DYN L23

September 24, 2019 9:21 AM

15 L23: Principle of Momentum and Impulse

Reading

15.1 Objectives

In this section, we will:

- Review the Principle of Impulse and Momentum and apply it to a rigid body for both linear and angular motion.
- Review conservation of linear and angular momentum and solve rigid body motion problems using this equation.
- Apply the principles of Impulse and Momentum to Impacts.

15.2 Context

So far we have looked at linear and angular relationships in the following contexts:

$\vec{v} = \frac{d\vec{x}}{dt}$

$\vec{a} = \frac{d\vec{v}}{dt}$

$a = \frac{dv}{dx} \cdot \frac{dx}{dt}$

$\Rightarrow a\,dx = v\,dv$

$\omega = \frac{d\theta}{dt}$

$\alpha = \frac{d\omega}{dt}$

$\alpha = \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt}$

$\Rightarrow \alpha\,d\theta = \omega\,d\omega$

Kinematics:

$\boldsymbol{x},\ \boldsymbol{v},\ \boldsymbol{a},\ t \qquad\qquad \boldsymbol{\theta},\ \boldsymbol{\omega},\ \boldsymbol{\alpha},\ t$

Newton's second law:

$\boldsymbol{F} = m\boldsymbol{a_g}$ or $\boldsymbol{F} - m\boldsymbol{a_g} = 0$ $\qquad\qquad$ $\boldsymbol{M} = I_G\boldsymbol{\alpha}$ or $\boldsymbol{M} - I_G\boldsymbol{\alpha} = 0$

Work and Energy:

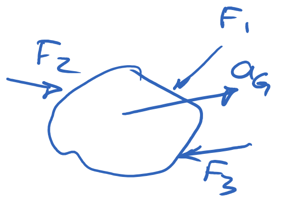
(Integrate NSL relationships over distance → SCALAR relationships

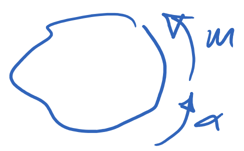
$T_{linear} = \frac{1}{2}mv_G^2 \qquad\qquad T_{rotational} = \frac{1}{2}I_G\omega^2$

$T_1 + V_1 = T_2 + V_2 = \text{constant}$ (cons. of energy)

$T_1 + V_1 + \underset{\text{non-cons.}}{\sum U_{1\to 2}} = T_2 + V_2$ (work-energy)

$U = \int (\vec{F} = m\vec{a})\,d\vec{s}$





15.3 Impulse and Momentum Definitions

For impulse and momentum, we integrate NSL relationships over time → VECTOR relationships:

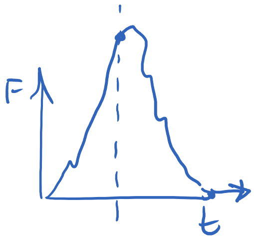
Definitions:

Linear impulse

$$= \text{force applied over time, } \Delta t \text{ (for constant mass)}$$
$$= \sum \int_{t_1}^{t_2} \boldsymbol{F} dt = \int_{t_1}^{t_2} m\boldsymbol{a_G} dt$$
$$= m(\boldsymbol{v_{G_2}} - \boldsymbol{v_{G_1}})$$
$$\Rightarrow m\boldsymbol{v_{G_1}} + \underbrace{\sum \int_{t_1}^{t_2} \boldsymbol{F} dt}_{\text{linear impulse}} = m\boldsymbol{v_{G_2}}$$

$\vec{F}$ is an "impulsive force"

Angular impulse



$$= \text{moment applied over time, } \Delta t \text{ (for constant inertia):}$$
$$= \sum \int_{t_1}^{t_2} \boldsymbol{M_G} dt = \int_{t_1}^{t_2} I_G \boldsymbol{\alpha} dt$$
$$= I_G(\boldsymbol{\omega_2} - \boldsymbol{\omega_1})$$
$$\Rightarrow I_G\boldsymbol{\omega_1} + \underbrace{\sum \int_{t_1}^{t_2} \boldsymbol{M_G} dt}_{\text{angular impulse}} = I_G\boldsymbol{\omega_2}$$

$\vec{M}$ is an "impulsive moment"

Linear momentum: vector $\boldsymbol{L} = m\boldsymbol{v_G}$ always CoG

Angular momentum: $\boldsymbol{H_G} = I_G\boldsymbol{\omega}$ CoG but other versions coming...

If there is **no linear impulse**, i.e., $\sum \int_{t_1}^{t_2} \boldsymbol{F} dt = 0$, then $\boldsymbol{L}$ is constant $\Rightarrow$ conservation of linear momentum:

$$\Rightarrow \boldsymbol{L} = m\boldsymbol{v_{G_1}} = m\boldsymbol{v_{G_2}}$$

Cons. of linear momentum (Σx, Σy)

(This is Newton's first law)

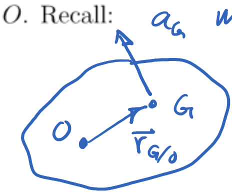
If there is **no angular impulse** i.e., $\sum \int_{t_1}^{t_2} \boldsymbol{M_G} dt = 0$, then $\boldsymbol{H_G}$ is constant $\Rightarrow$ conservation of angular momentum:

$$\Rightarrow \boldsymbol{H_G} = I_G\boldsymbol{\omega_1} = I_G\boldsymbol{\omega_2}$$

Often it is convenient to compute angular momentum about a point O. Recall:

$m\vec{a}_G = \Sigma \vec{F}$

$$\sum \boldsymbol{F} = m\boldsymbol{a_G}$$
$$\sum \boldsymbol{M_G} = I_G\boldsymbol{\alpha}$$



This can be written to find the equivalent moments about another point, O:

$$\sum \boldsymbol{M_O} = \sum \boldsymbol{M_G} + \underline{\boldsymbol{r_{G/O}} \times \sum \boldsymbol{F}}$$
$$\sum \boldsymbol{M_O} = \underline{I_G}\boldsymbol{\alpha} + \underbrace{\boldsymbol{r_{G/O}}}\times m\boldsymbol{a_G}$$

for any pt O on the body

familiar

We can again integrate with respect to time and show that:

$$\boldsymbol{H_O} = I_G\boldsymbol{\omega} + \boldsymbol{r_{G/O}} \times m\boldsymbol{v_G}$$
$$\Rightarrow \boldsymbol{H_O} = \boldsymbol{H_G} + \boldsymbol{r_{G/O}} \times \boldsymbol{L}$$

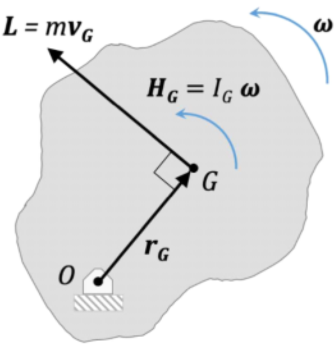
always CoG

ang. mom. about O

angular mom. about G

linear momentum

If point O is a **fixed point**:



$$\boldsymbol{v_O} = 0$$

$$\boldsymbol{H_O} = I_G\boldsymbol{\omega} + m\left[\boldsymbol{r_{G/O}} \times (\boldsymbol{\omega} \times \boldsymbol{r_{G/O}})\right]$$
$$= \left(\underbrace{I_G + mr^2_{G/O}}_{\text{parallel axes}}\right)\boldsymbol{\omega}$$
$$\Rightarrow \underline{\boldsymbol{H_O} = I_O\boldsymbol{\omega}}$$

$(\Sigma M_O = I_O\alpha)$ fixed pt.

ONLY IF PINNED (ICZV)

15.4 Principle of Impulse and Momentum for a System

The equations for impulse and moment can be applied to a system of connected bodies. This eliminates the need to include the reactive impulses (which cancel each other out – equal and opposite).

$\Sigma \vec{L}_{sys} = \vec{L}_1 + \vec{L}_2 + \cdots$

Thus one can write:

state 1 external state 2

$$\left(\sum \boldsymbol{L_{sys}}\right)_{t1} + \left(\sum \int_{t1}^{t2} \boldsymbol{F_{ext,sys}}dt\right) = \left(\sum \boldsymbol{L_{sys}}\right)_{t2}$$
$$\left(\sum \boldsymbol{H_{O,sys}}\right)_{t1} + \left(\sum \int_{t1}^{t2} \boldsymbol{M_{O,ext,sys}}dt\right) = \left(\sum \boldsymbol{H_{O,sys}}\right)_{t2}$$

$\Sigma H_{O\,sys} = H_{O1} + H_{O2} + \cdots$

external