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I, 14

Tutorial 4

1. $T(n) = 3T(\frac{n}{2}) + n^2$

$T(n) = aT(\frac{n}{b}) + f(n)$

$a \geq 1, b > 1$

On comparing

$a = 3, b = 2, f(n) = n^2$

Now,

$C = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = \theta(n^2)$

2. $T(n) = 4T(\frac{n}{2}) + n^2$

$a \geq 1, b > 1$

$a = 4, b = 2, f(n) = n^2$

$c = \log_2 4 = 2$

$\therefore n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \theta(n^2 \log n)$

3. $T(n) = T(\frac{n}{2}) + 2^n$

$a = 1, b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$ $T(n) = \theta(2^n)$

4. $T(n) = 2^n T(\frac{n}{2}) + n^n$

$a = 2^n$

$b = 2$, $f(n) = n^n$

$c = \log_b a = \log_2 2^n$

$= n$

$n^c = n^n$

$\therefore f(n) = n^c$

$T(n) = \theta(n^2 \log_2 n)$

5. $T(n) = 16T(\frac{n}{4}) + n$

$a = 16, b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = \theta(n^2)$

6. $T(n) = 2T(\frac{n}{2}) + n\log n$

$a = 2, b = 2$

$f(n) = n\log n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

$n\log n > n$

$\therefore f(n) > n^c$

$\therefore T(n) = \theta(n \log n)$

7. $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

$a=2$, $b=2$, $f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

Since $\frac{n}{\log n} \nless n$

$\because f(n) \nless n^c$

$T(n) = \Theta(n)$

9. $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$

$a = 0.5$, $b=2$

$a \geq 1$ but a is 0.5

So, we can't apply master theorem.

11. $4T\left(\frac{n}{2}\right) + \log n$

$a=4$, $b=2$, $f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$f(n) = \log n$

$\therefore \log n < n^2$

$\therefore f(n) < n^c$

$T(n) = \Theta(n^c)$

$= \Theta(n^2)$

8. $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a=2$, $b=4$, $f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$\therefore n^c = n^{0.5}$

Since, $n^{0.5} < n^{0.51}$

$f(n) > n^c$

$T(n) = \Theta(n^{0.51})$

10. $T(n) = 16T\left(\frac{n}{4}\right) + n!$

$a=16$, $b=4$, $f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \Theta(n!)$

12. $T(n) = \text{sqrt}(n)\, T\left(\frac{n}{2}\right) + \log n$

$a = \sqrt{n}$, $b = 2$

$c = \log_b a = \log_2 \sqrt{n}$

$= \frac{1}{2} \log_2 n$

$\therefore \frac{1}{2} \log n < \log(n)$

$\therefore f(n) > n^c$

$T(n) = \Theta(f(n))$

$= \Theta(\log n)$

13. $T(n) = 3T(\frac{n}{2}) + n$

$a = 3, b = 2, f(n) = n$

$c = \log_b^a = \log_2^3 = 1.584$

$\therefore n^c < n^{1.584}$

$n < n^{1.584}$

$\Rightarrow f(n) < n^c$

$T(n) = \theta(n^{1.584})$

14. $T(n) = 3T(\frac{n}{3}) +$ sqrt(n)

$a = 3, b = 3$

$c = \log_b a = \log_3^3 = 1$

$\therefore n^c = n^1 = n$

sqrt$(n) < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n)$

15. $T(n) = 4T(\frac{n}{2}) + cn$

$a = 4, b = 2$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$cn < n^2$ (for any constant)

$\therefore f(n) < n^c$

$T(n) = \theta(n^2)$

16. $T(n) = 3T(\frac{n}{4}) + n\log n$

$a = 3, b = 4$

$f(n) = n\log n$

$c = \log_b a = \log_4^3$

$= 0.792$

$n^c = n^{0.792}$

$\therefore n^{0.792} < n\log n$

$\therefore T(n) = \theta(n\log n)$

17. $T(n) = 3T(\frac{n}{3}) + \frac{n}{2}$

$a = 3, b = 3$

$c = \log_3 3 = 1$

$f(n) = n/2$

$n^c = n^1 = n$

$n/2 < n$

$\therefore T(n) = \theta(n)$

18. $T(n) = 6T(\frac{n}{3}) + n^2\log n$

$a = 6, b = 3$

$c = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

$n^{1.6309} < n^2\log n$

$\therefore T(n) = \theta(n^2\log n)$

19. $T(n) = 4T(\frac{n}{2}) + n/\log n$

$a = 4, b = 2, f(n) = \frac{n}{\log n}$

$c = \log_2 4 = 2$

$n^c = n^2$

$\therefore \frac{n}{\log n} < n^2$

$\therefore T(n) = \theta(n^2)$

20. $T(n) = 64T(\frac{n}{8}) - n^2 \log n$

$a = 64, b = 8$

$c = \log_8 64 = 2$

$n^c = n^2$

$\therefore n^2 \log n > n^2$

$\therefore T(n) = \theta(n^2 \log n)$

21. $T(n) = 7T(\frac{n}{3}) + n^2$

$a = 7, b = 3, f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$\therefore n^{1.7712} < n^2$

$\therefore T(n) = \theta(n^2)$

22. $T(n) = T(\frac{n}{2}) + n(2 - \cos n)$

$a = 1, b = 2$

$c = \log_b a = \log_2 1 = 0$

$\therefore n^c = n^0 = 1$

$\therefore n(2 - \cos n) > n^c$

$\therefore T(n) = \theta(n(2 - \cos n))$