Financial derivatives

Lecture 1: Overview of the derivatives market

Douglas Chung

What are derivatives?

What are derivatives?

Figure: Word cloud of the Wikipedia page “Derivative (finance)”

What are derivatives?

Derivatives are **securities** that **derive their values** from **one or multiple underlying asset(s)**. Common derivatives include:

- Forwards;
- Futures;
- Swaps;
- Options;
- Asset-backed securities;
- Structured products.

Underlying assets

There are a wide range of underlying assets in derivative markets.
Financial assets include:

- equities;
- fixed income securities; e.g. Fed Fund Rate futures
- interest rates;
- currencies; FX.
- credit. CDS (credit Default Swap).

Non-financial assets include:

- commodities; oil , coffee ,
- carbon emissions; carbin quota.
- weather;
- catastrophe.

Brief history of derivatives

Timeline of derivatives

Ancient time:

- Forwards on wheat in Egypt

1600s:

- Futures and options on tulips in Netherlands
- Futures on rice in Japan

1800s:

- Stock options at the London Stock Exchange
- Grain futures at the Chicago Board of Trade

1970s:

- Financial futures and stock options at the Chicago Mercantile Exchange
- Invention of the Black-Scholes option pricing model

Since 1980s:

- Explosive growth in the derivative market

Derivatives in the media


The Guardian

Receivers called in after £500m goes walkabout ☐ Mayhem feared in markets

Barings Bank goes bust

Figure: Barings Bank goes bust (Source: The Guardian)

Derivatives in the media

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AIG, Lehman Shock Hits World Markets

Focus Moves to Fate of Giant Insurer After U.S. Allows Investment Bank to Fail; Barclays in Talks to Buy Core Lehman Unit

The convulsions in the U.S. financial system sent markets across the globe tumbling, as two of Wall Street's biggest firms looked set to exit the scene and insurance titan American International Group Inc. turned to the Federal Reserve and the state of New York for assistance.

By Susanne Craig, Jeffrey McCracken, Jon Hilsenrath and Deborah Solomon

The U.S. stock market suffered its worst daily point plunge since the first day of trading after the Sept. 11, 2001, terrorist attacks. Financial markets were rattled by the rushed sale Sunday of Merrill Lynch & Co. and the bankruptcy-court filing of Lehman Brothers Holdings Inc., which scrambled Monday to sell its most-prized businesses before too many employees and customers walk out the door. (Please see related article on Page C1.)

All day Monday, top Lehman officials were huddled in Manhattan at their Seventh Avenue headquarters negotiating a sale of the U.S. investment bank—the core part of Lehman—to Barclays PLC of the U.K. People involved in the discussions were increasingly hopeful late Monday ing. For much of the day, the major U.S. market indexes were down 2%, which, while a good-sized decline, was smaller than many had thought would be the case. But in the final hour of trading, a wave of selling hit, driven by concerns about the fate of AIG. The Dow Jones Industrial Average ended down 504.48 points on Monday, off 4.4%, at its daily low of 10917.51, down 18% on the year. Of the Dow industrials' 30 components, all but one—Coca-Cola Co.—fell, led by a 60.8% plunge in AIG.

In Europe, London's FTSE 100 index dropped 3.9%. Several Asian markets, including Japan and China, were closed Monday due to holiday. By Tuesday, Tokyo shares were down 5.1% in early trading, and Hong Kong's Hang Seng index was down 6.1%.

Monday's action was the latest fallout in a widening financial crisis that began a year ago with the fall of American housing prices and is now reordering the U.S. financial system. Steps unveiled by the Federal Reserve to expand its emergency lending arsenal did little to snap the sense of gloom.

Plenty of potential land mines remain. Banks are increasingly hoarding cash, curbing lending at a time when the econ-

AIG Faces Cash Crisis As Stock Dives 61%

By Matthew Karnitschnig, Liam Pleven And Serena Ng

American International Group Inc. was facing a severe cash crunch last night as ratings agencies cut the firm's credit ratings, forcing the giant insurer to raise $14.5 billion to cover its obligations.

With AIG now tottering, a crisis that began with falling home prices and went on to engulf Wall Street has reached one of the world's largest insurance companies, threatening to intensify the financial storm and greatly complicate the government's efforts to contain it. The company, whose stock fell 61% yesterday, is such a big player in insuring risk for institutions around the world that its failure could shake the global financial system.

Figure: Financial crisis of 2007–2008 (Source: The Wall Street Journal)

Derivatives in the media

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U.S. to Take Over AIG in $85 Billion Bailout; Central Banks Inject Cash as Credit Dries Up

Emergency Loan Effectively Gives Government Control of Insurer; Historic Move Would Cap 10 Days That Reshaped U.S. Finance

The U.S. government seized control of American International Group Inc.—one of the world's biggest insurers—in an $85 billion deal that signaled the intensity of its concerns about the danger a collapse could pose to the financial system.

By Matthew Karnitschnig, Deborah Solomon, Liam Pleven and Jon E. Hilsenrath

The step marks a dramatic turnabout for the federal government, which had been strongly resisting overtures from AIG for an emergency loan or some intervention that would prevent the insurer from falling into bankruptcy. Just last weekend, the government essentially pulled the plug on Lehman Brothers Holdings Inc., allowing the big investment bank to go under instead of giving it financial support. This time, the government decided AIG truly was too big to fail.

The U.S. negotiators drove a hard bargain. Under terms hammered out Tuesday night, the Fed sets, including its profitable insurance businesses, giving the Fed some protection even if markets continue to sink. And if AIG rebounds, taxpayers could reap a big profit through the government's equity stake.

"This loan will facilitate a process under which AIG will sell certain of its businesses in an orderly manner, with the least possible disruption to the overall economy," the Fed said in a statement.

It puts the government in control of a private insurer—a historic development, particularly considering that AIG isn't directly regulated by the federal government. The Fed took the highly unusual step using legal authority granted in the Federal Reserve Act, which allows it to lend to nonbanks under "unusual and exigent" circumstances, something it invoked when Bear Stearns Cos. was rescued in March.

As part of the deal, Treasury Secretary Henry Paulson insisted that AIG's chief executive, Robert Willumstad, step aside. Mr. Paulson personally told Mr. Willumstad the news in a phone

housing and insurance industries, while Wall Street has watched two of its last four big independent brokerage firms exit the scene.

The U.S. on Sept. 6 took over mortgage-lending giants Fannie Mae and Freddie Mac as they teetered near collapse. This Sunday, the U.S. refused to bail out Wall Street pillar Lehman Brothers, which filed for bankruptcy-court protection and is now being sold off in pieces. That same day, another struggling Wall Street titan, Merrill Lynch & Co., agreed to sell itself to Bank of America Corp.

The AIG deal followed a day of high drama in Washington. The Treasury's Mr. Paulson and Federal Reserve Chairman Ben Bernanke convened in the early evening an unexpected meeting of top congressional leaders. Late in the trading day Tuesday, anticipation that the government might assist the insurer helped propel the Dow Jones Industrial Average to a 1.3% gain.

In bailing out AIG, the Federal Reserve appeared to be motivated in part by worries that Wall Street's financial crisis could be-

Urgent Mission

Plunging shares, soaring credit costs push the government to step in.

DJIA daily close

15000 14000 13000 12000 11000 10000

2007 '08

Overnight dollar Libor

7% 6 5 4 3 2 1 0

2007 '08

AIG's daily share price

$75

Fed chief Ben Bernanke

Getty Images

Treasury Secretary Hank Paulson

Getty Images

Lending Among Banks Freezes

By Carrick Mollenkamp, Mark Whitehouse And Neil Shah

Banks abruptly stopped lending to each other or charged exorbitantly high rates Tuesday, threatening to spread the troubles of American International Group Inc. and Lehman Brothers Holdings Inc. to a broad range of financial institutions and the global economy.

The breakdown came despite efforts by central bankers to keep money flowing. Central banks in the U.S., Europe and Japan pumped tens of billions of dollars each into the banking system. The Federal Reserve, while declining to lower its benchmark interest rate at a

Figure: Financial crisis of 2007–2008 (Source: The Wall Street Journal)

Building blocks

Forward

Forward buyer:

- The buyer is **committed to buy** the underlying asset at the **pre-specified forward price on a specified delivery (maturity) date**.

Forward seller:

- The seller has the **obligation to sell** the underlying asset **to the forward buyer** at the **pre-specified forward price on the delivery date**.

Futures and swap

Futures:

- It is a **standardized** forward contract with **daily settlement**.
- Unlike over-the-counter forwards, futures are **commonly traded** on **stock exchanges**.

Swap:

- It is an agreement between two counterparties to **exchange cash flows** for a certain period.
- Typical swap contracts involve exchanging **interest rates** or **foreign currencies**.
- Payoffs of a swap can be constructed as a **series of forward contracts**.

Call option

Call option buyer:

- The buyer has the **right to buy** the underlying asset at the pre-specified strike price.

Call option seller:
If the call option is **exercised** by the buyer:

- The seller has the **obligation to sell** the underlying asset **to the buyer** at the pre-specified strike price.

Put option

Put option buyer:

- The buyer has the **right to sell** the underlying asset at the pre-specified strike price.

Put option seller:
If the put option is **exercised** by the buyer:

- The seller has the **obligation to buy** the underlying asset **from the buyer** at the pre-specified strike price.

Why do we need derivative?

Use of derivatives

Risk management/Hedging:

- Reduce the uncertainty of a financial position.

Speculation:

- Make bets that are highly leveraged.

Reduce transaction costs:

- Use derivatives to create synthetic securities.

Regulatory arbitrage:

- Circumvent regulatory restrictions, taxes, and accounting rules by using derivatives.

Derivative markets

Exchange-traded derivative market

Exchange-traded derivatives are traded on **highly organized exchanges**. Their main characteristics are:

- **standardized contracts**;
- **highly regulated** to mitigate counterparty risk and promote liquidity;
- price is determined on the **competitive market**;
- positions can be **traded out**.

Over-the-counter derivative market

Over-the-counter (OTC) derivatives are **traded directly between buyers and sellers**. Their main characteristics are:

- **standardized or customized contracts**;
- **less transparent and less regulated**;
- price is **negotiated between market players**;
- positions has to be **transferred**.

Size and activity of the derivative market

To measure **size** and **activity** of the derivative market, we need to consider the following measures:

1. **Trading volume/Turnover**:
   - The number of the derivative that changes hands within a time period.
2. **Gross market value**:
   - The sum of the absolute market value of outstanding derivatives contracts.
3. **Notional amount**:
   - The value of a derivative's underlying assets at the market price.
4. **Open interest**:
   - The total number of contracts that are yet to be settled.

Size of the derivative market

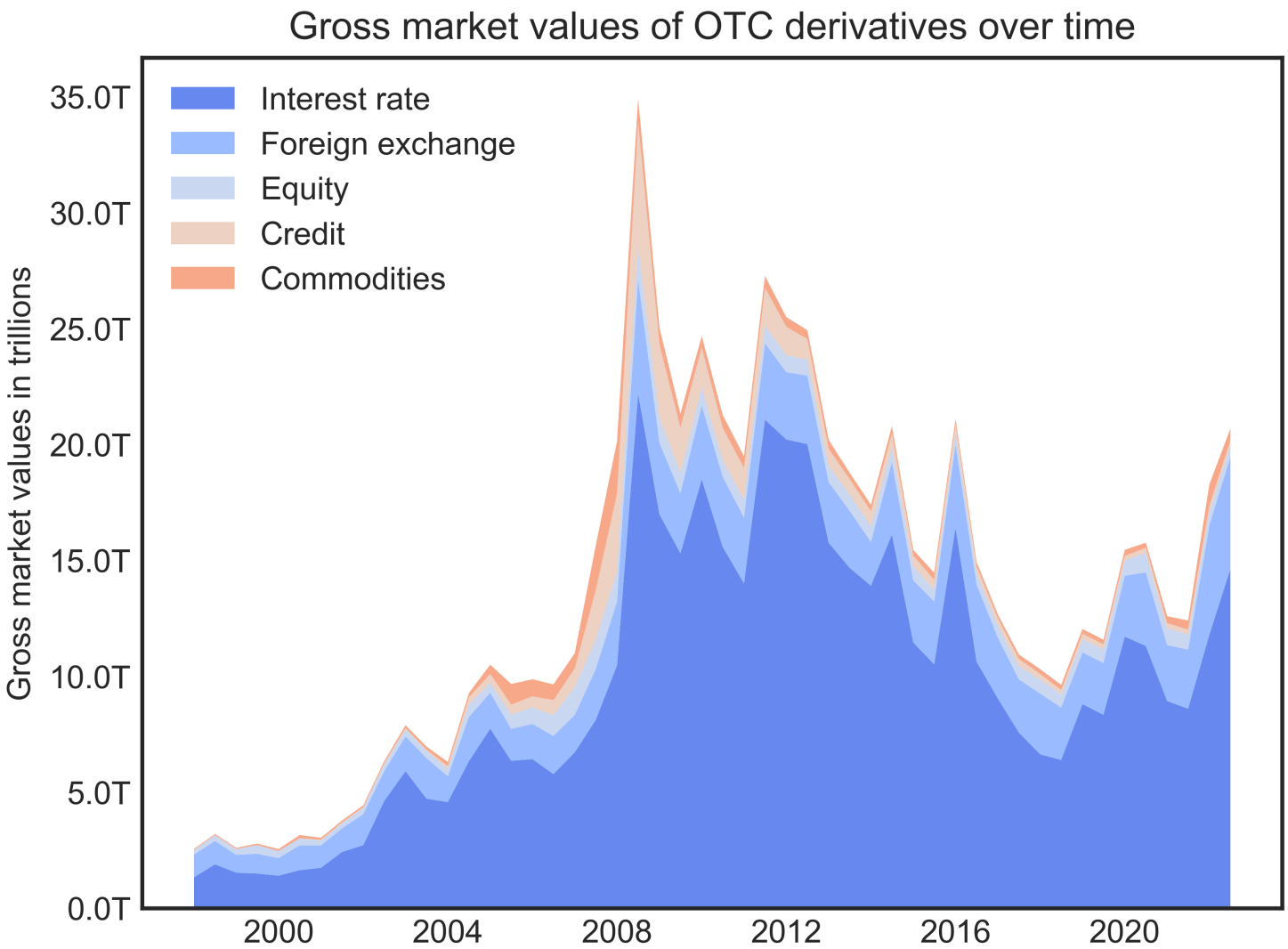


Figure: Gross market values of OTC derivatives (Source: Bank for International Settlements)

Size of the derivative market

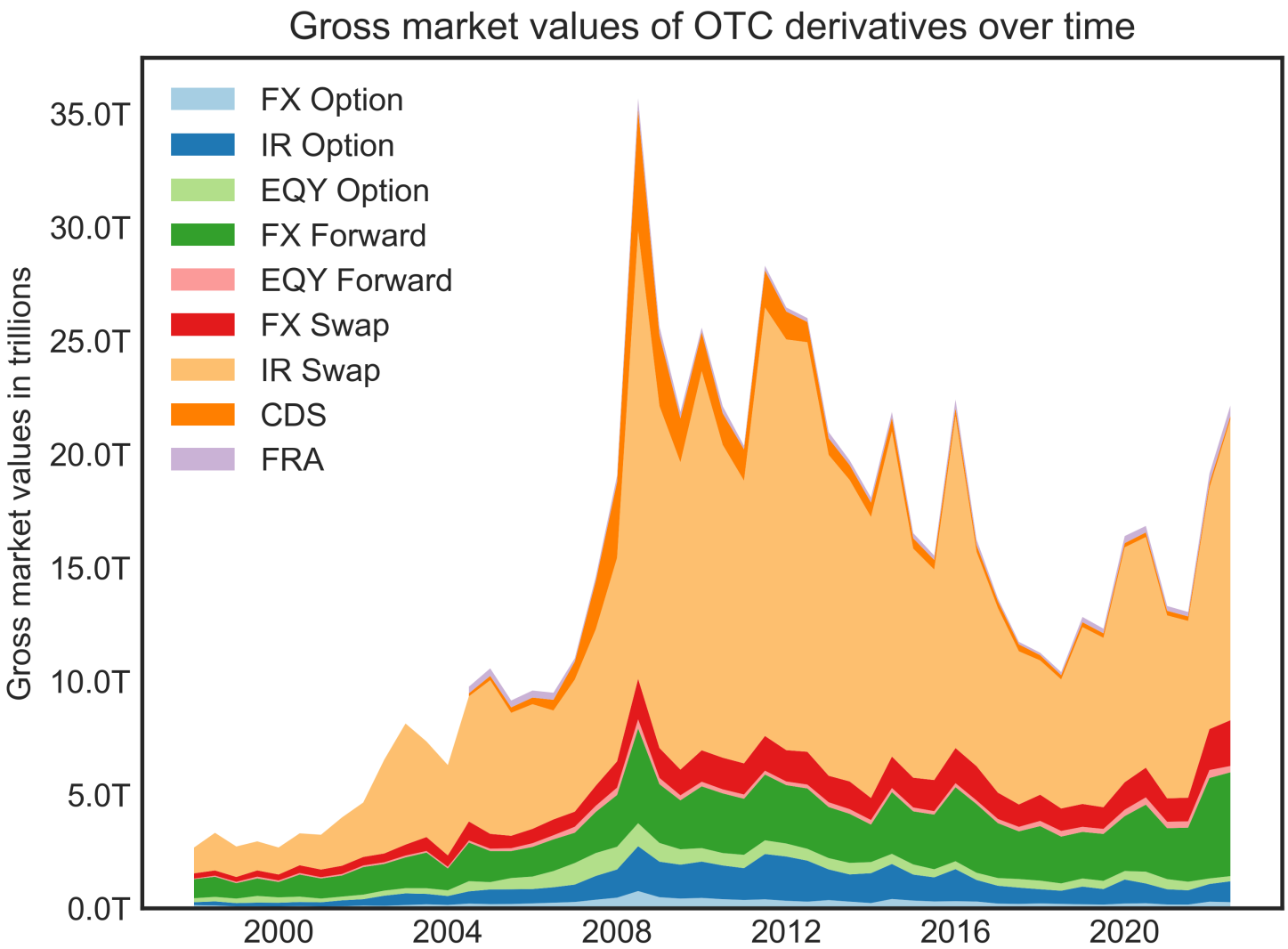


Figure: Gross market values of OTC derivatives by types (Source: Bank for International Settlements)

Size of the derivative market

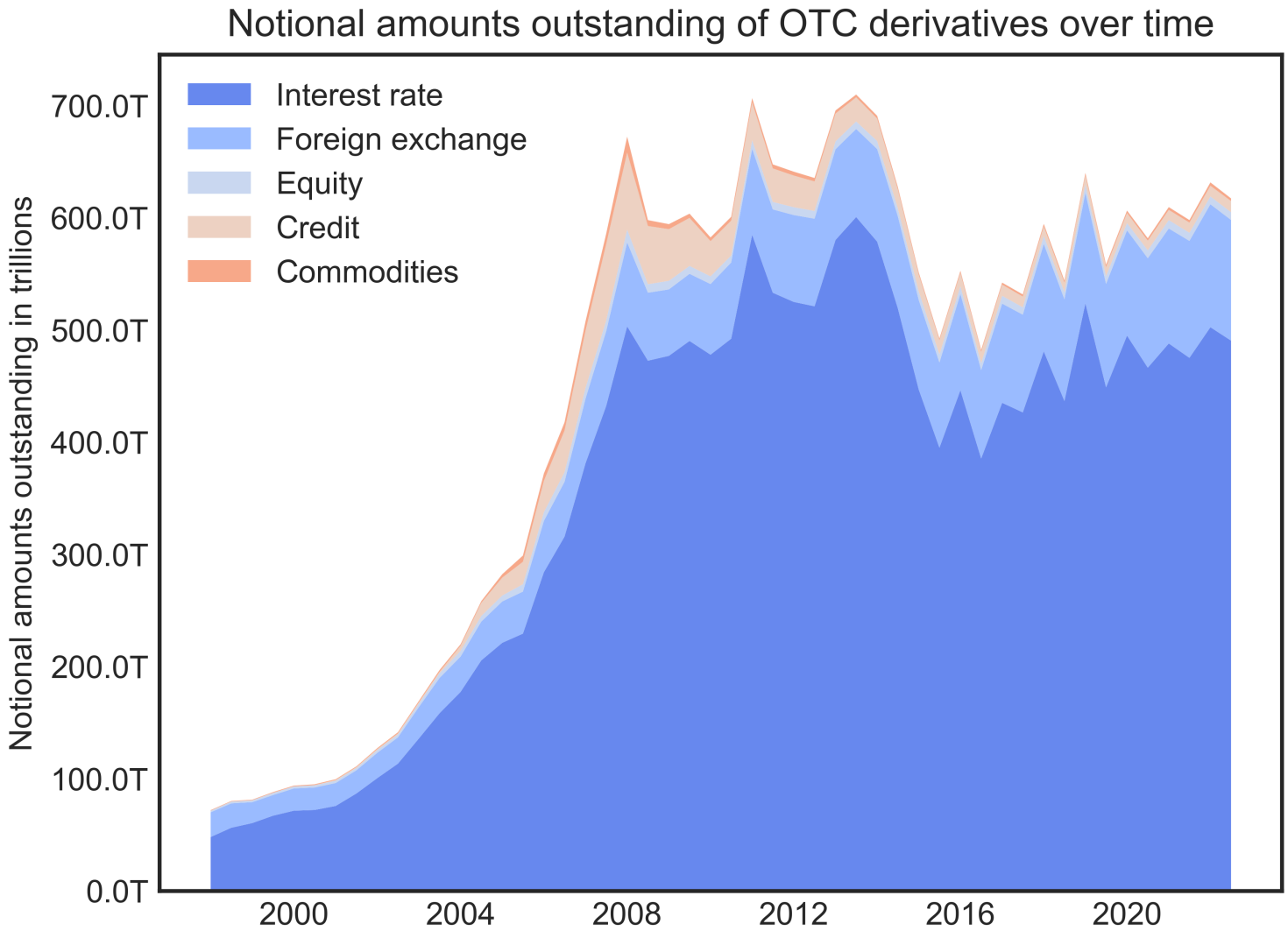


Figure: Notional amount of OTC derivatives
(Source: Bank for International Settlements)

Size of the derivative market

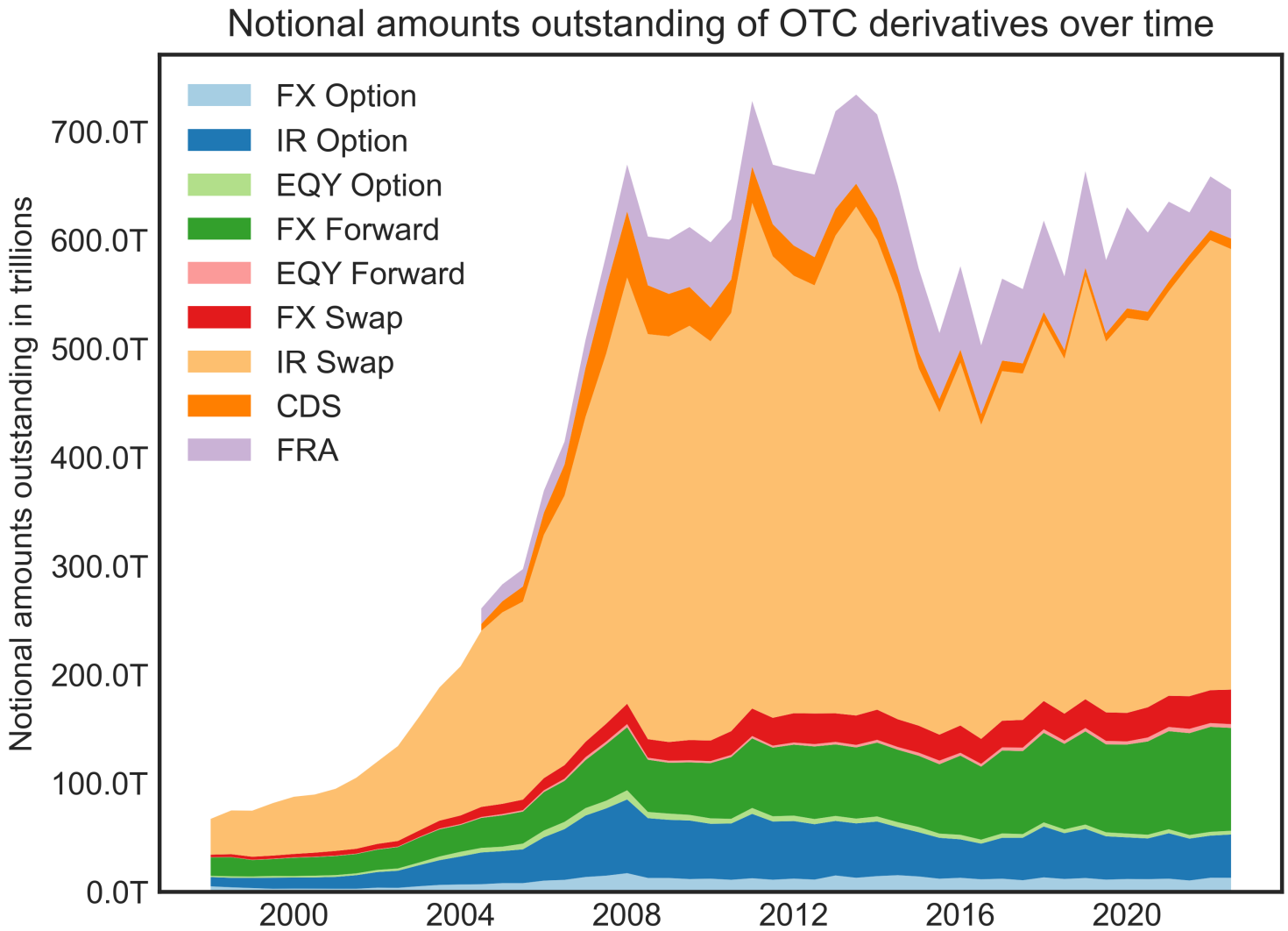


Figure: Notional amount of OTC derivatives by types
(Source: Bank for International Settlements)

Counterparties in derivative contracts

Zero sum game

There must be a buyer and a seller for each derivative contract:

- When you **long** a derivative, you are on the **buy side** of the contract.
- When you **short** a derivative, you are on the **sell side** of the contract.

Each derivative contract is a **zero-sum game**:

- When the buyer **gains**, the seller **losses**, and vice versa.
- The **net payoff** of each derivative contract is zero.

Counterparty risk

Counterparty risk is the **likelihood** that one of the parties involved in a financial contract might **default on its contractual obligation**. To reduce default risk, there are requirements on:

- **margin**;
- **collateral**;
- **credit ratings**.

Bilateral clearing versus central clearing

A **central clearing party (CCP)** is introduced by the **regulator** to manage **counter-party risk** in the OTC derivative market.

- **Promote transparency** through **netting down exposures** between **clearing members**.
- Impose **credit standards** and **margin/collateral requirements**.
- **Reduce systemic risk** by having **resolution framework** and **pooled guarantee fund**.

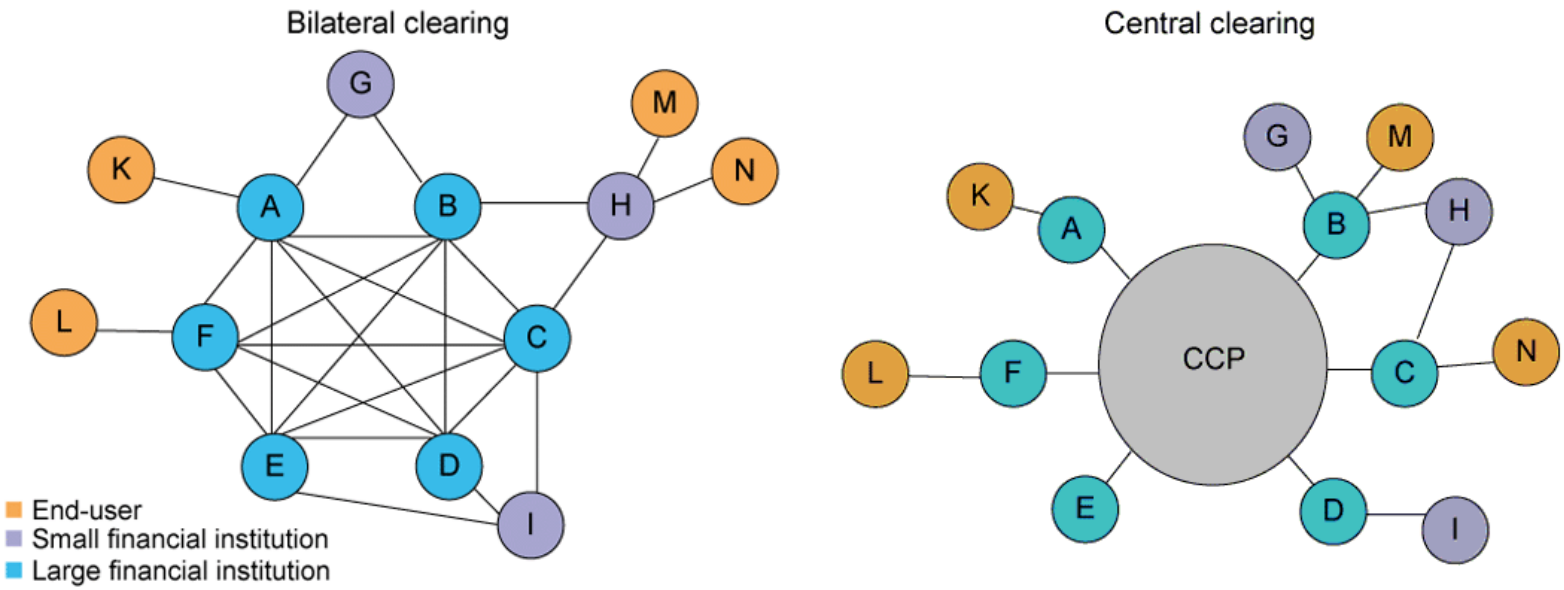


Figure: Bilateral clearing versus central clearing
(Source: Reserve Bank of Australia)

Derivative payoffs

Introduction to payoff diagram

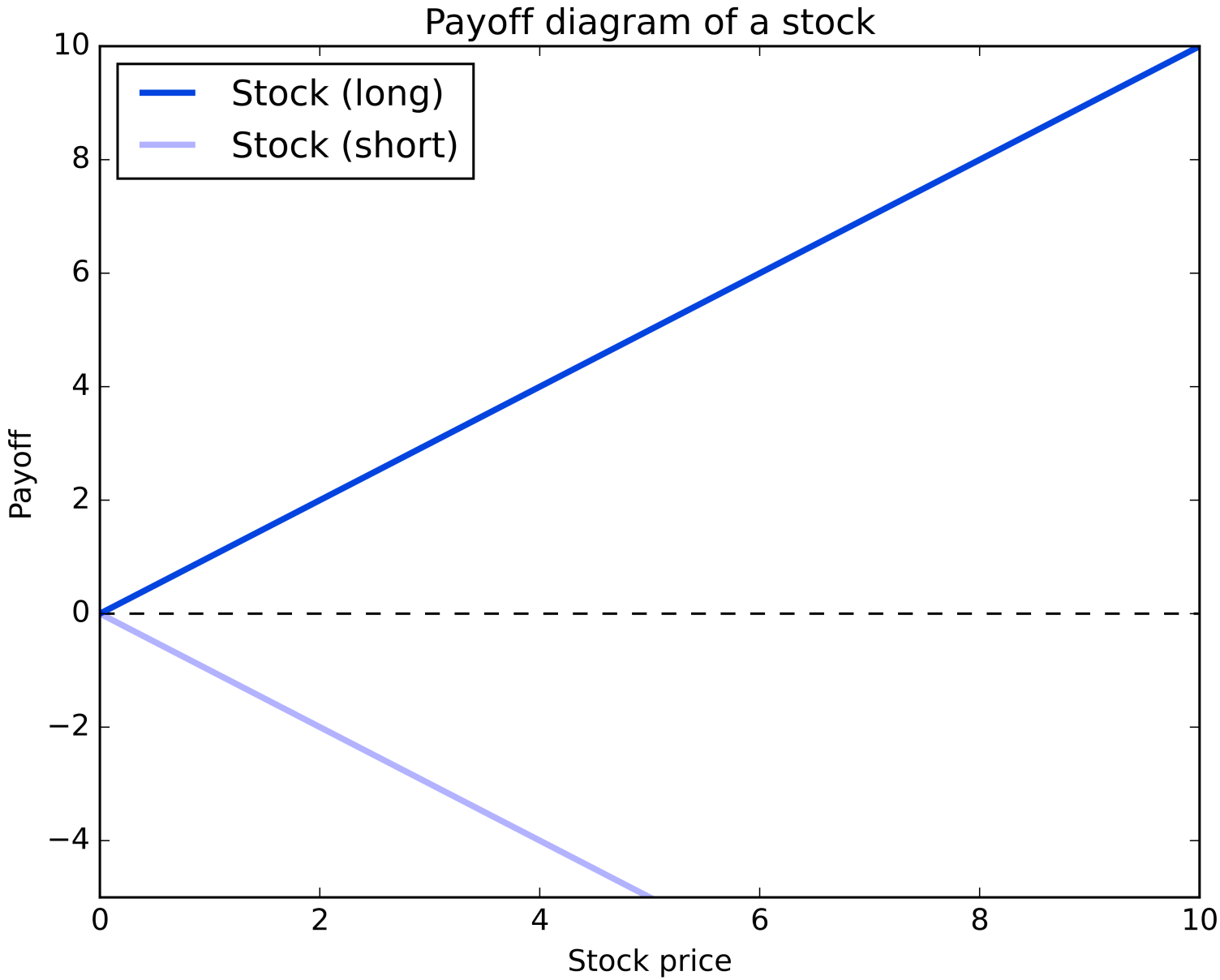
A **payoff diagram** is the graphical representation of a **financial position's potential outcomes**.

- The **x-axis** represents the **market value** of the **underlying asset** while the **y-axis** represents the **payoff**.

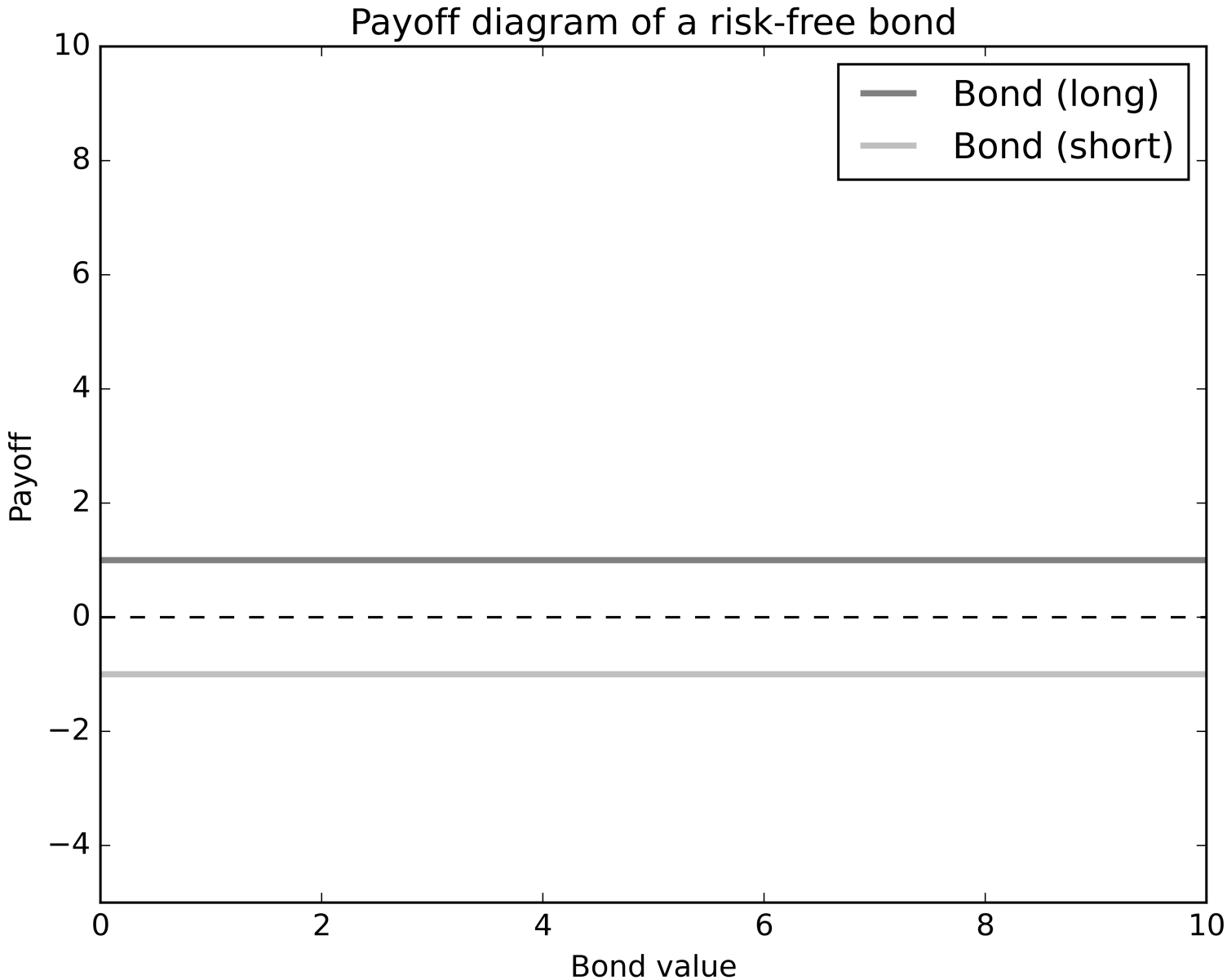
We will use the following mathematical notations for derivatives:

- S_t: **market price of the underlying asset** at time t.
- F: **pre-specified forward (futures)** price of a forward (futures).
- K: **pre-specified strike (exercise)** price of an option.
- $T-t$: **time to maturity** of the contract at time t.

Payoff diagram of a stock



Payoff diagram of a risk-free bond



Payoff diagram of forward contract

Forward buyer:

- At time T when the contract **expires**, the **payoff** is:

$$S_T - F$$

Forward seller:

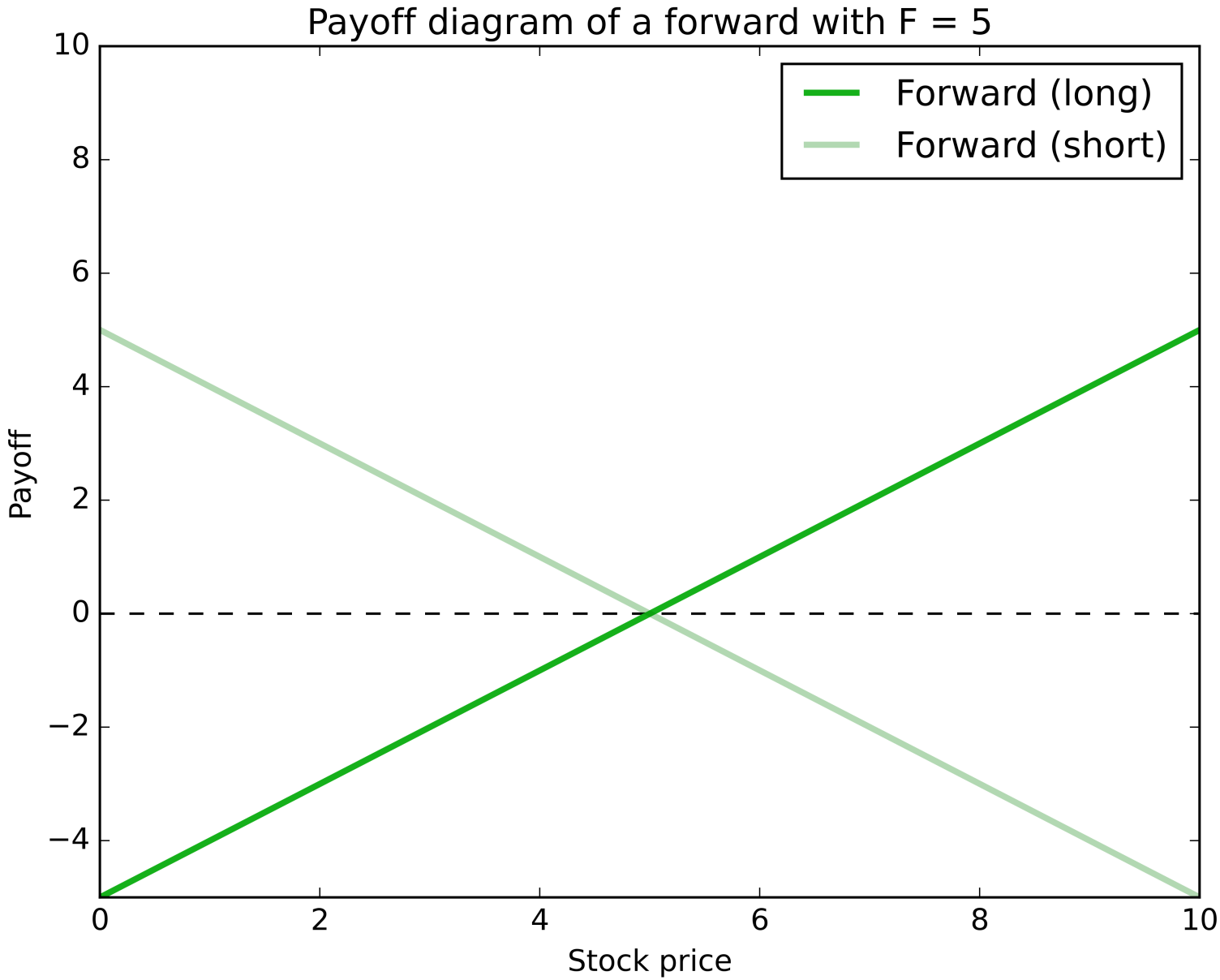
- At time T when the contract **expires**, the **payoff**is:

$$-(S_T - F) = F - S_T$$

Overall, the **net payoff** between buyer and seller is 0.

$$S_T - F + F - S_T = 0$$

Payoff diagram of forward contract



Payoff diagram of a call option

Call option buyer:

- At time t, the **payoff** of **buying a call option** is:

$$\max(S_t - K, 0) = (S_t - K)_+ = (S_t - K)\mathbb{1}_{S_t > K}$$

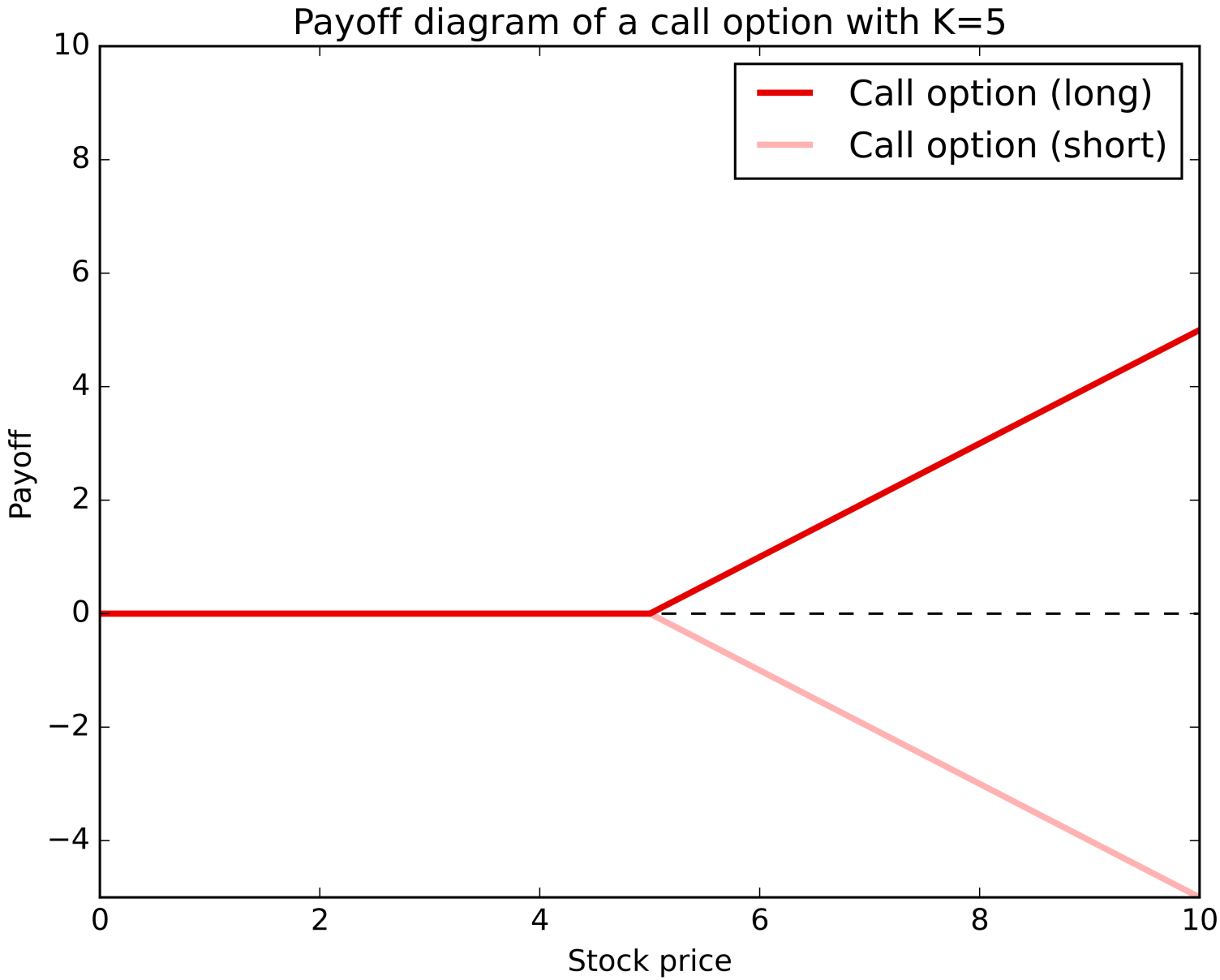
Call option seller:

- At time t, the **payoff** of **selling a call option** is:

$$-\max(S_t - K, 0) = -(S_t - K)_+ = (K - S_t)\mathbb{1}_{S_t > K}$$

Similarly, the **net payoff** between buyer and seller is 0.

Payoff diagram of a call option



Put option

Put option buyer:

- At time t, the **payoff** of **buying a put option** is:

$$\max(K - S_t, 0) = (K - S_t)_+ = (K - S_t)\mathbb{1}_{S_t<K}$$

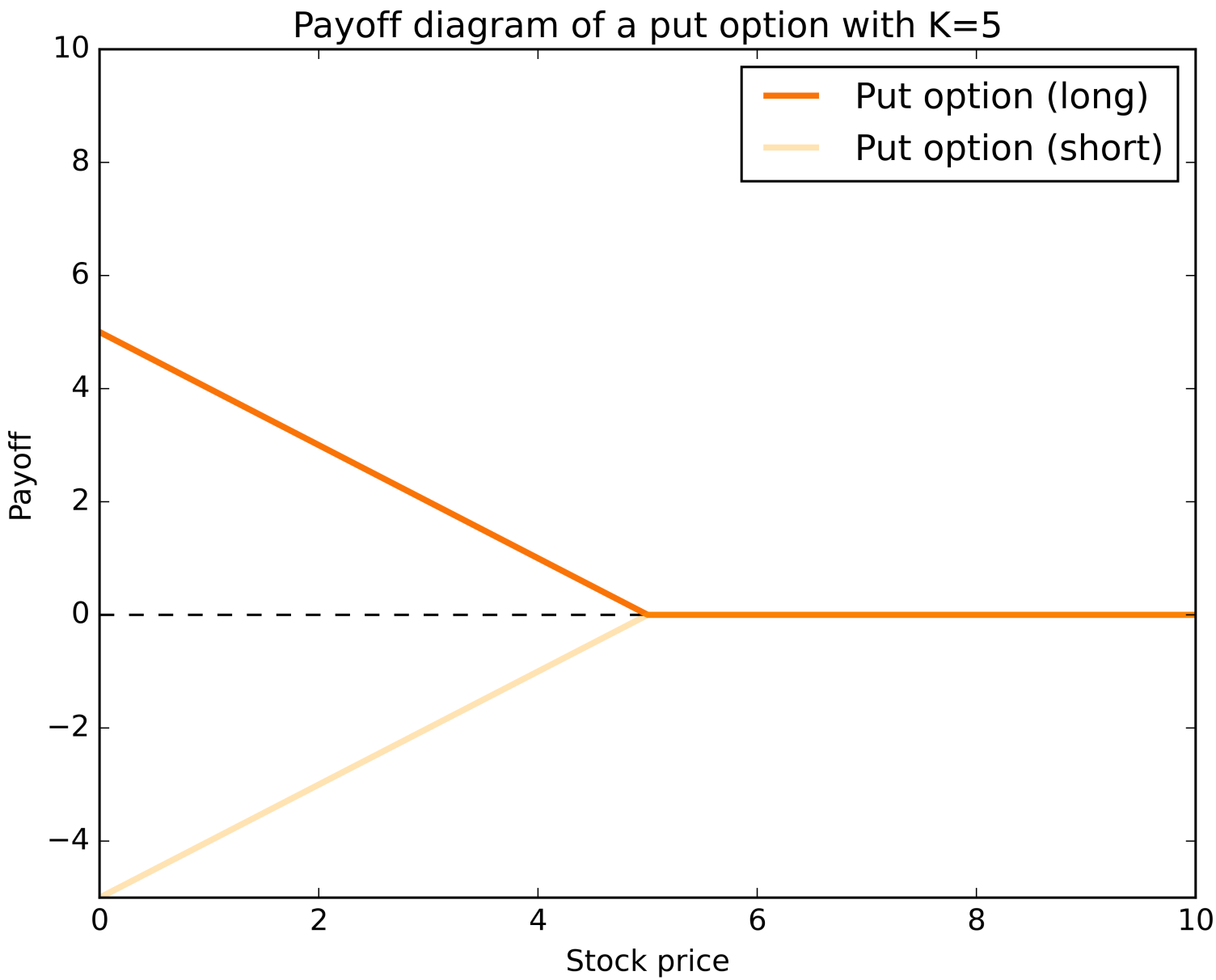
Put option seller:

- At time t, the **payoff** of **selling a put option** is:

$$-\max(K - S_t, 0) = -(K - S_t)_+ = (S_t - K)\mathbb{1}_{S_t<K}$$

Similarly, the **net payoff** between buyer and seller is 0.

Put option


Payoff diagram of a put option with K=5
Put option (long)
Put option (short)
Payoff
Stock price
10
8
6
4
2
0
−2
−4
0
2
4
6
8
10

Moneyness of options

In-the-money options:

- When the option payoff is **larger than zero** (**positive**), the option is said to be **in-the-money**.

At-the-money options:

- When the option payoff **equals zero**, the option is said to be **at-the-money**.

Out-of-the-money options:

- When the option payoff is **smaller than zero** (**negative**), the option is said to be **out-of-the-money**.

Derivative pricing

Idea behind derivative pricing

Figure: Bubble tea (Source: Icon by small.smiles - freepik.com)

Law of one price and no arbitrage condition

Law of one price (LOOP):

In the absence of **market frictions**, assets with **identical future payoffs** must have the **same present value**.

Hence, we can **price derivative securities** by **replicating their payoffs** with **existing assets**. In particular, we need to know:

- the **replicating portfolio** of the derivative;
- **market prices** of existing assets in the replicating portfolio.

If the LOOP is **violated**, a **risk-free arbitrage profit** is captured by:

- **buying** the relatively **underpriced** asset **(buy low)**;
- **short selling** the relatively **overpriced** asset **(sell high)**.

An example of arbitrage

Consider the current prices and the future payoffs of A and B for the low state and the high state in the following table:

Asset:	Price:	Low Payoff:	High Payoff:
A	$4	$5	$10
B	$9	$10	$20

We can see that the future payoffs of B are simply **twice** of that of A. Therefore, we can **replicate** the payoffs of B by buying two units of A with $8.

An example of arbitrage

To **exploit** this **arbitrage opportunity**, we can buy two units of A and short sell one unit of B. The resulting profit is:

Action:	Current CF:	Low CF:	High CF:
Long 2 A	-$8	+$10	+$20
Short 1 B	+$9	-$10	-$20
Net CF:	+$1	$0	$0

We can make $1 for each arbitrage position. Since the position is **self-financed**, we can in theory make an **infinite risk-free profit** if the **mispricing persists**. In **equilibrium**, we should expect the **price of A** to **go up** while the **price of B** to **go down**.

Interest rate

Compounding and interest rate

To compute **present value (PV)** or **future value (FV)**, we need to make use of the **discounting interest rate**. Given an **annual interest rate** of r, the relationship between PV and FV is:

$$\text{PV}(1+r) = \text{FV}$$

If we increase the compounding frequency to semi-annual:

$$\text{PV} = \frac{\text{FV}}{(1+r/2)^2} = \text{FV}(1+\frac{r}{2})^{-2}$$

Therefore, the general formula is:

$$\text{PV} = \text{FV}(1+\frac{r}{n})^{-nt}$$

where n is the **compounding frequency per time period** and t is the **number of periods**.

Continuously compounded interest rate

What if we allow $n \to \infty$? Recall the **exponential constant** is given by:

$$e = \lim_{n \to \infty} (1 + \frac{1}{n})^n$$

We need to find:

$$\text{PV} = \lim_{n \to \infty} \text{FV}(1 + \frac{r}{n})^{-nt}$$

Let $n = mr$, we have:

$$\text{PV} = \lim_{m \to \infty} \text{FV}(1 + \frac{1}{m})^{-mrt} = \text{FV}\left(\lim_{m \to \infty} (1 + \frac{1}{m})^m \right)^{-rt}$$

$$\text{PV} = \text{FV}e^{-rt}$$

Continuously compounded interest rate is commonly used in derivative pricing to give **closed-form expressions of pricing formulas**.