Ans 1. Size of $X^T X$: $X^T_{n \times m} \times X_{m \times n} \Rightarrow \underline{n \times n}$

Contradiction : Let $X^T X$ be invertible.

$\Rightarrow N(X^T X) \neq \{\vec{0}\}$

Let $\vec{v} \in N(X^T X)$

s.t. $\bar{v} \neq \bar{0}$ — (1)

$\therefore (X^T X)\vec{v} = \vec{0}$

$\Rightarrow \bar{v}^T X^T X \bar{v} = \bar{0}$

$\Rightarrow (X\bar{v})^T X\bar{v} = \bar{0}$

$\Rightarrow \|X\bar{v}\|^2 = \bar{0}$

$\Rightarrow X\bar{v} = \bar{0}$

[Alternatively, not all columns of $x^T x$ will be li. i.e. $<n$ l.i.
By rank theorem, $R(x^T x) + N(x^T x) = n$
$\therefore N(x^T x) > 1$]

Since X is invertible

$\Rightarrow N(X) = \{\bar{0}\}$ $\therefore \bar{v} = \vec{0}$ — (2)

(1) & (2) are a contradiction.

$\therefore X^T X$ must be invertible.

Ans 3. $X = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}$ $y = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 3 \\ 2 \end{bmatrix}$

$$\theta = (X^T X)^{-1} X^T y$$

$$\Rightarrow \theta = \left(\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \right)^{-1} X^T y$$

$$= \left(\begin{bmatrix} 5 & 13 \\ 13 & 39 \end{bmatrix} \right)^{-1} \left(\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 3 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} \right)$$

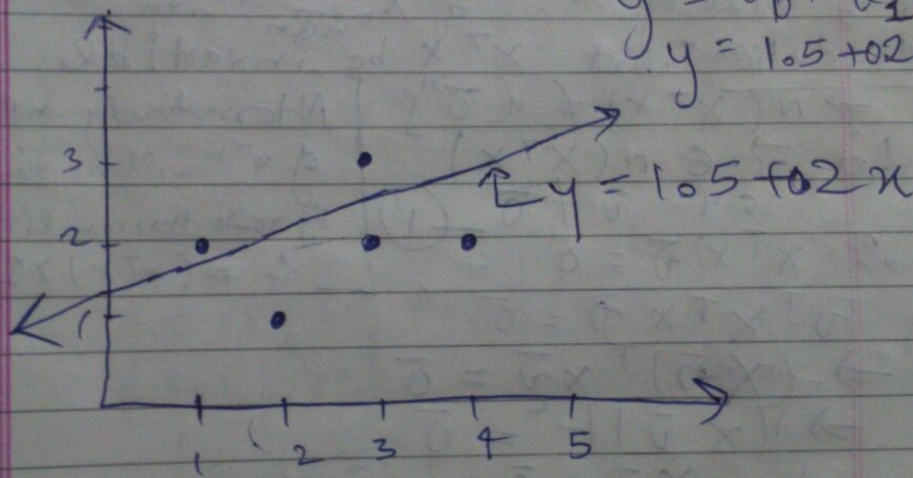
$$= \frac{1}{26} \begin{bmatrix} 39 & -13 \\ -13 & 5 \end{bmatrix} \left(\begin{bmatrix} 10 \\ 27 \end{bmatrix} \right)$$

$$= \frac{1}{26} \begin{bmatrix} 39 \\ 5 \end{bmatrix} \approx \begin{bmatrix} 1.5 \\ 0.2 \end{bmatrix}$$

i.e. $\theta_1 = 0.2$ $\theta_0 = 1.5$

$y = \theta_0 + \theta_1 x$

$y = 1.5 + 02x$



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(A) Computing inverse of $\begin{bmatrix} 5 & 13 \\ 13 & 39 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 5 & 13 & 1 & 0 \\ 13 & 39 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \to R_2/13} \left[\begin{array}{cc|cc} 5 & 13 & 1 & 0 \\ 1 & 3 & 0 & 1/13 \end{array}\right]$$

$$\downarrow R_2 \to R_2 - 1/5\, R_1$$

$$\left[\begin{array}{cc|cc} 5 & 13 & 1 & 0 \\ 0 & 1 & -1/2 & 5/2 \end{array}\right] \xleftarrow{R_2 \to \frac{5}{2} R_2} \left[\begin{array}{cc|cc} 5 & 13 & 1 & 0 \\ 0 & 2/5 & -1/5 & 1/13 \end{array}\right]$$

$$\downarrow R_1 \to R_1 - 13R_2$$

$$\left[\begin{array}{cc|cc} 5 & 0 & 15/2 & -5/2 \\ 0 & 1 & -1/2 & 5/26 \end{array}\right] \xrightarrow{R_1 \to \frac{1}{5} R_1} \left[\begin{array}{cc|cc} 1 & 0 & 3/2 & -1/2 \\ 0 & 1 & -1/2 & 5/26 \end{array}\right]$$

$\therefore$ inverse $= \begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 5/26 \end{bmatrix} = \frac{1}{26}\begin{bmatrix} 39 & -13 \\ -13 & 5 \end{bmatrix}$