_2 - K_1$	$K_2 + K_3 - K_1 - S_T$	$K_2 + K_3 - K_1 - K_4$

For a **symmetric** condor, we have $K_2 + K_3 = K_1 + K_4$:

Net cash flows	$C_{K_2,t} + C_{K_3,t} - C_{K_1,t} - C_{K_4,t}$	0	$S_T - K_1$	$K_2 - K_1$	$K_4 - S_T$	0

Calendar spreads

A **calendar spread** consists of:

1. a **long position** in the **call option** at K with maturity T_2;
2. a **short position** in the **call option** at K with maturity T_1 and $T_1 < T_2$;

or:

1. a **long position** in the **put option** at K with maturity T_2;
2. a **short position** in the **put option** at K with maturity T_1 and $T_1 < T_2$;

Essentially, we want to **buy a long maturity European** call (put) and **sell a short maturity European** call (put) at the same **strike price**.