# Basic (Low-Level) DIP
# -- Part I

Image Enhancement in the Spatial Domain

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1

## Introduction

- Objective:

  To process an image so that the result is more suitable than the original image for a **<u>specific</u>** application. → **Problem Oriented**
  - To suppress undesired distortions
  - Emphasize and sharpen image features for display and analysis
- There is no general theory of image enhancement. When an image is processed for visual interpretation, the viewer is the ultimate judge of how well a particular method works. → **Subjective Process**

2

- Two broad categories of image enhancement approaches
  - **Spatial Domain Methods**: Direct manipulate pixels in an image.
  - **Frequency Domain Methods**: Modify the Fourier or Wavelet transform of an image.
- Normally, enhancement techniques use various combinations of methods from these two categories.

3

- Image processing in spatial domain may be represented as

  $$g(x, y) = T(f(x, y))$$

  where $f(x, y)$ is the input image, $g(x, y)$ is the processed image, $T$ is the operator on $f(x, y)$.
- In general, the operator $T$ can operate on:
  - A single pixel (x, y) → point processing
  - A group of pixels → mask processing

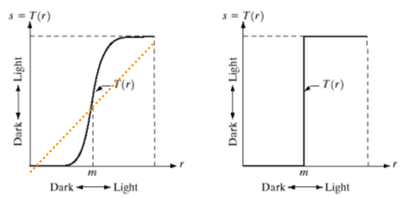
4

## Introduction to Point Processing

- Point processing modifies each pixel intensity independently based on a function $T$. Let the intensity of pixel before modification be $r$ and after modification be $s$, then $s = T(r)$.
- $T$ is often referred to as:

  **Gray-level (Intensity or Mapping) transformation function**.

5




(a) Contrast Stretching (b)Thresholding

a b FIGURE 3.2 Gray-level transformation functions for contrast enhancement.

What is the general effect after applying the (a) function?

Properties of $T$:
- **Single value and monotonically increasing** (or decreasing in the case to get the negative) in the interval $0 <= r <= L-1$
- **$0<=T(r) <= L-1$** for $0 <= r <= L-1$

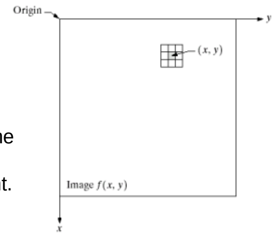
6

## Introduction to Mask Processing (Filtering)

- *T* operates on a neighborhood of pixels.
- Neighborhood about a point *(x,y)* normally is a square or rectangular subimage area centered at *(x, y).*
- The general approach is to use a function of the values of *f* in a predefined neighborhood of *(x, y)* to determine the value of *g* at *(x, y).*

7

**FIGURE 3.1** A 3 × 3 neighborhood about a point $(x, y)$ in an image.




Normally, square and rectangular masks are the most predominant ones used in the enhancement.

Ex2:

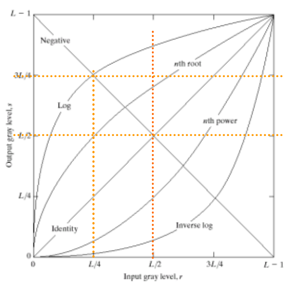
Mask (filter, kernel, template, or window)
Processing Function: Use a function of the values of *f* in a 3×3 neighborhood of *(x, y)* to determine the values of *g* at *(x, y)*

8

## Point Processing Methods
## -- Basic Gray Level Transformations

Basic Gray-level Transformation Function Used for Image Enhancement

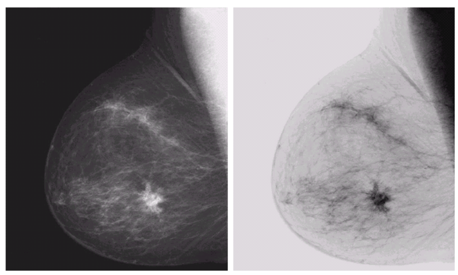
**FIGURE 3.3** Some basic gray-level transformation functions used for image enhancement.




9

The values of pixels, before and after processing, will be denoted by *r* and *s*, respectively.

1) Image Negative: $s = L - 1 - r$ ;



a b

**FIGURE 3.4** (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

10

2) Log Transformations:

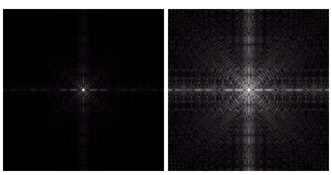
$s = c \log(1 + r)$ ;

What not $s = c \log r$ ?

where c is a constant.

a b

**FIGURE 3.5** (a) Fourier spectrum. (b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.



The shape of the log curve shows that this transformation maps a narrow range of low gray-level values in the input image into a wider range of output levels. ➔ Effect: Spreading/compressing of gray levels in an image.

The log function has the important characteristic that it compresses the dynamic range of images with large variations in pixel values.

11

3) Power-Law Transformation:

$s = cr^{\gamma}$

where c and γ are positive constants.

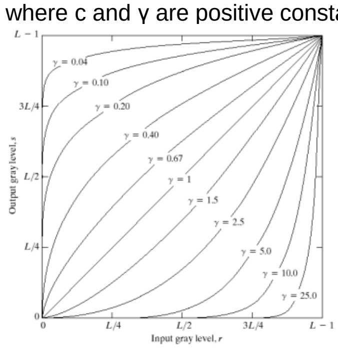
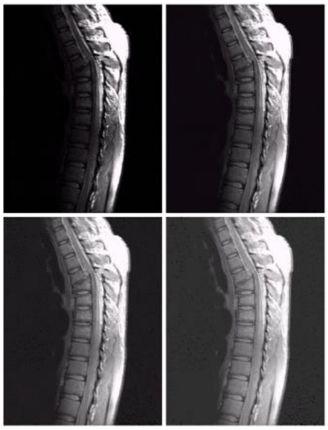



**FIGURE 3.6** Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases).

The curves generated with values of γ > 1 have exactly the opposite effect as those generated with values of γ < 1 .

12



a b
c d

FIGURE 3.8 (a) Magnetic resonance (MR) image of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4,$ and $0.3$, respectively. (Original image for this example courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Science, Van… Un… M…

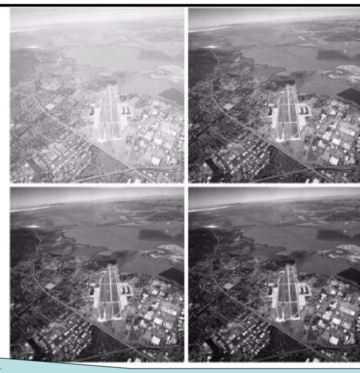
$\gamma$ = 0.6, 0.4 and 0.3.

1. Which one is the best enhancement in terms of contrast and discernable details?
2. What is the approximate limit for $\gamma$ for this enhancement?

13

a b
c d

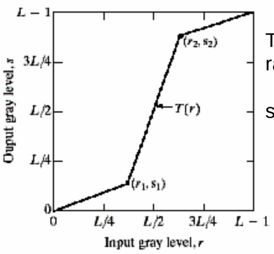
FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and $5.0$, respectively. (Original image for this example courtesy of NASA.)



1. Which one is the best enhancement in terms of contrast and discernable details?
2. What is the approximate limit for $\gamma$ for this enhancement?

14

## 4) Piecewise-Linear Transformation Functions

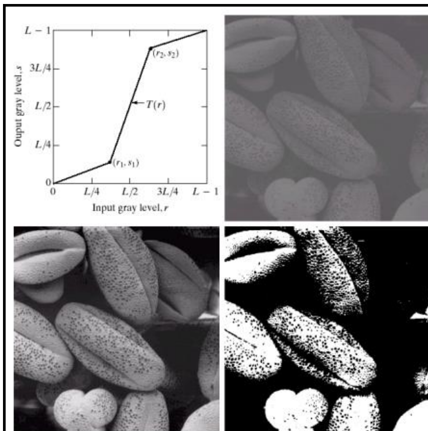



The goal is to increase the dynamic range.

$$s = \begin{cases} \alpha r & \text{for } 0 <= r < r1 \\ \beta(r - r1) + s1 & \text{for } r1 <= r < r2 \\ \gamma(r - r2) + s2 & \text{for } r2 <= r < L \end{cases}$$

1. How about r1 = s1 and r2 = s2?
2. How about r1 = r2, s1 = 0, and s2 = L-1?
3. How about r1 = Minimum gray level, s1 = 0 and r2 = maximum gray level, s2 = L-1?

5




a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

16

## 5) Bit-Plane Slicing (Bit Extraction): Highlight the contribution made to total image appearance by specific bits.

$U = K_{B-1}2^{B-1} + K_{B-2}2^{B-2} + \ldots + K_{B-n}2^{B-n} + \ldots + K_1 2 + K_0$ where $K_i = 0$ or $1$, and B is the total number of bits

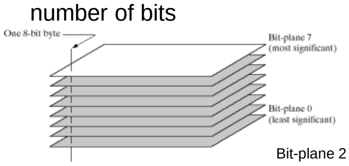



FIGURE 3.12 Bit-plane representation of an 8-bit image.

| | |
|---|---|
| 2 | 4 |
| 7 | 6 |

→

| | |
|---|---|
| 010 | 100 |
| 111 | 110 |

→

Bit-plane 2

| | |
|---|---|
| 0 | 1 |
| 1 | 1 |

Bit-plane 1

| | |
|---|---|
| 1 | 0 |
| 1 | 1 |

Bit-plane 0

| | |
|---|---|
| 0 | 0 |
| 1 | 0 |

17

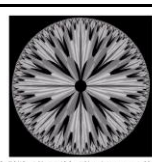


FIGURE 3.13 An 8-bit fractal image. (A fractal is an image generated from mathemat… expressions). (Courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, P…

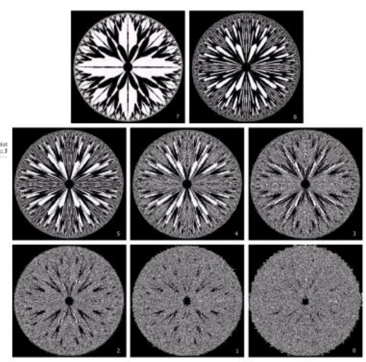


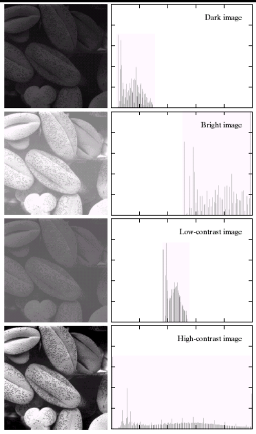

FIGURE 3.14 The eight bit planes of the image in Fig. 3.13. The number at the bottom, right of each image identifies the bit plane.

18

## Point Processing Methods
## -- Histogram Processing

- The **histogram** of a digital image with gray levels in the range [0, L-1] is a discrete function $h(r_k) = n_k$, where $r_k$ is the kth gray level and $n_k$ is the number of pixels in the image having gray level $r_k$.
- A **normalized histogram** is given by $p(r_k) = n_k/n$ for k = 0, 1, …, L-1 and n is the total number of pixels in the image. That is, $p(r_k)$ gives an estimate of the probability of occurrence of gray level $r_k$.

19




It is reasonable to conclude that an image whose pixels tend to **occupy the entire range** of possible gray levels and, in addition, tend to be **distributed uniformly**, will have an appearance of high contrast and will exhibit a large variety of gray tones.

The net effect will be an image that shows a great deal of gray-level detail and has high dynamic range.

20

## Point Processing Methods
## -- Properties of the Image Histogram

- Histogram clustered at the low end: Dark Image
- Histogram clustered at the high end: Bright Image
- Histogram with a small spread: Low contrast Image
- Histogram with a wide spread: High contrast Image

21

## Point Processing Methods
## -- Histogram Equalization

- Histogram equalization is to increase the dynamic range of an image by an **non-linear** intensity transformation function such that the histogram of the transformed image covers the whole dynamic range with equal probability.
  - ➔ Stretch the contrast by redistributing the gray-level values uniformly.
  - ➔ It is fully automatic compared to other contrast stretching techniques.

22

## Derivation of the Histogram Equalization

- Let *r* and *s* be the intensity before and after histogram equalization and *s = T(r).*
- Let the normalized histogram (probability density) before and after histogram equalization be *f(r)* and *g(s).*
- Let *Fs* and *Fr* be the cumulative density functions (CDF) for *s* and *r* respectively.
- The goal is to identify a function T such that *g(s)* = 1 for 0 < s < 1 [This derivation is given in the textbook in section 3.3.1]. In other word, s is uniformly distributed within its range (i.e., histogram is as smooth as possible).

23

## Derivation of the Histogram Equalization

Random variable

Its value

$$F_{\mathrm{r}}(r) = P(\mathrm{r} \le r) = P(T^{-1}(\mathrm{s}) \le r) = P(\mathrm{s} \le T(r))$$
$$= F_{\mathrm{s}}(T(r))$$

since $F_{\mathrm{s}}(s) = P(\mathrm{s} \le s) = \int_{t=0}^{s} g(t)dt = s.$ Thus,

$$F_{\mathrm{r}}(r) = F_{s}(T(r)) = T(r)$$

**This means the transfer function *T* is the CDF of *r*.**

However, the equalized histogram cannot be flat due to discrete approximation of the continuous density function by histogram.

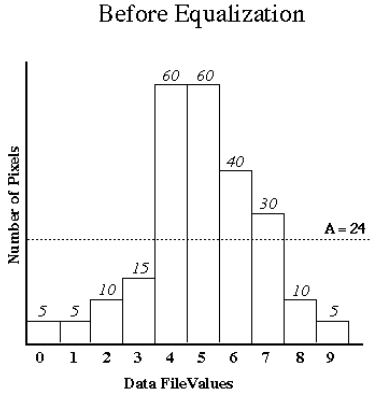
24

## Point Processing Methods -- Discrete Histogram Equalization

Given an image of M x N with the maximum intensity being Imax:

1. Obtain the histogram H(k), k = 0, 1, …, Imax
2. Compute the cumulative normalized histogram T(k): T(k) = $\sum_{i=0}^{k} H(i)/MN$
3. Compute the transformed intensity by:
   $g_k$ = (L-1) * T(k), where L is the maximum gray level of the new processed image
4. Scan the image and set the pixel with the intensity k to $g_k$.

25




26

Total Number of Pixels:
5+5+10+15+60+60+40+30+10+5 = 240.

Step 1: H(0) = 5 ; H(1) = 5 ; H(2) = 10 ;
H(3) = 15 ; H(4) = 60 ; H(5) = 60 ;
H(6) = 40 ; H(7) = 30 ; H(8) = 10 ;
H(9) = 5 ;

Step 2: T(0) = 5/240 ; T(1) = 10/240 ;
T(2) = 20/240 ; T(3) = 35/240 ;
T(4) = 95/240 ; T(5) = 155/240 ;
T(6) = 195/240 ; T(7) = 225/240 ;
T(8) = 235/240 ; T(9) = 240/240 ;

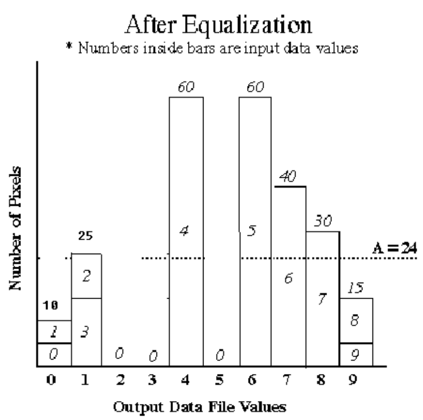
27

- Step 3: G0 = 0.1875 →0; G1 = 0.3750→0;
  G2 = 0.7500→1 ; G3 = 1.3125→1;
  G4 = 3.5625→4 ; G5 = 5.8125→6;
  G6 = 7.3125→7 ; G7 = 8.4375→8 ;
  G8 = 8.8125→9 ; G9 = 9.0000→9 ;
- Step 4:

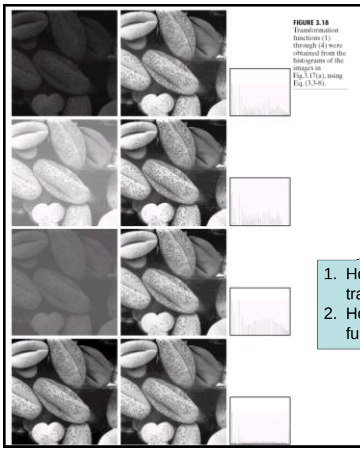
Original Intensity →New Intensity → Number

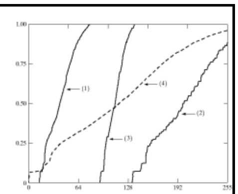
0→0→5; 1→0→5 ; 2→1→10 ;
3→1→15; 4→4→60; 5→6→60 ;
6→7→40; 7→8→30; 8→9→10;
9→9→5;

28



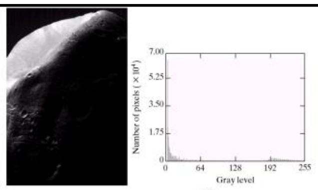

29





1. How to get the above transformation functions?
2. How to interpret the above functions?

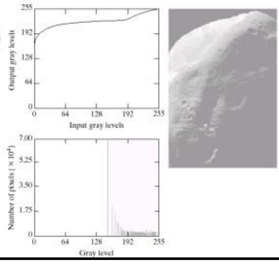
30



a b

**FIGURE 3.20** (a) Image of the Mars moon Ph *Surveyor.* (b) Histogram. (Original image courte

Is Histogram Equalization a general approach to enhance any image?

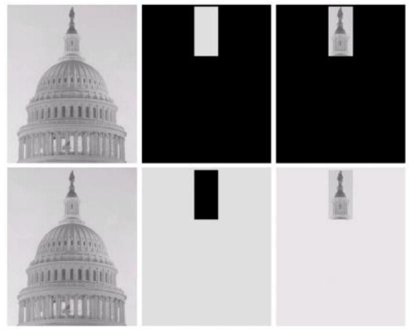


a b
c

**FIGURE 3.21** (a) Transformation function for histogram equalization. (b) Histogram-equalized image (note the washed-out appearance). (c) Histogram of (b).

## Mask Processing Methods

1) Logical Mask (AND, OR, and NOT)



a b c
d e f

**FIGURE 3.27** (a) Original image. (b) AND image mask. (c) Result of the AND operation on images (a) and (b). (d) Original image. (e) OR image mask. (f) Result of operation OR on images (d) and (e).

## Mask Processing Methods

2) Image Subtraction

- The difference between two images *f(x, y)* and *h(x, y),* expressed as:

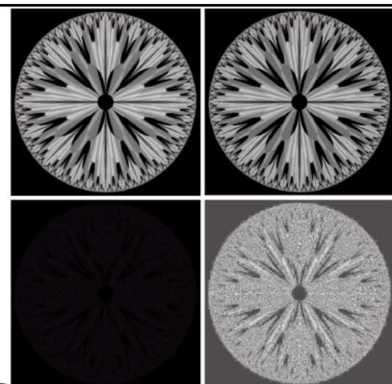
  $g(x, y) = f(x, y) - h(x, y)$

  is obtained by computing the difference between all pairs of corresponding pixels from *f* and *h*.
- Image subtraction is primarily used to detect objects (surveillance) or remove an object (background removal)
- It involves two images, one is called mask image (i.e., *h*) and the other is current image (i.e., *f*).

33

a b
c d

**FIGURE 3.28** (a) Original fractal image. (b) Result of setting the four lower-order bit planes to zero. (c) Difference between (a) and (b). (d) Histogram-equalized difference image. (Original image courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA).



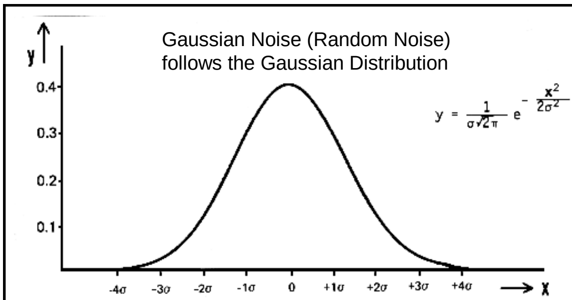
1. What is the range of the image subtraction?
2. How do we display the subtraction results?

## Mask Processing Methods

3) Image Averaging

- The purpose is to reduce image noise by averaging a sequence of images of the same scene but obtained at different times with small intervals.
- Assume the same noise characteristics (i.e., the noise is uncorrelated and has zero average value) for all images, averaging can therefore reduce noise.

35




The Gaussian distribution shows the probability *y* of finding a deviation *x* from the mean (*x* = 0), according to the equation stated, where *e* is the base of natural logarithms, and *σ* is the standard deviation. The probability of larger deviations can be seen to decrease rapidly.

36

Ideal Image

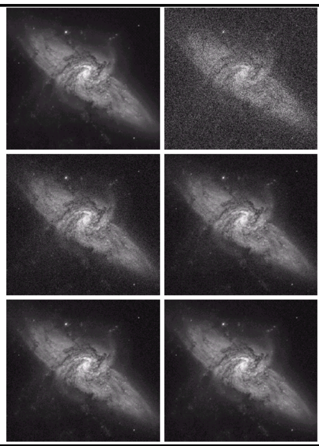
Corrupted Image

Average of 8 images

Average of 16 images

Average of 64 images

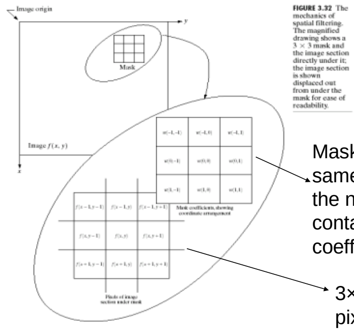
Average of 128 images





## Mask Processing Methods

4) Spatial Filtering



Mask (filter) with the same dimensions as the neighborhood contains the coefficients

3×3 window of the pixel f(x, y)



- Spatial filtering alters image intensity of a pixel based on its intensity, the intensities of the neighboring pixels, and the coefficients of the corresponding mask. That is:
  - For a linear spatial filtering, the response is given by a sum of products of the filter coefficients with the corresponding image pixels in the area spanned by the filter mask.
  - In general, linear filtering of an image *f* of size *M*×*N* with a filter mask of size m×n is given by the expression:

$$g(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\,f(x+s,y+t)$$

where $a=(m-1)/2$ and $b=(n-1)/2$

  - It is accomplished by convolving the mask with the original image.



Original Image:

| | | | | |
|---|---|---|---|---|
| 4 | 6 | 8 | 9 | 10 |
| 1 | 3 | 5 | 6 | 9 |
| 2 | 4 | 6 | 12 | 12 |
| 4 | 8 | 10 | 14 | 6 |

1. What happens when the center of the filter approaches the border of the image?
2. How to handle this situation?

Mask 1:

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

Convolution Result:

| | | | | |
|---|---|---|---|---|
| 14 | 27 | 37 | 47 | 34 |
| 20 | 39 | 59 | 77 | 58 |
| 22 | 43 | 68 | 80 | 59 |
| 18 | 34 | 54 | 60 | 44 |

Mask 2:

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 1 | 4 | 1 |
| 1 | 3 | 2 |

Convolution Result:

| | | | | |
|---|---|---|---|---|
| 31 | 56 | 77 | 95 | 82 |
| 47 | 84 | 124 | 160 | 119 |
| 51 | 94 | 137 | 174 | 116 |
| 40 | 74 | 114 | 138 | 74 |

Matlab Function filter2 can do this convolution!



Three types of Spatial Filters:

- Low-pass filter: Enhance low frequency components of the image while eliminating or reducing high frequency components.
- High-pass filter: Enhance high frequency components of the image while eliminating or reducing low frequency components.
- Band-pass filter: Enhance certain range of frequency components of the image while eliminating or reducing frequency components beyond the range.



## Mask Processing Methods

4.1) Smoothing Spatial Filters

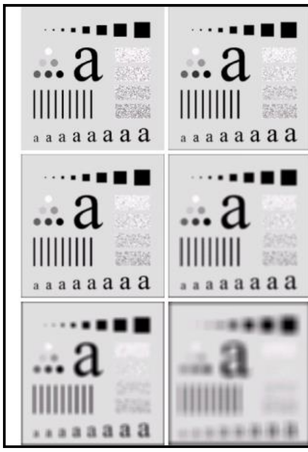
- It is a linear low-pass filter
- A standard averaging filter replaces the value of every pixel in an image by the average of the gray levels in the window defined by the filter mask. This process results in an image with reduced “sharp” transitions in gray levels.
- A weighted averaging filter uses different coefficients at different spatial locations.

$$\frac{1}{9}\times\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline\end{array} \qquad \frac{1}{16}\times\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline\end{array}$$

Standard vs. Weighted Averaging Filter

1. What is the sum of all the coefficients in the mask?
2. What is the characteristic of the weighted averaging filter in terms of the coefficient values?
3. What conclusion can you draw on the smoothing spatial filters?

43

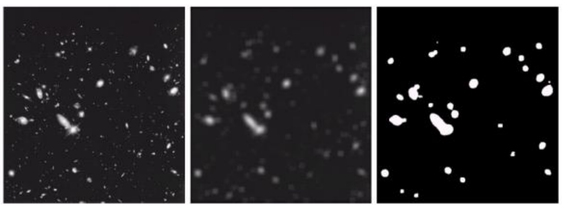


Averaging filter results with the mask size of 3x3, 5x5, 9x9, 15x15, and 35x35.

1. What is the desirable feature of the averaging filter?
2. What is the undesirable side effect of the averaging filter?
3. What is the effect of the size of the filter?
4. How to determine the best filter size for a specific image?

44

Ex1: Find the larger and brighter objects in the image



a b c

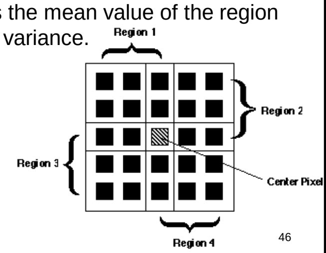
**FIGURE 3.36** (a) Image from the Hubble Space Telescope. (b) Image processed by a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

1. Image is processed by a 15×15 averaging mask.
2. Thresholding with a threshold value equal to 25% of the highest intensity in the blurred image

45

**How to reduce the blurring effect?**

1. One could replace a pixel value by the average value only if the pixel value deviates very much from the average. →Then the filter is changed to a non-linear filter.
2. **Kuwahara filter**: The output value of the center pixel in the window is the mean value of the region that has the smallest variance.



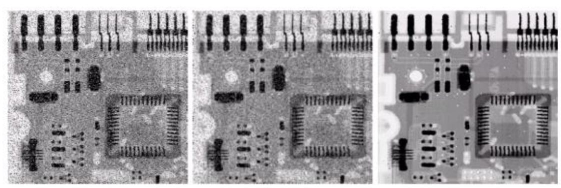

46

# Mask Processing Methods

## 4.2) Order-Statistics Filters

- It is a nonlinear low-pass filter
- Its filtering result is based on ordering (ranking) the pixels contained in the image area encompassed by the filter, and then replacing the value of the center pixel with the value determined by the ranking result.
  - Median filter: Replace center pixel with median of the gray levels in the window of that pixel (the original pixel value is included in the computation of the median).

47



a b c

**FIGURE 3.37** (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

1. What is the difference between the averaging filter and median filter results?
2. Which filter has more computational cost?
3. Which filter will you choose to remove the additive salt-and-pepper noise?

48

The filter can be shown as a convolution mask, for example, for an average 3x3 filter

What does this filter look like?

| image | average filter result | median filter result |
|---|---|---|
| 9 9 9 0 0 0 | . . . . . . | . . . . . . |
| 9 9 9 0 0 0 | . 8 5 3 0 . | . 9 9 0 0 . |
| 9 0 9 0 0 0 | . 8 5 4 1 . | . 9 9 0 0 . |
| 9 9 9 0 9 0 | . 8 5 4 1 . | . 9 9 0 0 . |
| 9 9 9 0 0 0 | . 9 6 4 1 . | . 9 9 0 0 . |
| 9 9 9 0 0 0 | . . . . . . | . . . . . . |



- **Determine the size of the mask**
  1. The size of the mask must be larger than the scale of the noise but smaller than the dimensions of any structure in the image that is important to subsequent analysis. That is, features (e.g. lines or spots) that are smaller than half the mask can be selectively eliminated as noise (or at least not features of interest).
  2. The larger the mask, the longer the ranking process (for the median filter) or the computational process (for the averaging filter) takes
- **Comparison between averaging and median filters:**

The median filter is superior to the smoothing filter in that it does not smooth or blur the boundaries of regions or features in the image**.**



# Mask Processing Methods

## 4.3) Sharpening Spatial Filters

- The goal of the sharpening is to highlight fine detail in an image or to enhance detail that has been blurred.
  - Sharpening could be accomplished by spatial differentiation.
  - Image differentiation enhances edges and other discontinuities (such as noise) and deemphasizes areas with slowly varying gray-level values.



The first-order derivative of a 1D function $f(x)$ is:

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

The second-order derivative of a 1D function $f(x)$ is:

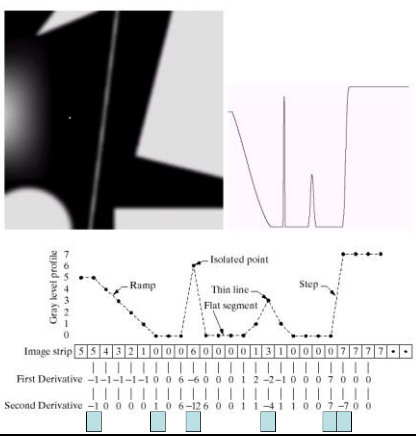
$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

- First derivative:
  1. Zero in flat segment
  2. Nonzero at the onset of a gray-level step or ramp
  3. Nonzero along the ramp.
- Second derivative:
  1. Zero in flag area.
  2. Nonzero at the onset and end of a gray-level step or ramp.
  3. Zero along ramps of constant slope.



a b
c

**FIGURE 3.38**
(a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point.
(c) Simplified profile (the points are joined by dashed lines to simplify interpretation).




- Compare the response between 1st and 2nd derivatives:
  - 1st derivatives generally produce thicker edges in an image.
  - 2nd derivatives have a stronger response to fine detail, such as thin lines and isolated points.
  - 1st derivatives generally have a stronger response to a gray-level step.
  - 2nd derivatives produce a double response at step changes in gray-level values in an image.
  - For similar changes in gray-level values in an image, the response from the 2nd derivative is stronger to a line than to a step, and to a point than to a line.

- The Laplacian Filter (2nd Derivatives)

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$
$$= [f(x+1, y) + f(x-1, y) - 2f(x, y)]$$
$$+ [f(x, y+1) + f(x, y-1) - 2f(x, y)]$$
$$= [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

1. What is the sum of the entries in the mask?
2. What is the difference between the sum of the smoothing and sharpening spatial filters?

(a)

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | −4 | 1 |
| 0 | 1 | 0 |

(b)

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | −8 | 1 |
| 1 | 1 | 1 |

(c)

| | | |
|---|---|---|
| 0 | −1 | 0 |
| −1 | 4 | −1 |
| 0 | −1 | 0 |

(d)

| | | |
|---|---|---|
| −1 | −1 | −1 |
| −1 | 8 | −1 |
| −1 | −1 | −1 |

a b
c d

**FIGURE 3.39** (a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.



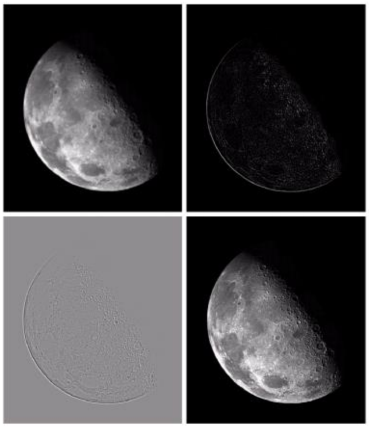
- Laplacian Filter is an isotropic filter. That is, its response is independent of the direction of the discontinuities in the image to which the filter is applied.
- Laplacian Filter is a linear operator since the derivatives of any order are linear operations.
- Laplacian Filter highlights gray-level discontinuities in an image and deemphasizes regions with slowly varying gray levels since it is a derivative operator.
- The basic way in which we use the Laplacian for image enhancement is as follows:

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) & \text{if the center coefficient of} \\ & \text{Laplacian mask is negative} \\ f(x, y) + \nabla^2 f(x, y) & \text{if the center coefficient of} \\ & \text{Laplacian mask is positive} \end{cases}$$



a b
c d

**FIGURE 3.40** (a) Image of the North Pole of the moon. (b) Laplacian-filtered image. (c) Laplacian image scaled for display purposes. (d) Image enhanced by using Eq. (3.7-5). (Original image courtesy of NASA.)





Simplify the Laplacian Enhancement Filter:

$$g(x, y) = f(x, y) - [f(x+1, y) + f(x-1, y)$$
$$+ f(x, y+1) + f(x, y-1)] + 4f(x, y)$$
$$= 5f(x, y) - [f(x+1, y) + f(x-1, y)$$
$$+ f(x, y+1) + f(x, y-1)]$$

Based on the above formulation, a composite Laplacian filter can be constructed. What does this composite filter look like?

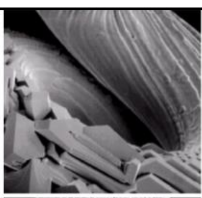


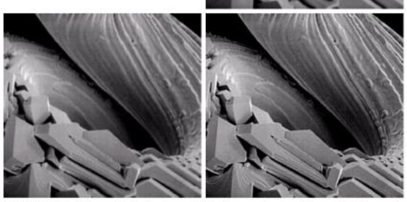
(a)

| | | |
|---|---|---|
| 0 | −1 | 0 |
| −1 | 5 | −1 |
| 0 | −1 | 0 |

(b)

| | | |
|---|---|---|
| −1 | −1 | −1 |
| −1 | 9 | −1 |
| −1 | −1 | −1 |





The results obtained with the mask containing the diagonal terms usually are a little sharper than those obtained with the basic mask.

a b c
d e

**FIGURE 3.41** (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)



- Unsharp Masking

Unsharp Masking: Subtract a blurred version of an image from the image itself to sharpen the image.

$$f_s(x, y) = f(x, y) - f_{blur}(x, y)$$

- High-Boost Filtering

High Boost Filtering is a slight further generalization of unsharp masking.

$$f_{hb}(x, y) = Af(x, y) - f_{blur}(x, y)$$
$$= (A-1)f(x, y) + f(x, y) - f_{blur}(x, y)$$
$$= (A-1)f(x, y) + f_s(x, y)$$
$$\text{where } A \geq 1$$

- The Gradient Enhancement – 1st Derivatives

The gradient of $f$ at coordinates $(x, y)$ is defined :

$$\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

The magnitude of this vector is given by :

$$mag(\nabla f) = [G_x^2 + G_y^2]^{1/2} = \left[ \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

$mag(\nabla f)$ can be approximated by using absolute values :

$$mag(\nabla f) \approx |G_x| + |G_y|$$

61

- The gradient of $f$ is a 2D column vector. The components of the gradient vector itself are linear operators, but the magnitude of this vector is not because of the squaring and square root operations.
- Since the magnitude of the gradient vector is isotropic, it often is referred to as the gradient.
- For computational simplicity, an absolute value approximation of the magnitude is used since it still preserves relative changes in gray levels.
- First derivative enhancement filters are also referred to as gradient edge detectors.

62

- Two Representative Filters (1st Derivatives)

a
b c
d e

**FIGURE 3.44** A 3 × 3 region of an image (the z's are gray-level values) and masks used to compute the gradient at point labeled $z_5$. All masks coefficients sum to zero, as expected of a derivative operator.

| | | |
|---|---|---|
| $z_1$ | $z_2$ | $z_3$ |
| $z_4$ | $z_5$ | $z_6$ |
| $z_7$ | $z_8$ | $z_9$ |

Roberts Cross-Gradient Operators

| | |
|---|---|
| −1 | 0 |
| 0 | 1 |

| | |
|---|---|
| 0 | −1 |
| 1 | 0 |

Gx=Z9 – Z5 ;

Gy = Z8 – Z6 ;

Sobel Operators

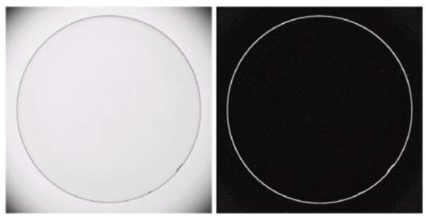
| | | |
|---|---|---|
| −1 | −2 | −1 |
| 0 | 0 | 0 |
| 1 | 2 | 1 |

| | | |
|---|---|---|
| −1 | 0 | 1 |
| −2 | 0 | 2 |
| −1 | 0 | 1 |

Gx=(z7+2Z8+z9)-(z1+2Z2+Z3) ;

Gy=(Z3+2Z6+Z9)-(Z1+2Z4+Z7) ;

63



a b

**FIGURE 3.45** Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

The edge defects are also quite visible, but with the added advantage that constant or slowly varying shades of gray have been eliminated.

The ability to enhance small discontinuities in an otherwise flat gray field is another important feature of the gradient

64

## Mask Processing Methods
## -- Local Histogram Equalization

- Instead of performing a histogram for the whole image, histogram equalization is performed locally on a small region of the image adaptively.
- This can be achieved using a window and moving the center of this window across the image from pixel to pixel. At each location, histogram equalization is performed on the portion of the image covered by the window. The pixel value at the center of the window is mapped to a new gray-level based on histogram equalization function. This process repeats until the last pixel of the image, with window moving one pixel at a time.

65

## Mask Processing Methods
## -- Local Enhancement by Statistics

### Several Important Statistical Concepts

Mean (Average): $m = \sum_{i=0}^{L-1} r_i p(r_i);$

The nth moment of r about its mean: $\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i);$

Variance $\mu_2(r) = \sigma^2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i).$

In a digital image, the mean is a measure of average gray level; the variance (or standard deviation, which is a square root of the variance) is a measure of average contrast

66

## Mask Processing Methods
## -- Local Enhancement by Statistics

The Interpretation of Some Important Statistical Concepts

- Variance (The 2nd Moment)
  - A small variance indicates a tight grouping.
  - A large variance indicates the opposite.

- The 3rd Moment
  - Indicate how many points on each side of the average
  - =0: symmetric about the mean
  - <0: A low bias with respect to the average
  - >0: A high bias with respect to the average

67

**FIGURE 3.24** SEM image of a tungsten filament and support, magnified approximately 130×. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).

Local enhancement by contrast manipulation is an ideal approach to try on problems where part of the image is acceptable, but other parts may contain hidden features of interest.

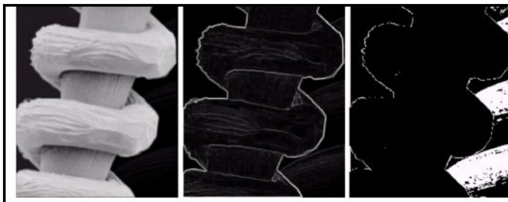
68

Let $f(x, y)$ represent the value of an image pixel at any image coordinates $(x, y)$,
Let $g(x, y)$ represent the corresponding enhanced pixel at those coordinates.
Then

$$g(x,y) = \begin{cases} E \times f(x,y) & \text{if } m_{S_{xy}} \le k_0 M_G \text{ and } k_1 D_G \le \sigma_{S_{xy}} \le k_2 D_G \\ f(x,y) & \text{otherwise} \end{cases}$$

where, $E, k_0, k_1,$ and $k_2$ are specified parameters;
$M_G$ is the global mean of the input image;
$D_G$ is its global standard deviation
$S_{xy}$ denotes a neighborhood of specified size, centered at $(x, y)$.

69



a b c

**FIGURE 3.25** (a) Image formed from all local means obtained from Fig. 3.24 using Eq. (3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq. (3.3-22). (c) Image formed from all multiplication constants used to produce the enhanced image shown in Fig. 3.26.

**FIGURE 3.26** Enhanced SEM image. Compare with Fig. 3.24. Note in particular the enhanced area on the right side of the image.

In this implementation:

E = 4.0, k0 = 0.4, k1 = 0.02, and k2 = 0.4.

70