Manifolds

```
M = Manifold(2, 'M');M
```

    2-dimensional differentiable manifold M

```
U = M.open_subset('U') ; U
```

    Open subset U of the 2-dimensional differentiable manifold M

```
V = M.open_subset('V');V
```

    Open subset V of the 2-dimensional differentiable manifold M

```
c_xy.<x,y> = U.chart() ; c_xy
```

    Chart (U, (x, y))

```
c_uv.<u,v> = V.chart();c_uv
```

    Chart (V, (u, v))

```
xy_to_uv = c_xy.transition_map(c_uv, (x+y, x-y), intersection_name='W',restrictions1= x>0,
restrictions2= u+v>0) ;xy_to_uv
```

    Change of coordinates from Chart (W, (x, y)) to Chart (W, (u, v))

```
uv_to_xy = xy_to_uv.inverse();uv_to_xy
```

    Change of coordinates from Chart (W, (u, v)) to Chart (W, (x, y))

```
W = U.intersection(V);W
```

    Open subset W of the 2-dimensional differentiable manifold M

```
eU = c_xy.frame() ;eU
```

    Coordinate frame (U, (d/dx,d/dy))

```
eV = c_uv.frame(); eV
```

    Coordinate frame (V, (d/du,d/dv))

```
a = M.diff_form(2,name = 'a') ; a
```

    2-form a on the 2-dimensional differentiable manifold M

```
a.parent()
```

    Module /\^2(M) of 2-forms on the 2-dimensional differentiable
    manifold M

```
a.degree()
```

    2

```
a[eU,0,1] = x*y^2 + 2*x
a.add_comp_by_continuation(eV, W, c_uv)
```

```
a.display(eU)
```

$$a = \left(xy^2 + 2\,x\right) \mathrm{d}x \wedge \mathrm{d}y$$

```
a.display(eV)
```

$$a = \left(-\frac{1}{16}\,u^3 + \frac{1}{16}\,uv^2 - \frac{1}{16}\,v^3 + \frac{1}{16}\,\left(u^2 - 8\right)v - \frac{1}{2}\,u\right) \mathrm{d}u \wedge \mathrm{d}v$$

```
a = M.one_form('a');a
```

    1-form a on the 2-dimensional differentiable manifold M

```
a.parent()
```

    Module /\^1(M) of 1-forms on the 2-dimensional differentiable
    manifold M

```
a.degree()
```

    1

```
a[eU,:] = [-y,x]
```

```
a.add_comp_by_continuation(eV,W,c_uv)
```

```
a.display(eU)
```

$a = -y\mathrm{d}x + x\mathrm{d}y$

```
a.display(eV)
```

$a = \frac{1}{2}\,v\mathrm{d}u - \frac{1}{2}\,u\mathrm{d}v$

```
da = a.exterior_derivative();da
```

```
2-form da on the 2-dimensional differentiable manifold M
```

```
da.display(eU)
```

$\mathrm{d}a = 2\mathrm{d}x \wedge \mathrm{d}y$

```
da.display(eV)
```

$\mathrm{d}a = -\mathrm{d}u \wedge \mathrm{d}v$

```
b = M.one_form('b');b
```

```
1-form b on the 2-dimensional differentiable manifold M
```

```
b[eU,:] = [1+x*y,x^2]
```

```
b.add_comp_by_continuation(eV,W,c_uv)
```

```
b.display(eU)
```

$$b = (xy + 1)\,\mathrm{d}x + x^2\mathrm{d}y$$

```
b.display(eV)
```

$$b = \left(\frac{1}{4}\,u^2 + \frac{1}{4}\,uv + \frac{1}{2}\right)\mathrm{d}u + \left(-\frac{1}{4}\,uv - \frac{1}{4}\,v^2 + \frac{1}{2}\right)\mathrm{d}v$$

```
s = a+b ; s
```

```
1-form a+b on the 2-dimensional differentiable manifold M
```

```
s.display(eU)
```

$$a \wedge b = \left(-2\,x^2y - x\right)\mathrm{d}x \wedge \mathrm{d}y$$

```
s.display(eV)
```

$$a \wedge b = \left(\frac{1}{8}\,u^3 - \frac{1}{8}\,uv^2 - \frac{1}{8}\,v^3 + \frac{1}{8}\,(u^2 + 2)v + \frac{1}{4}\,u\right)\mathrm{d}u \wedge \mathrm{d}v$$

```
s =  a .wedge(b) ;s
```

$$a \wedge b$$

```
s.display(eU)
```

$$a \wedge b = \left(-2\,x^2y - x\right)\mathrm{d}x \wedge \mathrm{d}y$$

```
s.display(eV)
```

$$a \wedge b = \left(\frac{1}{8}\,u^3 - \frac{1}{8}\,uv^2 - \frac{1}{8}\,v^3 + \frac{1}{8}\,(u^2 + 2)v + \frac{1}{4}\,u\right)\mathrm{d}u \wedge \mathrm{d}v$$

```
f = M.scalar_field({c_xy: (x+y)^2 , c_uv: u^2},name='f');f
```

```
Scalar field f on the 2-dimensional differentiable manifold M
```

```
s = f*a;s
```

```
1-form on the 2-dimensional differentiable manifold M
```

```
s.display(eU)
```

$$\left(-x^{2} y - 2 \, x y^{2} - y^{3}\right) \mathrm{d} x + \left(x^{3} + 2 \, x^{2} y + x y^{2}\right) \mathrm{d} y$$

```
s.display(eV)
```

$$\frac{1}{2} \, v^{3} \mathrm{d} u - \frac{1}{2} \, u v^{2} \mathrm{d} v$$