Financial derivatives

Lecture 2: Forward, futures, and hedging strategies

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Motivations behind hedging

Value and cash flow risks

In **monitoring** and **managing risk**, the manager of a company or an individual should **pay attention** to the following two things:

1. The **value** of a company (equity) or a portfolio of assets.
2. The **cash flows** of a company or a trading strategy if they have **value implications** due to **liquidity issues**.

If a company or an individual can **borrow** or **raise fund** at will, a cash flow shortfall does not pose serious risk.

Cash flow risk

However, when **external financing** is **costly**, cash flow shortfalls can destroy the value of a company or a portfolio.

- **Asymmetric information**:
  If **outside parties cannot evaluate** the **real cause** of cash shortfalls, they are unlikely to offer financing.
- **Direct costs**:
  When bank loans are not available, the **issuance** of bond or equity may entail **high cost**.

Forward and futures

Over-the-counter (OTC) forwards

A forward is an **OTC contract** that **exchanges an asset for a pre-specified price**.

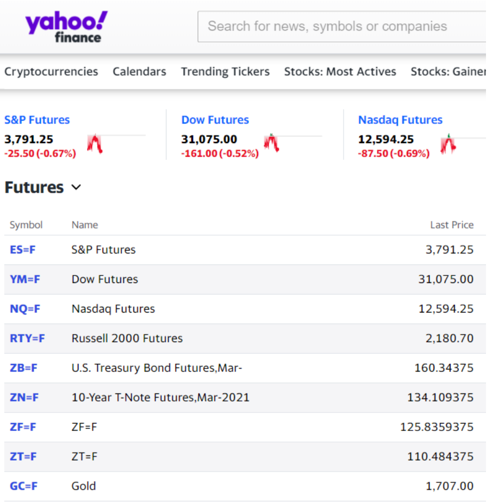
- Forward is **highly customizable** in terms of the **underlying asset, size, timing, delivery option, and settlement method**.
- To initiate the forward contract, there must be a **buy-side** and a **sell-side**. In other words, a buyer must find a seller as the **counter party**, and vice versa.
- Forward contract entails **higher counter party risk** due to its **bilateral nature**.
- OTC markets can be **opaque** and **illiquid**. Therefore, the regulator can introduce a **central clearing party (CCP)** to better **monitor and manage systemic risk**.

Futures

Futures are traded on **organized exchanges**.

- Futures are **standardized contracts** with pre-specified asset, size, delivery option.
- Buyers and sellers can trade easily on a **centralized exchange** and the **clearing house** acts as the **counter party to every contract**.
- Futures are **marked-to-market** daily. Together with **margin (collateral) requirements**, **counter party risk is eliminated**.
- The **delivery** of the underlying asset **rarely happens** as hedgers, speculators, and arbitragers only care about **cash flows**.
- Futures are **closed out** by taking an **offsetting position prior to the delivery date**.

Examples of futures contracts



yahoo! finance

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S&P Futures
3,791.25
-25.50 (-0.67%)

Dow Futures
31,075.00
-161.00 (-0.52%)

Nasdaq Futures
12,594.25
-87.50 (-0.69%)

Futures

Symbol	Name	Last Price
ES=F	S&P Futures	3,791.25
YM=F	Dow Futures	31,075.00
NQ=F	Nasdaq Futures	12,594.25
RTY=F	Russell 2000 Futures	2,180.70
ZB=F	U.S. Treasury Bond Futures,Mar-	160.34375
ZN=F	10-Year T-Note Futures,Mar-2021	134.109375
ZF=F	ZF=F	125.8359375
ZT=F	ZT=F	110.484375
GC=F	Gold	1,707.00

Figure: Examples of futures contracts (Source: Yahoo! Finance)

Examples of futures contracts

Underlying asset:

- Commodities (NYMEX, CBOT):
  - Metals: base, precious
  - Softs: coffee, sugar, cocoa
  - Grains: oil seeds, live stocks
  - Energy: crude oil, oil products, gas
- Financial (CME, LIFFE):
  - Interest rates: T-bills, T-bonds, Eurodollar
  - Foreign currencies: JPY, EUR, GBP
  - Equity indices: S&P500, DJIA, Nasdaq, Russell 2000
  - Individual stocks

Settlement method:

- Cash settlement
- Physical settlement

E-Mini S&P 500 futures - contract specification

CONTRACT UNIT	$50 x S&P 500 Index
PRICE QUOTATION	U.S. dollars and cents per index point
TRADING HOURS	CME Globex: Sunday - Friday 6:00 p.m. - 5:00 p.m. ET with trading halt 4:15 p.m. - 4:30 p.m. BTIC: Sunday - Friday 6:00 p.m. - 4:00 p.m. ET CME ClearPort: Sunday - Friday 6:00 p.m. - 5:00 p.m. ET TACO on CME Globex: Sunday - Friday 6:00 p.m. - 9:30 a.m. ET. Monday - Thursday 11:00 a.m. - 5:00 p.m. ET; no 11:00 a.m.- 5:00 p.m. ET session on Friday. Monday - Thursday 5:00 p.m. - 6:00 p.m. ET daily maintenance period. TACO on CME ClearPort: Sunday 6:00 p.m. - Monday 9:30 a.m. ET. Monday - Thursday 11:00 a.m. - 5:00 p.m. ET and 6:00 p.m. - 9:30 a.m. ET. Friday 11:00 a.m. - 5:00 p.m. ET.
MINIMUM PRICE FLUCTUATION	0.25 index points = $12.50 Calendar Spread: 0.05 index points = $2.50 BTIC: 0.05 index points = $2.50 TACO (ESQ): 0.05 index points = $2.50

Figure: Specification for E-Mini S&P 500 futures (Source: CME)

E-Mini S&P 500 futures - contract specification

PRODUCT CODE	CME Globex: ES CME ClearPort: ES Clearing: ES BTIC: "EST","ESQ"
LISTED CONTRACTS	Quarterly contracts (Mar, Jun, Sep, Dec) listed for 5 consecutive quarters
SETTLEMENT METHOD	Financially Settled
TERMINATION OF TRADING	Trading terminates at 9:30 a.m. ET on the 3rd Friday of the contract month. BTIC trading terminates at 4:00 p.m. ET on the Thursday before the 3rd Friday of contract month. TACO trading terminates at 9:30 a.m. ET on the Thursday before the 3rd Friday of the contract month.

Figure: Specification for E-Mini S&P 500 futures (Source: CME)

Marked-to-market

Forward versus futures

Settlements of forwards occur only on the **expiration date** T. The payoff of a long position in a forward contract is given by:

$$S_T - F_{0,T}$$

In contrast, futures are **settled daily**. If we get into a long futures position on day 0, the payoff of day 1 will be:

$$f_{1,T} - f_{0,T}$$

On day 2, the payoff will be:

$$f_{2,T} - f_{1,T}$$

Forward versus futures

If we **ignore interest and margin**, the total payoff from day 0 to day T will be:

$$f_{T,T} - f_{T-1,T} + \cdots + f_{2,T} - f_{1,T} + f_{1,T} - f_{0,T}$$

Since the futures price **converges toward** the spot price on day T, we have:

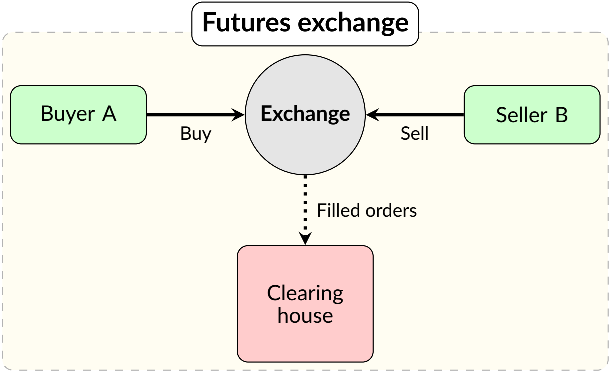
$$S_T - f_{0,T} = f_{T,T} - f_{T-1,T} + \cdots + f_{1,T} - f_{0,T}$$

Therefore, forward and futures provide **identical payoffs** if both contracts are **held until maturities**.

The mechanics of futures trading

Step 1: the initial trade ($t = 0$)

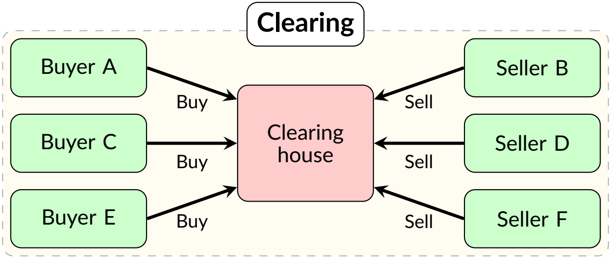
Seller B sells futures to Buyer A at the prevailing futures price.



The mechanics of futures trading

Step 2: the trade is cleared ($t = 0$)

All contracts established on the exchange are **cleared** by the exchange's clearing house. Both traders must **post margin** with the clearing house which is the counter party to both traders.

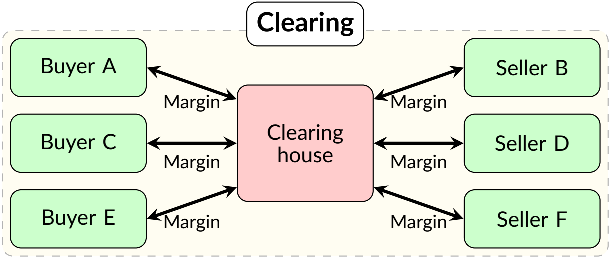


The **open interest** in this example is three contracts.

The mechanics of futures trading

Step 3: marked-to-market settlements ($t < T$)

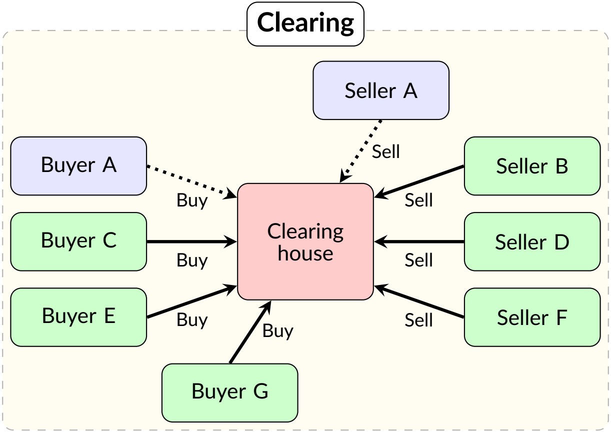
Every day, all open positions are marked-to-market. The profits and losses from the day's futures price change are **transferred** from the margin accounts of traders who lost to the winners. If any trader's margin account fall below the **maintenance margin** level, that trader will receive a **margin call** and must deposit enough additional funds to bring the account back up to the required initial margin level.



The mechanics of futures trading

Step 4: unwinding the position ($t < T$)

Buyer A unwinds his position by selling the same contract as Seller A to Buyer G. Trader A's **offsetting positions** are **canceled out**.



Marked-to-market

Margin account (good faith deposits/performance bonds):

- **Required margins** are either **cash deposit** or **marketable securities**.
- Margins minimize the possibility of **default on the contract.**
- The exchange **may increase** the margin amount in response to surges in daily volatility.

Daily settlement:

- The margin balance is **adjusted daily to account for changes** in futures price.
- A **margin call** occurs when the margin balance drops below the **maintenance margin**, the investor has to **replenish the account up to the required margin**. If **not**, the position will be **liquidated**.

Marked-to-market

When we account for the marked-to-market futures prices, we must know and compute the followings:

- **Daily settlement price**
- **Initial margin**
- **Maintenance margin**
- **Daily profit and loss**
- **Cumulative profit and loss**
- **Margin balance**
- **Margin call**

Hedging

Hedging with forward and futures

Forward and futures allow **hedgers** to **decrease the risk** of their **exposures to risk factors**. For instance:

- Forward:
  Transfer the risk to the counter party but entails **counter party risk**.
- Futures:
  Decrease the risk with a negatively correlated position but may face **basis risk (imperfect correlation)**, **roll-over risk**, and/or **marked-to-market risk**.

Hedging with forward and futures

There are two situations where hedging with forward and futures can be useful:

- If the firm value or the portfolio value **decreases** with the market value of the underlying asset:
  - We should take a **long position** in forward or futures.
- If the firm value or the portfolio value **increases** with the market value of the underlying asset:
  - We should take a **short position** in forward or futures.

Hedging with forward

In the absence of counter party risk, a forward contract can provide a **perfect hedge** to a certain exposure:

- We can **customize** the contract to suit our needs.
- The forward contract is written on the **cash position** to deliver or receive the **underlying asset**.
- **No basis risk** as the size, timing, and underlying of the forward contract are **identical to our exposure**.

However, we should be aware of the followings:

- **Counter party (credit) risk** may exist.
- We need to find counter party for the forward contract.
- Forward price might not be competitive.

Hedging with futures

A futures hedge involves two parallel trading positions:

1. A **cash position** to be transacted at the **market price** in the spot market.
2. A **futures position** that has to be **closed out** by taking an offsetting trade in the futures market.

The hedge works when a **loss on the underlying** is **offset by a profit on the futures hedging position**, and vice versa. To do so:

- The **quantity** and **timing** must be **matched**.
- The **spot price** is **perfectly correlated** with the **futures price**.

Nevertheless, a perfect hedge with futures may be **difficult**:

- Quantity, settlement, and maturity are **standardized**.
- The cash position **may differ** from the futures underlying.
- For **long horizon** hedges, we have to account for **tailing** due to marked-to-market daily settlements.

Hedging with futures

油價走跌，雖會產生燃油避險交易損失，但對航空公司而言，利大於弊。（記者王憶紅攝）

Figure: Fuel cost hedging by airlines (Source: Liberty Times Net)

Basis risk

Basis risk arises when there are mismatches in a hedging position:

- The hedge becomes **imperfect** as losses in the cash position are **not exactly offset** by the futures position, and vice versa.
- Standardized futures are **not available** for some assets.

Commodity basis risk ($S_T \neq f_{T,T}$):

- The **standard grades** of futures may not be the same as the grade of the asset being hedged.
- For example, **airlines** have to use **crude oil** futures to hedge their exposures in **jet fuel (kerosene)**.

Delivery basis risk ($S_t \neq f_{t,T}$):

- The **standard delivery dates** of futures may differ from the actual delivery date of the hedger.

Basis risk

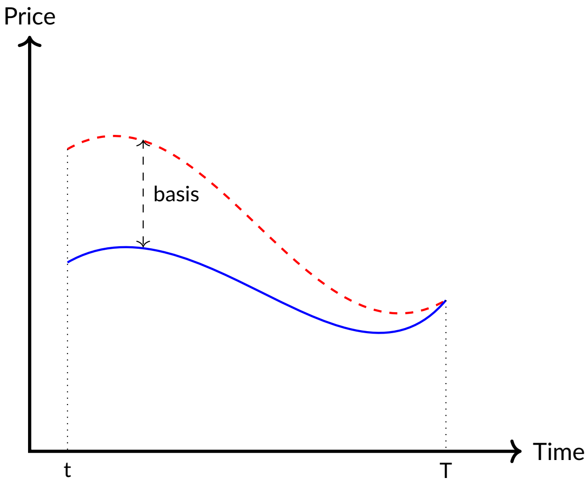


Figure: Basis risk

Risk exposures and hedging

Long hedge

A **long hedge** involves a **long position** in a forward contract or a futures contract. It is useful when:

- The company wants to **lock in the price now** for **buying a certain asset (input) in the future**.
- For example, airlines have to buy jet fuels and car manufacturers have to buy steel.

At time 0:

- Get into a long position in forward or futures.

At time T:

- Cash outflow of the cash position: $-S_T$.
- Cash flow of the long forward/futures position: $S_T - F_{0,T}$.
- Cash outflow of the hedged position: $-S_T + S_T - F_{0,T} = -F_{0,T}$.
- In effect, the hedger **pays** $F_{0,T}$ to **buy** the underlying asset.

Long hedge

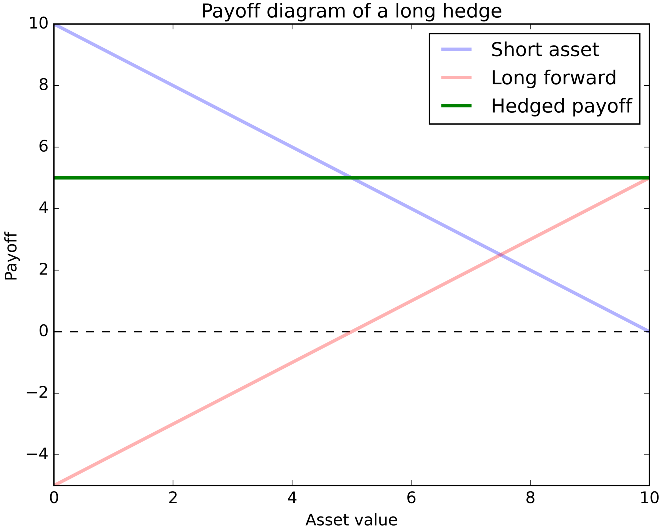


Figure: A long hedge

Short hedge

A **short hedge** involves a **short position** in a forward contract or a futures contract. It is useful when:

- The company wants to **lock in the price now** for **selling a certain asset (output) in the future**.
- For example, farmers want to ensure the selling prices of their crops in the future and oil producers want to protect themselves from oil price fluctuations.

At time 0:

- Get into a short position in forward or futures.

At time T:

- Cash inflow of the cash position: S_T.
- Cash flow of the short forward/futures position: $F_{0,T} - S_T$.
- Cash inflow of the hedged position: $S_T + F_{0,T} - S_T = F_{0,T}$.
- In effect, the hedger **receives** $F_{0,T}$ from **selling** the underlying asset.

Short hedge

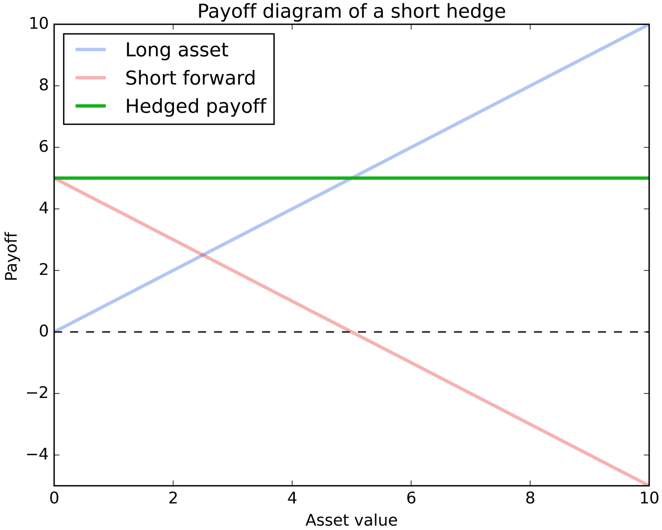


Figure: A short hedge

Cross hedging

Review of statistics

Given that a, b are constants while X and Y are random variables, we have the following properties:

1. $\mathbb{E}[a+X] = a + \mathbb{E}[X]$
2. $\mathbb{E}[aX] = a\mathbb{E}[X]$
3. $\mathbb{E}[aX+bY] = a\mathbb{E}[X] + b\mathbb{E}[Y]$
4. $\mathrm{Var}[a+X] = \mathrm{Var}[X]$
5. $\mathrm{Var}[aX] = a^2\mathrm{Var}[X]$
6. $\mathrm{Var}[aX+bY] = a^2\mathrm{Var}[X] + b^2\mathrm{Var}[Y] + 2ab\mathrm{Cov}[X,Y]$
7. $\mathrm{Cov}[a,X] = 0$
8. $\mathrm{Cov}[aX,bY] = ab\mathrm{Cov}[X,Y]$

Variance operator

Consider a discrete random variable X with an expected value of $\mathbb{E}[X] = \mu_X$, the variance operator is:

$$\begin{aligned}\mathrm{Var}[X] &= \mathbb{E}[(X - \mathbb{E}[X])^2] \\ &= \mathbb{E}[(X - \mu_X)^2] \\ &= \sum_{i=1}^{n} [(x_i - \mu_X)^2 \mathbb{P}(X = x_i)]\end{aligned}$$

Recall property 1, let's verify property 4:

$$\begin{aligned}\mathrm{Var}[a + X] &= \mathbb{E}[(a + X - \mathbb{E}[a + X])^2] \\ &= \mathbb{E}[(a - a + X - \mathbb{E}[X])^2] \\ &= \mathrm{Var}[X] \quad \blacksquare\end{aligned}$$

Variance operator

As for property 5:

$$\begin{aligned}\mathrm{Var}[aX] &= \mathbb{E}[(aX - \mathbb{E}[aX])^2] \\ &= \mathbb{E}[a^2(X - \mathbb{E}[X])^2] \\ &= a^2\mathrm{Var}[X] \quad \blacksquare\end{aligned}$$

Covariance operator

Consider discrete random variables X and Y with expected values of $\mathbb{E}[X] = \mu_X$ and $\mathbb{E}[Y] = \mu_Y$ respectively, the covariance operator is:

$$\begin{aligned}
\mathrm{Cov}[X,Y] &= \mathbb{E}[(X-\mathbb{E}[X])(Y-\mathbb{E}[Y])] \\
&= \mathbb{E}[(X-\mu_X)(Y-\mu_Y)] \\
&= \sum_{i=1}^{n}\sum_{j=1}^{m}[(x_i-\mu_X)(y_j-\mu_Y)\mathbb{P}(X=x_i, Y=y_j)]
\end{aligned}$$

To verify property 6:

$$\begin{aligned}
\mathrm{Var}[aX+bY] &= \mathbb{E}[(aX+bY-\mathbb{E}[aX+bY])^2] \\
&= \mathbb{E}[(aX-a\mathbb{E}[X]+bY-b\mathbb{E}[Y])^2] \\
&= \mathbb{E}[(a(X-\mathbb{E}[X])+b(Y-\mathbb{E}[Y]))^2] \\
&= \mathbb{E}[a^2(X-\mu_x)^2+b^2(Y-\mu_Y)^2+2ab(X-\mu_X)(Y-\mu_Y)] \\
&= a^2\mathbb{E}[(X-\mu_x)^2]+b^2\mathbb{E}[(Y-\mu_Y)^2]+2ab\mathbb{E}[(X-\mu_X)(Y-\mu_Y)] \\
&= a^2\mathrm{Var}[X]+b^2\mathrm{Var}[Y]+2ab\mathrm{Cov}[X,Y] \quad \blacksquare
\end{aligned}$$

Covariance operator

Recall a is a constant so its expected value is also a, let's verify property 7:

$$\begin{aligned}\mathrm{Cov}[a, X] &= \mathbb{E}[(a - \mathbb{E}[a])(X - \mathbb{E}[X])] \\ &= \mathbb{E}[(a - a)(X - \mu_X)] \\ &= 0 \quad \blacksquare\end{aligned}$$

Finally, let's verify property 8:

$$\begin{aligned}\mathrm{Cov}[aX, bY] &= \mathbb{E}[(aX - \mathbb{E}[aX])(bY - \mathbb{E}[bY])] \\ &= \mathbb{E}[a(X - \mathbb{E}[X])b(Y - \mathbb{E}[Y])] \\ &= ab\mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])] \\ &= ab\mathrm{Cov}[X, Y] \quad \blacksquare\end{aligned}$$

Correlations

Since the magnitude of covariance is influenced by the standard deviations of its underlying assets, we need to standardize covariance:

$$\mathrm{Corr}(x,y) = \rho_{x,y} = \frac{\mathrm{Cov}(x,y)}{\sigma_x \sigma_y}$$

Correlation is bounded by -1 and 1 so it is easier to compare the comovements between different pairs of assets.

The cash flow from a hedged position

∵ 在T之前要想辦法 protect.

Consider a hedger with a **commitment to buy** Q units of the asset on date T. In order to hedge, the hedger should:

1. Take a long futures position of size H at the futures price $f_{0,T}$ on date 0.
2. Close out the futures position on date T by taking a short position of size H.
3. Buy the committed quantity Q on the spot market on date T.

The **net cash outflow** on date T is thus:

Cost of buying at T ←

$$QS_T - H(f_{T,T} - f_{0,T})$$

long hedge.

The hedger has to choose H to **minimize the variance** of this cash outflow.

If S_T ↑, the long hedge on future will earn \$ to compensate the cost of buying spot on T.

The cash flow from a hedged position

E.g. Farmers.

Consider a hedger with a **commitment to sell** Q units of the asset on date T. In order to hedge, the hedger should:

1. Take a short futures position of size H at the futures price $f_{0,T}$ on date 0. look in selling price
2. Close out the futures position on date T by taking a long position of size H.
3. Sell the committed quantity Q on the spot market on date T.

The **net cash inflow** on date T is thus:

If $S_T \downarrow$, short on future will compensate.

$$\downarrow QS_T + H(\underset{\text{short}}{f_{0,T}} - f_{T,T}) \uparrow$$

Therefore, regardless of the direction of the exposure, the hedger faces the same problem in choosing H.

The no basis risk case (perfect hedge)

In the **absence of basis risk**, on date T, we must have:

$$S_T = f_{T,T} \quad \left(\text{Spot}_T \approx \text{Future}_T \right).$$

Therefore:

$$\begin{aligned} QS_T - H(\overset{\text{long}}{f_{T,T}} - f_{0,T}) &= QS_T - H(S_T - f_{0,T}) \\ &= (Q - H)\underbrace{S_T}_{\text{unknown.}} + \overbrace{Hf_{0,T}}^{\text{known.}} \end{aligned}$$

By setting $Q = H$, we can **eliminate** the variance of cash flows completely. Let's define the hedge ratio h as:

$$\boxed{h = \frac{H}{Q}}$$

Q: units of spot.

H: contract size.

For **perfect hedge**, we have an optimal hedge ratio $h^* = 1$.

The minimum-variance hedge ratio

To determine the minimize-variance hedge ratio:

原式 $\Rightarrow$

$$
\begin{aligned}
QS_T(-QS_0+QS_0)-H(f_{T,T}-f_{0,T}) &= Q(S_T-S_0)-H(f_{T,T}-f_{0,T})+QS_0\\
&= Q\Delta S - H\Delta f + QS_0\\
&= Q(\Delta S - h\Delta f)+QS_0
\end{aligned}
$$

$h = \frac{H}{Q}$

The variance of the hedged cash flow is given by:

const

$$
\begin{aligned}
\mathrm{Var}[Q(\Delta S-h\Delta f)+\cancel{QS_0}] &= Q^2\mathrm{Var}[\Delta S-h\Delta f]\\
&= Q^2[\sigma^2_{\Delta S}+h^2\sigma^2_{\Delta f}-2h\mathrm{Cov}(\Delta S,\Delta f)]
\end{aligned}
$$

objective func

$\mathrm{var}(ax+by) = a^2\,\mathrm{var}(x) + b^2\,\mathrm{var}(y) + 2ab\,\mathrm{cov}(x,y)$

The minimum-variance hedge ratio

To minimize the variance of the hedged cash flow:

$$\min_{h} Q^2[\sigma^2_{\Delta S} + h^2\sigma^2_{\Delta f} - 2h\mathrm{Cov}(\Delta S, \Delta f)]$$

The first order condition is given by: $\frac{d}{dh}$

$$2h\sigma^2_{\Delta f} - 2\mathrm{Cov}(\Delta S, \Delta f) = 0$$

Which gives the minimum-variance hedge ratio:

$$h^* = \frac{\mathrm{Cov}(\Delta S, \Delta f)}{\sigma^2_{\Delta f}} = \frac{\rho_{\Delta S,\Delta f} \cdot \sigma_{\Delta S} \cdot \cancel{\sigma_{\Delta f}}}{\sigma_{\Delta f}^{\cancel{2}}}$$

Or similarly:

$$h^* = \rho_{\Delta S,\Delta f}\frac{\sigma_{\Delta S}}{\sigma_{\Delta f}}$$

If $\rho_{\Delta S,\Delta f} = 1$.
$\Rightarrow$ perfect pos corr.
and if $\sigma_{\Delta S} = \sigma_{\Delta f}$

$\Rightarrow h^* = 1$, perfect hedge Ratio