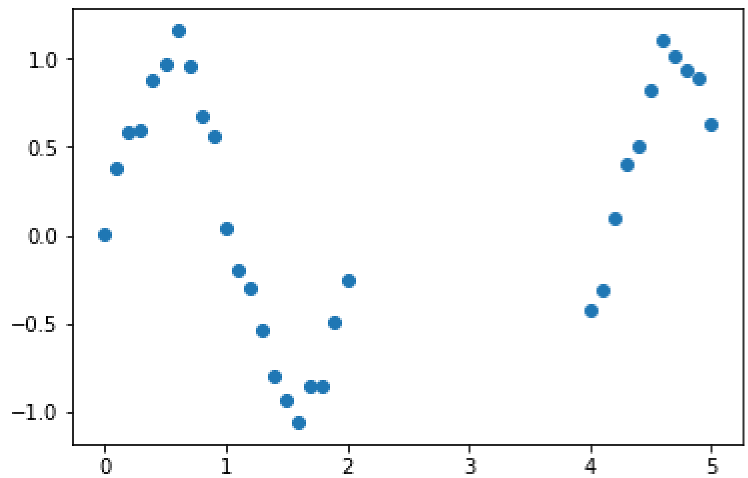
In [70]:
```
import numpy as np
from matplotlib import pyplot as plt
np.set_printoptions(suppress=True)
import seaborn as sns
from math import sin
import time
from sklearn.metrics import mean_squared_error as mse
import pickle as pkl
```

In [68]:
```
trainData=np.genfromtxt('Dataset/problem4a_train.csv',delimiter=',')
testData=np.genfromtxt('Dataset/problem4a_test.csv',delimiter=',')
x_train=trainData[:,0]
y_train=trainData[:,1]
x_test=testData
```

In [69]:
```
plt.scatter(x_train,y_train)
plt.show()
```



In [4]:
```
x_train=x_train.reshape(-1,1)
x_train.shape
```

Out[4]: (32, 1)

In [5]:
```
x_test=x_test.reshape(-1,1)
x_test.shape
```

Out[5]: (51, 1)

In [6]:
```
y_train.shape
```

Out[6]: (32,)

In [42]:
```
def get_y(x_test):

    y_test=np.sin(3*x_test)
    return y_test
y_test=get_y(x_test)
print(y_test.shape)
```

```
(51, 1)
```

In [7]:
```
from sklearn.gaussian_process import GaussianProcessRegressor
from sklearn.gaussian_process.kernels import RBF, ConstantKernel as C
from sklearn.gaussian_process.kernels import RationalQuadratic as RQ
from sklearn.gaussian_process.kernels import ExpSineSquared as ESS
```

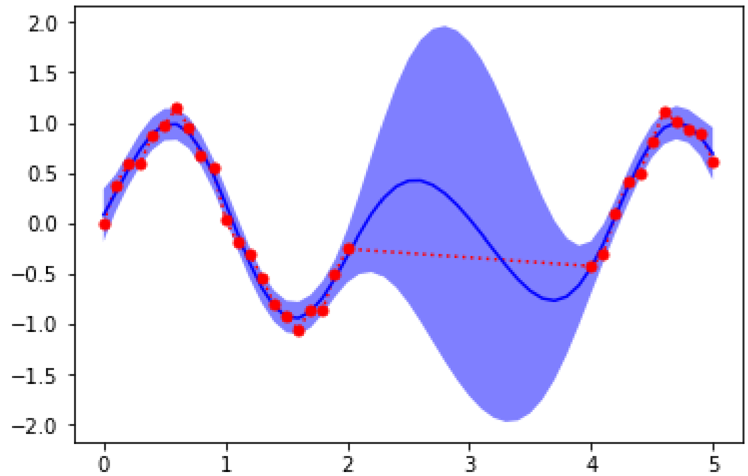
In [15]:
```
C1 = C(1.0, (1e-3, 1e3))
C2 = C(0.5, (1e-3, 1e3))
RBF1 = RBF(10, (1e-2, 1e2))
RBF2 = RBF(0.5, (1e-2, 1e2))
RQ1 = RQ(10, 0.5 ,(1e-2, 1e2))
ESS1 = ESS(1.0, 1.0, (1e-05, 100000.0), (1e-05, 100000.0))
kernel1 = C1 * RBF1 + C2
kernel2 = C1 * RBF1 + RBF2
kernel3 = C1 * RQ1 + RBF2
kernel4 = C1 * ESS1 + RBF2
```

In [16]:
```
gp = GaussianProcessRegressor(kernel=kernel2, alpha=0.5**5,n_restarts_optimizer=9)
gp.fit(x_train,y_train)
```

Out[16]:
```
GaussianProcessRegressor(alpha=0.03125, copy_X_train=True,
             kernel=1**2 * RBF(length_scale=10) + RBF(length_scale=0.5),
             n_restarts_optimizer=9, normalize_y=False,
             optimizer='fmin_l_bfgs_b', random_state=None)
```

In [17]:

```
y_pred, sigma = gp.predict(x_test, return_std=True)
plt.plot(x_train, y_train, 'r:', label=r'$f(x) = sin(3x)$')
plt.errorbar(x_train.ravel(), y_train, 0, fmt='r.', markersize=10, la
bel='Observations')
plt.plot(x_test, y_pred, 'b-', label='Prediction')
plt.fill(np.concatenate([x_test, x_test[::-1]]),
             np.concatenate([y_pred - 1.95 * sigma,
                            (y_pred + 1.95 * sigma)[::-1]]),
             alpha=.5, fc='b', ec='None', label='95% confidence inter
val')
plt.show()
```



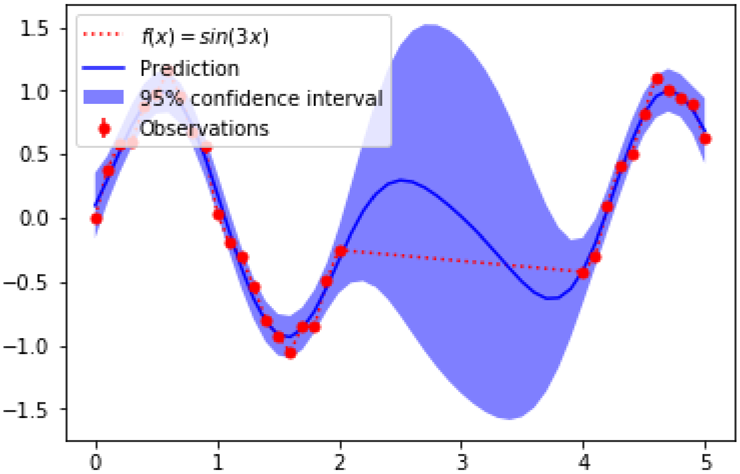
In [18]:
```
GP = []
for ndx, kernel in zip([1,2,3,4], [kernel1, kernel2, kernel3, kernel4
]):
    t = time.time()
    print('-------------------------------------------------------------
-----------------')
    print(f'time - {t} :: Fitting GP for kernel - {ndx}')
    gp = GaussianProcessRegressor(kernel=kernel, alpha=0.5 ** 5,
                                  n_restarts_optimizer=10)
    gp.fit(x_train, y_train)
    pkl.dump(fil)
    GP.append(gp)
    print(f'GP for Kernel - {ndx} Finished :: Elapsed Time - {time.ti
me()-t}')
    print('-------------------------------------------------------------
-----------------')
```

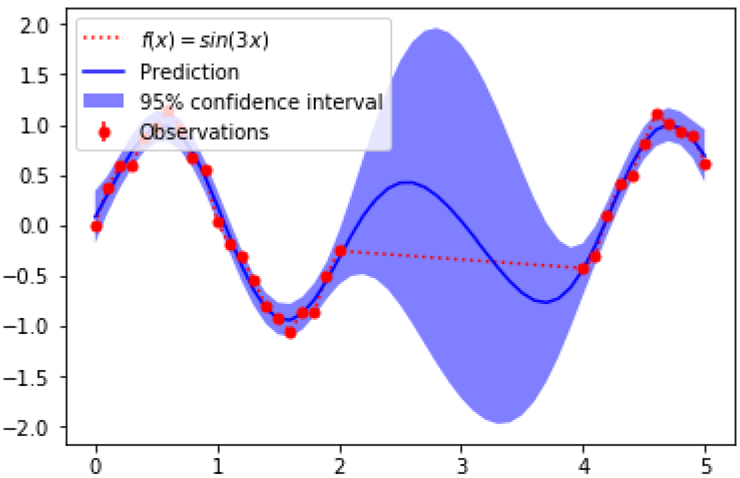
```
----------------------------------------------------------------------
------
time - 1571167899.3729181 :: Fitting GP for kernel - 1
GP for Kernel - 1 Finished :: Elapsed Time - 0.3065369129180908
----------------------------------------------------------------------
------
----------------------------------------------------------------------
------
time - 1571167899.67951 :: Fitting GP for kernel - 2
GP for Kernel - 2 Finished :: Elapsed Time - 0.23503851890563965
----------------------------------------------------------------------
------
----------------------------------------------------------------------
------
time - 1571167899.9147468 :: Fitting GP for kernel - 3
GP for Kernel - 3 Finished :: Elapsed Time - 0.3700573444366455
----------------------------------------------------------------------
------
----------------------------------------------------------------------
------
time - 1571167900.2848597 :: Fitting GP for kernel - 4
GP for Kernel - 4 Finished :: Elapsed Time - 0.289503812789917
----------------------------------------------------------------------
------
```

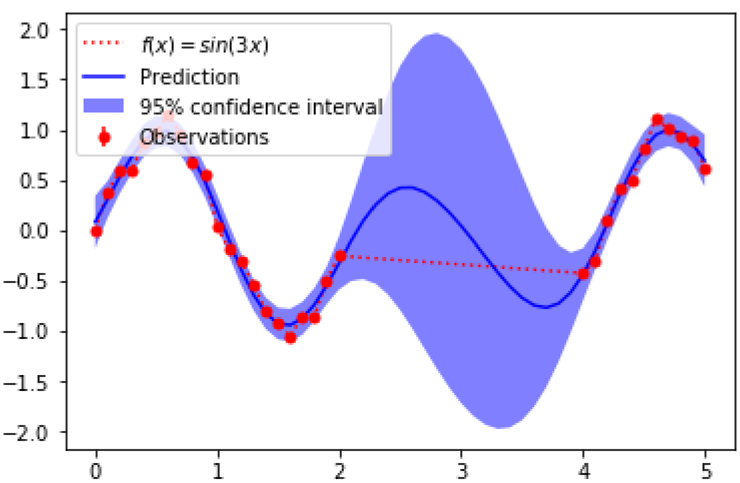
In [65]:
```
y_pred=[]
sigma=[]
for gp in (GP):
    pred1, sigma1 = gp.predict(x_test, return_std=True)
    print(gp.kernel_)
    y_pred.append(pred1)
    sigma.append(sigma1)
y_pred1=np.array(y_pred)
y_pred1=y_pred1.T
for i in range(0,y_pred1.shape[1]):
    print(f'MSE for kernel {i} is {mse(y_pred1[:,i],y_test)}')
```

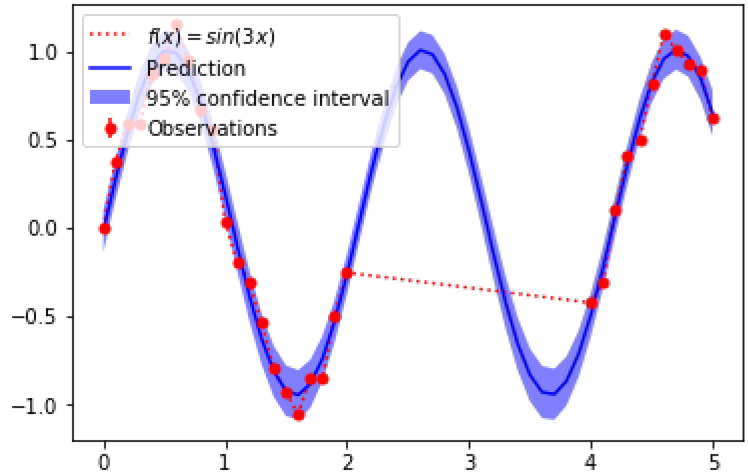
```
0.733**2 * RBF(length_scale=0.481) + 0.0316**2
0.0316**2 * RBF(length_scale=100) + RBF(length_scale=0.561)
0.0316**2 * RationalQuadratic(alpha=379, length_scale=100) + RBF(leng
th_scale=0.561)
3.11**2 * ExpSineSquared(length_scale=6.09, periodicity=2.09) + RBF(l
ength_scale=100)
MSE for kernel 0 is 0.07321080527806609
MSE for kernel 1 is 0.04596597190540239
MSE for kernel 2 is 0.045965971922065814
MSE for kernel 3 is 0.001168103027385105
```

In [66]:
```
for i in range(0,len(y_pred)):
    plt.plot(x_train, y_train, 'r:', label=r'$f(x) = sin(3x)$')
    plt.errorbar(x_train.ravel(), y_train, 0, fmt='r.', markersize=10
, label='Observations')
    plt.plot(x_test, y_pred[i], 'b-', label='Prediction')
    plt.fill(np.concatenate([x_test, x_test[::-1]]),
                 np.concatenate([y_pred[i] - 1.95 * sigma[i],
                                (y_pred[i] + 1.95 * sigma[i])[::-1
]]),
                 alpha=.5, fc='b', ec='None', label='95% confidence i
nterval')
    plt.legend(loc='upper left')
    plt.show()
```

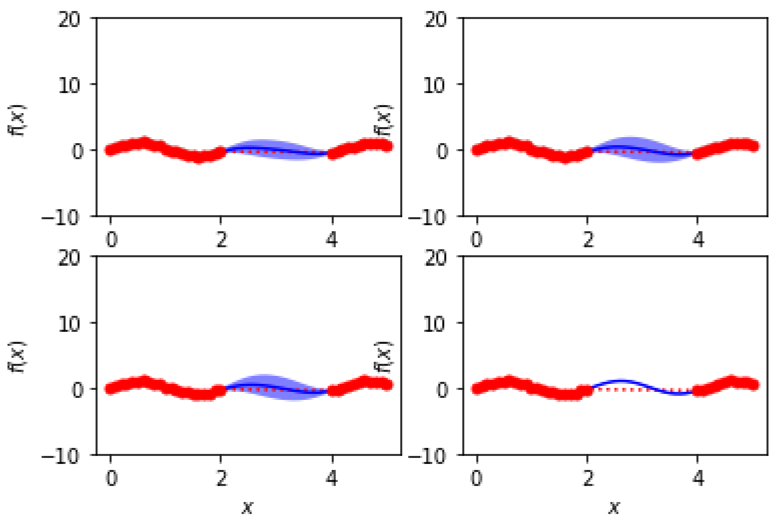

$f(x) = sin(3x)$
Prediction
95% confidence interval
Observations


$f(x) = sin(3x)$
Prediction
95% confidence interval
Observations


$f(x) = sin(3x)$
Prediction
95% confidence interval
Observations



In [67]:
```
f, axs = plt.subplots(2,2)
for ndx, gp, ax in zip([1,2,3,4], GP, [[0,0],[0,1],[1,0],[1,1]]):
    y_pred, sigma = gp.predict(x_test, return_std=True)
    axs[ax[0],ax[1]].plot(x_train, y_train, 'r:', label=r'$f(x) = sin
(3x)$')
    axs[ax[0],ax[1]].errorbar(x_train.ravel(), y_train,0, fmt='r.', m
arkersize=10, label='Observations')
    axs[ax[0],ax[1]].plot(x_test, y_pred, 'b-', label='Prediction')
    axs[ax[0],ax[1]].fill(np.concatenate([x_test, x_test[::-1]]),
             np.concatenate([y_pred - 1.95 * sigma,
                            (y_pred + 1.95 * sigma)[::-1]]),
             alpha=.5, fc='b', ec='None', label='95% confidence inter
val')
    axs[ax[0],ax[1]].set_xlabel('$x$')
    axs[ax[0],ax[1]].set_ylabel('$f(x)$')
    axs[ax[0],ax[1]].set_ylim(-10, 20)
    #axs[ax[0],ax[1]].legend(loc='upper left')
```



In [ ]: