EMORY

ROLLINS SCHOOL OF PUBLIC HEALTH

Introduction to Compartmental Models for Infectious Diseases

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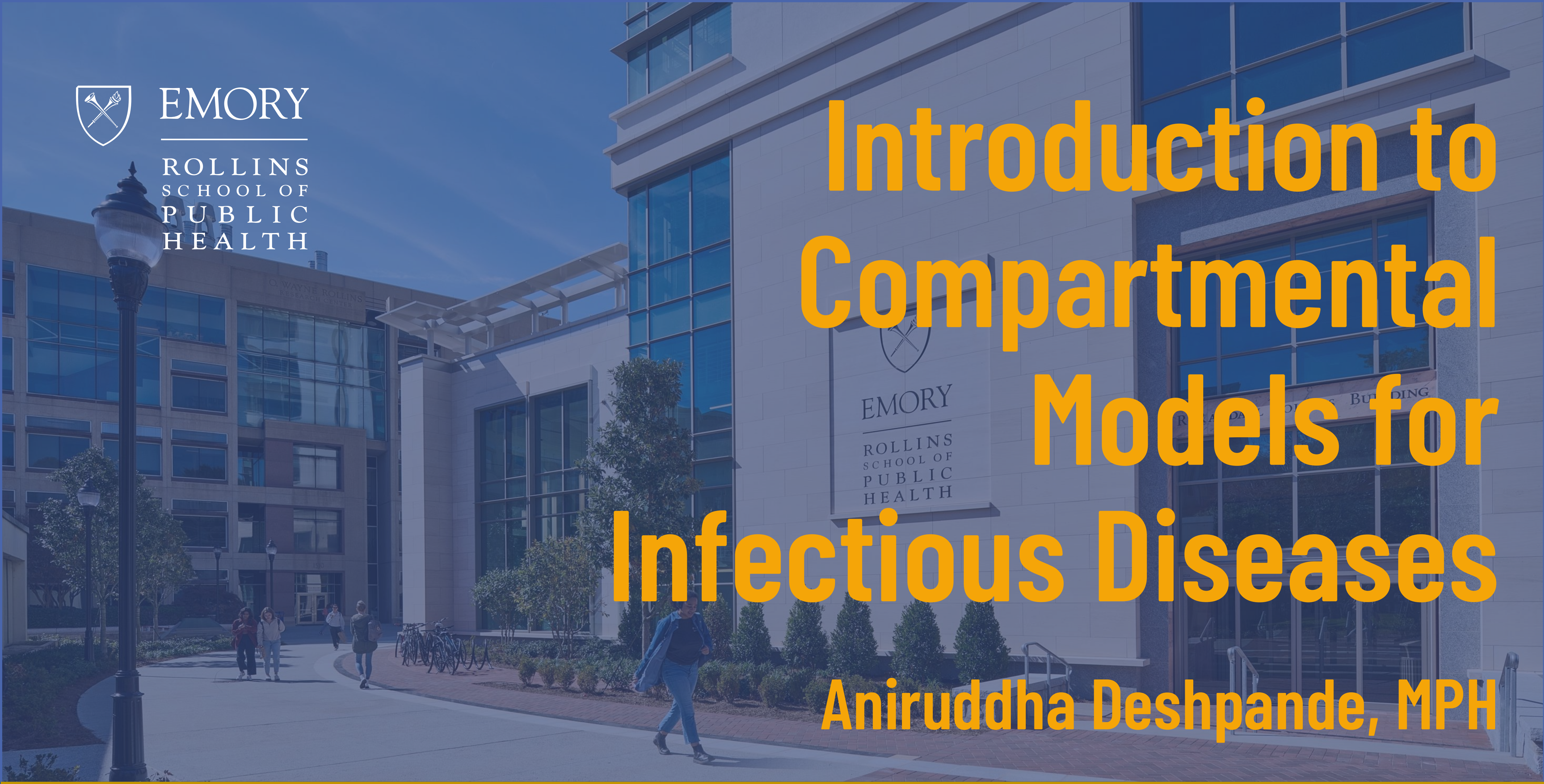


Overview: States & Rates

Structure

- Divide population into relevant compartments (<u>states</u>)
- People flow between states as per differential equations (<u>rates</u>)
  - Fixed parameters
  - Dynamic parameters: Parameter value changes with state value

Application

- Simulate scenarios
- Statistical inference to better understand disease dynamics

Defining the Model Structure

Two key features in infectious disease epidemiology:

1. Susceptibility/Immunity: What is the probability the subject will get infected if exposed to a pathogen?
2. Infectiousness: What is the probability a person will get infected if someone contacts the subject?

The model structure represents:

1. How the pathogen is transmitted in the population
2. Which dynamics are relevant to our research question

Common Model Structures: SI

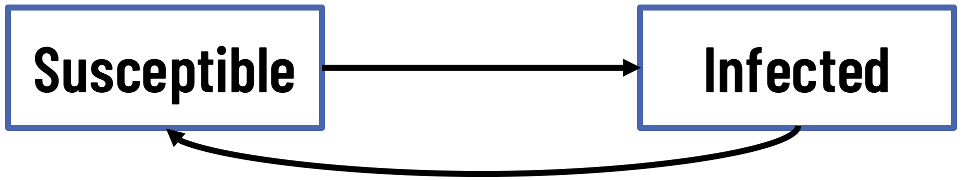
Description:

- Susceptible individuals become chronically infected

Potential Application:

- Equilibrium dynamics of uncontrolled HIV in a population

Common Model Structures: SIS



Description:

- Susceptible individuals are infected and become re-susceptible upon recovery from infection

Potential Application:

- Pathogens where infections confer short-term immunity (common cold)

Common Model Structures: SIR

Description:

- Susceptible individuals are infected and gain long-term immunity upon recovery

Potential Application:

- Measles

Common Model Structures: SIRS

Description:

- Susceptible individuals are infected, have temporary immunity which wanes over time.

Potential Application:

- Pathogens where infections confer “medium-term” immunity (S. Pneumoniae)

Common Model Structures: SEIR

Susceptible → Exposed → Infected → Recovered

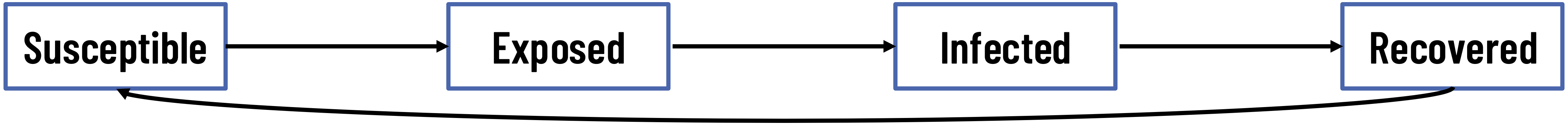
Description:

- Dynamics are similar to SIR but the "Exposed" class represents individuals with a latent infection. "Exposed" individuals are infected but do not transmit.

Potential Application:

- Varicella (Chickenpox)

Common Model Structures: SEIRS



Description:

- Same as SEIR, but immunity wanes.

Potential Application:

- RSV

The SIR Model

$$\boxed{\text{Susceptible}} \xrightarrow{\lambda_t} \boxed{\text{Infected}} \xrightarrow{\sigma} \boxed{\text{Recovered}}$$

- **Force of Infection** (λ_t): The average rate at which susceptible individuals become infected
- **Rate of Recovery** (σ): The average rate at which infected individuals recover from infection. **The reciprocal is the duration of infection!**

The SIR Model

$$\text{Susceptible} \xrightarrow{\lambda_t} \text{Infected} \xrightarrow{\sigma} \text{Recovered}$$

$$S_{t+1} = S_t - \lambda_t * S_t$$

$$I_{t+1} = I_t + \lambda_t * S_t - I_t * \sigma$$

$$R_{t+1} = R_t + I_t * \sigma$$

The SIR Model

$$\boxed{\text{Susceptible}} \xrightarrow{\lambda_t} \boxed{\text{Infected}} \xrightarrow{\sigma} \boxed{\text{Recovered}}$$

$$S_{t+1} = S_t - \lambda_t {*} S_t \qquad \frac{dS}{dt} = -\lambda_t {*} S_t$$

$$I_{t+1} = I_t + \lambda_t {*} S_t - I_t {*} \sigma \qquad \frac{dI}{dt} = \lambda_t {*} S_t - I_t {*} \sigma$$

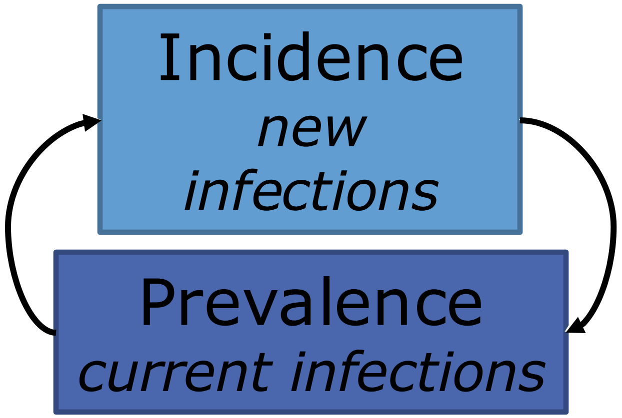
$$R_{t+1} = R_t + I_t {*} \sigma \qquad \frac{dR}{dt} = I_t {*} \sigma$$

The Force of Infection

The rate at which susceptible individuals become infected per unit time

$$\lambda_t = a * \beta * \frac{I}{N}$$

ɑ = number of contacts/unit time
β = probability of transmission per contact
N = population size



The Force of Infection

$$\boxed{S} \xrightarrow{\lambda} \boxed{I}$$

$$\lambda(t) = \alpha * \beta * I$$

Frequency Dependent

$$\lambda(t) = \alpha * \beta * \frac{I}{N}$$

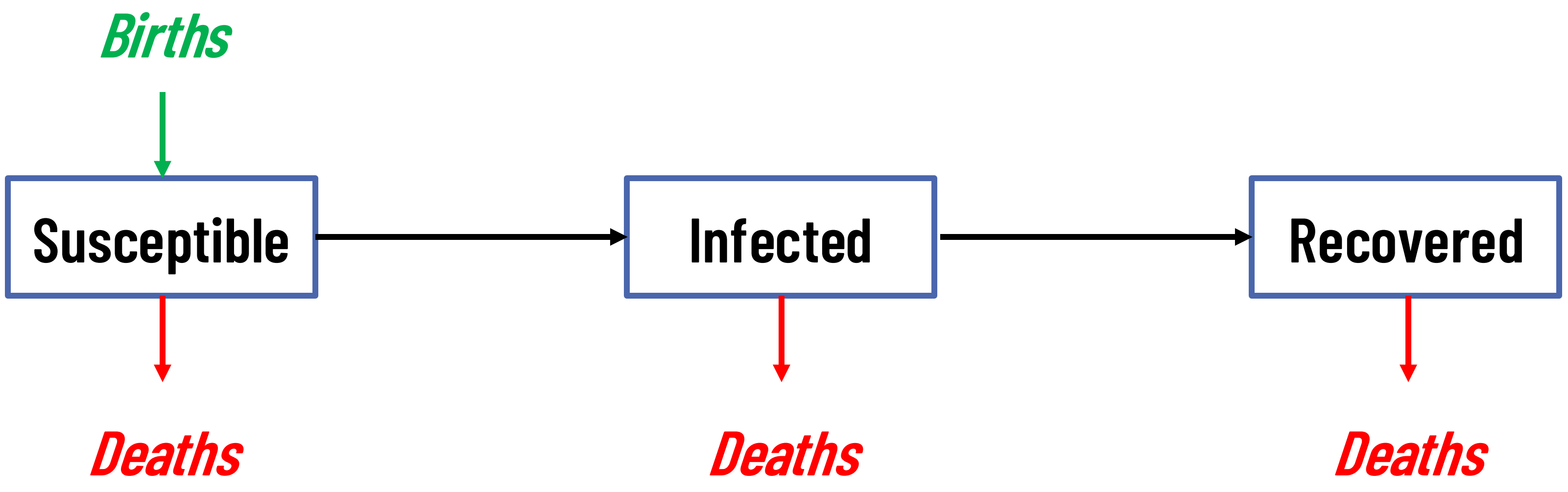
Density Dependent

Population Dynamics in Models of Infectious Disease

- Static populations are great for equilibrium dynamics
- Static populations make sense for:
  - Shorter time scales (<10 years)
  - Rare infections
  - Pathogens with low infectiousness
- In reality:
  - Populations age (India: 23.6→27.0 in 2010→2020)
  - Populations grow (India: 1.24 bn→1.40 bn in 2010→2020)

Population Dynamics in Models of Infectious Disease

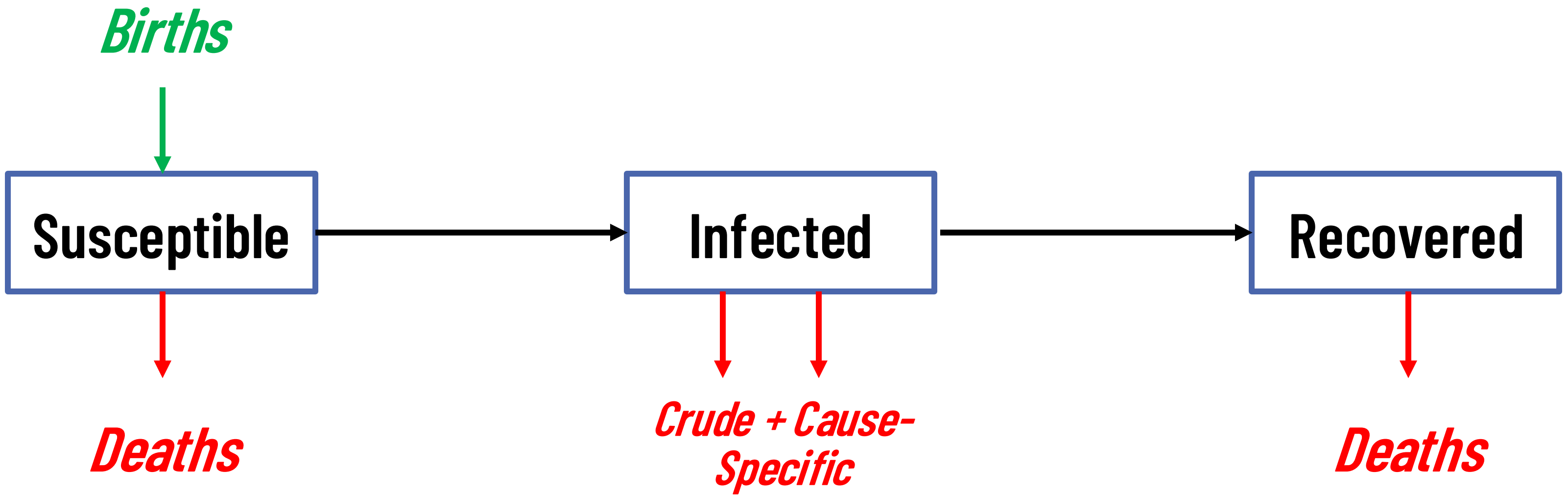
- Dynamic populations: births & deaths



- Birth rate: crude birth rate vs. fertility rate
- Deaths: crude mortality rate

Population Dynamics in Models of Infectious Disease

- Dynamic populations: births & deaths

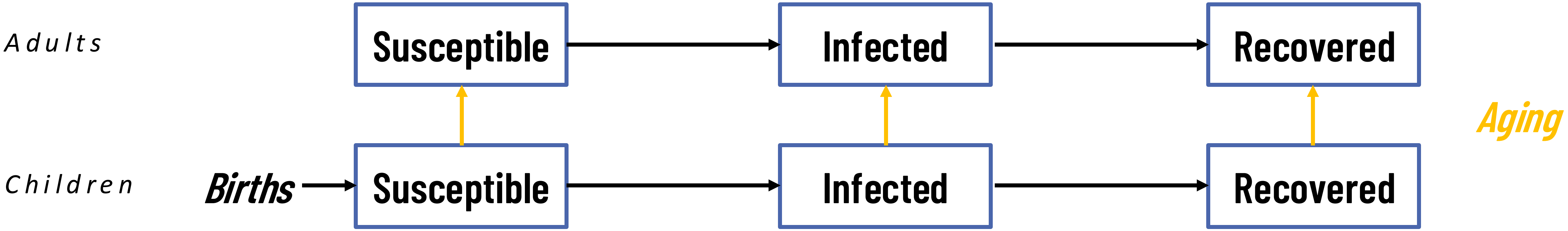


- Birth rate: crude birth rate vs. fertility rate
- Deaths: crude mortality rate or <u>cause specific mortality rate</u>

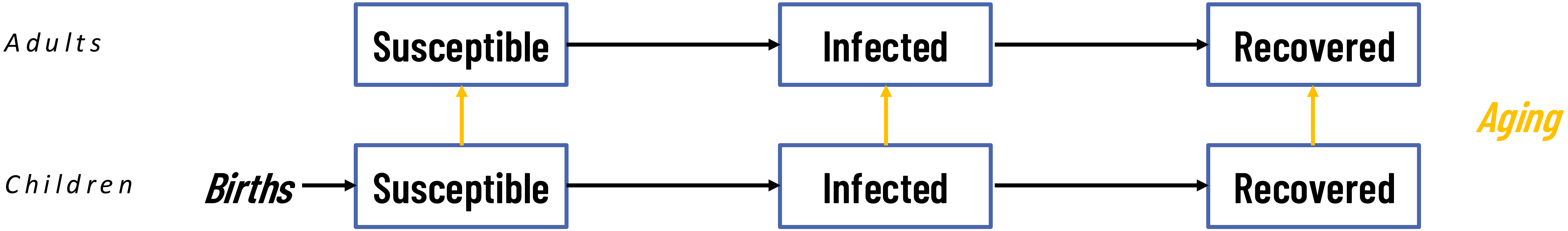
Age Structure in Models of Infectious Diseases

- Susceptibility to infection or disease can meaningfully vary across ages
  - Maternal immunity
  - Higher disease incidence amongst older adults

Age Structure in Models of Infectious Diseases



Age Structure in Models of Infectious Diseases



What is another factor across which susceptibility varies?