NEPAL COLLEGE OF INFORMATION TECHNOLOGY

Balkumari, Lalitpur

Affiliated to Pokhara University

ASSIGNMENT FOR DATA COMMUNICATION

TUTORIAL 2

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Data Communication

Tutorial 2

Q1) Define LTI systems. Explain with examples, how periodic and non-periodic signals are distinguished.

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LTI systems are systems that are linear (obey superposition) and time-variant (responses donot change with time).

Feature	Periodic signal	Non-Periodic signal
Definition	Repeats itself after a fixed time interval T.	Does not repeat over any interval.
Mathematical Form	$x(t) = x(t+T)$ for all t.	No such condition is satisfied.
Examples	$\sin(2\pi f t)$, $\cos(2\pi f t)$	e^{-t}, a speech signal
Frequency Domain	Discrete frequencies (harmonics of the fundamental frequency)	continuous or wideband spectrum.
Applications	communication systems, signal processing	Random signals, transient analysis.

Q2) Define energy and power signal.

→

A signal is an energy signal if it has finite energy but zero average power. The energy E of a signal $x(t)$ is given by,

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt < \infty$$

A signal is a power signal if it has finite average power but infinite energy. The average power P of a signal $x(t)$ is given by:

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt < \infty$$

Q3) How do you classify the signal?

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On the basis of function, the classification are as follows:

1. Continuous time signal & Discrete Time Signal

Continuous signal is a signal where the value of function is defined over all the period of time(t).

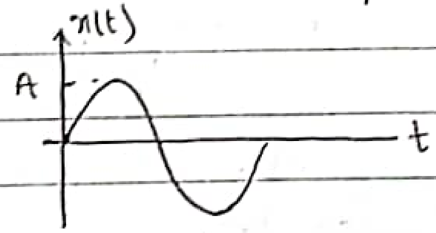


Fig. Continuous signal

Discrete time signal is a signal where the value of the function isn't defined over all the value of time.

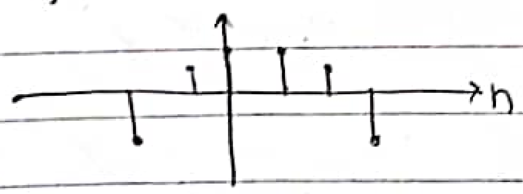


Fig. Discrete time signal

2. Even & Odd signal

A real valued signal $x(t)$ or $x[n]$ is referred to as an even signal if it satisfies the relation.

For continuous, $x(t) = x(-t)$, $\forall t$

For discrete, $x[n] = x[-n]$, $\forall n$

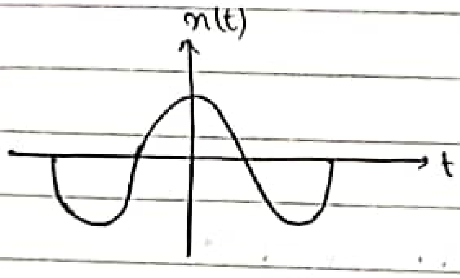


Fig. Even continuous signal.

Odd signal: For continuous, $x(t) = -x(-t)$, $\forall t$

For discrete, $x[n] = -x[-n]$, $\forall n$

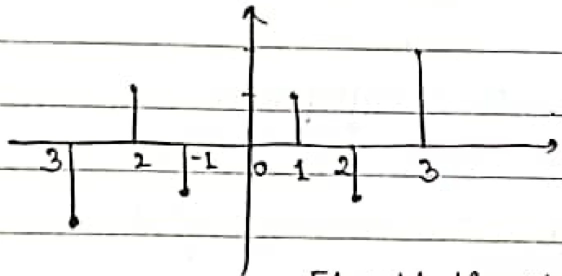


Fig. Odd discrete signal

3. Periodic & Aperiodic signal

- A continuous time signal $x(t)$ is periodic if $x(t) = x(t+T)$
- Periodic signal is unchanged by time shift 'T'
- Fundamental period $(T_0) = \frac{2\pi}{\omega_0}$

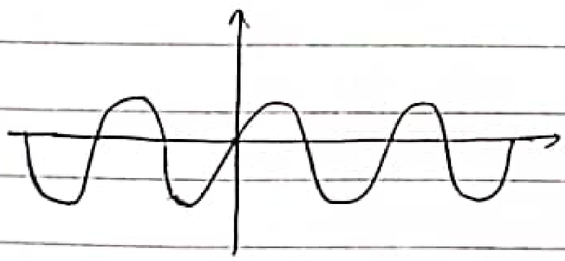


Fig. Periodic continuous Time signal

A signal that doesnot repeat itself after fixed period is aperiodic or Non-periodic signal.

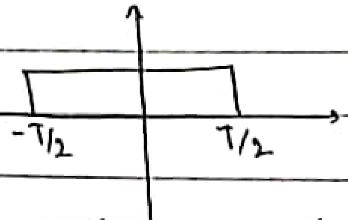


Fig. continuous aperiodic signal

4. Deterministic & Random signal

Any signal that can be uniquely described by mathematical expression, table of data or well defined rule is called deterministic. Signals that cannot be described to any reasonable degree of accuracy by mathematical formula are random signals.

5. Energy & Power signal

Energy signal : Finite energy & no average power

$$0 < E < \infty \, , P = 0$$

All practical non-periodic signal are energy signal.

Power signal : Finite power & infinite energy

$$0 < P < \infty \, , E = \infty$$

Periodic signal are power signal.

Q4) Check whether $x(t) = A\cos wt$ is a power or energy type signal.

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= \int_{-\infty}^{\infty} (A\cos wt)^2\, dt$$

$$= \int_{-\infty}^{\infty} A^2 \times \frac{\cos 2\omega t + 1}{2} dt$$

$$= \frac{A^2}{2} \left[\frac{\sin 2\omega t}{2\omega t} + t \right]_{-\infty}^{\infty}$$

$$= \infty$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2(t) \, dt$$

$$= \lim_{T \to \infty} \frac{A^2}{2T} \int_{-T}^{T} \frac{\cos 2\omega t + 1}{2} dt$$

$$= \lim_{T \to \infty} \frac{A^2}{4T} \left[\frac{\sin 2\omega t}{2\omega t} + t \right]_{-T}^{T}$$

$$= \lim_{T \to \infty} \frac{A^2}{4T} \left[\frac{\sin 2\omega T}{2\omega T} + \frac{\sin 2\omega T}{2\omega T} + T + T \right]$$

$$= \lim_{T \to \infty} \frac{A^2}{4T} \left[\frac{2 \sin 2\omega T}{2\omega T} + 2T \right]$$

$$= \lim_{T \to \infty} \frac{A^2}{4T} \left\{ \frac{2 \sin 4\pi}{4\pi} + 2T \right\}$$

$$= \lim_{T \to \infty} \frac{A^2}{4T} \times 2T$$

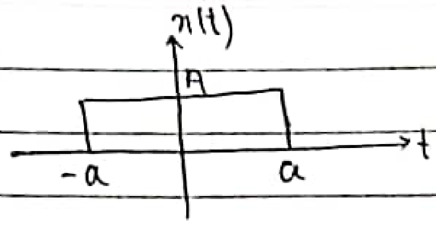
$$= \frac{A^2}{2}$$

Hence, $E = \infty$, $0 < P < \infty$

so, it is power signal. Ans

Q5) Determine whether the signal $x(t) = A[u(t+a) - u(t-a)]$ for $a > 0$ is a power or energy signal.

P.T.O.



$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= \int_{-a}^{a} A^2\, dt$$

$$= A^2 [t]_{-a}^{a} \qquad = A^2 2a \text{ Ans}$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-a}^{a} x^2(t)\, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-a}^{a} A^2\, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \times A^2 2a$$

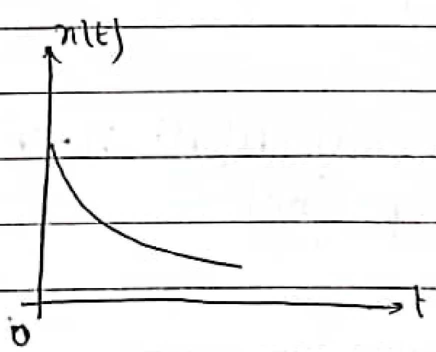
$$= 0$$

So, it is energy signal Ans

Q6) Find the energy and power of signal $x(t) = e^{-4t}\, u(t)$ and justify whether it is energy signal or power signal.

Soln

Here, $x(t) = e^{-4t}\, u(t)$



Energy, $E = \int_{-\infty}^{\infty} x^2(t)\,dt$

$$= \int_0^{\infty} e^{-8t}\,dt$$

$$= \left[\frac{e^{-8t}}{-8}\right]_0^{\infty}$$

$$= -\frac{1}{8}\left[\cancel{e^{-\infty}} - e^0\right]$$

$$\therefore E = -\frac{1}{8}$$

Now, Power $= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_0^{T} e^{-8t}\,dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left[\frac{e^{-8t}}{-8}\right]_0^{T}$$

$$= \lim_{T\to\infty} \frac{1}{2\times-8} \times \frac{1}{T} \times e^{-8T}$$

$$= \frac{1}{-16} \times \frac{1}{\infty} \times \cancel{e^{-\infty}}^{\,0}$$

$$= 0$$

As E is finite and P = 0. so, it is Energy signal. Ans

Q7) How would you say that a system is BIBO stable? comment about the stability for the following system:

$$y(n) = 2x(n+1) + [x(n-1)]^2$$

→

A system is Bounded-Input Bounded-Output (BIBO) stable if every bounded input produces a bounded output.

Mathematically,

If $|x(n)| \leq M_x$ (for some finite M_x) for all n,

Then $|y(n)| \leq M_y$ (for some finite M_y) for all n.

The given system is:

$$y(n) = 2x(n+1) + [x(n-1)]^2$$

Term 1 : $2x(n+1)$

If $x(n)$ is bounded, then $x(n+1)$ is also bounded.

$\therefore 2x(n+1)$ is bounded by $2B_x$.

Term 2 : $[x(n-1)]^2$

If $x(n)$ is bounded, then $x(n-1)$ is also bounded.

$\therefore [x(n-1)]^2$ is bounded by B_x^2.

Since the o/p $y(n)$ is the sum of these two terms,

$$y(n) \leq 2x(n+1) + [x(n-1)]^2$$

$$y(n) \leq 2B_x + B_x^2$$

So, the o/p $y(n)$ is bounded if the i/p $x(n)$ is bounded.

Therefore, system is BIBO stable.

Q8) Comment about the linearity, stability, time-invariance and causalty for the following system : $y(t) = t^2 x(t)$

→

For linearity,

$$y(t) = t^2 x(t)$$

$x(t) \longrightarrow$ system $\longrightarrow y(t) = t^2 x(t)$.

$x_1(t) \longrightarrow$ system $\longrightarrow y_1(t) = t^2 x_1(t)$

$x_2(t) \longrightarrow$ system $\longrightarrow y_2(t) = t^2 x_2(t)$

Now, $y_3(t) = y_1(t) + y_2(t)$
$= t^2 x_1(t) + t^2 x_2(t)$

$x_1(t) + x_2(t) \longrightarrow \text{system} \longrightarrow y_4(t) = t^2(x_1(t) + x_2(t))$
$= t^2 x_1(t) + t^2 x_2(t)$

Here, $y_3(t) = y_4(t)$
Hence, the system is linear.

For stability,

Lets test the stability by considering unit step signal.
Let, ~~$x(t) = u(t)$~~
$x(t) = u(t)$
So, $y(t) = t^2 y(t)$

For $x(t) = u(t)$, the input is bounded $(|x(t)| \leq 1)$
However, the output $y(t) = t^2$ grows without bound as $t \to \infty$.
Specifically when $t \geq 0$, $y(t)$ increases quadratically $y(t) = t^2$
This implies that o/p is unbounded even though i/p is bounded.

So, system is not stable because bounded i/p produces unbounded o/p.

For time-varience,

The response of system to $x(t-t_1)$; t_1 delay is
$y(t, t_1) = t^2 x(t-t_1)$

If we delay o/p by t_1, then t_1 is
$y(t-t_1) = (t-t_1) x(t-t_1)$

Here, $y(t,t_1) \neq y(t-t_1)$
so, it is time varient.

For causalty,
$$y(t) = t^2 x(t)$$
The o/p of system depends ~~only~~ on present i/p value.
so, the system is said to be causal system.

Q9) Check causalty and time-invariance of a signal.
$$y(t) = x(t) + u(t-5)$$
Sol^n

For causalty,

The term $u(t-5)$ is unit step function shifted by 5 unit to right, which is independent of $x(t)$ & is constant function for $t \geq 5$.
Since the o/p depend upon the present i/p value, the system is causal.

For time invariance,

The response of system to $x(t-t_1)$; t_1 delay is
$$y(t,t_1) = x(t-t_1) + u(t-t_1-5)$$

If we delay o/p by t_1, then o/p is
$$y(t-t_1) = x(t-t_1) + u(t-t_1-5)$$

As, $y(t,t_1) = y(t-t_1)$
so, it is time invarient.

Q10) Check whether the given system is linear, causal and time varient or not.

(i) $y(t) = \log x(t)$

Sol^n

For linearity,

$x(t) \longrightarrow \text{system} \longrightarrow y(t) = \log x(t)$

$x_1(t) \longrightarrow \text{system} \longrightarrow y_1(t) = \log x_1(t)$

$x_2(t) \longrightarrow \text{system} \longrightarrow y_2(t) = \log x_2(t)$

$x_1(t) + x_2(t) \longrightarrow \text{system} \longrightarrow y_3(t) = \log[(x_1(t) + x_2(t)]$

Now, $y_4(t) = y_1(t) + y_2(t)$

$= \log x_1(t) + \log x_2(t)$

Here, $y_3(t) \neq y_4(t)$.

Hence, the system is non-linear.

For causalty,

Since the o/p depends only on the present i/p value, the system is causal.

For time varience,

The response of system to $x(t-t_1)$; t_1 delay is

$y(t, t_1) = \log x(t-t_1)$

If we delay the o/p by t_1, then o/p

$y(t-t_1) = \log x(t-t_1)$

Here, $y(t, t_1) = y(t-t_1)$

So, it is time invarient.

(ii) $y(t) = t x(t)$

Sol^n

For linearity,

$x(t) \longrightarrow$ system $\longrightarrow y(t) = t x(t)$

$x_1(t) \longrightarrow$ system $\longrightarrow y_1(t) = t x_1(t)$

$x_2(t) \longrightarrow$ system $\longrightarrow y_2(t) = t x_2(t)$

$x_1(t) + x_2(t) \longrightarrow$ system $\longrightarrow y_3(t) = t(x_1(t) + x_2(t))$

$= t x_1(t) + t x_2(t)$

Now, $y_4(t) = y_1(t) + y_2(t)$

$= t x_1(t) + t x_2(t)$

As $y_3(t) = y_4(t)$,

the system is linear.

For causalty,

$y(t) = t x(t)$

since, the o/p depends only upon the present i/p value, ~~th~~ so the system is causal.

For time invarience,

The response of system to $x(t - t_1)$; t_1 delay is,

$y(t, t_1) = t\, x(t - t_1)$

If the o/p is delayed by t_1,

$y(t - t_1) = (t - t_1)\, x(t - t_1)$

As $y(t, t_1) \neq y(t - t_1)$

so, the system is time varient. Ans

Q11) Explain why a sinusoidal wave is periodic and deterministic signal.

→

A sinusoidal wave $x(t) = A\cos(\omega t + \phi)$ is both periodic and deterministic because of its mathematical structure and predictable behaviour.

(i) Periodic

A signal is periodic if it repeats its shape after a fixed time interval, called period T.

For a sinusoidal wave,

- The cosine or sine function repeats itself at a regular interval.
- The period T is related to the frequency f by $T = \frac{1}{f} = \frac{2\pi}{\omega}$

(ii) Deterministic

A signal is deterministic if its behavious is completely predictable at any time, meaning there is no randomness involved.

For a sinusoidal wave,

- The parameters A (amplitude), ω (frequency) and φ (phase) fully define the wave.
- At any point ~~on the~~ in time t, the value of the sinusoid can be calculated using the formula $x(t) = A\cos(\omega t + \phi)$ without any uncertainty.

Q12) Find whether the signal is energy or power signal : $\delta[n]$.

sol^n

P.T.O.

$\delta[n]$

Here, Energy $= \sum_{n=-\infty}^{\infty} x^2[n]$

So, $E = \sum_{n=-\infty}^{\infty} \{\delta[n]\}^2$,

As $\delta[n] = 1$ for $n=0$ and $\delta[n]=0$ for $n \neq 0$,

So, $E = 1$ //

Now,

Power $= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x^2[n]$

Hence,

$\therefore P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \{\delta[n]\}^2$

$= \lim_{N\to\infty} \frac{1}{2N+1} \times 1$

$= \frac{1}{\infty}$

$= 0$

As, E is finite and power is zero.
So, $\delta[n]$ is energy signal.