CS 474

Assignment 2

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Problem 1

Soln:

Let us use the symbol ψ to refer to the given formula.

$$\psi = (p \land (p \Rightarrow q)) \Rightarrow q$$

In order to show that ψ is valid, we can show that

$$\neg\psi = \neg\left((p \land (p \Rightarrow q) \Rightarrow q)\right)$$

is unsatisfiable. Rewriting the above formula:

$$\begin{aligned}
\neg\psi &= \neg\left((p \land (p \Rightarrow q) \Rightarrow q)\right) \\
&\equiv \neg\left((p \land (\neg p \lor q)) \Rightarrow q\right) \\
&\equiv \neg\left(\neg(p \land (\neg p \lor q)) \lor q\right) \\
&\equiv (p \land (\neg p \lor q)) \land \neg q
\end{aligned}$$

The last step is due to De Morgan's Law. We can now convert this to CNF, and construct a resolution refutation to show that it is unsatisfiable. To convert to CNF, we use the Tseitin transformation. We only need three new propositional variables, x_ψ, x_1, x_2, where x_ψ corresponds to ψ, x_1 corresponds to $(\neg p \lor q)$ and x_2 corresponds to $(p \land x_1)$. This gives us the following set of clauses:

$$\left\{\begin{array}{l}
\{\neg x_\psi\}, \\
\{\neg\neg x_\psi, x_2\}, \{\neg\neg x_\psi, \neg q\}, \{\neg x_\psi, \neg x_2, \neg\neg q\} \\
\{\neg x_2, p\}, \{\neg x_2, x_1\}, \{x_2, \neg p, \neg x_1\}, \\
\{x_1, \neg\neg p\}, \{x_1, \neg q\}, \{\neg x_1, \neg p, q\},
\end{array}\right\}$$

Simplifying the set by replacing $\neg\neg p$ with p for all propositional variables, we get:

$$\left\{\begin{array}{l}
\{\neg x_\psi\}, \\
\{x_\psi, x_2\}, \{x_\psi, \neg q\}, \{\neg x_\psi, \neg x_2, q\} \\
\{\neg x_2, p\}, \{\neg x_2, x_1\}, \{x_2, \neg p, \neg x_1\}, \\
\{x_1, p\}, \{x_1, \neg q\}, \{\neg x_1, \neg p, q\},
\end{array}\right\}$$

We can now create a resolution refutation to show that this set is unsatisfiable:

$$
\begin{aligned}
&1.\{\neg x_\psi\} \\
&2.\{x_\psi, x_2\} \\
&3.\{x_2\} \quad \text{Resolvent of 1 and 2} \\
&4.\{\neg x_2, p\} \\
&5.\{p\} \quad \text{Resolvent of 3 and 4} \\
&6.\{\neg x_1, \neg p, q\} \\
&7.\{\neg x_1, q\} \quad \text{Resolvent of 5 and 6} \\
&8.\{x_\psi, \neg q\} \\
&9.\{x_\psi, \neg x_1\} \quad \text{Resolvent of 7 and 8} \\
&10.\{\neg x_1\} \quad \text{Resolvent of 1 and 9} \\
&11.\{\neg x_2, x_1\} \\
&12.\{\neg x_2\} \quad \text{Resolvent of 3 and 11} \\
&13.\{\} \quad \text{Resolvent of 10 and 12}
\end{aligned}
$$

By creating this resolution refutation, we have shown that there is no valuation that can satisfy $\neg\psi$. Since $\neg\psi$ is unsatisfiable, we can conclude that ψ is valid.

Problem 2

Soln:

Problem 3

Soln:
