Miscellaneous Observations on Randomization

This version of the document is dated 2020-10-23.

Peter Occil

On a Binomial Sampler

Take the following sampler of a binomial(n, 1/2) distribution (where n is even), which is equivalent to the one that appeared in <<Bringmann et al. 2014|K. Bringmann, F. Kuhn, et al., “Internal DLA: Efficient Simulation of a Physical Growth Model.” In: *Proc. 41st International Colloquium on Automata, Languages, and Programming (ICALP'14)*, 2014.>>, and adapted to be more programmer-friendly.

1. Set m to floor(sqrt(n)) + 1.
2. (First, sample from an envelope of the binomial curve.) Generate unbiased random bits (zeros or ones) until a zero is generated this way. Set k to the number of ones generated this way.
3. Set s to an integer in [0, m) chosen uniformly at random, then set i to k*m + s.
4. Set *ret* to either $n/2+i$ or $n/2-i-1$ with equal probability.
5. (Second, accept or reject *ret*.) If $ret < 0$ or $ret > n2$, go to step 2.
6. With probability choose(n, *ret*)*m*$2^{k-(n+2)}$, return *ret*. Otherwise, go to step 2. (Here, choose(n, k) is a binomial coefficient.<<|choose(n, k) = n!/(k! * ($n - k$)!) is a binomial coefficient. It can be calculated, for example, by calculating $i/(n-i+1)$ for each integer i in [$n-k+1$, n], then multiplying the results (Yannis Manolopoulos. 2002. "Binomial coefficient computation: recursion or iteration?", SIGCSE Bull. 34, 4 (December 2002), 65–67. DOI: https://doi.org/10.1145/820127.820168). Note that for all m>0, choose(m, 0) = choose(m, m) = 1 and choose(m, 1) = choose(m, $m-1$) = m.>>)

This algorithm has an acceptance rate of 1/16 regardless of the value of n.

Notes

[Nothing here yet.]

1 License