CS 474

Assignment 2

Name: Adharsh Kamath UIN: 671259918 NetID: ak128

Problem 1

Soln:

Let us use the symbol ψ to refer to the given formula.

$$\psi = (p \wedge (p \Rightarrow q)) \Rightarrow q$$

In order to show that ψ is valid, we can show that

$$\neg\psi = \neg\left((p \wedge (p \Rightarrow q) \Rightarrow q)\right)$$

is unsatisfiable. Rewriting the above formula:

$$\begin{aligned}
\neg\psi &= \neg\left((p \wedge (p \Rightarrow q) \Rightarrow q)\right) \\
&\equiv \neg\left((p \wedge (\neg p \vee q)) \Rightarrow q\right) \\
&\equiv \neg\left(\neg(p \wedge (\neg p \vee q)) \vee q\right) \\
&\equiv (p \wedge (\neg p \vee q)) \wedge \neg q
\end{aligned}$$

The last step is due to De Morgan's Law. We can now convert this to CNF, and construct a resolution refutation to show that it is unsatisfiable. To convert to CNF, we use the Tseitin transformation. We only need three new propositional variables, x_ψ, x_1, x_2, where x_ψ corresponds to ψ, x_1 corresponds to $(\neg p \vee q)$ and x_2 corresponds to $(p \wedge x_1)$. This gives us the following set of clauses:

$$\left\{\begin{array}{l}
\{\neg x_\psi\}, \\
\{\neg\neg x_\psi, x_2\}, \{\neg\neg x_\psi, \neg q\}, \{\neg x_\psi, \neg x_2, \neg\neg q\} \\
\{\neg x_2, p\}, \{\neg x_2, x_1\}, \{x_2, \neg p, \neg x_1\}, \\
\{x_1, \neg\neg p\}, \{x_1, \neg q\}, \{\neg x_1, \neg p, q\},
\end{array}\right\}$$

Simplifying the set by replacing $\neg\neg p$ with p for all propositional variables, we get:

$$\left\{\begin{array}{l}
\{\neg x_\psi\}, \\
\{x_\psi, x_2\}, \{x_\psi, \neg q\}, \{\neg x_\psi, \neg x_2, q\} \\
\{\neg x_2, p\}, \{\neg x_2, x_1\}, \{x_2, \neg p, \neg x_1\}, \\
\{x_1, p\}, \{x_1, \neg q\}, \{\neg x_1, \neg p, q\},
\end{array}\right\}$$

We can now create a resolution refutation to show that this set is unsatisfiable:

$$
\begin{aligned}
&1.\{\neg x_\psi\} \\
&2.\{x_\psi, x_2\} \\
&3.\{x_2\} \quad \text{Resolvent of 1 and 2} \\
&4.\{\neg x_2, p\} \\
&5.\{p\} \quad \text{Resolvent of 3 and 4} \\
&6.\{\neg x_1, \neg p, q\} \\
&7.\{\neg x_1, q\} \quad \text{Resolvent of 5 and 6} \\
&8.\{x_\psi, \neg q\} \\
&9.\{x_\psi, \neg x_1\} \quad \text{Resolvent of 7 and 8} \\
&10.\{\neg x_1\} \quad \text{Resolvent of 1 and 9} \\
&11.\{\neg x_2, x_1\} \\
&12.\{\neg x_2\} \quad \text{Resolvent of 3 and 11} \\
&13.\{\} \quad \text{Resolvent of 10 and 12}
\end{aligned}
$$

By creating this resolution refutation, we have shown that there is no valuation that can satisfy $\neg\psi$. Since $\neg\psi$ is unsatisfiable, we can conclude that ψ is valid.

The above proof can be validated using the resolution tool provided as a part of the assignment. The initial set of clauses, and the resulting resolution proof object can be found at https://github.com/adharshkamath/cs474-hw/tree/main/resolution_proofs/p1

Problem 2
Soln:

We are given the formula:

$$\psi = (q \vee \neg r) \wedge (\neg p \vee r) \wedge (\neg q \vee r \vee p) \wedge (p \vee q \vee \neg q) \wedge (\neg r \vee q)$$

We can find resolvents in the following way: (Only resolutions that lead to clauses that do not already exist in the set of clauses are considered. Resolutions that result in "trivial" clauses are also considered.)

1.$\{q \lor \neg r\}$
2.$\{\neg p \lor r\}$
3.$\{q \lor \neg p\}$ Resolvent of 1 and 2
4.$\{\neg q \lor r \lor p\}$
5.$\{r \lor \neg r \lor p\}$ Resolvent of 1 and 4
6.$\{p \lor q \lor \neg q\}$
7.$\{p \lor q \lor \neg r\}$ Resolvent of 1 and 6
8.$\{r \lor \neg q\}$ Resolvent of 2 and 4
9.$\{r \lor \neg r\}$ Resolvent of 2 and 5
10.$\{r \lor p \lor \neg p\}$ Resolvent of 2 and 5
11.$\{r \lor q \lor \neg q\}$ Resolvent of 2 and 6
12.$\{r \lor q \lor \neg r\}$ Resolvent of 2 and 7
12.$\{p \lor q \lor \neg p\}$ Resolvent of 2 and 7
13.$\{...\}$

<u>Problem 3</u>
Soln:
