Financial derivatives

Lecture 3: Pricing of forward and futures

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Derivative pricing

Idea and assumptions behind derivative pricing

Equity Derivatives = { w.e. Stocks / 1-w.e. Bonds.

Law of one price:

- The price of a derivative is equal to the cost of its **replicating (synthetic) portfolio** that **provides identical payoffs**.

dynamic portfolio

No arbitrage condition:

- Arbitrage opportunities that **offer net cash inflows without any net cash outflows cannot persist** in an **efficient market.**

To simplify our analysis, we will make the following **assumptions**:

1. There are **no transaction cost**.
2. There are **no restrictions** on **short selling**. Short sellers can invest proceeds of short sales.
3. Investors can **borrow and lend** at the **common risk-free rate**.

Forward payoff

Payoff of a forward contract

If we long a **forward contract** at time $t = 0$, we will get the following **payoff** when it **matures** at time $t = T$:

Figure: Payoff of a forward contract

In other words, the payoff can be **broken down** into two parts:

1. the price of the **underlying asset** at **maturity**: S_T;
2. the price of the **forward contract** at **initiation**: $F_{0,T}$.

Pricing of equity forwards

Equity forward

Seller sell

An **equity forward contract** gives the buyer the **obligation to buy** the **underlying stock** at the **pre-specified price** on the **maturity date**. For simplicity, let's assume the following: F_0, T

- The underlying **non-dividend paying** stock is traded at S_0.
- The **continuously compounded risk-free interest rate** is r.
- The **maturity date** of the forward is T.

Replicating forward payoff

Table: Cash flow of replicating a forward

	Cash flow:	
Actions:	$t = 0$	$t = T$
1. Buy a forward (long)	0	$S_T - F_{0,T}$
2. Short the stock	S_0 pos cash from selling	$-S_T$ return stock
3. Invest the proceeds at r	$-S_0$ save it at risk free rate	$S_0 e^{rT}$
Net cash flow	0	$S_0 e^{rT} - F_{0,T} = 0$

In the **absence of arbitrage**, a portfolio with **zero net cash flow now** should have **zero net cash flow in the future.** Since r, T, and S_0 are known, the price of a forward contract is given by the **cost-of-carry** formula:

invest 0 today, get 0 in future

$$F_{0,T} = S_0 e^{rT} \quad \text{(continuous)}$$

Cost of carry formula: 持有成本 thm.

① pos: cost of storage, no income.

future price > spot price.

② neg: Dividend paying.

future price < spot price.

Replicating forward payoff

(Long)

In essence, we create a **replicating portfolio** of the **forward contract** at time $t = 0$ by:

1. buying the stock at S_0; $\Rightarrow S_T$
2. borrowing S_0 at the risk-free rate. $\Rightarrow -S_0e^{rT}$.

By doing so, we have a **current payoff** of $S_0 - S_0 = 0$. At time $t = T$, we have to **close our positions** by:

1. selling the stock at S_T;
2. returning S_0e^{rT} to the lender.

Thus, the synthetic **forward contract** has a **final payoff** of:

合成的

$$S_T - S_0e^{rT} = S_T - F_{0,T}.$$

Arbitrage from an overvalued forward

sell buy

What if $F_{0,T} > S_0 e^{rT}$? Mispricing

Table: Arbitrage from an overvalued forward

	Cash flow:	
Actions:	$t = 0$	$t = T$
1. Sell a forward sell (high)	0	$F_{0,T} - \cancel{S_T}$
2. Buy the stock	-S_0	$\cancel{S_T}$
3. Borrow S_0 at r	S_0	$-S_0 e^{rT}$
Net cash flow	0	$F_{0,T} - S_0 e^{rT} > 0$

0

- The **synthetic forward** is **relatively cheaper** than the **forward**.
- We can make an **arbitrage profit** by **buying the replicating portfolio** (buy low) and **selling the forward** (sell high).

Arbitrage from an undervalued forward

What if $F_{0,T} < S_0 e^{rT}$?

Table: Arbitrage from an undervalued forward

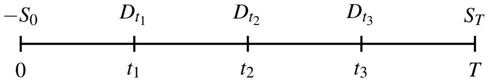
	Cash flow:	
Actions:	$t = 0$	$t = T$
1. Buy a forward	0	$S_T - F_{0,T}$
2. Short the stock	S_0	$-S_T$
3. Invest the proceeds at r	$-S_0$	$S_0 e^{rT}$
Net cash flow	0	$S_0 e^{rT} - F_{0,T} > 0$

- The **forward** is **relatively cheaper** than the **synthetic forward**.
- We can make an **arbitrage profit** by **buying the forward** **(buy low)** and **selling the replicating portfolio** **(sell high)**.

Stock dividend

Since the payoff only occurs on the **delivery date** for any forward contracts, we have to account for **intermediate cash flows** such as **dividends** or **coupons** distributed during the period $T-t$

Figure: Payoff of buying a dividend paying stock



At time T, the **sum** of **all three discrete dividends** will become:

$$\sum D_T = D_{t_1} e^{r(T-t_1)} + D_{t_2} e^{r(T-t_2)} + D_{t_3} e^{r(T-t_3)}$$

Stock dividend

Table: Cash flow of a synthetic forward on a dividend paying stock

	Cash flow:	
Actions:	$t=0$	$t=T$
1. Buy the stock	-S_0	S_T
2. Get dividend at t_1	0	$D_{t_1}e^{r(T-t_1)}$
3. Get dividend at t_2	0	$D_{t_2}e^{r(T-t_2)}$
4. Get dividend at t_3	0	$D_{t_3}e^{r(T-t_3)}$
5. Borrow S_0 at r	S_0	$-S_0e^{rT}$
Net cash flow	0	$S_T+\sum D_T-S_0e^{rT}$

Stock dividend

Under the **no arbitrage condition**, we have:

$$\begin{aligned} S_T - F_{0,T} &= S_T + \sum D_T - S_0 e^{rT} \\ F_{0,T} &= S_0 e^{rT} - \sum D_T \\ F_{0,T} &= S_0 e^{rT} - D_{t_1} e^{r(T-t_1)} - D_{t_2} e^{r(T-t_2)} - D_{t_3} e^{r(T-t_3)} \end{aligned}$$

Let δ be the stock's **continuously compounded dividend yield**:

$$F_{0,T} = S_0 e^{(r-\delta)T}$$

$$\sum_{i=1}^{3} D_{t_i} e^{r(T-t_i)} \approx S_0 \times e^{-\delta T}.$$

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Pricing of currencies forwards

Currencies

To price forwards on GBP/USD exchange rate, we first define the **spot foreign exchange rate** as:

$$S_0 = \frac{\$s_0}{£1}$$

USD : GBP

$= \$S_0 : £1.$

In other words, it takes s_0 USD to buy 1 GBP. Similarly, 1 USD is equivalent to $1/s_0$ GBP. Similarly, we define **the forward foreign exchange rate** as:

$$F_{0,T} = \frac{\$f_{0,T}}{£1}$$

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Covered interest rate parity

$USD : GBP$
$= 1 : S_0$

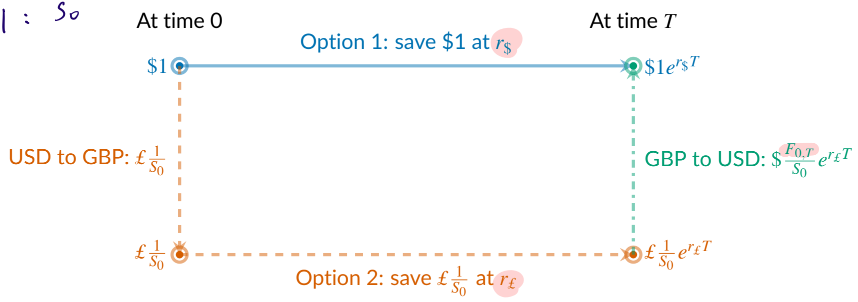

Figure: Covered interest rate parity

If we have 1 USD at time 0, we have two investment options:

1. Invest 1 USD at $r_\$$ until time T.
2. Exchange 1 USD for $1/S_0$ GBP at time 0 then invest it at $r_£$ until time T. Convert GBP back to USD using the forward.

Pricing of currencies forward

The forward price should **equalize** outcomes of the two investment options. Therefore:

$$\$\frac{F_{0,T}}{S_0}e^{r_{£}T} = \$1e^{r_{\$}T}$$

After moving terms:

$$F_{0,T} = S_0 e^{(r_{\$} - r_{£})T}$$

For discrete interest rates:

$$F_{0,T} = S_0 \frac{1 + r_{\$,0,T}}{1 + r_{£,0,T}}$$

$$\frac{F_{0,T}}{S_0}(1+r_{£,0,T}) = (1+r_{\$,0,T})$$

Pricing of commodity forwards

Commodities

The **cost-of-carry formula works well** in pricing financial forwards and futures:

- **Transaction costs** are **low**.
- **Short selling** is **permitted**.

The same **cannot be said** for commodities:

- Some underlying assets are **costly to store** (e.g. oil and electricity) or **impossible to trade** (e.g. weather).
- Producers receive **economic value (convenience yield)** from owning the commodity.

E.g. coffee bean.

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Storage cost

Table: Cash flow of a synthetic forward on a commodity

	Cash flow:	
Actions:	$t = 0$	$t = T$
1. Buy the commodity	-S_0	S_T
2. Borrow S_0 at r	S_0	$-S_0 e^{rT}$
3. Pay storage cost	0	$-C$
Net cash flow	0	$S_T - S_0 e^{rT} - C$

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Storage cost

payoff of. forward contract.

Under the **no arbitrage condition**, we have:

$$\cancel{S_T} - F_{0,T} = \cancel{S_T} - S_0 e^{rT} - C$$
$$F_{0,T} = S_0 e^{rT} + C$$

Let c be the storage costs per unit of commodity:

$$F_{0,T} = S_0 e^{(r+c)T}$$

Storage cost

What if $F_{0,T} > S_0 e^{(r+c)T}$?

Table: Arbitrage when $F_{0,T} > S_0 e^{(r+c)T}$

	Cash flow:	
Actions:	$t = 0$	$t = T$
1. Short the forward	0	$F_{0,T} - \cancel{S_T}$
2. Buy the commodity	$-S_0$	$\cancel{S_T}$
3. Borrow S_0 at r	S_0	$-S_0 e^{rT}$
4. Pay storage cost	0	$-C$
Net cash flow	0	$F_{0,T} - S_0 e^{(r+c)T} > 0$

impossible

Therefore, the **upper bound** of commodity forward is:

Hence,

$$S_0 e^{(r+c)T} \geq F_{0,T}$$

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Lease rate

What if $F_{0,T} < S_0 e^{(r+c)T}$?

- Owners of commodities are usually **reluctant to sell or lend** their holdings in the **spot market**.
- **Inventories** of commodities provide **flexibility** to change **production plans** and **insurance** against a **stock-out**.
- Arbitragers who wish to **short sell** a commodity have to pay a **lease rate** l to commodity owner.

borrow commodities.

since com.. provide benefit.

least rate (l) is like a cost.

Lease rate

What if $F_{0,T} < \cancel{S_0 e^{(r+c)T}}$? (和上面 opposite)

$S_0 e^{(r-l)T}.$

Table: Arbitrage when $F_{0,T} < S_0 e^{(r+c)T}$

	Cash flow:	
Actions:	$t = 0$	$t = T$
1. Buy the forward	0	$\cancel{S_T} - F_{0,T}$
2. Short the commodity 3. Lend S_0 at r 4. Pay lease rate	S_0 $+$ $-S_0$ 0	$\cancel{-S_T}$ $S_0 e^{rT}$ $-L$
Net cash flow	0	$S_0 e^{(r-l)T} - F_{0,T} > 0$

Therefore, the **lower bound** of commodity forward is:

$$F_{0,T} \geq S_0 e^{(r-l)T}$$

Convenience yield

c: storage cost.

l : lease rate (borrow cost) 租借

forward.

The **no-arbitrage region** of commodity forward is:

$$S_0 e^{(r+c)T} \geq F_{0,T} \geq S_0 e^{(r-l)T}$$

upper lower

To make the **observed commodity forward** price in line with the model, we have to introduce a **convenience yield** y where:

便利收益.

If $l+c \leq y$. lend at l, pay c, earn y

$$l + c \geq y \geq 0$$

$(l+c)$

(Spot. Demand ↑, y ↑)

So the commodity forward price is given by:

$$F_{0,T} = S_0 e^{(r+c-\underset{\text{yield}}{y})T}$$

持有 spot 商品時的附加收益.

Whenever the **spot supplies** are **tight** relative to the **consumption demand**, the **convenience yield** becomes **larger**.

Term structure of forwards

Backwardation and contango

Contango:

- Forward or futures price **increases** with maturity.
- **Typical** for commodity forward as holding commodities entails **storage costs** that are **increasing** with maturity.

$T \uparrow, \ c \uparrow.$

Backwardation:

- Forward or futures price **decreases** with maturity.
- Common for commodities with **large convenience yields**.

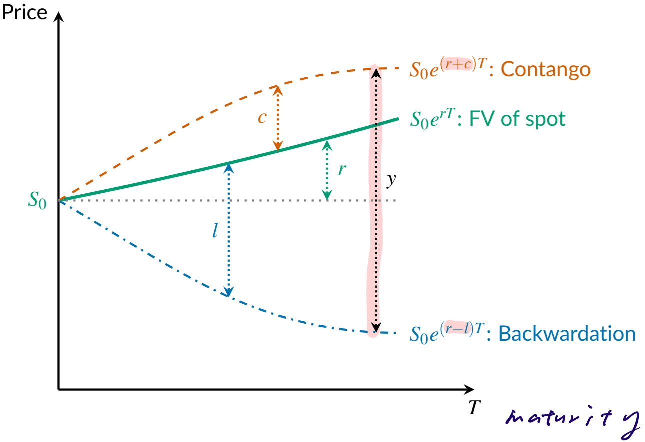
$y \uparrow$. (better off holding spot.)

Backwardation and contango

$F_{0,T} = S_0 e^{(r+c-y)T}$

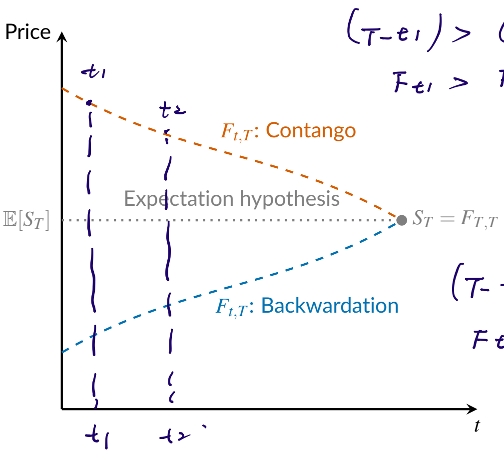
where, $l+c \geq y \geq 0$

Figure: Decomposition of the convenience yield



Backwardation and contango

Figure: Term structure of forwards



$(T - t_1) > (T - t_2)$

$F_{t_1} > F_{t_2}$.

$(T - t_1) > (T - t_2)$

$F_{t_1} < F_{t_2}$.

Pricing of interest rate forwards

Fixed income basics

Let r_T be the T-month **interbank interest rate**. The present value of a dollar due in d days is:

$$B(T) = \frac{1}{1 + r_T \frac{d}{360}}$$

For example, if the current three-month interest rate is 9%, the present value of a dollar receivable after three months (91 days) is:

$$B(3) = \frac{1}{1 + (0.09)\frac{91}{360}} = 0.9778$$

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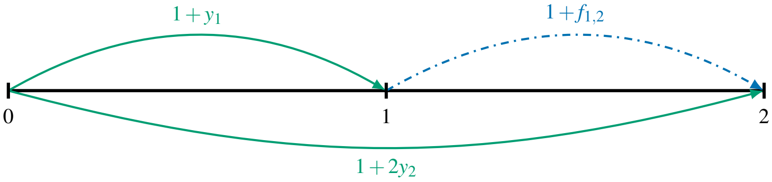
Forward rates

Let y_1 be the **annualized spot rate (YTM)** of a 1-year zero-coupon T-note from year 0 to 1 and y_2 be the annualized YTM of a 2-year zero-coupon T-note from year 0 to 2.



The price of a 1-year zero-coupon T-note:

$$B(12) = \frac{\$100}{1+y_1}$$

$$1+y_1 = \frac{100}{B(12)}$$

The price of a 2-year zero-coupon T-note:

$$B(24) = \frac{\$100}{(1+2y_2)}$$

$$(1+2y_2) = \frac{100}{B(24)}$$

Forward rates

Further, let $f_{1,2}$ be the **annualized forward rate** from year 1 to 2. We can then choose between two investment options:

1. Invest \$1 in 1-year zero-coupon T-note then reinvest:

   After two years: $\$1 \times (1+y_1) \times (1+f_{1,2})$ 分段.

2. Invest \$1 in 2-year zero-coupon T-note:

   After two years: $\$1 \times (1+2y_2)$ 一次.

By **equating** returns of the two options, we can **infer** the **expected future interest rate**:

$$f_{1,2} = \frac{1+2y_2}{1+y_1} - 1 = \frac{B(12)-B(24)}{B(24)}$$

介.

$$\frac{\frac{100}{B(24)}}{\frac{100}{B(12)}} - 1 = \left(\frac{\cancel{100}}{B(24)} \times \frac{B(12)}{\cancel{100}}\right) - 1$$

Forward rate agreements (FRAs)

Forward rate agreements (FRAs) are forward contracts issued on **interest rates** as underlying assets. For instance:

$$\underset{\text{start}}{t} \times \underset{\text{end.}}{T} \text{ FRA}$$

refers a $T - t$ investment period that begins in month t and ends in month T. The settings of a FRA are:

- **No actual exchange** of the **principal** P.
- The long side receives the **difference between a reference floating rate** r_t **and a fixed rate** k on time t.

$(r_t - k.)$

FRA payoffs

3/15.

We enter into a long position in a 4×7 FRA on March 15:

- Three month floating rate: r_t
- Principal amount: $P = \$5,000,000$
- Fixed rate: $k = 5\%$
- Beginning date: July 15 (four months from now) 7/15.
- Ending date: October 15 (seven months from now) 10/15

Suppose $r_t = 5.4\%$ on July 15, the payoff is:

$16+31+30+15 = 31\times2+30 = 92.$

$$\underbrace{(5.4\% - 5\%)}_{r_t - k} \times \frac{92}{360} \times \$5,000,000 = \$5,111.11$$

To discount it back to July 15, we will receive this amount from the seller on July 15:

$$\frac{\$5,111.11}{1 + (0.054)\frac{92}{360}} = \$5,041.54$$

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FRA payoffs

yearly.

Suppose $r_t = 4.7\%$ on July 15, the payoff is:

rt - k.

$$(4.7\% - 5\%) \times \frac{92}{360} \times \$5,000,000 = -\$3,833.33$$

To discount it back to July 15, w will give this amount from the seller on July 15:

$$\frac{-\$3,833.33}{1 + (0.047)\frac{92}{360}} = -\$3,787.83$$

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Pricing of FRAs

The payoff to the long position in FRA is thus:

$$\frac{(r_t - k) \times \frac{d}{360}}{1 + r_t \frac{d}{360}} \times P$$

By adding and subtracting $P/(1 + r_t \frac{d}{360})$, we have:

$$\frac{P + Pr_t \frac{d}{360} - P - Pk \frac{d}{360}}{1 + r_t \frac{d}{360}}$$

Which is the same as:

$$\frac{P(\cancel{1 + r_t \frac{d}{360}})}{\cancel{1 + r_t \frac{d}{360}}} - \frac{P(1 + k \frac{d}{360})}{1 + r_t \frac{d}{360}} = P - \frac{P(1 + k \frac{d}{360})}{1 + r_t \frac{d}{360}}$$

Pricing of FRAs

At time 0, we expect the payoffs at time t to be:

1. a **certain cash inflow** of P;
2. an **uncertain cash outflow** of $\frac{P(1+k\frac{d}{360})}{1+r_t\frac{d}{360}}$.

At time t, r_t is known so the uncertain cash outflow can be invested at the same rate. At time T, the cash outflow becomes:

$$\frac{P(1+k\frac{d}{360})}{\cancel{1+r_t\frac{d}{360}}} \times \cancel{(1+r_t\frac{d}{360})} = P(1+k\frac{d}{360})$$

Pricing of FRAs

As a result, the future payoffs of a FRA are:

1. a **certain cash inflow** of P at time t;
2. a **certain cash outflow** of $P(1+k\frac{d}{360})$ at time T.

The time 0 present value is given by:

$$B(t)P - B(T)P(1+k\frac{d}{360}) = 0$$

$$\frac{P}{1+t} \qquad \frac{1}{1+T_1} P(1+k\frac{d}{360})$$

So the arbitrage-free FRA price is:

$$k^* = \frac{B(t)-B(T)}{B(T)} \times \frac{360}{d}$$

$$B(t)\cancel{P} = B(T)\cancel{P}(1+k\frac{d}{360})$$

$$\left(\frac{B(t)}{B(T)}-1\right)\frac{360}{d} = k^*$$

Reference interbank interest rates

LIBOR reform

In the past, the **London interbank offered rate (Libor)** is the **standard reference interest rate** for interbank borrowing and lending on **eurodollar deposits** (USD deposits outside of the US).

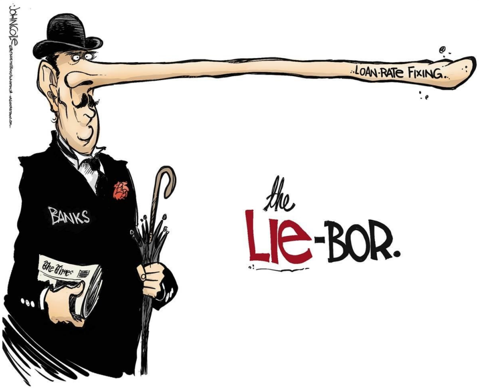


Figure: Source: John Cole/Scranton Times Tribune

LIBOR reform

By the end of 2021, new **benchmark interest rates** will replace Libor. Examples include:

- United States: Secured Overnight Financing Rate (SOFR)
- Euro Zone: Euro Overnight Index Average (Eonia)
- United Kingdom: Sterling Overnight Index Average (Sonia)
- Switzerland: Swiss Average Rate Overnight (Saron)
- Japan: Tokyo Interbank Offered Rate (Tibor) and Tokyo Overnight Average Rate (Tonar)

Pricing of futures

Pricing of futures

(interest rate future contracts)

can determine. time. location. quantity. quality ...

← Benefit seller. Buyer will anticipate lead to current $ ↓.

It is **difficult** to price futures contract **analytically**:

- The **delivery options** in futures create **uncertainty** about the **grade** of the **underlying asset** to be **delivered** at maturity.
- The **daily marked-to-market** procedure generates **uncertain intermediate cash flows**.

- Accounting. practice.
- adjust value by current market condition.
- Getting how much? if sold at current timing

Marked-to-market and pricing of futures

F: forward f: future. r: interest rate.

Since the **margin account** earns interest, we have the following relationships between forwards and futures:

- If $\mathrm{Corr}(\Delta f, \Delta r) > 0$, futures prices will be **higher** than forward prices. long future, if $\Delta f > 0$, margin ↑, interest payment ↑ $\Delta r > 0$.
- If $\mathrm{Corr}(\Delta f, \Delta r) < 0$, futures prices will be **lower** than forward prices. margin ↑, interest payment ↓.
- If $\mathrm{Corr}(\Delta f, \Delta r) = 0$, futures prices will be **identical to** forward prices. $(\Delta f = \Delta r + \Delta F.)$

For short-term contracts, the daily marked-to-market procedure has **little impact** on driving the price difference between forwards and futures. In SR, $\Delta r \approx 0$

Unbiased expectation hypothesis

Forward prices versus future spot prices

The no-arbitrage forward price for a **non-dividend paying stock**:

$$F_{0,T} = S_0 e^{rT}$$

Under the **unbiased expectation hypothesis**:

$$F_{0,T} = \mathbb{E}[S_T] \Rightarrow F_{T,T} = S_T$$

Which implies a risk-free rate of return:

$$\mathbb{E}[S_T] = S_0 e^{rT}$$

all constant.

no arbitrage price.

Yet, stock is a risky investment with an **expected return** of m. The forward price has a **systematic downward bias** in forecasting future spot price.

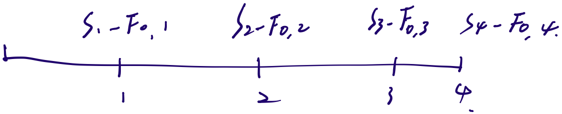
In real case (tend to be < true value)

based on Eg. CAPM

$$\Rightarrow \mathbb{E}[S_T] = S_0 e^{mT} > S_0 e^{rT} = F_{0,T}$$

Swaps

Swap



A **swap** is a contract between two counterparties for **exchanging cash flows or financial instruments** over a **pre-specified period**.

- It is essentially a series of **forwards** with different **maturities**.
- Swaps are usually **OTC** contracts between corporations or financial institutions.
- Swaps usually do not entail the exchange of **principal amounts**.
- Examples include interest rate swaps (IRS), currency swaps, commodity swaps, credit default swaps (CDS), and equity swaps.

Total Return Swaps.

→ leveraged / Inverse ETFs.

2X. -1X. -2X,

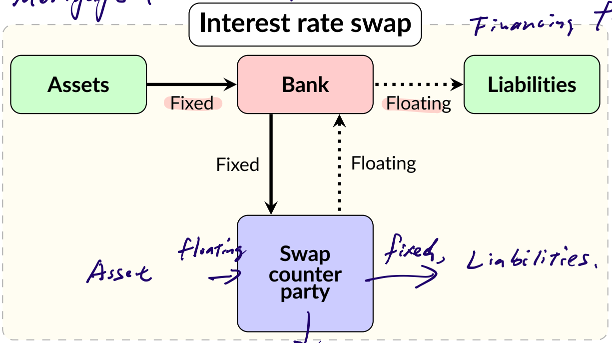
Interest rate swaps

Loans.

Mortgages (Asset 抵押)

Deposit,

Financing from other banks.



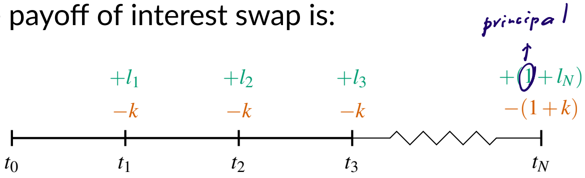
Asset floating

fixed, Liabilities.

(another bank)

Pricing of interest rate swaps

Let l_n be the **floating rate** observed at t_n and k be the **fixed swap rate**. The payoff of interest swap is:



receive floating

pay fixed.

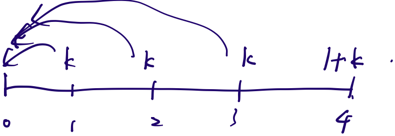
Furthermore, the **present value factor** is given by:

$$B(t_0, t_n) = \frac{\$1}{1 + r_{0,n}} \qquad (1+k)$$

Since the swap contract involves **exchanging cash flows** between two parties, it is similar to a combined position of:

- a **short** position in a **fixed rate bond with coupons** k;
- a **long** position in a **floating rate bond with coupons** l_n.

Pricing of interest rate swaps



The **present value at inception** for the **fixed rate part** is:

$$kB(t_0,t_1)+kB(t_0,t_2)+\cdots+\overset{(k+1)}{kB(t_0,t_N)+B(t_0,t_N)}$$

The **present value at inception** for the **floating rate part** is:

$$\frac{l_1}{1+l_1}+\frac{l_2}{(1+l_1)(1+l_2)}+\cdots+\frac{1+l_N}{\prod_{n=1}^{N}(1+l_n)}$$

$$=\frac{l_1\prod_{n=2}^{N}(1+l_n)+l_2\prod_{n=3}^{N}(1+l_n)+\cdots+(1+l_N)}{\prod_{n=1}^{N}(1+l_n)}$$

$$=\$1$$

<2 period>

$$PV(l_1)=\frac{l_1}{1+l_1}.$$

$$+)\quad PV(l_2)=\frac{1+\cancel{l_2}}{(1+l_1)(1+\cancel{l_2})}$$

1.

<3 period>

$$PV(\ell_1) = \frac{\ell_1}{1+\ell_1}$$

$$PV(\ell_2) = \frac{\ell_2}{(1+\ell_2)(1+\ell_1)}$$

$$+)\quad PV(\ell_3) = \frac{1+\cancel{\ell_3}}{(1+\cancel{\ell_3})(1+\ell_2)(1+\ell_1)}$$

$$\frac{1+\cancel{\ell_2}}{(1+\ell_1)(1+\cancel{\ell_2})} + \frac{\ell_1}{1+\ell_1} = \frac{1+\ell_1}{1+\ell_1} = 1 \quad \#$$

$\Rightarrow$ For n period. ($n \in$ arb pos. const)

The Floating rate is always one

Pricing of interest rate swaps

1 = PV (float Bond) = PV (Fixed Bond)

We need to find k such that the swap contract has zero value to both parties at inception. Both the **fixed rate part** and the **floating rate part** should be **priced at par** so:

$$\$1 = k[B(t_0,t_1) + \cdots + B(t_0,t_N)] + B(t_0,t_N)$$

The swap rate is thus:

$$k^* = \frac{1 - B(t_0,t_N)}{B(t_0,t_1) + B(t_0,t_2) + \cdots + B(t_0,t_N)}$$

target.

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