MTH5315 Midterm #1 (Due 02/19/19)

February 9, 2019

(1) Please submit a printed report about the homework.
(2) In the report, please attach the related computer code as well as the answers to the question, with all the necessary analysis and computer generated plots.

Question1.

Apply the first-order upwinding scheme to the wave equation $u_t + u_x = 0$ over domain $[0, 1]$ with periodic boundary condition and initial condition $u(x, 0) = \sin^2(2\pi x)$.

(a) Using $\Delta t = 0.75\Delta x$ and $\Delta x = 0.01$, plot the exact solution and the numerical solution at time = 2. Based on the dissipative property of the scheme, quantitatively explain the amount of damping in the numerical solution.
(b) Change the initial condition to $u(x, 0) = \sin^{20}(2\pi x)$ and redo the above test. Compare the numerical dissipation with that in (a). Explain the difference through the plot of the power spectral density of the initial condition.

Question2.

Apply scheme $U_k^{n+1} + \frac{R}{4}(U_{k+1}^{n+1} - U_{k-1}^{n+1}) = U_k^n - \frac{R}{4}(U_{k+1}^n - U_{k-1}^n)$ to the wave equation $u_t + u_x = 0$, with periodic boundary condition over domain $[0, 1]$ and initial condition $u(x, 0) = \sin^2(2\pi x)$. $R = \frac{\Delta t}{\Delta x}$.

(a) Using $\Delta t = 4\Delta x$ and $\Delta x = 0.01$, plot the analytical and numerical solution at time = 4.
(b) Based on the dispersive property of the scheme, quantitatively explain the error in the numerical solution.
(c) Discuss possible ways to reduce the dispersive error of the solution. Confirm your conclusion through numerical experiments.