Introduction to Hypothesis Testing

Definition:

A **statistical hypothesis** is an assumption about a **population parameter**.

For example, we may assume that the mean height of a male in the India is 70 inches. The assumption about the height is the ***statistical hypothesis*** and the true mean height of a male in the India is the *population parameter*.

A **hypothesis test** is a formal statistical test we use to reject or fail to reject a statistical hypothesis.

For example: z-test , p- value, t –test etc.

Need for Hypothesis Testing in Business

Businesses collect and analyze data to help managers make optimal decisions that maximize profit at minimum risk. This depends on the acceptance or rejection of a hypothesis. For example, suitable hypothesis formulation and testing can help in the situation described below:

The Two Types of Statistical Hypotheses

There are two types of statistical hypotheses:

- The **null hypothesis**, denoted as H_0, is the hypothesis that the sample data occurs purely from chance.
- The **alternative hypothesis**, denoted as H_1 or H_a, is the hypothesis that the sample data is influenced by some non-random cause.

A **hypothesis test** consists of five steps:

1. State the hypotheses.

2. Determine a significance level to use for the hypothesis.

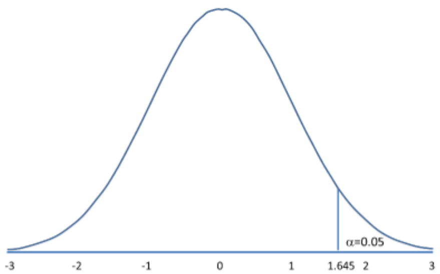
-Decide on a significance level. Common choices are .01, .05, and .1.

3. Find the test statistic.

4. Reject or fail to reject the null hypothesis.

5. Interpret the results.

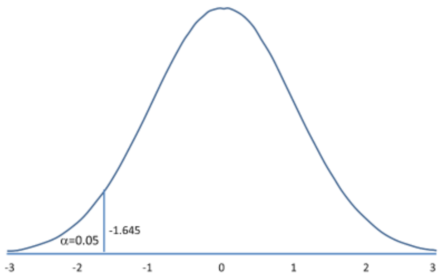
Right tailed vs left tailed vs two tailed



Rejection Region for Upper-Tailed Z Test (H_1: $\mu > \mu_0$) with α=0.05

The decision rule is: Reject H_0 if $Z \geq 1.645$.

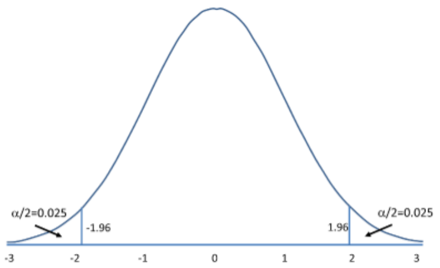
Upper-Tailed Test	
α	Z
0.10	1.282
0.05	1.645
0.025	1.960
0.010	2.326
0.005	2.576
0.001	3.090
0.0001	3.719



Rejection Region for Lower-Tailed Z Test (H_1: $\mu < \mu_0$) with α =0.05

The decision rule is: Reject H_0 if $Z \leq 1.645$.

Lower-Tailed Test	
a	Z
0.10	-1.282
0.05	-1.645
0.025	-1.960
0.010	-2.326
0.005	-2.576
0.001	-3.090
0.0001	-3.719



Rejection Region for Two-Tailed Z Test (H_1: $\mu \neq \mu_0$) with α =0.05

The decision rule is: Reject H_0 if $Z \leq -1.960$ or if $Z \geq 1.960$.

Two-Tailed Test	
α	Z
0.20	1.282
0.10	1.645
0.05	1.960
0.010	2.576
0.001	3.291
0.0001	3.819

Z test

- **Problem1:** A principal at a certain school claims that the students in his schools are above average intelligence. A random sample of thirty students IQ scores have a mean score of 112. Is there sufficient evidence to support the principal's claim? The mean population IQ is 100 with a standard deviation of 15. IQ scores are normally distributed.

Solution: $H0 : \mu=100$

$H1 : \mu > 100$ (right tailed)

$\alpha = 0.05$

z critical value = 1.65

z score = 4.56

compare zscore and zcritical values (z score > z critical)-> reject null hypothesis

- **Problem2:** In a college, the average score of girls is higher than 600 in the exam. We have the information that the standard deviation for girls' scores is 100. So, we collect the data of 20 girls by using random samples and record their marks. Consider the α value (significance level) to be 0.05. Mean Score for Girls is 641. Also find the p-value to accept or reject the null hypothesis

Solution:

- H0: $\mu=600$
- H1: $\mu> 600$
- $\alpha = 0.05$
- z critical value = 1.65
- z score = 1.8336
- p value = 0.033357

- if p < alpha (reject the null hypothesis)
- if p > alpha (accept the null hypothesis)

Note: When the test statistics is in critical region , p-value < alpha

Problem3:

Test the research hypothesis that the mean weight in men in 2006 is less than 191 pounds. Assume the sample data are as follows: $n=100$, x(bar) $=185.1$ and $s=25.6$.

Solution:

$H0: \mu = 191$

$H1: \mu < 191$

$\alpha = 0.05$

z critical value = ? (-1.645)

z score = ? (-2.30)

Problem4:

Test the research hypothesis that the mean weight in men in 2006 is not equal to 191 pounds. Assume the sample data are as follows: $n=100$, x(bar) $=185.1$ and $s=25.6$.

Solution:

The decision rule is: Reject H0 if $Z < -1.960$ or if $Z > 1.960$.

right side : $\alpha /2 = 0.025$

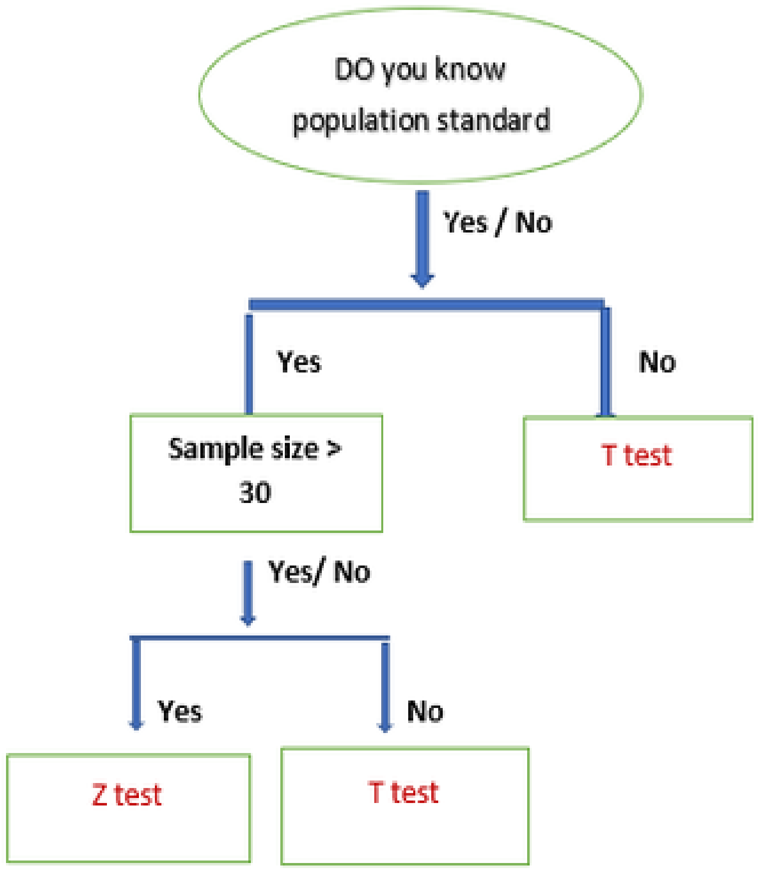
left side : $\alpha /2 = 0.025$

$H0: \mu = 191$

H1: μ != 191

z score = ? -2.30 / 2.30

z test vs t test



T-test

Problem:

Suppose we want to know whether or not the mean weight of a certain species of turtle is equal to 310 pounds. To test this, will perform a one-sample t-test at significance level $\alpha = 0.05$ using the following steps:

Step 1: Gather the sample data.

- Suppose we collect a random sample of turtles with the following information:
- Sample size $n = 25$
- Sample mean weight $x = 300$
- Sample standard deviation $s = 18.5$

Step 2: Define the hypotheses.

- We will perform the one sample t-test with the following hypotheses:
- **H_0:** $\mu = 310$ (population mean is equal to 310 pounds)
- **H_1:** $\mu \neq 310$ (population mean is not equal to 310 pounds)

T test

- Calculate the test statistic t.
  - $t = (x - \mu) / (s/\sqrt{n}) = (300-310) / (18.5/\sqrt{25}) =$

P-value

- Calculate the p-value of the test statistic t.
- ?

Conclusion

- Draw a conclusion.

Conclusions in Test of Hypothesis

	Do Not Reject H_0	Reject H_0
H_0 is True	Correct Decision	Type I Error
H_0 is False	Type II Error	Correct Decision

Anova test

State hypothesis: H0: M1 = M2 = M3 =…MK (k population mean)
H1: at least one mean is different (based on sample data)

Case 1:

Drug1	Drug2	Drug3
1	1	1
2	2	4
3	3	5
5	9	5
8	10	8
10	10	9
12	11	10

Note :1. variance within group remains even

2. variance between groups remains even

Conclusion:

Acc. to case 1, h0 is accepted

Case 2:

Note :1. variance within group remains even

2. variance between groups differs

Therefore, we reject the null hypothesis

Drug1	**Drug2**	**Drug3**
10	1	1
12	2	4
13	3	5
15	9	5
18	10	8
20	10	9
22	11	10

F-test (Snedecor-Fisher's test)

F Test to Compare Two Variances

A **Statistical F Test** uses an F Statistic to compare two variances, s_1 and s_2, by dividing them. The result is always a positive number (because variances are always positive).

The equation for comparing two variances with the f-test is:

$$F = s^2_1 / s^2_2$$

Note: Larger the ratio, the more likely it is the groups different mean

When to use which test

	Categorical	Continous
Categorical	Chisquare	One sample t-test, two sample t test, anova tets
Continous	Logistics regression	Correlation

Know this!

Nonparametric tests don’t require that your data follow the normal distribution. They’re also known as distribution-free tests .

Statistical hypothesis tests are more comfortable with parametric tests than nonparametric tests.

Parametric tests of means	**Nonparametric tests of medians**
1-sample t-test	Wilcoxon test
2-sample t-test	Mann-Whitney test
One-Way ANOVA	Kruskal-Wallis, Mood’s median test