CS 474
Assignment 2

Name: Adharsh Kamath   UIN: 671259918   NetID: ak128

Problem 1
Soln:

Let us use ψ to refer to the given formula.

$$\psi = (p \wedge (p \Rightarrow q)) \Rightarrow q$$

In order to show that ψ is valid, we can show that

$$\neg\psi = \neg\left((p \wedge (p \Rightarrow q) \Rightarrow q)\right)$$

is unsatisfiable. Rewriting the above formula:

$$\begin{aligned}
\neg\psi &= \neg\left((p \wedge (p \Rightarrow q) \Rightarrow q)\right) \\
&\equiv \neg\left((p \wedge (\neg p \vee q)) \Rightarrow q\right) \\
&\equiv \neg\left(\neg(p \wedge (\neg p \vee q)) \vee q\right) \\
&\equiv (p \wedge (\neg p \vee q)) \wedge \neg q
\end{aligned}$$

The last step is due to De Morgan's Law. We can now convert this to CNF, and use resolution to show that it is unsatisfiable. To convert to CNF, we use the Tseitin transformation. We only need three new propositional variables, x_ψ, x_1, x_2, where x_ψ corresponds to ψ, x_1 corresponds to $(\neg p \vee q)$ and x_2 corresponds to $(p \wedge x_1)$. This gives us the following set:

$$\left\{\begin{array}{l}
\{\neg x_\psi\}, \\
\{\neg\neg x_\psi, x_2\}, \{\neg\neg x_\psi, \neg q\}, \{\neg x_\psi, \neg x_2, \neg\neg q\} \\
\{\neg x_2, p\}, \{\neg x_2, x_1\}, \{x_2, \neg p, \neg x_1\}, \\
\{x_1, \neg\neg p\}, \{x_1, \neg q\}, \{\neg x_1, \neg p, q\},
\end{array}\right\}$$

We can now use resolution to show that this set is unsatisfiable. Simplifying the set by replacing $\neg\neg p$ with p for all propositional variables, we get:

$$\left\{\begin{array}{l}
\{\neg x_\psi\}, \\
\{x_\psi, x_2\}, \{x_\psi, \neg q\}, \{\neg x_\psi, \neg x_2, q\} \\
\{\neg x_2, p\}, \{\neg x_2, x_1\}, \{x_2, \neg p, \neg x_1\}, \\
\{x_1, p\}, \{x_1, \neg q\}, \{\neg x_1, \neg p, q\},
\end{array}\right\}$$

We can begin the resolution process by resolving the first two clauses:

1.$\{\neg x_\psi\}$
2.$\{x_\psi, x_2\}$
3.$\{x_2\}$ Resolvent of 1 and 2
4.$\{x_\psi, \neg q\}$ 5.$\{\neg x_\psi, \neg x_2, q\}$
6.$\{\neg x_2, q\}$ Resolvent of 4 and 5
7.$\{q\}$ Resolvent of 3 and 6   ...
...

Problem 2
Soln:

Problem 3
Soln: