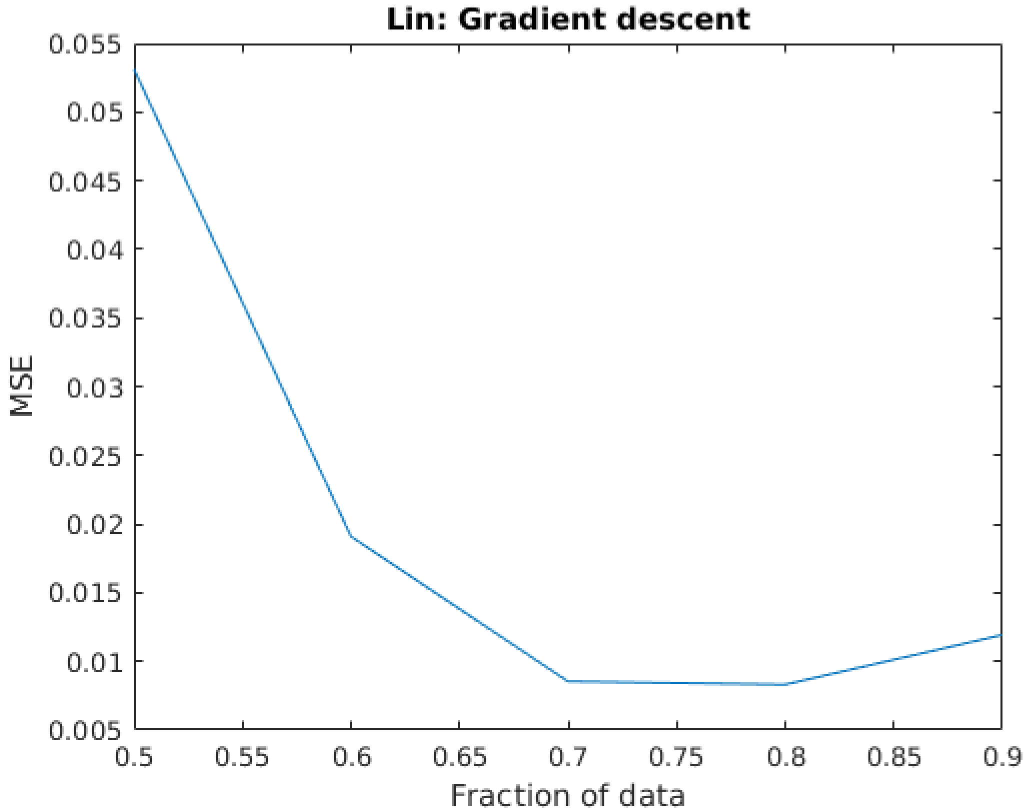
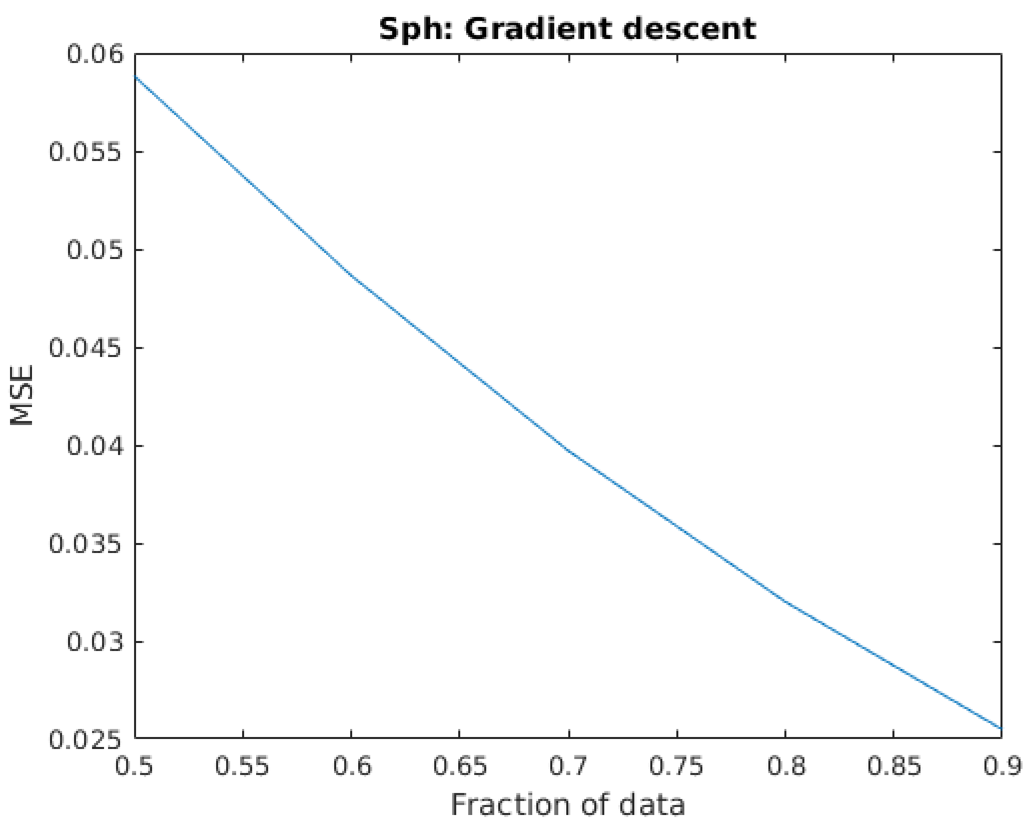
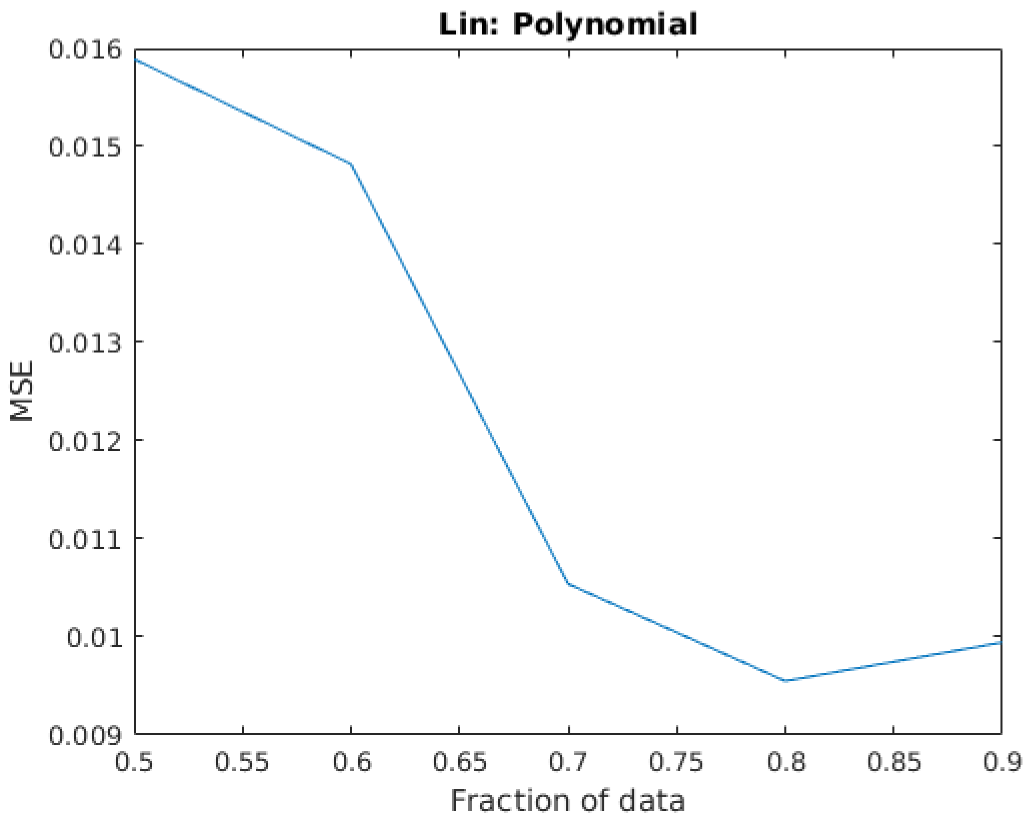
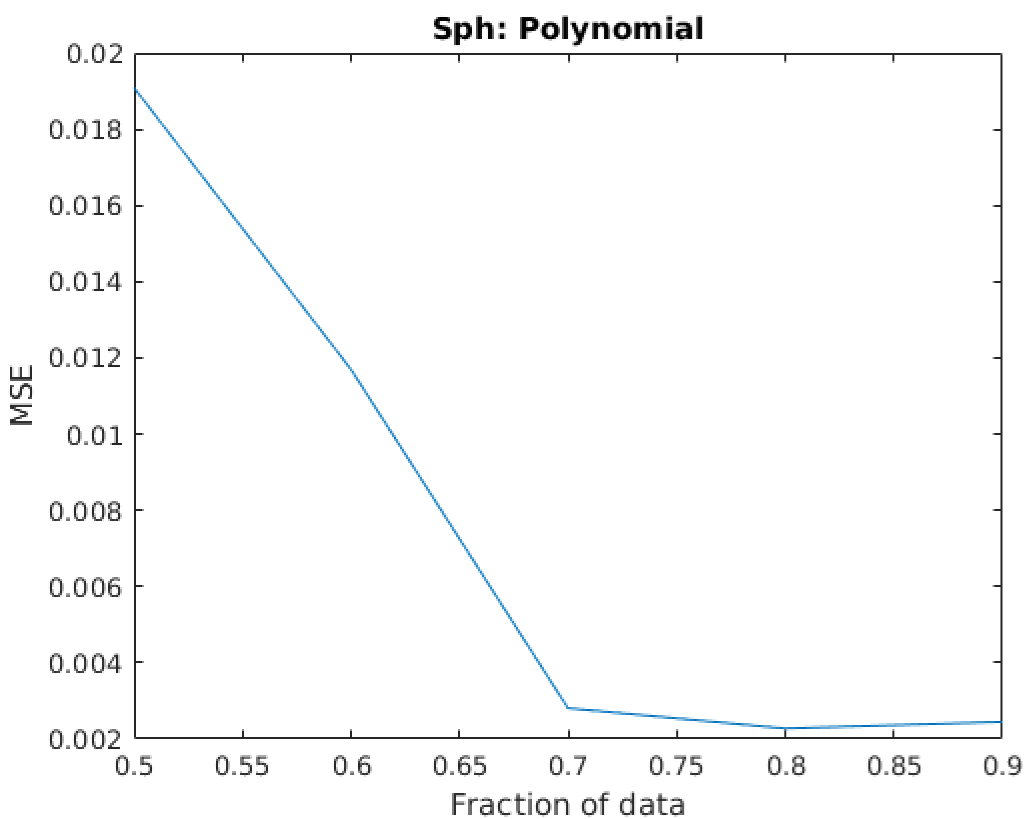
Ans 4. a.


Lin: Gradient descent
MSE
0.055
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
Fraction of data


Sph: Gradient descent
MSE
0.06
0.055
0.05
0.045
0.04
0.035
0.03
0.025
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
Fraction of data

Ans 4. b.

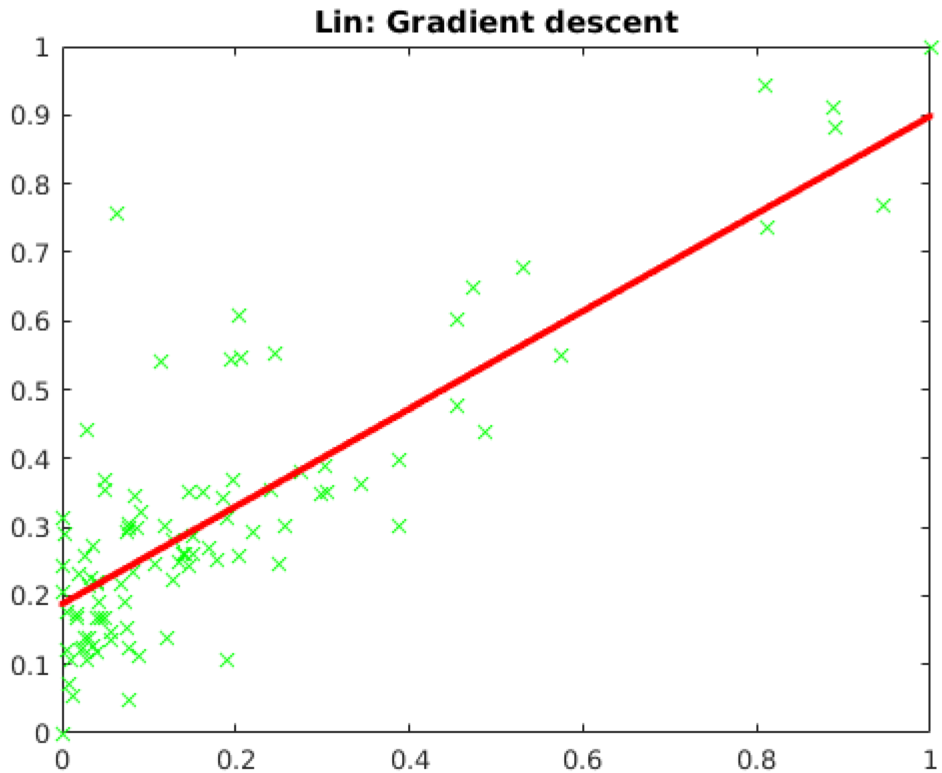

Lin: Polynomial
MSE
0.016
0.015
0.014
0.013
0.012
0.011
0.01
0.009
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
Fraction of data

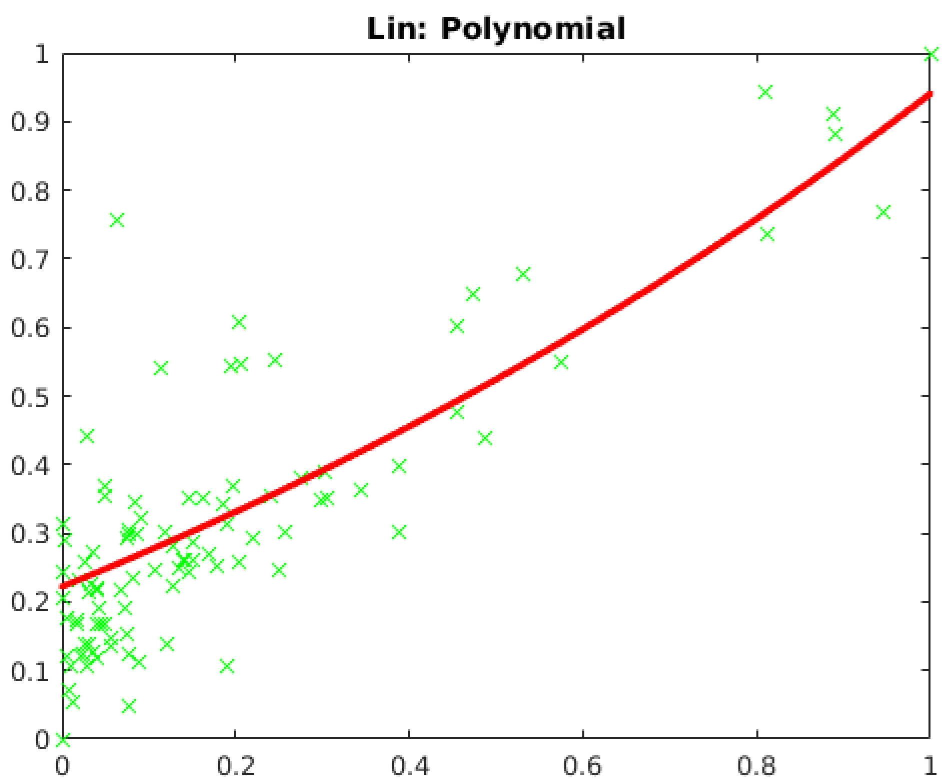

Sph: Polynomial
MSE
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
Fraction of data

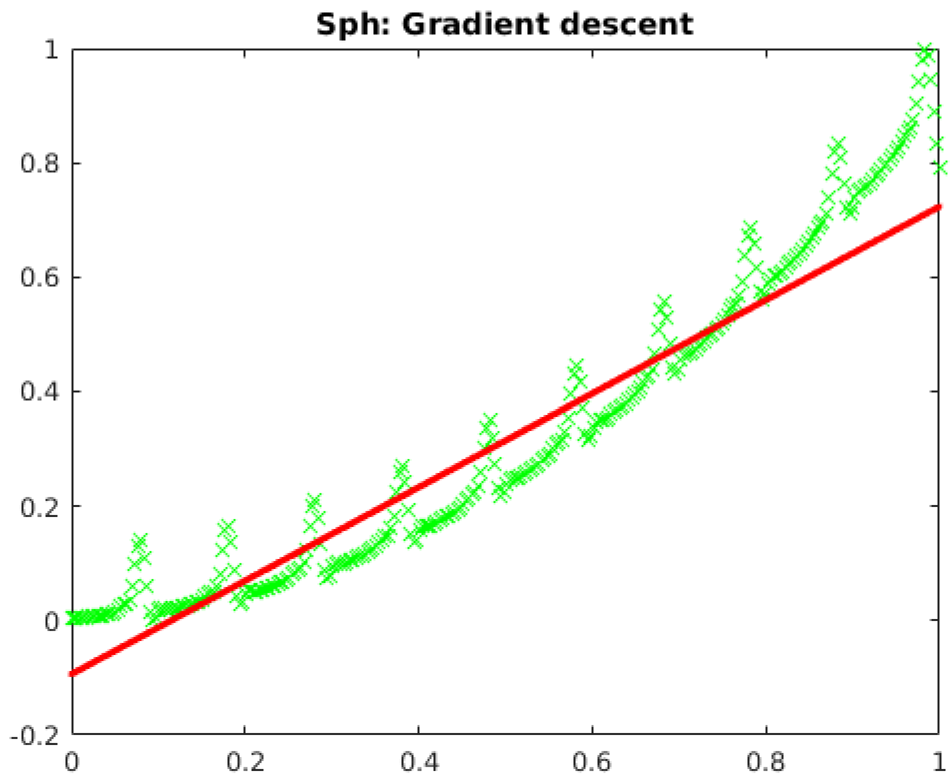
Visually, the best fit for the data seems to be quadratic. But, I went on with up to 3rd degree polynomial since it smooths out the curve a bit more and reduces the error even more. For prevention of over fitting, we can use ridge regression for proper fitting on the dataset.

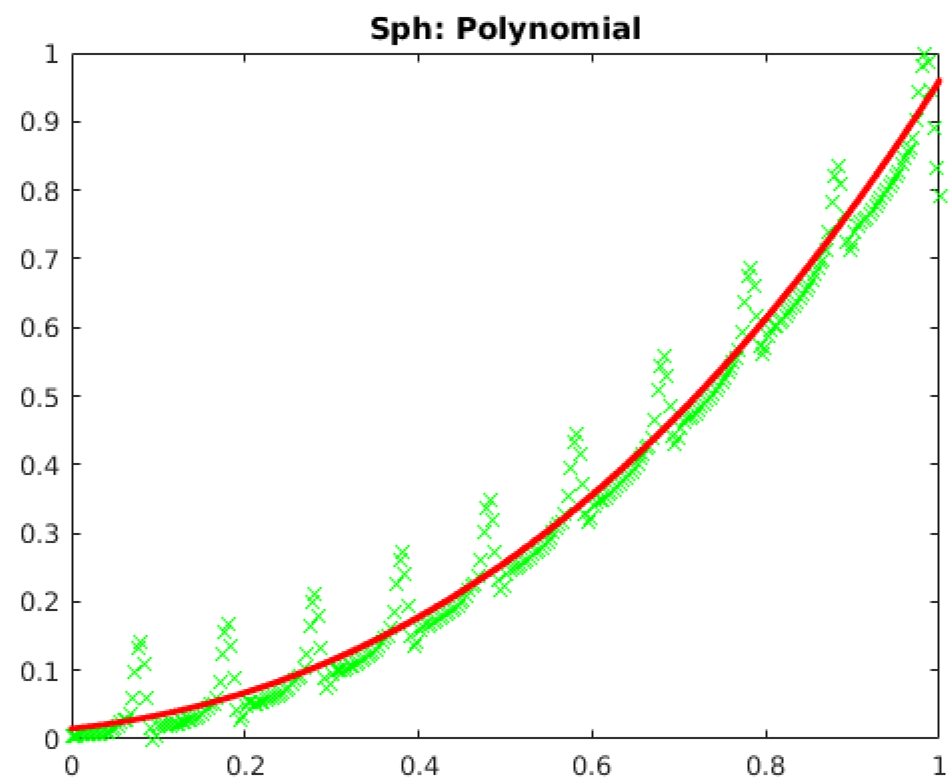
Also, for polynomial the error seems to be smaller(on avg) compared to gradient descent if the data is polynomial in nature i.e. for sph dataset.

Ans 4. c.




Sph: Gradient descent
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.2
0.4
0.6
0.8
1


Sph: Polynomial
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1

For lin dataset, polynomial fit doesn't seem to be a nice option. The data appears to be linear and hence is fit nicely by a non polynomial fit.

For sph dataset, the dataset seems to be polynomial visually and hence a $2^{nd}$ order or $3^{rd}$ order polynomial seems to be a better fit than a single line. In fact, a single line seems to be under fitting the data.

Ans 4. d.

Choosing the delta value:

Lin dataset:

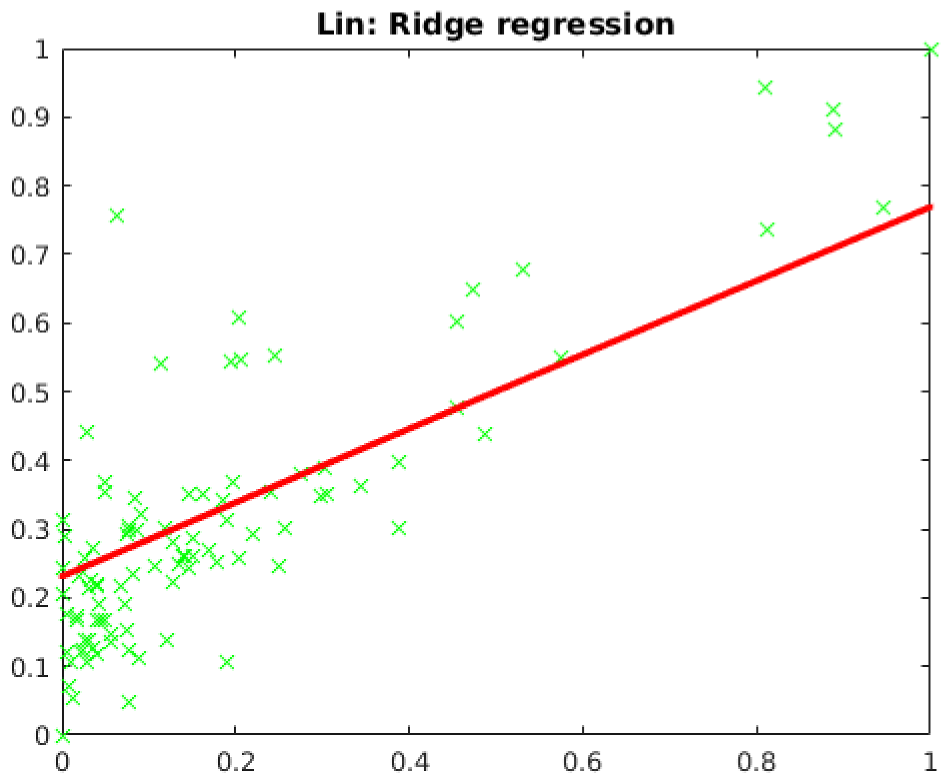
| | 80 % training data + 20 % test data | | | 90 % training data + 10 % test data | | |
|---|---|---|---|---|---|---|
| | Train Error | Test Error | Avg Error | Train Error | Test Error | Avg Error |
| 0.1 | 0.346640 | 0.014163 | 0.180401 | 0.185215 | 0.013989 | 0.099602 |
| 0.5 | 0.050667 | 0.017297 | 0.033982 | 0.149542 | 0.014900 | 0.082221 |
| 1 | 0.062826 | 0.007193 | 0.035009 | 0.031174 | 0.018482 | 0.024828 |
| 10 | 2.208664 | 0.008807 | 1.108736 | 1.922445 | 0.009766 | 0.966106 |
| 50 | 3.498616 | 0.011004 | 1.754810 | 2.168872 | 0.014136 | 1.091504 |

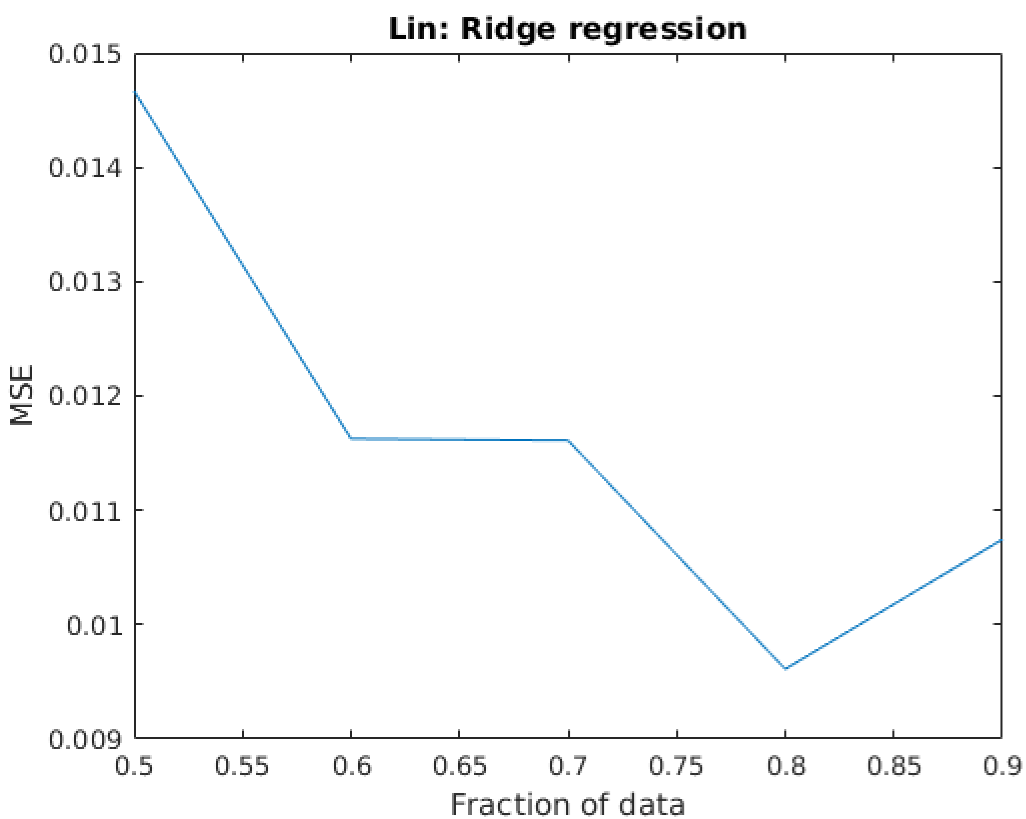
For 90% training example the test error seems to be minimized for delta = 0.1. It performs not so great compared to delta = 1(best avg error) but that is majorly due to poor performance in train error. Therefore, delta = 0.1

Sph dataset:

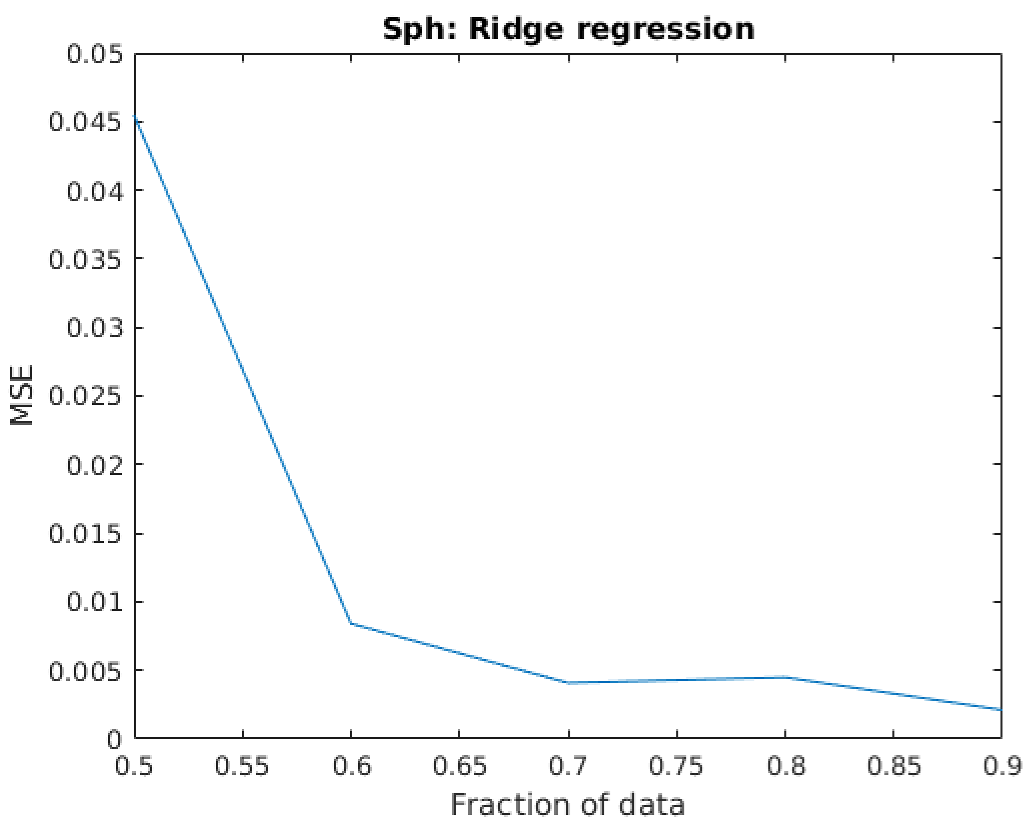
| | 80 % training data + 20 % test data | | | 90 % training data + 10 % test data | | |
|---|---|---|---|---|---|---|
| | Train Error | Test Error | Avg Error | Train Error | Test Error | Avg Error |
| 0.1 | 0.030096 | 0.002553 | 0.016325 | 0.031361 | 0.001729 | 0.016545 |
| 0.5 | 0.145685 | 0.005339 | 0.075512 | 0.153389 | 0.001715 | 0.077552 |
| 1 | 0.268837 | 0.003753 | 0.136295 | 0.293244 | 0.002082 | 0.147663 |
| 10 | 2.124107 | 0.024976 | 1.074542 | 2.301376 | 0.006795 | 1.154086 |
| 50 | 2.589090 | 0.020900 | 1.304995 | 13.872964 | 0.406893 | 7.139928 |

For 90% training example the test error seems to be minimized for delta = 0.5. It performs not so great compared to delta = 0.1(best avg error) but that is majorly due to poor performance in train error. Therefore, delta = 0.5

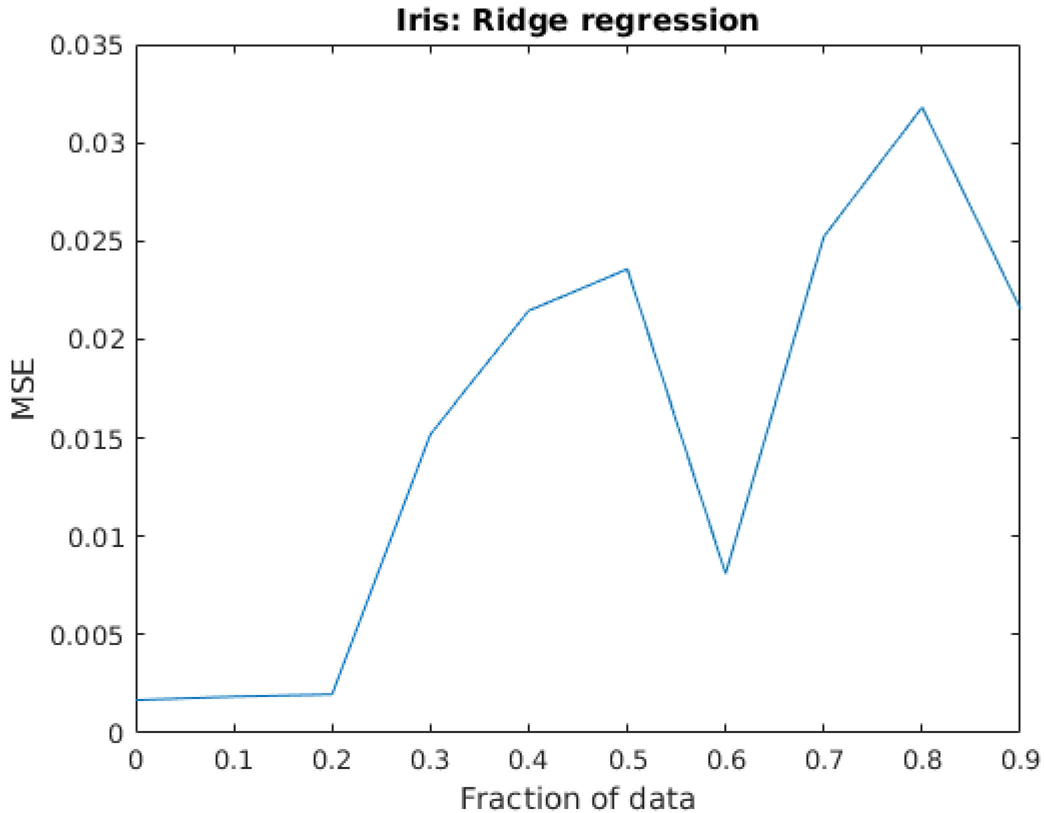

Lin: Ridge regression
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1


Lin: Ridge regression
MSE
0.015
0.014
0.013
0.012
0.011
0.01
0.009
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
Fraction of data

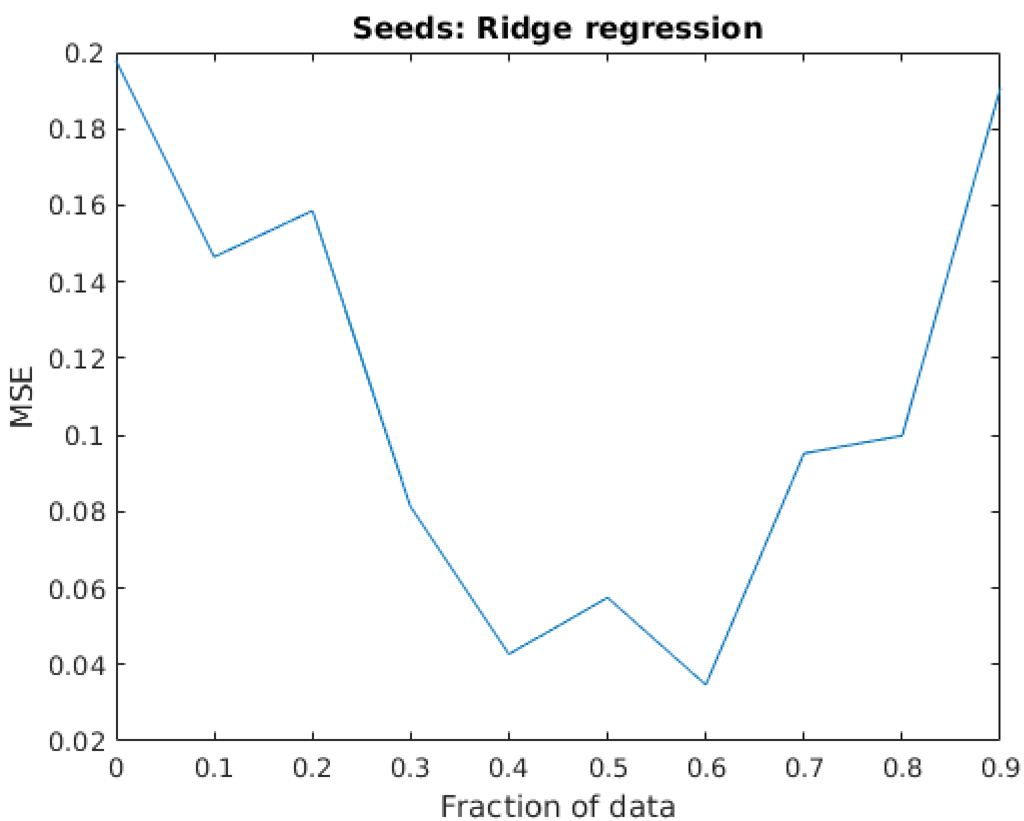

Sph: Ridge regression
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.2
0.4
0.6
0.8
1


Sph: Ridge regression
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
MSE
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
Fraction of data

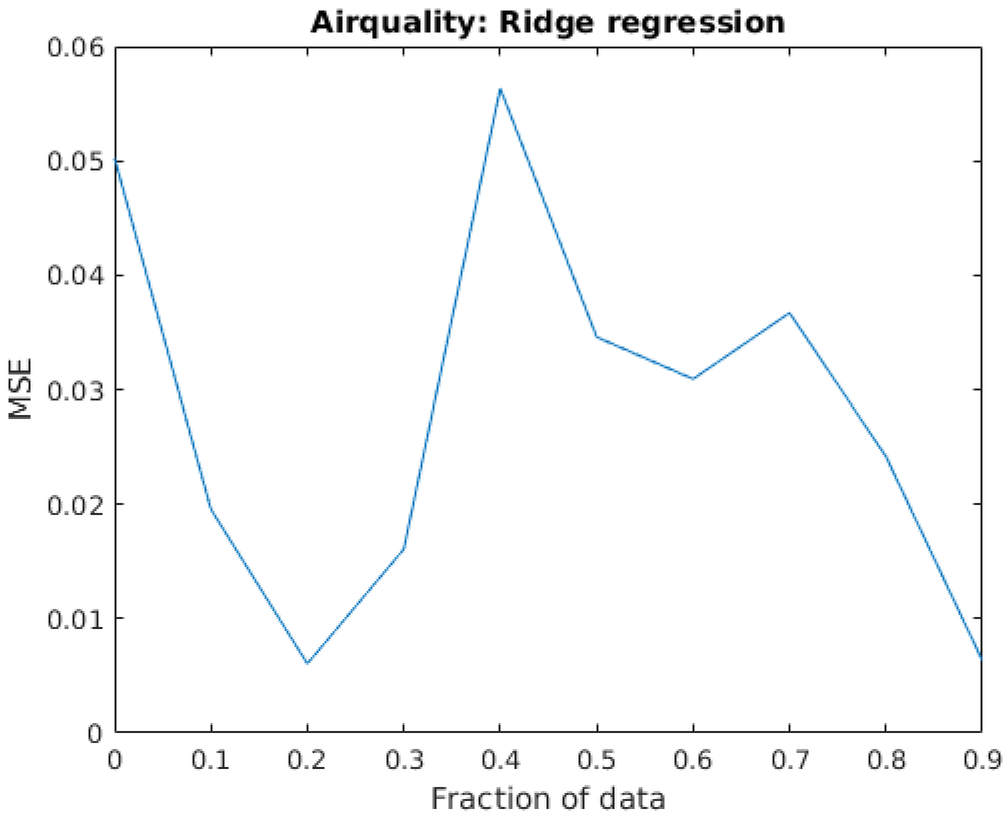
Ans 4. e. Error plots: (Fraction of data represents training data from 'fraction of data' to 'fraction of data'+10% data size)




Mean: 0.015237 Std deviation: 0.011118

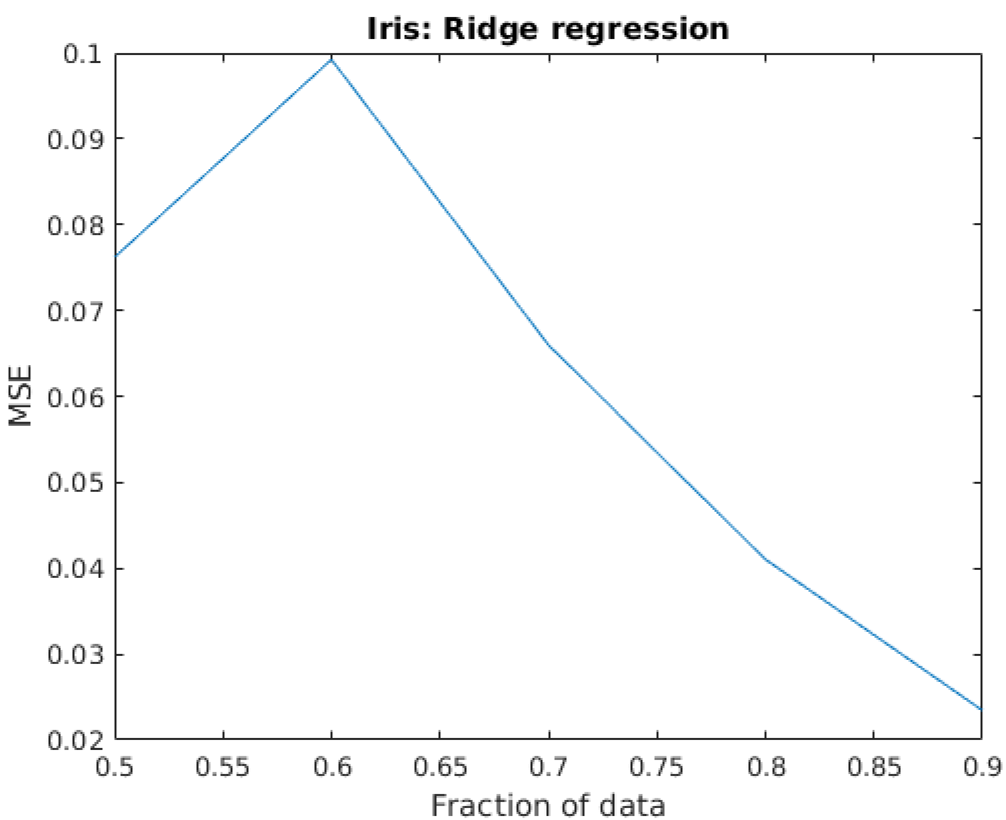


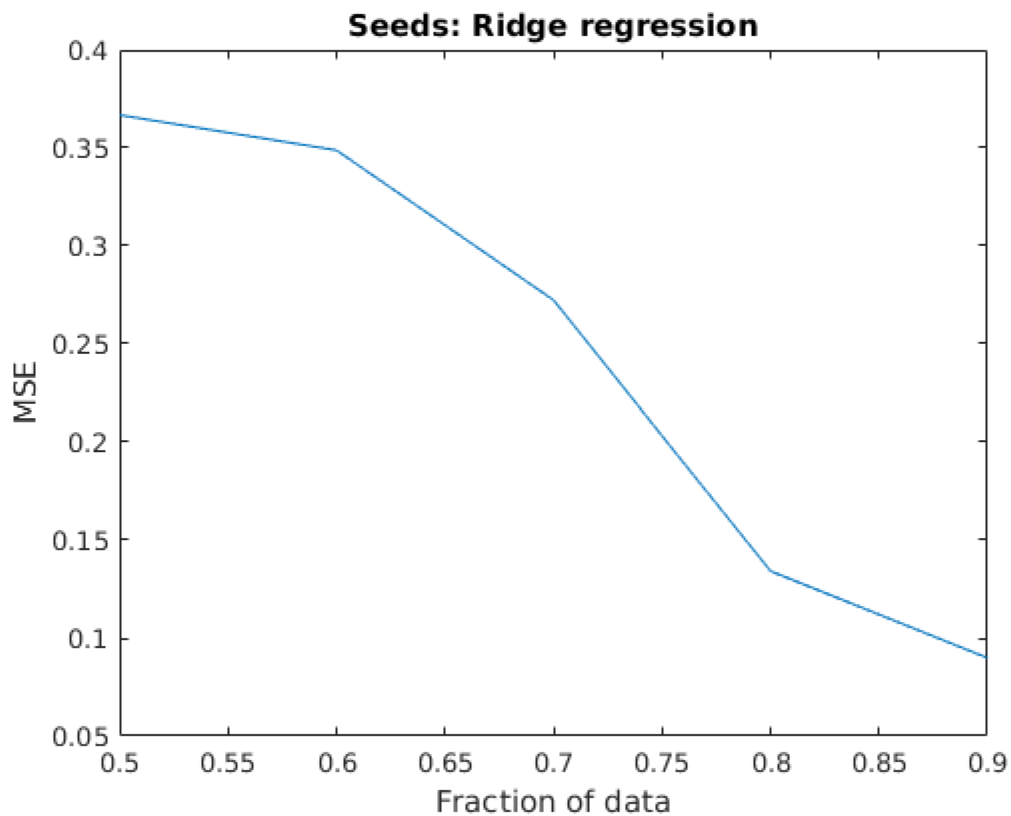
Mean: 0.110497 Std deviation: 0.059651

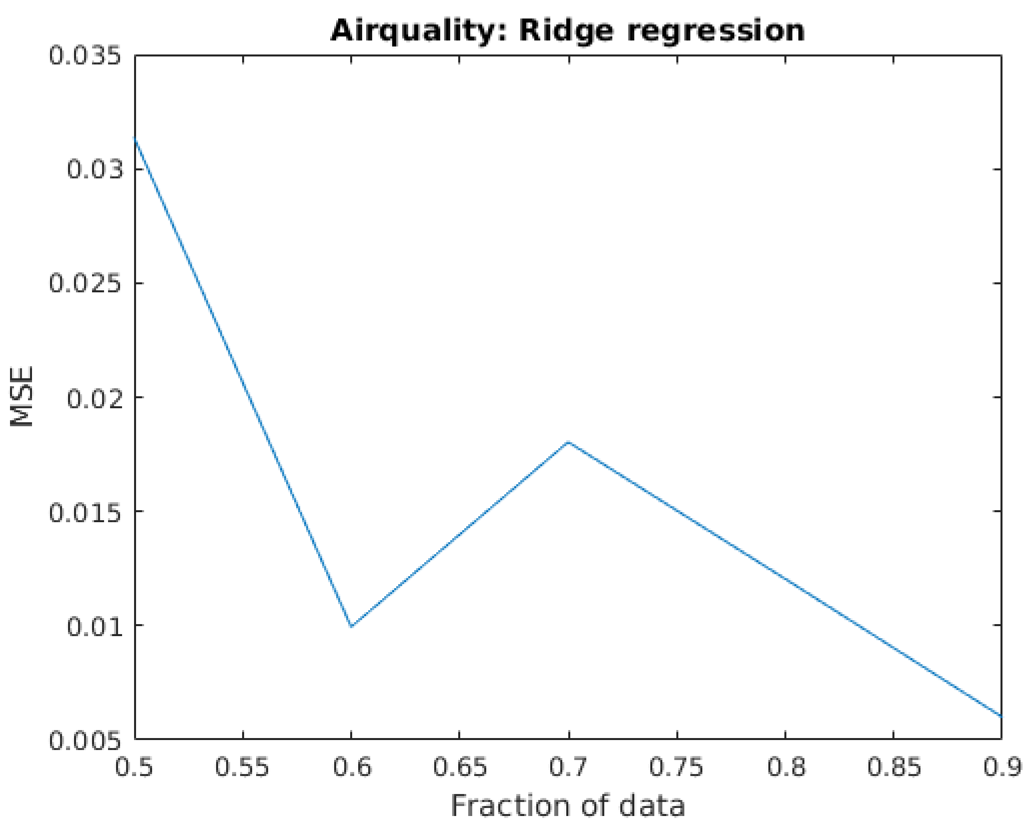



Mean: 0.028098 Std deviation: 0.017013

MSE vs fraction of data plots:




Seeds: Ridge regression
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
MSE
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
Fraction of data


Airquality: Ridge regression
0.035
0.03
0.025
0.02
0.015
0.01
0.005
MSE
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
Fraction of data