Cryptography

A (nearly) complete overview

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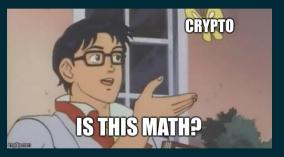
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Warning!

In this lesson we will use math!



It wasn't always like that though ...

Why cryptography?

Cryptography (from greek: kryptos "hidden, secret" and graphein, "to write")

```
problems Cruptographu
```

The science of secure communication

Cryptography yesterday





(a) Caesar Cipher

(b) Scitala

Cryptography today

The needs, as well as the available resources, have evolved and today we can divide cryptography into:

(EN DE)CRYPTION	ASYMMETRIC (RSA, ECC,) SYMMETRIC (DES, AES,)
KEY EXCHANGE	RSA, DH, ECDH,
AUTHENTICATION	RSA, DSA, ECDSA,
HASHING	MD5, SHA-1, SHA-256,

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What is a message?

A message is a sequence of symbols used to communicate

A symbol of the message is called character

The set of all the possibile characters is called alphabet

The set of all the possible (meaningful) messages is called dictionary

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ASCII encoding

ASCII = American Standard Code for Information Interchange char encoded in 7 bits + 1 bit for check (parity bit).

							ASC	II (197	7/1986)						
	_0	_1	_2	_3	_4	_5	_6	_7	_8	_9	_A	_B	_c	_D	_E	J.
0_	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	S0	SI
	0000	0001	0002	0003	0004	0005	0006	0007	0008	0009	000A	0008	000C	0000	000E	000F
1_	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
16	0010	0011	0012	0013	0014	0015	0016	0017	0018	0019	001A	0018	0010	0010	001E	001F
2_ 32	SP 0020	! 0021	0022	# 0023	\$ 0024	% 0025	& 0026	0027	(0028) 0029	* 002A	+ 0028	, 002C	- 0020	002E	/ 002F
3_	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
48		0031	0032	0033	0034	0035	0036	0037	0038	0039	003A	0038	003C	003D	003E	003F
4_	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	0
64	0040	0041	0042	0043	0044	0045	0046	0047	0048	0049	004A	0048	0040	0040	004E	004F
5_	P	Q	R	S	T	U	V	W	X	Y	Z	[\]	^	_
80	0050	0051	0052	0053	0054	0055	0056	0057	0058	0059	005A	005B	005C	005D	005E	005F
6_	0060	a	b	C	d	e	f	g	h	1	j	k	l	m	n	0
96		0061	0062	0063	0064	0065	0066	0067	0068	0069	006A	0068	0060	0060	006E	006F
7_	p	q	r	S	t	U	V	W	X	y	Z	{	007C	}	~	DEL
112	0070	0071	0072	0073	0074	0075	0076	0077	0078	0079	007A	0078		0070	007E	007F
	Letter	Nu	ımber	Pu	nctuati	on 📗	Symbo	ol 📗 (Other	und	efined	С	hange	d from	1963 v	ersion

 $0, \ldots, 31 + 127 \rightarrow \text{non-printable}$ chars (null, new line, tab, others) $32, \ldots, 126 \rightarrow \text{printable}$ chars (letters, digits, punctuation, others)

Extended ASCII \rightarrow char encoded in 8 bits (add 128 printable chars to standard ASCII)

Unicode encoding

Obviously 128 (or 256) characters are **not enough!** (Chinese, cyrillic, greek alphabets, emojis...)

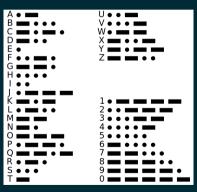
Different standards: UTF-8, UTF-16, UTF-32 and others



Currently assigned "only" 137993 characters

Morse code

(Audio) character encoding scheme used in (telegraph) telecommunication. Each character is encoded using a combination of short/long signal.



Braille code

(Tactile) character encoding scheme used for visually impaired people. Each characted is encoded using a 2×3 rectangle with "raised dots".

• 0 0 0	• o	• •	• • • •	• 0	• •	• •	• 0	0 •	0 • • • • •
a/1	b/2	c/3	d/4	e/5	f/6	g/7	h/8	i/9	j/O
• 0 0	• 0	• •	• •	• 0	• •	• •	• •	• 0	· •
k	- 1	m	n	0	р	q	r	s	t
• 0	• 0	• • • •	• • • •	• · ·					•••
u	V	х	у	Z					w

Base64

Group message in blocks of 6 bits.

Advantage: encode all the ASCII chars in printable characters

source ASCII (if <128)	М							а							n									
source octets		77 (0x4d)							97 (0x61)							110 (0x6e)								
Bit pattern		1	0	0	1	1	0	1	0	1	1	0	0	0	0	1	0	1	1	0	1	1	1	0
Index		19						2	2			5								46				
Base64-encoded		т				w					F						u							
encoded octets	84 (0x54)				87 (0x57)					70 (0x46)						117 (0x75)								

Valore	ASCII	Valore	ASCII	Valore	ASCII	Valore	ASCI
0	A	16	Q	32	g	48	W
1	В	17	R	33	h	49	Х
2	C	18	S	34	1	50	у
3	D	19	T	35	j	51	z
4	E	20	U	36	k	52	θ
5	F	21	V	37	ι	53	1
6	G	22	М	38	n	54	2
7	н	23	X	39	n	55	3
8	I	24	Y	40	0	56	4
9	3	25	Z	41	р	57	5
10	K	26	a	42	q	58	6
11	L	27	b	43	r	59	7
12	М	28	С	44	5	60	8
13	N	29	d	45	t	61	9
14	0	30	e	46	u	62	+
15	P	31	f	47	v	63	1

Message are padded with = (e.g. $flag \rightarrow ZmxhZwo=$)

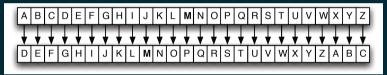
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Caesar cipher

Encrypt: right shift each letter of 3 positions

Decrypt: left shift each letter of 3 positions

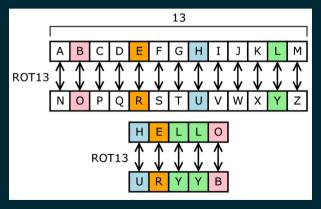


General cipher: shift letter of K positions

Attack: bruteforce all the possible K (only 26 values...)

ROT{13, 47}

ROT13: Caesar cipher with K=13 on alphabetic dictionary ROT47: Caesar cipher with K=47 on printable ASCII chars (33 - 126).



Why K = 13 (or K = 47)? Because Encrypt = Decrypt

Classical ciphers

Substitution ciphers

- Monoalphabetic ciphers: $C_{new} = P[C_{old}]$ (Where P is an alphabet permutation) (ROT-K is a monoalphabetic cipher where P is a cyclic rotation)
- Polialphabetic ciphers: multiple substitution alphabets (more than one dictionary permutation)

Transposition ciphers

Encryption systems where the positions held by units of plaintext (characters or groups of characters) are shifted according to a regular system.

E.g. We want to encrypt the message $WE\ ARE\ DISCOVERED.\ FLEE\ AT\ ONCE$ using the ${\bf route\ cipher}$:

Grid:

WRIORFEOE
EESVELANJ
ADCEDETCX
Cipher text: EJXCTEDECDAEWRIORFEONALEVSE

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Polialphabetic substituion cipher: Vigenère

The problem with monoalphabetic ciphers is that each character of the alphabet is replaced with always the same character in the ciphertext.

What can we do to solve this weakness? Stack more ciphers!

The Vigenère cipher is basically a sequence of Caesar ciphers with different shifts.



This table is called Vigenère table. It contains all the Caesar ciphers.

Encryption is performed character-by-character by accessing the cell of the table within the row corresponding to the current key letter and column to the plaintext letter.

Decryption works in the same way of the encryption but we use a transposed Vigenère table.

This simple technique earned the name of the indecipherable cipher, and resisted attacks for over 3 centuries (1553-1863)!

Vigenère cipher: an example

We want to encrypt the message THIS IS AN EXAMPLE with the key SECRET

First of all, we repeat the key until it has the same length of the plaintext:

```
m = THISISANEXAMPLE
k = SECRETSECRETSEC
```

In our example, the first letter of the plaintext is T and the first of the key is S, so we access the row S and column T, which yields L.

And so on until the end of the message:

```
c = LLKJMLSRGOEFHPG
```

Exercise:

```
c = UEGJEKTYVDSKWJWE
k = SECRET
m = ???
```

Transposition cipher: an example

The plaintext is written in a rectangular grid and then the columns are reshuffled, the encryption key is the column permutation

$$c = SHTINSIAMXEA=LPE$$

To recover the original message, just write the ciphertext in a grid again and apply the inverse permutation of columns

Exercise:

$$c = AGLF3RP\{T3UM5_1_D\}4B$$

 $m = ???$

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dcode.fr

https://www.dcode.fr/tools-list



Almost all possible classic ciphers (old and new), encoder/decoder, ...

Cryptanalysis

Often the vulnerability is not in the algorithm but in its application...

- ▶ Bad use of the key (too short, reused, badly generated, ...)
- ► Messages use a poorly distributed dictionary
- ▶ We know the message format (e.g.: $FLAG\{...\}$)

In particular we talk about statistical cryptanalysis when we force the cipher not from algorithmic point of view but from statistical one

For example in english the character E has a frequency of 12.02% while Z only 0.07%

Useful tool (for substitution ciphers): https://quipqiup.com

```
Puzzle: giuifg cei iprc tpnn du cei qprcni
-0.842 defend the east wall of the castle
1 -0.859 defend the east ball of the castle
2 -0.915 defend the east mall of the castle
```

Attack models

Classification of cryptographic attacks:

- ► Ciphertext-only attack: access only to the ciphertext, and has no access to the plaintext
- ► Known-plaintext attack: access to at least a limited number of pairs of plaintext and the corresponding enciphered text
- ► Chosen plaintext attack: able to choose a number of plaintexts to be enciphered and have access to the resulting ciphertext (encrypt oracle)
- ► Chosen ciphertext attack: able to choose arbitrary ciphertext and have access to plaintext decrypted from it and vice-versa (encryption/decrypt oracle)
- ► Side-channel attack: use of other informations to break the cipher (time, sound, power, error, ...).
- ▶ Bruteforce attack: try all the possible passwords/keys/... then wait, wait and wait.

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Symmetric-key cryptography

Symmetric ciphers are those where messages are encrypted and decrypted using the same key, which must be known only and exclusively to the two parts

$$C(m, k) = c$$
 (encrypt function)
 $D(c, k) = m$ (decrypt function)

Obviously:

$$\mathcal{D}(\mathcal{C}(m,k),k)=m$$

The original message is not altered during the communication

E.g. In the caesar cipher:

$$C(m, k) = \text{right shift of } k \text{ positions each character}$$

$$\mathcal{D}(c,k) = \text{left shift of } k \text{ positions each character}$$

Shannon principle

How to assess whether a cipher is robust enough? (Where robustness means its probability of being successfully attacked)

Shannon defines two key concepts:

- ► Confusion: the key must be well distributed in the encrypted message (each bit of the cipher should depend on each bit of the key with probability 50%)
- ▶ Diffusion: the message must be well distributed in the encrypted message (each bit of the cipher should depend on each bit of the message with probability 50%)

In the Caesar cipher we have no kind of diffusion and low confusion (why?)

XOR cipher

Consider the XOR (exclusive or) operation \oplus , the following properties are valid:

- $ightharpoonup 0 \oplus 0 = 1 \oplus 1 = 0$
- $ightharpoonup 0 \oplus 1 = 1 \oplus 0 = 1$
- $\blacktriangleright \ \ x \oplus y \oplus y = x$

We define the XOR cipher as:

Problem: the key k could be shorter than the message m Solution: reuse the key as $k' = k \cdot k \cdot \ldots \cdot k$ until ||k'|| > m

Example:

m = 01100011 01101001 01100001 01101111 (ciao in ASCII). k = 01111000 01111000 01111000 (x in ascii 4 times)

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One-time pad

The problem with the XOR cipher is that encrypting repeatedly reusing the same key can leak statistical informations of the original message

We call Vernam cipher (or one-time pad) a XOR cipher where the key has the same length of the message.

This cipher is called **perfect** because we have that:

$$P(M = m | C = c) = P(M = m)$$

The probability that M is a certain message m knowing that the cipher C is c is equal to the probability that M is a certain message not knowing the cipher (all ciphertexts are equiprobable, the encrypted message does not give us any information about the real message)

Nice in theory, but:

- ► The key must be exchanged using a secure method (exchange them by hand)
- ► The key must be generated randomly and not reused (otherwise a many-time pad attack is possible)

Many-time pad & XorTool

Nice article: the crowned.org/the-one-time-pad-and-the-many-time-pad-vulnerability

XorTool: tool for statistical analysis of encrypted messages:

```
rootEddCos:-/Desktop/xortool/xortool# xortool binary_xored
The most probable key lengths:
1: 9.6%
10: 21.7%
15: 9.3%
20: 13.6%
25: 6.0%
30: 9.1%
35: 4.2%
40: 6.6%
50: 5.0%
Key-Length can be 5*n
Most possible char is needed to guess the key!
```

Knowing the initial part, we can see words in the message:

Going by trial the final flag is reconstructed:

```
This is classified********
Do not share the s****

{FLG:ch3ck em@il}
```

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Block vs Stream ciphers

Block ciphers

- Works with fixed-length groups of bits (called blocks)
- ► More memory/time requirements
- ► High diffusion and confusion
- ► Error propagation
- Need to handle messages length (padding)

Famous ciphers:

DES

AES

BlowFish

Stream ciphers

- ► Works by encrypt digits one at the time
- ► Faster encryption/decryption
- ► Low diffusion
- ► Low propagation error
- ► Need a key stream (usually a shift register)

Famous ciphers:

ChaCha20

Salsa20

LFSR-based

Padding a message (PKCS#5 & PKCS#7)

How to handle messages of length not multiple of the block size? Idea: append "some chars" to the message (padding string)

PKCS#5: The padding string PS shall consist of $8 - (||M|| \mod 8)$ octets all having value $8 - (||M|| \mod 8)$

PKCS#7: For such algorithms, the method shall be to pad the input at the trailing end with $k - (l \mod k)$ octets all having value $k - (l \mod k)$, where l is the length of the input

Why $8 - (||M|| \mod 8)$ and not $(||M|| \mod 8)$?

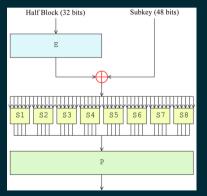
Padded 12345678 11 \rightarrow 12345678 11**666666** Padded 12345678 1111 \rightarrow 12345678 1111**4444** Padded 12345678 \rightarrow 12345678 **88888888**

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DES

Data Encryption Standard

Developed in 1975 by Feistel, encrypt blocks of 64 bits with a 56 bits key Implements the confusion and diffusion principle by 16 rounds of the Feistel function



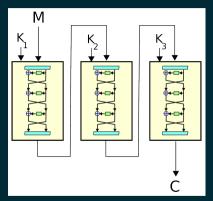
The Feistel function consists in 4 stages:

- ► 1. Expand the half block from 32 to 48 bits (E-Box)
- ► 2. Mix result and subkey using a XOR operation
- ➤ 3. Substitution of the 6-bits input with a 4-bits output according to a lookup table (S-Box)
- ► 4. Permutation of the result (P-Box)

Problem: DES is vulnerable to a (smart) bruteforce attack (only 56 bits for the key...)

3DES

What is three times more secure than DES? 3 DES!

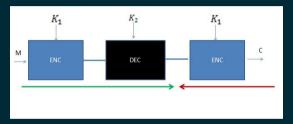


$$C = E_{K_3}(D_{K_2}(E_{K_1}(M)))$$

 $M = D_{K_1}(E_{K_2}(D_{K_3}(C)))$

From DES to 3DES... why not 2DES?

Two keys of 56 bit \rightarrow One key of 112 57 bits



$$C = E_{K_2}(E_{K_1}(M))$$

How to retrieve (K_1, K_2) ? Meet K_1 with K_2 !

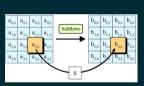
$$E_{K_1}(M)=D_{K_2}(C)$$

AES

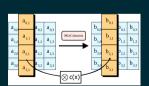
Advanced Encryption Standard

AES replace DES starting from 2001 and is currently the standard in secure communications (TLS1.3 support only AES and ChaCha20)

Based on a substitution-permutation network (the Feistel network equivalent of DES)

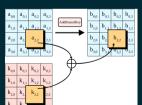


Step 1: Each byte is replaced with another according to a lookup table (S-Box)



Step 3: Linear mixing operation where each column is mapped into a new one





Step 2: Transposition of each rows by 0, 1, 2 or 3

Step 4: Each byte is XORed with the corresponding value of the subkey

Block cipher mode of operation

How to ciphers two or more blocks? Different modes, different features:

- ▶ Parallel encryption: encrypt different blocks at the same time
- ▶ Parallel decryption: decrypt different blocks at the same time
- ▶ Random read: decrypt any single block without decrypting the previous ones

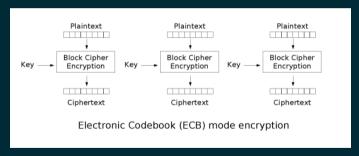
Mode	Parallel	Paralllel	Random
lviode	encryption	decryption	read
Electronic Code Book (ECB)	Yes	Yes	Yes
Cipher Block Chaining (CBC)	No	Yes	Yes
Propagating CBC (PCBC)	No	No	No
Cipher Feedback (CFB)	No	Yes	Yes
Output Feedback (OFB)	No	No	No
Counter (CTR)	Yes	Yes	Yes

ECB (Electronic Code Book)

The message is divided into blocks, and each block is encrypted/decrypted separately:

$$C_i = E_{Key}(M_i)$$

 $M_i = D_{Key}(C_i)$

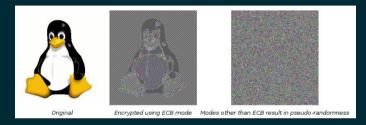


Problem: no diffusion, ECB encrypt same plaintext in same ciphertext

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How to break ECB (Chosen-prefix attack)

Problem: no diffusion, ECB encrypt same plaintext in same ciphertext



How to reverse: ae69a8467c46bd2f8d30db166ebfc135 ightarrow XXXXXXXX ?

ldea: AAAAAAA $^{
m Y}$ AAAAAAA $^{
m X}$ XXXXXXXP ightarrow c_0 c_1 c_2

To find X try all the possible value of Y until $c_0=c_1$

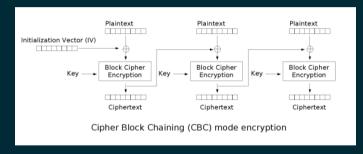
 Idea^2 : AAAAAAX $^{\mathsf{Y}}$ AAAAAAX $^{\mathsf{X}}$ XXXXXXPP $\mathsf{P} o c_0 \ c_1 \ c_2$

To find X try all the possible value of Y until $c_0 = c_1$

CBC (Cipher Block Chaining)

In CBC mode each plaintext block is XORed with the previous ciphertext block before being encrypted

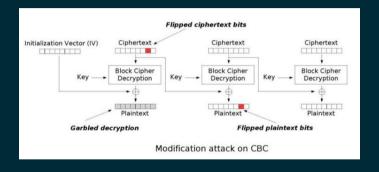
An initialization vector IV is needed for the first block (usually random generated)



$$C_0 = IV$$

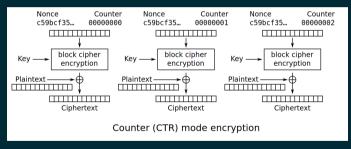
 $C_i = E_{Key}(M_i \oplus C_{i-1})$
 $M_i = D_{Kev}(C_i) \oplus C_{i-1}$

How to alter CBC messages? (bit-flipping attack)



CTR (Counter)

Counter mode turns a block cipher into a stream cipher. It generates the next keystream block by encrypting successive values of a "counter".



Stream cipher: LFSR

Linear-Feedback Shift Registers

Shift register whose input bit is a linear function of its previous state

```
class LFSR:
    def __init__(self, register, branches):
        self.register = register
        self.branches = branches
        self.n = len(register)
    def next_bit(self):
        ret = self.register[self.n - 1]
        new = 0
        for i in self.branches:
            new ^= self.register[i - 1]
        self.register = [new] + self.register[:-1]
        return ret
```

```
register = [0,1,1,0,1,0,0,0,1,1,1,1,0,0,1,1]
branches = [10,12,13,15]
gen = LFSR(register, branches)
for c in stream:
    print(c ^ gen.next_bit())
```

Vulnerability: we can retrieve register and branches if we know a portion of the key stream (via the Berlekamp-Massey algorithm)