Andy Dequin, Brandon Wong, Brooke Rethman, Jose Betances, Alexandra Valdepeñas

MPS 2: FACT 2 - Axis AFM Stage

All equations and variables can be found at: Functional Requirements MPS 2 FACT

Functional Requirements

Name	Description	Value	Range	Justification
Screw motion 1	Amount that will rotate per 5 degrees with constraint 1	3 mm/ 5 deg	± 2 mm / 5 deg	Enough translation so that the coupling between rotation and translation is visible. values are not a strict requirement but motion 1 and 2 should be significantly different
Screw motion 2	Amount that will rotate per 5 degrees with constraint 2	9 mm / 5 deg	± 2 mm / 5 deg	Enough translation so that the difference of the screw motion between constraint 1 and 2 is visible
Translational Stiffness - Constrained	Translational Stiffness in constrained translational DOFs	111,250 N/m	± 1112 N/m	5lbf moving it 0.2mm
Torsional Stiffness - Constrained	Torsional Stiffness in constrained torsional DOFs	350 Nm/rad	± 35 Nm/rad	Torque to turn a doorknob is about 3 Nm, rotating it 0.5 degrees
Center Axis translational Alignment	Center should stay aligned during screw motion	0	± 2 mm / 5 deg	5x Eye resolution when using a ruler. Same range as screw motions.
Center Axis rotational Alignment	Center should stay aligned during screw motion	0	±2 deg/ 5 deg	4x Visible change in angle and 40% of screw motion. As long as the unwanted motion is less than half the main rotation then the prototype concept should be clear
Max stress limit	Maximum stress the flexures can experience under loads due to people interacting with device	72 MPa	N/A	Flexure strength before breaking.
Range of Motion: Rotary	Amount the top platform of the device is able to rotate	5 deg	± 1 deg	Rotate enough to show screw motion
Range of Motion: Linear	Amount the top platform of the device is able to translate	5mm	± 2 mm	Translate enough to show screw motion

Pitch

The pitch is directly related to the angle θ that the "Z" flexure makes with the horizontal. When the upper plate is displaced δ_x, and neglecting the change in length of the Z flexure, the end of the flexure will also move δ_x in the x direction and $\delta_y = \frac{\delta_y}{tan(\theta)}$ in the y direction. δ_y can be related to the screw angle Φ by $\delta_y = 0.5\Phi W$. Rearranging, the pitch is $\frac{\delta_y}{\Phi} = 0.5W tan(\theta)$

Torsional stiffness in y and z

For torsional stiffness in z, we used $k = \frac{GJ}{L}$, where k represents the torsional stiffness in the z direction, G is the shear modulus (modulus of rigidity), J is the polar moment of inertia, and L is the length of the beam. We modeled the system as being supported by 5 beams, assuming that torsion only impacted the z components of the beams.

For torsional stiffness in y, we used $k = \frac{T}{\theta}$, where T represent torque and θ is the rotation in y. From the torque, we got the effective force on each beam, we can then model the elongation of both beams to relate force to the angle, and finally torque to the angle.

Translation and torsional stiffness in x

The stiffness in x was obtained by modeling the effective beams at the ends of the plate. We assumed we apply a force Fx to each end and can find the displacement of the ends of the beam by the beam bending equation: $\delta = \frac{FL^3}{3EI}$

Since stiffness is force divided by displacement, we can easily calculate stiffness using the previous equation.

For torsional stiffness in x, we again used $k = \frac{T}{\theta}$. To get torque, we are assuming that the biggest contribution to torque is the reaction force from bending applied from the Z flexure to the upper plate. This force can be calculated by the displacement of the flexure. This displacement is directly related to the translational displacement of the whole plate using a series of geometric relations between various displacements. θ can also be related to these displacements and we directly find a relationship between T and θ that gives us torsional stiffness.

Translational Stiffness in y and z

For translational stiffness in z, we used $k = \frac{FL}{EA}$ to get the stiffness in the individual beams which were then summed to get $k = \frac{4Eta}{\sqrt{H^2 + (w/2)^2}} + \frac{EtwH}{sin(\theta)}$ where the first component is the stiffness due to the two A-frame pieces and the second component is due to the pitch-modifying attachment.

For translational stiffness in y, we used $k = \frac{FL}{EA}$ for the 4 beams that make up the two A frames in the structure. For the translational stiffness in y in the pitch-modifying attachment we used $k = \frac{FL^3}{12EI}$. Summed together, this gives a total translational stiffness in y of $k = \frac{Ewt^3(sin\theta)^3}{H^3} + \frac{4Eat}{\sqrt{H^2 + (w/2)^2}}$.

Experimental verification

We placed a level on the top plate and lined it up with a ruler. Then, for each Z flexure, we displaced the top plate forward until the bubble in the level moved from the center section to the left section.

For the longer flexure, the displacement was around 1/16in. For the shorter flexure, the displacement needed for the same tilt angle was 3/16in.