Verifying of brownian motion through constant observation of 2μm and 1μm spheres in H_2O solution

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Abstract

In our study, we used 1 micron and 2 micron polystyreme orbs to determine Boltzmann's Constant. Our results determine that our average value for k_B was $1.61 \times 10^{-23} \pm 1.57 \times 10^{-25}$. While we were not within one standard deviation of the accepted value, we were fairly consistent with accepted literature.

1 Introduction

1.1 Physics Motivation

1.1.1 Kinetic Theory Perspective

The motion of a Brownian particle can be understood through the kinetic theory of gases. In a fluid, molecules move randomly and collide with other molecules and suspended particles [1].

$$\langle R^2 \rangle = \frac{4k_B T}{6\pi a\eta} t \tag{1}$$

Where:

- $\langle R^2 \rangle$ is the mean squared displacement of the molecule
- k_B is Boltzmann's constant
- T is the absolute temperature
- η is the viscosity of the liquid
- a is the radius of the particle
- t is time

1.2 Historical context

Brownian motion is one of the seminal concepts in the field of physics that bridges the gap between the macroscopic world we experience and the microscopic realm of atoms and molecules. The experimental observation of Brownian motion provided crucial evidence for the existence of atoms and molecules [1]

1.2.1 Robert Brown's Observations

The phenomenon now known as Brownian motion was first observed by the botanist Robert Brown in 1827. While examining pollen grains of the plant *Clarkia pulchella* suspended in water under a microscope, Brown noted that the tiny particles inside the pollen grains jiggled incessantly. Intriguingly, this motion persisted regardless of whether the water was still or in motion and was observed even when the water was purified to eliminate any living impurities.

1.2.2 Initial Theories and Confusion

For many years, the origin of this mysterious motion was not understood. Some believed it was related to the life force of the pollen, while others thought it might be due to convection currents in the fluid. Brown himself carried out a series of meticulous experiments and ruled out many potential causes, but the true nature of the phenomenon remained elusive.

1.2.3 Einstein's Contribution

The true significance and understanding of Brownian motion came about in the early 20th century with Albert Einstein's groundbreaking paper in 1905. Einstein provided a theoretical explanation, asserting that the motion observed was a direct result of molecular collisions. He derived a relationship between the measurable quantities like the mean square displacement of the particles and the unseen molecular parameters such as Avogadro's number.

Einstein's theoretical predictions were experimentally verified by the French physicist Jean Perrin in 1908. Through meticulous experiments, Perrin was able to confirm Einstein's predictions and provide one of the first direct evidences for the existence of atoms [2].

1.2.4 Legacy

The understanding of Brownian motion has had profound implications in various fields of science. It provided the necessary experimental evidence for the atomic theory of matter. Furthermore, its study led to the development of statistical mechanics and played a key role in our understanding of thermodynamics.

The importance of Brownian motion is not confined to theoretical physics. It finds applications in fields as varied as biology, where it can be used to study cellular processes, to finance, where it is used in the modeling of stock prices.

2 Theoretical Background

Brownian motion, named after the botanist Robert Brown who first observed the phenomenon in 1827, refers to the random motion of particles suspended in a fluid (either a liquid or a gas). This erratic movement arises from the continuous and random bombardment of the suspended particles by the molecules of the surrounding medium.

From a theoretical standpoint, the kinetic theory of gases postulates that thermal motion of gas molecules leads to random collisions with any suspended particles. Over time, due to the large number of collisions, the net force experienced by a particle averages to zero. However, the displacement from its original position doesn't, leading to the random walk characteristic of Brownian motion.

The study of Brownian motion has significant implications in statistical mechanics and thermodynamics. One notable outcome is the Einstein's relation, which links the diffusion coefficient of a particle undergoing Brownian motion to its temperature and viscous resistance. This relationship can be expressed as:

$$\langle R^2 \rangle = \frac{k_B T}{6\pi\eta a} t$$

where $\langle R^2 \rangle$ is the mean squared displacement, k_B is Boltzmann's constant, T is the absolute temperature, η is the viscosity of the fluid, and a is the radius of the spherical particle.

The observation and understanding of Brownian motion serves not only as a testament to the molecular nature of matter but also allows for the determination of important physical constants, like Boltzmann's constant.

3 Experimental setup

3.1 Apparatus

For our apparatus, we had solutions of 1μm and 2μm polystyrene spheres dissolved in water. We used one drop of the sphere's solution per 100mL of water. Furthermore, we used concave microscope slides and square cover slips to trap the solution in a bubble free environemnt.

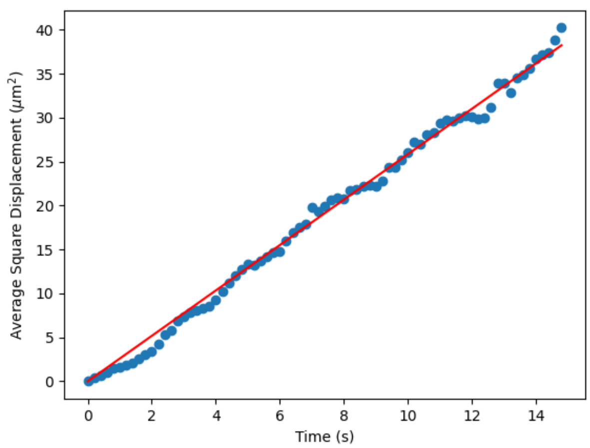

Figure 1: A plot of the square of the displacement versus time graph for a 1 micron polystyrene solution. The plot should look linear according to the theory of Brownian Motion.

Our microscope was an Olympus CX 41 Microscope set to 10x magnification. This magnification allowed us to clearly see many spheres, as well as clearly identify between spheres.

3.2 Data Collection

In order to collect data, we first put 3 to 4 drops of solution onto the back of a cover slip. We then upended the cover slip over the concavity in the slide to minimize the presence of bubbles. Ideally, there should be no bubbles in the concavity. If there are, there will be flow around the bubble, and the data is unusable. Then, we used ThorCAM software, combined with a Thorlabs CS165MU1 microscope camera to record .tif files of the motion. We took 12 frames per second, for five seconds, for each trial.

3.3 Data Analysis

To analyze our data, we added up the squares of the displacements of each particle, and plotted them as a function of time.

The Boltzmann constant, k_B, can be derived from the slope of the mean square displacement ($\langle \Delta R^2 \rangle$) of particles undergoing Brownian motion. The relationship is given by Eequation 1. By plotting $\langle \Delta R^2 \rangle$ against time and measuring the slope, k_B can be derived if the other parameters are known.

To determine the error in k_B when we have a slope with associated error, we use the method of propagation of error. Given:

$$k_B = \frac{kT}{6\pi\eta a} \times \text{slope}$$

where k is the Boltzmann constant, T is the absolute temperature, η is the viscosity of the fluid, and a is the radius of the particle. If Δslope is the error in the slope, the error in k_B (Δk_B) is:

$$\Delta k_B = \frac{kT}{6\pi\eta a} \times \Delta\text{slope}$$

Analyzing our different trials, we were able to obtain the average values for our different polystyrene sizes. We did so by using the temperature of the solution as 300K, and determining the viscosity of the water from this value as well. Our average value for k_B was $1.61{\times}10^{-23} \pm 1.57 \times 10^{-25}$

4 Results

Our average value for k_B was $1.61{\times}10^{-23} \pm 1.57 \times 10^{-25}$. Because the accepted value for k_B is $1.38{\times}10^{-23}$, we were not within one standard deviation of the accepted value. One way to improve this experiment would be to have a laser thermometer monitoring the solution at all times, in order to provide a clear and accurate measurement of the temperature and viscosity of the water as a function of time.

5 Summary and conlcusions

Our experiment made use of a microscope and camera, as well as 1 and 2 micron polystyrene orbs to determine the value of Boltzmann's constant. While we were not within one standard deviation, we were very close in our determination, as we were 16% away from the accepted value.

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