Python 基本機率函數繪圖

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Python 跟 R 是很常拿來作統計分析的程式語言，在 R 中常常使用 ggplot2 與 shiny 來繪製統計相關圖形，而在 Python 中則廣為使用 Matplotlib、Plotly 與 seaborn 來進行繪圖。在此文中，使用 Matplotlib 來進行一些基本機率函數圖形繪製，以幫助熟練使用套件 Matplotlib。另外，本文亦將在最後進行一個小專題練習。

1 基本機率函數繪圖

首先，本文在此介紹幾個基本機率函數圖形的繪製，以供之後利用套件 Matplotlib 進行繪圖之用。

- $f(x) = sin(x) + cos(x)$

```
import numpy as np
import matplotlib.pyplot as plt

x = np.linspace(-3*np.pi, 3*np.pi, 300)
y = np.sin(x) + np.cos(x)

fig, ax = plt.subplots(1)#畫布上一個圖形
ax.plot(x, y, color='skyblue')
ax.set_xticks(np.array([-3, -2, -1, 0, 1, 2, 3])*np.pi)
ax.set_xticklabels(['-3$\pi$', '-2$\pi$', '$\pi$', '0',
   '$\pi$','2$\pi$','3$\pi$'], \
        fontsize=10, color = 'b')
ax.set_ylim([-2, 2])
ax.grid(True)
plt.title("f(x)=sin(x)+cos(x)")
plt.show()
```

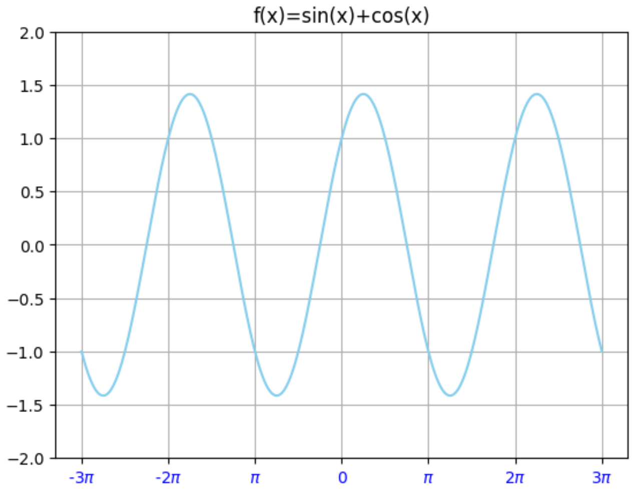

圖 1: f(x)=sin(x)+cos(x)

- $f(x) = (1 - exp(-2x))/(1 + exp(-2x))$

```
import matplotlib.pyplot as plt
import numpy as np

x = np.arange(-10,11,1)#type(x):array
# formulate a function f
f = lambda x : (1 - np.exp(-2*x)) / (1 + np.exp(-2*x))

fig = plt.figure(figsize=[6, 4])
plt.plot(x, f(x), color = 'r')
plt.grid(visible = True, color='g', linewidth=0.5)
plt.xlabel('X'), plt.ylabel('Y')
plt.ylim(-1.5,1.5)
plt.xlim(-10,10)
plt.axhline(1.01,color="b",linestyle="--")
plt.axhline(-1.01,color="b",linestyle="--")
plt.title('$f(x)=(1-exp(-2x))/(1+exp(-2x))$')
plt.show()
```

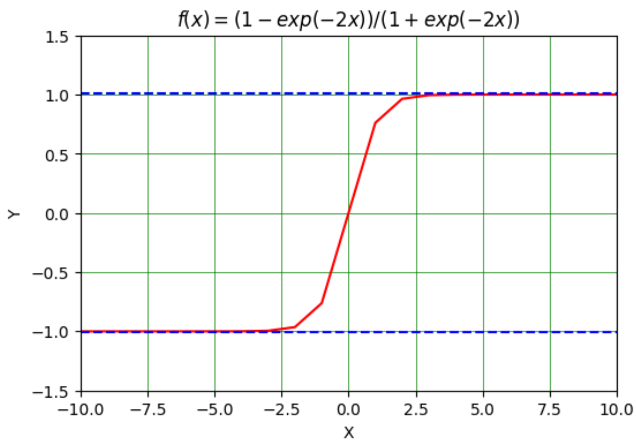

圖 2: f(x)=(1-exp(-2x))/(1+exp(-2x))

- $f(x) = \sqrt[3]{(4-x^3)/(1+x^2)}$

```
import matplotlib.pyplot as plt
import numpy as np

x = np.arange(-10,100,1)#type(x):array
# formulate a function f
f = lambda x : ((4-x**3)/(1+x**2))**(1/3)

fig = plt.figure(figsize=[6, 4])
plt.plot(x, f(x),  color = ’skyblue’,linewidth=3)
plt.grid(visible = True, color=’black’,  linewidth=0.3)
plt.xlabel(’X’), plt.ylabel(’Y’)
plt.title(’$f(x)=f(x)=\sqrt[3]{(4-x^3)/(1+x^2)}$’)
plt.show()
```

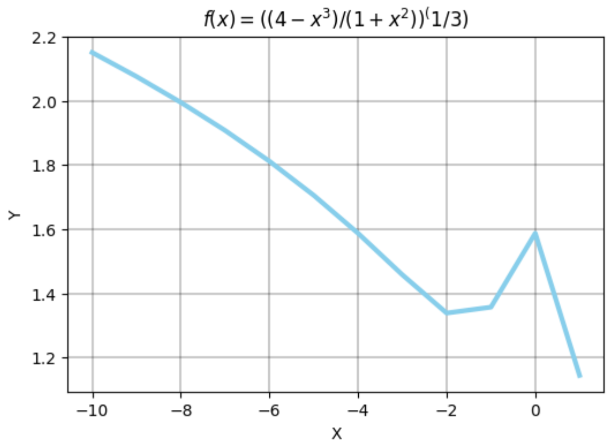

圖 3: $f(x) = \sqrt[3]{(4-x^3)/(1+x^2)}$

- $f(x) = 1/x$

```
import matplotlib.pyplot as plt
import numpy as np

x = np.arange(-50,50,1)#type(x):array
# formulate a function f
f = lambda x : 1/x

fig = plt.figure(figsize=[6, 4])
plt.plot(x, f(x),  color = ’green’,linewidth=2)
plt.grid(visible = True, color=’black’,  linewidth=0.3)
plt.xlabel(’X’), plt.ylabel(’Y’)
plt.title(’$f(x)=1/x$’)
plt.axhline(0,linestyle=”--”)
plt.axvline(0,linestyle=”--”)
plt.show()
```

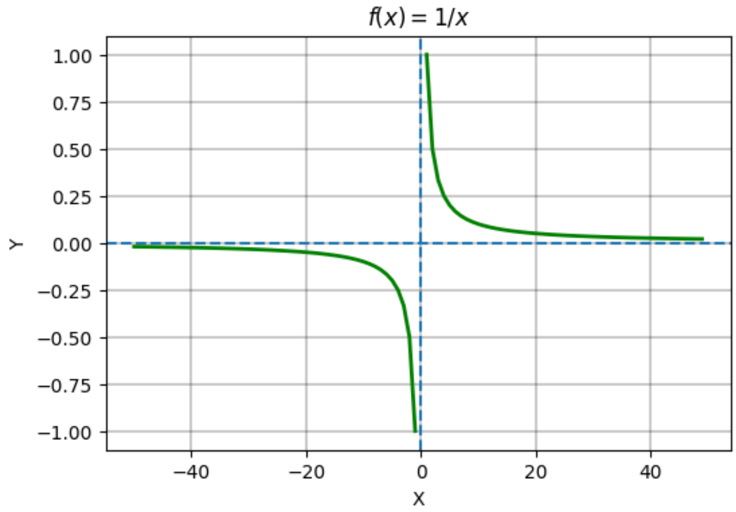

圖 4: $f(x) = 1/x$

- $f(x) = 1/(2\sqrt{2\pi})\,\exp(-(x-1)^2/8)$

```
import matplotlib.pyplot as plt
import numpy as np

x = np.arange(-20,20,1)#type(x):array
# formulate a function f
f = lambda x : 1/(2*np.sqrt(2*np.pi))*np.exp(-(x-1)**2/8)

fig = plt.figure(figsize=[6, 4])
plt.plot(x, f(x),  color = 'red',linewidth=2)
plt.grid(visible = True, color='black',  linewidth=0.3)
plt.xlabel('X'), plt.ylabel('Y')
plt.title('$f(x)=1/(2*sqrt(2*pi))exp(-(x-1)^2/8)$')
plt.show()
```

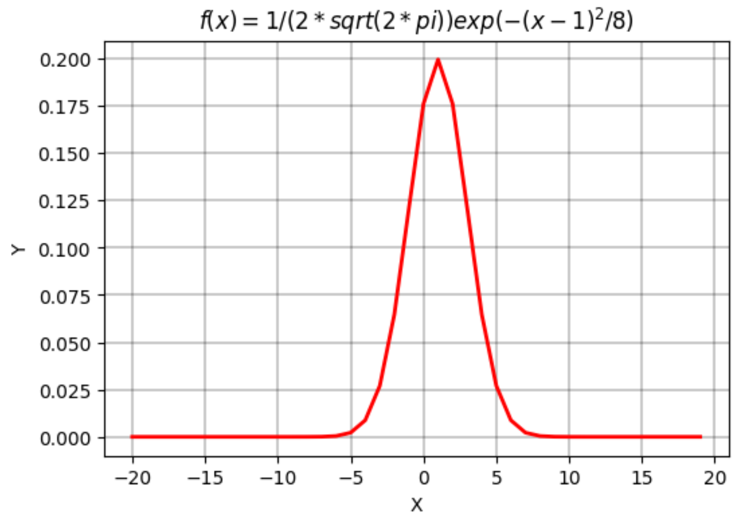

圖 5: $f(x) = 1/(2\sqrt{2\pi})\,\exp(-(x-1)^2/8)$

- $f(x) = x^{2/3}$

```
import matplotlib.pyplot as plt
import numpy as np

x = np.arange(0,50,1)#type(x):array
# formulate a function f
f = lambda x : x**(2/3)

fig = plt.figure(figsize=[6, 4])
plt.plot(x, f(x),  color = 'red',linewidth=2)
plt.grid(visible = True, color='black',  linewidth=0.3)
plt.xlabel('X'), plt.ylabel('Y')
plt.title('$f(x)=x**(2/3)$')
plt.show()
```

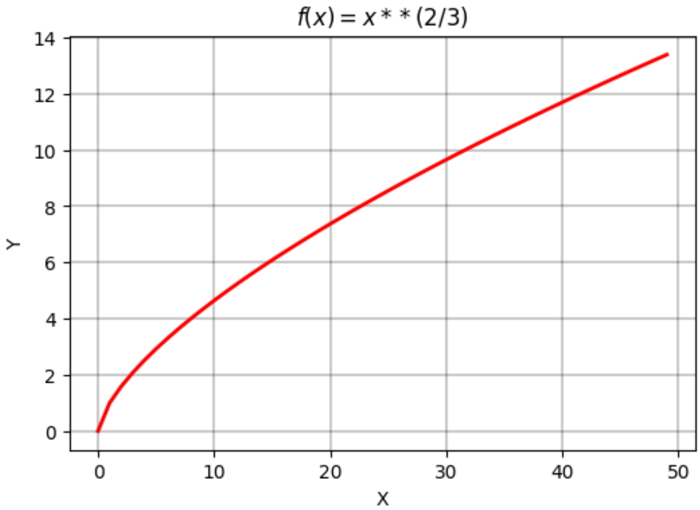

圖 6: $f(x) = x^{2/3}$

- $f(x) = 2x^3 - x^4$

```
import matplotlib.pyplot as plt
import numpy as np

x = np.arange(-35,35,1)#type(x):array
# formulate a function f
f = lambda x : 2*x**3-x**4

fig = plt.figure(figsize=[6, 4])
plt.plot(x, f(x),  color = 'red',linewidth=2)
plt.grid(visible = True, color='black',  linewidth=0.3)
plt.xlabel('X'), plt.ylabel('Y')
plt.xlim(-40,40)
plt.title('$f(x)=2*x^3-x^4$')
plt.show()
```

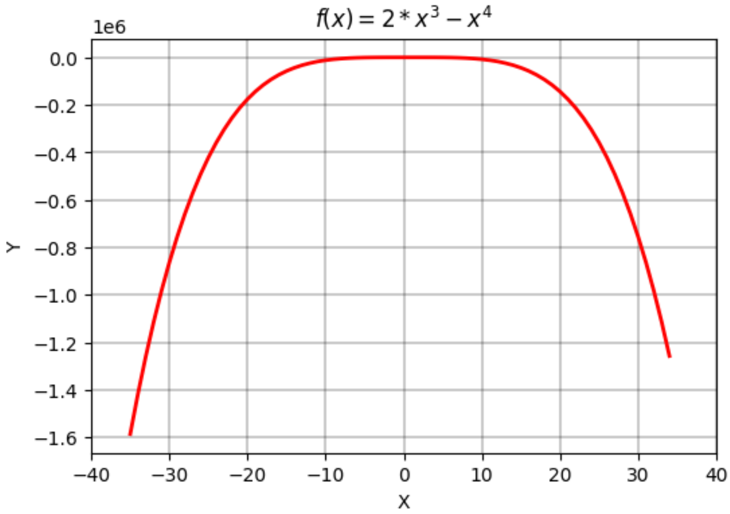

圖 7: $f(x) = 2x^3 - x^4$

- $f(x) = ln(x)/x^3$

```
import matplotlib.pyplot as plt
import numpy as np

x = np.arange(-35,35,1)#type(x):array
# formulate a function f
f = lambda x : np.log(x)/x**3

fig = plt.figure(figsize=[6, 4])
plt.plot(x, f(x),  color = 'blue',linewidth=2)
plt.grid(visible = True, color='black',  linewidth=0.3)
plt.xlabel('X'), plt.ylabel('Y')
plt.xlim(0,20)
plt.title('$f(x)=ln(x)/x**3$')
plt.show()
```

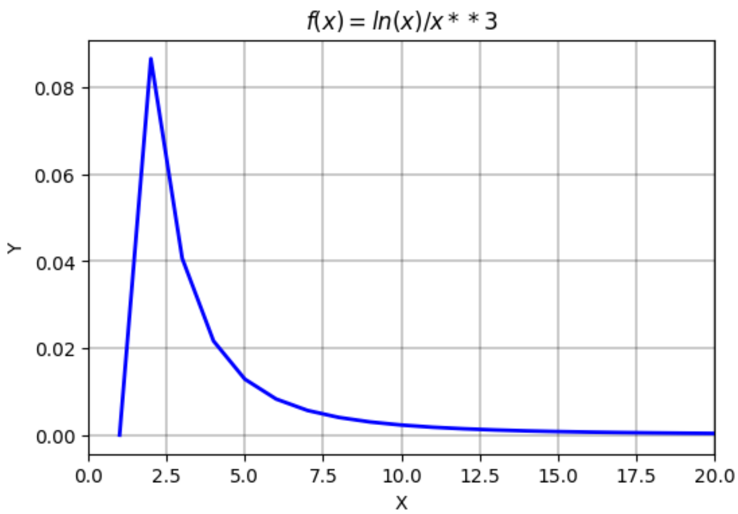

圖 8: $f(x) = ln(x)/x^3$

- $f(x) = 3, 1 \leq x \leq 5$

```
import matplotlib.pyplot as plt
import numpy as np
x=[1,5]
y=[3,3]
fig, ax = plt.subplots()
ax.hlines(y=3,xmin=1,xmax=5,linewidth=2)
plt.plot(x,y,marker="o")
plt.grid(visible = True, color='black',  linewidth=0.3)
plt.xlabel('X'), plt.ylabel('Y')
plt.xlim(0,6)
plt.ylim(0,6)
plt.title('$f(x)=3$')
plt.show()
```

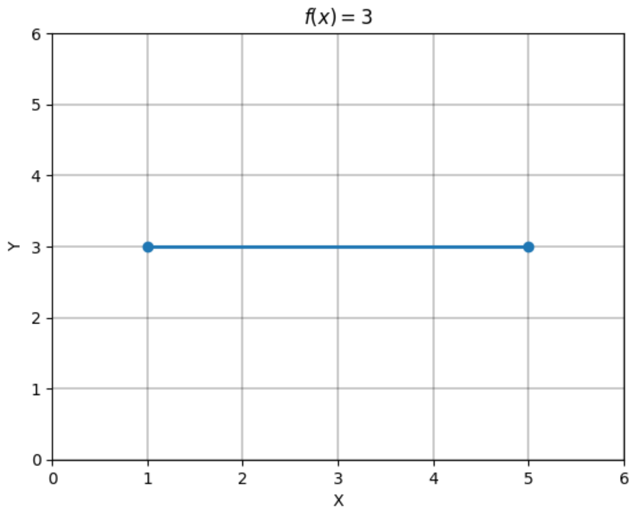

圖 9: $f(x) = 3$

- $x^2 + y^2 = 1$

```
import numpy as np
import matplotlib.pyplot as plt

t = np.linspace(0, 2*np.pi, 200)
r = 1
x = r*np.cos(t)
y = r*np.sin(t)

#刪掉邊線
ax=plt.gca()
ax.spines["right"].set_color("none")
ax.spines["top"].set_color("none")
#挪動x,y軸的位置
```

```
ax.spines["bottom"].set_position(("data",0))#data:將x軸綁
    定在y=0的位置
ax.spines["left"].set_position(("axes",0.5))#axes:將y軸綁
    定在x軸50%的位置
ax.set_aspect("equal")
plt.plot(x,y)
plt.xlim(-1.5,1.5)
plt.ylim(-1.5,1.5)
plt.title("Parametric Equation Circle")
plt.show()
```

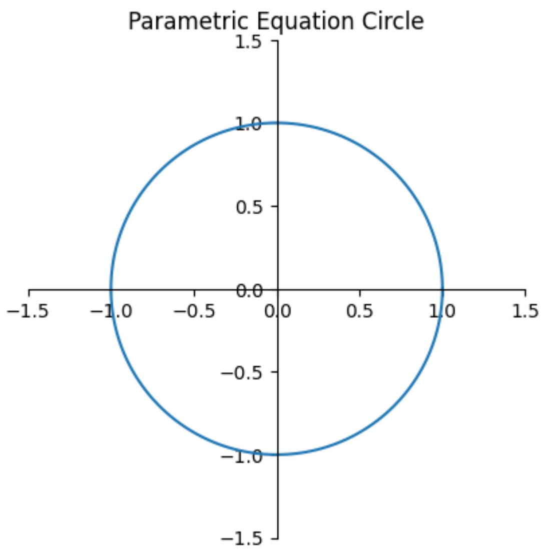

圖 10: $f(x) = 3$

- Square

```
import numpy as np
import matplotlib.pyplot as plt

fig, ax = plt.subplots()
ax = plt.gca()
ax.hlines(y=0.5, xmin=-0.5, xmax=0.5, linewidth=2)
ax.hlines(y=-0.5, xmin=-0.5, xmax=0.5, linewidth=2)
ax.vlines(x=0.5, ymin=-0.5, ymax=0.5, linewidth=2)
ax.vlines(x=-0.5, ymin=-0.5, ymax=0.5, linewidth=2)

ax.spines["right"].set_color("none")
ax.spines["top"].set_color("none")
ax.spines["bottom"].set_position(("data",0))#data:將x軸綁
    定在y=0的位置
ax.spines["left"].set_position(("axes",0.5))#axes:將y軸綁
    定在x軸50%的位置
ax.set_aspect("equal")
```

```
plt.xlim(-1,1)
plt.ylim(-1,1)
plt.xticks(np.arange(-1,1.5,0.5))
plt.yticks(np.arange(-1,1.5,0.5))
plt.title('A square of side 1')
plt.show()
```

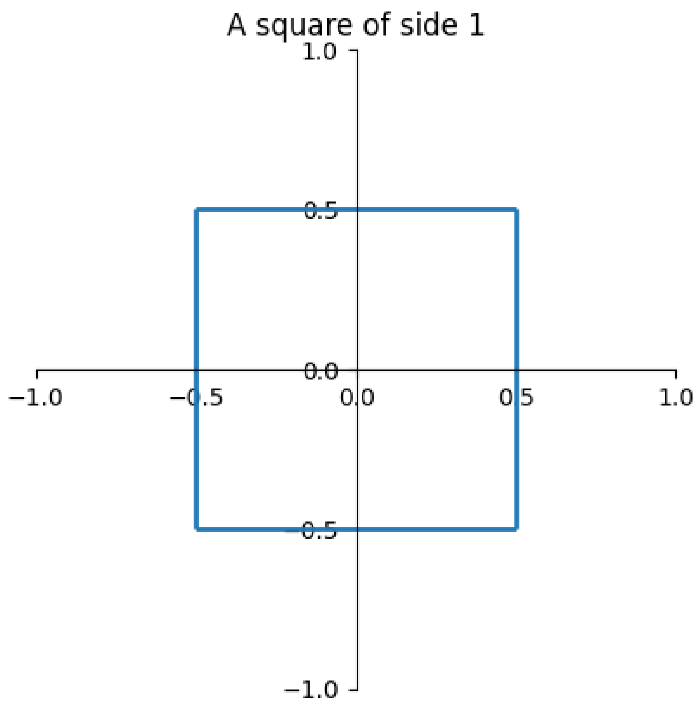

圖 11: *Square*

除了圖 11，亦可用其他方式來表達此圖，如下圖 12。

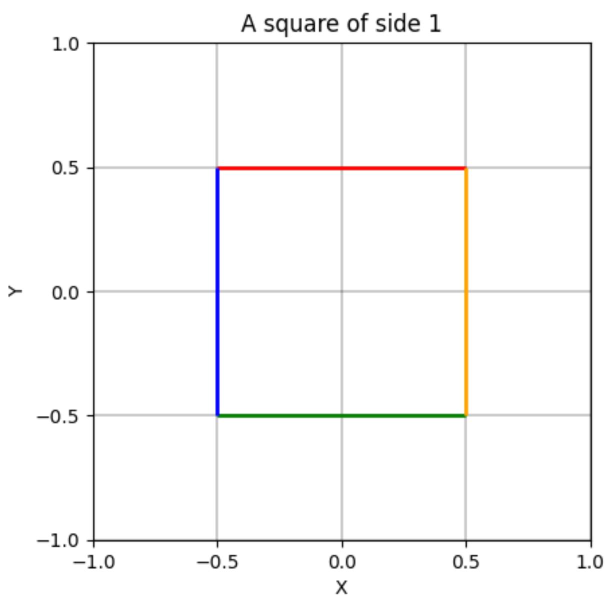

圖 12: *Square*

2 專題練習

接著本文列舉三個小專題來展示套件的應用。Let $S_n = \sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3+...+\frac{1}{n}}$

(a) Verify that $\lim_{n\to\infty} S_n$ diverges.

(b) Let γ_n denote the sum of the shade areas. Show that $\gamma_n - ln(n+1)$.

(c) Verify that $\frac{1}{2}(1-\frac{1}{n+1}) < \gamma_n < 1$.

Answer:

(a)

```
n = 10
a = np.arange(1,n+1)
print(a)
Sn=(1/a).sum()
print(Sn)

n = 10**np.arange(3)
print(np.arange(3))#[0 1 2]
Sn=np.zeros(len(n)) #挖空集合放不同的n得到的結果
print(n)#[1 10 100]
print(np.arange(len(n)))#[0 1 2]
print(np.arange(1,n[1]+1))#[1 2 3 4 5 6 7 8 9 10]

n = 10**np.arange(10)
Sn=np.zeros(len(n)) #挖空集合放不同的n得到的結果
for i in np.arange(len(n)):
    a = np.arange(1,n[i]+1)
    Sn[i] = (1/a).sum()
print(Sn)
print(n)

x = n
y = Sn
values = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
plt.plot(n,y)
plt.grid(visible = True, color='black',  linewidth=0.3)
plt.xlabel('n'), plt.ylabel('Sn')
plt.title('Sn diverge when n is infty')
#plt.xticks(n,values)
plt.show()
#a = np.arange(1,n[2]+1)
#print(a)
#Sn[2] = (1/a).sum()
#print(Sn[2])
```

根據以上程式碼我們可以得到下圖 13，故可得證。

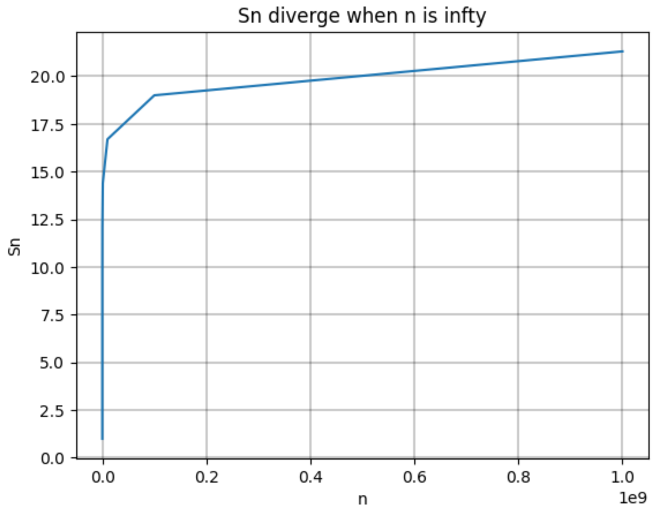

圖 13: S_n diverge when $n \to \infty$

(b)

```
import numpy as np
import matplotlib.pyplot as plt

x = np.arange(0,1000,0.1)#type(x):array
# formulate a function f
f = lambda x : 1/x

fig = plt.figure(figsize=[6, 4])
plt.plot(x, f(x),  color = 'blue',linewidth=2)
plt.grid(visible = True, color='black',  linewidth=0.3)
plt.xlabel('X'), plt.ylabel('Y')
plt.xlim(0,10)
plt.ylim(0,2)
plt.xticks(np.arange(0,11,1))
plt.title('$f(x)=1/x$')
plt.show()

def func(x):
    return 1/x
x = np.linspace(2,3,1000)
dx = (3-2)/1000
y = func(x)
area = np.sum(y*dx)
#print(func(2)-area)
#print(1-np.log(2))

n = np.arange(1,101)#[1 2 3 4 5 6 7 8 9 10]
gamma2=np.zeros(len(n))
def func(x):
    return 1/x
for i in np.arange(len(n)):#[0 1 2 3 4 5 6 7 8 9]
    x = np.linspace(n[i],n[i]+1,1000)
    dx = (2-1)/1000
```

```
    y = func(x)
    area = np.sum(y*dx)
    gamma2[i] = func(i+1)-area+gamma2[i-1]
print(gamma2)

n = np.arange(1,101)#[1 2 3 4 5 6 7 8 9 10]
Sn = np.zeros(len(n))
gamma1 = np.zeros(len(n))
for i in np.arange(len(n)):#[0 1 2 3 4 5 6 7 8 9]
    a = np.arange(1,n[i]+1)
    Sn[i] = (1/a).sum()
    gamma1[i] = Sn[i]-np.log(n[i]+1)
#print(Sn)
print(gamma1)
print(3/2-np.log(3))

fig, ax = plt.subplots(figsize = (9,6))
plt.subplot(1,2,1)
plt.plot(n,gamma1,color="red")
plt.grid(visible = True, color='black',  linewidth=0.3)
plt.xlabel('n'), plt.ylabel('gamma_n')
plt.title('gamma_n ')

plt.subplot(1,2,2)
plt.plot(n,gamma2,color="blue")
plt.grid(visible = True, color='black',  linewidth=0.3)
plt.xlabel('n'), plt.ylabel('Sn+ln(n+1)')
plt.title('Sn+ln(n+1)')
plt.show()
```

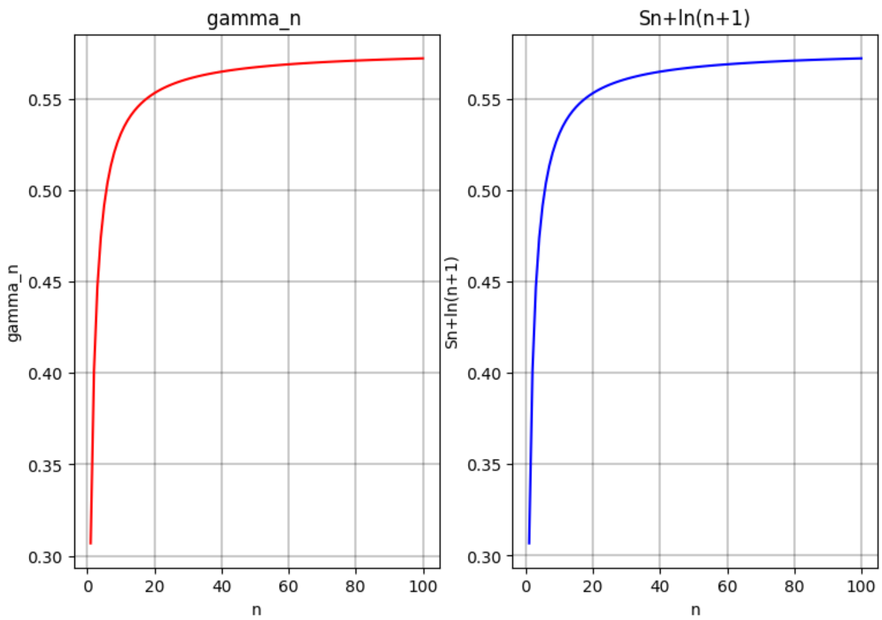

圖 14: γ_n and $S_n + ln(n+1)$

根據圖 14 以及以上程式碼我們可以求得從 $n=1$ 到 $n=100$ 時，γ_n 與 $S_n+ln(n+1)$ 個別的值，由此可以發現兩者的值基本趨近相同，故可得證。

另外，我們也可直接利用數學進行證明，其過程如下:

$$\gamma_n=\sum_{k=1}^{n}\int_{x=k}^{k+1}(\frac{1}{k}-\frac{1}{x})dx=\sum_{k=1}^{n}(\int_{k}^{k+1}\frac{1}{k}-\int_{k}^{k+1}\frac{1}{x}dx)=\sum_{k=1}^{n}(\frac{1}{k}-ln\,|k+1|+ln\,|k|)$$

$$=S_n-(ln2+ln3+...+ln(n)+ln(n+1))+(ln1+ln2+ln3+...+ln(n))$$

$$=S_n-ln(n+1)$$

(c)

```
import numpy as np
import matplotlib.pyplot as plt

x = np.arange(0,1000,0.1)#type(x):array
# formulate a function f
f = lambda x : 1/x

n = np.arange(1,101)#[1 2 3 4 5 6 7 8 9 10]
gamma2=np.zeros(len(n))
def func(x):
    return 1/x
for i in np.arange(len(n)):#[0 1 2 3 4 5 6 7 8 9]
    x = np.linspace(n[i],n[i]+1,1000)
    dx = (2-1)/1000
    y = func(x)
    area = np.sum(y*dx)
    gamma2[i] = func(i+1)-area+gamma2[i-1]

n = np.arange(1,101)
theta = np.zeros(len(n))
for i in np.arange(len(n)):
    theta[i] = 1/2*(1-1/(n[i]+1))


fig, ax = plt.subplots(figsize = (9,6))
plt.plot(n,gamma2,label="gamma_n",color="green")
plt.plot(n,theta,label="1/2(1-1/(n+1))",color="blue")
plt.axhline(1,color="red",label="1")
plt.ylim(0,1.1)
plt.xlim(0,100)
plt.xticks(np.arange(0,100,20))
plt.yticks(np.arange(0,1.1,0.1))
plt.grid(visible = True, color='black',  linewidth=0.3)
plt.xlabel('n'), plt.ylabel('Sn+ln(n+1)')

plt.legend(loc="lower right")
plt.show()
```

由上面程式碼與圖 15 我們即可得證 $\frac{1}{2}(1-\frac{1}{n+1}) < \gamma_n < 1$。

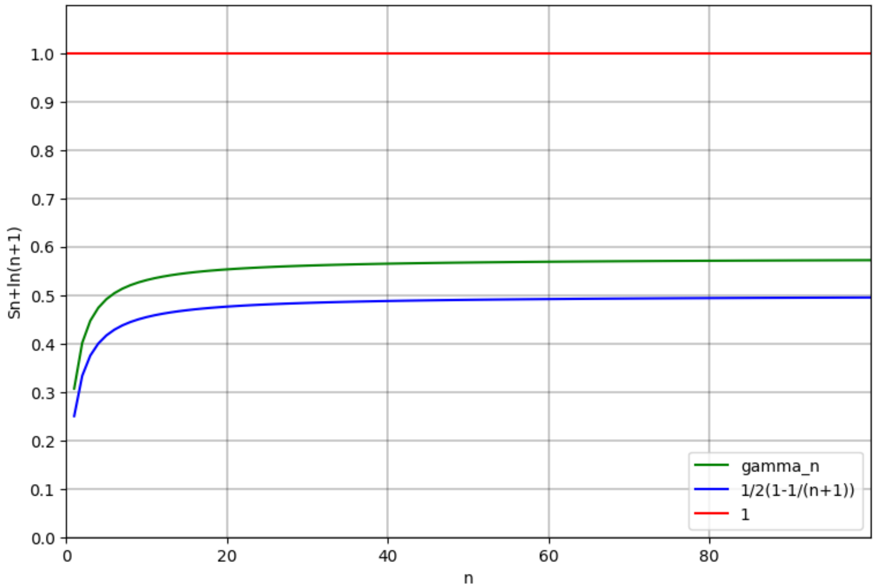

圖 15: $\frac{1}{2}(1-\frac{1}{n+1}) < \gamma_n < 1$

3 小結

在上面兩節，我們練習了套件 Matplotlib 的基本繪圖技巧，並也由此對機率函數有一些更深入的認知，望對以後使用 Python 進行繪圖有所幫助。