A Developer Diary

{about:"code learn and share"}

April 18, 2019 By Abhisek Jana — 24 Comments (Edit)

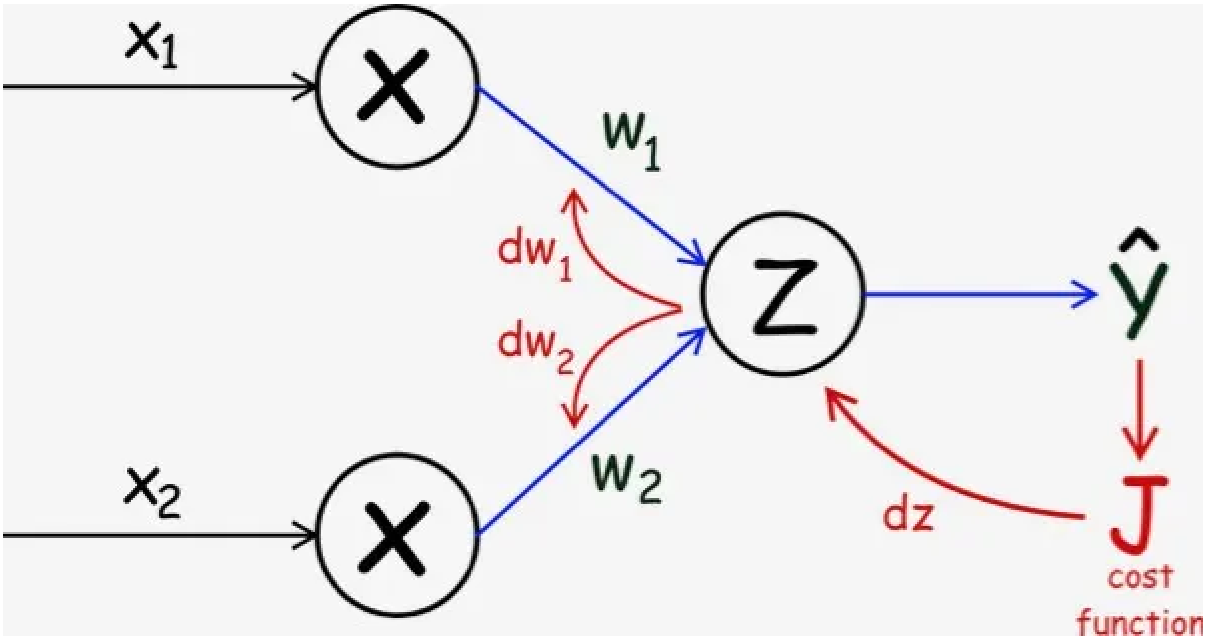
Understand and Implement the Backpropagation Algorithm From Scratch In Python

It’s very important have clear understanding on how to implement a simple Neural Network from scratch. In this Understand and Implement the Backpropagation Algorithm From Scratch In Python tutorial we go through step by step process of understanding and implementing a Neural Network. We will start from Linear Regression and use the same concept to build a 2-Layer Neural Network.Then we will code a N-Layer Neural Network using python from scratch.As prerequisite, you need to have basic understanding of Linear/Logistic Regression with Gradient Descent.

Let’s see how we can slowly move towards building our first neural network.

Linear Regression:

Here we have represented Linear Regression using graphical format (Bias `b` is not shown). As you see in the below diagram, we have two input features (x_1, x_2). `Z` represents the linear combination of the vectors `w`. The node with `Z` can also be named as hidden unit, since `X` & `Y` are visible (for training) and `Z` is something defined inside the model.



We can write the equation for predicting values using above linear regression as (this is shown using blue arrow),
$\hat{y} = z = b + x_1 w_1 + x_2 w_2$

So in order to find the best `w`, we need to first define the cost function `J`.To use gradient descent, take derivative of the cost function `J` w.r.t `w` and `b`, then update `w` and `b` by a fraction (learning rate) of `dw` and `db` until convergence (this is shown using red arrow).
We can write `dw` and `db` as follows (using chain rule).

$\frac{dJ}{dW} = \frac{dJ}{dZ}\frac{dZ}{dW}$
$\frac{dJ}{db} = \frac{dJ}{dZ}\frac{dZ}{db}$
And the gradient descent equation for updating `w` and `b` are,

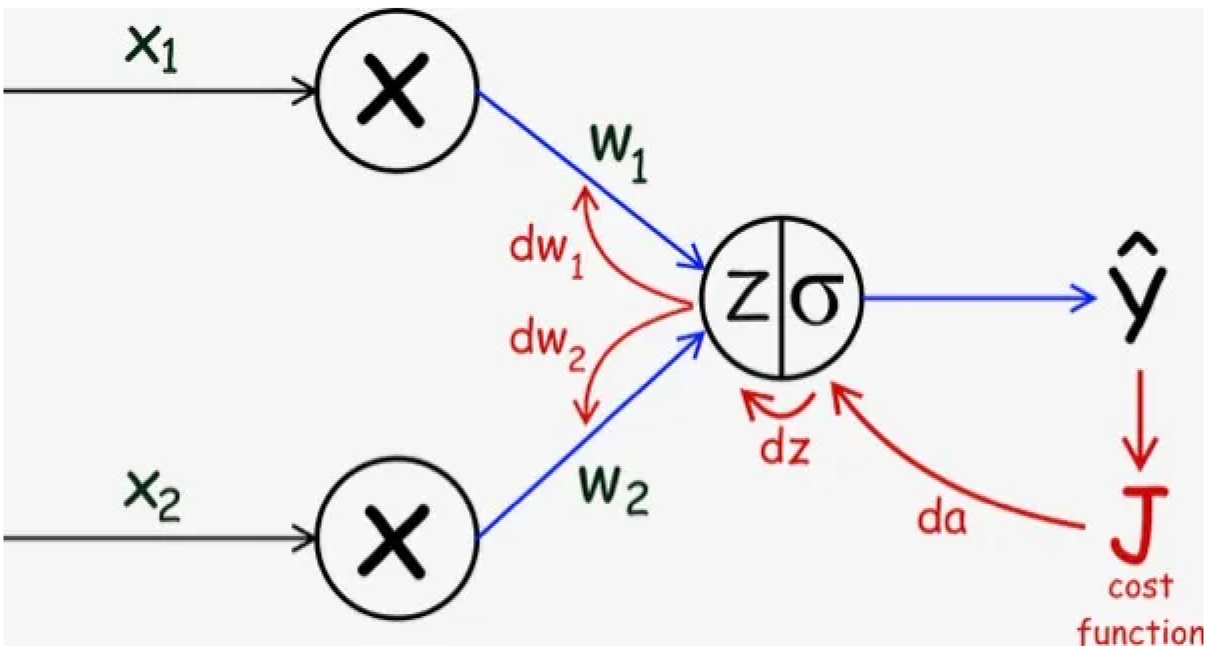
$W =: W - \alpha \frac{dJ}{dW}$
$b =: b - \alpha \frac{dJ}{db}$
In summary, first we predict the $\hat{y}$, then using this we calculate the cost, after that using gradient descent we adjust the parameters of the model. This happens in a loop and eventually we learn the best parameters (`w` and `b`) to be used in prediction. The above picture depicts the same.

Logistic Regression:

Here we will try to represent Logistic Regression in the same way. Mathematically Logistic regression is different than Linear Regression in two following ways:

- Logistic Regression has a different Cost Function `J`
- Apply a non-linear transformation (Sigmoid) on `Z` to predict probability of class label (Binary Classification)

As you see in the below diagram the blue arrow indicates the Forward Propagation.



Here are the steps of Forward Propagation in Logistic Regression. (Matrix Format)

$$Z = W^T X + b$$
$$\hat{y} = A = \sigma(Z)$$

The Gradient Descent (a.k.a Backpropagation) in Logistic Regression has an additional derivative to calculate.

$$\frac{dJ}{dW} = \frac{dJ}{dA} \frac{dA}{dZ} \frac{dZ}{dW}$$
$$\frac{dJ}{db} = \frac{dJ}{dA} \frac{dA}{dZ} \frac{dZ}{db}$$

The gradient descent equation for updating `w` and `b` will be exactly same as Linear Regression (They are same for Neural Network too),

$$W =: W - \alpha \frac{dJ}{dW}$$
$$b =: b - \alpha \frac{dJ}{db}$$

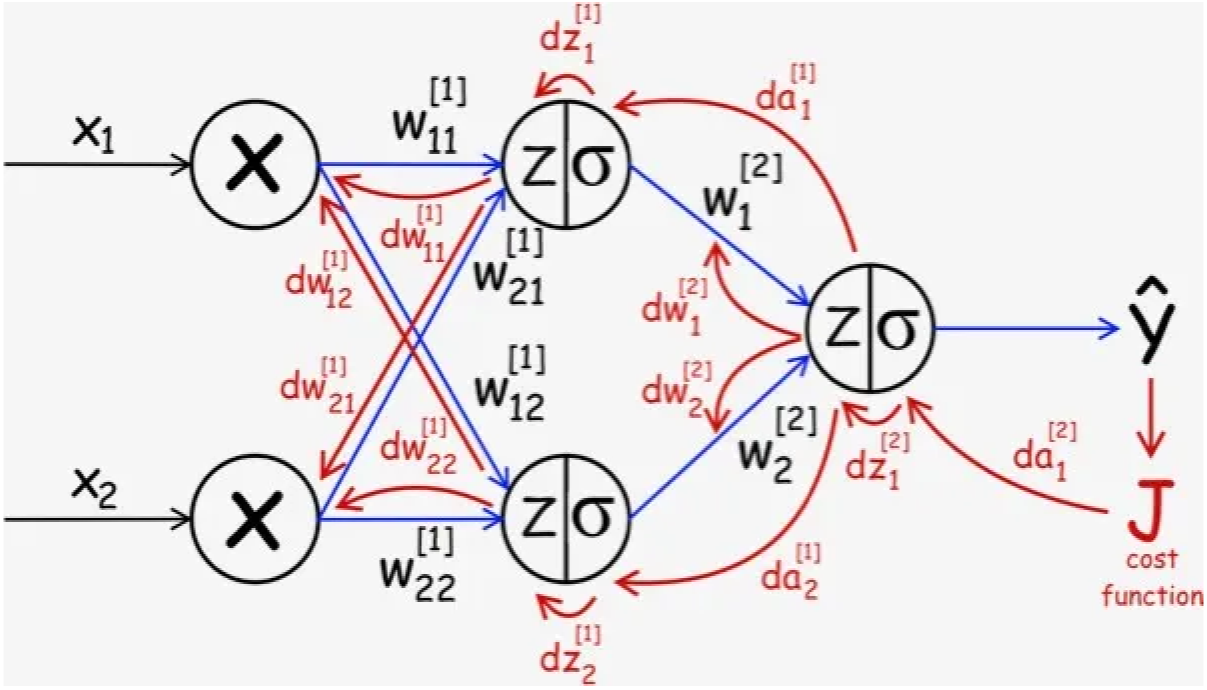
The process flow diagram is exactly the same for Logistic Regression too.

We can say that Logistic Regression is a 1-Layer Neural Network. Now we will extend the idea to a 2-Layer Neural Network.

2-Layer Neural Network:

Extend the same concept to a 2-Layer Neural Network. Refer the below diagram (bias term is not displayed). There are some minor notation changes, such as, the super-script now denotes the layer number. We have added two more hidden units to our model. The vector `W` will have different dimension for each hidden layer.

In case you are new to Neural Network, imagine that the output of the first layer used as input to the next layer. Earlier in case of Logistic Regression we didn’t have multiple layers. These intermediate hidden layers provides a way to solve complex tasks (a.k.a non-linearity).



We can write the forward propagation in two steps as (Consider uppercase letters as Matrix).

$$
\begin{aligned}
Z^{[1]} &= W^{[1]}X + b^{[1]} \\
A^{[1]} &= \sigma(Z^{[1]}) \\
Z^{[2]} &= W^{[2]}A^{[1]} + b^{[2]} \\
\hat{y} &= A^{[2]} = \sigma(Z^{[2]})
\end{aligned}
$$

Again, just like Linear and Logistic Regression gradient descent can be used to find the best `W` and `b`. The approach is basically same :

Define a cost function:

Take derivative `(dw, db)` of the cost function `J` w.r.t `w` and `b`.
Update `w` and `b` using `dw`, `db`.

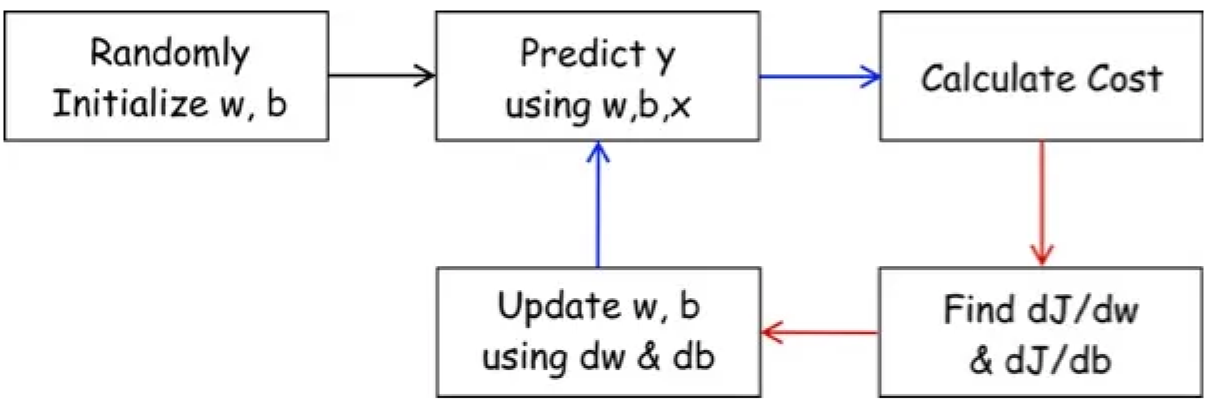
The back propagation has been shown in the above diagram using the red arrows. Let's find the `dw` and `db` using chain rule. This might look complicated, however if you just follow the arrows can you can then easily correlate them with the equation.

$$dW^{[2]} = \frac{dJ}{dW^{[2]}} = \frac{dJ}{dA^{[2]}} \frac{dA^{[2]}}{dZ^{[2]}} \frac{dZ^{[2]}}{dW^{[2]}}$$
$$db^{[2]} = \frac{dJ}{db^{[2]}} = \frac{dJ}{dA^{[2]}} \frac{dA^{[2]}}{dZ^{[2]}} \frac{dZ^{[2]}}{db^{[2]}}$$
$$dW^{[1]} = \frac{dJ}{dW^{[2]}} = \frac{dJ}{dA^{[2]}} \frac{dA^{[2]}}{dZ^{[2]}} \frac{dZ^{[2]}}{dA^{[1]}} \frac{dA^{[1]}}{dZ^{[1]}} \frac{dZ^{[1]}}{dW^{[1]}}$$
$$db^{[1]} = \frac{dJ}{dW^{[2]}} = \frac{dJ}{dA^{[2]}} \frac{dA^{[2]}}{dZ^{[2]}} \frac{dZ^{[2]}}{dA^{[1]}} \frac{dA^{[1]}}{dZ^{[1]}} \frac{dZ^{[1]}}{db^{[1]}}$$

Finally, we will update `w` and as following, (same as other algorithms)

$$W^{[1]} =: W^{[1]} - \alpha \frac{dJ}{dW^{[1]}}$$
$$b^{[1]} =: b^{[1]} - \alpha \frac{dJ}{db^{[1]}}$$
$$W^{[2]} =: W^{[2]} - \alpha \frac{dJ}{dW^{[2]}}$$
$$b^{[2]} =: b^{[2]} - \alpha \frac{dJ}{db^{[2]}}$$

As you see, technically the steps are same for Linear Regression, Logistic Regression and Neural Network.



In Artificial Neural Network the steps towards the direction of blue arrows is named as Forward Propagation and the steps towards the red arrows as Back-Propagation.

Backpropagation:

One major disadvantage of Backpropagation is computation complexity. Just for 2 layer Neural Network with 2 hidden unit in layer one, we already have pretty complex equation to solve. Imagine the computation complexity for a network having 100's of layers and 1000's of hidden units in each layer. In order to solve this problem we can use **dynamic programming**.

The high level idea is to express the derivation of $dw^{[l]}$ (where `l` is the current layer) using the already calculated values ($dA^{[l+1]}, dZ^{[l+1]} etc$) of layer `l+1`. In nutshell, this is named as Backpropagation Algorithm.

We will derive the Backpropagation algorithm for a 2-Layer Network and then will generalize for N-Layer Network.

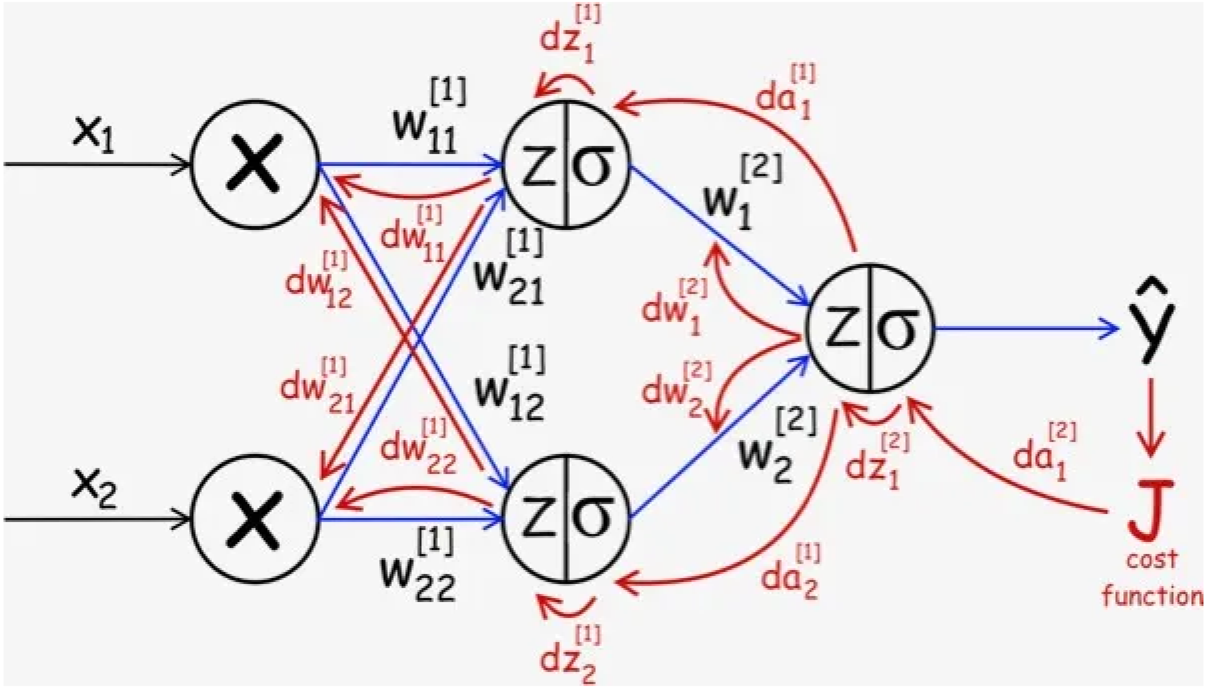
Derivation of 2-Layer Neural Network:

For simplicity propose, let's assume our 2-Layer Network only does binary classification. So the final Hidden Layer will be using a `Sigmoid Activation` function and our Cost function will be simply the `Binary Cross Entropy Error Function` used in Logistic Regression. The Activation function of the remaining hidden layer can be anything.

Why the above assumptions are important:

Since the Backpropagation starts from taking derivative of the cost/error function, the derivation will be different if we are using a different activation function such as `Softmax` (at the final hidden layer only). Softmax can be used for MultiClass Classification, I will have a separate post for that.

I will be referring the diagram above, which I drew to show the Forward and Backpropagation of the 2-Layer Network. So that you don't have to scroll up and down, I am having the same diagram here again.



Our first objective is to find $\frac{dJ}{dW^{[2]}}$ where `J` is the cost function and $W^{[2]}$ is a matrix of all the weights in the final layer. Using partial derivates we can define the following (follow the path (red color) of the Backpropagation in the picture above if you are confused)

$$\frac{dJ}{dW^{[2]}} = \frac{dJ}{dA^{[2]}}\frac{dA^{[2]}}{dZ^{[2]}}\frac{dZ^{[2]}}{dW^{[2]}}$$

Our `Cross Entropy Error Function` for binary classification is :

$$J = -\frac{1}{n}\Bigg(Y log\left(A^{[2]}\right) - (1-Y)\, log\left(1 - A^{[2]}\right) \Bigg)$$

Remember, in the above equation $a^{[2]}$ is nothing but $\hat{y}$

Now we can define our $\frac{dJ}{dW^{[2]}}$ as,

$$\frac{dJ}{dW^{[2]}} = \Bigg[-\frac{Y}{A^{[2]}} + \frac{1-Y}{1-A^{[2]}} \Bigg] \Bigg[A^{[2]}(1-A^{[2]}) \Bigg] \Bigg[A^{[2]} \Bigg]$$

Let's take a minute and understand what just happened here. The 1st part is the derivative of the Cost Function. As long as you know the derivate of log, you can see how this makes sense. (I have omitted the `1/n` factor here, we will ignore that for now, however during coding we will make sure to divide the result by `n`)

The 2nd part is the derivative of the `Sigmoid activation` function. Again, you can derive it by yourself just by knowing the derivate of e^x w.r.t `x`.

We already know $Z^{[2]}$ from our forward propagation,
$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$

The derivative of the above $Z^{[2]}$ w.r.t $W^{[2]}$ will simply be $A^{[1]}$.

Simplifying the equation, we get

$$\begin{aligned}\frac{dJ}{dW^{[2]}} &= \Bigg[-Y + \cancel{YA^{[2]}} + A^{[2]} - \cancel{YA^{[2]}} \Bigg] \Bigg[A^{[1]} \Bigg] \\ &= \Bigg[A^{[2]} - Y \Bigg] \Bigg[A^{[1]} \Bigg] \\ &= dZ^{[2]}A^{[1]}\end{aligned}$$

Just note that, (we will use this later)

$$dZ^{[2]} = \frac{dJ}{dZ^{[2]}} = \frac{dJ}{dA^{[2]}}\frac{dA^{[2]}}{dZ^{[2]}} = \Bigg[A^{[2]} - Y \Bigg]$$

Similarly we can define $\frac{dJ}{db^{[2]}}$ as,

$$\begin{aligned}\frac{dJ}{db^{[2]}} &= \frac{dJ}{dA^{[2]}}\frac{dA^{[2]}}{dZ^{[2]}}\frac{dZ^{[2]}}{db^{[2]}} \\ &= \Bigg[A^{[2]} - Y \Bigg] \Bigg[1 \Bigg] \\ &= \Bigg[A^{[2]} - Y \Bigg] \\ &= dZ^{[2]}\end{aligned}$$

We will now move to the first layer, (following the red arrows in the picture)

$$\begin{aligned}
\frac{dJ}{dW^{[1]}} &= \frac{dJ}{dA^{[2]}}\frac{dA^{[2]}}{dZ^{[2]}}\frac{dZ^{[2]}}{dA^{[1]}}\frac{dA^{[1]}}{dZ^{[1]}}\frac{dZ^{[1]}}{dW^{[1]}} \\
&= \frac{dJ}{dZ^{[2]}}\frac{dZ^{[2]}}{dA^{[1]}}\frac{dA^{[1]}}{dZ^{[1]}}\frac{dZ^{[1]}}{dW^{[1]}} \\
&= \bigg[A^{[2]} - Y\bigg]\bigg[W^{[2]}\bigg]\bigg[g'\Big(Z^{[1]}\Big)\bigg]\bigg[A^{[0]}\bigg] \\
&= dZ^{[2]}W^{[2]}g'\Big(Z^{[1]}\Big)A^{[0]} \\
&= dZ^{[1]}A^{[0]}
\end{aligned}$$

There are few points to note.

- First is reusability, the whole objective of dynamic programming is how to reuse already computed values in future computation. Thats the reason we are reusing $dZ^{[2]}$.
- $A^{[0]}$ here is nothing but our input X, however if you have more than 2 hidden layer, it will just be the activation output of the previous later.
- We can generalize this by equation for any layer except for the final hidden layer (The final layer equation depends on the Activaition of that layer).

Also need the following,

$$\begin{aligned}
\frac{dJ}{dA^{[1]}} &= \frac{dJ}{dA^{[2]}}\frac{dA^{[2]}}{dZ^{[2]}}\frac{dZ^{[2]}}{dA^{[1]}} \\
&= \frac{dJ}{dZ^{[2]}}W^{[2]} \\
&= dZ^{[2]}W^{[2]}
\end{aligned}$$

Same for $db^{[1]}$

$$\begin{aligned}
\frac{dJ}{db^{[1]}} &= \frac{dJ}{dA^{[2]}}\frac{dA^{[2]}}{dZ^{[2]}}\frac{dZ^{[2]}}{dA^{[1]}}\frac{dA^{[1]}}{dZ^{[1]}}\frac{dZ^{[1]}}{db^{[1]}} \\
&= \frac{dJ}{dZ^{[2]}}\frac{dZ^{[2]}}{dA^{[1]}}\frac{dA^{[1]}}{dZ^{[1]}}\frac{dZ^{[1]}}{db^{[1]}} \\
&= \bigg[A^{[2]} - Y\bigg]\bigg[W^{[2]}\bigg]\bigg[g'\Big(Z^{[1]}\Big)\bigg]\bigg[1\bigg] \\
&= dZ^{[2]}W^{[2]}g'\Big(Z^{[1]}\Big) \\
&= dZ^{[1]}
\end{aligned}$$

Since we have the required derivatives, $dW^{[2]}, db^{[2]}, dW^{[1]}, db^{[1]}$, it's time that we define the full algorithm.

N-Layer Neural Network Algorithm :

We will now define the full algorithm of a N-Layer Neural Network Algorithm by generalizing the equations we have derived for our 2-Layer Network.

- Initialize $W^{[1]} .. W^{[L]}, b^{[1]} \ldots b^{[L]}$
- Set $A^{[0]} = X$ (Input) , $L =$ Total Layers
- Loop epoch $= 1$ to max iteration
    - Forward Propagation
        - Loop $l = 1$ to $L - 1$
            - $Z^{[l]} = W^{[l]} A^{[l-1]} + b^{[l]}$
            - $A^{[l]} = g\left(b^{[l]}\right)$
            - Save $A^{[l]}, W^{[l]}$ in memory for later use
        - $Z^{[L]} = W^{[L]} A^{[L-1]} + b^{[L]}$
        - $A^{[L]} = \sigma\left(Z^{[L]}\right)$
    - Cost $J = -\frac{1}{n}\left(Y log\left(A^{[2]}\right) - (1 - Y)\, log\left(1 - A^{[2]}\right)\right)$
    - Backward Propagation
        - $dA^{[L]} = -\frac{Y}{A^{[L]}} + \frac{1 - Y}{1 - A^{[L]}}$
        - $dZ^{[L]} = dA^{[L]} \sigma'\left(dA^{[L]}\right)$
        - $dW^{[L]} = dZ^{[L]} dA^{[L-1]}$
        - $db^{[L]} = dZ^{[L]}$
        - $dA^{[L-1]} = dZ^{[L]} W^{[L]}$
        - Loop $l = L - 1$ to 1
            - $dZ^{[l]} = dA^{[l]} g'\left(dA^{[l]}\right)$
            - $dW^{[l]} = dZ^{[l]} dA^{[l-1]}$
            - $db^{[l]} = dZ^{[l]}$
            - $dA^{[l-1]} = dZ^{[l]} W^{[l]}$
    - Update W and b
        - Loop $l = 1$ to L
            - $W^{[l]} = W^{[l]} - \alpha . dW^{[l]}$
            - $b^{[l]} = b^{[l]} - \alpha . db^{[l]}$

The algorithm above is easy to understand. Just the generalized version of our previous derivation. Feel fee to ask me question in the comments section in case you have any doubt.

Python Implementation:

At this point technically we can directly jump into the code, however you will surely have issues with matrix dimension. Hence, let's make sure that we fully understand the matrix dimensions before coding. Once you do this coding should be very simple.

We will use MNIST dataset for our implementation.(You can google in case you are hearing about this dataset to know more about it.)
MNIST has `6000` `28x28` dimension gray scale image as training and total 10 different class, however since we will be focusing on binary classification here, we will choose all images with label 5 and 8 (Total `11272`). We will write a function which will return the data we need.

Each pixel will be a feature for us, so we will first flatten each image to `28x28 = 784` vector. The input dimension will be 11272 X 784.

In our Neural Network we will have total 2 layers, so it will be like `784 (input Layer)->196->1`.

Forward Propagation – Layer 1:

$$
\begin{aligned}
X &= (11272, 784) \\
W^{[1]} &= (196, 784) \\
b^{[1]} &= (196, 1) \\
A^{[0]} &= X^T \\
&= (784, 11272) \\
Z^{[1]} &= W^{[1]} A^{[0]} + b^{[1]} \\
&= (196, 784) * (784, 11272) + (196, 1) \\
&= (196, 11272) + (196, 1) \\
&= (196, 11272) \\
A^{[1]} &= g\left(Z^{[1]}\right) \\
&= (196, 11272)
\end{aligned}
$$

Forward Propagation – Layer 2:

$$
\begin{aligned}
W^{[2]} &= (1, 196) \\
b^{[2]} &= (1, 1) \\
Z^{[2]} &= W^{[2]} A^{[1]} + b^{[2]} \\
&= (1, 196) * (196, 11272) + (1, 1) \\
&= (1, 11272) + (1, 1) \\
&= (1, 11272) \\
A^{[2]} &= g\left(Z^{[2]}\right) \\
&= (1, 11272)
\end{aligned}
$$

Backward Propagation – Layer 2:

$$
\begin{aligned}
Y^T &= (1, 11272) \\
dA^{[2]} &= -\frac{Y^T}{A^{[2]}} + \frac{1 - Y^T}{1 - A^{[2]}} \\
&= (1, 11272) \\
dZ^{[2]} &= dA^{[2]} g'(Z^{[2]}) \\
&= (1, 11272) * (1, 11272) \\
&= (1, 11272) \\
dW^{[2]} &= dZ^{[2]} \left(A^{[1]}\right)^T \\
&= (1, 11272) * (11272, 196) \\
&= (1, 196) \\
db^{[2]} &= dZ^{[2]} \\
&= (1, 1) \\
dA^{[1]} &= \left(W^{[2]}\right)^T dZ^{[2]} \\
&= (196, 1) * (1, 11272) \\
&= (196, 11272)
\end{aligned}
$$

Backward Propagation – Layer 1:

$$
\begin{aligned}
dZ^{[1]} &= dA^{[1]} g'(Z^{[1]}) \\
&= (196, 11272) * (196, 11272) \\
&= (196, 11272) \\
dW^{[1]} &= dZ^{[1]} \left(A^{[0]}\right)^T \\
&= (196, 11272) * (11272, 784) \\
&= (196, 784) \\
db^{[1]} &= dZ^{[1]} \\
&= (196, 1)
\end{aligned}
$$

Two important points:

- I haven't fully explained the calculation for `b` above. We need need to sum over all the rows to make sure the dimension of $b^{[l]}$ and $db^{[l]}$ matches. We will use numpy's `axis=1` and `keepdims=True` option for this.
- We have completely ignore the divide by `n` calculation (It was part of our cost function). So as a practice, whenever we are calculating the derivative of `W` and `b`, we will divide the result by `n`.

We will be using a python library to load the MNIST data. It just helps us to focus on the algorithm. You can install it by running following command.

```
pip install python-mnist
```

We will create a class named ANN and have the following methods defined there.

```
ann = ANN(layers_dims)
ann.fit(train_x, train_y, learning_rate=0.1,
n_iterations=1000)
ann.predict(train_x, train_y)
ann.predict(test_x, test_y)
ann.plot_cost()
```

We will get the data then preprocess it and invoke our ANN class.Our main will look like this. Also we should be able to pass the number of layers we need in our model. We dont want to fix the number of layers, rather want to pass that as an array to our ANN class.

```
if __name__ == '__main__':
    train_x, train_y, test_x, test_y =
get_binary_dataset()

    train_x, test_x = pre_process_data(train_x, test_x)
```

```
    print("train_x's shape: " + str(train_x.shape))
    print("test_x's shape: " + str(test_x.shape))

    layers_dims = [196, 1]

    ann = ANN(layers_dims)
    ann.fit(train_x, train_y, learning_rate=0.1,
n_iterations=1000)
    ann.predict(train_x, train_y)
    ann.predict(test_x, test_y)
    ann.plot_cost()
```

The `get_binary_dataset()` function above will provide the Train and Test data. The dimension of the data will be as we have seen above. In the `pre_process_data()` function we will just normalize the data.

```
def pre_process_data(train_x, test_x):
    # Normalize
    train_x = train_x / 255.
    test_x = test_x / 255.

    return train_x, test_x
```

Below is the constructor of the ANN class. Here the layer size will be passed as an array.The `self.parameters` will be a dictonary object where we keep all the `W` and `b`.

```
def __init__(self, layers_size):
        self.layers_size = layers_size
        self.parameters = {}
        self.L = len(self.layers_size)
        self.n = 0
        self.costs = []
```

The `fit()` function will first call initialize_parameters() to create all the necessary `W` and `b` for each layer.Then we will have the training running in `n_iterations` times. Inside the loop first call the `forward()` function. Then calculate the cost and call the `backward()` function. Afterwards, we will update the `W` and `b` for all the layers.

```
def fit(self, X, Y, learning_rate=0.01,
n_iterations=2500):
        np.random.seed(1)

        self.n = X.shape[0]

        self.layers_size.insert(0, X.shape[1])

        self.initialize_parameters()
        for loop in range(n_iterations):
                A, store = self.forward(X)
                cost = np.squeeze(-(Y.dot(np.log(A.T)) +
(1 - Y).dot(np.log(1 - A.T))) / self.n)
                derivatives = self.backward(X, Y, store)

                for l in range(1, self.L + 1):
                        self.parameters["W" + str(l)] =
self.parameters["W" + str(l)] - learning_rate *
derivatives[
                                "dW" + str(l)]
                        self.parameters["b" + str(l)] =
self.parameters["b" + str(l)] - learning_rate *
derivatives[
                                "db" + str(l)]

                if loop % 100 == 0:
```

```
                print(cost)
                self.costs.append(cost)
```

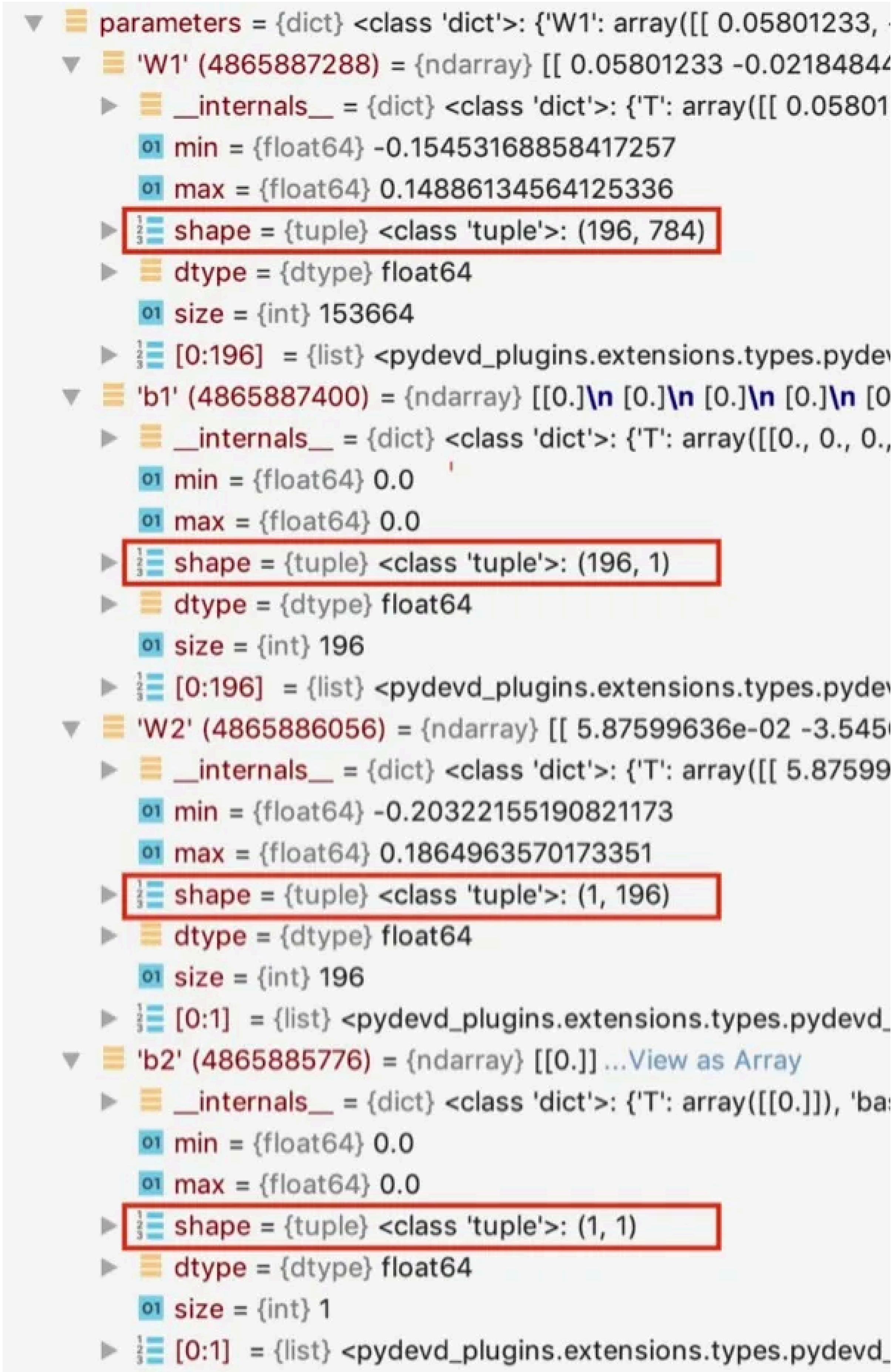
Since the `W1` parameter needs the number of features present in the training data, we will insert that in the `layers_size` array before invoking `initialize_parameters()`

In the `initialize_parameters()` function we loop through the `layers_size` array and store the parameters in the `self.parameters` dictionary.

```
def initialize_parameters(self):
        np.random.seed(1)

        for l in range(1, len(self.layers_size)):
                self.parameters["W" + str(l)] = 
np.random.randn(self.layers_size[l], self.layers_size[l - 
1]) / np.sqrt(
                        self.layers_size[l - 1])
                self.parameters["b" + str(l)] = 
np.zeros((self.layers_size[l], 1))
```

Once you run the code the `self.parameters` variable will look like this:


parameters = {dict} <class 'dict'>: {'W1': array([[0.05801233,
'W1' (4865887288) = {ndarray} [[0.05801233 -0.02184844
__internals__ = {dict} <class 'dict'>: {'T': array([[0.05801
min = {float64} -0.15453168858417257
max = {float64} 0.14886134564125336
shape = {tuple} <class 'tuple'>: (196, 784)
dtype = {dtype} float64
size = {int} 153664
[0:196] = {list} <pydevd_plugins.extensions.types.pydev
'b1' (4865887400) = {ndarray} [[0.]\n [0.]\n [0.]\n [0.]\n [0
__internals__ = {dict} <class 'dict'>: {'T': array([[0., 0., 0.,
min = {float64} 0.0
max = {float64} 0.0
shape = {tuple} <class 'tuple'>: (196, 1)
dtype = {dtype} float64
size = {int} 196
[0:196] = {list} <pydevd_plugins.extensions.types.pydev
'W2' (4865886056) = {ndarray} [[5.87599636e-02 -3.545
__internals__ = {dict} <class 'dict'>: {'T': array([[5.87599
min = {float64} -0.20322155190821173
max = {float64} 0.1864963570173351
shape = {tuple} <class 'tuple'>: (1, 196)
dtype = {dtype} float64
size = {int} 196
[0:1] = {list} <pydevd_plugins.extensions.types.pydevd_
'b2' (4865885776) = {ndarray} [[0.]] ...View as Array
__internals__ = {dict} <class 'dict'>: {'T': array([[0.]]), 'ba
min = {float64} 0.0
max = {float64} 0.0
shape = {tuple} <class 'tuple'>: (1, 1)
dtype = {dtype} float64
size = {int} 1
[0:1] = {list} <pydevd_plugins.extensions.types.pydevd_

The `forward()` function is very easy to understand. Even though we are using `Sigmoid Activation` function in all the layers, we will have the calculation for the final layer outside of the loop so that we can easily plugin a `Softmax` function there (Softmax is not covered in this tutorial).

We will also create a new `store` dictionary object and keep the `A`,`W` and `Z` for each layer so that we can use them during backpropagation.

```
def forward(self, X):
        store = {}

        A = X.T
        for l in range(self.L - 1):
                Z = self.parameters["W" + str(l + 1)].dot(A) + self.parameters["b" + str(l + 1)]
                A = self.sigmoid(Z)
                store["A" + str(l + 1)] = A
                store["W" + str(l + 1)] = self.parameters["W" + str(l + 1)]
                store["Z" + str(l + 1)] = Z

        Z = self.parameters["W" + str(self.L)].dot(A) + self.parameters["b" + str(self.L)]
        A = self.sigmoid(Z)
        store["A" + str(self.L)] = A
        store["W" + str(self.L)] = self.parameters["W" + str(self.L)]
        store["Z" + str(self.L)] = Z

        return A, store
```

Above in line 18, returned value `A` is basically the $\hat{y}$.

In the `backward()` function like we have in the derivation, first calculate the `dA`, `dW`, `db` for the `L'th` layer and then in the loop find all the derivatives for remaining layers.

The below code is the same as the derivations we went through earlier. We keep all the derivatives in the `derivatives` dictionary and return that to the `fit()` function.

```
def backward(self, X, Y, store):

        derivatives = {}

        store["A0"] = X.T

        A = store["A" + str(self.L)]
        dA = -np.divide(Y, A) + np.divide(1 - Y, 1 - A)

        dZ = dA * self.sigmoid_derivative(store["Z" + 
str(self.L)])
        dW = dZ.dot(store["A" + str(self.L - 1)].T) / 
self.n
        db = np.sum(dZ, axis=1, keepdims=True) / self.n
        dAPrev = store["W" + str(self.L)].T.dot(dZ)

        derivatives["dW" + str(self.L)] = dW
        derivatives["db" + str(self.L)] = db

        for l in range(self.L - 1, 0, -1):
                dZ = dAPrev * 
self.sigmoid_derivative(store["Z" + str(l)])
                dW = 1. / self.n * dZ.dot(store["A" + 
str(l - 1)].T)
```

```
                db = 1. / self.n * np.sum(dZ, axis=1,
keepdims=True)
                if l > 1:
                        dAPrev = store["W" +
str(l)].T.dot(dZ)

                derivatives["dW" + str(l)] = dW
                derivatives["db" + str(l)] = db

        return derivatives
```

Here is the code for the `sigmoid()` and `sigmoid_derivative()` function. In a later tutorial we will see how to use `ReLu` and `Softmax`.

```
def sigmoid(self, Z):
    return 1 / (1 + np.exp(-Z))

def sigmoid_derivative(self, Z):
    s = 1 / (1 + np.exp(-Z))
    return s * (1 - s)
```

In the `predict()` function we wil just use the current `W` and `b` and compute the probability usimng `forward()` function. Then we will convert the probability to a predicted class 0 or 1.

```
def predict(self, X, Y):
        A, cache = self.forward(X)
        n = X.shape[0]
        p = np.zeros((1, n))

        for i in range(0, A.shape[1]):
                if A[0, i] > 0.5:
                        p[0, i] = 1
```

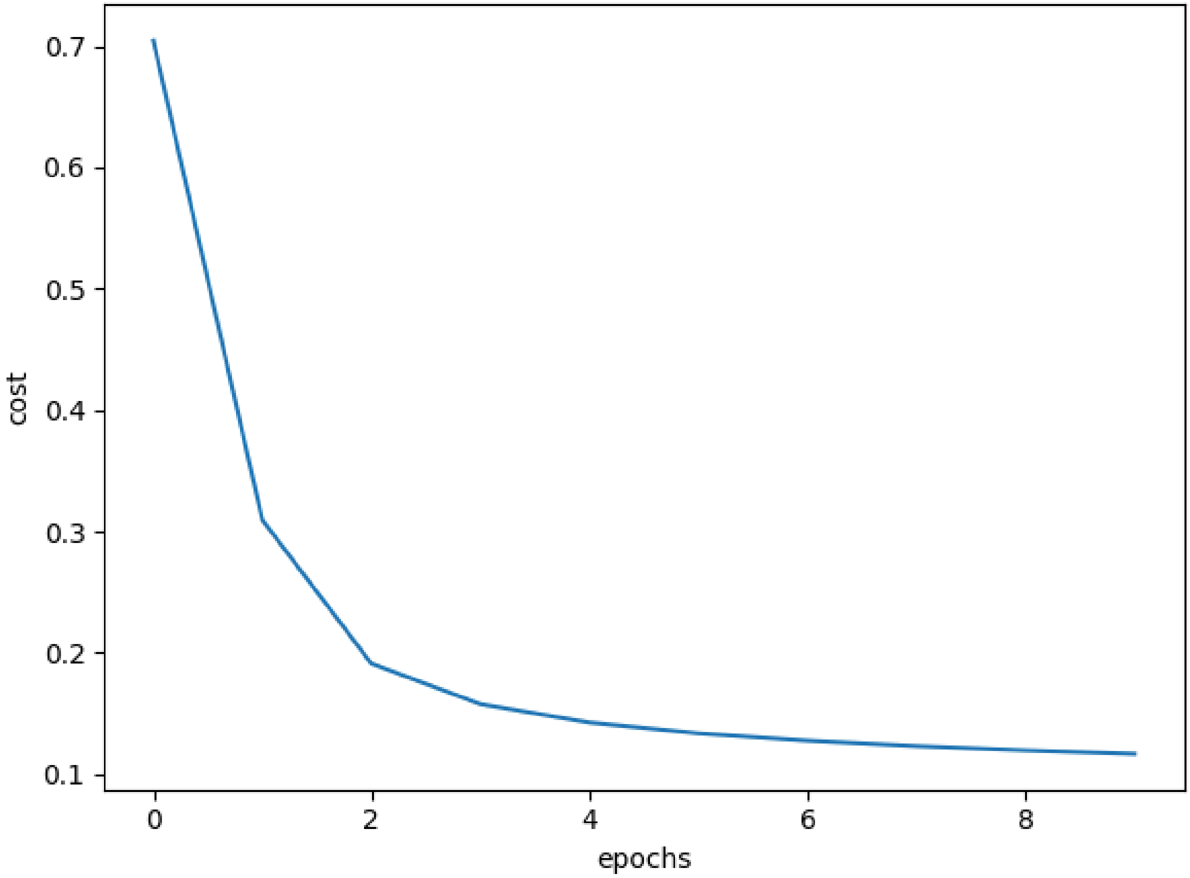
```
            else:
                    p[0, i] = 0

    print("Accuracy: " + str(np.sum((p == Y) / n)))
```

Let’s look at the outout. You will get around 96% Train and Test Accuracy.

```
train_x's shape: (11272, 784)
test_x's shape: (1866, 784)
0.7043777294167167
0.3094035971595143
0.19106252272122817
0.15772416612846746
0.14255528419489316
0.1336554279807337
0.12762011948747812
0.12313725638495653
0.11959735842488138
0.11667822494436252
Accuracy: 0.9599893541518808
Accuracy: 0.9598070739549842
```

The cost gradually does down as we run multiple iteration.



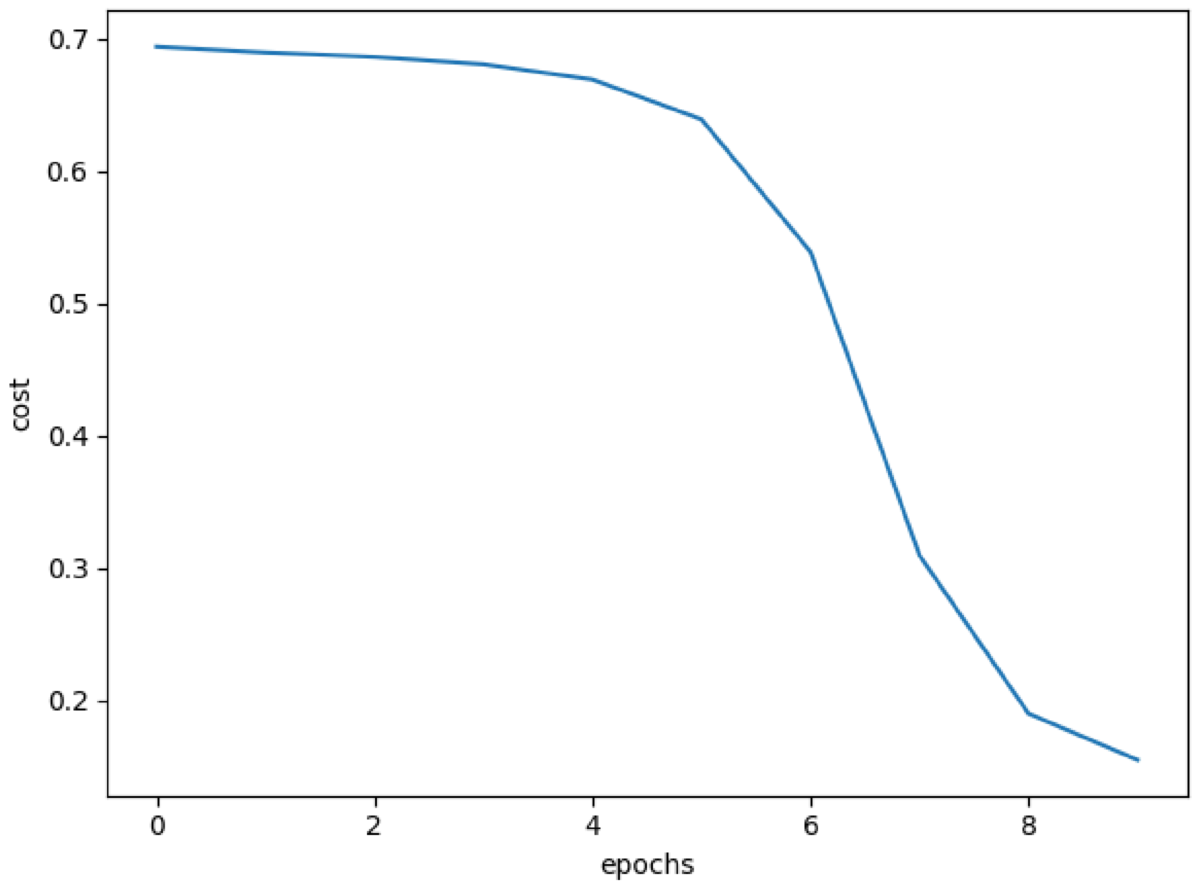
The best part of writing the code in a generic way is we can easily try using different layer size. Let’s try the following:

```
layers_dims = [392,196,98,1]
```

Here is the result.

```
train_x's shape: (11272, 784)
test_x's shape: (1866, 784)
0.6941917096801075
0.689779337934555
0.6864347273157968
0.680851445965145
0.6693297859482221
0.6392888056143693
0.5391389182596976
0.30972952941407295
```

```
0.1900953225522053
0.15499153620779857
Accuracy: 0.9491660752306601
Accuracy: 0.9555198285101825
```



Naturally, with the same data, iteration and learning rate the larger Network is performing poorly than the smaller one. If you were expecting a different result then let me know in the comment section and we can discuss about it.

Below is the full code of the ANN class:

```
import numpy as np
import datasets.mnist.loader as mnist
import matplotlib.pylab as plt

class ANN:
    def __init__(self, layers_size):
```

```
        self.layers_size = layers_size
        self.parameters = {}
        self.L = len(self.layers_size)
        self.n = 0
        self.costs = []

    def sigmoid(self, Z):
        return 1 / (1 + np.exp(-Z))

    def initialize_parameters(self):
        np.random.seed(1)

        for l in range(1, len(self.layers_size)):
            self.parameters["W" + str(l)] =
np.random.randn(self.layers_size[l], self.layers_size[l -
1]) / np.sqrt(
                self.layers_size[l - 1])
            self.parameters["b" + str(l)] =
np.zeros((self.layers_size[l], 1))

    def forward(self, X):
        store = {}

        A = X.T
        for l in range(self.L - 1):
            Z = self.parameters["W" + str(l + 1)].dot(A)
+ self.parameters["b" + str(l + 1)]
            A = self.sigmoid(Z)
            store["A" + str(l + 1)] = A
            store["W" + str(l + 1)] = self.parameters["W"
+ str(l + 1)]
            store["Z" + str(l + 1)] = Z
```

```
        Z = self.parameters["W" + str(self.L)].dot(A) +
self.parameters["b" + str(self.L)]
        A = self.sigmoid(Z)
        store["A" + str(self.L)] = A
        store["W" + str(self.L)] = self.parameters["W" +
str(self.L)]
        store["Z" + str(self.L)] = Z

        return A, store

    def sigmoid_derivative(self, Z):
        s = 1 / (1 + np.exp(-Z))
        return s * (1 - s)

    def backward(self, X, Y, store):

        derivatives = {}

        store["A0"] = X.T

        A = store["A" + str(self.L)]
        dA = -np.divide(Y, A) + np.divide(1 - Y, 1 - A)

        dZ = dA * self.sigmoid_derivative(store["Z" +
str(self.L)])
        dW = dZ.dot(store["A" + str(self.L - 1)].T) /
self.n
        db = np.sum(dZ, axis=1, keepdims=True) / self.n
        dAPrev = store["W" + str(self.L)].T.dot(dZ)

        derivatives["dW" + str(self.L)] = dW
        derivatives["db" + str(self.L)] = db
```

```
        for l in range(self.L - 1, 0, -1):
            dZ = dAPrev *
self.sigmoid_derivative(store["Z" + str(l)])
            dW = 1. / self.n * dZ.dot(store["A" + str(l -
1)].T)
            db = 1. / self.n * np.sum(dZ, axis=1,
keepdims=True)
            if l > 1:
                dAPrev = store["W" + str(l)].T.dot(dZ)

            derivatives["dW" + str(l)] = dW
            derivatives["db" + str(l)] = db

        return derivatives

    def fit(self, X, Y, learning_rate=0.01,
n_iterations=2500):
        np.random.seed(1)

        self.n = X.shape[0]

        self.layers_size.insert(0, X.shape[1])

        self.initialize_parameters()
        for loop in range(n_iterations):
            A, store = self.forward(X)
            cost = np.squeeze(-(Y.dot(np.log(A.T)) + (1 -
Y).dot(np.log(1 - A.T))) / self.n)
            derivatives = self.backward(X, Y, store)

            for l in range(1, self.L + 1):
                self.parameters["W" + str(l)] =
self.parameters["W" + str(l)] - learning_rate *
```

```
derivatives[
                    "dW" + str(l)]
                self.parameters["b" + str(l)] =
self.parameters["b" + str(l)] - learning_rate *
derivatives[
                    "db" + str(l)]

            if loop % 100 == 0:
                print(cost)
                self.costs.append(cost)

    def predict(self, X, Y):
        A, cache = self.forward(X)
        n = X.shape[0]
        p = np.zeros((1, n))

        for i in range(0, A.shape[1]):
            if A[0, i] > 0.5:
                p[0, i] = 1
            else:
                p[0, i] = 0

        print("Accuracy: " + str(np.sum((p == Y) / n)))

    def plot_cost(self):
        plt.figure()
        plt.plot(np.arange(len(self.costs)), self.costs)
        plt.xlabel("epochs")
        plt.ylabel("cost")
        plt.show()


def get_binary_dataset():
```

```
    train_x_orig, train_y_orig, test_x_orig, test_y_orig
= mnist.get_data()

    index_5 = np.where(train_y_orig == 5)
    index_8 = np.where(train_y_orig == 8)

    index = np.concatenate([index_5[0], index_8[0]])
    np.random.seed(1)
    np.random.shuffle(index)

    train_y = train_y_orig[index]
    train_x = train_x_orig[index]

    train_y[np.where(train_y == 5)] = 0
    train_y[np.where(train_y == 8)] = 1

    index_5 = np.where(test_y_orig == 5)
    index_8 = np.where(test_y_orig == 8)

    index = np.concatenate([index_5[0], index_8[0]])
    np.random.shuffle(index)

    test_y = test_y_orig[index]
    test_x = test_x_orig[index]

    test_y[np.where(test_y == 5)] = 0
    test_y[np.where(test_y == 8)] = 1

    return train_x, train_y, test_x, test_y

def pre_process_data(train_x, test_x):
    # Normalize
    train_x = train_x / 255.
```

```
    test_x = test_x / 255.

    return train_x, test_x



if __name__ == '__main__':
    train_x, train_y, test_x, test_y =
get_binary_dataset()

    train_x, test_x = pre_process_data(train_x, test_x)

    print("train_x's shape: " + str(train_x.shape))
    print("test_x's shape: " + str(test_x.shape))

    layers_dims = [196, 1]

    ann = ANN(layers_dims)
    ann.fit(train_x, train_y, learning_rate=0.1,
n_iterations=1000)
    ann.predict(train_x, train_y)
    ann.predict(test_x, test_y)
    ann.plot_cost()
```

You can access the full project here:

Github Project

Conclusion:

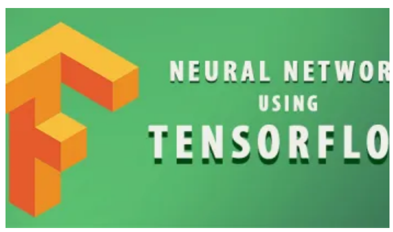
I hope that this tutorial provides a detail view on backpropagation algorithm. Since backpropagation is the backbone of any Neural Network, it’s important to understand in depth. We can make many optimization from this point onwards for improving the accuracy, faster computation etc. Next we will see how to implement the same using both Tensorflow and PyTorch.

Below are the articles on implementing the Neural Network using TensorFlow and PyTorch.

1. Understanding and implementing Neural Network with SoftMax in Python from scratch
2. Implement Neural Network using TensorFlow
3. Implement Neural Network using PyTorch

Related

Understanding and implementing Neural Network with SoftMax in Python from scratch
In "Data Science"



Implement Neural Network using TensorFlow
In "Data Science"

Implement Neural Network using PyTorch
In "Data Science"

Filed Under: Data Science, Machine Learning | Tagged With: Backpropagation, Backward Propagation, Deep Learning, Forward Propagation, Machine Learning, Neural Network, Python

Subscribe to stay in loop

* indicates required

Email Address *

Subscribe

Comments

Error in the equation says
April 8, 2020 at 2:28 am

(Edit)

In the section above i.e.

The Gradient Descent (a.k.a Backpropagation) in Logistic Regression has an additional derivative to calculate.

Following equation i.e.
dJ/dW=dJ/dA* dA/dZ* dZ/db is wrong.

It should be dj/db because it is calculating the partial derivative of cost function wrt bias

Reply

Abhisek Jana says
April 8, 2020 at 3:29 pm

(Edit)

Thanks a lot for indicating the typo. I have updated the equation.

Reply

Abdessalem says
April 10, 2020 at 10:59 am

(Edit)

Such a great post, thank you!

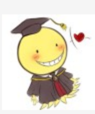
Reply

Abhisek Jana says
April 11, 2020 at 2:21 am

(Edit)

Thanks a lot for your feedback !!!

Reply



Daniel Agustian says
April 12, 2020 at 1:35 pm

(Edit)

Just wanna ask, when i try your code (I literally just download it and install the python-mnist), i got this error:

ModuleNotFoundError: No module named 'datasets'

Is the datasets need to be downloaded again? Do i need to download TensorFlow or something?
I use anaconda with python 3.6.
Thanks.
Reply

Abhisek Jana says
April 12, 2020 at 7:16 pm

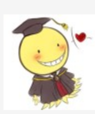
(Edit)

Hi Daniel,
You need to download the additional folder (link below) from github in your local. The best option is to download the entire project and then run the main.py.You need numpy and matplotlib library too. Install them using the package manager (pip/conda)

https://github.com/adeveloperdiary/blog/tree/master/datasets/mnist

Let me know whether this helps you to solve the issue.

Reply



Daniel Agustian says
April 13, 2020 at 4:12 am

(Edit)

There's another problem…

File "C:\Users\Daniel\Anaconda3\envs\Python 3.6\lib\gzip.py", line 163, in __init__ fileobj = self.myfileobj = builtins.open(filename, mode or 'rb')

FileNotFoundError: [Errno 2] No such file or directory: '../datasets/mnist/data_files\\train-labels-idx1-ubyte.gz'

Dunno why is that \\. I'm a noob on NN and python.

Reply

Abhisek Jana says
April 13, 2020 at 2:25 pm

(Edit)

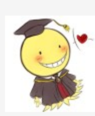
I am assuming you are using windows os. In that case, open the file `datasets/mnist/loader.py` and change the following line:

```
mndata =
MNIST('../datasets/mnist/data_files')
```

to

```
mndata =
MNIST('..\\datasets\\mnist\\data_files')
```

Reply



Daniel Agustian says
April 22, 2020 at 10:48 am

(Edit)

Sorry to bother you again, but it still doesnt work.

FileNotFoundError: [Errno 2] No such file or directory: '..\\datasets\\mnist\\data_files\\train-labels-idx1-ubyte.gz'

Do i need to replace the '..' with my path? BTW I'm using windows 10.

Reply

Abhisek Jana says

April 22, 2020 at 1:04 pm

(Edit)

You should try to replace the relative path with absolute path. Then should work.

Reply

Ayon says

May 13, 2020 at 11:00 pm

(Edit)

great post! Took me a while to grasp the math behind it, and yours is the only blog that breaks it down all the way. Thanks for this.

Reply

Abhisek Jana says
May 14, 2020 at 2:26 pm

(Edit)

Thank you for your feedback !!!

Reply

ss says
May 24, 2020 at 8:38 pm

(Edit)

Thank for the article. I might be wrong but I think the following equation should end with A1 instead of A2
$dJ/dW[2]=[-Y+YA[2]+A[2]-YA[2]][A[2]]=[A[2]-Y][A[2]]=dZ[2]A[2]$
thanks

Reply

Abhisek Jana says
June 1, 2020 at 1:53 am

(Edit)

Hi,
Thanks a lot for reading the post and letting me know about the typo!
It has been corrected.

Regards,

Abhisek Jana

Reply

matt berg says

July 12, 2020 at 8:39 am

(Edit)

Thanks for this very nice article!
I found some typos in the backpropagation algorithm, fixed them in latex editor, and I write the modified algorithm below,

- Initialize $W^{[1]} \dots W^{[L]}, b^{[1]} \dots b^{[L]}$
- Set $A^{[0]} = X$ (Input) , $L =$ Total Layers
- Loop epoch $= 1$ to max iteration
    - Forward Propagation
        - Loop $l = 1$ to $L - 1$
            - $Z^{[l]} = W^{[l]} A^{[l-1]} + b^{[l]}$
            - $A^{[l]} = g\left(Z^{[l]}\right)$
            - Save $A^{[l]}, Z^{[l]}$ in memory for later use
        - $Z^{[L]} = W^{[L]} A^{[L-1]} + b^{[L]}$
        - $A^{[L]} = \sigma\left(Z^{[L]}\right)$
    - Cost $J = -\frac{1}{n}\left(Y log\left(A^{[2]}\right) - (1 - Y)\, log\left(1 - A^{[2]}\right)\right)$
    - Backward Propagation
        - $dA^{[L]} = -\frac{Y}{A^{[L]}} + \frac{1 - Y}{1 - A^{[L]}}$
        - $dZ^{[L]} = dA^{[L]} \sigma'\left(A^{[L]}\right)$
        - $dW^{[L]} = dZ^{[L]} A^{[L-1]}$
        - $db^{[L]} = dZ^{[L]}$
        - $dA^{[L-1]} = dZ^{[L]} W^{[L]}$
        - Loop $l = L - 1$ to 1
            - $dZ^{[l]} = dA^{[l]} g'\left(Z^{[l]}\right)$
            - $dW^{[l]} = dZ^{[l]} A^{[l-1]}$
            - $db^{[l]} = dZ^{[l]}$
            - $dA^{[l-1]} = dZ^{[l]} W^{[l]}$
    - Update W and b
        - Loop $l = 1$ to L
            - $W^{[l]} = W^{[l]} - \alpha . dW^{[l]}$
            - $b^{[l]} = b^{[l]} - \alpha . db^{[l]}$

Reply

Ali says

December 20, 2020 at 12:22 am

(Edit)

Hello Sir! Thank you for the nice article! well done job!
I was going to ask if there is a proper citation to reference this article as I am going to use these formulas in my thesis.
Wish to hear from you ASAP and thank you again!

Reply

Abhisek Jana says
December 20, 2020 at 1:01 am

(Edit)

You are most welcome !

Please include the link of the article or the website name as citation.

Thanks

Reply

Ali says
December 20, 2020 at 2:17 am

(Edit)

Thank you so much, sir! You are AMAZING!!

Reply

Ali says

December 21, 2020 at 12:08 am

(Edit)

Hello again, sir! Correct me if I am wrong, please. At the beginning where you did the forward propagation, I see that you consider only one neuron in the hidden layer for calculating the output. Shouldn't the input for the output be the the activation value of each neuron in the hidden layer multiplied by their corresponding weights plus the bias of the output neuron. I mean may be it's not clear what I'm trying to ask. let me ask a different a question hoping that will answer the first. the network has 3 neurons that does computation, two in the hidden layer and one in the output layer; hence there should be 3 biases, correct, when you did the update using gradient descent, you just did it for b1 and b2. what about the other bias. Maybe if you can explain at the beginning the convention you followed to describe your network. This is will be very helpful. I wish to hear from you soon. Again, thank you for your time and consideration. I really enjoy your work.

Reply

Simon says
February 13, 2021 at 3:38 am

(Edit)

Thanks for a great post! I'm unable to implement this, and I thinks it's rooted in a small issue I'm unsure how to get around (I'm not very advanced with Python yet) –

This is the error I'm getting:
ModuleNotFoundError: No module named 'datasets.mnist'

is there a simple way of getting around this?

Many thanks,

Reply

Abhisek Jana says

February 23, 2021 at 1:25 am

(Edit)

Please find the file from the github repo. The file is provided there.

https://github.com/adeveloperdiary/blog/tree/master/datasets/mnist

Reply

Changsin Lee says

February 28, 2021 at 4:53 am

(Edit)

Thanks for the great post. Your article shows a great breakdown of the equations that actually match well with the code. I could be wrong on this, but I found an issue when running your code.

train_x's shape: (11272, 28, 28)
test_x's shape: (1866, 28, 28)
——————————————————————————
ValueError Traceback (most recent call last)
in ()
133
134 ann = ANN(layers_dims)
–> 135 ann.fit(train_x, train_y, learning_rate=0.1, n_iterations=1000)

136 ann.predict(train_x, train_y)
137 ann.predict(test_x, test_y)

1 frames
in forward(self, X)
29 A = X.T
30 for l in range(self.L – 1):
—> 31 Z = self.parameters[“W” + str(l + 1)].dot(A) + self.parameters[“b” + str(l + 1)]
32 A = self.sigmoid(Z)
33 store[“A” + str(l + 1)] = A

ValueError: operands could not be broadcast together with shapes (196,28,11272) (196,1)

The reason is that, unlike the blog post, the input is not flattened by the get_binary_dataset().

This line:
train_x = train_x_orig[index]

should be replaced with:
train_x = np.array([train_x_orig[id].flatten() for id in index])

The same thing goes with test_x.
After I flattened it, the code runs fine.
Reply

Kaliha says
March 5, 2021 at 5:55 pm

(Edit)

Hello! Thank you very much for the great article!
May I ask what kind of initialization you use, does it have a name or reference? (I mean the part with: ” / np.sqrt(self.layers_size[l – 1])”

Reply

Amir says
November 18, 2022 at 9:45 am

(Edit)

exactly this is one of my questions.. is there anyone can answer this?

Reply

Leave a Reply

Logged in as Abhisek Jana. Edit your profile. Log out? Required fields are marked *

Comment *

Post Comment