Homogenous coordinate

Consider the effect of general 2 by 2 transformation applied to the origin

So for the origin, Origin is invariant under general 2 by 2 transformation.

This limitation is overcome by homogenous coordinates

It is necessary to be able to modify position of origin i.e. to transform every point in 2 dimension plane.

This can be accomplished by translating origin or any other point in 2 dimension plane

If $x' = ax + by + m$
$y' = bx + dy + n$

In homogenous coordinate representation we add third coordinate to a point .

Instead of representing by a pair of number (x , y) each point is represented by a triple (x , y , h).

We say that 2 sets of homogenous coordinates (x, y, h) and (x', y', h) represent the same point if and only if one is multiple of another i.e. (2,3,6) , (4,6,12) are same points represented by different coordinate triples.

In order to transform a point (x, y) into homogenous representation we choose a non zero number 'h' and form a vector [hx , hy , h] and h is called scale factor or homogenous coordinate parameter.

For point [2,3] in 2 dimensional space it's representation in homogenous coordinated will be

[2, 3, 1] for h = 1
[4, 6, 2] for h = 2
[-2,-3,-1] for h = -1

Homogeneous coordinates are widely used in computer graphics because they enable
Affine Transformation
Affine Transformation: a transformation that preserves collinearity (i.e. points lying on a line initially still lie on a line after transformation) and ratios of distances
It is a combination of single transformations such as translation, rotation or reflection on an axis

Hence the general transformation matrix is of the form

$$[T] = \begin{bmatrix} a & b & m \\ c & d & n \\ 0 & 0 & 1 \end{bmatrix}$$

hence for translation
$P' = T(t_x, t_y) \; . \; P$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

now every point in 2 dimension plane every origin (x = y = 0) can be transformed.
Similarly, rotation transformation equation about coordinate origin EW
$P' = R(\theta) \; . \; P$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x.\cos\theta - y.\sin\theta \\ x.\sin\theta + y.\cos\theta \\ 1 \end{bmatrix}$$

Scaling transformation relative to coordinate origin is
$P' = S(s_x, s_y) \; . \; P$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x \, . \, s_x \\ y \, . \, s_y \\ 1 \end{bmatrix}$$