ASSIGNMENT

1. Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

a. Clearly define the decision variables

b. What is the objective function?

c. What are the constraints?

d. Write down the full mathematical formulation for this LP problem.

Solution:

Given:

Total shipment of material per week = 5000 sq. ft.
Material required for each Collegiate bag = 3 sq. ft.
Material required for each Mini bag = 2 sq. ft.
Sales forecast for Collegiate bag per week <= 1000 units
Sales forecast for Mini bag per week <= 1200 units
Cost Co-efficient:
    Unit profit for Collegiate = $32
    Unit profit for Mini = $24
Total labor hours = 35 labors * 40 hours = 1400 hours

1. **To define decision variable:**

Let, X_1 = Quantity of Collegiate Bags to be produced per week.
X_2 = Quantity of Mini Bags to be produced per week.

2. **Objective Function:**

Maximum Total Profit gained by Collegiate and Mini bags per week.

$$Max_z = 32X_1 + 24X_2$$

3. Constraints:

i. Total available Material

$3X_1 + 2X_2 <= 5000$ sq. ft.

ii. Sales Forecast

$X_1 <= 1000$ units

$X_2 <= 1200$ units

iii. Labor Hours

$(45/60)\ X_1 + (40/60)\ X_2 <= 1400$ hours - (45 min and 40 min are converted into hours)

4. Mathematical Formulation of Linear Programming Problem:

Let,

X_1 = Quantity of Collegiate Bags to be produced per week.

X_2 = Quantity of Mini Bags to be produced per week.

$Max_z = 32X_1 + 24X_2$

Subject to

$3X_1 + 2X_2 <= 5000$ sq. ft.

$X_1 <= 1000$ units

$X_2 <= 1200$ units

$(45/60)\ X_1 + (40/60)\ X_2 <= 1400$ hours

And

$X_1 >= 0,\ X_2 >= 0$

2. The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

a. Define the decision variables

b. Formulate a linear programming model for this problem.

Solution:

Given:

Cost Coefficient:

Net Profit of Products of Large Size = $420
Net Profit of Products of Medium Size = $360
Net Profit of Products of Small Size = $300

Excess capacity produced by Plants all sizes per day:

Plant 1 = 750 units
Plant 2 = 900 units
Plant 3 = 450 units

In process Storage space of:

Plant 1 = 13,000 sq. ft.
Plant 2 = 12,000 sq. ft.
Plant 3 = 5,000 sq. ft.

Units produced per day:

Large = 20 sq. ft.
Medium = 15 sq. ft.
Small = 12 sq. ft.

Sales forecast – Units sold per day if available

Large = 900 units

Medium = 1200 units

Small = 750 units

1. **Decision Variable:**

   Let,

   X_{ij} = Quantity of each sizes i of products produced by each plant j in Weigelt Corporation.

   i = Sizes of products produced by Weigelt Corporation (Large, Medium, Small), L, M, S

   j = Plants in which the products are produced by Weigelt Corporation. 1, 2, 3

2. **Mathematical representation of Liner Programming model:**

   Let,

   X_{ij} = Quantity of each sizes i of products produced by each plant j in Weigelt Corporation

   i = L, M, S

   j = 1, 2, 3

   $Max_z = 420 (X_{L1} + X_{L2} + X_{L3}) + 360 (X_{M1} + X_{M2} + X_{M3}) + 300 (X_{S1} + X_{S2} + X_{S3})$

   Subject To,

   $X_{L1} + X_{M1} + X_{S1} <= 750$ units

   $X_{L2} + X_{M2} + X_{S2} <= 900$ units

   $X_{L3} + X_{M3} + X_{S3} <= 450$ units

   $20X_{L1} + 15 X_{M1} + 12 X_{S1} <= 13,000$ sq. ft.

   $20X_{L2} + 15 X_{M2} + 12 X_{S2} <= 12,000$ sq. ft.

   $20X_{L3} + 15 X_{M3} + 12 X_{S3} <= 5,000$ sq. ft.

   $X_{L1} + X_{L2} + X_{L3} <= 900$ units

   $X_{M1} + X_{M2} + X_{M3} <= 1200$ units

   $X_{S1} + X_{S2} + X_{S3} <= 750$ units

   Same percentage of excess capacity to be used to produce new product thus the ratio of the excess capacities by every plant should be the same.

   $X_{L1} + X_{M1} + X_{S1}$: 750 units :: $X_{L2} + X_{M2} + X_{S2}$: 900 units

   $X_{L2} + X_{M2} + X_{S2}$: 900 units :: $X_{L3} + X_{M3} + X_{S3}$: 450 units

   $X_{L3} + X_{M3} + X_{S3}$: 450 units :: $X_{L1} + X_{M1} + X_{S1}$: 750 units

   Hence,

   $900(X_{L1} + X_{M1} + X_{S1}) = 750(X_{L2} + X_{M2} + X_{S2})$

   $450(X_{L2} + X_{M2} + X_{S2}) = 900(X_{L3} + X_{M3} + X_{S3})$

   $750(X_{L3} + X_{M3} + X_{S3}) = 450(X_{L1} + X_{M1} + X_{S1})$

$900(X_{L1} + X_{M1} + X_{S1}) - 750(X_{L2} + X_{M2} + X_{S2}) = 0$

$450(X_{L2} + X_{M2} + X_{S2}) - 900(X_{L3} + X_{M3} + X_{S3}) = 0$

$750(X_{L3} + X_{M3} + X_{S3}) - 450(X_{L1} + X_{M1} + X_{S1}) = 0$

$X_{ij} >= 0$