CS 474

Assignment 2

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Problem 1

Soln:

Let us use the symbol ψ to refer to the given formula.

$$\psi = (p \land (p \Rightarrow q)) \Rightarrow q$$

In order to show that ψ is valid, we can show that

$$\neg\psi = \neg\left((p \land (p \Rightarrow q) \Rightarrow q)\right)$$

is unsatisfiable. Rewriting the above formula:

$$\begin{aligned} \neg\psi &= \neg\left((p \land (p \Rightarrow q) \Rightarrow q)\right) \\ &\equiv \neg\left((p \land (\neg p \lor q)) \Rightarrow q\right) \\ &\equiv \neg\left(\neg(p \land (\neg p \lor q)) \lor q\right) \\ &\equiv (p \land (\neg p \lor q)) \land \neg q \end{aligned}$$

The last step is due to De Morgan's Law. We can now convert this to CNF, and construct a resolution refutation to show that it is unsatisfiable. To convert to CNF, we use the Tseitin transformation. We only need three new propositional variables, x_ψ, x_1, x_2, where x_ψ corresponds to ψ, x_1 corresponds to $(\neg p \lor q)$ and x_2 corresponds to $(p \land x_1)$. This gives us the following set of clauses:

$$\begin{Bmatrix} \{\neg x_\psi\}, \\ \{\neg\neg x_\psi, x_2\}, \{\neg\neg x_\psi, \neg q\}, \{\neg x_\psi, \neg x_2, \neg\neg q\} \\ \{\neg x_2, p\}, \{\neg x_2, x_1\}, \{x_2, \neg p, \neg x_1\}, \\ \{x_1, \neg\neg p\}, \{x_1, \neg q\}, \{\neg x_1, \neg p, q\}, \end{Bmatrix}$$

Simplifying the set by replacing $\neg\neg p$ with p for all propositional variables, we get:

$$\begin{Bmatrix} \{\neg x_\psi\}, \\ \{x_\psi, x_2\}, \{x_\psi, \neg q\}, \{\neg x_\psi, \neg x_2, q\} \\ \{\neg x_2, p\}, \{\neg x_2, x_1\}, \{x_2, \neg p, \neg x_1\}, \\ \{x_1, p\}, \{x_1, \neg q\}, \{\neg x_1, \neg p, q\}, \end{Bmatrix}$$

We can now create a resolution refutation to show that this set is unsatisfiable:

1.$\{\neg x_\psi\}$
2.$\{x_\psi, x_2\}$
3.$\{x_2\}$ Resolvent of 1 and 2
4.$\{\neg x_2, p\}$
5.$\{p\}$ Resolvent of 3 and 4
6.$\{\neg x_1, \neg p, q\}$
7.$\{\neg x_1, q\}$ Resolvent of 5 and 6
8.$\{x_\psi, \neg q\}$
9.$\{x_\psi, \neg x_1\}$ Resolvent of 7 and 8
10.$\{\neg x_1\}$ Resolvent of 1 and 9
11.$\{\neg x_2, x_1\}$
12.$\{\neg x_2\}$ Resolvent of 3 and 11
13.$\{\}$ Resolvent of 10 and 12

By creating this resolution refutation, we have shown that there is no valuation that can satisfy $\neg\psi$. Since $\neg\psi$ is unsatisfiable, we can conclude that ψ is valid.

The above proof can be validated using the resolution tool provided as a part of the assignment. The initial set of clauses, and the resulting resolution proof object can be found at https://github.com/adharshkamath/cs474-hw/tree/main/resolution_proofs/p1

Problem 2
Soln:

We are given the formula:

$$\psi = (q \lor \neg r) \land (\neg p \lor r) \land (\neg q \lor r \lor p) \land (p \lor q \lor \neg q) \land (\neg r \lor q)$$

We can find resolvents in the following way: (Only resolutions that lead to clauses that do not already exist in the set of clauses are considered. Resolutions that result in "trivial" clauses are also considered.)

1.$\{q \vee \neg r\}$
2.$\{\neg p \vee r\}$
3.$\{q \vee \neg p\}$ Resolvent of 1 and 2
4.$\{\neg q \vee r \vee p\}$
5.$\{r \vee \neg r \vee p\}$ Resolvent of 1 and 4
6.$\{p \vee q \vee \neg q\}$
7.$\{p \vee q \vee \neg r\}$ Resolvent of 1 and 6
8.$\{r \vee \neg q\}$ Resolvent of 2 and 4
9.$\{r \vee \neg r\}$ Resolvent of 2 and 5
10.$\{r \vee p \vee \neg p\}$ Resolvent of 2 and 5
11.$\{r \vee q \vee \neg q\}$ Resolvent of 2 and 6
12.$\{r \vee q \vee \neg r\}$ Resolvent of 2 and 7
12.$\{p \vee q \vee \neg p\}$ Resolvent of 2 and 7
13.$\{\}$

<u>Problem 3</u>
Soln:
