DYN L10 11

September 24, 2019 9:17 AM

8 L10/L11: Introduction to Rigid Body Kinetics

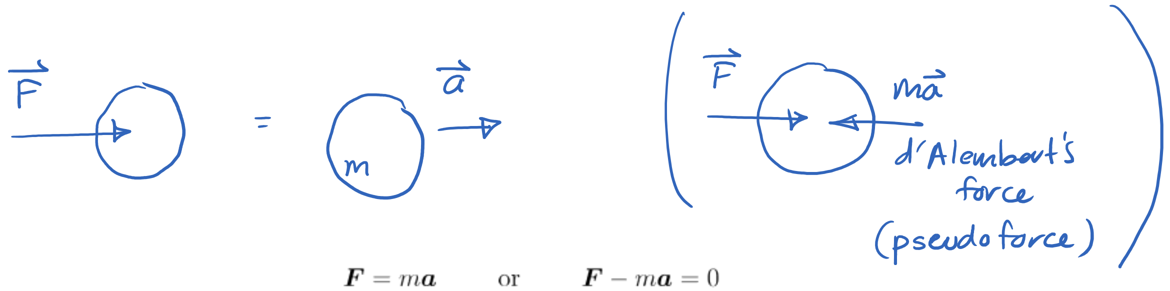
Readings

8.1 Objective

To apply Newton's Second Law to **planar** rigid bodies. This mean's that we need to account for the fact that the body be considered is NOT a point mass. We will use the concepts of Centre of Mass and Inertia, established in the previous lecture.

8.2 Review – Newton's Second Law

A particle acted on by an unbalanced force, $\boldsymbol{F}$, experiences an acceleration, $\boldsymbol{a}$, in the same direction as the force, and a magnitude that is directly proportional to the force.

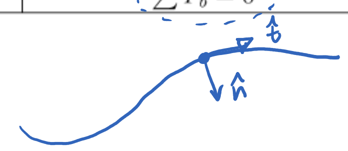


$$\boldsymbol{F} = m\boldsymbol{a} \qquad \text{or} \qquad \boldsymbol{F} - m\boldsymbol{a} = 0$$

$\boldsymbol{F}$ is a vector that represents the resultant of all forces acting on the body.

Thus, depending on the coordinate system selected, for a particle we would have the following set of equations:

Cartesian	Normal/Tangential	Polar/Cylindrical
$\sum F_x = ma_x$	$\sum F_n = ma_n$	$\sum F_r = ma_r$
$\sum F_y = ma_y$	$\sum F_t = ma_t$	$\sum F_\theta = ma_\theta$
$\sum F_z = ma_z$	$\sum F_b = 0$	$\sum F_z = ma_z$



no motion in "binormal" direction because particle only moving along path

$\hat{n} = f(x,y,z)$, $\hat{t} = f(x,y,z)$

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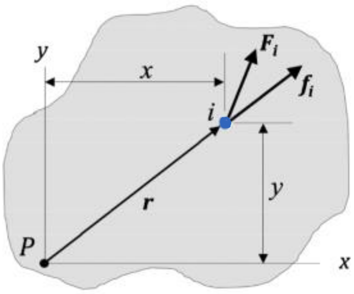
8.3 Equations of Motion for a Planar Rigid Body

Consider what this means for a planar body – which is a collection of particles.



resultant external force

resultant internal force (due to other particles in body)

For each particle on the body we have:

$$
\begin{aligned}
\text{particle 1 :}\quad & \boldsymbol{F_1} + \boldsymbol{f_1} = m\boldsymbol{a_1} \\
\text{particle 2 :}\quad & \boldsymbol{F_2} + \boldsymbol{f_2} = m\boldsymbol{a_2} \\
& \vdots \\
& \boldsymbol{F_n} + \boldsymbol{f_n} = m\boldsymbol{a_n} \\
\hline
& \sum_i \boldsymbol{F_{external}} + \sum_i \boldsymbol{f_{internal}} = \sum m_i \boldsymbol{a_i}
\end{aligned}
$$

Looking at each term:

BOOM!

$\sum_i \boldsymbol{f_{internal}}$ $= 0$ — Rigid body assumption - otherwise the body will explode

$\sum m_i \boldsymbol{a_i}$ $= m\vec{a}_G$ — Centre of mass equation:

$$\boldsymbol{r_G} = \frac{\sum_i \boldsymbol{r_i} \cancel{\Delta} m_i}{m}$$

$$\Rightarrow m\boldsymbol{r_G} = \sum_i \boldsymbol{r_i} \cancel{\Delta} m_i$$

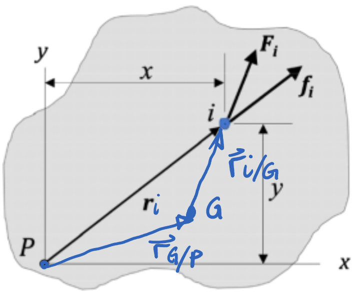
Differentiate twice:

$$\Rightarrow m\boldsymbol{a_G} = \sum_i \boldsymbol{a_i} \cancel{\Delta} m_i$$

Putting this together we have Newton's second law as applied to a rigid body. The important thing to note is where the acceleration is measured – AT THE CENTRE OF MASS.

$$\boxed{\sum \boldsymbol{F_{ext}} = m\boldsymbol{a_G}}$$ @ COG only

Now, lets look at the moment balance for this body. If we take moments about point P that is on the body:



$$\sum \boldsymbol{M_P} = \sum(\boldsymbol{r_i} \times \boldsymbol{F_{ext}}) = \sum(\boldsymbol{r_i} \times m_i\boldsymbol{a_i})$$

Let us reference this equation in terms of G, the centre of mass of the body.

$$\boldsymbol{r_i} = \boldsymbol{r_{G/P}} + \boldsymbol{r_{i/G}}$$ $\Rightarrow \vec{r}_{i/G} = \vec{r}_i - \vec{r}_G$

mult. by m_i, take sum

And we know from kinematics that at any point $\boldsymbol{a_i}$,

$$\sum_i m_i \vec{r}_{i/G} = \sum_i m_i (\underbrace{\vec{r}_i - \vec{r}_G}_{=0})$$

$$\boldsymbol{a_i} = \boldsymbol{a_G} + \boldsymbol{\alpha} \times \boldsymbol{r_{i/G}} - \omega^2\boldsymbol{r_{i/G}}$$

If we plug these last two equations into the RHS of our moment equation and simplify using the fact that,

$$\underbrace{\sum_i m_i\boldsymbol{r_{i/G}}}_{=0} = \sum_i m_i(\boldsymbol{r_i} - \boldsymbol{r_G}) = \sum_i m_i\boldsymbol{r_i} - m\boldsymbol{r_G} = 0$$

$\vec{r}_G = \vec{r}_{G/P}$

P is the origin

$\vec{r}_i = \vec{r}_{i/P}$

eqn' of centroid

$$\vec{r}_G = \frac{\sum m_i \vec{r}_i}{m}$$

$$m\vec{r}_G = \sum m_i \vec{r}_i$$

$$\sum m_i \vec{r}_i - m\vec{r}_G = 0$$

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we will get:

$$\sum \boldsymbol{M_P} = \sum_i m_i(\overbrace{\boldsymbol{r_{G/P}} + \boldsymbol{r_{i/G}}}^{\vec{r}_i}) \times (\overbrace{\boldsymbol{a_G} + \boldsymbol{\alpha} \times \boldsymbol{r_{i/G}} - \omega^2 \boldsymbol{r_{i/G}}}^{\vec{a}_i})$$

$$= \sum_i m_i[\boxed{\boldsymbol{r_{G/P}} \times \boldsymbol{a_G}} + \boldsymbol{r_{G/P}} \times (\boldsymbol{\alpha} \times \cancelto{0}{\boldsymbol{r_{i/G}}} - \omega^2 \cancelto{0}{\boldsymbol{r_{i/G}}}) + \boldsymbol{r_{i/G}} \times (\cancelto{0}{\boldsymbol{a_G}} + \boxed{\boldsymbol{\alpha} \times \boldsymbol{r_{i/G}}} - \omega^2 \cancelto{0}{\boldsymbol{r_{i/G}}})]$$

$\Sigma m_i \vec{r}_{i/G} = 0$

$\neq 0$

$\Sigma \underbrace{\vec{r}_{i/G} m_i}_{=0} \times \vec{a}_G = 0$

$\vec{r}_{i/G} \times \vec{r}_{i/G} = 0$

$$\boxed{\sum \boldsymbol{M_P} = m\boldsymbol{r_{G/P}} \times \boldsymbol{a_G} + \sum_i m_i \boldsymbol{r}_{i/G}^2 \boldsymbol{\alpha}}$$

The first part looks familiar – what we would expect if we simply multiplied our translational equation of motion by the torque arm around point P: $\boldsymbol{r_{G/P}} \times m\boldsymbol{a_G}$

The second part of this equation is the effect of the rotation of the entire rigid body – this is the part that relates to rotational mass – INERTIA

$$I_G = \sum_i m_i r_{i/G}^2$$

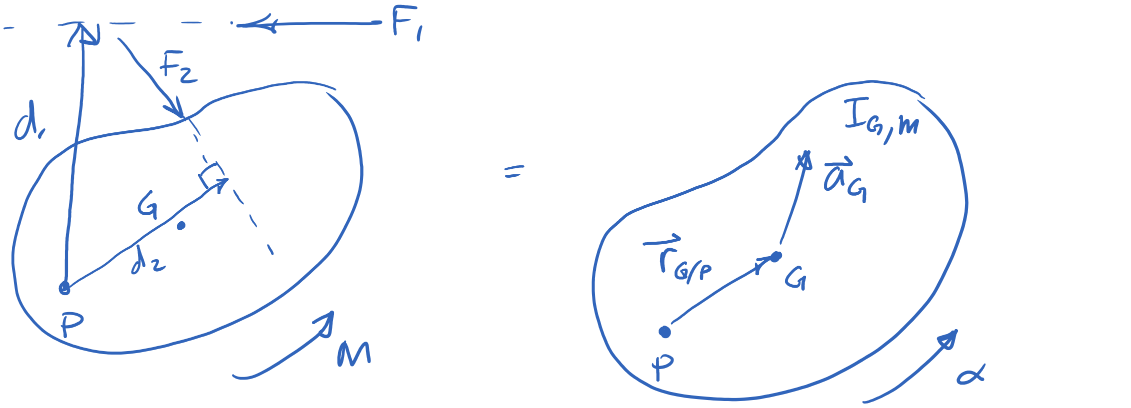
a_G & I_G @ COG.

So finally we have an equation for the effect of moments applied to a body, about a point P:

$$\boxed{\sum \boldsymbol{M_P} = m\boldsymbol{r_{G/P}} \times \boldsymbol{a_G} + I_G\boldsymbol{\alpha}}$$

$\Sigma M_G = I_G \alpha$

any point on rigid body



$$\Sigma M_P: \quad \vec{M} + \underbrace{\vec{d}_1 \times \vec{F}_1}_{F_1 d_1} + \underbrace{\vec{d}_2 \times \vec{F}_2}_{-F_2 d_2} = m(\vec{r}_{G/P} \times \vec{a}_G) + I_G \vec{\alpha}$$

8.4 Further Results

so far...

$$\sum \boldsymbol{F} = m\boldsymbol{a_G}$$

$$* \sum \boldsymbol{M_P} = m\boldsymbol{r_{G/P}} \times \boldsymbol{a_G} + I_G\boldsymbol{\alpha}$$

moment about P, but with terms wrt G

Applying our equation to measure the effect of Moments about at the Centre of Mass, G: $(\boldsymbol{r_{G/G}} = 0)$

if P = G

$$\boxed{\sum \boldsymbol{M_G} = I_G\boldsymbol{\alpha}}$$

For **other points on the rigid body**, we can also rewrite:

$$\boldsymbol{a_G} = \boldsymbol{a_P} - \omega^2\boldsymbol{r_{G/P}} + \boldsymbol{\alpha} \times \boldsymbol{r_{G/P}}$$

If we substitute this into our equation for $\sum \boldsymbol{M_P}$, we have:

$\vec{r}_{G/P} \times \vec{r}_{G/P} = 0$

$$\begin{aligned}\sum \boldsymbol{M_P} &= m\boldsymbol{r_{G/P}} \times (\boldsymbol{a_P} - \omega^2\boldsymbol{r_{G/P}} + \boldsymbol{\alpha} \times \boldsymbol{r_{G/P}}) + I_G\boldsymbol{\alpha} \\ &= m\boldsymbol{r_{G/P}} \times \boldsymbol{a_P} - m\omega^2(\boldsymbol{r_{G/P}} \times \boldsymbol{r_{G/P}}) + m\boldsymbol{r_{G/P}} \times (\boldsymbol{\alpha} \times \boldsymbol{r_{G/P}}) + I_G\boldsymbol{\alpha} \\ &= m\boldsymbol{r_{G/P}} \times \boldsymbol{a_P} + mr_{G/P}^2\boldsymbol{\alpha} + I_G\boldsymbol{\alpha}\end{aligned}$$

$$* \boxed{\sum \boldsymbol{M_P} = m\boldsymbol{r_{G/P}} \times \boldsymbol{a_P} + I_P\boldsymbol{\alpha}}$$

$I_G + m r_{G/P}^2 = I_P$

parallel axes

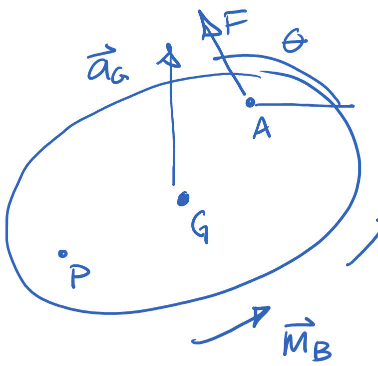
moment about P, expressed in terms of I_P and $\vec{a}_P$

Finally, if point P is PINNED (fixed to ground but body is allowed to ROTATE around P):

$\vec{a}_P = 0$ (pinned)

$$\boxed{\sum \boldsymbol{M_P} = I_P\boldsymbol{\alpha}}$$

pinned @ P



$* \quad \Sigma\vec{F} = m\vec{a}_G$ — always @ COG

$\Sigma F_x: \; F\cos\theta = m a_{Gx}$

$\Sigma F_y: \; F\sin\theta = m a_{Gy}$

$\vec{\alpha} \quad \Sigma M_G: \; \vec{M}_B + \vec{r}_{A/G} \times \vec{F} = I_G\vec{\alpha}$

OR

$* \quad \Sigma M_P: \; \vec{M}_B + \vec{r}_{A/P} \times \vec{F} = I_G\vec{\alpha} + m\vec{r}_{G/P} \times \vec{a}_G$

OR $= I_P\vec{\alpha} + m\vec{r}_{G/P} \times \vec{a}_P$

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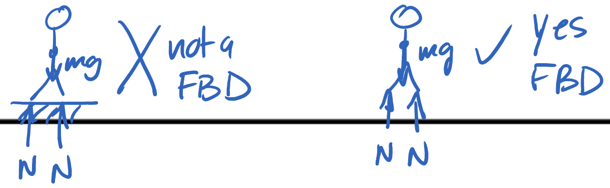


8.5 Method

FBD

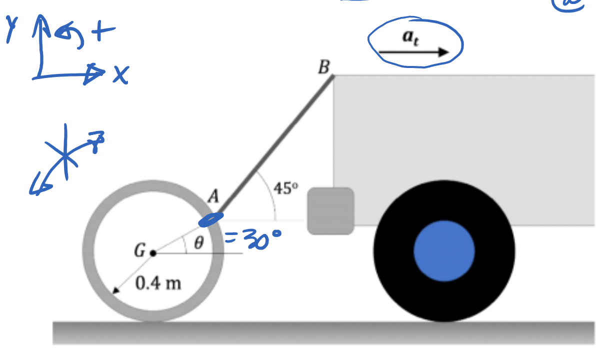
1. Draw the free body diagram for each separate body ← thinking
2. Write Equations of Motion
   - $\sum \boldsymbol{F} = m\boldsymbol{a}$ (for planar, two equations - in plane - such as $\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}$) 2
   - $\sum \boldsymbol{M}_G = I_G\boldsymbol{\alpha}$ (for planar, one equation – out of the plane $\hat{\boldsymbol{k}}$ coordinate) 1
   (can use a different moment equation if appropriate – i.e. for pinned rotation)

   $\Sigma M_P = \boxed{}$
3. Apply KINEMATIC CONSTRAINTS ← thinking
4. Solve

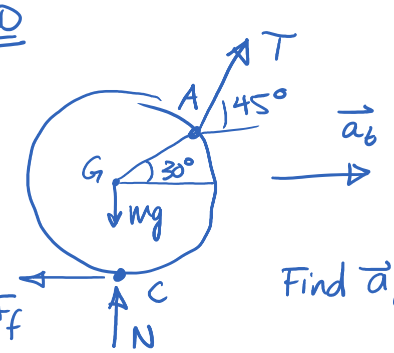
8.5.1 Example 1

The pipe has a mass of 800 kg and is being towed behind a truck. If the angle $\theta = 30°$, determine the acceleration of the truck and the tension in cable AB. The coefficient of kinetic friction between the pipe and the ground is $\mu_k = 0.1$.

@ steady state, dragging (not rolling)



FBD



Find $\vec{a}_t$ and $\vec{T}$

EOM

CoG

$\Sigma F_x: \; T\cos 45 - F_f = m a_{Gx}$ ①

$\Sigma F_y: \; T\cos 45 + N - mg = m \cancel{a_{Gy}} = 0$ ②

$\Sigma M_G: \; \vec{r}_{A/G} \times \vec{T} - \vec{r}_{C/G} \times \vec{F}_f = I_G \cancel{\alpha} = 0$

$\vec{r}_{A/G} = 0.4\left(\frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}\right)$ $\qquad \vec{F}_f = F_f(-\hat{\imath})$

$\vec{r}_{C/G} = 0.4(-\hat{\jmath})$ $\qquad \vec{T} = T\left(\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right)$

(Example continued)

$$\Sigma M_G: \quad 0.4T\left(\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right)\times\left(\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}\right)+(-0.4\hat{j})\times(-F_f\hat{i})=0$$

$$\Rightarrow 0.4T\left(\frac{\sqrt{3}}{2\sqrt{2}}\hat{k}-\frac{1}{2\sqrt{2}}\hat{k}\right)-0.4F_f\hat{k}=0 \quad (3)$$

$$F_f=\mu_k N$$

$$(2) \Rightarrow \quad N=mg-\frac{T}{\sqrt{2}}$$

$$F_f=\mu\left(mg-\frac{T}{\sqrt{2}}\right)$$

$$(2)+(3) \Rightarrow \quad 0.4T\left(\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)-0.4\overbrace{\mu\left(mg-\frac{T}{\sqrt{2}}\right)}^{F_f}=0$$

$$\Rightarrow T=2382\,N \quad (\text{mag.})$$

$$\boxed{\vec{T}=2382\left(\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}\right)N}$$

$$(1) \Rightarrow a_{Gx}=a_t=\frac{\frac{T}{\sqrt{2}}-F_f}{m}$$

$$(2) \Rightarrow F_f=\mu\left(mg-\frac{T}{\sqrt{2}}\right)=616\,N$$

$$\boxed{\vec{F}_f=-616\hat{i}\ N}$$

$$a_{Gx}=1.33\,m/s^2$$

$$\boxed{\vec{a}_t=1.33\hat{i}\ m/s^2}$$

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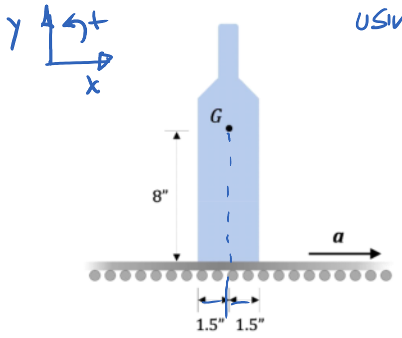
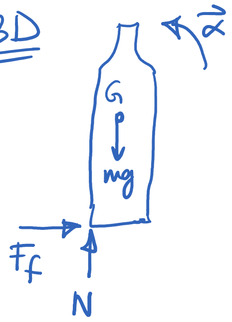
8.5.2 Example 2

The 2 lb bottle rests on a conveyor. If the coefficient of static friction is $\mu_s = 0.2$ determine the largest acceleration the conveyor can have without causing the bottle to slip or tip.

using μ_s for both condition (slip + tip)



FBD



NO TIP : $\vec{\alpha} = 0$

NO SLIP : $F_f \le \mu_s N$

EOM

ΣF_x : $F_f = m a_{Gx}$

ΣF_y : $N - mg = m \cancel{a_{Gy}}^{0} = 0 \Rightarrow N = mg$

ΣM_G : $-8\hat{j} \times F_f(\hat{i}) + (-1.5\hat{i}) \times N(\hat{j}) = I_G \cancel{\alpha}^{0} = 0$

SLIPPING : $F_f \le \mu_s mg$

$F_f \le 0.2 mg$

will TIP before it SLIPS

$\rightarrow$ use lower F_f to find $\vec{a}$

TIPPING : $8F_f - 1.5N = 0$

$\Rightarrow F_f = \frac{1.5}{8} N = 0.19 mg$ $\leftarrow$ can't exceed

$\Sigma F_x \Rightarrow F_f = \cancel{m} a_{Gx} \le 0.19 \cancel{m} g$

$\Rightarrow a_{Gx} \le 6.04 \ ft/s^2$

$$\boxed{\vec{a}_{max} = 6.04 \hat{i} \ ft/s^2}$$

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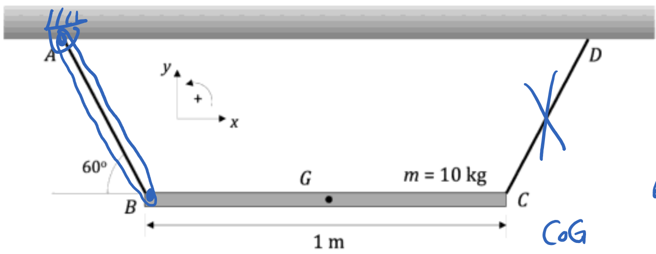
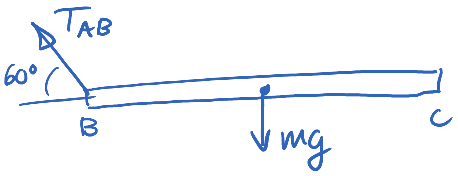
8.5.3 Example 3

Bar CD is supported by two cables AB and CD. Cable CD suddenly breaks. Find the angular acceleration of the bar, and the tension in the cable AB immediately after CD breaks. The mass of the bar is $10\,kg$ and is uniformly distributed over the length of $1\,m$.



FBD:



EOM:

$$\Sigma F_x: -T_{AB}\cos 60 = m a_x$$

$$\Sigma F_y: T_{AB}\sin 60 - mg = m a_y$$

$$\Sigma M_G: \underbrace{\left(-\tfrac{1}{2}\,m\,\hat{i}\right)}_{\vec{r}_{B/G}} \times T_{AB}\left(-\cos 60\,\hat{i} + \sin 60\,\hat{j}\right) = I_G\vec{\alpha}$$

$$\Rightarrow -\frac{T_{AB}}{2}\sin 60\,\hat{k} = I_G\,\alpha\,\hat{k}$$

(assume $\vec{\alpha} = \alpha\hat{k}$) $\vec{\alpha}$

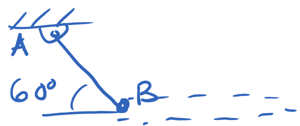
unknowns: $a_{Gx}, a_{Gy}, \alpha, T_{AB}$ [4]

eqns: 3

$\Rightarrow$ NEED ANOTHER EQN

KINEMATIC CONSTRAINT

Taut (tight) cable acts like a rigid body



$\vec{\alpha}_{AB} = \alpha_{AB}\hat{k}$

$$\vec{a}_B = \cancelto{0}{\vec{a}_A} + \vec{\alpha}_{AB}\times\vec{r}_{B/A} - \cancelto{0}{\omega_{AB}^2}\,\vec{r}_{B/A}$$

(starts from rest)

$$= \alpha_{AB}\hat{k}\times r_{B/A}\left(\tfrac{1}{2}\hat{i} - \tfrac{\sqrt{3}}{2}\hat{j}\right)$$

$$= \underbrace{\alpha_{AB}\, r_{B/A}}_{a_B = |\vec{a}_B|}\ \underbrace{\left(\tfrac{\sqrt{3}}{2}\hat{i} + \tfrac{1}{2}\hat{j}\right)}_{\text{unit vector dir. of } \vec{a}_B}$$

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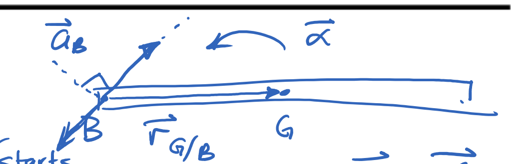
(Example continued)



$\vec\alpha = \vec\alpha_{BC}$

ANOTHER EQN USING $\vec a_B$

$$\vec a_G = \vec a_B + \vec\alpha_{BC}\times\vec r_{G/B} - \underbrace{\omega_{BC}^2}_{0 \text{ (starts from rest)}}\vec r_{G/B}$$

$$= a_B\left(\frac{\sqrt3}{2}\hat i + \frac12\hat j\right) + \alpha_{BC}\hat k\times\left(\frac12\hat i\right)$$

$$= a_B\left(\frac{\sqrt3}{2}\hat i + \frac12\hat j\right) + \frac{\alpha_{BC}}{2}\hat j = a_{Gx}\hat i + a_{Gy}\hat j$$

Two eqns ($\hat i$, $\hat j$)

add one unknown (a_B)

$$\Rightarrow \hat i:\ a_{Gx} = a_B\frac{\sqrt3}{2} \quad (4)$$

$$\hat j:\ a_{Gy} = \frac{a_B}{2} + \frac{\alpha}{2} \quad (5)$$

5 equations, 5 unknowns ($a_{Gx}, a_{Gy}, \alpha, T_{AB}, a_B$)

SOLVE

(1) + (4) $\Rightarrow -T_{AB}\cdot\frac12 = m a_B\frac{\sqrt3}{2} \Rightarrow T_{AB} = -m a_B\sqrt3$

(2) + (5) $\Rightarrow -mg + T_{AB}\frac{\sqrt3}{2} = m\left(\frac{a_B}{2} + \frac{\alpha}{2}\right)$

sub in $T_{AB} \Rightarrow -\cancel{m}g + (-\cancel{m}a_B\sqrt3)\frac{\sqrt3}{2} = \cancel{m}\left(\frac{a_B}{2} + \frac{\alpha}{2}\right)$

$$-g - 2a_B = \frac{\alpha}{2} \quad (6)$$

(3) $\Rightarrow -T_{AB}\frac{\sqrt3}{4} = I_G\,\alpha$ $\qquad I_G = \frac{1}{12}m l^2 = \frac{1}{12}m(1)^2 = \frac{m}{12}$

sub in $T_{AB} \Rightarrow \cancel{-}(\cancel{-}\cancel{m}a_B\sqrt3)\frac{\sqrt3}{\cancel{4}} = \frac{\cancel{m}}{\cancel{12}_3}\alpha \Rightarrow a_B = \frac{\alpha}{9}$

(6) $\Rightarrow -g - 2\left(\frac{\alpha}{9}\right) = \frac{\alpha}{2} \Rightarrow \alpha = -\frac{18}{13}g = -\frac{180}{13}\,\text{rad/s}^2 \Rightarrow \boxed{\vec\alpha = -\frac{180}{13}\,\text{rad/s}^2\,\hat k}$

$a_B = \frac{\alpha}{9} = -\frac{\cancel{180}^{20}}{13}\cdot\frac{1}{\cancel{9}} = -\frac{20}{13}\,\text{m/s}^2$, $\vec a_B = -\frac{20}{13}\left(\frac{\sqrt3}{2}\hat i + \frac12\hat j\right)\text{m/s}^2$

$\therefore T_{AB} = -m a_B\sqrt3 = -10\,\text{kg}\left(-\frac{20}{13}\,\text{m/s}^2\right)\sqrt3 = \frac{200\sqrt3}{13}\,\text{N}$

$$\boxed{\vec T_{AB} = \frac{200\sqrt3}{13}\left(-\frac12\hat i + \frac{\sqrt3}{2}\hat j\right)\text{N}}$$

$T_{AB} = \frac{[illegible]}{13}\left(-\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)N$

8.6 Rotation About a Fixed Axis

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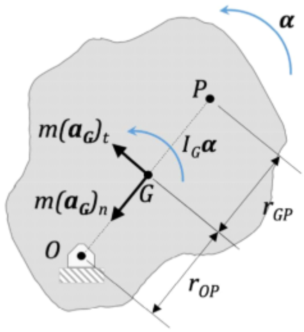
If a body is rotating about a fixed point (**pinned**) at O, then we can use this **kinematic constraint** to help solve the problem.

The equations of motion for any body are:

$$\sum \boldsymbol{F} = m\boldsymbol{a_G}$$
$$\sum \boldsymbol{M_G} = I_G\boldsymbol{\alpha}$$



$\vec{a}_0 = 0$

However, for this particular case we can use kinematics to write $\boldsymbol{a_G}$ in terms of vectors parallel and perpendicular to the line between the pin and the centre of mass, OG:

$$\boldsymbol{a_G} = \boldsymbol{\alpha} \times \boldsymbol{r_{G/O}} - \omega^2\boldsymbol{r_{G/O}}$$

Thus:

$$\sum \boldsymbol{F} = m\boldsymbol{\alpha} \times \boldsymbol{r_{G/O}} - m\omega^2\boldsymbol{r_{G/O}}$$

As well, we can use the equation for motion due to moments about a **pinned point**:

$$\sum \boldsymbol{M_O} = I_O\boldsymbol{\alpha}$$

parallel axes

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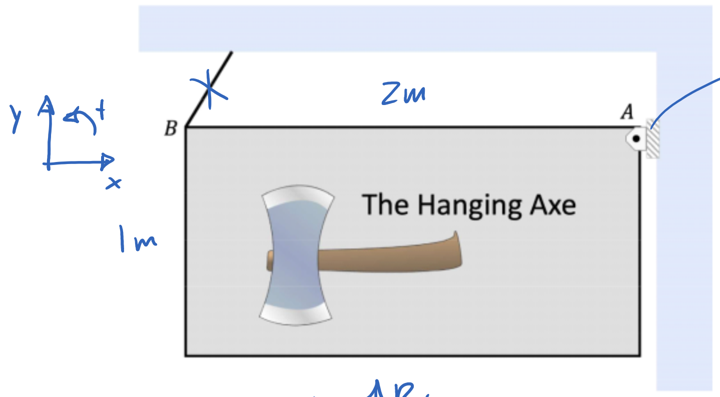
8.6.1 Example

The cable at point B supporting the $2\,m$ by $1\,m$ sign (mass $10\,kg$) is suddenly broken. Find the angular acceleration of the sign and the reaction forces at point A just after the cable breaks.



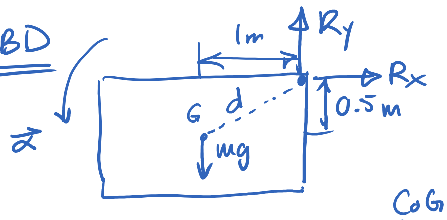
pinned

$\vec{a}_A = 0$

$\vec{\omega} = 0$ (starts from rest)

(different from

FBD



$I_A = I_G + md^2$

$= \frac{1}{12}m(2^2+1^2) + m(1^2+0.5^2)$

$= \frac{20}{12}\,m$

EOM

$\Sigma F_x:\ R_x = m a_{Gx}$ ① (CoG)

$\Sigma F_y:\ R_y - mg = m a_{Gy}$ ②

$\Sigma M_A:\ mg(1m) = I_A \alpha$ ③

unknowns: $\alpha, a_{Gx}, a_{Gy}, R_x, R_y$ (5)

3 eqns

KIN CONSTRAINTS

$\vec{a}_G = \cancel{\vec{a}_A}^{\,0} + \vec{\alpha} \times \vec{r}_{G/A} - \cancel{\omega}^{\,0\,2}\, \vec{r}_{G/A}$ (starts from rest)

$= \alpha\hat{k} \times (-1\hat{i} - 0.5\hat{j})$

$= \underbrace{-\alpha\hat{j}}_{=a_{Gy}} + \underbrace{0.5\alpha\,\hat{i}}_{=a_{Gx}}$ } 2 more eqns ④

SOLVE

③ $\Rightarrow \alpha = \frac{mg}{I_A} = \frac{\cancel{m}g}{\frac{20}{12}\cancel{m}} = \frac{12}{20}g$ $\boxed{\vec{\alpha} = \frac{3}{5}g\ \mathrm{rad/s^2}\ \hat{k}}$

①+④ $R_x = m\left(\underbrace{0.5\left(\frac{3}{5}g\right)}_{a_{Gx}}\right) = \frac{3}{10}mg$ ②+④ $\Rightarrow R_y = mg + m\underbrace{\left(\frac{-3}{5}g\right)}_{=-\alpha = a_{Gy}} = \frac{2}{5}mg$

$\Rightarrow \vec{R} = \frac{3}{10}mg\,\hat{i} + \frac{2}{5}mg\,\hat{j} = 30\,N\,\hat{i} + 40\,N$