ECO 530 - Introduction to Econometrics

Fall 2023

Exercise 2

100 points

Due Date: Friday, September 22

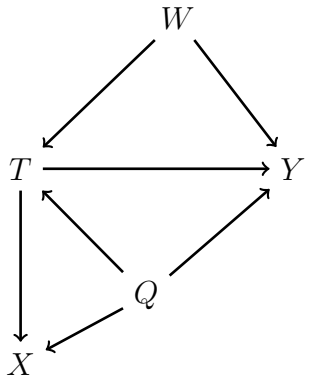
Instructions

Complete the exercises below. Be sure to show all of your work. For this assignment, you can submit:

- A PDF containing your written answers, tables, and figures along with the R script that generates them
- A R Markdown document containing your written answers with the R code embedded in the document.

Q1 - Directed Acyclic Graphs

Consider the Directed Acyclic Graph below.



Q1a Assume that you are interested in the relationship between T and Y and that T only takes values of zero or one. What would happen if you chose to estimate this relationship by comparing the sample average of Y among observations where $T = 1$ to the sample average of Y among observations where $T = 0$?

Q1b Using conditional expectations, write out the ideal comparison to identify the relationship between T and Y.

Q2 - Calculating Probabilities with Known Distributions

Q2a Consider a random variable X, where $X \sim N(0, 1)$. Use R to find $Pr(X \leq 1.64)$.

Q2b Consider a random variable X, where $X \sim N(42, 8)$. Use R to find $Pr(X \geq 30)$.

Q2c Consider a random variable X, where $X \sim N(0, 1)$. Use R to find $Pr(|X| \geq 1.64)$.

Q2d Consider a random variable X, where $X \sim N(0, 1)$. Use R to find $Pr(X \leq -1.64 \cup X \geq 2.5)$.

Q3 - The Age of UMaine Students

Download the "E2data.RData" data set and load it in R. Assume that the observations in the *universe* data frame represent the entire population of UMaine students. Assume that the true average age (μ) of a UMaine student is 22.

Q3a Consider the three estimators proposed below. Show whether each estimator is unbiased. Which one is the most efficient?

- $\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$
- $\tilde{y} = \frac{\sum_{i=1}^{5} y_i}{5}$
- $\hat{y} = y_1$ (The first observation you draw)

Q3b Draw a sample of 25 observations from the *universe* data frame. Using the sample, calculate an estimate of μ using $\bar{y}$, $\tilde{y}$ and $\hat{y}$. Report your estimates. Which one is the closest to the true value of μ?

Q4 - Hypothesis Testing

Imagine that you are interested the effect of going on a daily jog ($\hat{\beta}$) and life expectancy.

You estimate $\hat{\beta} = 6.25$ after analyzing a sample of 502 observations.

The standard error of your estimate is 1.43.

Q4a Test the following null hypotheses against the alternative, assuming you are willing to accept a 5% chance of committing a Type I error:

$H_0 : \beta = 9.25$

$H_0 : \beta = 2.1$

$H_0 : \beta = 3.5$

Q4b Still using the estimate from above, construct the following confidence intervals. Plot the confidence intervals in R and be sure to interpret each interval in words.

95% Confidence Interval

99% Confidence Interval

Q4c What is the smallest sized test for which you would reject the null hypotheses below?

$H_0 : \beta = 4$

Q5 - The Effect of Studying Hard on GPA

In this section, we will return to our assessment of the relationship between studying hard and GPA. Consider the estimator:

$$\hat{\gamma} = \bar{y}_1 - \bar{y}_2$$

Where:

- $\bar{y}_1$ is the average GPA among students who report studying hard.
- $\bar{y}_0$ is the average GPA among students who do not report studying hard.

The standard error of this estimator can be calculated as:

$$se(\hat{\gamma}) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_0^2}{n_0}}$$

Where:

- s_1^2 is the sample variance of GPA among students who study hard.
- n_1 is the number of students who study hard.
- s_0^2 and n_0 are similarly defined for students who did not study hard.

Q5a - Using the full sample, calculate an estimate of $\hat{\gamma}$.

Q5b - Calculate the standard error associated with your estimate.

Q5c - Test the hypothesis that studying hard has no effect on GPA

Q5d - Construct and discuss a 95 % confidence interval around your estimate of $\hat{\gamma}$

Q6...Conditional on the library

Repeat the tasks from **Q5a**, **Q5b**, and **Q5d**, but this time focus only on students who know where the library is. Compare and contrast your results to those you obtained in **Q5**.