Examples:

Q. Window port is given by (100, 100, 300, 300) and viewport is given by (50, 50, 150, 150). Convert the window port coordinate (200, 200) to the view port coordinate.

Solution:

Here,

$(xw_{min}, yw_{min}) = (100,100)$
$(xw_{max}, yw_{max}) = (300,300)$
$(xv_{min}, yv_{min}) = (50,50)$
$(xv_{max}, yv_{max}) = (150,150)$
$(xw, yw) = (200,200)$

Then we have

$$s_x = \frac{xv_{max} - xv_{min}}{xw_{max} - xw_{min}} = \frac{150-50}{300-100} = 0.5$$

$$s_y = \frac{yv_{max} - yv_{min}}{yw_{max} - yw_{min}} = \frac{150-50}{300-100} = 0.5$$

$$t_x = \frac{xw_{max} \cdot xv_{min} - xw_{min} \cdot xv_{max}}{xw_{max} - xw_{min}} = \frac{300 \times 50 - 100 \times 150}{300-100} = 0$$

$$t_y = \frac{yw_{max} \cdot yv_{min} - yw_{min} \cdot yv_{max}}{yw_{max} - yw_{min}} = \frac{300 \times 50 - 100 \times 150}{300-100} = 0$$

The equation for mapping window coordinate to view port coordinate is given by,

$$xv = s_x xw + t_x$$
$$yv = s_y yw + t_y$$

Hence,

$$xv = 0.5 \times 200 + 0 = 100$$
$$yv = 0.5 \times 200 + 0 = 100$$

The transformed viewport coordinate is (100, 100).

Search

Q. Find the normalization transformation matrix for window to viewport which uses the rectangle whose lower left corner is at (2, 2) and upper right corner is at (6, 10) as a window and the viewport that has lower left corner at (0, 0) and upper right corner at (1, 1).

Solution:

We have

$$s_x = \frac{xv_{max} - xv_{min}}{xw_{max} - xw_{min}} = \frac{1-0}{6-2} = 0.25$$

$$s_y = \frac{yv_{max} - yv_{min}}{yw_{max} - yw_{min}} = \frac{1-0}{10-2} = 0.125$$

$$t_x = \frac{xw_{max} \cdot xv_{min} - xw_{min} xv_{max}}{xw_{max} - xw_{min}} = \frac{6\times0-2\times1}{6-2} = -0.5$$

$$t_y = \frac{yw_{max} \cdot yv_{min} - yw_{min} \cdot yv_{max}}{yw_{max} - yw_{min}} = \frac{10\times0-2\times1}{10-2} = -0.25$$

The composite transformation matrix for transforming the window coordinate to viewport coordinate is given as

$$T = T_{(t_x,t_y)} S_{(s_x,s_y)}$$

$$= \begin{bmatrix} s_x & 0 & t_x \\ 0 & s_y & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.25 & 0 & -0.5 \\ 0 & 0.125 & -0.25 \\ 0 & 0 & 1 \end{bmatrix}$$

Q. A world coordinate & viewport have the following geometry:

Window (left, right, bottom, top) = (200, 600, 100, 400)

Viewport (left, bottom, width, height) = (0, 0, 800, 600)

The following vertices are drawn in the world:- P1: (356, 125), P2: (200, 354), P3: (230,

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

Q. A world coordinate & viewport have the following geometry:

Window (left, right, bottom, top) = (200, 600, 100, 400)

Viewport (left, bottom, width, height) = (0, 0, 800, 600)

The following vertices are drawn in the world:- P1: (356, 125), P2: (200, 354), P3: (230, 400), P4: (564, 200). What coordinate will each occupy in viewport.

Solution:

Here,

$(xw_{min}, yw_{min}) = (200,100)$

$(xw_{max}, yw_{max}) = (600,400)$

$(xv_{min}, yv_{min}) = (0,0)$

$(xv_{max}, yv_{max}) = (800,600)$

Then we have

$$S_x = \frac{xv_{max} - xv_{min}}{xw_{max} - xw_{min}} = \frac{800-0}{600-200} = 2$$

$$S_y = \frac{yv_{max} - yv_{min}}{yw_{max} - yw_{min}} = \frac{600-0}{400-100} = 2$$

$$t_x = \frac{xw_{max} \cdot xv_{min} - xw_{min} \cdot xv_{max}}{xw_{max} - xw_{min}} = \frac{600 \times 0 - 200 \times 800}{600-200} = -400$$

$$t_y = \frac{yw_{max} \cdot yv_{min} - yw_{min} \cdot yv_{max}}{yw_{max} - yw_{min}} = \frac{400 \times 0 - 100 \times 600}{400-100} = -200$$

The composite transformation matrix for transforming the window coordinate to viewport coordinate is given as

$$M = T_{(t_x,t_y)} S_{(s_x,s_y)}$$

$$= \begin{bmatrix} s_x & 0 & t_x \\ 0 & s_y & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -400 \\ 0 & 2 & -200 \\ 0 & 0 & 1 \end{bmatrix}$$

Now,

$$P1' = M.P1 = \begin{bmatrix} 2 & 0 & -400 \\ 0 & 2 & -200 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 356 \\ 125 \\ 1 \end{bmatrix} = \begin{bmatrix} 312 \\ 50 \\ 1 \end{bmatrix} = (312, 50)$$

$$P2' = M.P2 = \begin{bmatrix} 2 & 0 & -400 \\ 0 & 2 & -200 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 200 \\ 354 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 508 \\ 1 \end{bmatrix} = (0, 508)$$

$$P3' = M.P3 = \begin{bmatrix} 2 & 0 & -400 \\ 0 & 2 & -200 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 230 \\ 400 \\ 1 \end{bmatrix} = \begin{bmatrix} 60 \\ 600 \\ 1 \end{bmatrix} = (60, 600)$$

$$P4' = M.P4 = \begin{bmatrix} 2 & 0 & -400 \\ 0 & 2 & -200 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 564 \\ 200 \\ 1 \end{bmatrix} = \begin{bmatrix} 728 \\ 200 \\ 1 \end{bmatrix} = (728, 200)$$

❖ Clipping

The process of discarding those parts of a picture which are outside of a specified region or window is called **clipping**. The procedure using which we can identify whether the portions of the graphics object is within or outside a specified region or space is called **clipping algorithm.**

The region or space which is used to see the object is called window and the region on which the object is shown is called view port.

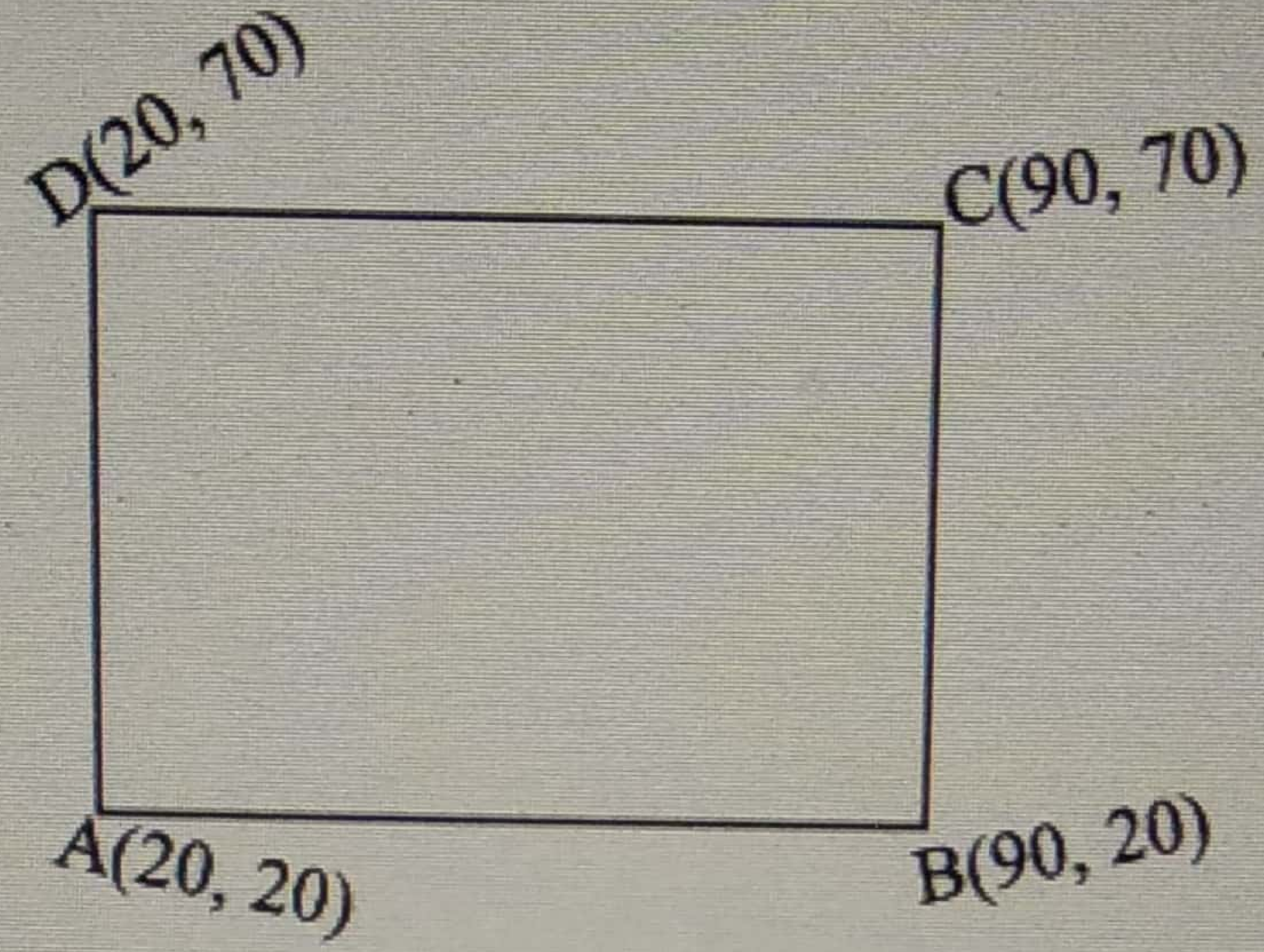
Clipping is necessary to remove those portions of the object which are not necessary for further operations. It excludes unwanted graphics from the screen. So, there are three cases:

1. The object may be completely outside the viewing area defined by the window port.
2. The object may be seen partially in the window port.

Examples:

Q. Consider a rectangle clipping window with A(20, 20), B(90, 20), C(90, 70) & D(20, 70). Clip the line P1P2 with P1(10, 30) & P2(80, 90) using Cohen-Sutherland line clipping algorithm.

Solution:



Here,

$xw_{min} = 20, xw_{max} = 90$

$yw_{min} = 20, yw_{max} = 70$

Region code for $P1(10, 30)$:

$x - xw_{min} = 10 - 20 = -10$	1	L
$xw_{max} - x = 90 - 10 = 80$	0	R
$y - yw_{min} = 30 - 20 = 10$	0	B
$yw_{max} - y = 70 - 30 = 40$	0	T

Region code for $P2(80, 90)$:

$x - xw_{min} = 80 - 20 = 60$ 0   L

$xw_{max} - x = 90 - 80 = 10$ 0   R

$y - yw_{min} = 90 - 20 = 70$ 0   B

$yw_{max} - y = 70 - 90 = -20$ 0   T

∴ Region code for P1= 1000

∴ Region code for P2= 0001

- Check if line P1P2 is completely inside or outside.

Take OR of 0001 & 1000

0001
1000
1001 → Which is not equal to 0000 i.e. line P1P1 is not completely inside.

Take AND

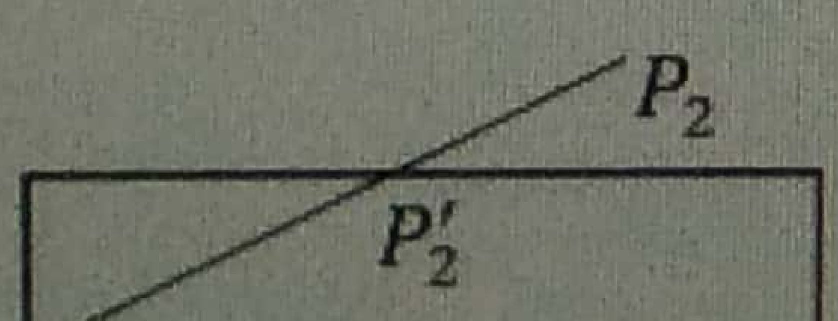
0001
1000
0000 → Which means P1P2 is not completely outside.

i.e. line P1P2 crosses the clipping window.

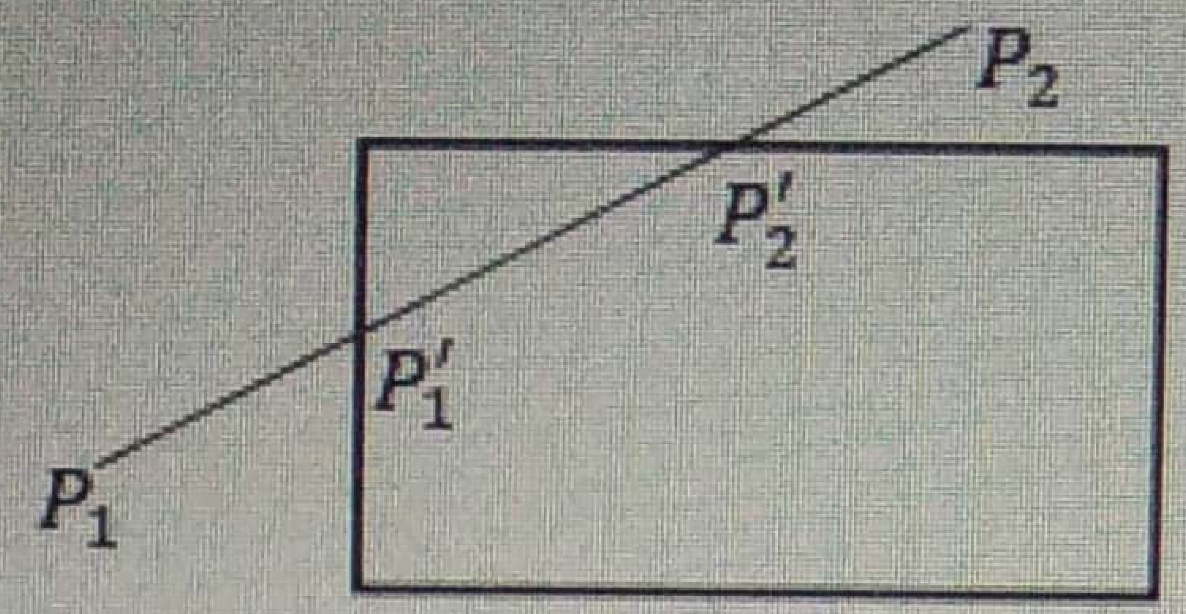
Now,

	T	B	R	L
P1:	0	0	0	1
P2:	1	0	0	0

The line crosses the top and left edge of the clipping window.



The line crosses the top and left edge of the clipping window.



Now find P_1' & P_2'.

$$\text{Slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{90-30}{80-10} = \frac{6}{7}$$

So for P_1'

$$x = 20$$

$$y = 30 + \frac{6}{7}(20 - 10) = 38.5$$

$\therefore P_1' = (20, 38.5)$

So for P_2'

$$y = 70$$

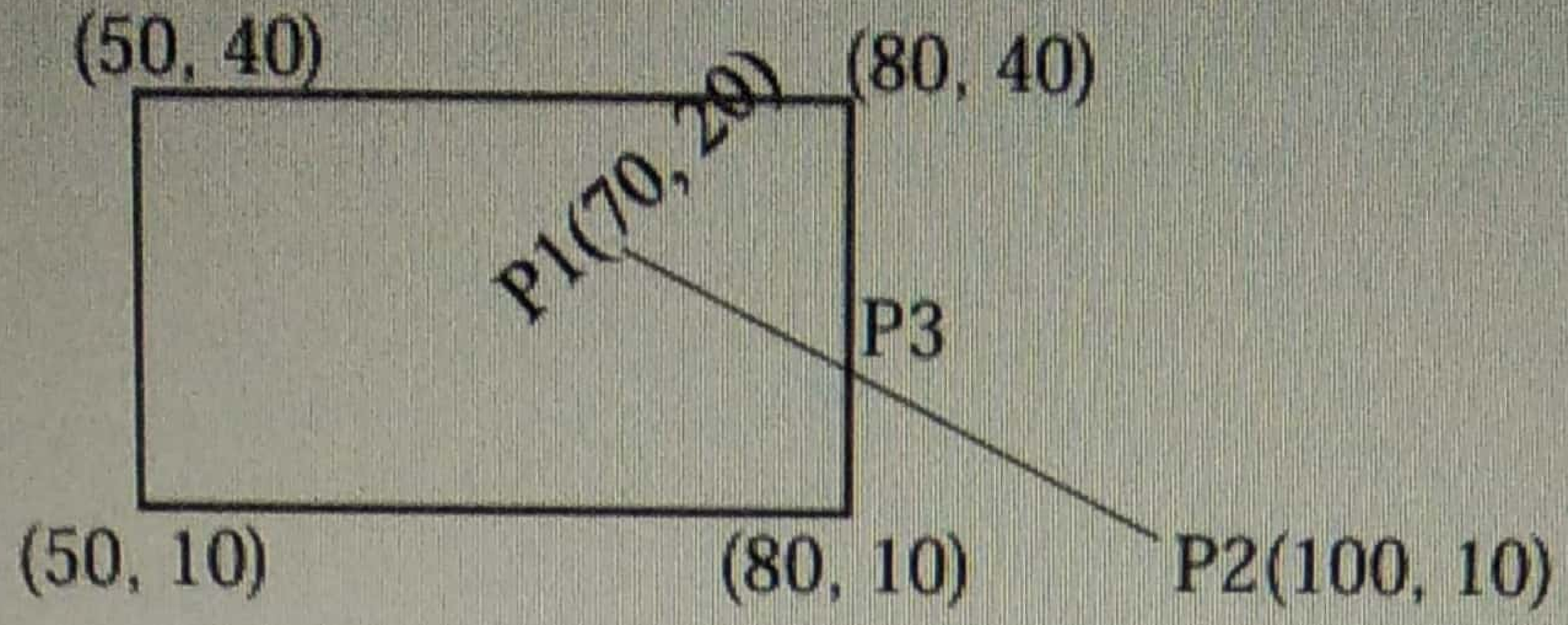
$$x = 10(90 - 60) \times \frac{7}{6} = 45$$

$\therefore P_2' = (45, 70)$

Thus the intersection point $P_1' = (20, 38.5)$ & $P_2' = (45, 70)$. So discarding the line segment that lie outside the boundary i.e. P_1P_1' & P_2P_2' , we get new line $P_1'P_2'$ with coordinate $P_1' = (20, 38.5)$ & $P_2' = (45, 70)$.

Q. Use the Cohen-Sutherland algorithm to clip the line P1(70, 20) and P2(100, 10) against a window lower left hand corner (50, 10) and upper right hand corner (80, 40).

Solution:



Assign 4 bit binary code to the two end point

P1=0000

P2=0010

Finding bitwise OR:

P1|P2=0000|0010=0010

Since P1|P2 !=0000, hence the two point doesn't lie completely inside the window.

Finding bitwise AND:

P1&P2=0000 & 0010=0000

Since P1&P2=0000, hence line is partially visible.

Now, finding the intersection of P1 and P2 with the boundary of window.

$p1(x_1, y_1) = (70, 20)$

$p2(x_2, y_2) = (100, 10)$

Slope $m = (10 - 20)/(100 - 70) = -1/3$

We have to find the intersection with right edge of window.

Here,

$x = 80$

Since P1|P2 !=0000, hence the two point doesn't lie completely inside the window.

Finding bitwise AND:

P1&P2=0000 & 0010=0000

Since P1&P2=0000, hence line is partially visible.

Now, finding the intersection of P1 and P2 with the boundary of window.

$p1(x_1, y_1) = (70, 20)$

$p2(x_2, y_2) = (100, 10)$

Slope $m = (10 - 20)/(100 - 70) = -1/3$

We have to find the intersection with right edge of window.

Here,

$x = 80$

$y = y_2 + m(x - x_2) = 10 + (-1/3)(80 - 100) = 10 + 6.67 = 16.67$

Thus the intersection point P3 = (80, 16.67). So, discarding the line segment that lie outside the boundary i.e. P3P2, we get new line P1P3 with coordinate P1(70, 20) and P3(80, 16.67).

Liang-Barsky Line Clipping Algorithm

#Q. Prove that two successive translations are additive.

Proof:
If two successive translation vector (t_{x1}, t_{y1}) & (t_{x2}, t_{y2}) is applied to coordinate position P, the final transformed location P' is calculated with the following composite transformation as,

$$T = T_{(t_{x2},t_{y2})} \cdot T_{(t_{x1},t_{y1})}$$

$$= \begin{bmatrix} 1 & 0 & t_{x2} \\ 0 & 1 & t_{y2} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & t_{x1} \\ 0 & 1 & t_{y1} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_{x1}+t_{x2} \\ 0 & 1 & t_{y1}+t_{y2} \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, $T_{(t_{x2},t_{y2})} \cdot T_{(t_{x1},t_{y1})} = T_{(t_{x1}+t_{x2},t_{y1}+t_{y2})}$ which demonstrates that two successive translations are additive.

#Q. Prove that two successive rotation are additive.

Proof:
Let P be the point anticlockwise rotated by angle θ_1 to point P' and again let P' be rotated by angle θ_2 to point P'', then the combined transformation can be calculated with the following composite matrix as:

$$T = R(\theta_2).R(\theta_1)$$

$$= \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_2 * \cos\theta_1 - \sin\theta_2 * \sin\theta_1 & -\cos\theta_2 * \sin\theta_1 - \sin\theta_2 * \cos\theta_1 & 0 \\ \sin\theta_2 * \cos\theta_1 + \cos\theta_2 * \sin\theta_1 & -\sin\theta_2 * \sin\theta_1 + \cos\theta_2 * \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

i.e. $R(\theta_2).R(\theta_1) = R(\theta_1+\theta_2)$ which demonstrates that two successive rotations are additive.

#Q. Prove that two successive scaling are multiplicative.

$$= \begin{bmatrix} sin\theta_2 & cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} . = \begin{bmatrix} sin\theta_1 & cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} cos\theta_2 * cos\theta_1 - sin\theta_2 * sin\theta_1 & -cos\theta_2 * sin\theta_1 - sin\theta_2 * cos\theta_1 & 0 \\ sin\theta_2 * cos\theta_1 + cos\theta_2 * sin\theta_1 & -sin\theta_2 * sin\theta_1 + cos\theta_2 * cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} cos(\theta_1 + \theta_2) & -sin(\theta_1 + \theta_2) & 0 \\ sin(\theta_1 + \theta_2) & cos(\theta_1 + \theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

i.e. $R(\theta_2).R(\theta_1) = R(\theta_1 + \theta_2)$ which demonstrates that two successive rotations are additive.

#Q. Prove that two successive scaling are multiplicative.

Proof:
Let point P is first scaled with scaling factors s_{x1}, s_{y1} to P' and again let P' be scaled by scaling factors s_{x2}, s_{y2} to point P", then the combined transformation can be calculated with the following composite matrix

$$T = S_{(S_{x2}, S_{y2})} . S_{(S_{x1}, S_{y1})}$$

$$= \begin{bmatrix} s_{x2} & 0 & 0 \\ 0 & s_{y2} & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} s_{x1} & 0 & 0 \\ 0 & s_{y1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} s_{x1}s_{x2} & 0 & 0 \\ 0 & s_{y1}s_{y2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

i.e. $S_{(S_{x2}, S_{y2})} . S_{(S_{x1}, S_{y1})} = S_{(S_{x1}S_{x2}, S_{y1}S_{y2})}$ which demonstrates that two successive scaling are multiplicative.

❖ General 2D pivot rotation

- Suppose the pivot point is located at (x_r, y_r).
- To rotate about arbitrary point, we have to perform the following transformation:
  a) Translate the object so that pivot point position is moved to coordinate origin.
  b) Rotate the object about coordinate origin.
  c) Translate the object so that pivot point is returned to original position.

Matrix representation:

$$\begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}$$

Q. Find the scaled triangle with vertices A(0, 0), B(1, 1) & C(5, 2) after it has been magnified twice its size.

Solution:

Here, $s_x = 2$ & $s_y = 2$

Now,

$A' = S.A$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = (0,0)$$

$B' = S.B$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = (2,2)$$

$C' = S.C$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 4 \\ 1 \end{bmatrix} = (10,4)$$

Hence the final coordinate points are $A'(0,0)$, $B'(2,2)$, $C'(10,4)$.

Q. Rotate a triangle A(0, 0), B(2, 2), C(4, 2) about the origin by the angle of 45 degree.

Solution:

The given triangle ABC can be represented by a matrix formed from homogenous coordinates of vertices.

$$\begin{bmatrix} 0 & 2 & 4 \\ 0 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

Also, we have

$$R_{45^0} = \begin{bmatrix} cos45^0 & -sin45^0 & 0 \\ sin45^0 & cos45^0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So the coordinates of the rotated triangle ABC are

$$R_{45^0}[ABC] = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 4 \\ 0 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \sqrt{2} \\ 0 & 2\sqrt{2} & 3\sqrt{2} \\ 1 & 1 & 1 \end{bmatrix}$$

Hence the final coordinate points are $A'(0,0)$, $B'(0,2\sqrt{2})$, $C'(\sqrt{2}, 3\sqrt{2})$.

Q. *Rotate a triangle (5, 5), (7, 3), (3, 3) about fixed point (5, 4) in counter clockwise by 90 degree.*

Solution:

The required steps are:

1. Translate the fixed point to origin.
2. Rotate about the origin by 90 degree.
3. Reverse the translation as performed earlier.

Thus, the composite matrix is given by

$M = T_{(x_f, y_f)} R_\theta T_{(-x_f, -y_f)}$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos90 & -sin90 & 0 \\ sin90 & cos90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 4 \\ 1 & 0 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 9 \\ 1 & 0 & -1 \end{bmatrix}$$

The required steps are:

1. Translate the fixed point to origin.
2. Rotate about the origin by 90 degree.
3. Reverse the translation as performed earlier.

Thus, the composite matrix is given by

$M = T_{(x_f,y_f)} R_\theta T_{(-x_f,-y_f)}$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos90 & -sin90 & 0 \\ sin90 & cos90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 4 \\ 1 & 0 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 9 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence the required coordinate can be calculated as:

$P' = M * P$

$$= \begin{bmatrix} 0 & -1 & 9 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 7 & 3 \\ 5 & 3 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 & 6 \\ 4 & 6 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

Hence, the new required coordinates are (4, 4), (6, 6), (6, 2).

Q. Rotate a triangle A(7, 15), B(5, 8) & C(10, 10) by 45 degree clockwise about origin and scale it by (2, 3) about origin.

Solution:

The steps required are:

1. Rotate by 45 clockwise
2. Scale by $s_x = 2$ & $s_y = 3$.

Thus the composite matrix is given by;

$M = S(2,3).R_{45}$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-45) & -\sin(-45) & 0 \\ \sin(-45) & \cos(-45) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{\sqrt{2}} & \frac{2}{\sqrt{2}} & 0 \\ -\frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The transformation points are

$A' = M.A$

$$= \begin{bmatrix} \frac{2}{\sqrt{2}} & \frac{2}{\sqrt{2}} & 0 \\ -\frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 15 \\ 1 \end{bmatrix} =$$

$B' = M.B$

$$= \begin{bmatrix} \frac{2}{\sqrt{2}} & \frac{2}{\sqrt{2}} & 0 \\ -\frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \\ 1 \end{bmatrix} =$$

$C' = M.C$

$$= \begin{bmatrix} \frac{2}{\sqrt{2}} & \frac{2}{\sqrt{2}} & 0 \\ -\frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 1 \end{bmatrix} =$$

Q. A square with vertices A(0, 0), B(2, 0), C(2, 2) & D(0, 2) is scaled 2 units in x & y direction about the fixed point (1, 1). Find the coordinates of the vertices of new square.

Solution:

Here,

$s_x = 2$ & $s_y = 2$

$x_f = 1$ & $y_f = 1$

Composite matrix is,

$M = T_{(x_f, y_f)} . S . T_{(-x_f, -y_f)}$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -2 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, the transformation points are

$A' = M.A$

$$= \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = (-1, -1)$$

$B' = M.B$

$$= \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} = (3, -1)$$

$C' = M.C$

$$= \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix} = (3, 3)$$

$D' = M.D$

$$= \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} = (-1, 3)$$

Q. A triangle having vertices A(3, 3), B(8, 5) & C(5, 8) is first translated by 2 units about fixed point (5, 6) & finally rotated 90 degree anticlockwise about pivot point (2, 5). Find the final position of triangle.

Solution:

$$T = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 7 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

Composite matrix,

$M = R.S.T$

$$= \begin{bmatrix} 0 & -1 & 7 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 & 13 \\ 2 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, the transformation points are;

$A' = M.A$

$$= \begin{bmatrix} 0 & -2 & 13 \\ 2 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 1 \end{bmatrix} = (7, 8)$$

$B' = M.B$

$$= \begin{bmatrix} 0 & -2 & 13 \\ 2 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 18 \\ 1 \end{bmatrix} = (3, 18)$$

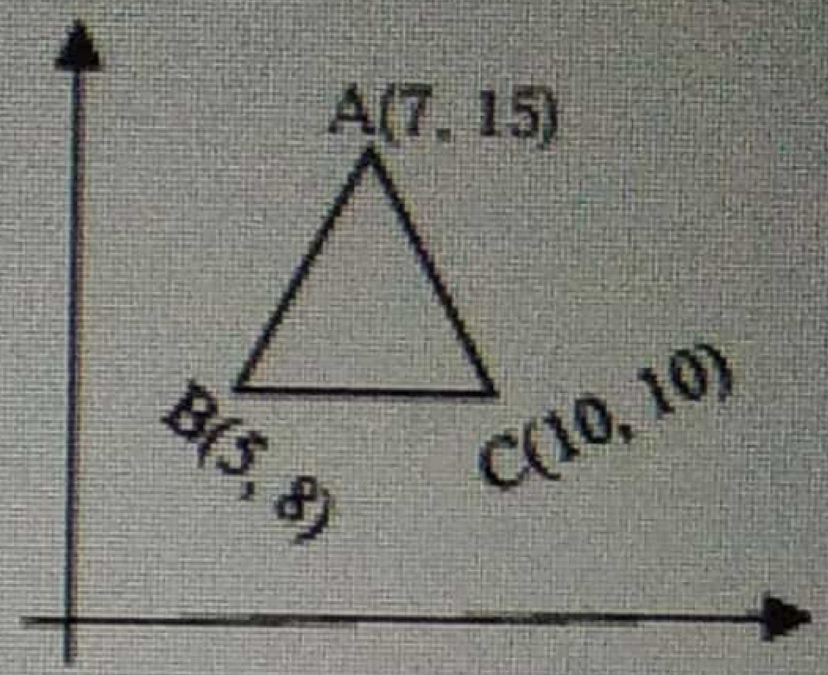
$C' = M.C$

$$= \begin{bmatrix} 0 & -2 & 13 \\ 2 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 12 \\ 1 \end{bmatrix} = (-3, 12)$$

$$= \begin{bmatrix} 2 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \end{bmatrix} = \begin{bmatrix} 12 \\ 1 \end{bmatrix} = (-3, 12)$$

Q. Rotate the ΔABC by $90°$ anti-clock wise about (5, 8) and scale it by (2, 2) about (10, 10).

Solution:



Step 1:
T(-5, -8)

Step 2:
R(90°)

Step 3:
T(5, 8)

Step 4:
T(-10, -10)

Step 5:
S(2, 2)

Step 6:
T(10, 10)

The composite matrix is given by

M= T(10,10).S(2,2).T(-10,-10).T(5,8).R(90) .T(-5,-8)

Complete urself.

❖ Reflection

Providing a mirror image about an axis of an object is called reflection.

Reflection about x-axis (y=0)

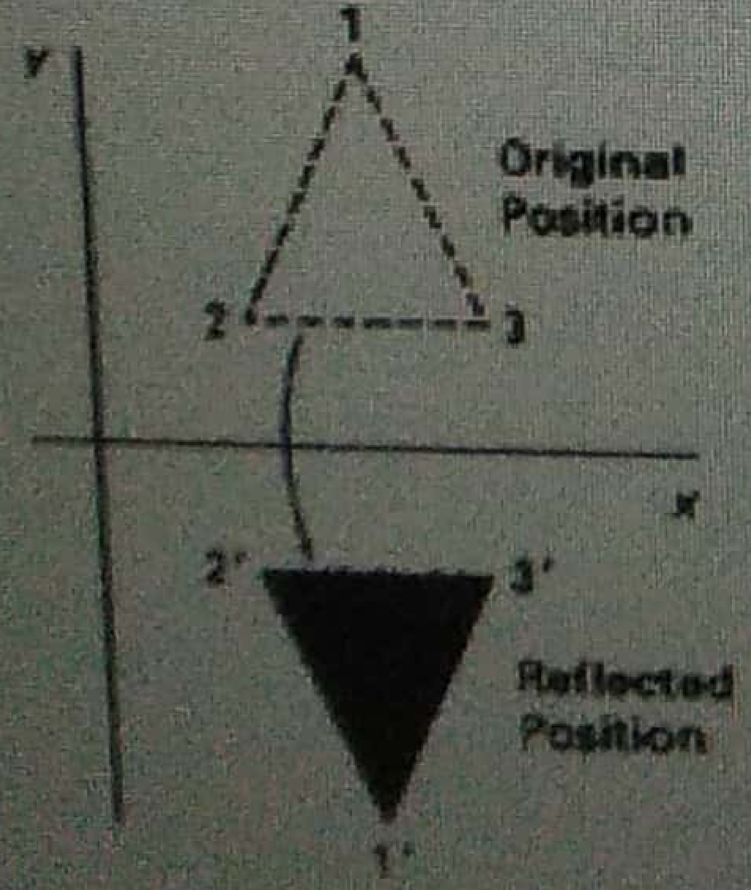
The reflection of a point P(x, y) on x-axis, changes the y-coordinate sign i.e. P(x, y) changes to P'(x, -y).

In matrix form,

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$P' = R_{fx}.P$

R_{fx}= Reflection matrix about x-axis.



Reflection about y=mx+c

Perform the following transformation:

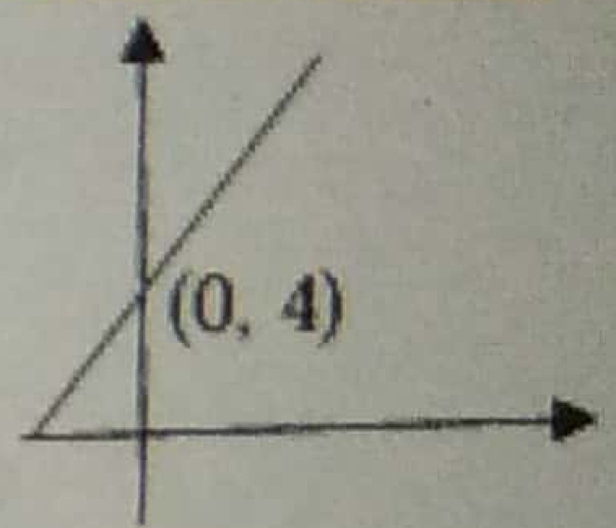
a) Translate the line so that it passes through origin.
b) Rotate the line so that it coincides with any coordinate axis.
c) Reflect object about that axis.
d) Perform reverse rotation.
e) Perform reverse translation so that line is placed to its original position.

Q. What is the basic purpose of composite transformation?

→ The basic purpose of composing transformation is to gain efficiency by applying a single composed transformation to a point, rather than applying a series of transformation, one after another.

Q. A triangle having vertices A(2, 3), B(6, 3) & C(4,8) is reflected about y=3x+4. Find the final position of triangle.

Solution:



Here,

y=3x+4

m=3, c=4

$(t_x, t_y) = (0, c) = (0, 4)$

$\theta = tan^{-1}(m) = tan^{-1}(3) = 71.56$

Now,

$$T(-t_x, -t_y) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(-\theta) = \begin{bmatrix} \cos(-71.56) & -\sin(-71.56) & 0 \\ \sin(-71.56) & \cos(-71.56) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.32 & 0.95 & 0 \\ -0.95 & 0.32 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{fx} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[cos(71.56) −sin(71.56) 0] [0.32 −0.95 0]

$\theta = tan^{-1}(m) = tan^{-1}(3) = 71.56$

Now,

$$T(-t_x, -t_y) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(-\theta) = \begin{bmatrix} \cos(-71.56) & -\sin(-71.56) & 0 \\ \sin(-71.56) & \cos(-71.56) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.32 & 0.95 & 0 \\ -0.95 & 0.32 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{fx} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(\theta) = \begin{bmatrix} \cos(71.56) & -\sin(71.56) & 0 \\ \sin(71.56) & \cos(71.56) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.32 & -0.95 & 0 \\ 0.95 & 0.32 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(t_x, t_y) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, composite transformation matrix is;

$M = T(t_x, t_y).R(\theta).R_{fx}.R(-\theta).T(-t_x, -t_y)$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.32 & -0.95 & 0 \\ 0.95 & 0.32 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.32 & 0.95 & 0 \\ -0.95 & 0.32 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.32 & -0.95 & 0 \\ 0.95 & 0.32 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.32 & 0.95 & -3.8 \\ -0.95 & 0.32 & -1.28 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.32 & -0.95 & 0 \\ 0.95 & 0.32 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.32 & 0.95 & -3.8 \\ 0.95 & -0.32 & 1.28 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.8001 & 0.608 & -2.432 \\ 0.608 & 0.8001 & -3.2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.8001 & 0.608 & -2.432 \\ 0.608 & 0.8001 & 0.7996 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, the transformation points are;

$A' = M.A$

$$= \begin{bmatrix} -0.8001 & 0.608 & -2.432 \\ 0.608 & 0.8001 & 0.7996 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -2.21 \\ 4.42 \\ 1 \end{bmatrix} = (-2.21, 4.42)$$

$B' = M.B$

$$[-0.8001 \quad 0.608 \quad -2.432][6] \quad [-5.41]$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.8001 & 0.608 & -2.432 \\ 0.608 & 0.8001 & -3.2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.8001 & 0.608 & -2.432 \\ 0.608 & 0.8001 & 0.7996 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, the transformation points are;

$A' = M.A$

$$= \begin{bmatrix} -0.8001 & 0.608 & -2.432 \\ 0.608 & 0.8001 & 0.7996 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -2.21 \\ 4.42 \\ 1 \end{bmatrix} = (-2.21, 4.42)$$

$B' = M.B$

$$= \begin{bmatrix} -0.8001 & 0.608 & -2.432 \\ 0.608 & 0.8001 & 0.7996 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -5.41 \\ 6.85 \\ 1 \end{bmatrix} = (-5.41, 6.85)$$

$C' = M.C$

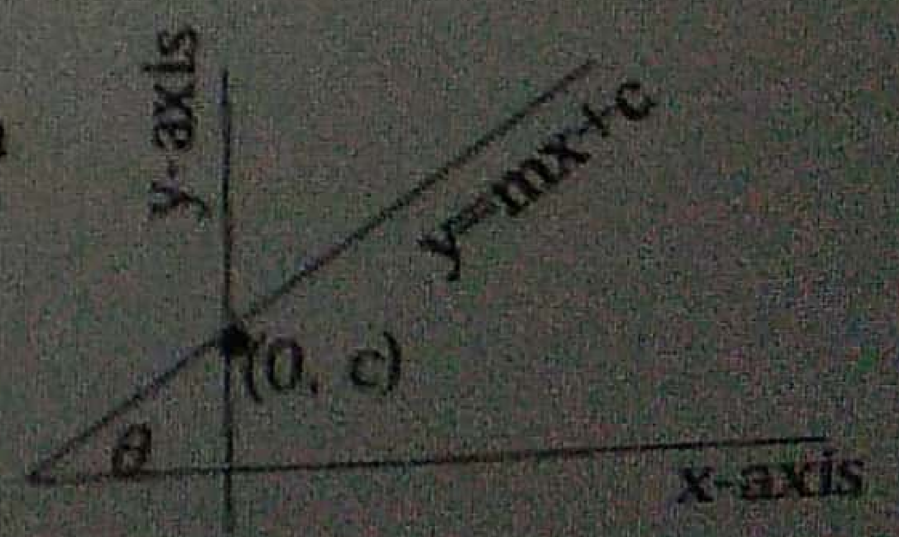
$$= \begin{bmatrix} -0.8001 & 0.608 & -2.432 \\ 0.608 & 0.8001 & 0.7996 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.77 \\ 9.63 \\ 1 \end{bmatrix} = (-0.77, 9.63)$$

Q. Derive the composite matrix for reflecting an object about any arbitary line y=mx+c.

Solution:

In order to reflect an object about any line y=mx+c, we need to perform Composite transformation as below

$T = T_{(0,c)}.R_{(\theta)}.R_{fx}.R_{(-\theta)}.T_{(0,-c)}$



And

Slope $m = \tan\theta$

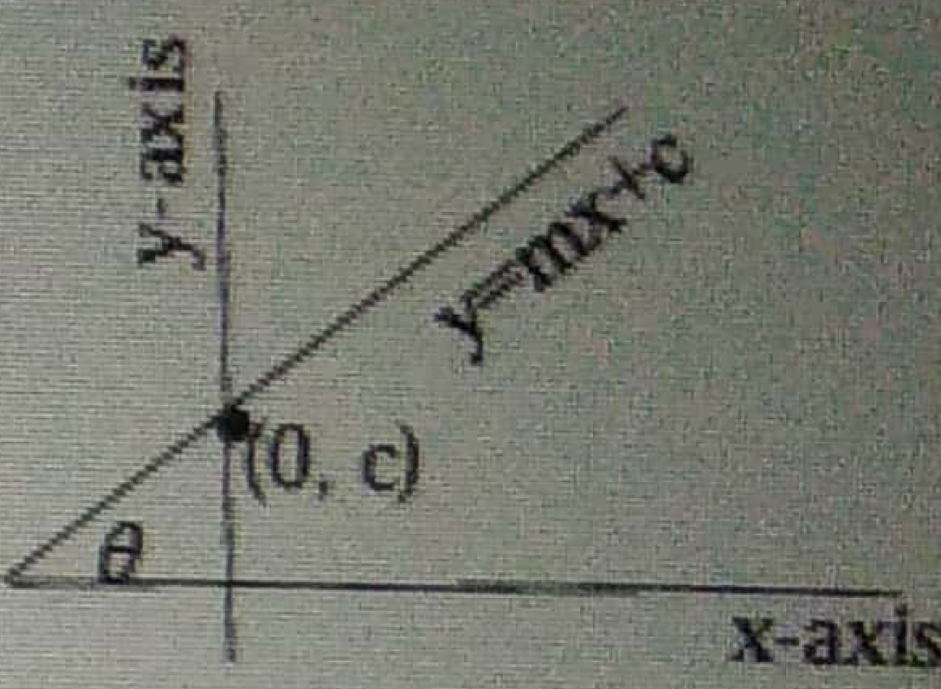
Also we have,

$\cos^2\theta =$

Q. Derive the composite matrix for reflecting an object about any arbitary line y=mx+c.
Solution:

In order to reflect an object about any line y=mx+c, we need to perform Composite transformation as below

$T = T_{(0,c)}.R_{(\theta)}.R_{fx}.R_{(-\theta)}.T_{(0,-c)}$



And

Slope m=$tan\theta$

Also we have,

$$cos^2\theta = \frac{1}{tan^2\theta+1} = \frac{1}{m^2+1}$$

$$\therefore cos\theta = \frac{1}{\sqrt{m^2+1}}$$

Also we have,

$$sin^2\theta + cos^2\theta = 1$$

$$sin^2\theta = 1 - cos^2\theta = 1 - \frac{1}{m^2+1} = \frac{m^2}{m^2+1}$$

$$\therefore sin\theta = \frac{m}{\sqrt{m^2+1}}$$

So,

$$T = T_{(0,c)}.R_{(\theta)}.R_{fx}.R_{(-\theta)}.T_{(0,-c)}$$

$$= \begin{bmatrix}1 & 0 & 0\\0 & 1 & c\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}cos\theta & -sin\theta & 0\\sin\theta & cos\theta & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}cos\theta & sin\theta & 0\\-sin\theta & cos\theta & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 1 & -c\\0 & 0 & 1\end{bmatrix}$$

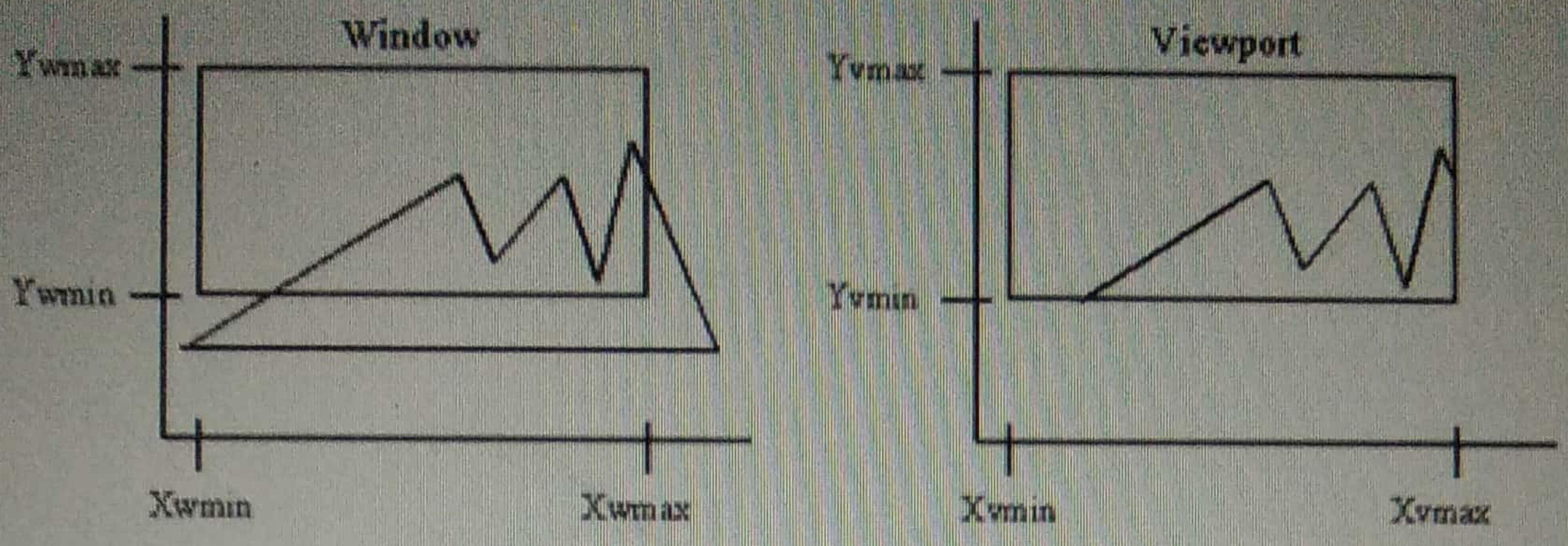
$$= \begin{bmatrix}1 & 0 & 0\\0 & 1 & c\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}cos\theta & -sin\theta & 0\\sin\theta & cos\theta & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}cos\theta & sin\theta & -csin\theta\\-sin\theta & cos\theta & -ccos\theta\\0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0 & 0\\0 & 1 & c\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}cos\theta & -sin\theta & 0\\sin\theta & cos\theta & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}cos\theta & sin\theta & -csin\theta\\sin\theta & -cos\theta & ccos\theta\\0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0 & 0\\0 & 1 & c\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{m^2+1}} & \frac{-m}{\sqrt{m^2+1}} & 0\\\frac{m}{\sqrt{m^2+1}} & \frac{1}{\sqrt{m^2+1}} & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{m^2+1}} & \frac{m}{\sqrt{m^2+1}} & \frac{-cm}{\sqrt{m^2+1}}\\\frac{m}{\sqrt{m^2+1}} & \frac{-1}{\sqrt{m^2+1}} & \frac{c}{\sqrt{m^2+1}}\\0 & 0 & 1\end{bmatrix}$$

❖ 2D Viewing

The process of mapping the world coordinate scene to device coordinate is called viewing transformation or windows to view port transformation.



World Coordinates　　　　Device Coordinates

- A world co-ordinate area selected for display is called a window and an area on the display device to which a window is mapped is called a view port.
- The window defines what is to be viewed and the viewport defines where it is to be displayed.
- Window deals with object space whereas viewport deals with image space.

Transformations from world to device coordinate involves translation, rotation and scaling operations, as well as procedure for deleting those parts of the picture that are outside the limits of selected display area i.e. clipping.
To make the viewing process independent of the requirements of any output device, graphics systems convert object description to normalized coordinates.

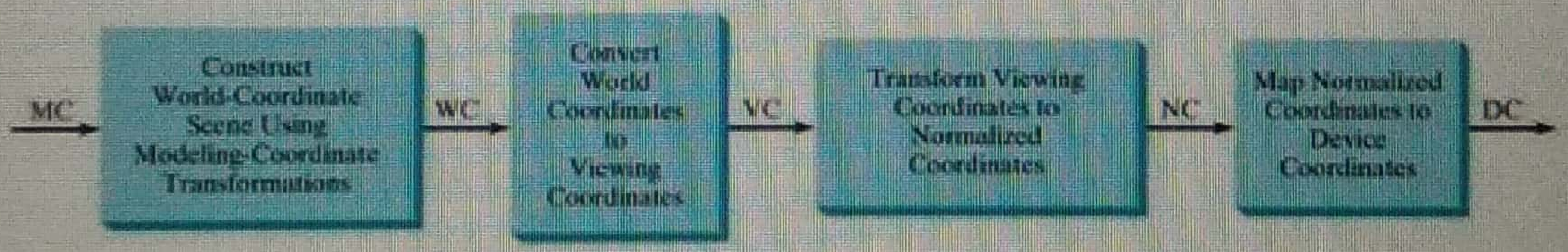


Fig. (c) Two-dimensional viewing transformation pipeline

Applications

- By changing the position of the view port, we can view objects at different positions on the display area of an output device.
- By varying the size of view ports, we can change size of displayed objects.
- Zooming effects can be obtained by successively mapping different-sized windows on a fixed-sized view port.
- Panning effects (Horizontal scrolling) are produced by moving a fixed-sized window across the various objects in a scene.

Example

Q. Digitize the ellipse with $r_x = 8, r_y = 6$ and center at (3, 5).

Solution:

For region 1

The initial point for the ellipse at origin

$$(x_0, y_0) = (0, r_y) = (0, 6)$$

The initial decision parameter

$$p1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2 = 6^2 - 8^2 \times 6 + \frac{1}{4} \times 8^2 = -332$$

From midpoint algorithm, for region 1 we know,

If $p1_k < 0$ then

$x_{k+1} = x_k + 1, \quad y_{k+1} = y_k$ and $p1_{k+1} = p1_k + 2r_y^2 x_{k+1} + r_y^2$

If $p1_k \geq 0$ then

$x_{k+1} = x_k + 1, \quad y_{k+1} = y_k - 1$ and $p1_{k+1} = p1_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_y^2$.

k	$p1_k$	(x_{k+1}, y_{k+1})	$2r_y^2 x_{k+1}$	$2r_x^2 y_{k+1}$
0	-332	(1, 6)	72	768
1	-224	(2, 6)	144	768
2	-44	(3, 6)	216	768
3	208	(4, 5)	288	640
4	-108	(5, 5)	360	640
5	288	(6, 4)	432	512
6	244	(7, 3)	504	384

Now, we move out of region 1 since $2r_y^2 x_{k+1} > 2r_x^2 y_{k+1}$.

1st quadrant	2nd quadrant	3rd quadrant	4th quadrant
(4, 11)	(-4, 11)	(-4, -11)	(4, -11)
(5, 11)	(-5, 11)	(-5, -11)	(5, -11)
(6, 11)	(-6, 11)	(-6, -11)	(6, -11)
(7, 10)	(-7, 10)	(-7, -10)	(7, -10)
(8, 10)	(-8, 10)	(-8, -10)	(8, -10)
(9, 9)	(-9, 9)	(-9, -9)	(9, -9)
(10, 8)	(-10, 8)	(-10,- 8)	(10, -8)

k	p_k	(x_{k+1}, y_{k+1})	$2x_{k+1}$	$2y_{k+1}$
0	-9	(1, 10)	2	20
1	-6	(2, 10)	4	20
2	-1	(3, 10)	6	20
3	6	(4, 9)	8	18
4	-3	(5, 9)	10	18
5	8	(6, 8)	12	16
6	5	(7, 7)	14	14

Q. Digitize the circle with radius r =10 centered (3, 4) in first octant.

Solution:

> *Note: When center is not origin, we first calculate the octants points of the circle in the same way as the center at origin then add the given circle center on each calculated pixel.*

Here,

Center = (3, 4)

Radius (r) =10

Initial point = (0, r) = (0, 10)

Initial decision parameter $p_0 = 1 - r = 1 - 10 = -9$

From mid-point circle algorithm we have;

If $p_k < 0$;

Plot $(x_k + 1, y_k)$ and $p_{k+1} = p_k + 2x_{k+1} + 1$

$p_k \geq 0$;

Plot $(x_k + 1, y_k - 1)$ and $p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$

k	p_k	(x_{k+1}, y_{k+1}) at (0, 0)	$2x_{k+1}$	$2y_{k+1}$	(x_{k+1}, y_{k+1}) at (3, 4)
0	-9	(1, 10)	2	20	(4, 14)
1	-6	(2, 10)	4	20	(5, 14)
2	-1	(3, 10)	6	20	(6, 14)
3	6	(4, 9)	8	18	(7, 13)
4	-3	(5, 9)	10	18	(8, 13)
5	8	(6, 8)	12	16	(9, 12)
6	5	(7, 7)	14	14	(10, 11)

Q. Digitize the circle with radius r =5 centered (2, 3).

Solution:

Here

$$x = x + x_c, \quad y = y + y_c$$

6. Repeat step 3 through 5 until $x \geq y$.

Examples

Q. Digitize the circle $x^2 + y^2 = 100$ in first octant.

Solution:

Here,
Center = (0, 0)
Radius (r) =10
Initial point = (0, r) = (0, 10)
Initial decision parameter $p_0 = 1 - r = 1 - 10 = -9$
From mid-point circle algorithm we have;
If $p_k < 0$;
Plot $(x_k + 1, y_k)$ and $p_{k+1} = p_k + 2x_{k+1} + 1$
$p_k \geq 0$;
Plot $(x_k + 1, y_k - 1)$ and $p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$

k	p_k	(x_{k+1}, y_{k+1})	$2x_{k+1}$	$2y_{k+1}$
0	-9	(1, 10)	2	20
1	-6	(2, 10)	4	20
2	-1	(3, 10)	6	20
3	6	(4, 9)	8	18
4	-3	(5, 9)	10	18
5	8	(6, 8)	12	16
6	5	(7, 7)	14	14

Q. Digitize the circle with radius r =10 centered (3, 4) in first octant.

Search

Advantages of Bresenham's line algorithm (BLA) over DDA:

- In DDA algorithm each successive point is computed in floating point, so it required more time and memory space. While in BLA each successive point is calculated in integer value or whole number. So it requires less time and less memory
- In DDA, since the calculated point value is floating point number, it should be rounded at the end of calculation but in BLA it does not need to round, so there is no accumulation of rounding error.
- Due to rounding error, the line drawn by DDA algorithm is not accurate, while in BLA line is accurate.
- DDA algorithm cannot be used in other application except line drawing, but BLA can be implemented in other application such as circle, ellipse, and other curves.

Q. A homogenous coordinate point P(3, 2, 1) is translated in x, y, z direction by -2, -2 & -2 unit respectively followed by successive rotation of 60⁰ about x- axis. Find the final position of homogenous coordinate.

Solution:

Here,

$t_x = -2$

$t_y = -2$

$t_z = -2$

$$T = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x(60^0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos60 & -sin60 & 0 \\ 0 & sin60 & cos60 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Composite transformation

$$R_x(60^0).T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & -1+\sqrt{3} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & -\sqrt{3}-1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now,

$$P' = M.P = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & -1+\sqrt{3} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & -\sqrt{3}-1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix} =$$

Bezier surfaces have the same properties as Bezier curves, and they provide a conve
method for interactive design applications.

Q. Construct Bezier curve for control points (4, 2), (8, 8) and (16, 4).

Solution:

Given control points

$p_0 = (x_0, y_0) = (4,2)$

$p_1 = (x_1, y_1) = (8,8)$

$p_2 = (x_2, y_2) = (16,4)$

Here, degree (or order) n= 2

We have basis function as

$$P(u) = \sum_{k=0}^{n} p_k BEZ_{k,n}(u), \qquad 0 \le u \le 1$$

$$= \sum_{k=0}^{2} p_k BEZ_{k,2}(u), \qquad 0 \le u \le 1$$

$$\therefore\ P(u) = p_0 BEZ_{0,2}(u) + p_1 BEZ_{1,2}(u) + p_2 BEZ_{2,2}(u)$$

Parametric equations are;

$$x(u) = x_0 BEZ_{0,2}(u) + x_1 BEZ_{1,2}(u) + x_2 BEZ_{2,2}(u) \ \ldots\ldots\ldots\ldots \text{(i)}$$
$$y(u) = y_0 BEZ_{0,2}(u) + y_1 BEZ_{1,2}(u) + y_2 BEZ_{2,2}(u) \ \ldots\ldots\ldots\ldots \text{(ii)}$$

Now,

$$BEZ_{0,2}(u) = C(2,0)u^0(1-u)^{2-0} = \frac{2!}{0!(2-0)!} \times (1-u)^2 = (1-u)^2$$

$$BEZ_{1,2}(u) = C(2,1)u^1(1-u)^{2-1} = \frac{2!}{1!(2-1)!} \times u(1-u)^1 = 2u(1-u)$$

$$BEZ_{2,2}(u) = C(2,2)u^2(1-u)^{2-2} = \frac{2!}{2!(2-2)!} \times u^2(1-u)^0 = u^2$$

Putting these values in eq. (i) & (ii) we get;

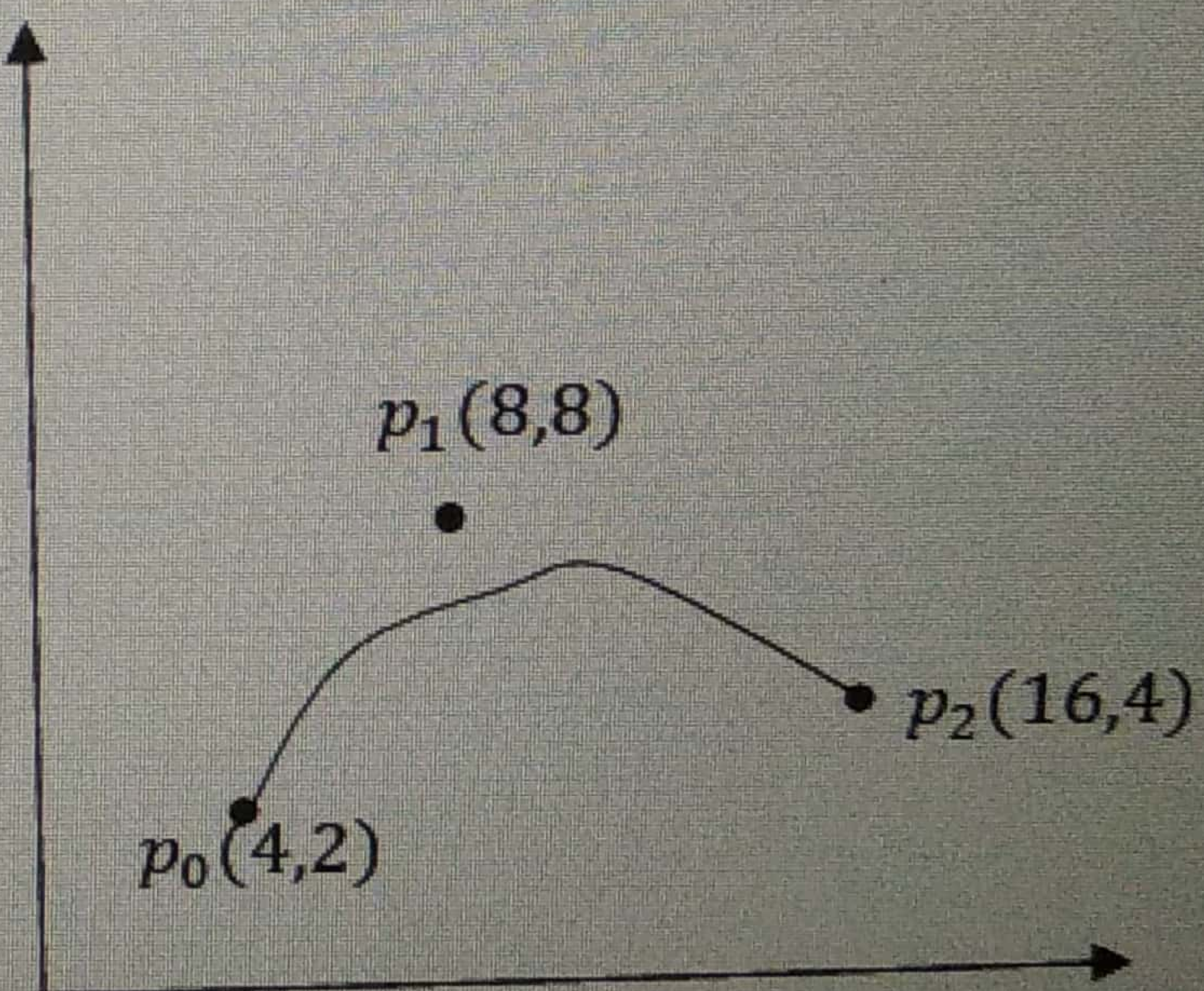
$$x(u) = x_0(1-u)^2 + x_1 2u(1-u) + x_2u^2 = 4u^2 + 8u + 4$$

$$y(u) = y_0(1-u)^2 + y_1 2u(1-u) + y_2u^2 = -10u^2 + 12u + 2$$

Now,

	$x(u)$	$y(u)$
$u=0$	4	2
$u=0.2$	5.76	4.0
$u=0.4$	7.84	5.20
$u=0.6$	10.24	5.6
$u=0.8$	12.96	5.2
$u=1$	16	4

Drawing these points we get:



Q. Construct the Bezier curve of order 3 with 4 vertices of the control polygon $p_0(0,0)$, $p_1(1,2)$, $p_2(3,2)$ & $p_3(2,0)$. Generate at least 5 points on the curve.

Q. Construct the Bezier curve of order 3 with 4 vertices of the control polygon $p_0(0,0)$, $p_1(1,2)$, $p_2(3,2)$ & $p_3(2,0)$. Generate at least 5 points on the curve.

Solution:

Given control points

$p_0 = (x_0, y_0) = (0,0)$

$p_1 = (x_1, y_1) = (1,2)$

$p_2 = (x_2, y_2) = (3,2)$

$p_3 = (x_3, y_3) = (2,0)$

Here, degree n= 3

We have basis function as

$$P(u) = \sum_{k=0}^{n} p_k BEZ_{k,n}(u), \qquad 0 \le u \le 1$$

$$= \sum_{k=0}^{3} p_k BEZ_{k,3}(u), \qquad 0 \le u \le 1$$

$$\therefore\ P(u) = p_0 BEZ_{0,3}(u) + p_1 BEZ_{1,3}(u) + p_2 BEZ_{2,3}(u) + p_3 BEZ_{3,3}(u)$$

Parametric equations are;

$$x(u) = x_0 BEZ_{0,3}(u) + x_1 BEZ_{1,3}(u) + x_2 BEZ_{2,3}(u) + x_3 BEZ_{3,3}(u) \ \ldots\ldots\ldots\ (i)$$
$$y(u) = y_0 BEZ_{0,3}(u) + y_1 BEZ_{1,3}(u) + y_2 BEZ_{2,3}(u) + y_3 BEZ_{3,3}(u) \ \ldots\ldots\ldots\ (ii)$$

Now,

$$BEZ_{0,3}(u) = C(3,0)u^0(1-u)^{3-0} = \frac{3!}{0!(3-0)!} \times (1-u)^3 = (1-u)^3$$
$$BEZ_{1,3}(u) = C(3,1)u^1(1-u)^{3-1} = \frac{3!}{1!(3-1)!} \times u(1-u)^2 = 3u(1-u)^2$$
$$BEZ_{2,3}(u) = C(3,2)u^2(1-u)^{3-2} = \frac{3!}{2!(3-2)!} \times u^2(1-u)^1 = 3u^2(1-u)$$
$$BEZ_{3,3}(u) = C(3,3)u^3(1-u)^{3-3} = \frac{3!}{3!(3-3)!} \times u^3(1-u)^0 = u^3$$

Putting these values in eq. (i) & (ii) we get;

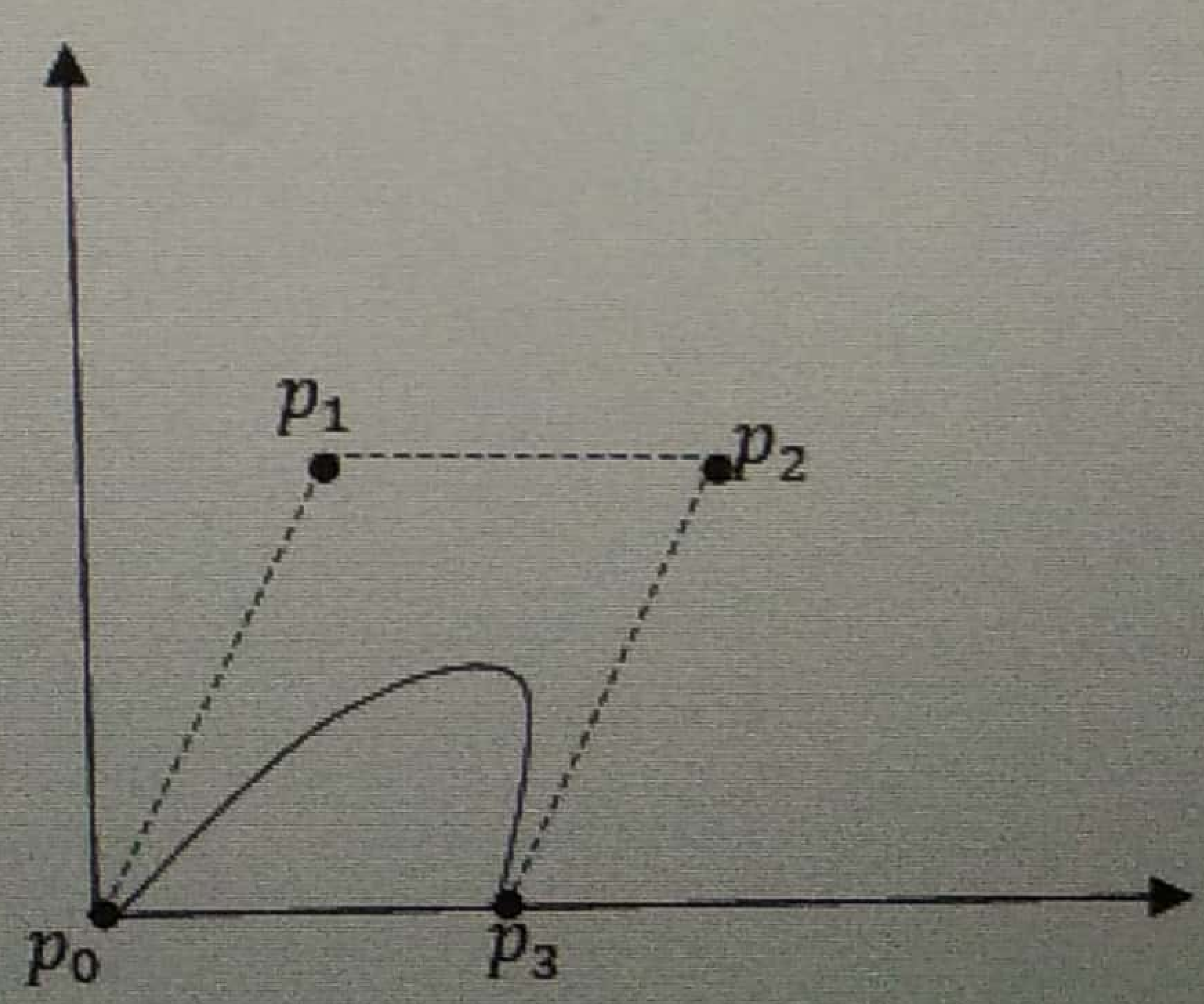
$$x(u) = x_0(1-u)^3 + x_1 3u(1-u)^2 + x_2 3u^2(1-u) + x_3 u^3 = -4u^3 + 3u^2 + 3u$$

$$y(u) = y_0(1-u)^3 + y_1 3u(1-u)^2 + y_2 3u^2(1-u) + y_3 u^3 = -6u^2 + 6u$$

Now,

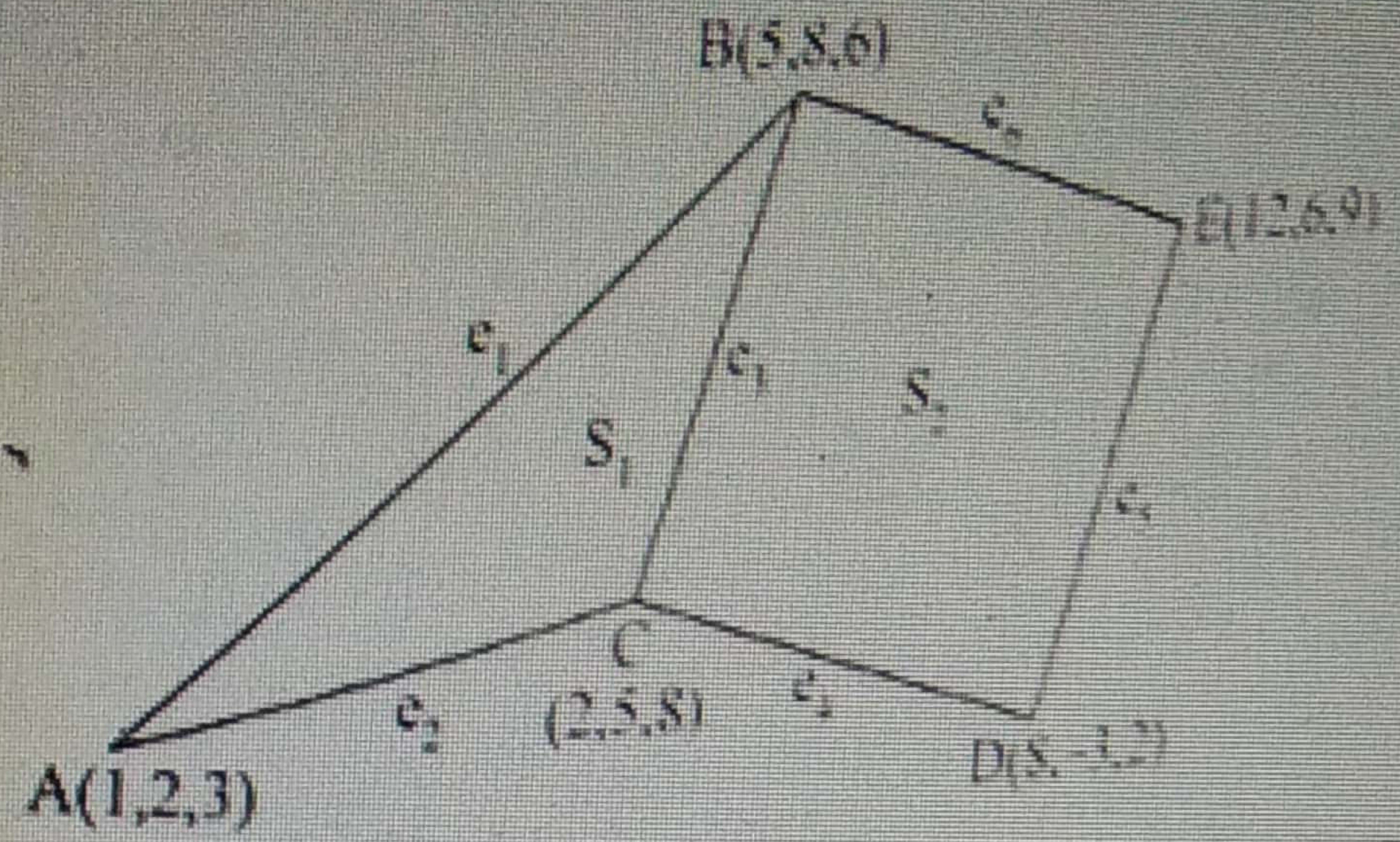
	$x(u)$	$y(u)$
$u=0$	0	0
$u=0.15$	0.50	0.76
$u=0.35$	1.24	1.36
$u=0.5$	1.75	1.50
$u=0.65$	2.12	1.36
$u=0.85$	2.14	0.76
$u=1$	2	0

Plotting the graph;



8. B-spline Curve

1. ***Represent the following surfaces by polygon table method. Find the normal of surface S_1.*** *[2076 Ashwin Back]*



Vertex table
A: 1,2,3
B: 5,8,6
C: 2,5,8
D: 8,–3,2
E: 12,6,9

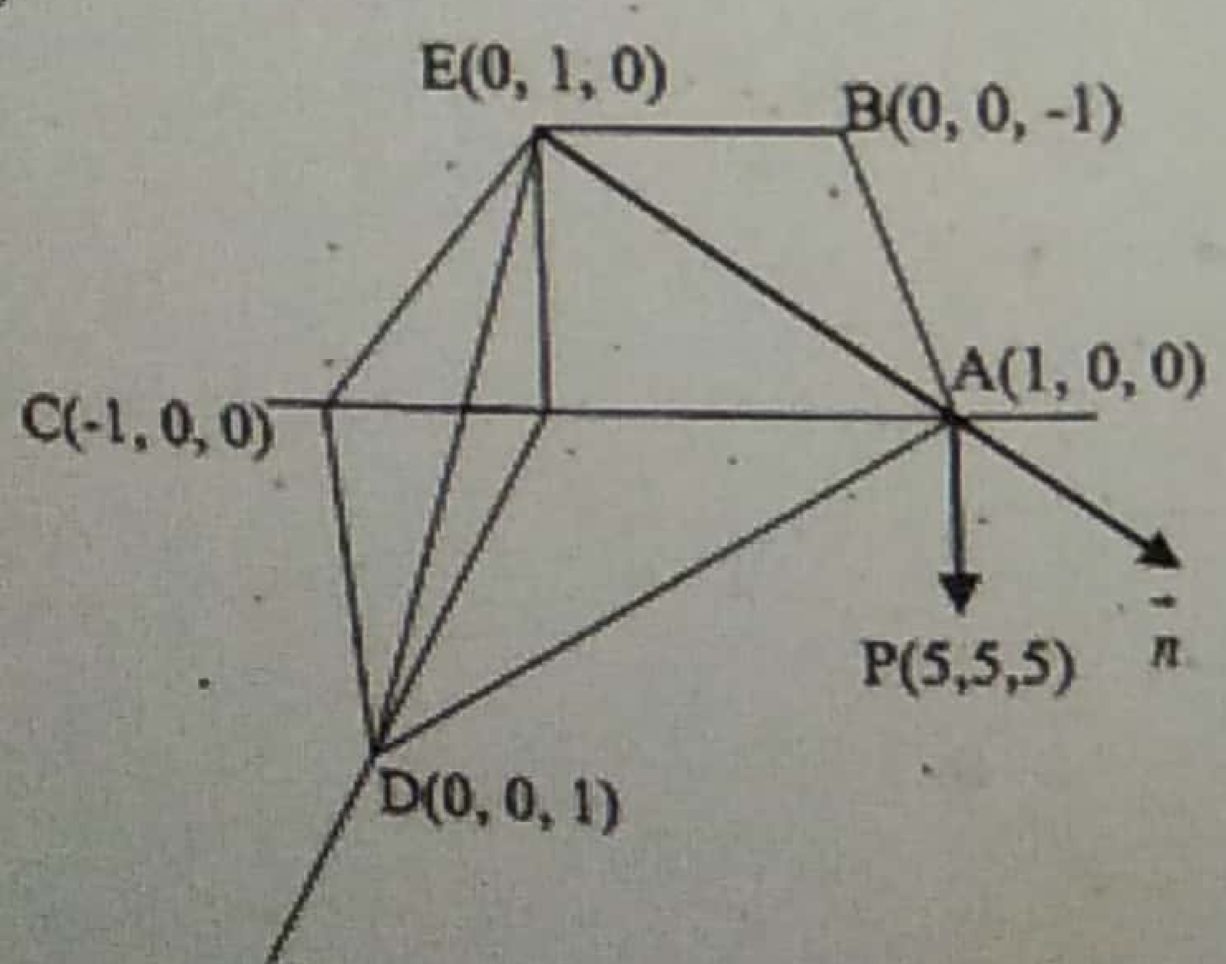
Edge table
e_1: A, B
e_2: A, C
e_3: C, B
e_4: C, D
e_5: D, E
e_6: E, B

Polygon -surface table
S_1: e_1, e_2, e_3
S_2: e_3, e_4, e_5, e_6

We can find the normal of surface S_1 by selecting any three vertices in counter clockwise direction. While viewing from outside.

$$\vec{N} = \overrightarrow{AC} \times \overrightarrow{AB}$$
$$= [(2-1)i + (5-2)j + (8-3)k] \times [(5-1)i + (8-2)j + (6-3)k]$$
$$= (i + 3j + 5k) \times (4i + 6j + 3k)$$
$$= \begin{bmatrix} i & j & k \\ 1 & 3 & 5 \\ 4 & 6 & 3 \end{bmatrix}$$
$$= (9-30)i - (20-3)j + (6-12)k$$
$$= -27i - 17j - 6k$$

2. *Find the visibility for the surface AED where observer at P(5,5,5)*



Solution:

Step 1:

Find the normal vector n for AED surface (always take anti-clockwise direction convention)

i.e. AE*AD, NOT AD*AE

$$AE = E - A$$
$$= (0 - 1)i + (0 - 0)j + (0 - 0)k$$
$$= -i + j$$
$$N = AE * AD$$
$$= (-i + j) \times (-i + k)$$
$$\begin{bmatrix} i & j & k \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$
$$-i(1 - 0) - j(-1 + 0) + k(0 + 1)$$
$$= i + j + k$$

Step 2:

The observer is at P(5,5,5) so we can construct the view vector V from surface to view point A(1,0,0) as

$$V = PA = (1 - 5)i + (0 - 5)j + (0 - 5)K$$
$$= -4i - 5j - 5k$$

Steps 3:

To find the visibility of the object, we use dot product of view vector and normal vector N as

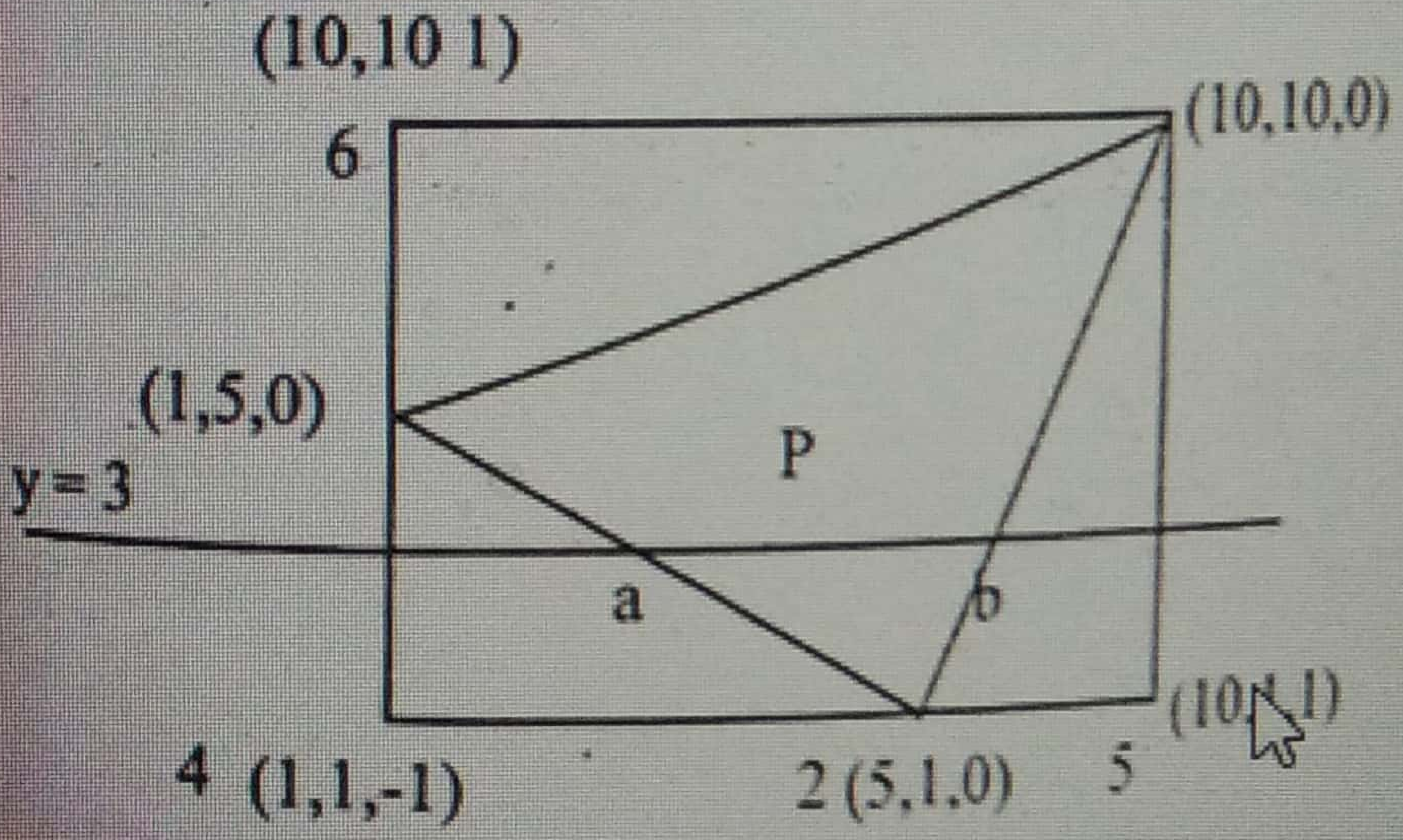
$$\overline{V.N} = (-4i - 5j - 5k).(i + j + k)$$
$$= -4 - 5 - 5$$
$$= -14 < 0$$

This shows that the surface is visible for observe

- method.
- It is expensive rendering method.

SOLVED NUMERICALS

1. ***Find out the intensity of light reflected from the midpoint P on scan line y = 3 in the above given figure using Gouraud shading model. Consider a single point light source located at positive intensity on z-axis and assume vector to eye as (1,1,1). Given d = 0, k = 1, la = 1, l_l = 10, K_s = 2, K_a = K_d = 0.8 for use in a simple illumination model.***



Solution:

Step 1

Calculate unit normal vectors $\hat{N}_1$, $\hat{N}_2$, $\hat{N}_3$ at vertices 1, 2 and 3 respectively.

The normal vectors at the vertices can be approximated by averaging the cross product of all the edges that terminate at the vertices. It is important that the order of vectors should be so chosen that the cross product yields outward normal vectors only.

The normal vectors at 1,

$$\vec{N}_1' = V_1V_2 \times V_1V_3 + V_1V_3 \times V_1V_6 + V_1V_4 \times V_1V_2$$

$$= (4\hat{i}-4\hat{j})\times(9\hat{i}+5\hat{j}) + (9\hat{i}+5\hat{j}) \times (5\hat{j}+\hat{k}) + (-4\hat{j}-\hat{k}) \times (4\hat{i}-4\hat{j})$$

$$= \hat{i}-13\hat{j}+117\hat{k}$$

The unit normal at 1,

$$\hat{N}_1 = \frac{\vec{N}_1'}{|\vec{N}_1'|}$$

$$= \frac{\hat{i}-13\hat{j}-117\hat{k}}{\sqrt{1^2+(-13)^2+117^2}}$$

$$= 0.01\hat{i}+0.11\hat{j}+0.99\hat{k}$$

Similar at 2,

$$\vec{N}_2' = V_2V_3 \times V_2V_1 + V_2V_1 \times V_2V_4 + V_2V_5 \times V_2V_3$$

$$= 13\hat{i}+\hat{j}+117\hat{k}$$

$$\hat{N}_2 = \frac{\vec{N}_2'}{|\vec{N}_2'|}$$

$$= -0.11\hat{i}+0.001\hat{j}+0.99\hat{k}$$

Similarly at 3,

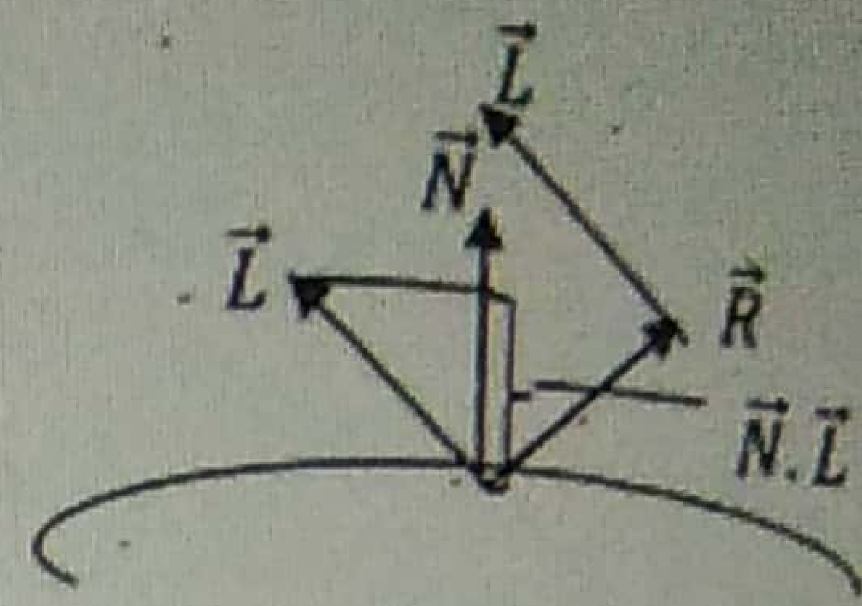
$$\vec{N}_3' = -4\hat{i}-4\hat{j}+146\hat{k}$$

$$\hat{N}_3 = \frac{\vec{N}_3'}{|\vec{N}_3'|}$$

$$= -0.03\hat{i}-0.03\hat{j}-0.99\hat{k}$$

Step 2,

Calculate unit reflection vectors $\vec{R}_1, \vec{R}_2, \vec{R}_3$ at vertices 1, 2, and 3 respectively



The projection of $\vec{L}$ onto the direction of the normal vector is obtained with the dot product $\vec{N}.\vec{L}$.

So,

$$\vec{R} + \vec{L} = (2\vec{N}.\vec{L})\vec{N}$$

$$\vec{R} = (2\vec{N}.\vec{L})\vec{N} - \vec{L}$$

$$\vec{R}_1 = 2(\vec{N}_1.\vec{L})\,\vec{N}_1 - \vec{L}$$

$$= 2((0.01\hat{i} - 0.11\hat{j} + 0.99\hat{k}).\hat{k})(0.01\hat{i} - 0.11\hat{j} + 0.99\hat{k}) - \hat{k}$$

$$= (0.02\hat{i} - 0.22\hat{j} + 0.96\hat{k})$$

$$\vec{R}_2 = 2(\vec{N}_2.\vec{L})\vec{N}_2 - \vec{L}$$

$$= (0.02\hat{i} - 0.22\hat{j} + 0.96\hat{k})$$

$$\vec{R}_3 = 2(\vec{N}_3.\vec{L})\,\vec{N}_3 - \vec{L}$$

$$= (-0.06\hat{i} - 0.06\hat{j} + 0.96\hat{k})$$

Step 3,

Calculate intensities I_1, I_2, I_3 at vertices 1, 2, and 3 respectively

$$I_1 = I_aK_a + \frac{Il(K_d(\vec{N}_1.\vec{L})+K_s)\vec{R}_1.\vec{V})^{ns})}{K+d}$$

$\vec{N}_1.\vec{L} = (0.01\hat{i} - 0.11\hat{j} + 0.99\hat{k}).\hat{k}$
$= 0.99$

$\vec{R}_1.\vec{V} = (0.02\hat{i} - 0.22\hat{j} + 0.96\hat{k}).(0.58\hat{i} + 0.58\hat{j} + 0.58\hat{k})$
$= 0.44$

$I_1 = (1)(0.10)+(10.1)((0.10)(0.99)+(0.80)(0.44)^2)$
$= 2.64$

$I_2 = I_a K_a + \frac{Il(Kd(N2L)+Ks)R2.V)^{ns)}}{K+d}$

$\vec{N}_2.\vec{L} = (0.01\hat{i} - 0.11\hat{j} + 0.99\hat{k}).\hat{k}$
$= 0.99$

$\vec{R}_2.\vec{V} = (0.02\hat{i} - 0.22\hat{j} + 0.96\hat{k}).(0.58\hat{i} + 0.58\hat{j} + 0.58\hat{k})$
$= 0.44$

$I_2 = (1)(0.10)+(10.1)((0.10)(0.99)+(0.80)(0.44)^2)$
$= 2.64$

Similarly, calculate $\vec{N}_3.\vec{L}$, $\vec{R}_3.\vec{V}$ and we get

$I_3 = 3.09$

Step 4

Interpolate intensities I_a, I_b and I_p at a, b, p respectively

Referring the figure,

The scan line y = 3 containing point p intersects the edges 1-2 and 3-2 respectively at a and b.

$$\frac{x_1 - x_a}{x_1 - x_2} = \frac{y_1 - y}{y_1 - y_2}$$

$$\frac{x_2 - x_b}{x_2 - x_3} = \frac{y_2 - y}{y_2 - y_3}$$

Using the slope of the edges the co-ordinates of a and b are found to be (3,3,0) and (6.11,3,0) respectively.

The coordinates of p, the midpoint of a b is found (4.56, 3,0). Now we have apply 3 stage interpolation technique to determine I_p

$$\frac{I_1 - I_a}{I_1 - I_2} = \frac{y_1 - y}{y_1 - y_2}$$

$I_a = 2.64$ $\qquad$ $I_b = 2.74$

$x_a = 3$ $\qquad$ $x_b = 6.11$

$x_p = 4.555$

$$\frac{I_a - I_p}{I_a - I_b} = \frac{x_a - x_p}{x_a - x_b}$$

$I_p = 2.69$