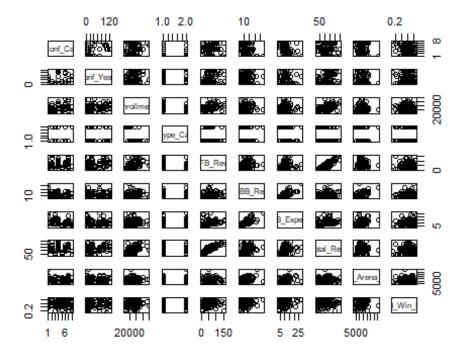
project final copy

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```
library(olsrr)
##
## Attaching package: 'olsrr'
## The following object is masked from 'package:datasets':
##
##
       rivers
library(readx1)
data_win = read_excel("proj_data_win.xlsx", na="-99")
data=data win
summary(data)
##
       Conf Cat
                      Conf_Years
                                       Enrollment
                                                        Type_Cat
##
   Min.
          :1.000
                         : 2.00
                                     Min.
                                            : 3200
                                                     Min.
                                                             :1.000
                    Min.
                    1st Qu.: 10.00
                                     1st Ou.:17593
   1st Qu.:2.000
                                                     1st Qu.:1.000
## Median :4.000
                    Median : 31.00
                                     Median :27950
                                                     Median :1.000
                         : 48.98
## Mean
                                     Mean
                                            :28946
           :4.417
                    Mean
                                                     Mean
                                                             :1.323
                                     3rd Qu.:36947
                                                     3rd Qu.:2.000
## 3rd Qu.:6.250
                    3rd Qu.: 90.00
                                     Max.
## Max.
           :8.000
                    Max.
                           :126.00
                                            :79232
                                                     Max.
                                                             :2.000
##
##
        FB_Rev
                         MBB Rev
                                        MBB Expense
                                                           Total Rev
## Min.
          : 1.235
                      Min.
                           : 2.690
                                       Min. : 2.690
                                                         Min.
                                                                : 20.78
    1st Qu.: 35.850
                      1st Qu.: 8.458
                                       1st Qu.: 7.594
                                                         1st Qu.: 48.84
##
   Median : 52.242
                      Median :12.227
                                       Median : 9.992
                                                        Median :108.46
##
   Mean
         : 58.358
                      Mean
                             :13.247
                                       Mean
                                             :10.343
                                                        Mean
                                                                :106.63
##
    3rd Qu.: 78.697
                      3rd Qu.:16.262
                                       3rd Qu.:12.416
                                                         3rd Qu.:144.20
## Max.
           :161.533
                      Max.
                             :45.109
                                       Max.
                                              :28.020
                                                        Max.
                                                                :246.61
##
   NA's
           :15
##
   MBB Arena Cap
                     MBB Win Perc
##
   Min.
          : 3104
                           :0.0968
                    Min.
   1st Qu.: 9454
                    1st Qu.:0.4801
## Median :12494
                    Median :0.5942
##
   Mean
           :12817
                    Mean
                           :0.5802
##
   3rd Qu.:15390
                    3rd Qu.:0.7154
##
         :35446
                           :0.8919
   Max.
                    Max.
##
pairs(data)
```



```
fit = lm(MBB_Win_Perc~., data=data)
summary(fit)
##
## Call:
## lm(formula = MBB_Win_Perc ~ ., data = data)
##
## Residuals:
##
                  1Q
                       Median
                                     3Q
                                             Max
        Min
## -0.44357 -0.09320 0.00254 0.13578
                                         0.29816
##
## Coefficients:
##
                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                  4.407e-01
                              1.287e-01
                                          3.424
                                                 0.00103 **
## Conf_Cat
                                          0.450
                  4.518e-03
                              1.005e-02
                                                 0.65442
## Conf_Years
                  2.257e-05
                              5.537e-04
                                          0.041
                                                 0.96759
## Enrollment
                  2.555e-09
                              1.738e-06
                                          0.001
                                                 0.99883
## Type Cat
                  1.596e-02
                              5.645e-02
                                          0.283
                                                 0.77817
                                         -0.834
## FB_Rev
                 -1.022e-03
                              1.226e-03
                                                 0.40727
## MBB_Rev
                  8.970e-03
                              5.133e-03
                                          1.747
                                                 0.08488
## MBB_Expense
                 -3.520e-03
                             7.637e-03
                                         -0.461
                                                 0.64628
## Total Rev
                  7.857e-04
                              1.007e-03
                                          0.780
                                                 0.43795
## MBB_Arena_Cap -1.725e-06
                             4.732e-06
                                         -0.365
                                                 0.71650
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1674 on 71 degrees of freedom
```

```
## (15 observations deleted due to missingness)
## Multiple R-squared: 0.1151, Adjusted R-squared:
## F-statistic: 1.026 on 9 and 71 DF, p-value: 0.4284
anova(fit)
## Analysis of Variance Table
## Response: MBB_Win_Perc
                Df Sum Sq Mean Sq F value Pr(>F)
##
## Conf Cat
                 1 0.00390 0.003905 0.1394 0.70997
## Conf_Years
                 1 0.03670 0.036699 1.3104 0.25618
                 1 0.00577 0.005772 0.2061 0.65123
## Enrollment
## Type_Cat
                 1 0.01626 0.016259 0.5806 0.44862
## FB Rev
                 1 0.00197 0.001975 0.0705 0.79138
## MBB Rev
                 1 0.17394 0.173940 6.2107 0.01503 *
                 1 0.00023 0.000231 0.0083 0.92782
## MBB Expense
## Total_Rev
                 1 0.01602 0.016022 0.5721 0.45193
## MBB Arena Cap 1 0.00372 0.003723 0.1329 0.71650
## Residuals
                71 1.98848 0.028007
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
fit best = ols step best subset(lm(MBB Win Perc~., data=data))
final = data.frame(x=fit_best$predictors, r2=fit_best$rsquare, adjr2 =
fit best$adjr,
                  AIC=fit best$aic, BIC=fit best$sbc,
r2press=fit_best$predrsq, Cp=fit_best$cp)
final
##
Х
## 1
MBB Rev
## 2
Type Cat MBB Rev
## 3
Conf_Cat Type_Cat MBB_Rev
                                                          Conf Cat
## 4
Conf_Years Type_Cat MBB_Rev
                                              Conf_Cat Conf_Years Type_Cat
## 5
MBB Rev MBB Expense
                                   Conf Cat FB Rev MBB Rev MBB Expense
## 6
Total_Rev MBB_Arena_Cap
                          Conf_Cat Type_Cat FB_Rev MBB_Rev MBB_Expense
## 7
Total Rev MBB Arena Cap
## 8
               Conf_Cat Conf_Years Type_Cat FB_Rev MBB_Rev MBB_Expense
Total Rev MBB Arena Cap
## 9 Conf Cat Conf Years Enrollment Type Cat FB Rev MBB Rev MBB Expense
Total_Rev MBB_Arena_Cap
##
           r2
                    adjr2
                                AIC
                                          BIC
                                                  r2press
                                                                  Ср
```

```
## 1 0.1029232 0.093379792 -73.39222 -65.69918 0.06739601 -4.2247161
## 2 0.1091770 0.090019531 -72.06382 -61.80643 0.05015541 -2.8366298
## 3 0.1122669 0.083319121 -70.39739 -57.57564 0.03465980 -1.1389661
## 4 0.1130322 0.074044600 -68.48018 -53.09409 0.01385079 0.7861566
## 5 0.1133328 0.064073514 -66.51272 -48.56228 -0.01340984 2.7567434
## 6 0.1137947 0.041940162 -54.29030 -35.13471 -0.07828786 4.1010231
## 7 0.1150331 0.030173247 -52.40358 -30.85354 -0.12876150 6.0016622
## 8 0.1150538 0.016726421 -50.40547 -26.46098 -0.16017053 8.0000022
## 9 0.1150538 0.002877527 -48.40547 -22.06653 -0.18417435 10.0000000
```

Best R^2 : model 8 & 9 (highest) Best R^2_{adj} : model 1 (highest) Best R^2_{press} : model 1 (highest) Best C_n : model 8 (closest to k+1) Best AIC: model 1 (lowest) Best BIC: model 1 (lowest)

We choose model 1 as our best model because it fits 4/6 optimality criterion.

```
fit_step = ols_step_both_p(lm(MBB_Win_Perc~., data=data), prem=0.30,
pent=0.15, details=T)
## Stepwise Selection Method
## Candidate Terms:
##
## 1. Conf Cat
## 2. Conf Years
## 3. Enrollment
## 4. Type_Cat
## 5. FB_Rev
## 6. MBB_Rev
## 7. MBB Expense
## 8. Total Rev
## 9. MBB Arena Cap
## We are selecting variables based on p value...
##
##
## Stepwise Selection: Step 1
##
## - MBB_Rev added
##
##
                        Model Summary
## ------
                       0.321 RMSE
0.103 Coef. Var
0.093 MSE
## R
                                                   0.162
## R-Squared
                                                 27.871
                    0.093
## Adj. R-Squared
                                                   0.026
## Pred R-Squared
                      0.067
                                  MAE
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
```

## ##			AN	OVA				
## ##		Sum of						
##		Squares		Mean Square		_		
## ##	Regression Residual	0.282 2.458	1 94	0.282 0.026	10.785			
	Total							
##				Parameter Esti	mates 			
low	ver upper			r Std. Beta				
##	(Intercept) 115 0.550	0.482	0.03	4	14.177	0.000		
## 0.0	MBB_Rev 003 0.012			2 0.321		0.001		
## ## ## ## ##	No more varia	Output						
## ##								
##	R-Squared Adj. R-Square			Coef. Var MSE	27.8 0.0	026		
## ##	RMSE: Root Mean Square Error							
##								
##								
## ##			AN					
## ##		Sum of Squares				Sig.		

```
## Residual 2.458 94
                      0.026
                95
## Total
         2.740
##
                  Parameter Estimates
    model Beta Std. Error Std. Beta t
                                Sig
lower upper
## -----
## (Intercept) 0.482
                           14.177 0.000
               0.034
0.415 0.550
## MBB_Rev 0.007 0.002 0.321 3.284 0.001
0.003 0.012
## ------
```

Stepwise selection chose model 1

```
fit_back = ols_step_backward_p(lm(MBB_Win_Perc~., data=data),prem=0.15,
details=T)
## Backward Elimination Method
## -----
##
## Candidate Terms:
##
## 1 . Conf_Cat
## 2 . Conf_Years
## 3 . Enrollment
## 4 . Type Cat
## 5 . FB Rev
## 6 . MBB_Rev
## 7 . MBB_Expense
## 8 . Total Rev
## 9 . MBB Arena Cap
## We are eliminating variables based on p value...
## - Enrollment
##
## Backward Elimination: Step 1
##
## Variable Enrollment Removed
##
##
                          Model Summary
                          0.339 RMSE
## R-Squared
                     0.115 Coef. Var 28.691
```

```
## Adj. R-Squared 0.017 MSE
                                          0.028
## Pred R-Squared
                            MAE
                  -0.160
                                          0.127
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##
                      ANOVA
##
           Sum of
                  8 0.032 1.17 0.3290
72 0.028
##
                   DF Mean Square F
          Squares
                                            Sig.
## -----
## Regression 0.259
## Residual 1.988
## Total
            2.247
                      80
##
##
                         Parameter Estimates
##
               Beta Std. Error Std. Beta
       model
                                        t
                                               Sig
lower upper
             0.441 0.115
## (Intercept)
                                        3.824
                                               0.000
0.211 0.671
    Conf_Cat
              0.005 0.010
                                 0.060
##
                                        0.464
                                               0.644
0.015
     0.024
## Conf_Years
              0.000
                   0.001
                                 0.006 0.041
                                              0.967 -
0.001
     0.001
                                               0.751
    Type_Cat
             0.016
                       0.050
                                0.038 0.319
-0.219 -0.845
                        0.001
                                               0.401
             0.009
                                0.398
                                       1.791
                        0.005
                                               0.078
0.001 \quad 0.0\overline{19}
## MBB_Expense -0.004 0.007 -0.097 -0.473
                                              0.637
0.018
    0.011
    Total Rev
             0.001
                       0.001
                                0.230 0.823
                                               0.413
0.001
     0.003
## MBB Arena_Cap
              0.000
                        0.000 -0.048 -0.367 0.714
0.000
     0.000
##
##
## - Conf_Years
## Backward Elimination: Step 2
```

## Variable Con	ıf_Years Rem	oved			
## ##		Model Summ	arv		
##			-		
## R		0.339		0.165	
## R-Squared			Coef. Var		
	## Adj. R-Squared ## Pred R-Squared			0.027	
## Pred R-Square				0.127	,
## RMSE: Root M					
## MSE: Mean Sq	•				
## MAE: Mean Ab	solute Erro	r			
##		ANO./A			
## ##		ANOVA			
	Sum of				
##	Squares	DF	Mean Square	F S	Sig.
##					
## Regression				1.356 0.2	2372
## Residual ## Total	2.247		0.027		
##					
##					
##			Parameter Esti	mates	
##					
## model	Beta	Std. Err	or Std. Bet	a †	Sig
lower upper	. Deca	Sed. Elli	o.	ч	3-6
##					
## (Intercept) 0.213 0.669	0.441	0.1	14	3.854	0.000
## Conf_Cat	9.005	0.0	9 9.96	2 0.520	0.605 -
0.013 0.023				_	0.000
## Type_Cat	0.016	0.0	50 0.03	8 0.320	0.750 -
0.083 0.114					0.000
## FB_Rev 0.003 0.001	-0.001	0.0	01 -0.22	0 -0.860	0.393 -
## MBB_Rev	0.009	0.0	05 0.40	1 1.935	0.057
0.000 0.018	3,332				
## MBB_Expense	-0.004	0.0	07 -0.09	9 -0.511	0.611 -
0.018 0.010					
## Total_Rev 0.001 0.003	0.001	0.0	01 0.23	3 0.868	0.388 -
## MBB_Arena_Cap	9.999	0.0	00 -0.04	9 -0.382	0.704
0.000 0.000					0.,01
##					
## ##					
ππ					

```
## - Type Cat
##
## Backward Elimination: Step 3
## Variable Type_Cat Removed
##
##
                    Model Summary
## ------
                   0.337 RMSE0.114 Coef. Var0.042 MSE
                                          28.321
## R-Squared
                   0.042
## Adj. R-Squared
                                           0.027
## Pred R-Squared
                -0.078
                            MAE
                                           0.127
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##
                       ANOVA
## ------
##
             Sum of
## Squares DF Mean Square F Sig. ##
                  6
74
## Regression 0.256
## Residual 1.991
                               0.043 1.584 0.1639
                                0.027
## Total
             2.247
##
##
                            Parameter Estimates
               Beta Std. Error Std. Beta t Sig
        model
lower upper
_____
## (Intercept)
              0.465
                        0.084
                                           5.520
                                                 0.000
0.297 0.633
## Conf_Cat 0.004 0.009 0.055 0.471 0.639
0.013 0.022
## FB_Rev -0.001
0.003 0.001
## MBB_Rev 0.009
0.000 0.018
      FB_Rev -0.001
                     0.001 -0.230 -0.914
                                                 0.364
                               0.403 1.956 0.054
                         0.005
                               -0.094 -0.492
## MBB_Expense -0.003
                         0.007
                                                 0.624
0.017 0.011
                                  0.236 0.886
     Total Rev
               0.001
                          0.001
                                                 0.378
0.001
      0.003
## MBB_Arena_Cap
               0.000
                          0.000
                               -0.058 -0.463 0.645
0.000 0.000
```

```
##
##
## - MBB_Arena_Cap
## Backward Elimination: Step 4
##
## Variable MBB_Arena_Cap Removed
##
##
                  Model Summary
                 0.334 RMSE
0.111 Coef. Var
0.052 MSE
## R
                                       0.163
## R-Squared
                                      28.172
## Adj. R-Squared
                                       0.027
## Pred R-Squared -0.044 MAE
                                        0.127
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##
                    ANOVA
## -----
           Sum of
       Squares DF Mean Square F Sig.
## -----
                  5
75
## Regression 0.250
                            0.050 1.877 0.1084
           1.997
## Residual
                             0.027
## Total
            2.247
                    80
##
##
                        Parameter Estimates
    model Beta Std. Error Std. Beta t
                                           Sig
     upper
## -----
## (Intercept) 0.445 0.071
                                     6.279 0.000
0.303 0.586
                0.009 0.049 0.428 0.670
## Conf_Cat 0.004
0.014 0.021
##
    FB Rev
           -0.001
                 0.001
                             -0.220 -0.880 0.381
0.003 0.001
## MBB_Rev
          0.008
                     0.004
                          0.363 1.954 0.054
0.000 0.017
## MBB Expense -0.003
                     0.007 -0.080 -0.423
                                           0.673
0.017 0.011
## Total_Rev
          0.001
                     0.001 0.234 0.882
                                           0.380
0.001 0.003
```

```
##
##
## - MBB_Expense
## Backward Elimination: Step 5
##
## Variable MBB_Expense Removed
##
##
                  Model Summary
                 0.330 RMSE
0.109 Coef. Var
0.062 MSE
## R
                                      0.162
## R-Squared
                                     28.020
## Adj. R-Squared
                                       0.026
                      MAE
## Pred R-Squared -0.012
                                       0.127
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##
                    ANOVA
## -----
           Sum of
       Squares DF Mean Square F Sig.
## -----
## Regression 0.245
                 4
                            0.061 2.327 0.0638
                    76
## Residual
           2.002
                            0.026
## Total
           2.247
                    80
##
##
                        Parameter Estimates
    model Beta Std. Error Std. Beta t
                                           Sig
     upper
## -----
## (Intercept) 0.438 0.069
                                    6.372 0.000
0.301 0.575
## Conf_Cat 0.004 0.009 0.049 0.434 0.665
0.013 0.021
                0.001
##
  FB Rev -0.001
                             -0.186 -0.791 0.432
0.003 0.001
## MBB Rev 0.007
                0.003 0.306 2.402 0.019
0.001 0.013
## Total Rev 0.001
                     0.001 0.195
                                    0.788
                                          0.433
0.001
     0.002
##
##
```

```
## - Conf Cat
##
## Backward Elimination: Step 6
## Variable Conf_Cat Removed
##
##
                  Model Summary
## -----
                0.327 RMSE
0.107 Coef. Var
0.072 MSE
                                    27.872
## R-Squared
## Adj. R-Squared
                                       0.026
                        MAE
## Pred R-Squared
                 0.007
                                       0.127
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##
                     ANOVA
## ------
##
           Sum of
## Squares DF Mean Square F Sig. ##
                 3
77
## Regression 0.240
## Residual 2.007
                          0.080 3.072 0.0326
                            0.026
## Total
            2.247
##
##
                       Parameter Estimates
##
             Beta Std. Error Std. Beta t
     model
                                            Sig
lower upper
-----
## (Intercept) 0.456 0.054
                                     8.415 0.000
0.348 0.564
## FB_Rev -0.001 0.001 -0.183 -0.781 0.437
0.003 0.001
## MBB_Rev 0.007
                 0.003 0.289
                                    2.396 0.019
0.001 0.012
## Total_Rev 0.001 0.001 0.199 0.812 0.419
0.001
     0.002
##
##
## - FB_Rev
## Backward Elimination: Step 7
```

```
## Variable FB Rev Removed
##
##
                 Model Summary
## -----
                0.322 RMSE
0.104 Coef. Var
0.085 MSE
0.047 MAE
## R
                                     0.162
## R-Squared
                                   28.005
## Adj. R-Squared
                0.085
                                     0.026
## Pred R-Squared
                                     0.128
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##
                    ANOVA
##
          Sum of
        Squares DF Mean Square F Sig.
##
## -----
## Regression 0.285 2 0.142 5.392 0.0061
## Residual 2.456 93 0.026
## Total
           2.740
                95
##
##
                       Parameter Estimates
## -----
     model Beta Std. Error Std. Beta t
lower upper
## -----
-----
## (Intercept) 0.489 0.040
                                  12.276 0.000
0.410 0.568
## MBB_Rev 0.008
                    0.003 0.337 3.038 0.003
0.003 0.013
## Total Rev 0.000
                    0.000 -0.035 -0.319 0.750
0.001 0.001
##
##
## - Total Rev
## Backward Elimination: Step 8
##
## Variable Total Rev Removed
##
                 Model Summary
## R
                 0.321
                        RMSE
                                     0.162
## R-Squared 0.103 Coef. Var 27.871
```

```
## Adj. R-Squared 0.093 MSE
                                       0.026
## Pred R-Squared 0.067
                         MAE
                                       0.129
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##
                     ANOVA
## -----
##
           Sum of
          Squares DF Mean Square F Sig.
##
## -----
## Regression 0.282 1 0.282 10.785 0.0014
## Residual 2.458 94 0.026
## Total 2.740 95
##
##
                   Parameter Estimates
## -----
           Beta Std. Error Std. Beta t
##
     model
                                          Sig
lower upper
## -----
## (Intercept) 0.482 0.034
                                  14.177 0.000
0.415 0.550
           0.007 0.002 0.321 3.284 0.001
## MBB_Rev
0.003 0.012
##
##
## No more variables satisfy the condition of p value = 0.15
##
##
## Variables Removed:
##
## - Enrollment
## - Conf_Years
## - Type_Cat
## - MBB_Arena_Cap
## - MBB Expense
## - Conf_Cat
## - FB Rev
## - Total Rev
##
##
## Final Model Output
## -----
```

```
##
##
                 Model Summary
## -----
## R 0.321 RMSE 0.162
## R-Squared 0.103 Coef. Var 27.871
## Adj. R-Squared 0.093 MSE 0.026
## Pred R-Squared 0.067 MAE 0.129
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##
                     ANOVA
           Sum of
          Squares DF Mean Square F Sig.
## -----
## Regression 0.282 1 0.282 10.785 0.0014
## Residual 2.458 94 0.026
## Total 2.740 95
                  Parameter Estimates
## model Beta Std. Error Std. Beta t Sig
lower upper
## -----
## (Intercept) 0.482 0.034
                                  14.177 0.000
0.415 0.550
## MBB_Rev 0.007 0.002 0.321 3.284 0.001
0.003 0.012
## -----
```

Backward Regression chose Model 1

#New Model

```
## Coefficients:
             Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 0.482490 0.034033 14.177 < 2e-16 ***
## MBB Rev 0.007378
                        0.002247 3.284 0.00144 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.1617 on 94 degrees of freedom
## Multiple R-squared: 0.1029, Adjusted R-squared:
## F-statistic: 10.78 on 1 and 94 DF, p-value: 0.001438
anova(new_fit)
## Analysis of Variance Table
## Response: MBB Win Perc
           Df Sum Sq Mean Sq F value
                                        Pr(>F)
##
## MBB Rev 1 0.28204 0.282045 10.785 0.001438 **
## Residuals 94 2.45830 0.026152
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

#Hypothesis Test on Full vs Reduced Model H_0 : Reduced model is adequate. H_a : Full model is adequate.

```
test_F = ((2.2458-1.988)/7)/(1.988/(96-9))
crit_F = qf(0.05,7,87,lower.tail=F)
```

F test statistic 1.6117 > F critical value 2.1167 so we fail to reject the null hypothesis. We do not have evidence to show the full model is adequate.

#Hypothesis Test for Significance of Estimates H_0 : $B_j = 0$; the j^{th} predictor is not significant. H_a : $B_j \neq 0$; the j^{th} predictor is significant. Predictor MBB_Rev has a p-value of 0.00144, so we reject H_0 and find that it is a significant predictor of men's basketball winning percentage.

#Predict using NOVA!

Prediction was 63.38%, actual was 78.95% (19.7% error)

#Confidence Interval

```
predict(new_fit, test, interval='confidence')
## fit lwr upr
## 1 0.6338475 0.5877517 0.6799433
```

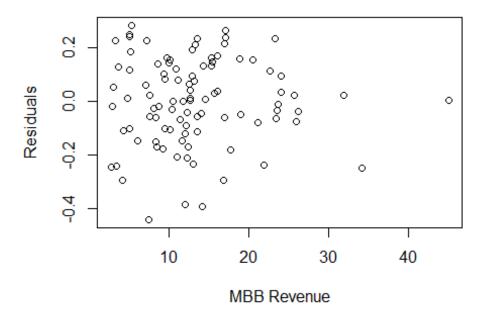
#Prediction Interval

```
predict(new_fit, test, interval='prediction')
## fit lwr upr
## 1 0.6338475 0.3094646 0.9582305
```

CHECKING ASSUMPTIONS

#Linearity

```
ei = new_fit$residuals
plot(data$MBB_Rev, ei, xlab='MBB Revenue', ylab='Residuals')
```

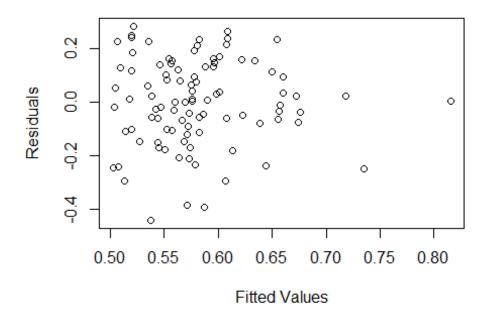


No obvious

systematic pattern

#Equal Variances

```
plot(new_fit$fitted.values, ei, xlab='Fitted Values', ylab='Residuals')
```



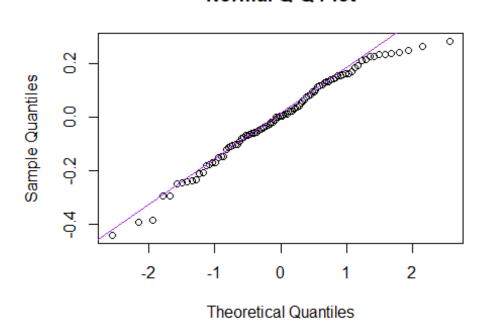
no systematic

pattern!

#Normality

```
qqnorm(ei)
qqline(ei, col='purple')
```

Normal Q-Q Plot



```
shapiro.test(ei)

##

## Shapiro-Wilk normality test

##

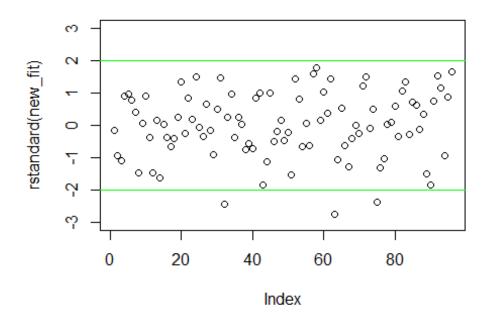
## data: ei

## W = 0.97759, p-value = 0.09907
```

normality satisfied: fit relatively close to line, but Wilk-Shapiro has high p-value

#Outliers

```
plot(rstandard(new_fit), ylim=c(-3,3))
abline(h=c(-2,2), col='green')
```



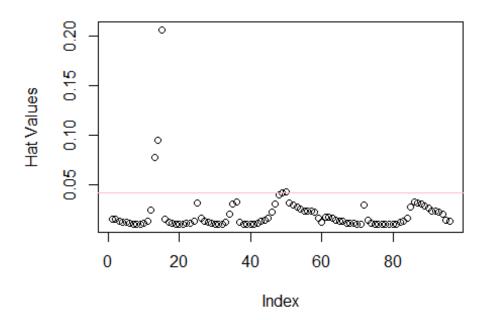
```
rstandard(new_fit)
##
                           2
                                       3
   -0.16565469 -0.93513415 -1.09452815
                                           0.91214351
                                                        0.98045774
##
             7
                           8
                                        9
                                                    10
                              0.06089641
    0.40095665 -1.45693715
                                           0.91819383 -0.36993419 -1.48671249
##
             13
                         14
                                       15
                                                   16
                                                                17
    0.16265360 -1.62304083
                              0.03593494
                                          -0.37046013 -0.65879494
##
##
             19
                         20
                                       21
                                                   22
                                                                 23
                1.32658953 -0.26820597
##
    0.25873186
                                           0.83570112
                                                        0.18934777
                                                                     1.50115902
##
             25
                         26
                                       27
                                                   28
                                                                 29
##
   -0.06154995 -0.34155576
                              0.65507122
                                          -0.16996456
                                                       -0.89830574
                                                                     0.48134842
                                                   34
##
                         32
                                       33
##
    1.46534058 -2.44623815
                              0.23548828
                                           0.97257887
                                                       -0.38627398
                                                                     0.23278821
             37
                                       39
##
                          38
                             -0.55034628
##
    0.01563358 -0.74298486
                                          -0.70629160
                                                        0.83422502
                                                                     1.00974414
            43
                         44
                                      45
                                                   46
                                                                47
                              1.00876432
   -1.83137205 -1.11822235
                                         -0.49834265
                                                       -0.19752419
                                                                     0.15900894
            49
##
                         50
                                       51
                                                   52
                                                                 53
   -0.46742218 -0.23661781 -1.53499415
                                           1.43925444
                                                        0.81156231
                                                                    -0.67492159
##
##
             55
                         56
                                       57
                                                   58
                                                                 59
    0.07058356 -0.62469835
                              1.57577439
                                           1.78450386
                                                        0.14986120
                                                                     1.02469449
                                                                 65
##
             61
                         62
                                       63
                                                   64
    0.37718601
                 1.42715241 -2.75007043 -1.05978022
                                                        0.52405838
                                                                    -0.62672817
                                       69
                                                                71
             67
                         68
                                                   70
   -1.29594075
               -0.41276877
                              0.01240990 -0.25654706
                                                        1.20465082
             73
                         74
                                       75
                                                   76
                                                                 77
```

```
## -0.10564947 0.51091592 -2.38578690 -1.30082878 -1.04086527 0.03176607
##
                        80
            79
                                     81
                                                 82
                                                              83
                                                                          84
##
    0.07518434
                0.60604093 -0.34154453
                                         1.05970469
                                                     1.34154135 -0.29458850
##
            85
                        86
                                     87
                                                 88
                                                              89
##
    0.72087388
                0.60705113 -0.12270238
                                         0.33829631 -1.51261605 -1.84596184
##
            91
                                                 94
                                                              95
                                     93
    0.73236695
               1.53135257 1.15892635 -0.92566428
##
                                                     0.87205561
                                                                 1.65974477
```

3 outliers: 32, 63, 75

#Influential Points

```
bm = 2*(2/96) ## about 0.0417
plot(hatvalues(new_fit), ylab='Hat Values')
abline(h=bm, col='pink')
```



```
hatvalues(new_fit)
                        2
##
            1
                                    3
                                                4
                                                           5
                                                                       6
7
## 0.01565888 0.01507526 0.01348651 0.01247856 0.01237500 0.01155578
0.01053713
                        9
                                   10
##
                                              11
                                                          12
                                                                      13
14
## 0.01043160 0.01076945 0.01129327 0.01311760 0.02471560 0.07804894
0.09481776
##
                                                                      20
           15
                       16
                                   17
                                              18
                                                          19
21
```

```
## 0.20637368 0.01500454 0.01225123 0.01110330 0.01048075 0.01041741
0.01053651
##
           22
                      23
                                  24
                                             25
                                                        26
                                                                    27
28
## 0.01122337 0.01155608 0.01332303 0.03125653 0.01653882 0.01342981
0.01207250
##
           29
                      30
                                 31
                                             32
                                                        33
                                                                    34
35
## 0.01088989 0.01041780 0.01043446 0.01057906 0.01195294 0.02060944
0.03040268
##
           36
                      37
                                  38
                                             39
                                                        40
                                                                    41
42
## 0.03296646 0.01186961 0.01071175 0.01063264 0.01043369 0.01063134
0.01121400
##
           43
                      44
                                 45
                                             46
                                                        47
                                                                    48
49
## 0.01290921 0.01418862 0.01661066 0.02244698 0.03101089 0.04015078
0.04159246
##
           50
                      51
                                  52
                                             53
                                                        54
                                                                    55
56
## 0.04316034 0.03192797 0.02992246 0.02814541 0.02595972 0.02409252
0.02361232
##
           57
                      58
                                  59
                                             60
                                                        61
                                                                    62
63
## 0.02345930 0.02276279 0.01660066 0.01282704 0.01794433 0.01737776
0.01680651
                      65
                                 66
                                             67
##
           64
                                                        68
                                                                    69
70
## 0.01478947 0.01319988 0.01318618 0.01144843 0.01109681 0.01083223
0.01062489
##
           71
                      72
                                 73
                                             74
                                                        75
                                                                    76
77
## 0.01044104 0.02987701 0.01448383 0.01124530 0.01070260 0.01060999
0.01056634
##
           78
                      79
                                 80
                                             81
                                                        82
                                                                    83
84
## 0.01049634 0.01047699 0.01044297 0.01043908 0.01195443 0.01314731
0.01691302
##
           85
                      86
                                 87
                                             88
                                                        89
                                                                    90
91
## 0.02754733 0.03313068 0.03132941 0.03051853 0.02905039 0.02626363
0.02355034
##
                      93
                                  94
                                             95
           92
                                                        96
## 0.02329697 0.02300220 0.02028668 0.01464149 0.01322485
```

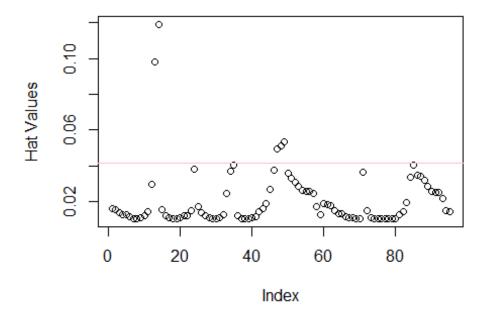
#Remove 15 - high influence point

```
data_2 <- data[-c(15),]
new_fit_2 = lm(MBB_Win_Perc~MBB_Rev, data=data_2)
summary(new_fit_2)</pre>
```

```
##
## Call:
## lm(formula = MBB_Win_Perc ~ MBB_Rev, data = data_2)
## Residuals:
##
                      Median
        Min
                  1Q
                                   3Q
                                           Max
## -0.44114 -0.10043
                     0.00515 0.13168
                                       0.28503
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.482953
                         0.036590 13.199 < 2e-16 ***
              0.007338
                         0.002522
                                    2.909
                                           0.00453 **
## MBB Rev
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1626 on 93 degrees of freedom
## Multiple R-squared: 0.08342,
                                  Adjusted R-squared:
## F-statistic: 8.464 on 1 and 93 DF, p-value: 0.00453
```

#Recheck influential points

```
bm2 = 2*(2/95) ## about 0.0421
plot(hatvalues(new_fit_2), ylab='Hat Values')
abline(h=bm, col='pink')
```



hatvalues(new_fit_2)

## 7	1 2	2 3	4	5	6	
## 0.0162486	1 0.01556911	0.01373737	0.01259673	0.01248103	0.01158156	
	8 9	9 10	11	12	13	
14 ## 0.0105271 0.11917020	1 0.0112115	0.01199049	0.01452514	0.02977247	0.09790702	
## 1	5 16	5 17	18	19	20	
21 ## 0.0154869	7 0.01234321	0.01110507	0.01054027	0.01056437	0.01083004	
0.01188933 ## 2	2 23	3 24	25	26	27	
28 ## 0.0123663 0.01089067	9 0.01480383	0.03821642	0.01727795	0.01367264	0.01214521	
## 2 35	9 36	31	32	33	34	
## 0.0105424 0.04041603	3 0.01062459	0.01090397	0.01292446	0.02443609	0.03711702	
## 3 42	5 37	7 38	39	40	41	
## 0.0119221 0.01424153	4 0.01072172	0.01065192	0.01062256	0.01099133	0.01187572	
## 4 49	3 44	45	46	47	48	
## 0.0159703 0.05348466	8 0.01919131	0.02682871	0.03790021	0.04963304	0.05147869	
## 5 56	9 51	L 52	53	54	55	
## 0.0356717 0.02548932	8 0.03325248	0.03111234	0.02848549	0.02624705	0.02567236	
## 5 63	7 58	59	60	61	62	
## 0.0246567 0.01523747	5 0.01735048	0.01298840	0.01893152	0.01826374	0.01759208	
## 6 70	4 65	66	67	68	69	
## 0.0134107 0.01052641	7 0.01339519	0.01146665	0.01109841	0.01083467	0.01064534	
## 7 77	1 72	2 73	74	75	76	
## 0.0364398 0.01054901	1 0.01488370	0.01125212	0.01071342	0.01063288	0.01059785	
## 7 84	8 79	80	81	82	83	
## 0.0105383 0.03343477	5 0.01052659	0.01063634	0.01292654	0.01456550	0.01959038	
## 8 91	5 86	5 87	88	89	90	

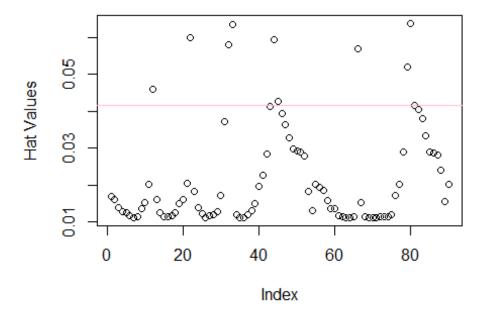
```
## 0.04062714 0.03494931 0.03397111 0.03220177 0.02885034 0.02559822 0.02529519 ## 92 93 94 95 ## 0.02494282 0.02170598 0.01506607 0.01467075
```

high influence points: 13, 14, 47, 48, 49

#Remove influence points and recheck again!

```
data_3 <- data_2[-c(13,14,47,48,49),]
new_fit_3 = lm(MBB_Win_Perc~MBB_Rev, data=data_3)

bm3 = 2*(2/90) ## about 0.0444
plot(hatvalues(new_fit_3), ylab='Hat Values')
abline(h=bm, col='pink')</pre>
```



#Compare first reduced model & model without 6 influential points

```
summary(new_fit)
##
## Call:
## lm(formula = MBB_Win_Perc ~ MBB_Rev, data = data)
##
## Residuals:
        Min
                       Median
                                            Max
##
                  1Q
                                    3Q
## -0.44098 -0.10004
                      0.00514 0.13058
                                        0.28528
##
```

```
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
##
                         0.034033 14.177 < 2e-16 ***
## (Intercept) 0.482490
                         0.002247
                                    3.284 0.00144 **
## MBB Rev
             0.007378
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.1617 on 94 degrees of freedom
## Multiple R-squared: 0.1029, Adjusted R-squared: 0.09338
## F-statistic: 10.78 on 1 and 94 DF, p-value: 0.001438
summary(new_fit_2)
##
## Call:
## lm(formula = MBB Win Perc ~ MBB Rev, data = data 2)
##
## Residuals:
##
       Min
                 10
                      Median
                                   3Q
                                           Max
## -0.44114 -0.10043 0.00515 0.13168 0.28503
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
                         0.036590 13.199 < 2e-16 ***
## (Intercept) 0.482953
## MBB_Rev
              0.007338
                         0.002522
                                   2.909 0.00453 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.1626 on 93 degrees of freedom
## Multiple R-squared: 0.08342, Adjusted R-squared: 0.07357
## F-statistic: 8.464 on 1 and 93 DF, p-value: 0.00453
summary(new_fit_3)
##
## Call:
## lm(formula = MBB Win Perc ~ MBB Rev, data = data 3)
## Residuals:
                      Median
       Min
                 1Q
                                   3Q
                                           Max
## -0.43439 -0.09733 0.00161 0.13247 0.29680
##
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
                         0.041400 11.097
                                            <2e-16 ***
## (Intercept) 0.459415
## MBB Rev
              0.009579
                         0.003127
                                    3.064
                                            0.0029 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.1642 on 88 degrees of freedom
```

```
## Multiple R-squared: 0.09639, Adjusted R-squared: 0.08612
## F-statistic: 9.387 on 1 and 88 DF, p-value: 0.0029
```