```
This version of the document is dated 2020-08-23.
Help on module randomgen:
NAME
   randomgen
DESCRIPTION
   Sample code for the article "Randomization and Sampling Methods"
   [https://www.codeproject.com/Articles/1190459/Random-Number-Generation-Methods]
(https://www.codeproject.com/Articles/1190459/Random-Number-Generation-Methods)
   Written by Peter 0.
   Any copyright is released to the Public Domain.
   [https://creativecommons.org/publicdomain/zero/1.0/]
(https://creativecommons.org/publicdomain/zero/1.0/)
CLASSES
   builtins.object
       AlmostRandom
       BinaryExpansion
       BringmannLarsen
       ConvexPolygonSampler
       {\tt DensityInversionSampler}
       DensityTiling
       FastLoadedDiceRoller
       KVectorSampler
       OptimalSampler
       PascalTriangle
       RandomGen
       RatioOfUniformsTiling
       SortedAliasMethod
       VoseAlias
   class AlmostRandom(builtins.object)
       Methods defined here:
       __init__(self, randgen, list)
           Initialize self. See help(type(self)) for accurate signature.
       choose(self)
       ______
       Data descriptors defined here:
       __dict
           dictionary for instance variables (if defined)
         weakref
           list of weak references to the object (if defined)
   class BinaryExpansion(builtins.object)
       Methods defined here:
       __init__(self, arr, zerosAtEnd=False)
           Binary expansion of a real number in [0, 1], initialized
           from an array of zeros and ones expressing the binary
           expansion.
           The first binary digit is the half digit, the second
           is the quarter digit, the third is the one-eighth digit,
```

```
and so on. Note that the number 1 can be
        expressed by passing an empty array and specifying
        zerosAtEnd = False, and the number 0 can be
        expressed by passing an empty array and specifying
        zerosAtEnd = True.
       arr - Array indicating the initial digits of the binary
        expansion.
        zerosAtEnd - Indicates whether the binary expansion
        is expressed as 0.xxx0000... or 0.yyy1111... (e.g., 0.1010000...
       vs. 0.1001111.... Default is the latter case (False).
   entropy(self)
   eof(self)
       Returns True if the end of the binary expansion was reached; False otherwise.
        Creates a binary expansion object from a 64-bit floating-point number in the
        interval [0, 1].
    fromFraction(f)
        Creates a binary expansion object from a fraction in the
        interval [0, 1].
        Creates a binary expansion object from a fraction, 'int', or
        'float' in the interval [0, 1]; returns 'f' unchanged, otherwise.
    get0rReset(f)
        Creates a binary expansion object from a fraction, 'int', or
        'float' in the interval [0, 1]; resets 'f' (calls its reset method) otherwise.
    nextbit(self)
       Reads the next bit in the binary expansion.
    reset(self)
       Resets this object to the first bit in the binary expansion.
   value(self)
   Data descriptors defined here:
   __dict
       dictionary for instance variables (if defined)
   __weakref
       list of weak references to the object (if defined)
class BringmannLarsen(builtins.object)
   Implements Bringmann and Larsen's sampler, which chooses a random number in [0, n)
   where the probability that each number is chosen is weighted. The 'weights' is the
   list of weights each 0 or greater; the higher the weight, the greater
   the probability. This sampler supports only integer weights.
   This is a succinct (space-saving) data structure for this purpose.
   Reference:
   K. Bringmann and K. G. Larsen, "Succinct Sampling from Discrete
   Distributions", In: Proc. 45th Annual ACM Symposium on Theory
 | of Computing (STOC'13), 2013.
 | Methods defined here:
```

```
init (self, weights)
       Initialize self. See help(type(self)) for accurate signature.
   next(self, randgen)
   Data descriptors defined here:
    __dict
       dictionary for instance variables (if defined)
    __weakref
       list of weak references to the object (if defined)
class ConvexPolygonSampler(builtins.object)
   A class for uniform random sampling of
   points from a convex polygon. This
    class only supports convex polygons because
   the random sampling process involves
    triangulating a polygon, which is trivial
    for convex polygons only. "randgen" is a RandomGen
   object, and "points" is a list of points
    (two-item lists) that make up the polygon.
   Methods defined here:
   __init__(self, randgen, points)
       Initialize self. See help(type(self)) for accurate signature.
    sample(self)
        Choose a random point in the convex polygon
        uniformly at random.
   Data descriptors defined here:
        dictionary for instance variables (if defined)
     weakref
       list of weak references to the object (if defined)
class DensityInversionSampler(builtins.object)
  A sampler that generates random samples from
      a continuous distribution for which
      only the probability density function (PDF) is known,
      using the inversion method. This sampler
      allows quantiles for the distribution to be calculated
      from pregenerated uniform random numbers in [0, 1].
    - pdf: A function that specifies the PDF. It takes a single
      number and outputs a single number. The area under
      the PDF need not equal 1 (this sampler works even if the
      PDF is only known up to a normalizing constant).
    - bl, br - Specifies the sampling domain of the PDF. Both
      bl and br are numbers giving the domain,
      which in this case is [bl, br]. For best results, the
      probabilities outside the sampling domain should be
       negligible (the reference cited below uses cutoff points
       such that the probabilities for each tail integrate to
       about ures*0.05 or less).
```

```
- ures - Maximum approximation error tolerable, or
      "u-resolution". Default is 10^-8. This error tolerance
      "does not work for continuous distributions [whose PDFs
      have] high and narrow peaks or poles".
      Reference:
      Gerhard Derflinger, Wolfgang Hörmann, and Josef Leydold,
      "Random variate generation by numerical inversion when
      only the density is known", ACM Transactions on Modeling
      and Computer Simulation 20(4) article 18, October 2010.
   Methods defined here:
    init (self, pdf, bl, br, ures=1e-08)
        Initialize self. See help(type(self)) for accurate signature.
    codegen(self, name='dist')
       Generates standalone Python code that samples
                (approximately) from the distribution estimated
                in this class. Idea from Leydold, et al.,
                "An Automatic Code Generator for
                Nonuniform Random Variate Generation", 2001.
        - name: Distribution name. Generates Python methods called
           sample X (samples one random number), and quantile X
           (finds the quantile
           for a uniform random number in [0, 1]),
           where X is the name given here.
    quantile(self, v)
       Calculates quantiles from uniform random numbers
             in the interval [0, 1].
        - v: A list of uniform random numbers.
       Returns a list of the quantiles corresponding to the
        uniform random numbers. The returned list will have
        the same number of entries as 'v'.
    sample(self, rg, n=1)
       Generates random numbers that (approximately) follow the
              distribution modeled by this class.
        - n: The number of random numbers to generate.
        Returns a list of 'n' random numbers.
   Data descriptors defined here:
    dict
       dictionary for instance variables (if defined)
       list of weak references to the object (if defined)
class DensityTiling(builtins.object)
    Produces a tiling of a probability density function (PDF)
         for the purposes of random number generation. The PDF is
         decomposed into tiles; these tiles will either cross the PDF
         or go below the PDF. In each recursion cycle, each tile is
         split into four tiles, and tiles that end up above the PDF are
         discarded.
    - pdf: A function that specifies the PDF. It takes a single
      number and outputs a single number. The area under
      the PDF need not equal 1 (this class tolerates the PDF even if
```

```
the PDF should be bounded from above (that is, it should be free of poles, or
points
          that approach infinity). If the PDF does contain a pole, this class
          may accommodate the pole by sampling from a modified version of the PDF,
          so that points extremely close to the pole may be sampled
          at a higher or lower probability than otherwise (but not in a way
          that significantly affects the chance of sampling points
          outside the pole region).
        - bl, br - Specifies the sampling domain of the PDF. Both
          bl and br are numbers giving the domain,
          which in this case is [bl, br].
        - cycles - Number of recursion cycles in which to split tiles
           that follow the PDF. Default is 8.
         Additional improvements not yet implemented: Hörmann et al.,
         "Inverse Transformed Density Rejection for Unbounded Monotone Densities", 2007.
         Fulger, Daniel and Guido Germano. "Automatic generation of
         non-uniform random variates for arbitrary pointwise computable
         probability densities by tiling",
         arXiv:0902.3088v1 [cs.MS], 2009.
       Methods defined here:
        __init__(self, pdf, bl, br, cycles=8)
            Initialize self. See help(type(self)) for accurate signature.
        codegen(self, name, pdfcall=None)
           Generates Python code that samples
                    (approximately) from the distribution estimated
                    in this class. Idea from Leydold, et al.,
                    "An Automatic Code Generator for
                    Nonuniform Random Variate Generation", 2001.
            - name: Distribution name. Generates a Python method called
               sample X where X is the name given here (samples one
               random number).
            - pdfcall: Name of the method representing pdf (for more information,
               see the __init__ method of this class). Optional; if not given
               the name is pdf X where X is the name given in the name parameter.
        maybeAppend(self, pdfevals, newtiles, xmn, xmx, ymn, ymx)
        sample(self, rq, n=1)
           Generates random numbers that (approximately) follow the
                  distribution modeled by this class.
            - n: The number of random numbers to generate.
           Returns a list of 'n' random numbers.
       Data descriptors defined here:
           dictionary for instance variables (if defined)
         weakref
           list of weak references to the object (if defined)
    class FastLoadedDiceRoller(builtins.object)
     | Implements the Fast Loaded Dice Roller, which chooses a random number in [0,\ n)
     where the probability that each number is chosen is weighted. The 'weights' is the
```

it is only known up to a normalizing constant). For best results,

```
list of weights each 0 or greater; the higher the weight, the greater
   the probability. This sampler supports only integer weights.
   Reference: Saad, F.A., Freer C.E., et al. "The Fast Loaded Dice Roller: A
   Near-Optimal Exact Sampler for Discrete Probability Distributions", in
    _AISTATS 2020: Proceedings of the 23rd International Conference on Artificial
   Intelligence and Statistics, Proceedings of Machine Learning Research 108,
   Palermo, Sicily, Italy, 2020.
   Methods defined here:
   init (self, weights)
       Initialize self. See help(type(self)) for accurate signature.
   codegen(self, name='sample discrete')
       Generates standalone Python code that samples
               from the distribution modeled by this class.
               Idea from Leydold, et al.,
               "An Automatic Code Generator for
               Nonuniform Random Variate Generation", 2001.
       - name: Method name. Default: 'sample discrete'.
   next(self, randgen)
   ______
   Data descriptors defined here:
       dictionary for instance variables (if defined)
     weakref
       list of weak references to the object (if defined)
class KVectorSampler(builtins.object)
   A K-Vector-like sampler of a continuous distribution
   with a known cumulative distribution function (CDF).
   Uses algorithms
   described in Arnas, D., Leake, C., Mortari, D., "Random
   Sampling using k-vector", Computing in Science &
   Engineering 21(1) pp. 94-107, 2019, and Mortari, D.,
   Neta, B., "k-Vector Range Searching Techniques".
   Methods defined here:
    init (self, cdf, xmin, xmax, pdf=None, nd=200)
       Initializes the K-Vector-like sampler.
       Parameters:
        - cdf: Cumulative distribution function (CDF) of the
          distribution. The CDF must be
          monotonically nondecreasing everywhere in the
          interval [xmin, xmax] and must output values in [0, 1];
          for best results, the CDF should
          be increasing everywhere in [xmin, xmax].
       - xmin: Maximum x-value to generate.
       - xmax: Maximum x-value to generate. For best results,
          the range given by xmin and xmax should cover all or
          almost all of the distribution.
       - pdf: Optional. Distribution's probability density
          function (PDF), to improve accuracy in the root-finding
       - nd: Optional. Size of tables used in the sampler.
          Default is 200.
```

```
quantile(self, uniforms)
            Returns a list of 'n' numbers that correspond
            to the given uniform random numbers and follow
            the distribution represented by this sampler. 'uniforms'
            is a list of uniform random values in the interval
            [0, 1]. For best results, this sampler's range
            (xmin and xmax in the constructor)
            should cover all or almost all of the desired distribution and
            the distribution's CDF should be monotonically
            increasing everywhere (every number in the distribution's
            range has nonzero probability of occurring), since
            among other things,
            this method maps each uniform value to the
            range of CDFs covered by this distribution (that is,
            [0, 1] is mapped to [minCDF, maxCDF]), and
            uniform values in "empty" regions (regions with
            constant CDF) are handled by replacing those
            values with the minimum CDF value covered.
        sample(self, rq, n)
            Returns a list of 'n' random numbers of
            the distribution represented by this sampler.
            - rg: A random generator (RandGen) object.
       Data descriptors defined here:
        dict
            dictionary for instance variables (if defined)
            list of weak references to the object (if defined)
    class OptimalSampler(builtins.object)
        Implements a sampler which chooses a random number in [0, n)
        where the probability that each number is chosen is weighted. The 'weights' is the
        list of weights each 0 or greater; the higher the weight, the greater
        the probability. This sampler supports only integer weights, but the sampler is
        entropy-optimal as long as the sum of those weights is of the form 2<sup>k</sup> or 2<sup>k</sup>-2<sup>m</sup>.
       Reference: Feras A. Saad, Cameron E. Freer, Martin C. Rinard, and Vikash K.
Mansinghka.
       Optimal Approximate Sampling From Discrete Probability Distributions. Proc.
        ACM Program. Lang. 4, POPL, Article 36 (January 2020), 33 pages.
       Methods defined here:
        __init__(self, m)
            Initialize self. See help(type(self)) for accurate signature.
        codegen(self, name='sample discrete')
            Generates standalone Python code that samples
                    from the distribution modeled by this class.
                    Idea from Leydold, et al.,
                    "An Automatic Code Generator for
                    Nonuniform Random Variate Generation", 2001.
            - name: Method name. Default: 'sample discrete'.
        next(self, rg)
        nextFromMatrix(self, pm, rg)
```

```
Data descriptors defined here:
    __dict
       dictionary for instance variables (if defined)
       list \overline{\text{of}} weak references to the object (if defined)
class PascalTriangle(builtins.object)
    Generates the rows of Pascal's triangle, or the
   weight table for a binomial(n,1/2) distribution.
   Methods defined here:
    __init__(self)
       Initialize self. See help(type(self)) for accurate signature.
   aliasinfo(self, desiredRow)
    getrow(self, desiredRow)
       Calculates an arbitrary row of Pascal's triangle.
        Generates the next row of Pascal's triangle, starting with
        row 0. The return value is a list of row-number-choose-k
        values.
    nextto(self, desiredRow)
       Generates the row of Pascal's triangle with the given row number,
        skipping all rows in between. The return value is a list of
        row-number-choose-k values.
    row(self)
        Gets the row number of the row that will be generated
       the next time _next_ is called.
   Data descriptors defined here:
       dictionary for instance variables (if defined)
       list of weak references to the object (if defined)
class RandomGen(builtins.object)
   A class that implements many methods for
    random number generation and sampling. It takes
   an underlying RNG as specified in the constructor.
   Methods defined here:
    init (self, rng=None)
       Initializes a new RandomGen instance.
       NOTES:
       1. Assumes that 'rng' implements
       a 'randint(a, b)' method that returns a random
        integer in the interval [a, b]. Currently, this
        class assumes 'a' is always 0.
```

```
2. 'rndint' (and functions that ultimately call it) may be
    slower than desirable if many random numbers are
    needed at once. Ways to improve the performance
    of generating many random numbers at once include
    vectorization (which is often PRNG specific) and multithreading
    (which is too complicated to show here).
ball point(self, dims, radius=1)
    Generates an independent and uniform random point inside a 'dims'-dimensional
    ball (disc, solid sphere, etc.) centered at the origin.
bernoulli(self, p)
    Returns 1 at probability p, 0 otherwise.
beta(self, a, b, nc=0)
    Generates a beta-distributed random number.
    `a` and `b` are the two parameters of the beta distribution,
    and `nc` is a parameter such that `nc` other than 0
    indicates a noncentral distribution.
binomial(self, trials, p, n=None)
binomial int(self, trials, px, py)
cauchy(self)
choice(self, list)
derangement(self, list)
    Returns a copy of list with each of its elements
    moved to a different position.
derangement algorithm s(self, list)
    Returns a copy of 'list' with each of its elements
    moved to a different position (a derangement),
    but with the expected number of cycle lengths
    in probability, even though the list
    need not be a uniformly randomly
    chosen derangement. Uses importance sampling.
    Reference:
    J.R.G. Mendonça, "Efficient generation of
    random derangements with the expected
    distribution of cycle lengths", arXiv:1809.04571v4
    [stat.CO], 2020.
derangement algorithm t(self, list)
    Returns a copy of 'list' with each of its elements
    moved to a different position (a derangement),
    but with the expected number of cycle lengths
    in probability, even though the list
    need not be a uniformly randomly
    chosen derangement. Reference:
    J.R.G. Mendonça, "Efficient generation of
    random derangements with the expected
    distribution of cycle lengths", arXiv:1809.04571v4
    [stat.CO], 2020.
diceRoll(self, dice, sides=6, bonus=0)
dirichlet(alphas)
discretegen(self, probs)
```

Generates a random integer in [0, n), where the probability of drawing each integer is specified as a list of probabilities that sum to 1, where n is the number of probabilities. This method is optimal, or at least nearly so, in terms of the number of random bits required to generate the number on average. This method implements a solution to exercise 3.4.2 of chapter 15 of Luc Devroye's Non-Uniform Random Variate Generation , 1986. - probs. List of probability objects, where for each item in the probability list, the integer 'i' is chosen with probability 'probs[i]'. Each probability object provides access to a binary expansion of the probability, which must be a real number in the interval [0, 1]. The binary expansion is a sequence of zeros and ones expressed as follows: The first binary digit is the half digit, the second is the quarter digit, the third is the one-eighth digit, and so on. Note that any probability with a terminating binary expansion (except 0) can be implemented by "subtracting" 1 from the expansion and then appending an infinite sequence of ones at the end. The probability object must implement the following three methods: - reset(): Resets the probability object to the first digit in the binary expansion. - nextbit(): Gets the next digit in the binary expansion. - eof(): Gets whether the end of the binary expansion was reached (True or False), meaning the rest of the digits in the expansion are all zeros. The probability object will have to be mutable for this method to work. The BinaryExpansion class is a convenient way to express numbers as probability objects that meet these criteria. Each probability object can also be a float, int, or Fraction in the interval [0, 1]. expoNumerator(self, denom) Generates the numerator of an exponential random number with a given denominator, using von Neumann's algorithm ("Various techniques used in connection with random digits", 1951). expoRatio(self, base, rx=1, ry=1) Generates an exponential random number (in the form of a ratio, or two-element list) given the rate `rx`/`ry` and the base `base`. The number will have the denominator `base\*rx`. exponential(self, lamda=1.0) exprandfill(self, a, bits) Fills the unsampled bits of the given exponential random number 'a' as necessary to make a number whose fractional part has 'bits' many bits. If the number's fractional part already has that many bits or more, the number is rounded using the round-to-nearest, ties to even rounding rule. Returns the resulting number as a multiple of 2^'bits'. exprandless(self, a, b) Determines whether one partially-sampled exponential number is less than another; returns True if so and False otherwise. During

```
the comparison, additional bits will be sampled in both numbers
            if necessary for the comparison.
        exprandnew(self, lamdanum=1, lamdaden=1)
            Returns an object to serve as a partially-sampled
            exponential random number with the given
            rate 'lamdanum'/'lamdaden'. The object is a list of five numbers:
            the first is a multiple of 1/(2^X), the second is X, the third is the integer
            part (initially -1 to indicate the integer part wasn't sampled yet),
            and the fourth and fifth are the lamda parameter's
            numerator and denominator, respectively. Default for 'lamdanum'
            and 'lamdaden' is 1.
            The number created by this method will be "empty"
            (no bits sampled yet).
        frechet(self, a, b, mu=0)
        gamma(self, mean, b=1.0, c=1.0, d=0.0)
            Generates a random number following a gamma distribution.
        gaussian copula(self, cov)
        geoellipsoid point(self, a=6378.137, invf=298.2572236)
            Generates an independent and uniform random
            point on the surface of a geoellipsoid. The
            geoellipsoid uses the following parameters:
            a - semimajor axis (distance from the center of
               the geoellipsoid to the equator). The default
               is the WGS 84 ellipsoid's semimajor axis
               in kilometers.
            invf - inverse flattening. The default is the
               WGS 84 ellipsoid's inverse flattening.
        geometric(self, p)
        qumbel(self, a, b)
        hypercube point(self, dims, sizeFromCenter=1)
            Generates an independent and uniform random point on the surface of a 'dims'-
dimensional
            hypercube (square, cube, etc.)
            centered at the origin.
        hypergeometric(self, trials, ones, count)
        hypersphere point(self, dims, radius=1)
            Generates an independent and uniform random point on the surface of a 'dims'-
dimensional
            hypersphere (circle, sphere, etc.)
            centered at the origin.
        integersWithSum(self, n, total)
            Returns a list of 'n' integers 0 or greater that sum to 'total'.
            The combination is chosen uniformly at random among all
            possible combinations.
        integers_from_pdf(self, pdf, mn, mx, n=1)
            Generates one or more random integers from a discrete probability
            distribution expressed as a probability density
            function (PDF), which is also called the probability mass
            function for discrete distributions. The random integers will be in the interval [mn, mx]. `n` random integers will be
```

```
for that parameter, a weight indicating the relative likelihood
    that a random integer will equal that parameter.
    The area under the "curve" of the PDF need not be 1.
   By default, `n` is 1.
integers from u01(self, u01, pmf)
    Transforms one or more random numbers into numbers
    (called quantiles) that
    follow a discrete distribution, assuming the distribution
          produces only integers 0 or greater.
          - `u01` is a list of uniform random numbers, in [0, 1].
          - `pmf` is the probability mass function (PMF)
          of the discrete distribution; it takes one parameter and returns,
          for that parameter, the probability that a random number is
          equal to that parameter (each probability is in the interval [0, 1]).
          The area under the PMF must be 1: it
          is not enough for the PMF to be correct up to a constant.
intsInRangeSortedWithSum(self, numSamples, numPerSample, mn, mx, sum)
    Generates one or more combinations of
     'numPerSample' numbers each, where each
     combination's numbers sum to 'sum' and are listed
     in sorted order, and each
     number is in the interval '[mn, mx]'.
     The combinations are chosen uniformly at random.
         'mn', 'mx', and
     'sum' may not be negative. Returns an empty
     list if 'numSamples' is zero.
     The algorithm is thanks to a Stack Overflow
    answer (`questions/61393463`) by John McClane.
   Raises an error if there is no solution for the given
    parameters.
intsInRangeWithSum(self, numSamples, numPerSample, mn, mx, sum)
    Generates one or more combinations of
     'numPerSample' numbers each, where each
     combination's numbers sum to 'sum' and are listed
     in any order, and each
     number is in the interval '[mn, mx]'.
      The combinations are chosen uniformly at random.
         'mn', 'mx', and
     'sum' may not be negative. Returns an empty
     list if 'numSamples' is zero.
     The algorithm is thanks to a Stack Overflow
    answer (`questions/61393463`) by John McClane.
    Raises an error if there is no solution for the given
    parameters.
intsInRangesWithSum(self, numSamples, ranges, total)
    Generates one or more combinations of
     'len(ranges)' numbers each, where each
     combination's numbers sum to 'total', and each number
     has its own valid range. 'ranges' is a list of valid ranges
     for each number; the first item in each range is the minimum
     value and the second is the maximum value. For example,
     'ranges' can be [[1,4],[3,5],[2,6]], which says that the first
     number must be in the interval [1, 4], the second in [3, 5],
     and the third in [2, 6].
     The combinations are chosen uniformly at random.
         Neither the integers in the 'ranges' list nor
     'total' may be negative. Returns an empty
```

generated. `pdf` is the PDF; it takes one parameter and returns,

```
list if 'numSamples' is zero.
      This is a modification I made to an algorithm that
       was contributed in a Stack Overflow
    answer ('questions/61393463') by John McClane.
    Raises an error if there is no solution for the given
    parameters.
kth smallest of n u01(self, k, n)
    Generates the kth smallest number among n random numbers
    in the interval [0, 1].
kthsmallest(self, n, k, b)
    Generates the 'k'th smallest 'b'-bit uniform random
    number out of 'n' of them.
kthsmallest_urand(self, n, k)
    Generates the 'k'th smallest 'b'-bit uniform random
    number out of 'n' of them; returns the result in
    the form of a "u-rand", or a partially sampled uniform
    random number (Karney, "Sampling exactly from the normal distribution").
latlon(self)
    Generates an independent and uniform random latitude and
    longitude, in radians. West and south coordinates
    are negative.
lognormal(self, mu=0.0, sigma=0.0)
lower bound copula(self)
mcmc(self, pdf, n)
    Generates 'n' random numbers that follow
    the probability density given in 'pdf' using
    a Markov-chain Monte Carlo algorithm, currently
    Metropolis--Hastings. The resulting random numbers
    are not independent, but are often close to
    being independent. 'pdf' takes one number as
    a parameter and returns a number 0 or greater.
    The area under the curve (integral) of 'pdf'
    need not be equal to 1.
mcmc2(self, pdf, n)
    Generates 'n' pairs of random numbers that follow
    the probability density given in 'pdf' using
    a Markov-chain Monte Carlo algorithm, currently
    Metropolis--Hastings. The resulting random pairs
    are not independent, but are often close to
    being independent. 'pdf' takes one parameter,
    namely, a list of two numbers giving a sampled
    point and returns a number 0 or greater.
    The volume under the surface (integral) of 'pdf'
    need not be equal to 1.
monte_carlo_integrate(self, func, bounds, samples=1000)
    Estimates the integral (volume) of a function within the
    given bounds using Monte Carlo integration, which generates
    an estimate using the help of randomization.
    func - Function to integrate. Takes the same number
       of parameters as the length of bounds.
    bounds - Bounds of integration at each dimension.
       An N-length array of arrays. Each array in turn
       contains two items: the lower bound and upper bound
```

```
for that dimension.
    samples - Number of times to sample the bounds of
       integration randomly. The default is 1000 samples.
    Returns an array containing two items: the estimated
    integral and the standard error.
moyal(self, mu=0, sigma=1)
    Sample from a Moyal distribution, using the
    method given in C. Walck, "Handbook on
    Statistical Distributions for Experimentalists",
    pp. 93-94.
multinomial(self, trials, weights)
multinormal(self, mu, cov)
multinormal n(self, mu, cov, n=1)
multipoisson(self, firstmean, othermeans)
    Multivariate Poisson distribution (as found in Mathematica).
multivariate t(self, mu, cov, df)
    Multivariate t-distribution, mu is the mean (can be None),
    cov is the covariance matrix, and df is the degrees of freedom.
negativeMultinomial(self, succ, failures)
    Negative multinomial distribution.
    Models the number of failures of one or more
    kinds before a given number of successes happens.
    succ: Number of successes.
    failures: Contains probabilities for each kind of failure.
    The sum of probabilities must be less than 1.
    Returns: A list containing a random number
    of failures of each kind of failure.
negativebinomial(self, successes, p)
negativebinomialint(self, successes, px, py)
    Generates a negative binomial random number, defined
    here as the number of failures before 'successes' many
    successful trials, where the probability of success in
    each trial is px/py.
nonzeroIntegersWithSum(self, n, total)
    Returns a list of 'n' integers greater than 0 that sum to 'total'.
    The combination is chosen uniformly at random among all
    possible combinations.
normal(self, mu=0.0, sigma=1.0)
    Generates a normally-distributed random number.
numbersWithSum(self, count, sum=1.0)
numbers from cdf(self, cdf, mn, mx, n=1)
    Generates one or more random numbers from a continuous probability
    distribution by numerically inverting its cumulative
    distribution function (CDF).
    - cdf: The CDF; it takes one parameter and returns,
    for that parameter, the probability that a random number will
    be less than or equal to that parameter.
```

```
- mn, mx: Sampling domain. The random number
           will be in the interval [mn, mx].
            - n: How many random numbers to generate. Default is 1.
       numbers from dist(self, pdf, mn=0, mx=1, n=1, bitplaces=53)
           Generates 'n' random numbers that follow a continuous
            distribution in an interval [mn, mx]. The distribution's
           PDF (probability density function) must be bounded from above
            (have a finite value) and be continuous almost everywhere
           in the interval. Implements section 4 of Devroye and Gravel,
            "The expected bit complexity of the von Neumann rejection
           algorithm", arXiv:1511.02273v2 [cs.IT], 2016.
            - 'n' is the number of random numbers to generate. Default is 1.
            - 'pdf' is a procedure that takes three arguments: xmin, xmax, bitplaces,
              and returns an array of two items: the greatest lower bound of f(x) anywhere
              in the interval [xmin, xmax] (where f(x) is the PDF), and the least upper
              bound of f(x) anywhere there. Both bounds are multiples of 2^--bitplaces.
            - 'bitplaces' is an accuracy expressed as a number of bits after the
              binary point. The random number will be a multiple of 2^-bitplaces,
              or have a smaller granularity. Default is 53.
            - 'mn' and 'mx' express the interval. Both are optional and
              are set to 0 and 1, respectively, by default.
       numbers from dist inversion(self, icdf, n=1, digitplaces=53, base=2)
           Generates 'n' random numbers that follow a continuous
            or discrete probability distribution, using the inversion method.
            Implements section 5 of Devroye and Gravel,
            "Sampling with arbitrary precision", arXiv:1502.02539v5 [cs.IT], 2015.
            - 'n' is the number of random numbers to generate. Default is 1.
            - 'icdf' is a procedure that takes three arguments: u, ubits, digitplaces,
              and returns a number within base^-digitplaces of the True inverse
              CDF (inverse cumulative distribution function, or quantile function)
              of u/base^ubits, and is monotonic for a given value of `digitplaces`.
            - 'digitplaces' is an accuracy expressed as a number of digits after the
              point. Each random number will be a multiple of base^-digitplaces,
              or have a smaller granularity. Default is 53.
            - base is the digit base in which the accuracy is expressed. Default is 2
               (binary). (Note that 10 means decimal.)
       numbers from pdf(self, pdf, mn, mx, n=1, steps=100)
           Generates one or more random numbers from a continuous probability
           distribution expressed as a probability density
            function (PDF). The random number
           will be in the interval [mn, mx]. `n` random numbers will be
           generated. `pdf` is the PDF; it takes one parameter and returns,
            for that parameter, a weight indicating the relative likelihood
            that a random number will be close to that parameter. `steps`
            is the number of subintervals between sample points of the PDF.
           The area under the curve of the PDF need not be 1.
           By default, `n` is 1 and `steps` is 100.
       numbers from u01(self, u01, pdf, cdf, mn, mx, ures=None)
           Transforms one or more random numbers into numbers
            (called quantiles) that follow a continuous probability distribution, based on
its PDF
            (probability density function) and/or its CDF (cumulative distribution
            function).
            - u01: List of uniform random numbers in [0, 1] that will be
           transformed into numbers that follow the distribution.
            - pdf: The PDF; it takes one parameter and returns,
            for that parameter, the relative probability that a
```

```
the PDF need not be 1 (this method works even if the PDF
            is only known up to a normalizing constant). Optional if a CDF is given.
            - cdf: The CDF; it takes one parameter and returns,
            for that parameter, the probability that a random number will
            be less than or equal to that parameter. Optional if a PDF is given.
            For best results, the CDF should be
            monotonically nondecreasing everywhere in the
            interval [xmin, xmax] and must output values in [0, 1];
            for best results, the CDF should
            be increasing everywhere in [xmin, xmax].
            - mn, mx: Sampling domain. The random number
           will be in the interval [mn, mx]. For best results,
            the range given by mn and mx should cover all or
           almost all of the distribution.
            - ures - Maximum approximation error tolerable, or
            "u-resolution". Default is 10^-8. Currently used only if a
           PDF is given.
        pareto(self, minimum, alpha)
        partialshuffle(self, list, k)
           Does a partial shuffle of
           a list's items (stops when 'k' items
           are shuffled); the shuffled items
           will appear at the end of the list.
           Returns 'list'.
        piecewise_linear(self, values, weights)
        piecewise linear n(self, values, weights, n=1)
        poisson(self, mean)
           Generates a random number following a Poisson distribution.
        poissonint(self, mx, my)
            Generates a random number following a Poisson distribution with mean mx/my.
        polya_int(self, sx, sy, px, py)
            Generates a negative binomial (Polya) random number, defined
            here as the number of failures before 'successes' many
            successful trials (sx/sy), where the probability of success in
           each trial is px/py.
        powerlognormal(self, p, sigma=1.0)
            Power lognormal distribution, as described in NIST/SEMATECH
            e-Handbook of Statistical Methods, [http://www.itl.nist.gov/div898/handbook/,]
(http://www.itl.nist.gov/div898/handbook/,)
           accessed Jun. 9, 2018, sec. 1.3.6.6.14.
        powernormal(self, p)
            Power normal distribution, as described in NIST/SEMATECH
            e-Handbook of Statistical Methods, [http://www.itl.nist.gov/div898/handbook/,]
(http://www.itl.nist.gov/div898/handbook/,)
           accessed Jun. 9, 2018, sec. 1.3.6.6.13.
       product_copula(self, n=2)
       quantile urands(self, icdf, urands, digitplaces=53)
           Finds the quantile of 'n' uniform random numbers expressed as "u-rands", or
partially-sampled uniform random numbers (Karney, "Sampling exactly from the normal
distribution"). Implements section 5 of Devroye and Gravel, "Sampling with arbitrary
```

random number close to that number is chosen. The area under

```
precision", arXiv:1502.02539v5 [cs.IT], 2015.
            - 'urands' is a list of "u-rands", or partially-sampled uniform random numbers.
Each u-rand is a list of two items, namely a multiple of 1/2^X, followed by X. For example,
the following generates a list of five empty
            u-rands: `[[0,0] for i in range(5)]`.
            - 'icdf' is a procedure that takes three arguments: u, ubits, digitplaces,
               and returns a number within 2^-digitplaces of the True inverse
               CDF (inverse cumulative distribution function, or quantile function)
               of u/2^ubits, and is monotonic for a given value of `digitplaces`.
            - 'digitplaces' is an accuracy expressed as a number of bits after the
               point. Each quantile will be a multiple of 2^-digitplaces,
               or have a smaller granularity. Default is 53.
            Example: The following example generates the maximum of 10
            random numbers, to an accuracy of 2<sup>53</sup>.
            ur=randgen.kthsmallest urand(10. 10)
            maxrand=randgen.guantile urands(icdf, [ur], 53)[0]
        randbit(self)
        randbits(self, n)
            Generates an n-bit random integer.
        randomwalk posneg1(self, n)
            Random walk of uniform positive and negative steps.
        randomwalk u01(self, n)
            Random walk of uniform 0-1 random numbers.
        rayleigh(self, a)
            Generates a random number following a Rayleigh distribution.
        rndint(self, maxInclusive)
        rndint fastdiceroller(self, maxInclusive)
        rndintexc(self, maxExclusive)
        rndintexcrange(self, minInclusive, maxExclusive)
        rndintrange(self, minInclusive, maxInclusive)
        rndrange(self, minInclusive, maxInclusive)
        rndrangemaxexc(self, minInclusive, maxExclusive)
        rndrangeminexc(self, mn, mx)
        rndrangeminmaxexc(self, mn, mx)
        rndu01(self)
        rndu01oneexc(self)
        rndu01zeroexc(self)
       rndu01zerooneexc(self)
       sample(self, list, k)
       sattolo(self, list)
```

```
Puts the elements of 'list' in random order, choosing
            from among all cyclic permutations (Sattolo's algorithm).
            Returns 'list'.
        shell point(self, dims, outerRadius=1, innerRadius=0.5)
            Generates an independent and uniform random point inside a 'dims'-dimensional
            spherical shell (donut, hollow sphere, etc.)
            centered at the origin.
        shuffle(self, list)
            Puts the elements of 'list' in random order (does an
            in-place shuffle). Returns 'list'.
        simplex point(self, points)
            Generates an independent and uniform random point on the surface of an N-
dimensional
            simplex (line segment, triangle, tetrahedron, etc.)
           with the given coordinates.
        slicesample(self, pdf, n, xstart=0.1)
            Slice sampling of R. M. Neal.
            Generates 'n' random numbers that follow
            the probability density given in 'pdf' using
            slice sampling. The resulting random numbers
            are not independent, but are often close to
              being independent. 'pdf' takes one number as
              a parameter and returns a number 0 or greater.
              The area under the curve (integral) of 'pdf'
              need not be equal to 1. 'xstart' should be
            chosen such that `pdf(xstart)>0`.
       spsa minimize(self, func, guess, iterations=200, constrain=None, a=None, c=None,
acap=None)
            Tries to find a choice of parameters that minimizes the value
            of a scoring function, also called the objective function or loss
            function, starting from an initial guess. This method uses an
            algorithm called "simultaneous perturbation
            stochastic approximation", which is a randomized
            search for the minimum value of the objective function.
            func - Objective function, a function that calculates a score for the
            given array of parameters and returns that score. The score is a
             single number; the lower the score, the better.
            The score can be negative. (Note that the problem of maximizing
            the score is the same as minimizing it except
             that the score's sign is reversed at the end.)
            guess - Initial guess for the best choice of parameters. This is an
            array of parameters, each of which is a number. This array has
             as many items as the array passed to 'func'.
            iterations - Maximum number of iterations in which to run the
             optimization process. Default is 200.
            constrain - Optional. A function that takes the given array of
            parameters and constrains them to fit the bounds of a valid
            array of parameters. This function modifies the array in place.
            a - Optional. A setting used in the optimization process; greater than \theta.
            c - Optional. A setting used in the optimization process; greater than 0. As a
quideline,
              'c' is about equal to the "standard deviation of the measurement noise"
              for several measurements at the initial guess, and is a "small positive
              number" if measurements are noise-free (Spall 1998). Default
              is 0.001.
            acap - Optional. A setting used in the optimization process; an
              integer greater than 0.
```

```
stable(self, alpha, beta)
            Generates a random number following a stable distribution.
        stable0(self, alpha, beta, mu=0, sigma=1)
           Generates a random number following a 'type 0' stable distribution.
        surface point(self, f, bounds, ngrad, gmax)
           Generates a uniform random point on
               a parametric surface, using a rejection
               approach developed by Williamson, J.F.,
               "Random selection of points distributed on
               curved surfaces", Physics in Medicine & Biology 32(10), 1987.
            - f: Takes two parameters (u and v) and returns
              a 3-element array expressing
              a 3-dimensional position at the given point.
            - bounds: Two 2-element arrays expressing bounds
              for u and v. Of the form [[umin, umax], [vmin,
              vmax]].
            - ngrad: Takes two parameters (u and v) and returns
              the norm of the gradient (stretch factor)
              at the given point. Can be None, in which
              the norm-of-gradient is calculated numerically.
            - gmax: Maximum norm-of-gradient
              for entire surface.
        t copula(self, cov, df)
            Multivariate t-copula. 'cov' is the covariance matrix
            and 'df' is the degrees of freedom.
       triangular(self, startpt, midpt, endpt)
        truncnormal(randgen, a, b)
            Samples from a truncated normal distribution in [a, b]; this method is
            designed to sample from either tail of that distribution.
            Reference:
            Botev, Z. and L'Ecuyer, P., 2019. Simulation from the Tail of the
           Univariate and Multivariate Normal Distribution. In Systems
           Modeling: Methodologies and Tools (pp. 115-132). Springer, Cham.
        upper bound copula(self, n=2)
       vonmises(self, mean, kappa)
       weibull(self, a, b)
            Generates a Weibull-distributed random number.
       weighted choice(self, weights)
       weighted choice inclusion(self, weights, n)
            Chooses a random sample of `n` indices from a list of items (whose weights are
given as `weights`), such that the chance that index `k` is in the sample is given as
weights[k]*n/Sum(weights)`. It implements the splitting method found in pp. 73-74 in
"Algorithms of sampling with equal or unequal probabilities",
www.eustat.eus/productosServicios/52.1 Unequal prob sampling.pdf .
       weighted choice n(self, weights, n=1)
       wiener(self, st, en, step=1.0, mu=0.0, sigma=1.0)
            Generates random numbers following a Wiener
            process (Brownian motion). Each element of the return
```

```
value contains a timestamp and a random number in that order.
        zero or one(self, px, py)
           Returns 1 at probability px/py, 0 otherwise.
        zero_or_one_exp_minus(self, x, y)
            Generates 1 with probability exp(-px/py); 0 otherwise.
           Reference:
           Canonne, C., Kamath, G., Steinke, T., "The Discrete Gaussian
            for Differential Privacy", arXiv:2004.00010v2 [cs.DS], 2020.
        zero or one power(self, px, py, n)
           Generates 1 with probability (px/py)^n (where n can be positive, negative, or
zero); 0 otherwise.
        zero or one power_ratio(self, px, py, nx, ny)
           Generates 1 with probability (px/py)^(nx/ny) (where nx/ny can be positive,
negative, or zero); 0 otherwise.
       Data descriptors defined here:
        __dict
           dictionary for instance variables (if defined)
       weakref
           list of weak references to the object (if defined)
       Data and other attributes defined here:
       FPPRECISION = 53
        FPRADIX = 2
       MINEXPONENT = -1074
    class RatioOfUniformsTiling(builtins.object)
       Produces a tiling for the purposes
            of fast sampling from a probability distribution via the
             ratio of uniforms method.
        - pdf: The probability density function (PDF); it takes one parameter and returns,
           for that parameter, the relative probability that a
           random number close to that number is chosen. The area under
           the PDF need not be 1; this method works even if the PDF
           is only known up to a normalizing constant, and even if
           the distribution has infinitely extending tails to the left and/or right.
          However, for the ratio of uniforms method to work, both pdf(x) and
          x*x*pdf(x) must be bounded from above (thus, if the distribution has
           tails, they must drop off at a faster than quadratic rate).
        - mode: X-coordinate of the PDF's highest peak or one of them,
          or a location close to it. Optional; default is 0.
        - y0, y1: Bounding coordinates for the ratio-of-uniforms tiling.
           For this class to work, y0 \le min(x*sqrt(pdf(x))) and
          y1 >= max( x*sqrt(pdf(x)) ) for all x. Optional; the default is y0=-10, y1=10.
        - cycles - Number of recursion cycles in which to split tiles
           for the ratio-of-uniforms tiling. Default is 8.
         Additional improvements not yet implemented:
         Generalized ratio-of-uniforms in Hörmann et al., "Automatic
         Nonuniform Random Variate Generation", 2004.
```

```
Section IV.7 of Devroye, L., "Non-Uniform Random Variate Generation", 1986.
     Section 4.5 of Fulger, D., "From phenomenological modelling of anomalous
     diffusion through continuous-time random walks and fractional
     calculus to correlation analysis of complex systems", dissertation,
     Philipps-Universität Marburg, 2009.
   Methods defined here:
    __init__(self, pdf, mode=0, y0=-10, y1=10, cycles=8)
        Initialize self. See help(type(self)) for accurate signature.
    codegen(self, name, pdfcall=None)
       Generates Python code that samples
                (approximately) from the distribution estimated
                in this class. Idea from Leydold, et al.,
                "An Automatic Code Generator for
                Nonuniform Random Variate Generation", 2001.
        - name: Distribution name. Generates a Python method called
           sample X where X is the name given here (samples one
           random number).
        - pdfcall: Name of the method representing pdf (for more information,
           see the init method of this class). Optional; if not given
           the name is pdf X where X is the name given in the name parameter.
   maybeAppend(self, newtiles, xmn, xmx, ymn, ymx, depth=0)
    sample(self, rg, n=1)
       Generates random numbers that (approximately) follow the
             distribution modeled by this class.
        - n: The number of random numbers to generate.
       Returns a list of 'n' random numbers.
    svg(self)
   Data descriptors defined here:
   __dict
       dictionary for instance variables (if defined)
       list of weak references to the object (if defined)
class SortedAliasMethod(builtins.object)
 | Implements a weighted sampling table
   where each weight must be in sorted
   order (ascending or descending).
   When many entries are in the table,
   the initialization is faster than with
    FastLoadedDiceRoller or VoseAlias. Reference:
   K. Bringmann and K. Panagiotou, "Efficient Sampling
   Methods for Discrete Distributions." In: Proc. 39th
    International Colloquium on Automata, Languages,
    and Programming (ICALP'12), 2012.
    - p: List of weights, in sorted order (ascending or
       descending).
   Methods defined here:
   __init__(self, p)
```

```
next(self, rg)
       ______
       Data descriptors defined here:
           dictionary for instance variables (if defined)
           list of weak references to the object (if defined)
   class VoseAlias(builtins.object)
    | Implements Vose's version of the alias sampler, which chooses a random number in [0,
n)
       where the probability that each number is chosen is weighted. The 'weights' is the
       list of weights each 0 or greater; the higher the weight, the greater
       the probability. This sampler supports integer or non-integer weights.
       Vose, Michael D. "A linear algorithm for generating random numbers with a given
       distribution." IEEE Transactions on software engineering 17, no. 9 (1991): 972-975.
       Methods defined here:
       __init__(self, weights)
           Initialize self. See help(type(self)) for accurate signature.
       next(self, randgen)
       ______
       Data descriptors defined here:
       __dict
           dictionary for instance variables (if defined)
           list of weak references to the object (if defined)
FUNCTIONS
   numericalTable(func, x, y, n=100)
   urandfill(rg, a, bits=53)
       Fills the unsampled bits of the given u-rand 'a' as necessary to
             make a number with 'bits' many bits. If the u-rand already has
             that many bits or more, the u-rand is rounded using the round-to-nearest,
             ties to even rounding rule. Returns the resulting number as a
             multiple of 2^'bits'. Default for 'bits' is 53.
       - rg: An object that must supply a 'randbit' method that generates an
         unbiased random bit.
   urandgreater(rg, a, b)
       Determines whether the first u-rand is greater than another u-rand; returns
             True if so and False otherwise. During
             the comparison, additional bits will be sampled in both u-rands if necessary
             for the comparison.
       - rg: An object that must supply a 'randbit' method that generates an
         unbiased random bit.
        - a, b: The first and second u-rands.
   urandless(rg, a, b)
```

Initialize self. See help(type(self)) for accurate signature.

```
True if so and False otherwise. During
              the comparison, additional bits will be sampled in both u-rands if necessary
              for the comparison.
        - rg: An object that must supply a 'randbit' method that generates an
          unbiased random bit.
         - a, b: The first and second u-rands.
    urandnew()
        Returns an object to serve as a partially-sampled uniform random
        number called a "u-rand" (Karney, "Sampling exactly from the normal distribution").
        A u-rand is a list of two numbers: the first is a multiple of 1/(2^X), and the second
is X.
        The urand created by this method will be "empty" (no bits sampled yet).
FTLF
    /home/rooster/Documents/SharpDevelopProjects/peteroupc.github.io/randomgen.py
Help on module fixed:
NAME
    fixed
CLASSES
    builtins.object
        Fixed
    class Fixed(builtins.object)
       Fixed-point numbers, represented using integers that store multiples
       of 2^-BITS. They are not necessarily faster than floating-point numbers, nor
        do they necessarily have the same precision or resolution of floating-point
        numbers. The main benefit of fixed-point numbers is that they improve
       determinism for applications that rely on non-integer real numbers (notably
       simulations and machine learning applications), in the sense that the operations
       given here deliver the same answer for the same input across computers,
       whereas floating-point numbers have a host of problems that make repeatable
        results difficult, including differences in their implementation, rounding
        behavior, and order of operations, as well as nonassociativity of
        floating-point numbers.
       The operations given here are not guaranteed to be "constant-time"
        (non-data-dependent and branchless) for all relevant inputs.
       Any copyright to this file is released to the Public Domain. In case this is not
        possible, this file is also licensed under Creative Commons Zero version 1.0.
       Methods defined here:
       __abs__(self)
       __add__(a, b)
        __cmp__(self, other)
        __div__(a, b)
        __eq__(self, other)
           Return self==value.
       __float__(a)
        floordiv (a, b)
```

Determines whether the first u-rand is less than another u-rand; returns

```
ge_(self, other)
    Return self>=value.
 __gt__(self, other)
    Return self>value.
__init__(self, i)
    Initialize self. See help(type(self)) for accurate signature.
__int__(a)
__le__(self, other)
    Return self<=value.
__lt__(self, other)
    Return self<value.
mod (a, b)
__mul__(a, b)
 ne (self, other)
    Return self!=value.
__neg__(self)
__pos__(self)
__rdiv__(a, b)
__repr__(self)
    Return repr(self).
__rtruediv__(a, b)
str (self)
    Return str(self).
sub (a, b)
__truediv__(a, b)
acos(a)
    Calculates an approximation of the inverse cosine of the given number.
    Calculates an approximation of the inverse sine of the given number.
atan2(y, x)
    Calculates the approximate measure, in radians, of the angle formed by the
    X axis and a line determined by the origin and the given coordinates of a 2D
    point. This is also known as the inverse tangent.
cos(a)
     Calculates the approximate cosine of the given angle; the angle is in radians.
    For the fraction size used by this class, this method is accurate to within
    1 unit in the last place of the correctly rounded result for all inputs
    in the range [-pi*2, pi*2].
    This method's accuracy decreases beyond that range.
exp(a)
```

```
Calculates an approximation of e (base of natural logarithms) raised
    to the power of this number. May raise an error if this number
    is extremely high.
floor(a)
    Calculates an approximation of the natural logarithm of this number.
    Calculates an approximation of this number raised to the power of another number.
round(a)
    Calculates the approximate sine of the given angle; the angle is in radians.
    For the fraction size used by this class, this method is accurate to within
    1 unit in the last place of the correctly rounded result for all inputs
    in the range [-pi*2, pi*2].
    This method's accuracy decreases beyond that range.
    Calculates an approximation of the square root of the given number.
    Calculates the approximate tangent of the given angle; the angle is in radians.
    For the fraction size used by this class, this method is accurate to within
    2 units in the last place of the correctly rounded result for all inputs
    in the range [-pi*2, pi*2].
    This method's accuracy decreases beyond that range.
Static methods defined here:
v(i)
    Converts a string, integer, Decimal, or other number type into
    a fixed-point number. If the parameter is a Fixed, returns itself.
    If the given number is a non-integer, returns the closest value to
    a Fixed after rounding using the round-to-nearest-ties-to-even
    rounding mode. The parameter is recommended to be a string
    or integer, and is not recommended to be a `float`.
Data descriptors defined here:
__dict
    dictionary for instance variables (if defined)
    list of weak references to the object (if defined)
Data and other attributes defined here:
ArcTanBitDiff = 9
ArcTanFrac = 29
ArcTanHTable = [0, 294906490, 137123709, 67461703, 33598225, 16782680,...
ArcTanTable = [421657428, 248918914, 131521918, 66762579, 33510843, 16...
```

```
BITS = 20
        ExpK = 648270061
       HALF = 524288
       HalfPiArcTanBits = 843314856
       HalfPiBits = 1647099
        HalfPiHighRes = 130496653328243011213339889301986179
        HighResFrac = 116
        Ln2ArcTanBits = 372130559
        Log2Bits = 726817
        LogMin = 157286.4
        MASK = 1048575
        PiAndHalfHighRes = 391489959984729033640019667905958538
        PiArcTanBits = 1686629713
        PiBits = 3294199
        PiHighRes = 260993306656486022426679778603972359
        QuarterPiArcTanBits = 421657428
        SinCosK = 326016435
        TwoTimesPiArcTanBits = 3373259426
       TwoTimesPiBits = 6588397
       TwoTimesPiHighRes = 521986613312972044853359557207944718
        _{\rm hash} = None
FILE
    /home/rooster/Documents/SharpDevelopProjects/peteroupc.github.io/fixed.py
Help on module bernoulli:
NAME
    bernoulli
CLASSES
    builtins.object
        Bernoulli
        DiceEnterprise
    class Bernoulli(builtins.object)
     | This class contains methods that generate Bernoulli random numbers,
           (either 1 or heads with a given probability, or 0 or tails otherwise).
           This class also includes implementations of so-called "Bernoulli factories",
algorithms
     | that turn coins biased one way into coins biased another way.
     | Written by Peter 0.
```

```
References:
       - Flajolet, P., Pelletier, M., Soria, M., "On Buffon machines and numbers",
       arXiv:0906.5560v2 [math.PR], 2010.
       - Huber, M., "Designing perfect simulation algorithms using local correctness",
       arXiv:1907.06748v1 [cs.DS], 2019.
       - Huber, M., "Nearly optimal Bernoulli factories for linear functions",
       arXiv:1308.1562v2 [math.PR], 2014.
       - Huber, M., "Optimal linear Bernoulli factories for small mean problems",
       arXiv:1507.00843v2 [math.PR], 2016.
       - Łatuszyński, K., Kosmidis, I., Papaspiliopoulos, O., Roberts, G.O., "Simulating
       events of unknown probabilities via reverse time martingales", arXiv:0907.4018v2
       [stat.CO], 2009/2011.
        - Goyal, V. and Sigman, K. 2012. On simulating a class of Bernstein
       polynomials. ACM Transactions on Modeling and Computer Simulation 22(2),
       Article 12 (March 2012), 5 pages.
       - Morina, G., Łatuszyński, K., et al., "From the Bernoulli Factory to a Dice
       Enterprise via Perfect Sampling of Markov Chains",
       arXiv:1912.09229v1 [math.PR], 2019.
        - Shaddin Dughmi, Jason D. Hartline, Robert Kleinberg, and Rad Niazadeh.
       2017. Bernoulli Factories and Black-Box Reductions in Mechanism Design.
       In Proceedings of 49th Annual ACM SIGACT Symposium on the Theory
       of Computing_, Montreal, Canada, June 2017 (STOC'17).
        - Gonçalves, F. B., Łatuszyński, K. G., Roberts, G. O. (2017). Exact Monte
       Carlo likelihood-based inference for jump-diffusion processes.
        - Vats, D., Gonçalves, F. B., Łatuszyński, K. G., Roberts, G. O. Efficient
       Bernoulli factory MCMC for intractable likelihoods, arXiv:2004.07471v1
       [stat.CO], 2020.
        - Mendo, Luis. "An asymptotically optimal Bernoulli factory for certain
       functions that can be expressed as power series." Stochastic Processes and their
       Applications 129, no. 11 (2019): 4366-4384.
        - Canonne, C., Kamath, G., Steinke, T., "The Discrete Gaussian
       for Differential Privacy", arXiv:2004.00010v2 [cs.DS], 2020.
       - Lee, A., Doucet, A. and Łatuszyński, K., 2014. Perfect simulation using
       atomic regeneration with application to Sequential Monte Carlo,
       arXiv:1407.5770v1 [stat.CO]
       Methods defined here:
        __init__(self)
            Creates a new instance of the Bernoulli class.
       add(self, f1, f2, eps=Fraction(1, 20))
           Addition Bernoulli factory: B(p), B(q) \Rightarrow B(p+q) (Dughmi et al. 2017)
            - f1, f2: Functions that return 1 if heads and 0 if tails.
            - eps: A Fraction in (0, 1). eps must be chosen so that p+q <= 1 - eps,
             where p and q are the probability of heads for f1 and f2, respectively.
       alt series(self, f, series)
            Alternating-series Bernoulli factory: B(p) \rightarrow B(s[0] - s[1]*p + s[2]*p^2 - ...)
            (Łatuszyński et al. 2011).
            - f: Function that returns 1 if heads and 0 if tails.
            - series: Object that generates each coefficient of the series starting with the
first.
              Each coefficient must be less than or equal to the previous and all of them
              be 1 or less.
              Implements the following two methods: reset() resets the object to the first
              coefficient; and next() generates the next coefficient.
       arctan n div n(self, f)
            Arctan div N: B(p) \rightarrow B(arctan(p)/p). Uses a uniformly-fast special case of
```

must

```
the two-coin Bernoulli factory, rather than the even-parity construction in
    Flajolet's paper, which is not uniformly fast.
   Reference: Flaiolet et al. 2010.
     - f: Function that returns 1 if heads and 0 if tails.
bernoulli x(self, f, x)
    Bernoulli factory with a given probability: B(p) \Rightarrow B(x) (Mendo 2019).
        Mendo calls Bernoulli factories "non-randomized" if their randomness
        is based entirely on the underlying coin.
    - f: Function that returns 1 if heads and 0 if tails.
    - x: Desired probability, in [0, 1].
bernstein(self, f, alpha)
    Bernstein polynomial Bernoulli factory: B(p) => B(Bernstein(alpha))
         (Goyal and Sigman 2012).
    - f: Function that returns 1 if heads and 0 if tails.
    - alpha: List of Bernstein coefficients for the Bernstein polynomial,
       whose degree is this list's length minus 1.
       For this to work, each coefficient must be in [0, 1].
    Convenience method to generate a function that returns
    1 (heads) with the given probability c (which must be in [0, 1])
   and 0 (tails) otherwise.
complement(self, f)
    Complement (NOT): B(p) \Rightarrow B(1-p) (Flajolet et al. 2010)
    - f: Function that returns 1 if heads and 0 if tails.
conditional(self, f1, f2, f3)
    Conditional: B(p), B(q), B(r) \Rightarrow B((1-r)*q+r*p) (Flajolet et al. 2010)
    - f1, f2, f3: Functions that return 1 if heads and 0 if tails.
cos(self. f)
    Cosine Bernoulli factory: B(p) \Rightarrow B(cos(p)). Special
    case of Algorithm3 of reverse-time martingale paper.
disjunction(self, f1, f2)
   Disjunction (OR): B(p), B(q) \Rightarrow B(p+q-p*q) (Flajolet et al. 2010)
    - f1, f2: Functions that return 1 if heads and 0 if tails.
divoneplus(self, f)
   Divided by one plus p: B(p) \Rightarrow B(1/(1+p)), implemented
            as a special case of the two-coin construction. Prefer over even-parity
            for being uniformly fast.
    - f: Function that returns 1 if heads and 0 if tails.
   Note that this function is slow as the probability of heads approaches 1.
eps div(self, f, eps)
    Bernoulli factory as follows: B(p) -> B(eps/p) (Lee et al. 2014).
    - f: Function that returns 1 if heads and 0 if tails.
    - eps: Fraction in (0, 1), must be chosen so that eps < p, where p is
      the probability of heads.
evenparity(self, f)
    Even parity: B(p) \Rightarrow B(1/(1+p)) (Flajolet et al. 2010)
    - f: Function that returns 1 if heads and 0 if tails.
   Note that this function is slow as the probability of heads approaches 1.
exp minus(self, f)
    Exp-minus Bernoulli factory: B(p) -> B(exp(-p)) (Łatuszyński et al. 2011).
    - f: Function that returns 1 if heads and 0 if tails.
```

```
exp minus ext(self, f, c=0)
            Extension to the exp-minus Bernoulli factory of (Łatuszyński et al. 2011):
            B(p) \rightarrow B(exp(-p - c))
            To the best of my knowledge, I am not aware
                   of any article or paper that presents this particular
                   Bernoulli factory (before my articles presenting
                   accurate beta and exponential generators).
            - f: Function that returns 1 if heads and 0 if tails.
            - c: Integer part of exp-minus. Default is 0.
        fill geometric bag(self, bag, precision=53)
        geometric bag(self, u)
            Bernoulli factory for a uniformly-distributed random number in (0, 1)
            (Flajolet et al. 2010).
            - u: List that holds the binary expansion, from left to right, of the uniformly-
              distributed random number. Each element of the list is 0, 1, or None (meaning
              the digit is not yet known). The list may be expanded as necessary to put
              a new digit in the appropriate place in the binary expansion.
        linear(self, f, cx, cy=1, eps=Fraction(1, 20))
            Linear Bernoulli factory: B(p) \Rightarrow B((cx/cy)*p) (Huber 2016).
            - f: Function that returns 1 if heads and 0 if tails.
            - cx, cy: numerator and denominator of c; the probability of heads (p) is
multiplied
              by c. c must be 0 or greater. If c > 1, c must be chosen so that c*p <= 1
eps.
            - eps: A Fraction in (0, 1). If c > 1, eps must be chosen so that c*p <= 1 - eps.
        linear lowprob(self, f, cx, cy=1, m=Fraction(249, 500))
            Linear Bernoulli factory which is faster if the probability of heads is known
                to be less than half: B(p) \Rightarrow B((cx/cy)*p) (Huber 2016).
            - f: Function that returns 1 if heads and 0 if tails.
            - cx, cy: numerator and denominator of c; the probability of heads (p) is
multiplied
              by c. c must be 0 or greater. If c > 1, c must be chosen so that c*p <= m <
1/2.
            - m: A Fraction in (0, 1/2). If c > 1, m must be chosen so that c*p <= m < 1/2.
        linear_power(self, f, cx, cy=1, i=1, eps=Fraction(1, 20))
            Linear-and-power Bernoulli factory: B(p) \Rightarrow B((p*cx/cy)^i) (Huber 2019).
            - f: Function that returns 1 if heads and 0 if tails.
            - cx, cy: numerator and denominator of c; the probability of heads (p) is
multiplied
              by c. c must be 0 or greater. If c > 1, c must be chosen so that c*p <= 1
eps.
            - i: The exponent. Must be an integer and 0 or greater.
            - eps: A Fraction in (0, 1). If c > 1, eps must be chosen so that c*p <= 1 - eps.
        logistic(self, f, cx=1, cy=1)
            Logistic Bernoulli factory: B(p) \rightarrow B(cx*p/(cy+cx*p)) or
                B(p) \rightarrow B((cx/cy)*p/(1+(cx/cy)*p)) (Morina et al. 2019)
            - f: Function that returns 1 if heads and 0 if tails. Note that this function
can
              be slow as the probability of heads approaches 0.
            - cx, cy: numerator and denominator of c; the probability of heads (p) is
multiplied
              by c. c must be in (0, 1).
        mean(self, f1, f2)
            Mean: B(p), B(q) \Rightarrow B((p+q)/2) (Flajolet et al. 2010)
```

```
- f1, f2: Functions that return 1 if heads and 0 if tails.
        old linear(self, f, cx, cv=1, eps=Fraction(1, 20))
            Linear Bernoulli factory: B(p) \Rightarrow B((cx/cy)*p). Older algorithm given in (Huber
2014)
            - f: Function that returns 1 if heads and 0 if tails.
            - cx, cy: numerator and denominator of c; the probability of heads (p) is
multiplied
              by c. c must be 0 or greater. If c > 1, c must be chosen so that c*p < 1 - eps.
            - eps: A Fraction in (0, 1). If c > 1, eps must be chosen so that c*p < 1 - eps.
        one div pi(self)
            Generates 1 with probability 1/pi.
            Reference: Flajolet et al. 2010.
        power(self, f, ax, ay=1)
            Power Bernoulli factory: B(p) \Rightarrow B(p^(ax/ay)). (case of (0, 1) provided by
             Mendo 2019).
            - f: Function that returns 1 if heads and 0 if tails.
            - ax, ay: numerator and denominator of the desired power to raise the probability
             of heads to. This power must be 0 or greater.
        powerseries(self, f)
            Power series Bernoulli factory: B(p) \Rightarrow B(1 - c(0)*(1-p) + c(1)*(1-p)^2 +
              c(2)*(1-p)^3 + ...), where c(i) = c[i]/sum(c) (Mendo 2019).
            - f: Function that returns 1 if heads and 0 if tails.
            - c: List of coefficients in the power series, all of which must be
              non-negative integers.
        probgenfunc(self, f, rng)
            Probability generating function Bernoulli factory: B(p) \Rightarrow B(E[p^x]), where x is
rng()
             (Dughmi et al. 2017). E[p^x] is the expected value of p^x and is also known
             as the probability generating function.
            - f: Function that returns 1 if heads and 0 if tails.
            - rng: Function that returns a non-negative integer at random.
              Example (Dughmi et al. 2017): if 'rng' is Poisson(lamda) we have
              an "exponentiation" Bernoulli factory as follows:
              B(p) \Rightarrow B(exp(p*lamda-lamda))
        product(self, f1, f2)
            Product (conjunction; AND): B(p), B(q) \Rightarrow B(p*q) (Flajolet et al. 2010)
            - f1, f2: Functions that return 1 if heads and 0 if tails.
            Generates a random bit that is 1 or 0 with equal probability.
        rndintexc(self, maxexc)
            Returns a random integer in [0, maxexc).
        sin(self, f)
            Sine Bernoulli factory: B(p) \Rightarrow B(\sin(p)). Special
            case of Algorithm3 of reverse-time martingale paper.
        square(self, f1, f2)
            Square: B(p) \Rightarrow B(1-p). (Flajolet et al. 2010)
            - f1, f2: Functions that return 1 if heads and 0 if tails.
        twocoin(self, f1, f2, c1=1, c2=1, beta=1)
            Two-coin Bernoulli factory: B(p), B(q) \Rightarrow
                       B(c1*p*beta / (beta * (c1*p+c2*q) - (beta - 1)*(c1+c2)))
                 (Goncalves et al. 2017, Vats et al. 2020; in Vats et al.,
```

```
C1,p1 corresponds to cy and C2,p2 corresponds to cx).
                Logistic Bernoulli factory is a special case with q=1, c2=1, beta=1.
            - fl. f2: Functions that return 1 if heads and 0 if tails.
            - c1, c2: Factors to multiply the probabilities of heads for f1 and f2,
respectively.
            - beta: Early rejection parameter ("portkey" two-coin factory).
              When beta = 1, the formula simplifies to B(c1*p/(c1*p+c2*q)).
        twofacpower(self, fbase, fexponent)
            Bernoulli factory B(p, q) \Rightarrow B(p^q).
            Based on algorithm from (Mendo 2019),
            but changed to accept a Bernoulli factory
            rather than a fixed value for the exponent.
            To the best of my knowledge, I am not aware
            of any article or paper that presents this particular
            Bernoulli factory (before my articles presenting
            accurate beta and exponential generators).
            - fbase, fexponent: Functions that return 1 if heads and 0 if tails.
              The first is the base, the second is the exponent.
        zero or one(self, px, py)
            Returns 1 at probability px/py, 0 otherwise.
        zero or one arctan n div n(self, x, y=1)
            Generates 1 with probability arctan(x/y)*y/x; 0 otherwise.
               x/y must be in [0, 1]. Uses a uniformly-fast special case of
            the two-coin Bernoulli factory, rather than the even-parity construction in
            Flajolet's paper, which is not uniformly fast.
            Reference: Flajolet et al. 2010.
        zero or one exp minus(self, x, y)
            Generates 1 with probability exp(-x/y); 0 otherwise.
           Reference: Canonne et al. 2020.
        zero or one log1p(self, x, y=1)
            \overline{Generates} 1 with probability log(1+x/y); 0 otherwise.
           Reference: Flajolet et al. 2010. Uses a uniformly-fast special case of
            the two-coin Bernoulli factory, rather than the even-parity construction in
            Flajolet's paper, which is not uniformly fast.
        zero or one pi div 4(self)
            Generates 1 with probability pi/4.
           Reference: Flajolet et al. 2010.
        zero or one power(self, px, py, n)
            Generates 1 with probability (px/py)^n (where n can be
            positive, negative, or zero); 0 otherwise.
        zero or one power ratio(self, px, py, nx, ny)
            Generates 1 with probability (px/py)^(nx/ny) (where nx/ny can be
            positive, negative, or zero); 0 otherwise.
       Data descriptors defined here:
           dictionary for instance variables (if defined)
         weakref
            list of weak references to the object (if defined)
```

```
turning loaded dice with unknown bias into loaded dice
       with a different bias. Specifically, it supports specifying
       the probability that the output die will land on a given
        number, as a polynomial function of the input die's bias.
       The case of biased coins to biased coins is also called
        the Bernoulli factory problem; this class allows the output
        coin's bias to be specified as a polynomial function of the
        input coin's bias.
        Reference: Morina, G., Łatuszyński, K., et al., "From the
       Bernoulli Factory to a Dice Enterprise via Perfect
       Sampling of Markov Chains", arXiv:1912.09229v1 [math.PR], 2019.
        Example:
       >>> ent=DiceEnterprise()
       >>> # Example 3 from the paper
       >>> ent.append_poly(1,[[math.sqrt(2),3]])
       >>> ent.append poly(0,[[-5,3],[11,2],[-9,1],[3,0]])
       >>> coin=lambda: 1 if random.random() < 0.60 else 0
       >>> print([ent.next(coin) for i in range(100)])
       Methods defined here:
        init (self)
            Initialize self. See help(type(self)) for accurate signature.
        append poly(self, result, poly)
            Appends a probability that the output die will land on
            a given number, in the form of a polynomial.
            result - A number indicating the result (die roll or coin
              flip) that will be returned by the _output_ coin or _output_
              die with the probability represented by this polynomial.
              Must be an integer 0 or greater. In the case of dice-to-coins
              or coins-to-coins, must be either 0 or 1, where 1 means
              heads and 0 means tails.
            poly - Polynomial expressed as a list of terms as follows:
              Each term is a list of two or more items that each express one of
              the polynomial's terms; the first item is the coefficient, and
              the remaining items are the powers of the input die's
              probabilities. The number of remaining items in each term
              is the number of faces the _input_ die has. Specifically, the
              term has the following form:
              In the case of coins-to-dice or coins-to-coins (so the probabilities are 1-p
and p,
              where the [unknown] probability that the input coin returns 0
              is 1 - p, or returns 1 is p):
                       term[0] * p**term[1] * (1-p)**term[2].
              In the case of dice-to-dice or dice-to-coins (so the probabilities are p1, p2,
etc.,
              where the [unknown] probability that the _input_ die returns
              0 is p1, returns 1 is p2, etc.):
                       term[0] * p1**term[1] * p2**term[2] * ... * pn**term[n].
              For example, [3, 4, 5] becomes:
                      3 * p**4 * (1-p)**5
              As a special case, this list can contain two items and a zero is
              squeezed between the first and second item.
              For example, [3, 4] is the same as [3, 0, 4], which in turn becomes:
                       3 * p**4 * (1-p)**0 = 3 * p **4
```

Implements the Dice Enterprise algorithm for

```
For best results, the coefficient should be a rational number
              (such as int or Python's Fraction).
              Each term in the polynomial must have the same number of items (except
              for the special case given above). For example, the following is not a valid
              way to express this parameter:
                       [[1, 1, 0], [1, 3, 4, 5], [1, 1, 2], [2, 3, 4]]
              Here, the second term has four items, not three like the rest.
            Returns this object.
       augment(self, count=1)
            Augments the degree of the function represented
            by this object, which can improve performance in some cases
            (for details, see the paper).
            - count: Number of times to augment the ladder.
           Returns this object.
        next(self, coin)
            Returns the next result of the flip from a coin or die
            that is transformed from the given input coin or die by the function
            represented by this Dice Enterprise object.
            coin - In the case of coins-to-dice or coins-to-coins (see the "append poly"
method),
               this specifies the input coin , which must be a function that
               returns either 1 (heads) or 0 (tails). In the case of dice-to-dice or dice-
to-coins.
               this specifies an _input die_ with _m_ faces, which must be a
               function that returns an integer in the interval [0, m), which
               specifies which face the input die lands on.
       Data descriptors defined here:
        __dict
           dictionary for instance variables (if defined)
           list of weak references to the object (if defined)
FILE
    /home/rooster/Documents/SharpDevelopProjects/peteroupc.github.io/bernoulli.py
Help on module interval:
NAME
    interval
DESCRIPTION
    # Implements interval numbers and interval arithmetic, backed
    # by Decimal values.
    # Written by Peter O. Any copyright to this file is released to the Public Domain.
    # In case this is not possible, this file is also licensed under Creative Commons Zero
    # (https://creativecommons.org/publicdomain/zero/1.0/).
CLASSES
    builtins.object
        Interval
    class Interval(builtins.object)
```

```
An interval of two Decimal values.
Methods defined here:
__add__(self, v)
| __max__(a, b)
| __min__(a, b)
| __mul__(self, v)
 __neg__(self)
 __pow__(self, v)
  __radd__(self, v)
  __repr__(self)
    Return repr(self).
 __rmul__(self, v)
  __rsub__(self, v)
  __rtruediv__(self, v)
  __sub__(self, v)
  __truediv__(self, v)
abs(self)
  ceil(self)
  clamp(self, a, b)
  clampleft(self, a)
  cos(self)
  exp(self)
  floor(self)
  isAccurateTo(self, v)
  log(self)
  magnitude(self)
  mignitude(self)
  pi(prec=56)
  pow(self, v)
  rem(self, v)
  sin(self)
 sqrt(self)
```

```
tan(self)
       width(self)
           NOTE: Not rigorous!
       Static methods defined here:
        new (cl, v, sup=None, prec=None)
            Create and return a new object. See help(type) for accurate signature.
       Data descriptors defined here:
        dict
           dictionary for instance variables (if defined)
           list of weak references to the object (if defined)
    /home/rooster/Documents/SharpDevelopProjects/peteroupc.github.io/interval.py
Help on module moore:
    moore
DESCRIPTION
    # Implements the Moore Rejection Sampler.
    # Written by Peter O. Any copyright to this file is released to the Public Domain.
    # In case this is not possible, this file is also licensed under Creative Commons Zero
    # (https://creativecommons.org/publicdomain/zero/1.0/).
CLASSES
    builtins.object
       MooreSampler
    class MooreSampler(builtins.object)
       Moore rejection sampler, for generating independent samples
       from continuous distributions in a way that minimizes error,
        if the distribution's PDF (probability density function)
       uses "well-defined" arithmetic expressions.
       It can sample from one-dimensional or multidimensional
       distributions. It can also sample from so-called "transdimensional
       distributions" if the distribution is the union of several component
       distributions that may have different dimensions and are associated
       with one of several _labels_.
       Parameters:
        - pdf: A function that specifies the PDF. It takes a single parameter that
            differs as follows, depending on the case:
            - One-dimensional case: A single Interval. (An Interval is a mathematical
              object that specifies upper and lower bounds of a number.)
            - Multidimensional case: A list of Intervals, one for each dimension.
            - Transdimensional case (numLabels > 1): A list of two items: the Interval
              or Intervals, followed by a label number (an integer in [0, numLabels)).
           This function returns an Interval. For best results,
```

FILE

NAME

the function should use interval arithmetic throughout. The area under the PDF need not equal 1 (this sampler works even if the PDF is only known up to a normalizing constant).

- mn, mx: Specifies the sampling domain of the PDF. There are three cases:
  - One-dimensional case: Both mn and mx are numbers giving the domain, which in this case is [mn, mx].
  - Multidimensional case: Both mn and mx are lists giving the minimum and maximum bounds for each dimension in the sampling domain.
     In this case, both lists must have the same size.
  - Transdimensional case: Currently, this class assumes the component distributions share the same sampling domain, which is given depending on the preceding two cases.

For this sampler to work, the PDF must be "locally Lipschitz" in the sampling domain, meaning that the function is continuous everywhere in the domain, and has no slope that tends to a vertical slope anywhere in that domain.

- numlabels: The number of labels associated with the distribution, if it's a transdimensional distribution. Optional; the default is 1.
- bitAccuracy: Bit accuracy of the sampler; the sampler will sample from a distribution (truncated to the sampling domain) that is close to the ideal distribution by 2^-bitAccuracy. The default is 53.

## Reference:

Sainudiin, Raazesh, and Thomas L. York. "An Auto-Validating, Trans-Dimensional, Universal Rejection Sampler for Locally Lipschitz Arithmetical Expressions." Reliable Computing 18 (2013): 15-54.

The following reference describes an optimization, not yet implemented here: Sainudiin, R., 2014. An Auto-validating Rejection Sampler for Differentiable Arithmetical Expressions: Posterior Sampling of Phylogenetic Quartets. In Constraint Programming and Decision Making (pp. 143-152). Springer, Cham.

Methods defined here:

```
__init__(self, pdf, mn, mx, numLabels=1, bitAccuracy=53)
    Initialize self. See help(type(self)) for accurate signature.

acceptRate(self)

sample(self)

Samples a number or vector (depending on the number of dimensions)
    from the distribution and returns that sample.

If the sampler is transdimensional (the number of labels is greater than 1),
    instead returns a list containing the sample and a random label in the
    interval [0, numLabels), in that order.
```

-----

Data descriptors defined here:

FTLE

/home/rooster/Documents/SharpDevelopProjects/peteroupc.github.io/moore.py