Homework 4

CSE 571 Fall 2020

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Exercise 1.1 (10pt)

Prompt:

Suppose the agent has progressed to the point shown below, having perceived nothing in [1,1], a breeze in [2,1], and a stench in [1,2], and is now concerned **only** with the contents of [1,3],[2,2], and [3,1]. Each of these can contain a pit, and *most one can contain a wumpus*.

A. (4pt) Following the example shown in class, construct the set of possible worlds. (You should find 32 of them).

B (3pt) Mark the worlds in which the KB is true and those in which each of the following sentences is true:

$\alpha 2$ = "There is no pit in [2,2]."

$\alpha 3$ = "There is a wumpus in [1,3]." Hence show that KB $\models$ $\alpha 2$ and KB $\models$ $\alpha 3$.

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 ok	2,2	3,2	4,2
1,1 A,ok	2,1 ok	3,1	4,1

C (3pt) Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences?

a. $B \lor C$.

b. $\neg A \lor \neg B \lor \neg C \lor \neg D$.
c. $(A \Rightarrow B) \land A \land \neg B \land C \land D$.

Response:

First, let's visualize this environment.It's worth noting there may be only one Wumpus in this world.

1,4	2,4	3,4	4,4
1,3 ?	2,3	3,3	4,3
1,2 S	2,2 ?	3,2	4,2
1,1 OK	2,1 B	3,1 ?	4,1

A. Constructing a set of all possible worlds. To do this, we need to use some basic mathematics. We can determine the number of possible worlds by calculating
   a. The possible locations of the wumpus can be [1,3],[2,2],[3,1], or none at all. Therefore, all possible combinations leaves us with 4 possible states..
   b. The possible locations of the pit can be in either [1,3],[2,2],[3,1],or none at all. All combinations here leave 8 possible states.
   c. Multiplying the states 4 * 8 leaves us with 32 possible worlds.
B. Truth Table Construction and KB Table Below

Wumpus World Truth Table (Parts A & B)						
World Number	State of [1,3]	State of [2,2]	State of [3,1]	α2	α3	Valid KB
1				T	F	
2			W	T	F	
3		W		T	F	
4	W			T	T	T
5			P	T	F	
6			W/P	T	F	
7		W	P	T	F	

8	W		P	T	T	T
9		P		F	F	
10		P	W	F	F	
11		W/P		T	F	
12	W	P		F	T	
13	P			T	F	
14	P		W	T	F	
15	P	W		T	F	
16	W/P			T	T	
17		P	P	F	F	
18		P	W/P	F	F	
19		W/P	P	F	F	
20	W	P	P	F	T	
21	P	P		F	F	
22	P	P	W	F	F	
23	P	W/P		T	F	
24	W/P	P		F	T	
25	P		P	T	F	
26	P		W/P	T	F	
27	P	W	P	T	F	
28	W/P		P	T	T	
29	P	P	P	F	F	
30	P	P	W/P	F	F	
31	P	W/P	P	F	F	
32	W/P	P	P	F	T	

C. Vocabulary Modeling

a. For B V C, we can use the following truth table. There are only 3 models that satisfy B V C.

B	**C**	**B V C**
¬B	¬C	False
¬B	C	True
B	¬C	True
B	C	True

b. ¬A ∨ ¬B ∨ ¬C ∨ ¬D.
Given each symbol has two states [true/false] and there are four symbols, this is equivalent to 4^2 combinations, or 16 states. All combinations below satisfy ¬A ∨ ¬B ∨ ¬C ∨ ¬D, except for number 1. Therefore, there are only 15 models.

Number	A	B	C	D	¬A ∨ ¬B ∨ ¬C ∨ ¬D
1	A	B	C	D	False
2	A	B	C	¬D	True
3	A	B	¬C	D	True
4	A	B	¬C	¬D	True
5	A	¬B	C	D	True
6	A	¬B	C	¬D	True
7	A	¬B	¬C	D	True
8	A	¬B	¬C	¬D	True
9	¬A	B	C	D	True
10	¬A	B	C	¬D	True
11	¬A	B	¬C	D	True
12	¬A	B	¬C	¬D	True
13	¬A	¬B	C	D	True
14	¬A	¬B	C	¬D	True
15	¬A	¬B	¬C	D	True
16	¬A	¬B	¬C	¬D	True

c. (A ⇒ B) ∧ A ∧ ¬B ∧ C ∧ D.
If A implies B, then A ∧ ¬B will never be satisfied. Therefore, the number of models is zero.

Exercise 1.2 (12pt)

Prompt:

Prove the validity of the following sequents, assuming the binding priority of the connectives covered in class ***and using ONLY the basic natural deduction rules.*** **Hint: all of them have disjunction elimination as a key block.**

a. $p \rightarrow q$, $r \rightarrow s \vdash p \vee r \rightarrow q \vee s$
b. $(p \vee (q \rightarrow p)) \wedge q \vdash p$
c. $p \rightarrow (q \vee r)$, $q \rightarrow s$, $r \rightarrow s \vdash p \rightarrow s$

Response:

A. $p \rightarrow q$, $r \rightarrow s \vdash p \vee r \rightarrow q \vee s$

Line	Argument	Proof
1	$p \rightarrow q$	premise
2	$r \rightarrow s$	premise
3	$p \vee r$	assumption
4	p	assumption
5	q	→e 1,4
6	$q \vee s$	∨ i 3,5
7	r	assumption
8	s	→e 2,7
9	$q \vee s$	∨ i 3,7
10	$q \vee s$	∨ i 3-6,7-9
11	$p \vee r \rightarrow q \vee s$	→ i 3-10

B. $(p \vee (q \rightarrow p)) \wedge q \vdash p$

Line	Argument	Proof
1	$(p \vee (q \to p)) \wedge q$	premise
2	$q \to p$	premise
3	q	assumption
4	p	$\to$e 2,3
5	$p \vee q$	$\to$e 1,4
6	$p \wedge q \vdash p$	$\to$e 3,4
7	$(p \vee (q \to p)) \wedge q$	$\to$ i 3-6

C. $p \to (q \vee r), q \to s, r \to s \vdash p \to s$

Line	Argument	Proof
1	$p \to (q \vee r)$	premise
2	$q \to s$	premise
3	$r \to s$	premise
4	p	assumption
5	$(q \vee r)$	$\to$e 1,4
6	s	$\to$ i 2-3,5
7	$p \to (q \vee r), q \to s, r \to s \vdash p \to s$	$\to$ i 4-6

Exercise 1.3 (12pt)

Prompt:

P? P	OK		
B OK A	~~P?~~ OK A	BGS OK A	
OK A	S OK A	W	

Given the Wumpus world, that the following actions of the agents [1,1] → [1,2](up) →[1,1] →[2,1](right), and the sensor readings as shown in the figure (B for Breeze, S for Stench, and G for Glitter). Use natural deduction, similar to that shown in our lecture, to show that $W_{3,1}$ (Wumpus is at the location [3,1]) is true. Clearly write out the (necessary) premises first and then the proof step. **Hint: this will be very similar to what we discussed in lectures.**

Response:

Line	Argument	Proof
1	¬P1,1	Premise
2	¬W1,1	Premise
3	¬P2,2	Premise
4	¬W2,2	
5	B1,2	Premise
6	S2,1	Premise
7	B1,2 ⇔P1,3 ∨ P2,2	Premise (rules)

8	S2,1 ⇔ W1,1 ∨ W2,2 ∨ W3,1	Premise (rules)
9	¬W3,1	assumption
10	W3,1 ∨ W2,2	→e 7,8
11	W2,2	→e 8,9
12	⊥	¬e4,11
13	¬W2,2	→ i 9-12
14	S2,1 →¬W2,2 ∨ W3,1	→ i 8,9
15	W3,1	→ i 9-14

Exercise 1.4 (10pt)

Prompt:

A. (4pt) Represent the following sentences in first-order logic, using a vocabulary which you must define.
    a. There is an agent who sells policies only to people who are not insured.
    b. Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.
B. (3pt) Assume the environment in Exercise 1.3, using **propositional logic**, express the following sentences:
    a. There is only one Wumpus in the world.
    b. Locations that are adjacent to the Wumpus are smelly
    c. If a Breeze is detected, a Pit must be around.
C. (3pt) Assume the environment in Exercise 1.3, using **first-order logic**, express the same sentences in b.

Response:

A. Defining the sentences using first-order logic
    a. Let us first define in terms of x,y,z. Agent(x) sells Policy(y) to a Person(z) that is not Insured(z).
    $\exists x\ Agent(x) \wedge \forall y, Policy(y) \wedge S(x,y,z) \Rightarrow Person(z) \wedge \neg Insured(z)$
    b. Defined as: All politicians Politician(x) can fool Fool(x,y,t) all people Person(y) some of the time. All politicians cannot fool all people all the time

$\forall x\ \forall y\ \exists t \Rightarrow Fool(x, y, t) \wedge$
$\forall x\ \forall y\ \forall t \Rightarrow \neg Fool(x, y, t) \wedge$
$\forall x\ \exists y\ \forall t \Rightarrow Fool(x, y, t)$

B. Using Propositional Logic

a. Using the environment described in exercise 1.3, we know

Line	Argument	Explanation
1	S2,1 ⇔ W1,1 ∨ W2,2 ∨ 3,1	We know a Wumpus is in a surrounding stench tile
2	S2,1	We know tile 2,1 has a stench
3	¬W1,1	No Wumpus at 1,1
4	¬W2,2	No Wumpus at 2,2
5	∀x∀y ¬S(x,y) ⇒¬W(x,y)	If there is not a stench on a tile, there is not a wumpus
6	∃W ⇔1 ∧ 2 ∧ 3 ∧ 4 ∧ 5	There exists a wumpus tile that satisfies all conditions above. [3,1]

b. To prove that locations adjacent to the Wumpus are smells:
W(3,1) ⇔ S(2,1) ∧ S(3,2) ∧ S(4,1).

c. Similar to proving there is a stench surrounding a Wumpus, we can use the same idea of pits and breeze. The difference being there may be one or more pits for each breeze. For 1.3, this is:
B(1,2) ⇔ P(1,3) V P(2,2)
B(3,2) ⇔ P(3,1) V P(2,2) V P(3,3) V P(4,2)

C. Using First Order Logic

a. There is only one Wumpus in the world.
$\exists !x\ \exists !y\ W(x, y)$

b. ∃x∃y W(X,Y) ⇔ S(x,y-1) ∧ S(x-1,y) ∧ S(x,y+1) ∧ S(x+1,y)
In other words, for the x,y coordinate of a wumpus, there exists smelly locations down, left, up, and right of the wumpus. (Generic Solution)

c. ∃x∃y B(x,y) ⇔ P(x-1,y) V P(x,y+1) V P(x+1,y) V P(x,y-1)
This generic solution says that if there exists a breeze, there will be a P left, up, right, or below the breeze.

Exercise 1.5 (6pt)

Prompt:

A. (3pt) In the Forward Chaining algorithm, after the algorithm stops, prove that for those atoms that are not assigned to true during the inference process, there exists a model in the KB in which the atom is true ***and*** there exists a model in the KB in which the atom is false.

B. (3pt) Prove that the resolution process discussed in the lecture (also attached below) is correct using entailment (semantics of propositional logic), or that ***the top entails the bottom.***

$$\frac{l_1 \vee \cdot\cdot\cdot \vee l_k, \qquad m_1 \vee \cdot\cdot\cdot \vee m_n}{l_1 \vee \cdot\cdot\cdot \vee l_{i-1} \vee l_{i+1} \vee \cdot\cdot\cdot \vee l_k \vee l_k \vee m_1 \vee \cdot\cdot\cdot \vee m_{j-1} \vee m_{j+1} \vee \cdot\cdot\cdot \vee m_n}$$

Response:

A. Using the forward chaining algorithm described in class, we can give an example where an atom is true, but inside the KB, the KB becomes false
$C \Leftrightarrow (A \vee \neg B)$ when the atom B is false
$C \Leftrightarrow (A \vee B)$
For as long as the atom A is true, the value of atom B does not matter.
Therefore,there exists a model of a KB where the value of B can be true or false.

B. Here the proof is to have the top ⊨ (entail) bottom. To do that, we can use proof by resolution. If all values on the top are true for l1 to lk and m1 to mk, then we know that the bottom must also be true. If li is false, then mj must be true.