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MPS Four Writeup

Functional Requirements

Name	**Description**	**Numbers (value and range)**	**Justification**
Negligible mass for beams	The mass of the mass at the center should be ~10 times greater than the mass of the flexures	10:1	By having the mass be heavier than the flexures, we can neglect their mass in calculations and they won't have a significant effect on the overall motion of the energy harvester.
Natural Frequency	The natural frequency that our system should be able to operate at given the nodes are at a certain angle.	0.015-30 Hz	The device will be tested by shaking it by hand. It's reasonable for a human to comfortably shake something at a rate of about 3 Hz. We want our device to perform optimally over a range of 2000x, so having a safety factor of 1 on our upper frequency yields this range.
Flexure stiffness in xy	The stiffness of the flexures in the xy direction	< 1500 N/m	We calculated they need to be 1000 N/m to not break given a maximum force of 10N from the person testing and with a displacement of 1 cm. We then applied a safety factor of 1.5
Max stress	Max stress at any flexure	< 2.9 x 10^7 Pa	Given by the max stress for ABS

Stress

We have that the maximum stress occurs at the end of the flexures and it is given by:

$$S_{max} = Mc/2I$$

We calculated the force at each node and then the maximum stress at each flexure. We made sure that all stress was lower than the maximum stress for ABS (2.9 x 10^7 Pa).

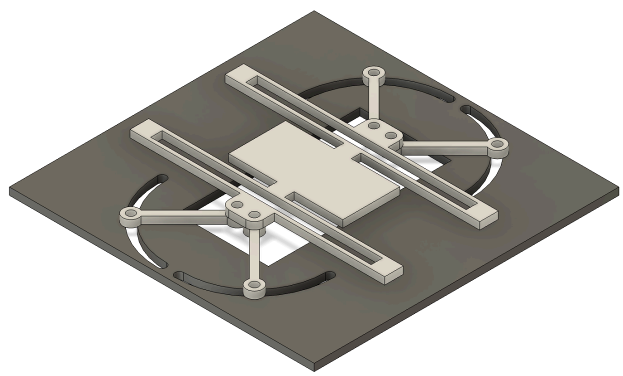
Modeling

To model our system, we used the stiffness matrix technique. For each beam, we used the rotational matrix transformations to convert a 6 DOF beam stiffness matrix from local coordinates to the global coordinate system. This gave us a 33-by-33 matrix, K. From there we could reduce it to 21-by-21 because the 4 nodes with variable angles have fixed displacements. From there we could use the inverse of K and the known force, F, which we planned to apply at the mass, to solve for the displacements at all remaining nodes. This allowed us to solve for effective stiffness, k, at the mass in the vertical direction by computing F over the vertical displacement at node 6.

The next step was to iterate over our design space to see which parameters gave the largest range of k. We used a bound of 0-90 degrees for the 4 angles we could change. For the beams we knew the width-to-length ratio should greatly favor length, so we used bounds of ⅛ -½ inches for width and 2-5 inches for length. We compared sets of parameters by looking at the maximum difference of possible k values.

Unfortunately, the lowest possible k value we could achieve was $8.6 * 10^5$ N/m. This means the lowest w_n we could achieve is around $2.1 * 10^4$ Hz. The upper bound was $1.1 * 10^6$ N/m for k. This means our range of w_n is only about plus or minus 4000 Hz. This is not the 2000x range we were trying to achieve. (We would need a range on k of about plus or minus 4 million N/m).

These calculations were done in MATLAB. CAD was done in Fusion 360 and used for modeling.



Experimental Verification of Performance

To verify our model works, we can apply an impulse to the system (configured at a specified top and bottom angle). The frequency that the mass then shakes at is the natural frequency of the system for that set of angles.
Unfortunately, the materials provided caused the flexures to behave like linkages, so we were not wholly able to verify our model.

Reflections

The primary thing we took away from this is the difference between theoretical feasibility and practical feasibility is often farther away than we think. On paper, we had the tools to achieve the desired range of natural frequency but could not make that device in the physical world. However, it was a very beneficial exercise in building and manipulating stiffness matrices in a practical setting.