DYN L2

September 19, 2019 3:36 PM

2 L2: Relative Plane Motion Analysis - Velocity

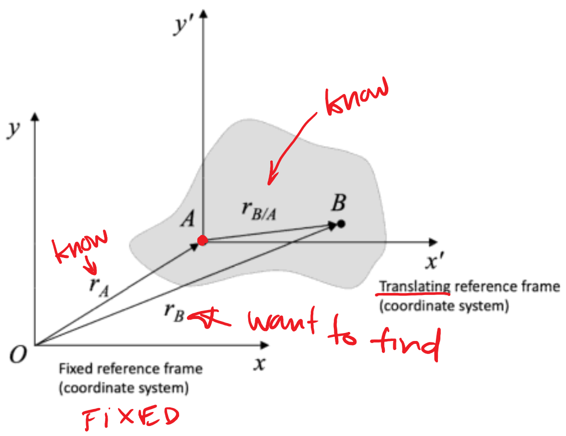
Readings

2.1 Objective

To describe the motion of a point on a rigid body using a translating reference frame attached to the body.

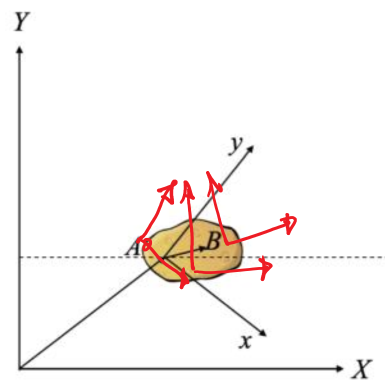
2.2 Fixed and Translating Reference Frames

In this section we consider a frame $x'y'$ attached to a body at a point A as if by a pin – i.e.: translating but NOT rotating with the body. xy is a fixed frame (attached to ground).



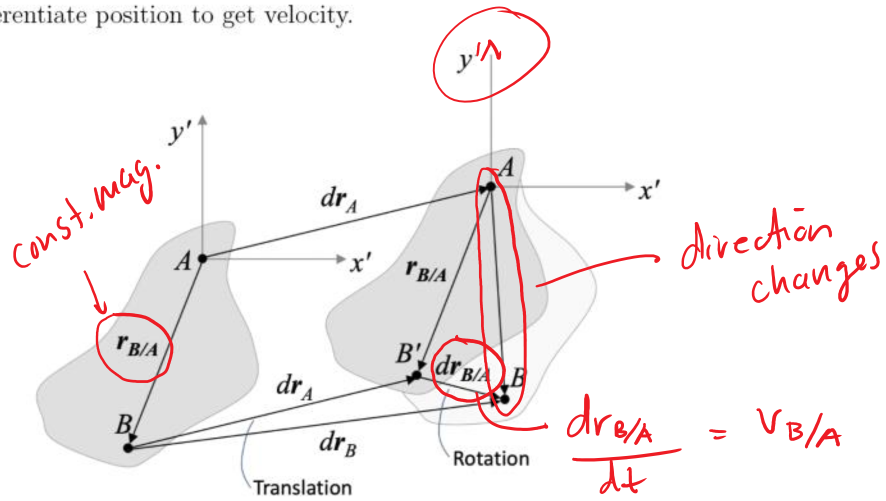
r_A	+	$r_{B/A}$	=	r_B
Describes the location of the translating reference frame $x'y'$ with respect to the fixed xy frame	+	Describes the location of point B with respect to translating reference frame $x'y'$	=	Describes the location of point B with respect to the fixed frame xy

IMPORTANT NOTE: "with respect to" means "measured from". BUT we must make sure we are using the **same axis directions** to do the measurements.



choose a point for translating frame that has known motion

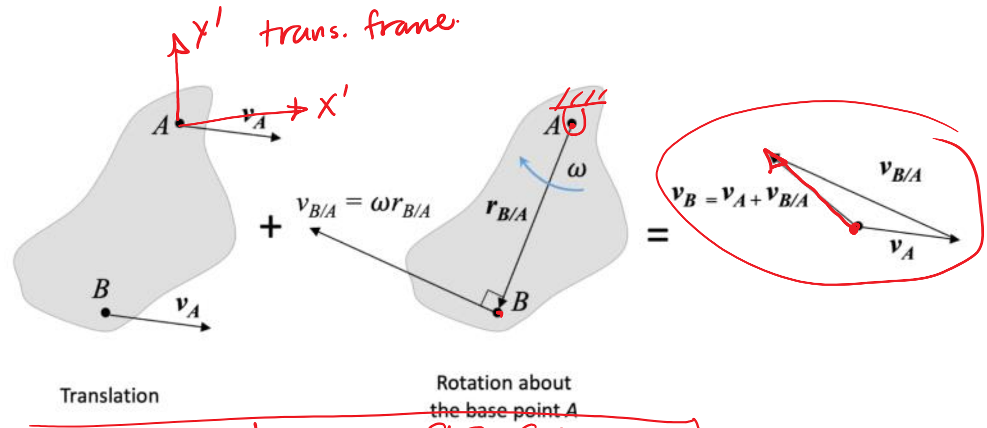
As before, we differentiate position to get velocity.



Since the body is rigid (i.e. does not change size), the magnitude of $\boldsymbol{r}_{B/A}$ must be constant. Therefore, $\boldsymbol{v}_{B/A} = \dfrac{d\boldsymbol{r}_{B/A}}{dt}$ is due only to the **rotation** of point B about point A. That is, the motion follows Chasles' theorem:

change in dir.

$$\boxed{\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{\omega} \times \boldsymbol{r}_{B/A}}$$



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2.2.1 Example

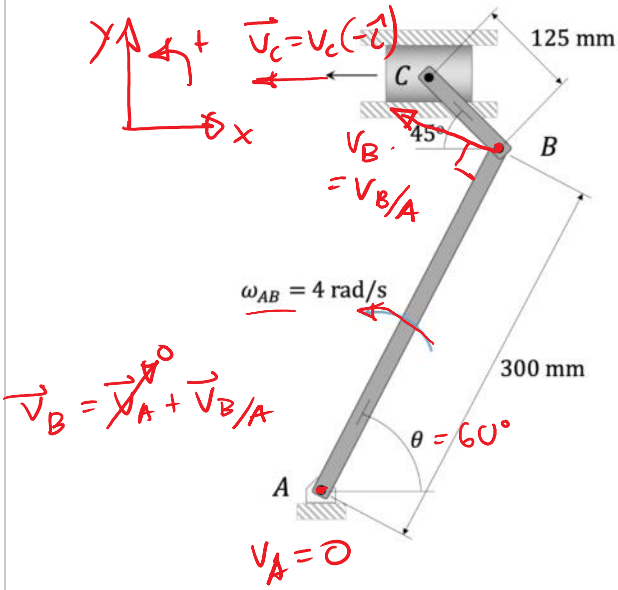
Find ω_{BC} and v_C at the instant shown. $\theta = 60°$.

1. choose coord.
2. draw vector diagram

* KINEMATIC CONSTRAINTS *

3. write eqns
4. solve

use your brain



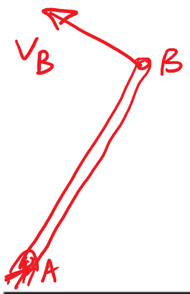
Given:

$r_{AB} = 0.3\text{m}$

$r_{BC} = 0.125\text{m}$

$\theta = 60°$

$\vec{\omega}_{AB} = 4\text{ rad } \hat{k}$



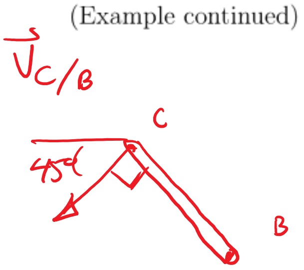
$$\vec{v}_B = \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$\|\vec{v}_B\| = |\vec{\omega}_{AB}|\,|\vec{r}_{B/A}|$$

$$= |4|\,|0.3|\ \text{m/s}$$

$$= 1.2\text{m/s}$$

(Example continued)

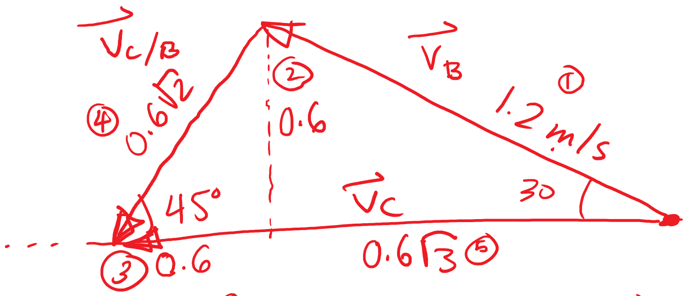


$$\vec{V}_{C/B} = \vec{\omega}_{BC} \times \vec{r}_{C/B}$$

?

$$\vec{V}_C = \vec{V}_B + \vec{V}_{C/B} = \vec{V}_B + \vec{\omega}_{BC} \times \vec{r}_{C/B} = V_C(-\hat{i})$$

BY VECTOR DIAGRAM:



$\cos 30 = \frac{\sqrt{3}}{2}$

$\sin 30 = \frac{1}{2}$

$$\vec{V}_C = \overset{③}{0.6} + \overset{⑤}{0.6\sqrt{3}} = 1.64(-\hat{i})\,\text{m/s}$$

$$|\vec{V}_{C/B}| = 0.6\sqrt{2} = |\vec{\omega}_{BC}||\vec{r}_{C/B}|$$

$$|\vec{\omega}_{BC}| = \frac{0.6\sqrt{2}\ \text{m/s}}{0.125\,\text{m}} = 6.7\,\text{rad/s}\ \circlearrowleft$$

BY MATH/SYS. OF EQNS

($\vec{V}_A = 0$, pinned)

$$\vec{V}_C = V_C\hat{i} = \vec{V}_B + \vec{V}_{C/B} = \vec{\omega}_{AB} \times \vec{r}_{B/A} + \vec{\omega}_{BC} \times \vec{r}_{C/B}$$

$$V_C\hat{i} = \underbrace{4\,\text{rad/s}\,\hat{k}}_{\vec{\omega}_{AB}} \times \underbrace{0.3\,\text{m}\left(\tfrac{1}{2}\hat{i} + \tfrac{\sqrt{3}}{2}\hat{j}\right)}_{\vec{r}_{B/A}}$$

$$+\ \omega_{BC}\hat{k} \times 0.125\,\text{m}\left(-\tfrac{1}{\sqrt{2}}\hat{i} + \tfrac{1}{\sqrt{2}}\hat{j}\right)$$

$$V_C\hat{i} = \left[-0.6\sqrt{3}\,\hat{i} + 0.6\,\hat{j} - \omega_{BC}\left(\frac{0.125}{\sqrt{2}}\right)\hat{i} - \omega_{BC}\left(\frac{0.125}{\sqrt{2}}\right)\hat{j}\right]$$

By components:

$\hat{i}$: $V_C = -0.6\sqrt{3} - \omega_{BC}\left(\frac{0.125}{\sqrt{2}}\right)$ m/s (A)

$\hat{j}$: $0 = 0.6 - \omega_{BC}\left(\frac{0.125}{\sqrt{2}}\right)$ m/s (B)

B $\omega_{BC} = \dfrac{0.6\sqrt{2}}{0.125} = 6.7\ \text{rad/s}$ $\vec{\omega}_{BC} = 6.7\ \text{rad/s}\ \hat{k}$

$(\vec{\omega}_{BC} = \omega_{BC}\hat{k})$

A : $V_C = -0.6\sqrt{3} - 0.6\ \text{m/s} = -1.64\ \text{m/s}$

$\vec{V}_C = -1.64\ \text{m/s}\ \hat{\imath}$