DYN L4

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4 L4: Relative Plane Motion, Acceleration - Fixed Frame

Readings

4.1 Objective

To describe the planar acceleration of **any point** on a rigid body that is both translating and rotating. general plane motion

4.2 Reference Frame - Translating, but NOT Rotating

We have already established the following relationships for fixed reference frame O_{xyz} and translating reference frame $A_{x'y'z'}$.

Position:

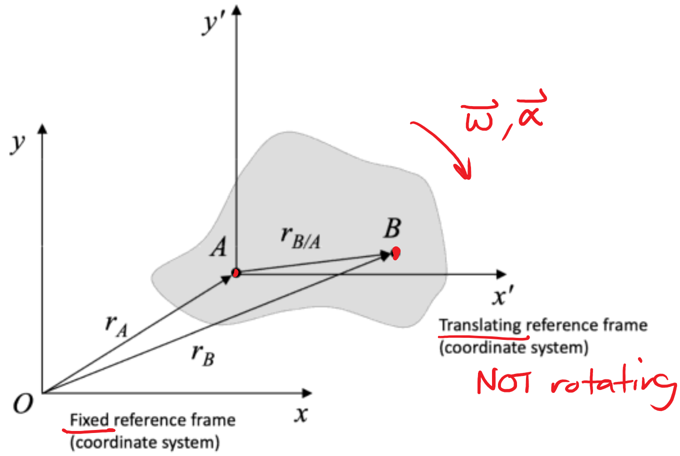
$$r_B = r_A + r_{B/A}$$

$\downarrow \frac{d}{dt}$

Velocity:

$$v_B = v_A + \omega \times r_{B/A}$$

Chasles' theorem

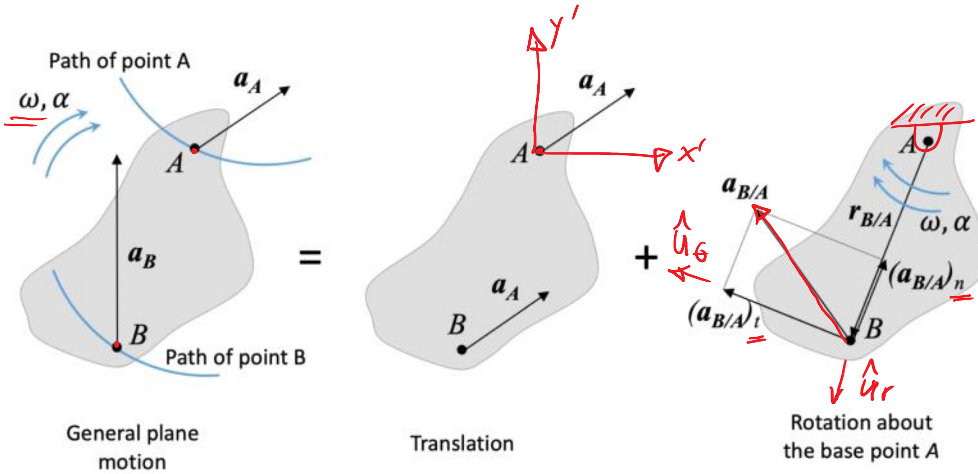


Once again, we need to differentiate to get acceleration: $\frac{d(\vec v_B)}{dt}$

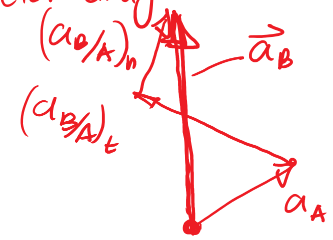
$$a_B = \frac{dv_B}{dt} = \frac{dv_A}{dt} + \frac{d\omega \times r_{B/A}}{dt}$$

$$\underbrace{\frac{d(\vec\omega)}{dt}}_{\vec\alpha} \times \vec r_{B/A} + \vec\omega \times \underbrace{\frac{d(\vec r_{B/A})}{dt}}_{(\vec\omega \times \vec r_{B/A})}$$

$$a_B = a_A + \underbrace{\alpha \times r_{B/A}}_{(\vec a_{B/A})_t} + \underbrace{\boxed{\omega \times (\omega \times r_{B/A})}}_{(\vec a_{B/A})_n}$$



Vector diagram



$$\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_n + (\vec{a}_{B/A})_t$$

What about that third term, $\omega \times (\omega \times r_{B/A})$?

$\vec{\omega} = \omega \hat{k}$

$\hat{i}$ & $\hat{j}$ components

$$\omega \hat{k} \times (\omega \hat{k} \times (r_{B/A})\hat{u}_r)$$

$\hat{i}$ & $\hat{j}$ components

$$= \omega \hat{k} \times (\omega r_{B/A})\hat{u}_\theta$$

$$= \omega^2 r_{B/A} (-\hat{u}_r)$$

$\vec{r}_{B/A} = r_{B/A} \hat{u}_r$

$$= -\omega^2 \vec{r}_{B/A}$$

PLANAR MTN ONLY $\vec{r}_{B/A} \perp \vec{\omega}$ at all times

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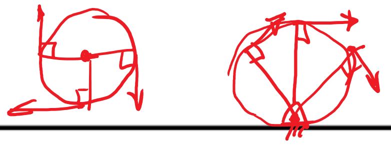
Final result for acceleration with a translating reference frame:

$$\boxed{a_B = a_A + \alpha \times r_{B/A} - \omega^2 r_{B/A}}$$

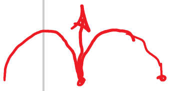
$$\vec{v}_B = \vec{v}_a + \vec{\omega} \times \vec{r}_{B/A}$$

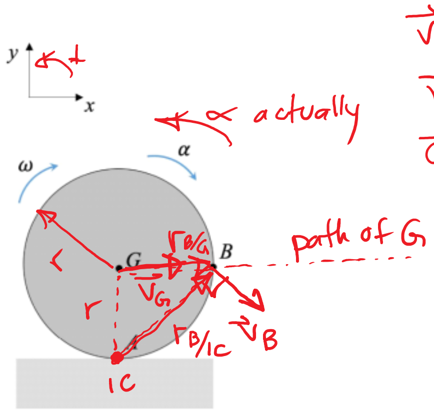
general plane motion



4.2.1 Example 1

Consider a wheel rolling without slipping. Find the velocity and acceleration of point G in the xyz frame shown. Also, find the velocity and acceleration of a point B at the front of the wheel. $r = 1\,m$, $\boldsymbol{\omega} = -1\,rad/s\,\hat{\boldsymbol{k}}$, $\boldsymbol{\alpha} = 1\,rad/s^2\,\hat{\boldsymbol{k}}$.





$\vec{v}_A = 0$ no slipping

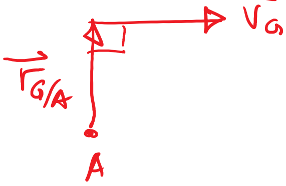
$\vec{v}_G = v_G\,\hat{i}$
$\vec{a}_G = a_G\,\hat{i}$ } path of G is straight line toward x

(OR $\vec{v}_G = \cancel{\vec{v}_A}^{\,0} + \vec{\omega} \times \vec{r}_{G/A}$)

$\underline{\vec{v}_G ?}$ $\vec{v}_G = \vec{\omega} \times \vec{r}_{G/A}$

$-1\,rad/s\,\hat{k} \times 1m\,\hat{j}$

$\vec{v}_G = \underline{1\,m/s\,\hat{i}}$



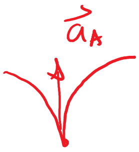
non-zero

$\underline{\vec{a}_G ?}$: $\vec{a}_G = \vec{a}_A + \vec{\alpha} \times \vec{r}_{G/A} - \omega^2 \vec{r}_{G/A}$

scalar

$\vec{\omega} \times (\vec{\omega} \times \vec{r}_{G/A}) = -\omega^2 \vec{r}_{G/A}$

$a_G\,\hat{i} = a_A\,\hat{j} + \alpha\,\hat{k} \times 1m\,\hat{j} - \omega^2 (1m\,\hat{j})$

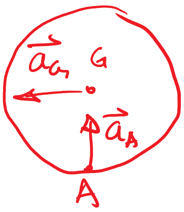


$a_{Ax} = 0$
$a_{Ay} \neq 0$

$\hat{i}$: $a_G = -1\ m/s^2$ $\quad \underline{\vec{a}_G = -1\,m/s\,\hat{i}}$

$\hat{j}$: $0 = a_A - \omega^2 (1m)$

$a_A = \omega^2 r$ $\quad \vec{a}_A = 1\,m/s^2\,\hat{j}$



$\underline{\vec{v}_B}$: $\vec{v}_B = \vec{\omega} \times \vec{r}_{B/IC} = -1\,rad/s\,\hat{k} \times (1\,\hat{i} + 1\,\hat{j})m$

$= \underline{(1\,\hat{i} - 1\,\hat{j})\,m/s}$

$\vec{r}_{B/G} = 1\,m\,\hat{i}$

$\underline{\vec{a}_B}$: $\vec{a}_B = \vec{a}_G + \vec{\alpha} \times \vec{r}_{B/G} - \omega^2 \vec{r}_{B/G}$

$= -1\,m/s^2\,\hat{i} + (1\,\hat{k})\,rad/s^2 \times 1\,\hat{i}\,m - (-1\,rad/s)^2 (1\,m\,\hat{i})$

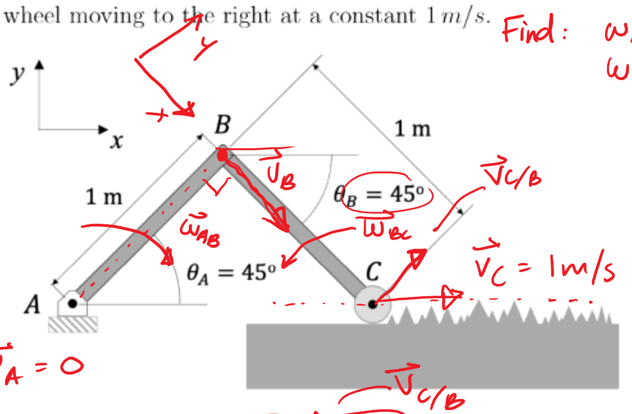
$= \underline{(-2\,\hat{i} + 1\,\hat{j})\,m/s^2}$

($\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$) 2 components ($\hat{i}, \hat{j}$)

4.2.2 Example 2

Consider a planar robot pushing a grinding wheel over a surface (robotic de-burring). Find the angular velocity and acceleration of each link (at the instant shown) necessary to keep the grinding wheel moving to the right at a constant $1\,m/s$.

Find: ω_{AB}, α_{AB}, ω_{BC}, α_{BC}

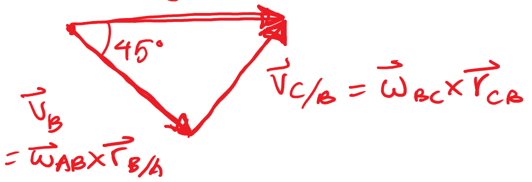


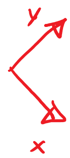
$\vec{v}_C = 1\,m/s$ constant, $\vec{a}_C = 0$

$\vec{v}_A = 0$

$$\vec{v}_C = \vec{v}_B + \underbrace{\vec{\omega}_{BC} \times \vec{r}_{C/B}}_{\vec{v}_{C/B}} \qquad \vec{v}_B = \underbrace{\vec{\omega}_{AB} \times \vec{r}_{B/A}}$$

$$= \vec{\omega}_{AB} \times \vec{r}_{B/A} + \vec{\omega}_{BC} \times \vec{r}_{C/B}$$





$\vec{r}_{B/A} = 1\,m\,\hat{j}$

$\vec{r}_{C/B} = 1\,m\,\hat{i}$

$\vec{v}_C = \left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right) m/s$

$$\vec{v}_C = \left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right) m/s = -\omega_{AB}\hat{k} \times (1\,m\,\hat{j}) + \omega_{BC}\hat{k} \times (1\,m\,\hat{i})$$

$$= \omega_{AB}\hat{i} + \omega_{BC}\hat{j}$$

$\hat{i}: \quad \frac{1}{\sqrt{2}} = \omega_{AB}$

$\hat{j}: \quad \frac{1}{\sqrt{2}} = \omega_{BC}$

$$\boxed{\begin{aligned} \vec{\omega}_{AB} &= -\frac{1}{\sqrt{2}}\,rad/s\,\hat{k} \\ \vec{\omega}_{BC} &= \frac{1}{\sqrt{2}}\,rad/s\,\hat{k} \end{aligned}}$$

$$\vec{\alpha}_{AB} = \alpha_{AB}\hat{k}$$
$$\vec{\alpha}_{BC} = \alpha_{BC}\hat{k}$$

$$\vec{a}_C = \overbrace{\left(\vec{\alpha}_{AB}\times\vec{r}_{B/A} - \omega_{AB}^2\vec{r}_{B/A}\right)}^{\vec{a}_B} + \vec{\alpha}_{BC}\times\vec{r}_{C/B} - \omega_{BC}^2\vec{r}_{C/B}$$

(Example continued)

$$\cancel{\vec{a}_C}^{\,0} = \vec{a}_B + \vec{\alpha}_{BC}\times\vec{r}_{C/B} - \omega_{BC}^2\vec{r}_{C/B}$$

$$\vec{a}_B = \cancel{\vec{a}_A}^{\,0} + \vec{\alpha}_{AB}\times\vec{r}_{B/A} - \omega_{AB}^2\vec{r}_{B/A}$$

$\vec{r}_{B/A}$ $\vec{r}_{C/B}$

$$\vec{a}_C = 0 = \overbrace{\left(\alpha_{AB}\hat{k}\times 1\hat{j}\,m\right) - \left(\frac{-1}{\sqrt{2}}\frac{rad}{s}\right)^2 \cdot 1m\hat{j}}^{\vec{a}_B} + \left(\alpha_{BC}\hat{k}\times 1m\hat{i}\right) - \left(\frac{1}{\sqrt{2}}\frac{rad}{s}\right)^2 1m\hat{i}$$

$$= -\alpha_{AB}\hat{i} - \frac{1}{2}\hat{j} + \alpha_{BC}\hat{j} - \frac{1}{2}\hat{i}$$

components:

$\hat{i}: \; 0 = -\alpha_{AB} - \frac{1}{2} \Rightarrow \alpha_{AB} = -\frac{1}{2}\,rad/s^2$

$$\boxed{\vec{\alpha}_{AB} = -\frac{1}{2}\,rad/s^2\,\hat{k}}$$

$\hat{j}: \; 0 = -\frac{1}{2} + \alpha_{BC} \Rightarrow \alpha_{BC} = \frac{1}{2}\,rad/s^2$

$$\boxed{\vec{\alpha}_{BC} = \frac{1}{2}\,rad/s^2\,\hat{k}}$$