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A Computationally Efficient Solution to the Simultaneous Localisation and Map Building (SLAM) Problem

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Abstract

The theoretical basis of the solution to the simultaneous localisation and map building (SLAM) problem where an autonomous vehicle starts in an unknown location in an unknown environment and then incrementally build a map of landmarks present in this environment while simultaneously using this map to compute absolute vehicle location is now well understood [3]. Although a number of SLAM implementations have appeared in the recent literature [2, 1], the need to maintain the knowledge of the relative relationships between all the landmark location estimates contained in the map makes SLAM computationally intractable in implementations containing more than few tens of landmarks. In this paper, the theoretical basis and a practical implementation of a computationally efficient solution to SLAM is presented. The paper shows that it is indeed possible to remove a large percentage of the landmarks from the map without making the map building process statistically inconsistent. Furthermore, it is shown that the efficiency of the SLAM can be maintained by judicious selection of landmarks, to be preserved in the map, based on their information content.

1 Introduction

In the past year, there has been rapid and substantial progress on the estimation-theoretic solution to the simultaneous localisation and map building (SLAM) problem. A recent session at the International Symposium on Experimental Robotics (ISER) was devoted to describing progress by a number of research groups working on the SLAM problem. These presentations described theoretical proofs of convergence of the SLAM problem [3], experimental verification of adapting robot behaviour to improve the accuracy of SLAM [4], and two different implementations of SLAM algorithms on outdoor and indoor vehicles [3, 1]. Together, these papers have set the foundations for a comprehensive and pervasive solution to the combined localisation and map building problem. The challenge now facing researchers in this area is to deploy a substantial implementation of a SLAM system in a large-scale unstructured environment: The ability to deploy a vehicle in an unknown environment and have it construct a map of this environment while simultaneously using this map to compute it's location would truely make such a vehicle autonomous.

Although the theoretical basis of SLAM is now well understood, a number of critical theoretical and practical issues need to be resolved before SLAM can be demonstrated in large unstructured environments. These include issues of computational efficiency, map management methods, local and global convergence properties of the map estimator, data association and sensor management. This paper is concerned with the computational efficiency of the SLAM process.

The existence of a non-divergent estimation theoretic solution to the SLAM problem and the general structure of SLAM navigation algorithms is described in [3]. It was shown that the uncertainty in the estimates of relative landmark locations reduces monotonically, that these uncertainties converge to zero, and that the uncertainty in vehicle and absolute map locations achieves a lower bound. It was also shown that it is the cross-correlations in the map covariance matrix which maintain knowledge of the relative relationships between landmark location estimates and which in turn underpin the exhibited convergence properties. Thus propagation of the full map covariance matrix was shown to be essential to the solution of the SLAM problem.

However, the use of the full map covariance matrix at each step in the map building problem causes substantial computational problems. As the number of landmarks N increases, the computation required at each step increases as N^3, and required map storage increases as N^2. As the range over which it is desired to operate a SLAM algorithm increases (and thus the number of landmarks increases), it becomes essential to develop a computationally tractable version of the SLAM algorithm which maintains the properties of being consistent and non-divergent.

The key contribution of this paper is a map management strategy that results in a computationally efficient solution to SLAM. Firstly, it shows that any landmark and associated elements of the map covariance matrix can be deleted during the SLAM process without compromising the statistical consistency of the underlying Kalman filter. Secondly, it defines the information content of a landmark and devises a strategy to select landmarks for deletion from the map, while minimising the loss of information during this process. Finally, it demonstrates and evaluates the implementation of the proposed algorithm in an indoor environment using a scanning laser range finder.

Section 2 of this paper summarises the mathematical structure of the estimation-theoretic SLAM problem as defined in [3]. Section 3 then establishes the theoretical basis of the map management strategy. Section 4 provides a practical demonstration of an implementation of the proposed algorithm. It is shown that the proposed strategy has a minimal effect on the convergence properties of the SLAM algorithm whilst substantially improving its computational efficiency. Discussion and conclusions are provided in Section 5.

2 The Estimation-Theoretic solution to the SLAM Problem

This section summarises the mathematical framework employed in the study of the SLAM problem in [3]. This framework is provided here for completeness and to facilitate the discussion in the later sections. An interested reader is referred to [3] for a more comprehensive description.

The setting for the SLAM problem is that of a vehicle with a known kinematic model, starting at an unknown location, moving through an environment containing a population of features or landmarks. The terms feature and landmark will be used synonymously. The vehicle is equipped with a sensor that can take measurements of the relative location between any individual landmark and the vehicle itself as shown in Figure 1. The absolute locations of the landmarks are not available.

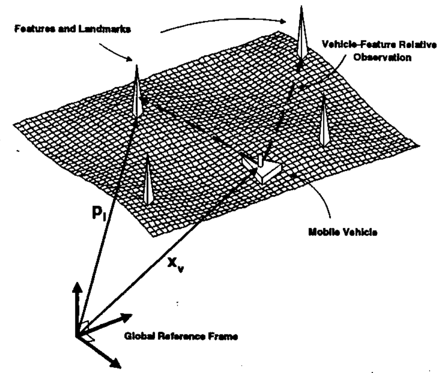

Figure 1: The Structure of the SLAM problem. A vehicle takes relative observations of environment landmarks. The absolute location of landmarks and vehicle are unknown.

The state of the system of interest consists of the position and orientation of the vehicle together with the position of all landmarks. The state of the vehicle at a time step k is denoted $\mathbf{x}_v(k)$. A linear (synchronous) discrete-time model of the evolution of the vehicle and the observations of landmarks is adopted. Although vehicle motion and the observation of landmarks is almost always non-linear and asynchronous in any real navigation problem, the use of linear synchronous models does not affect the validity of the discussion in Section 3 other than to require the same linearisation assumptions as those normally employed in the development of an extended Kalman filter.

The motion of the vehicle through the environment is modeled by a conventional linear discrete-time state transition equation or process model of the form

$$\mathbf{x}_v(k+1) = \mathbf{F}_v(k)\mathbf{x}_v(k) + \mathbf{u}_v(k+1) + \mathbf{v}_v(k+1), \tag{1}$$

where $\mathbf{F}_v(k)$ is the state transition matrix, $\mathbf{u}_v(k)$ a vector of control inputs, and $\mathbf{v}_v(k)$ a vector of temporally uncorrelated process noise errors with zero mean and covariance $\mathbf{Q}_v(k)$. The location of the i^{th} landmark is denoted $\mathbf{p}_i$. The "state transition equation" for the i^{th} landmark is

$$\mathbf{p}_i(k+1) = \mathbf{p}_i(k) = \mathbf{p}_i, \tag{2}$$

since landmarks are assumed stationary.

The augmented state transition model for the complete system may now be written as

$$\begin{bmatrix} \mathbf{x}_v(k+1) \\ \mathbf{p}_1 \\ \vdots \\ \mathbf{p}_N \end{bmatrix} = \begin{bmatrix} \mathbf{F}_v(k) & 0 & \dots & 0 \\ 0 & \mathbf{I}_{p_1} & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \mathbf{I}_{p_N} \end{bmatrix} \begin{bmatrix} \mathbf{x}_v(k) \\ \mathbf{p}_1 \\ \vdots \\ \mathbf{p}_N \end{bmatrix} + \begin{bmatrix} \mathbf{u}_v(k+1) \\ \mathbf{0}_{p_1} \\ \vdots \\ \mathbf{0}_{p_N} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_v(k+1) \\ \mathbf{0}_{p_1} \\ \vdots \\ \mathbf{0}_{p_N} \end{bmatrix} \quad (3)$$

$$\mathbf{x}(k+1) = \mathbf{F}(k)\mathbf{x}(k) + \mathbf{u}(k+1) + \mathbf{v}(k+1) \quad (4)$$

where N is the number of landmarks, $\mathbf{I}_{p_i}$ is the $\dim(p_i) \times \dim(p_i)$ identity matrix and $\mathbf{0}_{p_i}$ is the $\dim(p_i) \times \dim(p_i)$ null matrix.

2.1 The Observation Model

The vehicle is equipped with a sensor that can obtain observations of the relative location of landmarks with respect to the vehicle. The observation model for the i^{th} landmark is written in the form

$$\begin{aligned} \mathbf{z}_i(k) &= \mathbf{H}_i\mathbf{x}(k) + \mathbf{w}_i(k) && (5) \\ &= \mathbf{H}_{p_i}\mathbf{p} - \mathbf{H}_v\mathbf{x}_v(k) + \mathbf{w}_i(k) && (6) \\ & && (7) \end{aligned}$$

where $\mathbf{w}_i(k)$ is a vector of temporally uncorrelated observation errors with zero mean and variance $\mathbf{R}_i(k)$. The structure of Equation 5 reflects the fact that the observations are "relative" between the vehicle and the landmark, often in the form of relative location, or relative range and bearing (see Section 4).

2.2 The Estimation Process

In the estimation-theoretic formulation of the SLAM problem, the Kalman filter is used to provide estimates of vehicle and landmark location. The Kalman filter recursively computes estimates for a state $\mathbf{x}(k)$ which is evolving according to the process model and which is being observed according to the observation model. Due to space limitations, the equations used in the Kalman filter are not provided here.

3 Map management by deleting landmarks

This section examines in detail, the evolution of the state covariance matrix $\mathbf{P}(k)$ that results from the special structure of the matrix $\mathbf{H}_i(k)$ give in Equation 5. It is shown that when a landmark is no longer visible from the current robot location, it can be deleted from the map without affecting the statistical consistency of the underlying estimation process. It is also shown that once deleted, all information accumulated so far about the landmark need to be discarded and that the landmark need to be re-initialised when it comes back into view. A process for selecting the landmarks for removal is proposed.

3.1 Evolution of the state covariance matrix

The state covariance matrix at any instant k may be written as

$$\mathbf{P(k/k)} = \sum_{j=v,1,2,..n} \sum_{l=v,1,2,..,n} \mathbf{P(k/k)}_{jl} \quad (8)$$

where $\mathbf{P}_{vv}$ is the error covariance matrix associated with the vehicle state estimate, $\mathbf{P}_{jl}, j = 1..n, l = 1..n$ are the covariances associated with the landmark state estimates, and $\mathbf{P}_{vl}, l = 1..n$ are the cross-covariances between vehicle and landmark states. Dropping the index k for clarity and using Equation 5, the innovation covariance after i^{th} landmark is observed, is given by

$$\mathbf{S}_i = \mathbf{H}_v\mathbf{P}_{vv}\mathbf{H}_v^T - \mathbf{H}_{pi}\mathbf{P}_{vi}\mathbf{H}_v^T - \mathbf{H}_v\mathbf{P}_{vi}\mathbf{H}_{pi}^T + \mathbf{H}_{pi}\mathbf{P}_{vi}\mathbf{H}_{pi}^T$$

Clearly $\mathbf{S}_i$ is a function of the covariances and cross-covariance of the vehicle and landmark i, and is independent of all other landmark covariances and cross-covariances.

During the update stage that follows the observation of feature i, the Kalman gains and the change to the state covariance matrix can be written as

$$\mathbf{W}_{ij} = \left[-\mathbf{P}_{vj}\mathbf{H}_v^T + \mathbf{P}_{ij}\mathbf{H}_{pi}^T\right]\mathbf{S}_i^{-1}, j = v, 1, 2, ..., n$$

$$\delta\mathbf{P}_{jl} = \left[-\mathbf{P}_{vj}\mathbf{H}_v^T + \mathbf{P}_{ji}\mathbf{H}_{pi}^T\right]\mathbf{S}_i^{-1}\left[-\mathbf{H}_v\mathbf{P}_{vl} + \mathbf{H}_{pi}\mathbf{P}_{li}\right]$$

Effect of the observation of a given landmark i on the evolution of the vehicle and landmarks states and the state covariance matrix can now be summerised as follows

1. The Kalman gains associated with the vehicle state and all the landmark states are non-zero. Therefore the estimate of all the landmark states are updated even when many of these landmarks are not visible from the current robot location.

2. All the elements of the state covariance matrix need to be modified after each observation. It is not sufficient to update the elements of the state covariance matix corresponding to the vehicle and the landmark currently being observed. This is the main reason for the computational complexity of the SLAM process.

3. The Kalman gain and the changes to the elements of the state covariance matrix associated with the j^{th} landmark are a function of the covariances and cross-covariances associated with the vehicle state and the landmark states i and j. These updates are independent of covariances and cross-covariances of all other landmark states.

4. Updates of the vehicle and landmark states and the associated covariance matrix after the observation of a landmark i are unaffected by the removal of the state corresponding to any landmark j and the corresponding rows and columns of the state covariance matrix. Therefore any landmark that is currently not being observed can be removed from the map without affecting the statistical consistancy of the map building process.

5. When a removed landmark is observed again, the vehicle location and the states of other landmarks can not be updated using the state and the associated covariances of the landmark at the instant it was removed as these are no longer valid as updates mentioned in steps 1 and 2 are not performed after deletion. The landmark need to be reinitialised.

3.2 Procedure for selecting landmarks for removal

The prime objective of the SLAM process is to maintain a good estimate of the vehicle location by observing and estimating locations of landmarks. The information available at any instant for localisation depends on the accuracy of the sensor and the accuracy of the location estimates of the landmarks visible from the vehicle. The geometrical distribution of the these landmarks are also of importance. For example, when the number of landmarks to be maintained in the map is to be restricted, it is more advantageous to keep landmarks that are physically far apart than those that are close to each other. Also, the landmark location estimates that are less correlated provide more information. In the limiting case, when two landmark location estimates are fully correlated, there is no advantage to be gained by maintaining both these landmarks in the map except for geometric considerations. Most importantly, the computational effort involved with selecting the landmarks to be removed need to be efficient so as not to offset the computational efficiency gained in the SLAM process by deleting these landmarks.

The following two step process is therefore proposed for selecting the landmarks for removal from the map.

- Collect the set of landmarks S_L that changed from visible to not visible over a time interval during which the vehicle travelled a predetermined distance d_v. Select one landmark to be preserved in the map and remove all the remaining landmarks in S_L. This will result in an even distribution of landmarks over the area traversed by the vehicle. The landmark density of the final map obtained will be closely related to d_v. Suitable choice for d_v is a function of the range, accuracy and the speed of the sensor used, extent of process noise and the tradeoff between the localisation accuracy required and the computational efficiency. Such an incremental process of deletion, to a large extent, eliminates the need to consider effect of geometry in evaluating the landmarks for their information content.

- The next step is to select the landmark that has the maximum information content from the set of landmarks S_L. This landmark now serves to localise the vehicle when the vehicle revisits the region surrounding the landmark. The selected landmark should provide the best information to improve the location of the vehicle when the landmark is observed from any position within this region. Clearly, lower the uncertainty of the location estimate of the landmark, better the effect of observing this landmark on the estimate of the vehicle location. Therefore, it is proposed to use the reciprocal of the trace of the covariance matrix $\mathbf{P}_{jj}$ of the of the landmark location to define the information content of the landmark j under consideration.

Various other information measures such as the Shannon or Fisher information that correspond to the covariance matrix of the landmarks being evaluated, can also be used. However experiments indicated that the effect of the strategy used to evaluate the merit of the landmarks does not have a significant effect on the accuracy of the vehicle location estimates.

4 Experimental Results

In this section a practical implementation of the proposed simultaneous localisation and map building (SLAM) algorithm on an indoor robot is presented. The robot is equipped with a Laser range finder which provides a two-dimensional range scans. A feature detector [5] is used to extract location of landmarks with respect to the vehicle. This implementation serves to highlight the effectiveness of the proposed map management strategy.

Figure 2 shows the test vehicle; a three wheeled vehicle fitted with a laser range finder as the primary sensor used in the experiments. Encoders are fitted to the drive shafts and to the steering motor to provide a measure of the vehicle speed and vehicle heading.

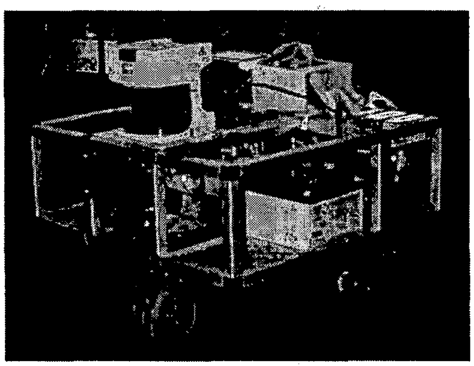

Figure 2: The test vehicle

The vehicle is driven manually. Range scans are logged together with encoder and steer information by an on-board computer system. Range scans are processed to obtain point landmarks correspond to foreground points and corners in the environment. A detailed description of the feature detector can be found in [5]. In the evaluation of the SLAM algorithm, this information is employed without any *a priori* knowledge of landmark location to deduce estimates for both vehicle position and landmark locations.

Figure 3 shows the map of the environment consisting of 148 landmarks and the path of the vehicle computed using the full SLAM algorithm that maintains the complete set of landmarks in the state vector. In this experiment the robot starts at the origin and is driven along a corridor. Total duration of the test run is about 800 seconds. Although the true path of the vehicle was not available, the map was verified by measuring the relative locations of some of the landmarks. Furthermore, the full SLAM algorithm has previously been verified on a number of different environments [3, 2].

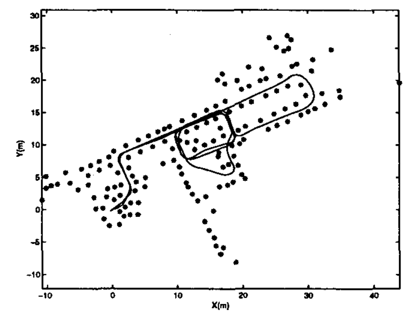

Figure 3: Vehicle path and the map obtained using the full SLAM algorithm

Figure 4 shows one instance of the feature map maintained, by the algorithm described in this paper. In this example, landmarks that change from visible to invisible were collected over a time period during which the vehicle traveled 5 m. Visibility range of the sensor was set to 15 m. The number of landmarks maintained in the map changes as the vehicle moves. On the average, this strategy results in a map that contains about 70 landmarks resulting in about an 8 fold reduction in the computational effort.

Figure 5 that compares the vehicle location estimates obtained using the full slam and the proposed algorithm shows that the differences are hardly noticeable. Furthermore, Figure 6 shows that the increase in the uncertainty of the vehicle location estimates due to the removal of landmarks is relatively small.

5 Conclusions

In this paper a map management strategy to increase the computational efficiency of the solution to SLAM is presented. It is shown that deleting landmarks from the map does not compromise the statisti-

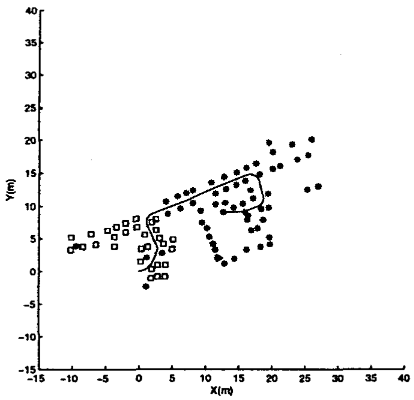

Figure 4: Vehicle path and the map obtained using the proposed computationally efficient SLAM algorithm after 200 seconds. The '*' show the landmarks currently active whereas 'o' show the landmarks that are inactive

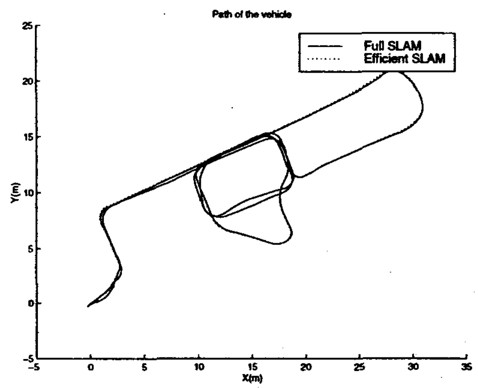

Figure 5: Vehicle path obtained using the full SLAM algorithm and the proposed computationally efficient SLAM algorithm

cal consistency of the SLAM algorithm. The information content of a landmark is quantified and a strategy to select landmarks to be removed is described. Experimental results show that removing suitably selected landmarks does not significantly increase the errors in the estimated vehicle location. However, the computational efficiency of the SLAM process is significantly reduced. Further challenges in this area remains a more rigorous analysis of the information content of landmarks, giving due consideration to the geometrical effects as well as the uncertainty of the landmark location estimate.

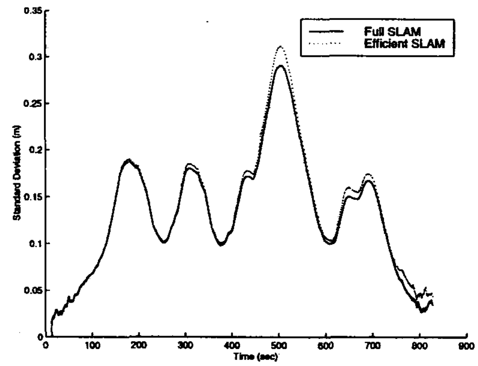

Figure 6: Standard deviation of the position estimated by the full SLAM algorithm and the proposed computationally efficient SLAM algorithm

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