CS 474
Homework 4

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Problem 1
Soln: A group is defined as a set S, along with a binary operation $\cdot : S \times S \to S$ such that

1. $\forall a\, \forall b\, \forall c\, (a \cdot b) \cdot c = a \cdot (b \cdot c)$
2. $\exists e\, \forall a\, (e \cdot a) = (a \cdot e) = a$
3. $\forall a\, \exists b\, (a \cdot b) = (b \cdot a) = e$

From these axioms, we can see that e is the identity element.

Task 1: Prove that under group axioms, there is no other identity.

$$\forall e\, \forall e'\, \forall a\, (((e \cdot a) = (a \cdot e) = a)\ \wedge ((e' \cdot a) = (a \cdot e') = a)) \implies e = e'$$

Task 2: Prove that under group axioms, every element has only one inverse.

$$\forall a\, \forall b\, \forall b'\, (((a \cdot b) = (b \cdot a) = e)\ \wedge ((a \cdot b') = (b' \cdot a) = e)) \implies b = b'$$

Problem 2
Soln:

The given formula φ is

$$\varphi = y \leq x \wedge x \leq y \wedge f(y) = f(7) \wedge x \leq 5$$

Problem 3
Soln:

(a)
Given $f : 2^{\natural} \to 2^{\natural}$ that is defined as

$$f(S) = \{2\} \cup \{y | y = 2x, x \in S\}$$
