ECO 530 - Introduction to Econometrics

Fall 2023

Exercise 1

50 points

Due Date: Friday, September 8 2023

Instructions

Complete the exercises below. Be sure to show all of your work. For this assignment, you can submit:

- A PDF containing your written answers, tables, and figures along with the R script that generates them
- A R Markdown document containing your written answers with the R code embedded in the document.

Q1

Consider the function:

$$y = f(x) = 8 \cdot x - 3$$

a - Evaluate $f(x)$ at $x = 3$

b - What is the slope of the function?

c - How does the slope of the function at $x = 3$ compare to the slope of the function at $x = 6$

Q2

Consider the function:

$$y = f(x) = 3 - x + 2 \cdot x^2$$

a - Evaluate $f(x)$ at $x = 3$

b - What is the slope of the function?

c - How does the slope of the function at $x = 3$ compare to the slope of the function at $x = 6$

d - Does this function have a maximum or a minimum? What is it?

Q3

Consider the function:

$$y = f(x, z) = 100 + 3 \cdot x^2 + 2 \cdot z - 5 \cdot x \cdot z$$

a - Define and derive the two elements below:

$$\frac{\partial y}{\partial x} \qquad \frac{\partial y}{\partial z}$$

b - Define and derive the two elements below:

$$\frac{\partial^2 y}{\partial x^2} \qquad \frac{\partial^2 y}{\partial z \partial x}$$

Q4

Evaluate the following expressions. Show your work where necessary.

a - $\sum_{y=1}^{10} y$

b - $\sum_{i=1}^{10} 5$ **(or, more generally, any constant** c**)**

c - $\frac{df}{dx}$ **where** $f(x) = (12x + 3)(6x^2 + 8x - x^3)$

d - $\frac{df}{dx}$ **where** $f(x) = \frac{12x+3}{(6x^2+8x-x^3)}$

e - $\frac{df}{dx}$ **where** $f(x) = e^{-6x+2}$

Q5

Indicate whether the following expressions are correct or incorrect. If incorrect, briefly explain why.

a $log(x^{\beta}) = \beta \cdot log(x)$

b $log(0) = 1$

c $log(6x) = 6 \cdot log(x)$

d $log(xyz) = log(x) + log(y) + log(z)$

e $log(x^2) = log(x)log(x)$

f $\sum_{i=1}^{100} (X_i + Y_i + Z_i) = \sum_{i=1}^{100} X_i + \sum_{i=1}^{100} Y_i + \sum_{i=1}^{100} Z_i$

g $\sum_{i=1}^{100} (X_i \cdot Y_i + Z_i) = \sum_{i=1}^{100} X_i \cdot \sum_{i=1}^{100} Y_i + \sum_{i=1}^{100} Z_i$

h $\lim_{n\to\infty} \frac{6}{n} = 0$

i $\lim_{n\to\infty} \frac{6n^2+n}{n^2} = 0$

j $\prod_{i=1}^{5} y_i = y_1 \cdot y_2 \cdot y_3 \cdot y_4 \cdot y_5$

k $\prod_{i=1}^{5} e^{y_i} = e^{(y_1+y_2+y_3+y_4+y_5)}$

For the Questions that Follow:

Assume that the X,Y, and Z are independent random variables with expected values μ_X, μ_Y, and μ_Z and variances σ_X^2, σ_Y^2, and σ_Z^2 respectively.

Q6

Let W be a new random variable defined as:

$$W = 6X + Y - Z$$

a What is $E[W]$?

b What is σ_W^2?

Q7

Let R be a new random variable defined as:

$$R = Y - 12$$

a What is $E[R]$?

b What is σ_R^2?

c Assume that Y was normally distributed. Sketch the probability density functions for Y and R.

Q8

Let Q be a new random variable defined as:

$$Q = \frac{Z - \mu_Z}{\sigma_Z}$$

a What is $E[Q]$?

b What is σ_Q^2?

c Assume that Y was normally distributed. Sketch the probability density function for Z and Q.

1 R Exercises

Write a script that allows you to answer the questions below. Submit both a text version of your answers and the (heavily commented) script that you used to generate your answers.

R1

a. Load the tidyverse and vtable libraries.

b. Set pathways to the "data", "scripts", and "tables and figures" folders associated with Exercise 1.

c. Change the directory to the "data" folder and read in the "cars.csv" data file.

d. Use the summary table command (*st()*) to report the contents of the data.

R2

Starting with the "cars" data frame:

a. Use group_by() and summarize() to create a data frame called "summary" containing the average price, mpg, and weight of Foreign/Domestic cars in the data.

b. Use kable() to make a nicely formatted version of your "summary()" data frame.

c. Make a new data frame containing only Domestic Cars called "domestic.cars"

d. Add a variable (using the mutate() function) to the domestic.cars data frame that is equal to "price" divided by "mpg".

R3

Generate vectors containing 250 draws from each of the following normal distributions:

- $var1 \sim N(3, 1)$
- $var2 \sim N(-1, 2)$
- $var3 \sim N(2, 3)$

Place all three variables in a data frame together called "random.draws". Include in your data frame a variable called "id" that indicates an observations row number.

R4

Generate a scatter plot of $var1$. Make sure that your scatter plot has a title and informative labels on the axes.

R5

Generate a density plot of $var2$. Choose a fill color different than ggplot()'s default.

R6

Create a new variable in your data frame called $var4$ that is equal to the sum of the other three.

a. Where do you expect it's density to be centered when you plot it?

b. Create a density plot of your new variable, placing a dashed, vertical line at it's expected value.

R7

Create a new variable in your data frame called $var5$ by subtracting one from each element in $var3$ and dividing each element by 2.

a. Where do you expect the new variable's density to be centered?

b. How else do you expect it to change?

c. Create a density plot of your new variable placing a dashed, vertical line at it's expected value.