DYN L3

September 19, 2019 3:37 PM

3 L3: Instantaneous Centres of Zero Velocity

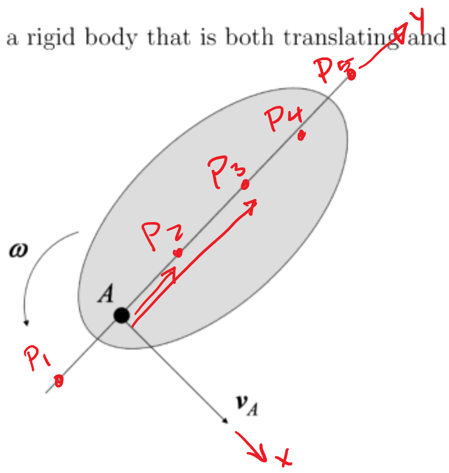
Readings

3.1 Objective

To be able to identify the ICZV of a rigid body, and to use the ICZV to quickly solve planar motion problems.

3.2 Locating the Instantaneous Centre

Consider a rigid body that is both translating and rotating.



If we draw a line perpendicular to $\boldsymbol{v_A}$ and passing through A, all points P_i on this line will have velocity:

$$\boldsymbol{v_{P_i}} = \boldsymbol{v_A} + \boldsymbol{\omega} \times \boldsymbol{r_{P_i/A}}$$

If we set up a coordinate system at point A with x along $\cancel{\boldsymbol{v_A}}$ V_A and y along the line perpendicular to $\boldsymbol{v_A}$, then:

$\boldsymbol{v_A} = V_A \hat{i}$

$\boldsymbol{\omega} = \omega \hat{k}$

$\boldsymbol{r_{P_i/A}} = \overline{AP_i}\, \hat{j}$

$\overline{AP_i} > 0$ P_i +ve y from A

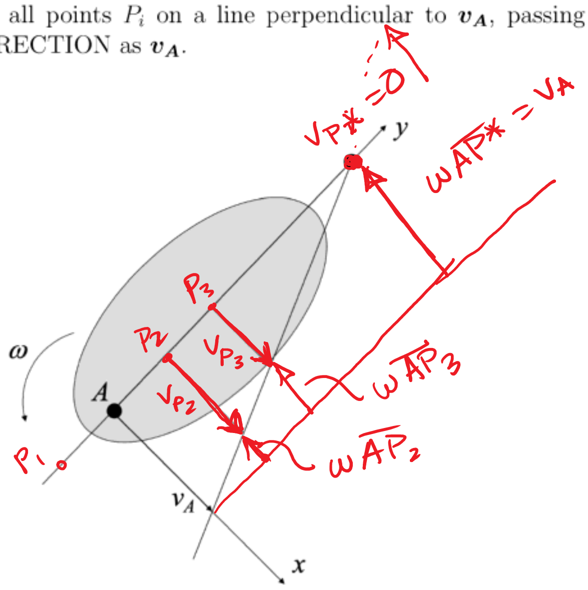
$\overline{AP_i} < 0$ P_i -ve y from A

Now: $\boldsymbol{v_{P_i}} = v_A\hat{\boldsymbol{i}} + \omega\hat{\boldsymbol{k}} \times (AP_i)\hat{\boldsymbol{j}}$ $\hat{k} \times \hat{j} = -\hat{i}$

$\boldsymbol{v_{P_i}} =$ $[v_A - \omega(\overline{AP_i})]\,\hat{i}$

$v_A = v_{P_i} + \omega\,\overrightarrow{AP_i}$

Therefore, all points P_i on a line perpendicular to $\boldsymbol{v_A}$, passing through A, have velocity in the SAME DIRECTION as $\boldsymbol{v_A}$.

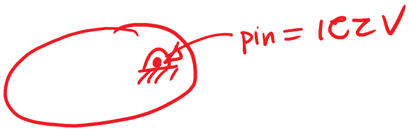


At some point along the line P^*, $AP^*\omega = v_A$

At this point, $v_{P^*} = v_A - (AP^*)\omega = 0$!

This point is known as the **Instantaneous Centre of Zero Velocity, or ICZV**.

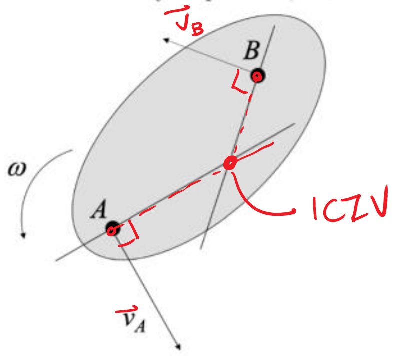
True for pure rotation



True for general plane motion too

3.3 Comments about the ICZV

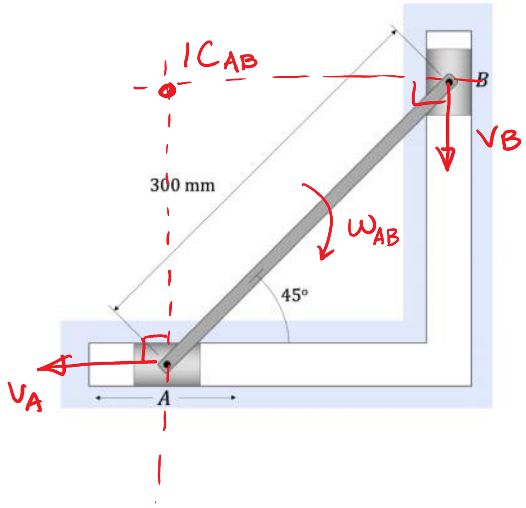
- The ICZV is always on a line passing through any point, P, on the body that is perpendicular to the absolute velocity of point P, $\boldsymbol{P}$.



appears to rotate about this point AT THIS INSTANT

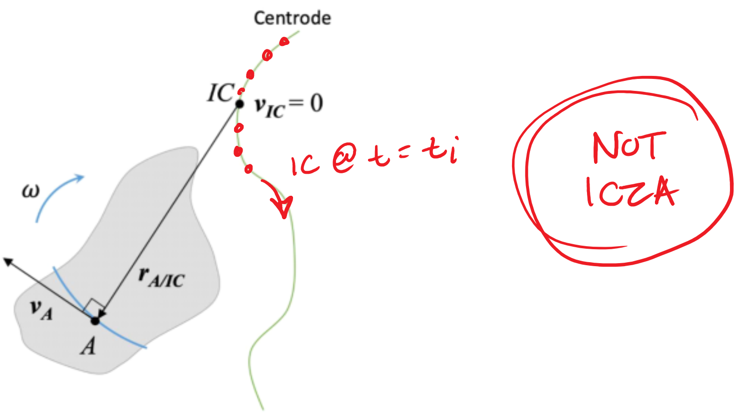
location changes as object moves

- The ICZV is NOT necessarily located on the body!



independent of which points you pick

- For general plane motion (translation plus rotation), the location of the ICZV changes over time.



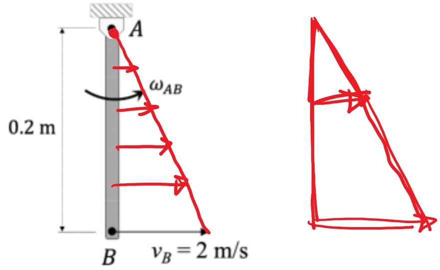
- For bodies that are **pinned to a fixed location**, the ICZV is at the pin.

$$v_B = \omega r_{B/A}$$

$$v_P = \omega r_{P/A}$$

$$\frac{v_P}{v_B} = \frac{r_{P/A}}{r_{B/A}}$$

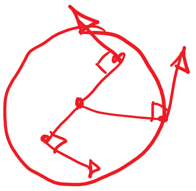
} similar triangles

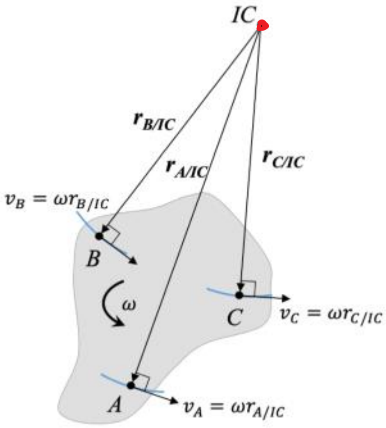


corollary of being able to pick any two pts.

- If the location of the ICZV is known, the velocity of any other point, P, on the body is:

$$\boldsymbol{v_P} = \boldsymbol{\omega} \times \boldsymbol{r_{P/ICZV}}$$





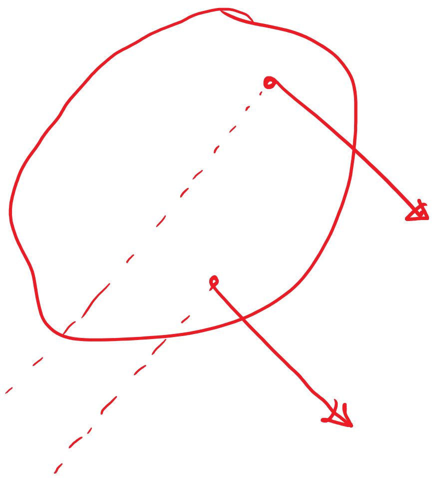
acting like pinned @ IC

$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/IC}$

$\vec{V}_B = \vec{\omega} \times \vec{r}_{B/IC}$

$\vec{\omega}$ is the same for each pt. on a rigid body

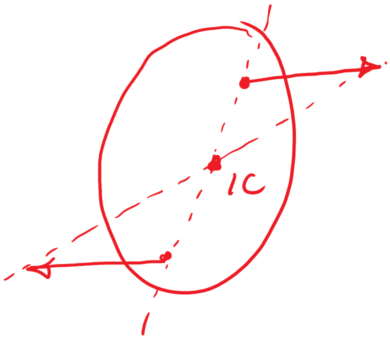
- If the velocities (magnitudes and directions) of two points on the rigid body are the same, then the body is NOT rotating, $\omega = 0$, $\boldsymbol{r_{IC}} = \infty$.



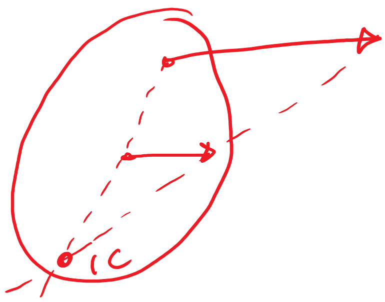
same mag + dir

- If the velocities of two points on the rigid body are co-linear (along the same direction, but different magnitude and/or sense), the method for finding the ICZV differs. There are three cases:

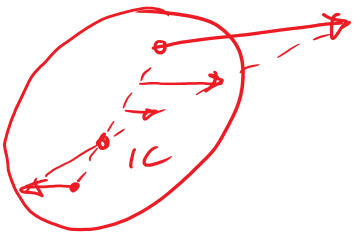
same dir, same mag, different sense



//, same sense, diff mag



//, diff sense, diff mag



Need BOTH magnitude + direction of two vectors to find ICZV if they are parallel

(can find with direction alone if not parallel) www.menti.com 42 88 62

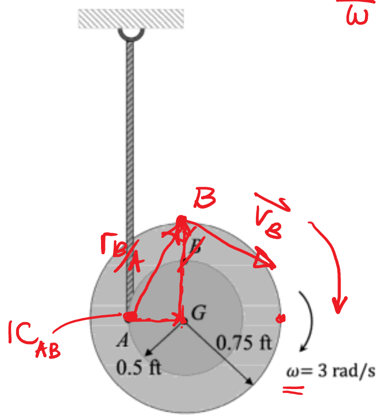
3.3.1 Example 1

Consider a yo-yo unraveling, without the string slipping. Find $\boldsymbol{v_A}$ and $\boldsymbol{v_B}$ at the instant shown.

$\vec{\omega} = -3 \text{ rad/s } \hat{k}$

ICZV? @ A

i) $\vec{V}_A = 0$



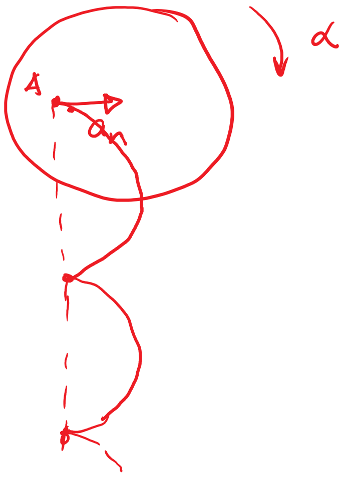
ii) $\vec{V}_B = \vec{\omega} \times \vec{r}_{B/A}$

$-3 \text{ rad/s } \hat{k} \times (0.5\hat{i} + 0.75\hat{j}) \text{ ft}$

$= (2.25\hat{i} - 1.5\hat{j}) \text{ ft/s}$

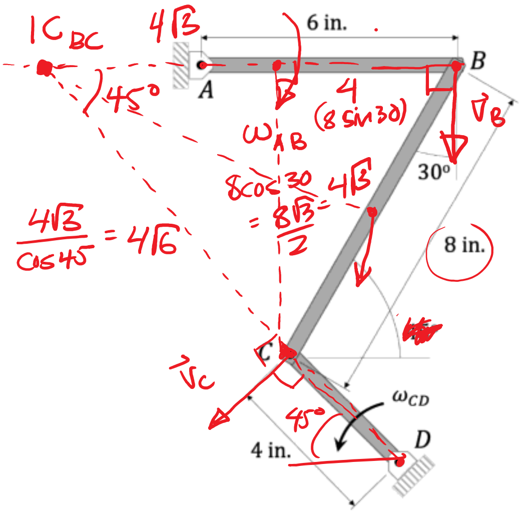
$|\vec{a}_{nA}| = \omega^2 r_{A/G}$

$\vec{a}_A = \vec{a}_G + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/G})}_{\omega^2 r} + \vec{\alpha} \times \vec{r}_{A/G}$



3.3.2 Example 2

Consider the mechanism shown. Find the ICZV of bar BC and the angular velocity ($\vec{\omega}_{CD}$) of bar CD at the instant shown, given $\boldsymbol{\omega_{AB}} = -1\,rad/s\,\hat{\boldsymbol{k}}$.



$\vec{v}_A = 0 \qquad \vec{v}_D = 0$

CD, AB pure rotation

CB is general plane motion

$\sin 30 = \cos 60 = \frac{1}{2}$

$\cos 30 = \sin 60 = \frac{\sqrt{3}}{2}$

$\cos 45 = \sin 45 = \frac{1}{\sqrt{2}}$

$|\vec{r}_{C/IC}| = 4\sqrt{6}$

$|\vec{r}_{B/IC}| = 4\sqrt{3} + 4$

From AB:

$\vec{v}_B = -1\,rad/s\,\hat{k} \times 6\,in\,\hat{i} = -6\,in/s\,\hat{j}$

From BC:

$\vec{v}_B = \vec{\omega}_{BC} \times \vec{r}_{B/IC}$ pure rot about IC

$$|\vec{\omega}_{BC}| = \frac{|\vec{v}_B|}{|\vec{r}_{B/IC}|} = \frac{6\,in/s}{(4\sqrt{3}+4)\,in} = \frac{3}{2}\,\frac{1}{1+\sqrt{3}}\,\frac{rad}{s}$$

Find $\vec{v}_C$:

$$\vec{v}_C = \vec{\omega}_{BC} \times \vec{r}_{C/IC} = \frac{3}{2}\,\frac{1}{1+\sqrt{3}} \cdot 4\sqrt{6} = \frac{6\sqrt{6}}{1+\sqrt{3}}\,in/s$$

Find $\vec{\omega}_{CD}$

$$\vec{v}_C = \vec{\omega}_{CD} \times \vec{r}_{C/D} \Rightarrow |\vec{\omega}_{CD}| = \frac{|\vec{v}_C|}{|\vec{r}_{C/D}|} = \frac{\frac{6\sqrt{6}}{1+\sqrt{3}}\,in/s}{4\,in} = \frac{3\sqrt{6}}{2(1+\sqrt{3})}\,\frac{rad}{s}$$

(Example continued)

Can we do this w/o ICZV? Yes!

1. Find $\vec{v}_B$ (same as above)

2. $$\underset{\substack{m\ \times \\ d\ \checkmark}}{\vec{v}_C} = \underset{\substack{m\ \checkmark \\ d\ \checkmark}}{\vec{v}_B} + \underset{\substack{m\ \times \\ d\ \checkmark}}{\vec{\omega}_{BC}} \times \underset{\substack{m\ \checkmark \\ d\ \checkmark}}{\vec{r}_{C/B}}$$

components $\hat{i}, \hat{j}$

2 unknowns

2 eqn

... solve for $\vec{v}_C$, $\vec{\omega}_{BC}$

3. $\vec{v}_C = \vec{\omega}_{CD} \times \vec{r}_{C/D}$ (same as above)

... solve for $\vec{\omega}_{CD}$