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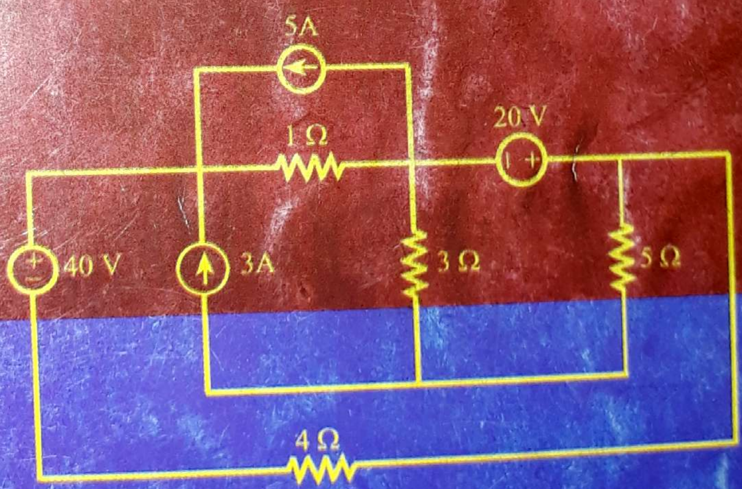
Revised Edition

A Course in

BASIC ELECTRICAL ENGINEERING



Er. Manish Pyakurel

Er. Manisha Maharjan

A Course in

BASIC ELECTRICAL ENGINEERING

B.E. First Year

Features of the book

- Balanced coverage provided of both theory & numerical.
- Subject matter written in lucid, direct & easily understandable manner.
- Simple diagrams, large number of solved examples & additional questions at the end of each chapter.
- Solution of old questions of past IOE exams.

By

Er. Manish Pyakurel
Lecturer, KEC, Kalimati

Er. Manisha Maharjan
Lecturer, KEC, Kalimati

G.L. Book House Pvt. Ltd.
Maitighar, Kathmandu (In front of Thapathali Campus)

ANIL

Preface to the Second Edition

We are very thankful to our readers for the welcome given to the previous edition of this book and giving us useful and constructive suggestions for the improvement of this book. We expect that this revised edition will prove even more useful to our readers.

We are very thankful to our publishers, seniors, colleagues, students for their support during the preparation of this edition. Our deepest appreciation goes to Associate prof. Jayaiswer Man Pradhan, Er. Archana Maharjan, Er.Nischal Guruwacharya, Er.Mukesh Gautam, Er. Nripesh Ayer, Er. Umesh Dahal, Er.Prajina Tandukar, Er.Abhilasha Bajracharya, Er. Binod Bhandari, Er. Kanchan Bohara, Er. Mahendra Kumar Das, Er. Saroj Khanal, Er. Sailesh Wasti, Er. Bikash Poudel, Er. Karshan Maharjan, Er. Binod Ghimire, Mr. Chitra Bhadur Khadka, Yadu kumar, Bashu Kalauni, Dipa Bhandari and Laxmi Man Shrestha for the advice, patience, understanding and professionalism that they demonstrated throughout the process of bringing the manuscript into print.

Any suggestions for the improvement of the book will be well appreciated and receive our best attention.

Authors

pyakurelmanis@gmail.com
maharjan.maneesha@gmail.com

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Preface to the First Edition

"*Basic Electrical Engineering*" is a compulsory course in the first year for BE students of all disciplines of engineering. This text is meant for the use as introductory course in electrical engineering. It covers the syllable of "Basic Electrical Engineering" as prescribed for IOE students.

This book is organized into seven chapters. In first four chapters, we shall deal with circuit solving techniques in DC network whereas in remaining portions we shall discuss AC analysis of circuits. Throughout this text, the basic concepts involved in electrical engineering are emphasized step by step procedures for solving the problems are described well. The problems have been worked out in sufficient details so that the reader can follow the procedures easily. The detailed solutions of past questions of IOE examinations are presented chapter wise in this book. Additional questions chapter and also listed at the end of this book to help the readers strengthen their understanding of the course.

It is earnestly hoped that this text will facilitate students not only to score better in examinations but also to gain clear knowledge of the subject matter discussed. Every care has been taken to avoid misprints and errors. We would be grateful to readers for bringing into notice any of such errors or misprints they may come across while going through the book.

We would like to express our deep gratitude to Kathmandu Engineering college (KEC) Kalimati for providing us platform and exposure in the education field as lecturers. We are very thankful to the publishers of "G.L. Book House Pvt. Ltd.", Maitighar, Kathmandu for giving us this marvelous opportunity and bringing out this book in such a short time. We appreciate the sincere efforts of Yadu Kumar, Rekha Baral, Hemanta Shrestha and Bashu Kalauni for shaping this book in presentable form and being patient with us while developing the book.

Without great support of Department of Electrical Engineering, KEC it would not have been possible to write this text timely. Our seniors and colleagues have always been very responsive in providing necessary advice and without their generous support and inspiration our work would have lacked proper supervision. We express our warm thanks to them for their guidance. We would also like to thank our family, friends and students for their valuable support and companionship.

Lastly would like to dedicate the first edition of this text to all our students who have encouraged us to develop this book.
Best wishes to all our readers.

pyakurelmanis@gmail.com
maharjan.maneesha@gmail.com

Authors

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GENERAL ELECTRIC SYSTEM

1.1 Electrical Current

Electric current may be defined as the time rate of net motion of electric charge across a cross sectional area in definite direction. At normal state, there is only random motion of free electrons in a metal which doesn't constitute an electric current because there is no net transfer of charge across a cross sectional area.

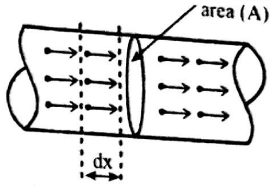


Fig. 1.1

Let, n = number of free electrons in a metal (per m^3)

v = drift velocity of electrons (in meter per sec)

A = cross-sectional area of the conductor

dx = distance moved by electrons in a small time interval of dt

= vdt

e = charge on an electron

Volume swept in time $dt = dx \times A$

∴ Net number of electrons transfer across dx section in dt sec

$= nAvdt$

∴ Net charge transfer across dx section in dt sec is given by

$dq = neAvdt.$

Now, Electric current, $i = \frac{dq}{dt} = \frac{neAvdt}{dt}$

∴ $i = neAv$ (coulombs/sec or ampere)

Current density, $J = \frac{i}{A} = nev$ (A/m^2)

1.2 Electric Circuit

An electric circuit is a closed path composed of various components through which electric current completes its path.

Different constituent parts of an electrical system are

(i) Source; provides energy to circuit such as battery, generators.
(ii) Conductor; used to carry current such as wires, cables.
(iii) Safety devices; for protection such as fuses, breakers.
(iv) Controlling devices; for control such as switch.
(v) Load; elements that utilizes electrical energy such as resistors.

1.3 Electromotive force and potential difference

Electromotive force (e.m.f) is the force that causes a current of electricity to flow. The potential difference (p.d) V, between two points in a circuit is the electrical pressure or voltage required to drive the current between them. The volt is unit of p.d and e.m.f.

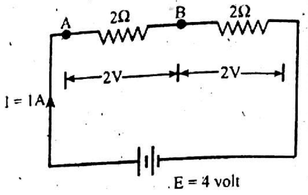


Fig. 1.2

E.m.f of device, say a battery, is a measure of the energy the battery gives to each coulomb of charge. Thus, if a battery supplies 4 joules of energy per coulomb, we say that it has an e.m.f of 4 volts.

The potential difference between two points in an electrical circuit is that difference in their electrical state which tends to cause flow of electric current between them. Here, the potential difference between point A and B is 2 volts, it means that each coulomb will give up an energy of 2 joules in moving from A to B.

1.4.1 Electrical units

Quantity	Unit	Symbol
Electric current	ampere	A
Power	watt	W, J/s
Quantity of electricity	coulomb	C, A/s
Potential, pd, emf	volt	V
Electric field strength	volt per metre	V/m
Resistance	ohm	Ω, V/A
Capacitance	farad	F
Magnetic flux	weber	wb
Inductance	henry	H
Magnetic flux density	tesla	T

1.4.2 Prefix and symbol

Factor by which units is multiplied	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hectro	h
10	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

1.5.1 Ohm's law

For a fixed metal conductor, the temperature and other conditions remaining constant, the current (I) through it is proportional to the potential difference (V) between its ends.

Mathematically,

$$\frac{V}{I} = \text{constant}$$

or, $$\frac{V}{I} = R$$

Where, R is the resistance of the conductor between the two points considered.

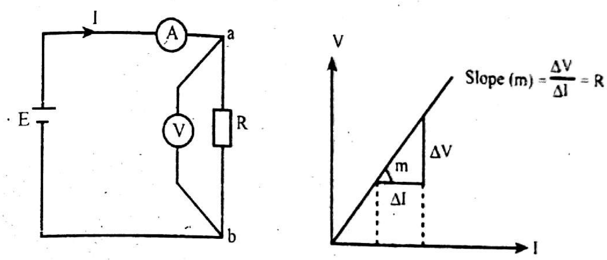


Fig. 1.3 Verification of Ohm's law

1.5.2 Ohmic and non-ohmic conductor

The conductor which exactly satisfy the ohm's law i.e. voltage/ current relationship is linear is called ohmic conductor or linear resistor. Example: Metals.

The conductor which doesn't exactly satisfy the ohm's law i.e. voltage/current relationship is not linear is called non-ohmic conductor or non linear resistor. Example: Diode.

1.6 Resistors, resistivity

Resistance may be defined as, the property of material by virtue of which it opposes the flow of electrons through the material. The circuit element having this property is known as resistor.

The resistance of a conducting wire is found to be :

(i) directly proportional to its length ($R \propto \ell$)

(ii) inversely proportional to its area of cross sectional ($R \propto \frac{1}{a}$)

(iii) depends upon the nature of the conducting material and

(iv) depends upon temperature.

i.e. $R \propto \frac{\ell}{a}$

$\therefore \quad R = \rho \frac{\ell}{a}$

Where, ρ (rho) is a constant of material called specific resistance or resistivity of a material. If l = 1m and a = 1m^2 then ρ = R. Hence, the resistivity of a material (ρ) is the resistance offered by 1m of its length and having a cross-section of 1m^2.

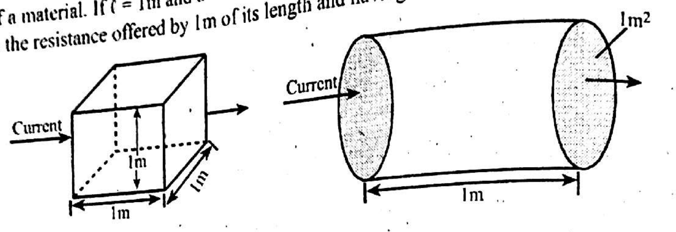


Fig. 1.4 Definition of resistivity of a material

The S.I unit of resistivity is ohm-metre.

1.7.1 Effect of temperature on resistance

The resistance of all pure metals and alloys increases with increase in temperature. Over the normal range of operating temperature the variation of resistance with temperature is linear. The temperature coefficient for pure metals and alloys is positive, whereas semi-conductors and insulating material have negative temperature coefficient. Thus, the resistance of semi-conductors and insulators decreases with increase in temperature. The temperature coefficient of metals decreases with increase in temperature.

1.7.2 Temperature coefficient of resistance

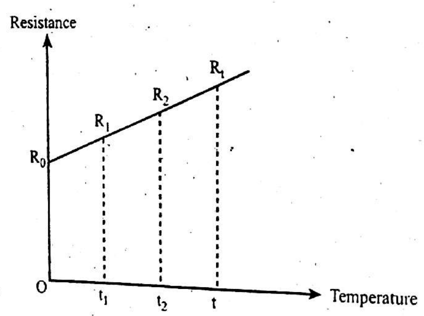


Fig. 1.5

If the resistance of any pure metal is plotted on a temperature base, the graph is as shown in figure.

Here, R_o = resistance at 0^0C

R_t = resistance at t^0C

Since, Change in resistance is directly proportional to the initial resistance and to the rise in temperature. So,

$$(R_t - R_o) \propto R_0 \quad \text{—— (i)}$$

$$(R_t - R_o) \propto t \quad \text{—— (ii)}$$

Combining equations (i) and (ii), we get

$$(R_t - R_o) \propto R_0 t$$

$$\therefore \quad (R_t - R_o) = \alpha_o R_o t \quad \text{—— (iii)}$$

Where α_o is the temperature coefficient of resistance at 0^0C whose value depends upon the nature of material and temperature.

From equation (iii),

$$R_t = R_o + \alpha_o R_0 t$$

$$\therefore \quad R_t = R_o [1 + \alpha_o t]$$

Also,

From equation (iii)

$$\alpha_o = \frac{R_t - R_o}{R_0 t}$$

Temperature coefficient of resistance may be defined as the ratio of increase in resistance per degree of rise of temperature to the original resistance.

Note: **If the resistance of a conductor is R_2 at t_2^0 C and R_1 at t_1^0C ($t_1 < t_2$), then**

$$R_2 = R_1 [1 + \alpha_1 (t_2 - t_1)]$$

Example: Show that if α_1 is the **resistance temperature coefficient of a conductor at t_1^0C then resistance temperature coefficient at t_2^0 C is given by**

$$\frac{1}{\frac{1}{\alpha_1} + (t_2 - t_1)}$$

Solution:

Let R_1, R_2 and R_3 be the resistances of a conductor at temperatures t_1, t_2 and t_3°C respectively.

Then,

$$R_2 = R_1 [1 + \alpha_1 (t_2 - t_1)] \quad \text{—— (i)}$$

$$R_3 = R_1 [1 + \alpha_1 (t_3 - t_1)] \quad \text{—— (ii)}$$

Also,

$$R_3 = R_2 [1 + \alpha_2 (t_3 - t_2)]$$

or, $$\frac{R_3}{R_2} = [1 + \alpha_2 (t_3 - t_2)] \quad \text{—— (iii)}$$

Now, We divide equation (ii) by equation (i)

$$\frac{R_3}{R_2} = \frac{R_1 [1 + \alpha_1 (t_3 - t_1)]}{R_1 [1 + \alpha_1 (t_2 - t_1)]}$$

or, $$\frac{R_3}{R_2} = \frac{1 + \alpha_1(t_2 - t_1) + \alpha_1(t_3 - t_1) - \alpha_1(t_2 - t_1)}{1 + \alpha_1(t_2 - t_1)}$$

or, $$\frac{R_3}{R_2} = 1 + \frac{\alpha_1(t_3 - t_2)}{1 + \alpha_1(t_2 - t_1)} \quad \text{—— (iv)}$$

Comparing equation (iii) and equation (iv), we get,

or, $$1+\alpha_2(t_3-t_2) = 1+\frac{\alpha_1(t_3-t_2)}{1+\alpha_1(t_2-t_1)}$$

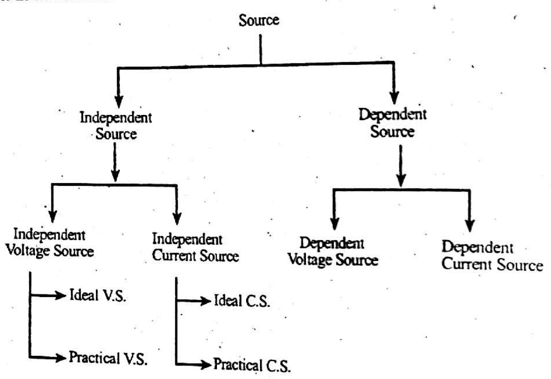
or, $$\alpha_2(t_3-t_2) = \frac{\alpha_1(t_3-t_2)}{1+\alpha_1(t_2-t_1)}$$

or, $$\alpha_2 = \frac{\alpha_1}{1+\alpha_1(t_2-t_1)}$$

$\therefore$ $$\alpha_2 = \frac{1}{\frac{1}{\alpha_1}+(t_2-t_1)}$$

1.8 Source

It is one of the constituent of an electrical system responsible for the flow of current in the circuit.



1.8.1 Ideal and practical voltage sources

An ideal voltage source is that which can give a constant terminal voltage across the load over infinite variation in load or load current. An ideal voltage source possess zero internal resistance or impedance.

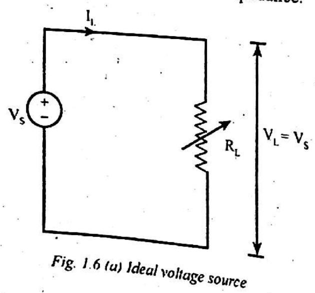


Fig. 1.6 (a) Ideal voltage source

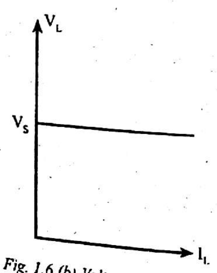


Fig. 1.6 (b) Volt - amp characteristics of ideal voltage source

Realization of such voltage source in practice is not possible. An actual voltage source will have some internal resistance. Hence, the terminal voltage across load will decrease according to load current due to voltage drop in internal resistance. A real voltage source can be treated mathematically as an ideal voltage source in series with its internal resistance.

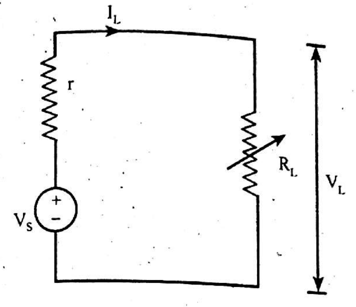


Fig. 1.7 (a) Real voltage source

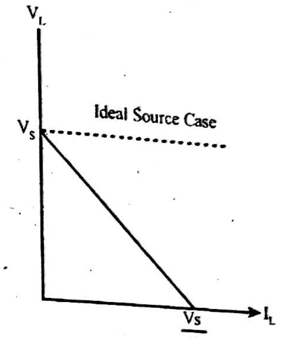


Fig. 1.7 (b) Volt - amp characteristics of real voltage source

The relation between source voltage and terminal load voltage is given by,

$$V_L = V_S - I_L r$$

When $I_L = 0$ then $V_L = V_S$

When $I_L = I_{sc} = \frac{V_s}{r}$ then $V_L = V_s - I_L r = V_s - \frac{V_s}{r}\times r = 0$

1.8.2 Ideal and practical current sources

An ideal current source is that source which gives a fixed constant load current despite infinite variation in load or load voltage. An ideal current source posses infinite internal resistance or impedance. For an ideal source load current, $I_L = I_s$, whatever the load voltage may be.

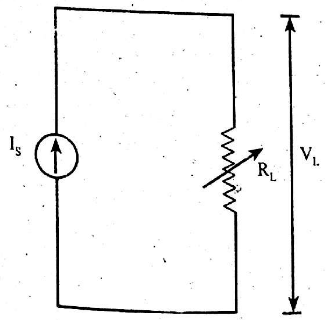


Fig. 1.8 (a) Ideal current source

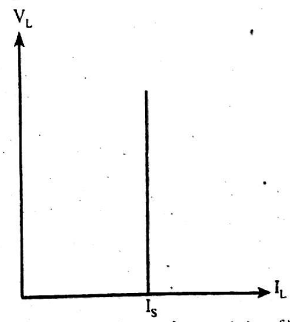


Fig. 1.8 (b) Volt-amp characteristics of ideal current source

For a practical current source, the load current is given by,

$$I_L = I_s - \frac{V_L}{R_s}$$

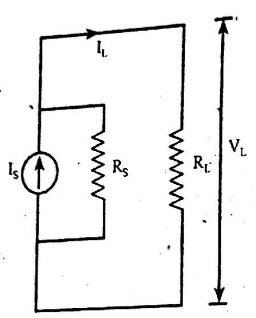


Fig. 1.9 (a) Practical current source

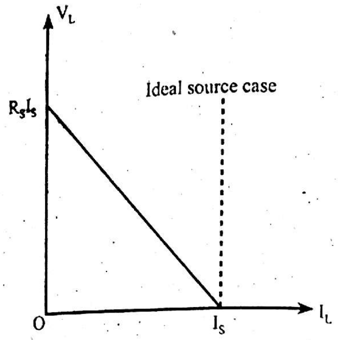


Fig. 1.9 (b) Volt-amp characteristics of practical current source

Here, the two extreme cases are,

when $R_L = 0$, $V_L = 0$ then $I_L = I_S$

when $R_L = \infty$, $I_L = 0$ then $V_L = I_S R_S$

1.9 Source transformation

Two sources are defined as being equivalent if they produce identical values of terminal voltage V_L and load current I_L connected to an identical load resistance R_L.

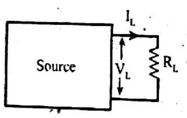


Fig. 1.10 (a) A source connected to load

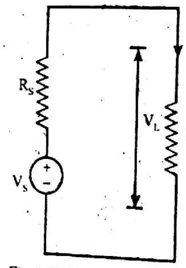


Fig. 1.10 (b) Voltage source representation

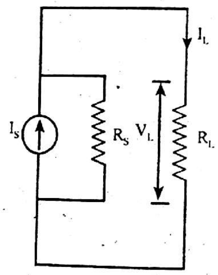


Fig. 1.10 (c) Current source representation

Example:

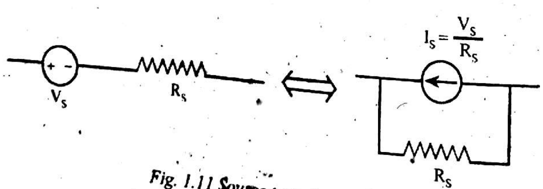


Fig. 1.11 Source transformation

Exam Solutions

1. **An aluminium wire 7.5 m long is connected in parallel with a copper wire 6 m long. When a current of 5A is passed through the parallel combination, it is found that the current in the aluminium wire is 3A. The diameter of aluminium wire is 1mm. Determine the diameter of copper wire, the resistivity of copper is 0.017 μΩ-m and that of aluminium is 0.028 μΩ-m.**

Solution: **[2065 Kartik]**

Using subscript symbols 1 and 2 for aluminium (Al) and copper (Cu) respectively.

We have,

$$\ell_1 = 7.5 \text{ m}; \; a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(1 \times 10^{-3})^2 = 7.854 \times 10^{-7} \text{ m}^2$$

$$\rho_1 = 0.028 \; \mu\Omega\text{-m}$$

$$I = 5A, \; I_1 = 3A, \; I_2 = I - I_1 = 2A$$

$$\ell_2 = 6\text{m}, \; \rho_2 = 0.017 \; \mu\Omega\text{-m}.$$

$$\therefore \quad R_1 = \rho_1 \frac{\ell_1}{a_1}$$

$$= 0.028 \times 10^{-6} \times \frac{7.5}{7.854 \times 10^{-7}}$$

$$= 0.2673 \; \Omega$$

$\therefore$ Voltage drop across the two ends of Al wire,

$$V_1 = I_1 R_1 = 3 \times 0.2673 = 0.8022 \text{ V}$$

$$R_2 = \rho_2 \frac{\ell_2}{a_2}$$

$$= \frac{0.017 \times 10^{-6} \times 6}{a_2} \; \Omega$$

$\therefore$ Voltage drop through the two ends of Cu wire,

$$V_2 = I_2 R_2 = 2 \times \frac{0.017 \times 10^{-6} \times 6}{a_2}$$

$$= \frac{2.04 \times 10^{-7}}{a_2} \text{ V}$$

Since the wires are connected in parallel. So $V_1 = V_2$ i.e.,

$$0.8022 = \frac{2.04 \times 10^{-7}}{a_2}$$

$$\text{or, } a_2 = \frac{2.04 \times 10^{-7}}{0.8022}$$

$$\text{or, } \frac{\pi d_2^2}{4} = 2.54 \times 10^{-7}$$

$$\therefore d_2 = 0.5[illegible] \times 10^{-3}\text{m}$$

Hence, diameter [illegible] copper wire is 0.569×10^{-3}m or 0.569 mm.

2. 1 km of wire having a diameter of 11.7 mm and of resistance 0.031 Ω is drawn so that its diameter becomes 5mm. What does it resistance become? [2066 Kartik]

Solution:

Using subscript symbols 1 and 2 for the wire before and after it is drawn.

$R_1 = 0.031\,\Omega$

$r_1 = \frac{1}{2} \times 11.7 = 5.85 \times 10^{-3}m$

$r_2 = \frac{1}{2} \times 5 = 2.5 \times 10^{-3}m$

Now,

$R = \rho\frac{\ell}{a} = \rho\frac{\frac{V}{a}}{a} = \rho\frac{V}{a^2}$ (Where, V = volume of wire)

Since, volume of wire is constant and the materials same, so:

$R \propto \frac{1}{a^2}$

or, $\frac{R_2}{R_1} = \left(\frac{a_1}{a_2}\right)^2$

or, $R_2 = R_1\left(\frac{a_1}{a_2}\right)^2$

or, $R_2 = R_1\left(\frac{\pi r_1^2}{\pi r_2^2}\right)^2$

or, $R_2 = R_1\left(\frac{r_1}{r_2}\right)^4$

or, $R_2 = 0.031 \times \left(\frac{5.85 \times 10^{-3}}{2.5 \times 10^{-3}}\right)^4$

$\therefore\ R_2 = 0.928\,\Omega$

3. A coil is connected across a constant dc source of 120V. It draws a current of 12 Amp at room temperature of 25°C. After 5 hours of operation, its temperature rises to 65°C and current reduces to 8 Amp. Calculate:

(i) Current when its temperature has increased to 80°C.

(ii) Temperature coefficient of resistance at 30°C. [2067 Mangsir]

Solution:

Here, Resistance of coil at 25°C, $R_{25} = \frac{120}{12} = 10\,\Omega$

Resistance of coil at 65°C, $R_{65} = \frac{120}{8} = 15\,\Omega$

$\because R_{65} = R_{25}[1 + \alpha_{25}(65 - 25)]$

or, $15 = 10[1 + \alpha_{25} \times 40]$

$\therefore\ \alpha_{25} = 0.0125/^0C$

(i) Resistance of coil at 80°C

$R_{80} = R_{25}[1 + \alpha_{25}(80 - 25)]$

$= 10[1 + 0.0125 \times 55]$

$= 16.875\,\Omega$

Current at this temperature of $80^0C = \frac{120}{16.875}$

$= 7.111$ A

(ii) Temperature coefficient of resistance at 30° C,

$\alpha_{30} = \frac{1}{\frac{1}{\alpha_{25}} + (30 - 25)} = \frac{1}{\frac{1}{0.0125} + 5}$

$\therefore\ \alpha_{30} = 0.01176/^0C$

4. **The temperature rise of a machine field winding was determined by the measurement of the winding resistance. At 20°C, the field resistance was 150 Ω. After running the m/c for 6 hours at full load, the resistance was 175 Ω. The temperature coefficient of resistance of the copper winding is 4.3×10^{-3}/K at 0°C. Determine the temperature rise of the m/c.** **[2068 Baishakh]**

Solution:

Resistance at 20°C, $R_{20} = 150\,\Omega$

Resistance at full load, $R_t = 175\,\Omega$

Temperature coefficient of resistance at 0°C, $\alpha_0 = 4.3 \times 10^{-3}$/K

Temperature rise, t – 20 = ?

We know,

$\alpha_{20} = \frac{1}{\frac{1}{\alpha_0} + (20 - 0)}$

$= \frac{1}{\frac{1}{4.3 \times 10^{-3}} + 20}$

$\therefore\ \alpha_{20} = 3.959 \times 10^{-3}$/K

Since, $R_t = R_{20}[1 + \alpha_{20}(t - 20)]$

or, $175 = 150[1 + 3.959 \times 10^{-3} \times (t - 20)]$

$\therefore\ t - 20 = 42.098\ ^0C$

Hence, temperature rise is 42.098°C.

5. **The coil of a relay takes a current of 0.12A when it is at the room temperature of 15°C and connected across a 60V supply. If the minimum operating current of the relay is 0.1A, calculate the temperature above which the relay will fail to operate when connected to the same supply. Resistance temperature coefficient of the coil material is 0.0043 per °C at 6°C.** **[2068 Bhadra]**

Solution:

Resistance at 15^0C, $R_{15} = \frac{60}{0.12} = 500\ \Omega$

Resistance at t^0C, $R_t = \frac{60}{0.1} = 600\ \Omega$

Temperature coefficient of the coil at 6^0C, $\alpha_6 = 0.0043/^oC$

∴ Temperature coefficient of the coil at 15^0C,

$$\alpha_{15} = \frac{1}{\frac{1}{\alpha_6} + (15-6)} = \frac{1}{\frac{1}{0.0043} + (15-6)}$$

$$\therefore \quad \alpha_{15} = 4.1397 \times 10^{-3}/^oC$$

Since,

$$R_t = R_{15}[1 + \alpha_{15}(t - 15)]$$

or, $600 = 500\,[1 + 4.1397 \times 10^{-3}\,(t - 15)]$

$\therefore t - 15 = 48.31\ ^0C$

Hence, for the temperature rise of 48.31^0C, the relay will operate.

6. A 230V metal filament lamp has its filament 50cm long with cross sectional area of 3×10^{-6} cm². Specific resistance of the filament metal at 20^0C is $4 \times 10^{-6}\ \Omega$ cm. If the working temperature of the filament is 2000^0C, find the wattage of the lamp. Temperature coefficient of resistance of the filament material at 20^0C is 0.0055 per degree centigrade. [2069 Bhadra]

Solution:

Resistance of filament at 20^0C,

$$R_{20} = \rho_{20}\frac{\ell}{A}$$

$$= 4 \times 10^{-6} \times 10^{-2} \times \frac{50 \times 10-2}{3 \times 10^{-6} \times 10^{-4}} = 66.6667\ \Omega$$

Temperature coefficient of resistance at 20^0C, $\alpha_{20} = 0.0055/^0C$.

We know,

Resistance of filament at 2000^0C,

$R_{2000} = R_{20}\,[1 + \alpha_{20}(2000 - 20)]$

or, $R_{2000} = 66.6667[1 + 0.0055 \times 1980]$

∴ $R_{2000} = 792.6671\ \Omega$

∴ Wattage of the lamp,

$$P = \frac{V^2}{R_{2000}} = \frac{230^2}{792.6671} = 66.7368 \text{ watt}$$

7. A lead wire and an iron wire are connected in parallel. Their specific resistances are in the ratio 49:24. The former carries 80% more current than the latter and the latter is 47% longer than the former. Determine the ratio of their cross-sectional areas. [2069 Ashad]

Solution:

Let suffix 1 represent lead and suffix 2 represent iron, we are given,

$$\frac{\rho_1}{\rho_2} = \frac{49}{24}$$

If $i_2 = 1, i_1 = 1.8$; if $\ell_1 = 1, \ell_2 = 1.47$

Now,

$$R_1 = \rho_1\frac{\ell_1}{A_1} \text{ and } R_2 = \rho_2\frac{\ell_2}{A_2}$$

Since the two wires are in parallel,

$$i_1 = \frac{V}{R_1} \text{ and } i_2 = \frac{V}{R_2}$$

$$\therefore \quad \frac{i_2}{i_1} = \frac{R_1}{R_2} = \frac{\rho_1\ell_1}{A_1} \times \frac{A_2}{\rho_2\ell_2}$$

$$\therefore \quad \frac{A_2}{A_1} = \frac{i_2}{i_1} \times \frac{\rho_2\ell_2}{\rho_1\ell_1} = \frac{1}{1.8} \times \frac{24}{49} \times 1.47 = 0.4$$

8. The resistivity of a metal alloy is $50 \times 10^{-8}\ \Omega$ m. A sheet of material 15cm long, 6 cm wide and 0.014 cm thick. Calculate the resistance in the direction:

(a) along the length and

(b) along the thickness. [2069 chaitra]

Solution:

(a) along the length

Here,

$\ell = 15$ cm $= 0.15$ m

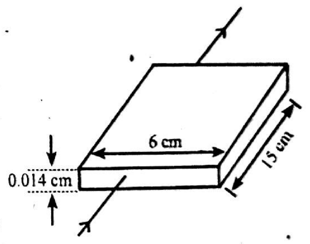
$A = 6 \times 10^{-2} \times 0.014 \times 10^{-2}$

$= 8.4 \times 10^{-6}$m²

$\therefore \quad R = \rho\frac{\ell}{a}$

$= 50 \times 10^{-8} \times \frac{0.15}{8.4 \times 10^{-6}}$

$= 8.928 \times 10^{-3}\ \Omega$



(b) along the thickness

Here,

$\ell = 0.014$ cm $= 1.4 \times 10^{-4}$ m

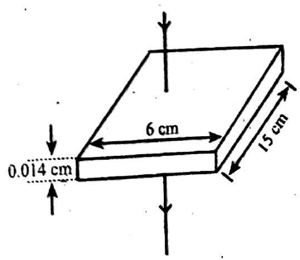
$a = 6 \times 10^{-2} \times 15 \times 10^{-2}$

$= 9 \times 10^{-3}$m²

$\therefore \quad R = \rho\frac{\ell}{a}$

$= 50 \times 10^{-8} \times \frac{1.4 \times 10^{-4}}{9 \times 10^{-3}}$

$= 7.77 \times 10^{-9}\ \Omega$



9. Explain the method for converting practical current source in to practical voltage source. **[2069 Chaitra]**

Ans:

A practical current and voltage source is represented in the figure below. Practical sources takes into account the internal resistance.

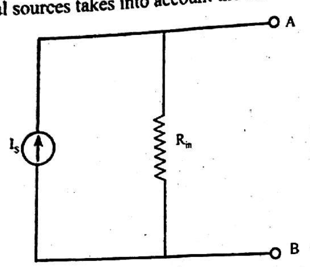


Fig. (a) Practical current source

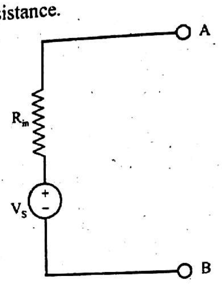


Fig. (b) Practical Voltage Source

When there is no any load connected in figure (a), then open circuit voltage across AB is,

$V_{AB} = I_s R_{in}$ ....(i)

Also,

When there is no any load connected in figure (b), then open circuit voltage across AB is,

$V_{AB} = V_s$ ....(ii)

It is to be noted that a current source parallel resistance combination is equivalent to a voltage source series resistance combination if and only if both combinations can produce identical values of terminal voltage $V_{AB} = V_L$ and load current I_L connected to an identical load resistance R_L.

So, from (i) and (ii), we write

$V_S = I_S R_{in}$

Where, R_{in} = internal resistance

Here,

$$R_{in} = \frac{V_S}{I_S} = \frac{\text{Open-circuit voltage}}{\text{Short circuit current}}$$

Hence, current source I_S and source resistance R_{in} can be replaced by a voltage source, $V_S = I_S R_{in}$ and a source resistance R_{in} in series with V_S.

10. A conductor material has a free electron density of 10^{24} electrons per m^3. When a voltage is applied a constant drift velocity of 1.5×10^{-2} m/s is attained by the electrons. If the cross sectional area of the material is 1 cm^2, calculate the magnitude of the current. **[2070 Ashad]**

Solution:

Electron density, $n = 10^{24}$

Electron charge, $e = 1.6 \times 10^{-19}C$

Drift velocity, $v = 1.5 \times 10^{-2}$ m/s = 0.015 m/s

Cross - sectional area, $A = 1\ cm^2 = 1 \times 10^{-4}\ m^2$

Magnitude of the current, i = ?

We know,

$$i = nAev$$

$$= 10^{24} \times 1 \times 10^{-4} \times 1.6 \times 10^{-19} \times 0.015 = 0.24\ A$$

11. A piece of resistance wire, 15.6m long of cross-sectional area $12mm^2$ at a temperature of 0^0C, passes a current of 7.9A when connected to DC supply at 240V. Calculate:

(i) resistivity of the wire

(ii) the current when the temperature rises to 55°C. The temperature coefficient of the wire is 0.00029/°C. **[2070 magh]**

Solution:

Resistance of wire, $R = \frac{V}{I} = \frac{240}{7.9} = 30.38\ \Omega$

But, $R = \rho\frac{\ell}{a} = \rho \times \frac{15.6}{12 \times 10^{-6}}$

or, $30.38 = \rho \times \frac{15.6}{12 \times 10^{-6}}$

or, $\frac{30.38 \times 12 \times 10^{-6}}{15.6} = \rho$

$\therefore$ $\rho = 2.34 \times 10^{-5}\ \Omega$ m.

Since, $R_{55} = R_0[1 + \alpha_0(55 - 0)]$

or, $R_{55} = 30.38[1 + \alpha_0 \times 55]$

or, $R_{55} = 30.38[1 + 0.00029 \times 55]$

$\therefore$ $R_{55} = 30.86\ \Omega$.

$\therefore$ Current, $I = \frac{V}{R_{55}} = \frac{240}{30.86} = 7.78\ A$

12. Two resistors made of different materials have temperature coefficients of resistance $\alpha_1 = 0.004\ ^0C^{-1}$ and $\alpha_2 = 0.005^0C^{-1}$ are connected in parallel and consume equal power at 15^0C. What is the ratio of power consumed in resistance R_2 to that in R_1 at 70°C? **[2070 Chaitra]**

Solution:

Power consumed in a resistor $= \frac{V^2}{R}$. Since the two resistors are connected in parallel the voltage across each resistor is the same.

At 15^0C;

$P_1 = P_2$

or, $\frac{V^2}{R_1} = \frac{V^2}{R_2}$

or, $R_1 = R_2$

or, $R_{01}(1 + 15\alpha_1) = R_{02}(1 + 15\alpha_2)$

or, $\frac{R_{01}}{R_{02}} = \frac{1 + 15\alpha_2}{1 + 15\alpha_1}$ ——— (i)

Now, At 70^0C;

$R_1 = R_{01}(1 + 70\alpha_1)$; $R_2 = R_{02}(1 + 70\alpha_2)$

Power ratio at 70^0C is ;

$$\frac{P_2}{P_1} = \frac{\left(\frac{V^2}{R_2}\right)}{\left(\frac{V^2}{R_1}\right)} = \frac{R_1}{R_2} = \frac{R_{01}(1 + 70\alpha_1)}{R_{02}(1 + 70\alpha_2)}$$

Using equation (i)

$$\frac{P_2}{P_1} = \frac{(1 + 15\alpha_2)(1 + 70\alpha_1)}{(1 + 15\alpha_1)(1 + 70\alpha_2)}$$

$$= \frac{(1 + 15 \times 0.005)(1 + 70 \times 0.004)}{(1 + 15 \times 0.004)(1 + 70 \times 0.005)} = 0.962$$

13. A coil has a resistance of 18 Ω when its mean temperature is 20^0C and 20Ω when its mean temperature is 50° C. Find its mean temperature rise when its resistance is 21 Ω and the ambient temperature is 15^0C. [2071 Bhadra]

Solution:

Here,

Resistance of coil at $20^{\circ}C$, $R_{20} = 18\ \Omega$

Resistance of coil at $50^{\circ}C$, $R_{50} = 20\ \Omega$

Ambient temperature , $t_{amb} = 15°C$

Let, t be the mean temperature when the resistance is 21 Ω i.e. $R_t = 21\Omega$

we know,

$R_{20} = R_{15}[1 + \alpha_{15}(20 - 15)]$

or, $18 = R_{15}[1 + 5\alpha_{15}]$ .... (i)

Also,

$R_{50} = R_{15}[1 + \alpha_{15}(50 - 15)]$

or, $20 = R_{15}[1 + 35\alpha_{15}]$ .... (ii)

dividing equation (i) by equation (ii)

we get,

$$\frac{18}{20} = \frac{1 + 5\alpha_{15}}{1 + 35\alpha_{15}}$$

or, $0.9(1 + 35\alpha_{15}) = 1 + 5\alpha_{15}$

or, $0.9 + 31.5\alpha_{15} = 1 + 5\alpha_{15}$

or, $26.5\alpha_{15} = 0.1$

∴ $\alpha_{15} = \frac{1}{265}$ /°C

Now,

Putting the value of α_{15} in eq (i), we get ,

$$R_{15} = \frac{18}{1 + 5 \times \frac{1}{265}}$$

$= 17.67\ \Omega$

Since, $R_t = R_{15}[1 + \alpha_{15}(t - 15)]$

or, $21 = 17.67\left[1 + \frac{1}{265}(t - 15)\right]$

∴ $t - 15 = 49.94$ °C

Hence, mean temperature rise is 49.94°C

14. A coil connected to a constant dc supply of 100V drew a current of 13A at room temperature of 25°C. After sometime, its temperature increased to 70°C and current fell to 8.5 A. Find the current it will draw when its temperature will further rise to 80°C. Also find the temperature coefficient of resistance of the coil at 20°C. [2071 Magh]

Solution:

Here,

Resistance of coil at 25°C, $R_{25} = \frac{100}{13} = 7.692\Omega$

Resistance of coil at 70°C, $R_{70} = \frac{100}{8.5} = 11.765\Omega$

we know,

$R_{25} = R_0[1 + \alpha_0(25 - 0)]$ .... (i)

$R_{70} = R_0[1 + \alpha_0(70 - 0)]$ .... (ii)

where , R_0 = resistance of coil at 0°C

α_0 = temperature coefficient of resistance at 0°C.

dividing equation (i) by equation (ii), we get

$$\frac{R_{25}}{R_{70}} = \frac{1 + 25\alpha_0}{1 + 70\alpha_0}$$

or, $\frac{7.692}{11.765} = \frac{1 + 25\alpha_0}{1 + 70\alpha_0}$

or, $0.654(1 + 70\alpha_0) = 1 + 25\alpha_0$

or, $0.654 + 45.766\alpha_0 = 1 + 25\alpha_0$

∴ $\alpha_0 = 0.0167$/°C

Using equation (i)

$$R_0 = \frac{R_{25}}{1 + 25\alpha_0} = \frac{7.692}{1 + 25 \times 0.0167} = 5.426\Omega$$

Resistance of coil at 80°C

$R_{80} = R_0[1 + \alpha_0(80 - 0)]$

$= 5.426[1 + 0.0167 \times 80]$

$= 12.675\Omega$

Temperature coefficient of resistance of coil at 20°C,

$$\alpha_{20} = \frac{1}{\frac{1}{\alpha_0} + (20 - 0)} = \frac{1}{\frac{1}{0.0167} + 20} = 0.0125/°C$$

15. The field winding of d.c. motor connected across 230V supply takes 1.15A at room temperature of 20°C. After working for some hours the current falls to 0.26A, the supply voltage remaining constant. Calculate the final working temperature of field winding. Resistance temperature coefficient of copper at 20°C is $\frac{1}{254.5}$. [2071 chaitra]

Solution,

Here,

Resistance at 20°C, $R_{20} = \frac{230}{1.15} = 200\Omega$

Resistance at t°C, $R_t = \frac{230}{0.26} = 884.615\Omega$

Resistance temperature coefficient of copper at 20°C , $\alpha_{20} = \frac{1}{254.5}$ /°C

Since, $R_t = R_{20}[1 + \alpha_{20}(t-20)]$

or, $884.615 = 200\left[1 + \frac{1}{254.5}(t-20)\right]$

or, $4.423 = 1 + \frac{1}{254.5}(t-20)$

or, $3.423 = \frac{1}{254.5}(t-20)$

or, $t - 20 = 871.154$

$\therefore$ $t = 891.154°C$

Hence, final working temperature of field winding is 891.154°C

Additional Problems

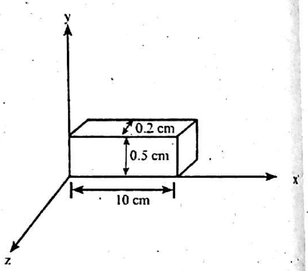
1. A rectangular metal strip has the dimensions x = 10 cm, y = 0.5 cm, z = 0.2 cm. Determine the ratio of the resistances Rx, Ry and Rz between the respective pair of opposite faces.

Solution

$\ell_x = 10$ cm; $a_x = 0.5\text{cm} \times 0.2\text{cm} = 0.1\text{cm}^2$

$\ell_y = 0.5$cm; $a_y = 10\text{cm} \times 0.2\text{cm} = 2\text{cm}^2$

$\ell_z = 0.2$cm; $a_z = 10\text{cm} \times 0.5\text{cm} = 5\text{cm}^2$



$R_x = \rho\frac{l_x}{a_x} = \rho\frac{10}{0.1} = 100\,\rho$

$R_y = \rho\frac{l_y}{a_y} = \rho\frac{0.5}{2} = 0.25\,\rho$

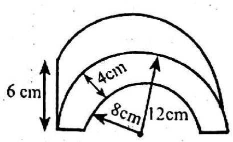
$R_z = \rho\frac{l_z}{a_z} = \rho\frac{0.2}{5} = 0.04\,\rho$

$\therefore$ $R_x : R_y : R_z = 100 : 0.25 : 0.04 = 10000 : 25 : 4$

2. A semicircular ring of copper has an inner radius of 80mm, radial thickness 40 mm and axial thickness 60 mm. Calculate the resistance of the ring at 50°C between its two end faces. Resistivity of copper at 20°C is 1.724×10^{-8} Ωm. Resistance temperature coefficient of copper at 0°C is 0.0043/°C.

Solution:

The semicircular ring is shown in figure



Mean radius of the ring $= \frac{80 + 120}{2}$

$= 100\text{mm} = 10\text{cm}.$

Mean length of the ring $= 10\pi$ cm $= 31.416$cm.

Area of cross section of the ring $= 6 \times 4 = 24\text{cm}^2$

$\alpha_0 = 0.0043$ / °C

$$\alpha_{20} = \frac{1}{\frac{1}{0.0043} + (20-0)} = 0.00396/^0C$$

$\rho_{20} = 1.724 \times 10^{-8}\,\Omega$ m

$\therefore$ $\rho_{50} = \rho_{20}[(1 + \alpha_{20}(50-20))] = 1.724 \times 10^{-8}[1 + 0.00396 \times 30]$

$= 1.929 \times 10^{-8}\,\Omega$m.

$R_{50} = \rho_{50} \times \frac{\ell}{a} = 1.929 \times 10^{-8} \times \frac{31.416 \times 10^{-2}}{24 \times 10^{-4}} = 2.525 \times 10^{-6}\,\Omega$

3. A specimen of copper wire has a specific resistance of 1.7×10^{-8} Ωm at 0°C and has temperature coefficient of 1/254.5 per degree celsius at 20°C. Find the specific resistance and temperature coefficient at 70° C.

Solution:

$\rho_0 = 1.7 \times 10^{-8}\,\Omega$ m

$\alpha_{20} = \frac{1}{254.5}/^0C$

$$\alpha_0 = \frac{1}{\frac{1}{\left(\frac{1}{254.5}\right)} + (0-20)} = \frac{2}{469}/^0C$$

$\rho_{70} = \rho_0\,[1 + \alpha_0(70-0)] = 1.7 \times 10^{-8}\left[1 + \frac{2}{469} \times 70\right] = 2.2074 \times 10^{-8}\,\Omega$
m

$\therefore$ Temperature coefficient at 70°C,

$$\alpha_{70} = \frac{1}{\frac{1}{\alpha_{20}} + (70-20)} = \frac{1}{\left(\frac{1}{254.5}\right) + 50} = \frac{2}{609}/^0C = \frac{1}{304.5}/^0C$$

4. A 60W, 240V incandescent filament lamp is switched on at 20°C. The operating temperature of the filament is 2000°C. Determine the current taken by the lamp at the instant of switching on. The temperature coefficient of resistance of filament material is 0.0045/°C.

Solution:

Let R_1 and R_2 be the resistance of the filament of the lamp at 20° C and 2000°C respectively.

Here, Temperature coefficient at 20°C, $\alpha_{20} = 0.0045$/°C

$\because$ $P = \frac{V^2}{R}$

$\therefore$ $R = \frac{V^2}{P}$

Then, $R_{2000} = \frac{(240)^2}{60} = 960\,\Omega$

$\because$ $R_{2000} = R_{20}[1 + \alpha_{20}(2000-20)]$

or, $960 = R_{20}[1 + 0.0045 \times 1980]$

$\therefore$ $R_{20} = 96.9\,\Omega$

Therefore, current taken by the lamp at the instant of switching on $= \frac{V}{R_{20}} = \frac{240}{96.9} = 2.48$ A

5. At 30°C, two coils connected in series having resistance of 800Ω and 400 Ω respectively and temperature coefficient of resistance Coil 1 and coil 2 are,

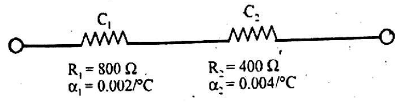
α_{C1} = 0.002 /°C at 30°C

α_{C2} = 0.004 /°C at 30°C

(i) Find the resistance of combination at 60° C.

(ii) Effective temperature coefficient of resistance of combination at 30°C.

Solution:



For coil 1;

$$R_{60} = R_{30}[1 + \alpha_{30}(60 - 30)] = 800[1 + 0.002 \times 30] = 848\ \Omega$$

For coil 2;

$$R_{60} = R_{30}[1 + \alpha_{30}(60 - 30)] = 400[1 + 0.004 \times 30] = 448\ \Omega$$

(i) Resistance of combination at 60°C,

$$R_{60\,(comb)} = 848\ \Omega + 448\ \Omega = 1296\ \Omega$$

(ii) $R_{30\,(comb)} = 800 + 400 = 1200\ \Omega$

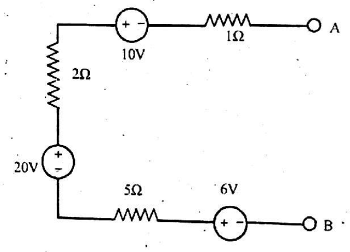
$\because$ $R_{60\,(comb)} = R_{30\,(comb)}[1 + \alpha_{30\,(comb)}(60 - 30)]$

or, $1296 = 1200[1 + \alpha_{30\,(comb)} \times 30]$

$\therefore$ $\alpha_{30\,(comb)} = 2.67 \times 10^{-3}/°C$.

Hence, temperature coefficient of resistance of combination at 30° C is $2.67 \times 10^{-3}/°C$.

6. **Find the equivalent current source for the circuit shown in figure.**



Solution:

The equivalent voltage source is obtained as follows:

V_{eq} = 6 + 20 − 10 = 16 V

R_{eq} = 5 + 2 + 1 = 8 Ω

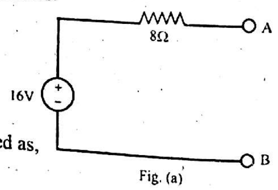


Fig. (a)

Fig (a) represents the single equivalent voltage source. The equivalent current source can be obtained as,

$$I_{eq} = \frac{16}{8} = 2A ; R = 8\ \Omega$$

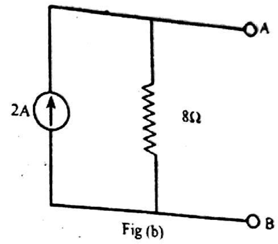
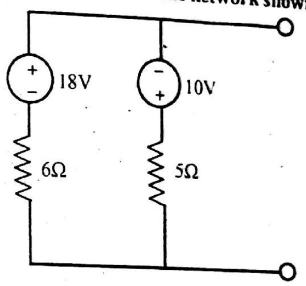


Fig (b)

Fig (b) represents the required equivalent current source.

7. **Obtain a single current source for the network shown in figure.**



Solution:

Let us first convert the voltage sources to equivalent current source [fig (a)].

Here, $I_1 = \frac{18}{6} = 3\ A ; r_1 = 6\ \Omega$

$I_2 = \frac{10}{5} = 2\ A ; r_2 = 5\ \Omega$

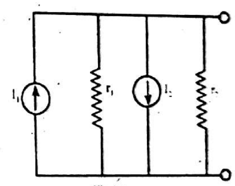


Fig (a)

The equivalent current source is obtained as,

$I_{eq} = I_1 - I_2 = 1A$

$$R = \frac{r_1 r_2}{r_1 + r_2} = \frac{6 \times 5}{6 + 5} = 2.73\ \Omega$$

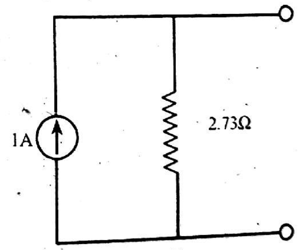
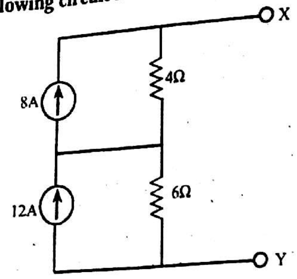


Fig (b)

Hence, fig(b) represents the equivalent current source.

8. **Convert the following circuit into a single voltage source.**



Solution:

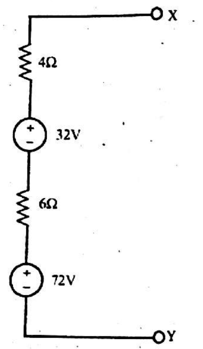


Fig. (a)

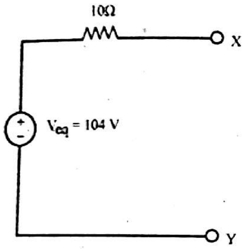


Fig. (b)

Let us first convert the multiple current sources to equivalent voltage sources as shown in fig (a). Next, these voltage sources are transformed into single voltage source fig(b),

Where,

$V_{eq} = 32 + 72 = 104$ V

and $R = 4 + 6 = 10\ \Omega$

❁ ❁ ❁

2

DC CIRCUITS

2.1 DC Circuit

A dc circuit is an electric circuit that consists of any combination of constant voltage sources, constant current sources and resistors. Direct current is the unidirectional flow of electric charge.

2.2 Series Circuits

When the resistors are connected end to end so that they form only one path for the flow of current then resistors are said to be connected in series and such circuits are known as series circuits.

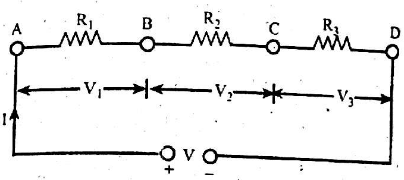


Fig. 2.1 (a): Resistors in Series

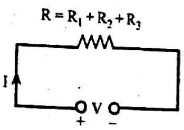


Fig. 2.1 (b): Equivalent resistance

In figure 2.1 (a), A and D are the free ends of three resistors AB, BC and CD connected in series. Let R_1, R_2 and R_3 be the respective resistance, R = resistance of combination, V = total p.d across the resistors and I = current strength. Then,

$V = IR$ .... (i)

But, V = sum of the individual p.d across R_1, R_2 and R_3

$\therefore$ $V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$ .... (ii)

Using equations (i) and (ii), we get

$IR = IR_1 + IR_2 + IR_3$

$\therefore$ $R = R_1 + R_2 + R_3$ Also, $\frac{1}{G} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3}$

i.e. If a number of resistors are connected in series, then combined resistance of the system equals the sum of the individual resistances.

2.2.1 Voltage divider rule

$V_1 = IR_1 = \frac{V}{R_1 + R_2} \times R_1 = \frac{V}{G_1 + G_2} \times G_2$

(Where, $G = \frac{1}{R}$ = conductance)

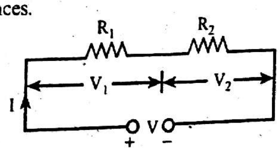


Fig. 2.2 Two resistors in series

$$V_2 = IR_2 = \frac{V}{R_1 + R_2} \times R_2 = \frac{V}{G_1 + G_2} \times G_1$$

Also,

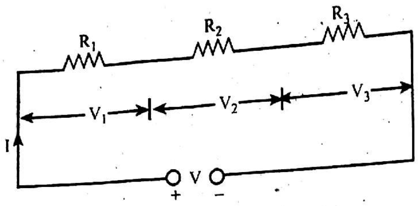


Fig. 2.3 Three resistors in series

$$V_1 = IR_1 = \frac{V}{R_1 + R_2 + R_3} \times R_1 = \frac{V}{G_1 G_2 + G_2 G_3 + G_3 G_1} \times G_2 G_3$$

$$V_2 = IR_2 = \frac{V}{R_1 + R_2 + R_3} \times R_2 = \frac{V}{G_1 G_2 + G_2 G_3 + G_3 G_1} \times G_3 G_1$$

$$V_3 = IR_3 = \frac{V}{R_1 + R_2 + R_3} \times R_3 = \frac{V}{G_1 G_2 + G_2 G_3 + G_3 G_1} \times G_1 G_2$$

2.3 Parallel Circuits

When a number of resistors are connected in such a way that one end of each of them is joined to a common point and the other ends being joined to another common point then resistors are said to be connected in parallel and such circuits are known as parallel circuits.

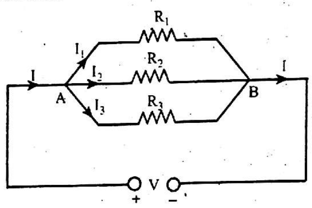


Fig. 2.4 (a) Resistors in parallel

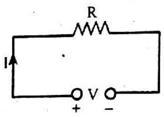


Fig. 2.4 (b) Equivalent resistance

In fig 2.4 (a), three resistors of resistance R_1, R_2 and R_3 are connected between the common points A and B. Same p.d (V) exists between the ends of each resistors but the amount of current passing through each is different, depending upon their resistances. Suppose the main current I is divided into I_1, I_2 and I_3 through resistors R_1, R_2 and R_3. If R is the combined resistance between A and B, then

$$I = \frac{V}{R} \quad \text{.........} \ (i)$$

Since the main current (I), which enters the combination, must also come out as such,

so, $I = I_1 + I_2 + I_3$

or, $$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \quad \text{.........} \ (ii)$$

Using equations (i) and (ii), we get

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\therefore \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_1} + \frac{1}{R_3} \qquad \text{Also, } G = G_1 + G_2 + G_3$$

i.e. If a number of conductors are connected in parallel, the reciprocal of the combined resistance is equal to the sum of the reciprocal of the individual resistances.

2.3.1 Current divider rule

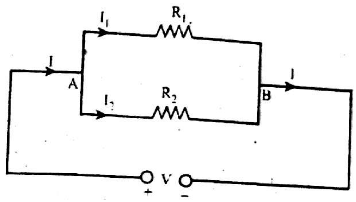


Fig. 2.5 Two resistors in parallel

$$I_1 = \frac{V}{R_1} = \frac{IR}{R_1} \quad \text{[R is the equivalent resistance of combination]}$$

or, $$I_1 = \frac{I \times \left(\frac{1}{R_1}\right)}{\frac{1}{R}}$$

or, $$I_1 = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}} \times I$$

or, $$I_1 = \frac{I}{R_1 + R_2} \times R_2 = \frac{I}{G_1 + G_2} \times G_1$$

Similarly, $$I_2 = \frac{I}{G_1 + G_2} \times G_2$$

Also,

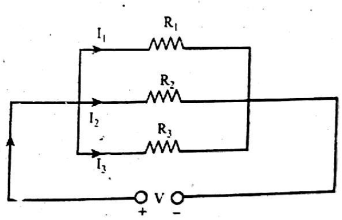


Fig. 2.6 Three resistors in parallel

$$I_1 = \frac{V}{R_1} = \frac{IR}{R_1} \quad \text{[R is the equivalent resistance of combination]}$$

$$\text{or,}\quad I_1 = \frac{I \times \left(\frac{1}{R_1}\right)}{\frac{1}{R}}$$

$$\text{or,}\quad I_1 = \frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} \times I$$

$$\therefore\quad I_1 = \frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} \times I = \frac{G_1}{G_1 + G_2 + G_3} \times I$$

$$\text{Similarly,}\quad I_2 = \frac{G_2}{G_1 + G_2 + G_3} \times I$$

$$I_3 = \frac{G_3}{G_1 + G_2 + G_3} \times I$$

2.4 Circuit containing series and parallel connections

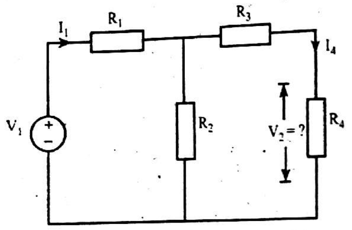


Fig. 2.7 Resistors in series and parallel

The total resistance seen from the source is

$$R = \frac{V_1}{I_1} = R_1 + R_2 \parallel (R_3 + R_4)$$

$$\text{or,}\quad \frac{V_1}{I_1} = R_1 + \frac{R_2 (R_3 + R_4)}{R_2 + R_3 + R_4}$$

$$\text{or,}\quad \frac{V_1}{I_1} = \frac{R_1 (R_2 + R_3 + R_4) + R_2 (R_3 + R_4)}{R_2 + R_3 + R_4}$$

$$\text{Therefore,}\quad I_1 = \frac{(R_2 + R_3 + R_4) V_1}{R_1 (R_2 + R_3 + R_4) + R_2 (R_3 + R_4)}$$

$$= \frac{(R_2 + R_3 + R_4) V_1}{R_1 R_2 + R_3 R_1 + R_1 R_4 + R_2 R_3 + R_2 R_4}$$

Using the current division formula, we can find,

$$I_4 = \frac{I_1}{R_2 + R_3 + R_4} \times R_2$$

Finally,

Since, $V_2 = I_4 R_4$

$$\text{We have,}\quad V_2 = \frac{R_2 R_4 I_1}{R_2 + R_3 + R_4}$$

2.4.1 Summary of series and parallel circuit

Circuit	Potential difference	Current	Resistance
Series	Each load uses a portion of the total p.d. supplied by the battery.	The current is the same throught the series circuit.	The current decreases when more resistors are added because resistance increases.
Parallel	Each load uses all the potential difference supplied by the battery.	The current divides into different paths.	Adding resistors in parallel decreases the total resistance of the circuit.

2.5 Kirchhoff's law

In simple circuits, we can carry out the analysis of current, voltage and resistance simply with the help of Ohm's law. However, in complex circuit or network, the calculations can be done with the help of Kirchhoff's laws, which are stated as follows:

(i) Kirchhoff's current law

(ii) Kirchhoff's voltage law

(i) Kirchhoff's current law

This law is also called as Kirchhoff's first law or point law or junction rule. According to this law, at any node of a several circuit elements, the sum of currents entering the node must equal the sum of currents leaving it. This law is based on conservation of charge.

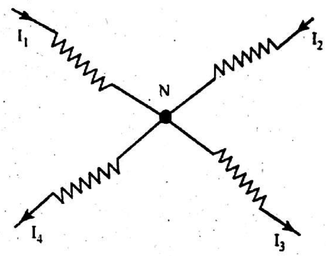


Fig. 2.8

In this figure, the currents directed towards node N are I_1, I_2, $-I_3$ and $-I_4$.

So, $I_1 + I_2 + (-I_3) + (-I_4) = 0$

$$\sum_{k=1}^{n=4} I_k = 0$$

Where n is the total number of branches with current flowing towards or away from the node.

(ii) Kirchhoff's voltage law

This law is also called Kirchhoff's second law or Kirchhoff's loop (or mesh) rule. According to this law, in any closed electrical circuit or mesh, the algebraic sum of all the electromotive forces (e.m.fs) and voltage drops in resistors is equal to zero. This law is based on conservation of energy.

$\therefore$ In any closed circuit or mesh,

Algebraic sum of e.m.fs + Algebraic sum of voltage drops = 0

Applying KVL in above mesh;

$$E + (-V_1) + (-V_2) + (-V_3) = 0$$

$$\therefore \quad E = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$$

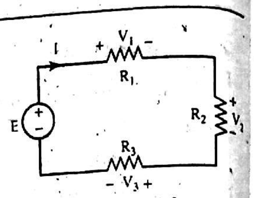


Fig. 2.9

Sign rules for KVL

- Give positive sign to all rise in voltage and negative to voltage drops. Thus, if we move from negative (–ve) terminal of a battery to positive (+ve) terminal, a positive sign should be given, since there is a rise in voltage. On the other hand, if we go from positive (+ve) terminal to negativem (–ve) terminal, a negative sign should be given, since there is a drop in voltage.

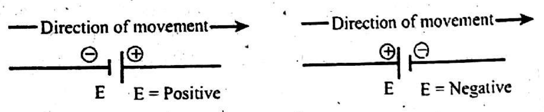


Fig. 2.10

- For a conductor of resistance R, if the direction of current is same/opposite as the direction of movement, then voltage (= IR) should be taken negative/positive, since current flows from higher voltage to lower one and hence there is a voltage drop/voltage rise while crossing the conducting element. This is illustrated below;

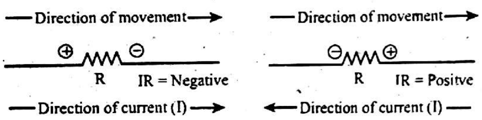


Fig. 2.11

2.6 Power and energy in dc circuit

As a charge carrier q move around a circuit and drops an amount of potential V in time t, it loses an amount of potential energy qV. The power or the rate at which it loses energy is $\frac{Vq}{t}$. Since the current I is equal to $\frac{q}{t}$, the power can be expressed as,

$$P = \frac{qV}{t} = VI \quad \text{joules/sec or watts}$$

We can combine the equations for power and Ohm's law to get expressions for power in terms of resistance.

$$P = I^2R = \frac{V^2}{R} \quad \text{joules/sec or watts}$$

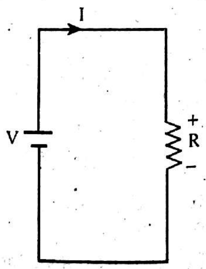


Fig. 2.12

As current flows through a resistor, the resistors heats up. The heat in joules is given by,

$$H = I^2Rt = Pt \quad \text{joules}$$

Where t is the time in seconds. In other words,a resistor heats up more when there is a high current running through a strong resistor over a long stretch of time.

2.7 Open Circuit and Short Circuit

2.7.1 Open Circuit

Two points are said to be open circuited if there is no direct connection between them. It represents a break in the continuity of the circuit. Due to this break,

i. Resistance R between two points is infinite.

ii. There is no flow of current between two points.

Example:

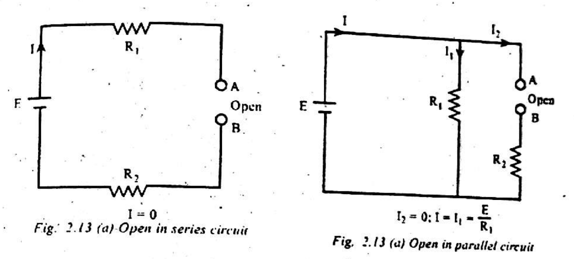


Fig. 2.13 (a) Open in series circuit *Fig. 2.13 (a) Open in parallel circuit*

2.7.2 Short Circuit

When two points are connected together by a thick metallic wire (resistance less wire), they are said to be short circuited. Due to this,

i. Resistance R between points is zero.

ii. No voltage can exist across it, V = IR = 0

iii. Current through it is very large (theoretically infinite) called short circuited current.

Example:

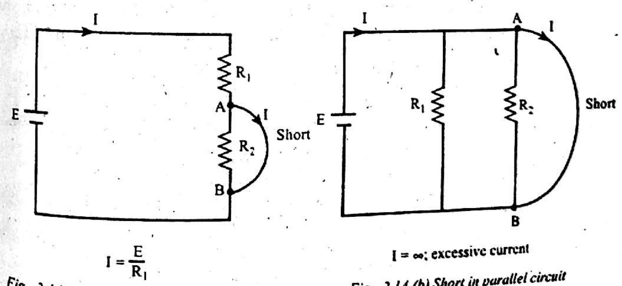


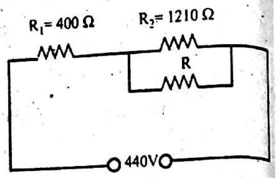
Fig. 2.14 (a) Short in series circuit *Fig. 2.14 (b) Short in parallel circuit*

Exam Solutions

1. **It is proposed to work in series two lamps at their related power of 100W, 200V and 40W, 220V on a 440V supply by connecting a resistor R in parallel with 40 watt lamp. Find the value of R if total power drawn from the source is 400 watt.** [2064 Poush]

Solution:

Considering lamp resistance constant



Resistance of first lamp, $R_1 = \frac{200^2}{100} = 400\ \Omega$

Resistance of second lamp, $R_2 = \frac{220^2}{40} = 1210\ \Omega$

Since, Power drawn from source is 400 watt

So, $P = \frac{V^2}{R_{eq}}$

or, $400 = \frac{440^2}{R_{eq}}$

or, $R_{eq} = \frac{440^2}{400}$

or, $400 + (1210 \parallel R) = \frac{440^2}{400}$

or, $400 + \frac{1210 \times R}{1210 + R} = \frac{440^2}{400}$

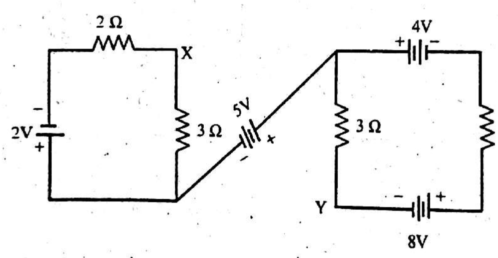
or, $400 + \frac{1210 \times R}{1210 + R} = 484$

or, $\frac{1210\ R}{1210 + R} = 84$

or, $1210\ R = 101640 + 84\ R$

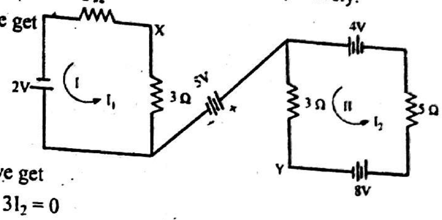
$\therefore$ $R = 90.27\ \Omega$

2. **What is the difference of potential between X and Y in the network shown in figure below.** [2067 Mangsir]



Solution:

Let I_1 and I_2 be the mesh current of mesh I and mesh II respectively.



Applying KVL on mesh I, we get

$2 - 3I_1 - 2I_1 = 0$

or, $2 - 5I_1 = 0$

$\therefore$ $I_1 = \frac{2}{5}$ A

Applying KVL on mesh II, we get

$8 - 5\ I_2 + 4 - 3I_2 = 0$

or, $12 - 8I_2 = 0$

$\therefore$ $I_2 = \frac{3}{2}$ A.

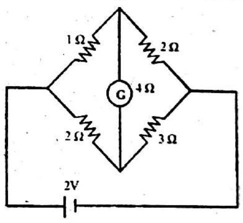
Now, $V_{XY} = V_X - V_Y$

$= 3I_2 - 5 - 3I_1$ [Write KVL equation; move from Y to X]

$= 3 \times \frac{3}{2} - 5 - 3 \times \frac{2}{5} = -1.7$ volt.

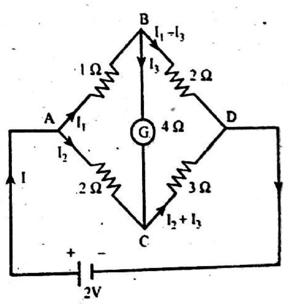
Which indicates that Y is at higher potential with respect to X.

3. **Calculate the current through the galvanometer in the bridge circuit as shown in figure given below using Kirchhoff's laws.** [2067 Mangsir]



Solution:

Let the current distribution in the network be as shown in figure



Applying KVL to mesh ABCA, we get

$-I_1 - 4I_3 + 2I_2 = 0$ .... (i)

Also,

Applying KVL to mesh BCDB, we get

$-4\ I_3 - 3(I_2 + I_3) + 2(I_1 - I_3) = 0$

or, $-4\ I_3 - 3I_2 - 3I_3 + 2I_1 - 2I_3 = 0$

or, $2I_1 - 3I_2 - 9I_3 = 0$ .... (ii)

and,

Applying KVL to mesh ACDA, we get,

$-2I_2 - 3(I_2 + I_3) + 2 = 0$

or, $-2I_2 - 3I_2 - 3I_3 + 2 = 0$

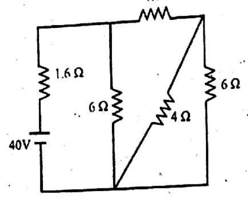
or, $-5I_2 - 3I_3 = -2$ .... (iii)

solving equations (i), (ii) and (iii), we get,

$$I_1 = \frac{15}{22}\text{A},\ I_2 = \frac{17}{44}\text{A},\ I_3 = \frac{1}{44}\text{A}$$

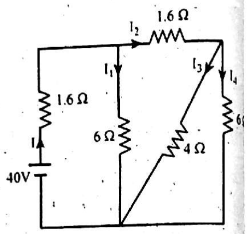
Hence, The current through the galvanometer is $\frac{1}{44}$ A

4. Find the current through 4 Ω resistance. [2068 Bhadra]



Solution:

Equivalent resistance of the given circuit



$$R_{eq} = [(6 \parallel 4) + 1.6] \parallel 6 + 1.6$$

$$= \left[\frac{6 \times 4}{6+4} + 1.6\right] \parallel 6 + 1.6$$

$$= 4 \parallel 6 + 1.6 = \frac{4 \times 6}{4+6} + 1.6 = 4\Omega$$

$\therefore$ current, $I = \frac{V}{R_{eq}} = \frac{40}{4} = 10$ A

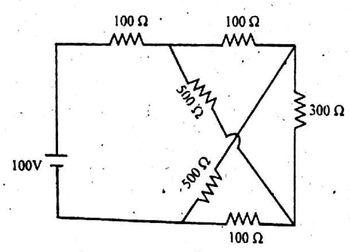
Using current division rule

$$I_2 = \frac{I}{6 + \left(1.6 + \frac{4 \times 6}{4+6}\right)} \times 6 = \frac{10}{6+4} \times 6 = 6\text{A}$$

$$\therefore I_3 = \frac{I_2}{4+6} \times 6 = \frac{6}{10} \times 6 = 3.6\text{ A}$$

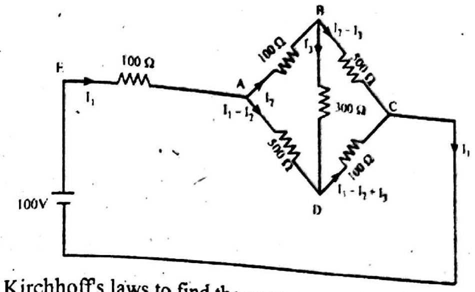
Hence, the current through 4 Ω resistance is 3.6 A.

5. Determine the current supplied by the battery in the circuit shown in figure below. [2068 Bhadra]



Solution:

Redrawing the circuit,



Using Kirchhoff's laws to find the current supplied by the battery we consider current distribution in the network be as shown in figure:

Applying KVL on mesh EADCE, we get

$$100 - 100\,I_1 - 500\,(I_1 - I_2) - 100(I_1 - I_2 + I_3) = 0$$

or, $-700\,I_1 + 600\,I_2 - 100\,I_3 = -100$ .... (i)

Applying KVL on mesh ABDA, we get,

$$-100\,I_2 - 300\,I_3 + 500\,(I_1 - I_2) = 0$$

or, $500\,I_1 - 600\,I_2 - 300\,I_3 = 0$ .... (ii)

Applying KVL on mesh BCDB, we get,

$$-500\,(I_2 - I_3) + 100\,(I_1 - I_2 + I_3) + 300\,I_3 = 0$$

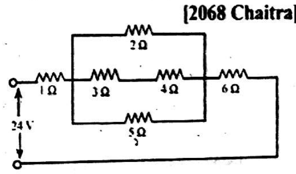
or, $100\,I_1 - 600\,I_2 + 900\,I_3 = 0$ .... (iii)

solving equations (i), (ii) and (iii), we get

$$I_1 = \frac{3}{10}\text{A},\ I_2 = \frac{1}{5}\text{A},\ I_3 = \frac{1}{10}\text{A}$$

Hence, the current supplied by the battery in the circuit is $\frac{3}{10}$A.

6. Find the equivalent resistance in the figure shown and power dissipated in the 5 Ω resistor. **[2068 Chaitra]**



Solution:

Equivalent resistance of the circuit,

$$R = 1 + (2 \parallel 7 \parallel 5) + 6$$

$$= 1 + \left\{\left(\frac{2 \times 7}{2+7}\right) \parallel 5\right\} + 6 = 1 + \left(\frac{14}{9} \parallel 5\right) + 6$$

$$= 1 + \frac{70}{59} + 6 = 8.1864\ \Omega$$

Total current, $I = \frac{24}{8.1864} = 2.9317$ A

Now, Using current division rule,

Current flowing through 5 Ω resistor,

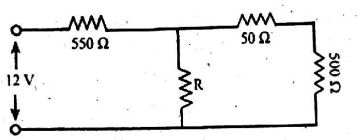
$$I_{5\Omega} = \frac{G_{5\Omega}}{G_{2\Omega} + G_{7\Omega} + G_{5\Omega}} \times I$$

$$= \frac{\frac{1}{5}}{\frac{1}{2}+\frac{1}{7}+\frac{1}{5}} \times 2.9317 = 0.6957 \text{ A}$$

$\therefore$ Power dissipated in 5 Ω resistor $= I_{5\Omega}^2 R$

$= (0.6957)^2 \times 5 = 2.4197$ watt

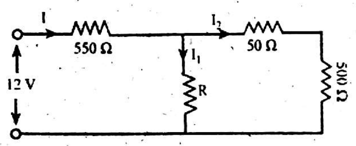
7. Determine the value of unknown resistor 'R' in the circuit below if the voltage drop across 500 Ω resistor is 2.5 volts. [2068 Shrawan]



Solution:

We have,

Voltage drop across 500 Ω resistor = 2.5 volt



So, Current flowing through 500 Ω resistor.

$$I_2 = \frac{2.5}{500}$$

$\therefore$ $I_2 = 0.005$ A.

$\because$ Voltage drop across R = voltage drop across (50 + 500) Ω

or, $I_1 R = 0.005 \times (50 + 500)$

or, $I_1 R = 2.75$ .... (i)

Voltage drop across 550 Ω resistor $= 12 - I_2 (50 + 500)$

or, $I \times 550 = 12 - 0.005 \times (50 + 500)$

or, $I \times 550 = 9.25$

$\therefore$ $I = 0.0168$ A

Simply by KCL,

$I = I_1 + I_2$

or, $0.0168 = I_1 + 0.005$

$\therefore$ $I_1 = 0.0168 - 0.005 = 0.0118$ A

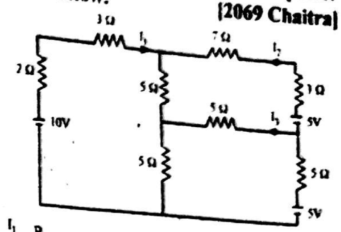
We know, Voltage drop across R = $I_2 (50 + 500)$

or, $I_1 \times R = 0.005 \times 550$

or, $0.0118 \times R = 2.75$

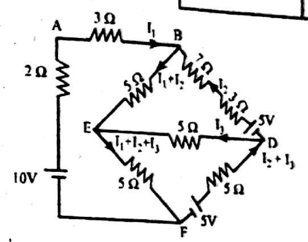
$\therefore$ $R = 233.0508\ \Omega$

8. Find the curents I_1, I_2, I_3 using Kirchhoff's law and also find the power output of each voltage source of figure below. [2069 Chaitra]



Solution:

Redrawing the given circuit,



Let the current distribution in the circuit be as shown in the figure.

Applying KVL on mesh ABEFA, we get,

$$-3I_1 - 5(I_1 + I_2) - 5(I_1 + I_2 + I_3) + 10 - 2I_1 = 0$$

or, $-3I_1 - 5I_1 - 5I_2 - 5I_1 - 5I_2 - 5I_3 - 2I_1 = -10$

or, $-15I_1 - 10I_2 - 5I_3 = -10$ .... (i)

Applying KVL on mesh BDEB, we get

$$7I_2 + 3I_2 - 5 - 5I_3 + 5(I_1 + I_2) = 0$$

or, $10I_2 - 5 - 5I_3 + 5I_1 + 5I_2 = 0$

or, $5I_1 + 15I_2 - 5I_3 = 5$ .... (ii)

Applying KVL on mesh DEFD, we get

$$-5I_3 - 5(I_1 + I_2 + I_3) + 5 - 5(I_2 + I_3) = 0$$

or, $-5I_3 - 5I_1 - 5I_2 - 5I_3 + 5 - 5I_2 - 5I_3 = 0$

or, $-5I_1 - 10I_2 - 15I_3 = -5$ .... (iii)

Solving equations (i), (ii) and (iii), we get

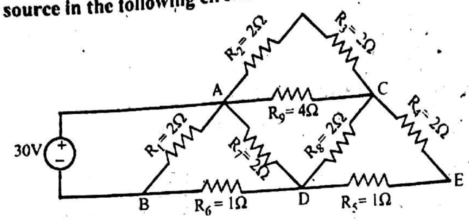
$$I_1 = \frac{13}{24} \text{ A}, \ I_2 = \frac{1}{6} \text{ A}, \ I_3 = \frac{1}{24} \text{ A}$$

Now, Power output of 10V source, $P_{10} = 10 \times \frac{13}{24} = 5.4167$ W

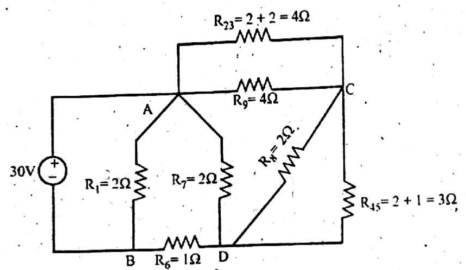
Power output of 5 V source, $P_5 = 5 \times \frac{1}{6} = 0.8333$ W

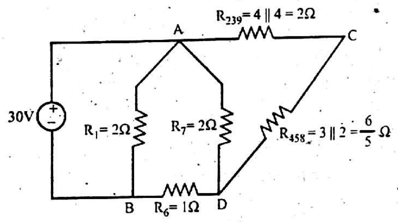
Power output of 5 V source, $P_5' = 5 \times \left(\frac{1}{6} + \frac{1}{24}\right) = 1.0417$ W

9. Using series-parallel combination of resistances find the current delivered by the source in the following circuit. **[2069 Ashad]**



Solution:



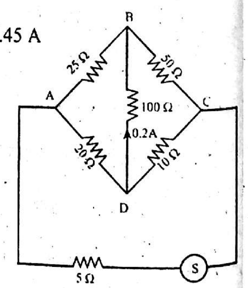


∴ Equivalent resistance of combination is,

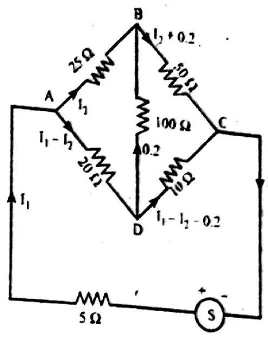
$$R_{eq} = \left[\left(2+\frac{6}{5}\right) \parallel 2 + 1\right] \parallel 2 = \left[\left(\frac{16}{5} \parallel 2\right) + 1\right] \parallel 2 = \frac{29}{13} \parallel 2 = \frac{58}{55}\,\Omega$$

Hence, current delivered by the source $= \frac{30}{\frac{58}{55}} = 28.45$ A

10. Use Kirchhoff's laws determine the magnitude of source current and polarity of the source S, if the current flowing through branch BD is 0.2A from D to B in the circuit shown below. **[2069 Ashad]**



Solution:



Let the polarity of S be as shown. Then,

Applying KVL to mesh ABDA, we get

$$-25I_2 + 0.2 \times 100 + 20(I_1 - I_2) = 0$$

or, $-25I_2 + 20 + 20I_1 - 20I_2 = 0$

or, $20I_1 - 45I_2 = -20$ .... (i)

Applying KVL to mesh BCDB, we get

$$-50(I_2 + 0.2) + 10(I_1 - I_2 - 0.2) - 0.2 \times 100 = 0$$

or, $-50I_2 - 10 + 10I_1 - 10I_2 - 2 - 20 = 0$

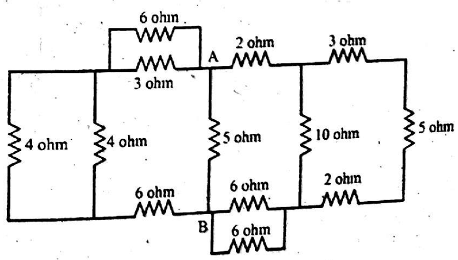
or, $10I_1 - 60I_2 = 32$ .... (ii)

solving equations (i) and (ii), we get

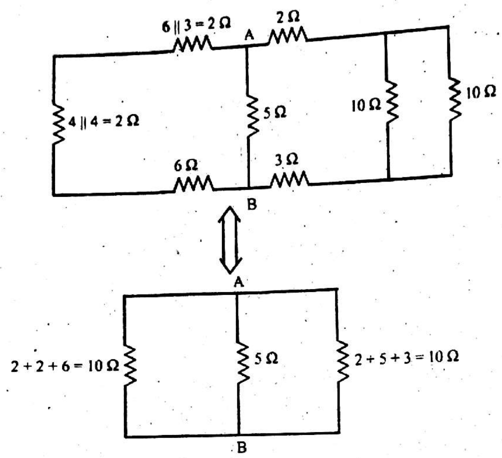
$I_1 = -3.52$ A, $I_2 = -1.12$ A

The magnitude of current I_1 indicates that polarity of source S is reverse of the shown one.

11. Find the equivalent resistance across the terminals A and B, R_{AB}. **[2070 Ashad]**



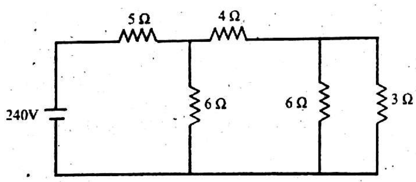
Solution:



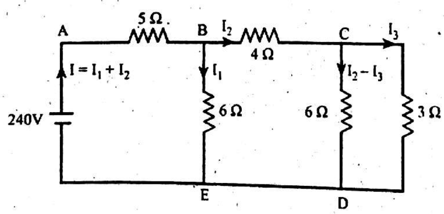
$$\therefore\ R_{AB} = \frac{10 \times 5 \times 10}{10 \times 5 + 5 \times 10 + 10 \times 10} = 2.5\ \Omega$$

Hence, the equivalent resistance across the terminals is A and B is 2.5 Ω.

12. Find the circuit current and current through each branch using branch current method. **[2070 Bhadra]**



Solution:



Let the current distribution be as shown in the figure.

Applying KVL on mesh ABEA, we get

$$240 - 5(I_1 + I_2) - 6I_1 = 0$$

or, $\quad -5I_1 - 5I_2 - 6I_1 + 240 = 0$

or, $\quad -11I_1 - 5I_2 = -240$ .... (i)

Applying KVL on mesh BCDEB, we get,

$\quad -4I_2 - 6(I_2 - I_3) + 6I_1 = 0$

or, $\quad -4I_2 - 6I_2 + 6I_3 + 6I_1 = 0$

or, $\quad 6I_1 - 10I_2 + 6I_3 = 0$ .... (ii)

Applying KVL on mesh CDC, we get

$\quad -3I_3 + 6(I_2 - I_3) = 0$

or, $\quad -3I_3 + 6I_2 - 6I_3 = 0$

or, $\quad 6I_2 - 9I_3 = 0$ .... (iii)

solving equations (i), (ii) and (iii), we get I_1 = 15A, I_2 = 15A, I_3 = 10A

Therefore,

Current in branch BAE, $I = I_1 + I_2 = 30$ A

Current in branch BE, I_1 = 15 A

Current in branch BC, I_2 = 15 A

Current in branch CD containing 6 Ω resistor, $I_2 - I_3$ = 5A

Current in branch CD containing 3 Ω resistor, I_3 = 10 A.

13. What is the total cost of using the following at Rs. 7 per kilowatt hour?

(i) A 1200W toaster for 30 minutes

(ii) Six 50W bulbs for 4 hours

(iii) A 400W washing machine for 45 minutes

(iv) A 4800W electric clothes dryer for 20 minutes **[2068 Chaitra]**

Solution

Total energy consumption,

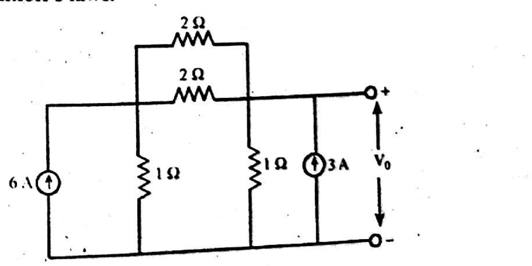
$$= \frac{1200}{1000} \times \frac{30}{60} + 6 \times \frac{50}{1000} \times 4 + \frac{400}{1000} \times \frac{45}{60} + \frac{4800}{1000} \times \frac{20}{60}$$

$$= 0.6 + 1.2 + 0.3 + 1.6$$

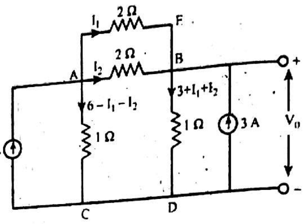
$$= 3.7 \text{ kWhr}$$

$\therefore$ Total cost = 7 × 3.7 = Rs. 25.9

14. Calculate the output voltage, V_0 for the circuit shown in figure below using Kirchhoff's laws. **[2070 Chaitra]**



Solution:



Let the current distribution be as shown in the figure.

Applying KVL on mesh AEBDCA, we get,

$$-2I_1 - 1(3 + I_1 + I_2) + 1(6 - I_1 - I_2) = 0$$

or, $-2I_1 - 3 - I_1 - I_2 + 6 - I_1 - I_2 = 0$

or, $-4I_1 - 2I_2 = -3$ .... (i)

Also, Applying KVL on mesh ABDCA, we get

$$-2I_2 - 1(3 + I_1 + I_2) + 1(6 - I_1 - I_2) = 0$$

or, $-2I_2 - 3 - I_1 - I_2 + 6 - I_1 - I_2 = 0$

or, $-2I_1 - 4I_2 = -3$ .... (ii)

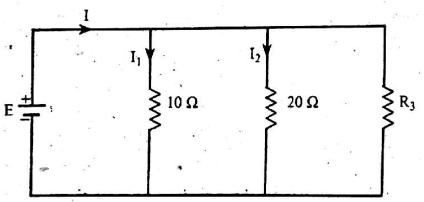
solving equations (i) and (ii), we get

$$I_1 = \frac{1}{2}\text{ A},\quad I_2 = \frac{1}{2}\text{ A}$$

Since, V_o = voltage drop in branch BD

$$= (3 + I_1 + I_2) \times 1 = \left(3 + \frac{1}{2} + \frac{1}{2}\right) \times 1 = 4 \text{ volt.}$$

15. Given the information provided in figure, calculate R_3, E, I and I_2. Equivalent resistance of the circuit is 4Ω.



[2071 Magh]

Solution:

Let, Req be the equivalent resistance of the given circuit.

We know,

$$\frac{1}{Req} = \frac{1}{10} + \frac{1}{20} + \frac{1}{R_3}$$

or, $\frac{1}{4} = \frac{1}{10} + \frac{1}{20} + \frac{1}{R_3}$ [∵ Req = 4Ω (given)]

∴ $R_3 = 10\Omega$

Here, resistors are connected in parallel.

By Ohm's law,

$$E = I\,Req = I_1 R_1 = I_2 R_2 = I_3 R_3$$

Using current division rule,

$$I_2 = \frac{G_2}{G_1 + G_2 + G_3} \times I = \frac{\frac{1}{20}}{\frac{1}{10} + \frac{1}{20} + \frac{1}{10}} \times I$$

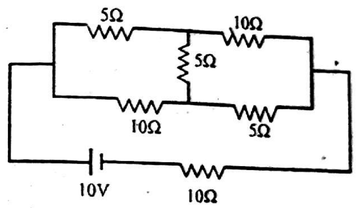
$$= 0.2I$$

Suppose, E = 6V (say)

Then, $I = \frac{E}{Req} = \frac{6}{4} = 1.5$ A

$I_2 = 0.2 \times 1.5 = 0.3$ A

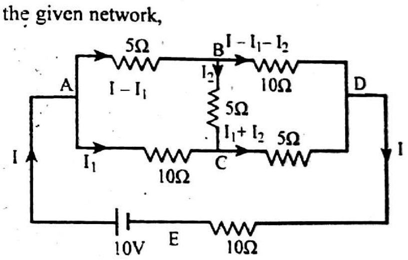
16. Find equivalent resistance of the given network.



[2071 Bhadra]

Solution:

Redrawing the given network,



Let the current distribution in the network be as shown in figure. Here, we will find the equivalent resistance using Kirchhoff's laws.

Applying KVL on mesh ABCA, we get

$$-5(I - I_1) - 5I_2 + 10I_1 = 0$$

or, $-5I + 5I_1 - 5I_2 + 10I_1 = 0$

or, $15I_1 - 5I_2 - 5I = 0$ .... (i)

Applying KVL on mesh BDCB, we get

$$-10(I - I_1 - I_2) + 5(I_1 + I_2) + 5I_2 = 0$$

or, $-10I + 10I_1 + 10I_2 + 5I_1 + 5I_2 + 5I_2 = 0$

or, $15I_1 + 20I_2 - 10I = 0$ .... (ii)

Applying KVL on mesh ACDEA, we get

$$-10I_1 - 5(I_1 + I_2) - 10I + 10 = 0$$

or, $-10I_1 - 5I_1 - 5I_2 - 10I + 10 = 0$

or, $15I_1 + 5I_2 + 10I = 10$ .... (iii)

Solving equations (i), (ii) and (iii), we get

$$I_1 = \frac{4}{17}\text{ A},\ I_2 = \frac{2}{17}\text{ A},\ I = \frac{10}{17}\text{ A}$$

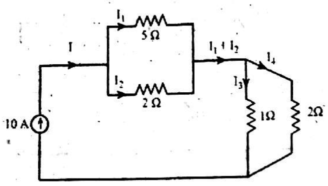
∴ Equivalent resistance of given network.

$$\text{Req} = \frac{\text{Emf of battery}}{\text{Current supplied by battery}}$$

$$= \frac{10}{\frac{10}{17}} = 17\ \Omega$$

Additional Problems

1. **Find I_1, I_2 I_3 and I_4 in given figure:**



Solution:

Using the current division method,

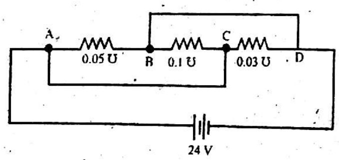
$$I_1 = \frac{2}{5+2}\times I = \frac{2}{7}\times 10 = 2.857\text{ A}$$

$$I_2 = \frac{5}{5+2}\times I = \frac{5}{7}\times 10 = 7.143\text{ A}$$

$$I_3 = \frac{2}{1+2}\times I = \frac{2}{3}\times 10 = 6.67\text{ A}$$

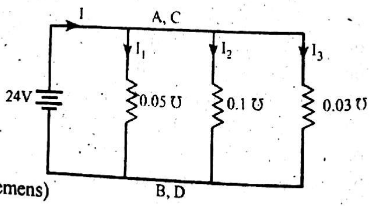
$$I_4 = \frac{1}{1+2}\times I = \frac{1}{3}\times 10 = 3.33\text{ A}$$

2. **For the circuit shown below, calculate the power consumed by the 0.1℧ resistor.**



Solution:

The given circuit can be redrawn as,



Equivalent conductance,

$$G = G_1 + G_2 + G_3 = 0.05 + 0.1 + 0.03$$

$$= 0.18\ ℧\ \text{(or mho or siemens)}$$

Since, $I = GV$

So, $I = 0.18 \times 24$

∴ $I = 4.32$ A

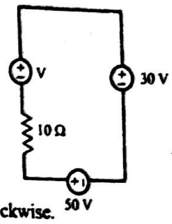
Now, current flowing through 0.1 ℧

$$I_2 = \frac{G_2}{G_1 + G_2 + G_3}\times I = \frac{0.1}{0.05 + 0.1 + 0.03}\times 4.32 = 2.4\text{ A}$$

∴ Power consumed by the 0.1 ℧ resistor

$$= V I_2 = 24 \times 2.4 = 57.6\text{ watt}$$

3. **Find V such that 100 C charge is injected to the 50 V source in 1 min in the circuit.**



Solution:

In order to inject the 100 C charge to the 50 V source, the current in the loop must be anti–clockwise.

But, $I = \frac{Q}{t} = \frac{100}{60} = 1.67$ A [∵ Q = 100C; t = 1 min = 60 sec]

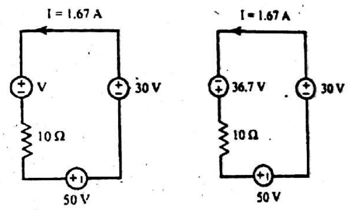


Fig. (a) *Fig. (b)*

Applying KVL in loop [Fig (a)]

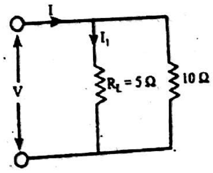
$$V - 30 + 50 + 10\,I = 0$$

Or, $V = 30 - 50 - 10 \times 1.67$

∴ $V = -36.7$ V

Thus, the voltage of the given source must be 36.7 V with polarities as shown in Fig (b).

4. **Calculate the supply current (I) in the following network, if 5 ohms resistor dissipates energy at the rate of 20 W.**



Solution:

Energy dissipated in 5 Ω resistor,

$$I_1^2 \times 5 = 20$$

∴ $I_1 = 2$ A

But, $I_1 = \frac{1}{5+10}\times 10 = \frac{10\,I}{15} = \frac{2}{3}I$

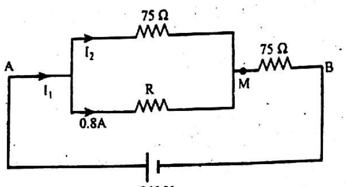
$$I = \frac{3}{2} I_1 = \frac{3}{2} \times 2$$

$\therefore$ $I = 3$ A

5. **A 150 Ω resistance coil AB is connected across 240 V.d.c. supply. Calculate the value of the resistance which, when connected between the midpoint of AB and end A, will carry a current of 0.8 A.**

Solution:

The circuit is shown in figure. Let M be the midpoint of AB and R be the resistance connected between M and A. The current in R is 0.8A.



Voltage drop across R_{MB}, $V_{MB} = I_1 R_{MB} = 75 I_1$

$V_{AM} + V_{MB} = V_{AB}$

$V_{AM} = V_{AB} - V_{MB} = 240 - 75 I_1$ .... (i)

By KCL at point A,

$$I_1 = I_2 + 0.8 = \frac{V_{AM}}{75} + 0.8$$

or, $$I_1 = \frac{240 - 75 I_1}{75} + 0.8 \text{ [using (i)]}$$

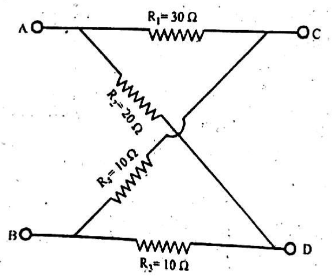
or, $$75 I_1 = 240 - 75 I_1 + 75 \times 0.8$$

$\therefore$ $$I_1 = \frac{300}{150} = 2 \text{ A}$$

From (i) $$V_{AM} = 240 - 75 \times 2 = 90 \text{ V}$$

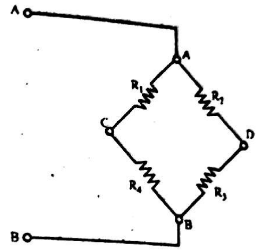
$\therefore$ $$R = \frac{V_{AM}}{0.8} = \frac{90}{0.8} = 112.5\ \Omega$$

6. **Find the input resistance at AB for the network shown in figure, when terminal CD are (a) open - circuited and (b) short circuited.**



Solution:

Redrawing the given network,



(a) When the terminals C and D are open circuited the network can be redrawn as shown in figure below:

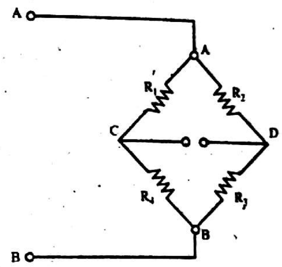


Fig. a

Here, R_1 and R_4 are in series. Similarly, R_2 and R_3 are in series.

$R_{AB} = (R_2 + R_3) \parallel (R_1 + R_4)$

$= (20 + 10) \parallel (30 + 10)$

$= 30 \parallel 40$

$$= \frac{30 \times 40}{30 + 40}$$

$= 17.143\ \Omega$

(b) When the terminals C and D are short circuited the network becomes as shown in figure.

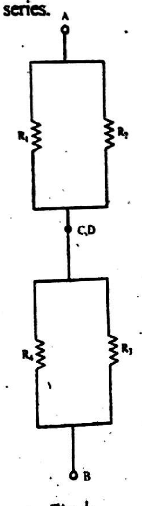


Fig. b

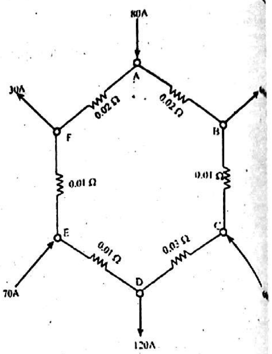
Here, $$R_{AB} = (R_1 \parallel R_2) + (R_3 \parallel R_4)$$

$$= \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4}$$

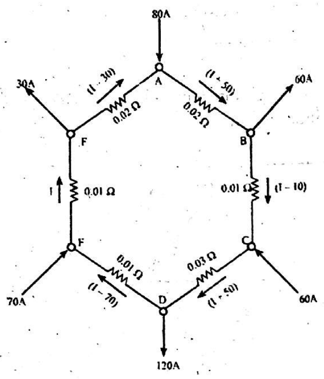
$$= \frac{30 \times 20}{30 + 20} + \frac{10 \times 10}{10 + 10}$$

$= 17\ \Omega$

7. **Find the current in the branch AB of the network shown below.**



Solution:



Let the current in arm EF be I A. By using KCL at junctions A,B,C,D,E and F the currents in all six branches are expressed in terms of I as depicted.

Now, Applying KVL to the loop ABCDEFA, we get,

$-0.02\,(I + 50) - 0.01\,(I - 10) - 0.03\,(I + 50) - 0.01\,(I - 70) - 0.01 \times I - 0.02\,(I - 30) = 0$,

On simplification, we get $I = -11$ A

$\therefore$ Current in branch AB $= I + 50 = -11 + 50 = 39$A (A to B)

8. **In a residential house, the following are the loads connected:**

(i) 60 watt lamps – 4 Nos., switched on for 4 hours a day

(ii) 40 watt lamps – 4 Nos., switched on for 6 hours a day

(iii) 1,000 watt heater –1 No, working for 2 hours /day

(iv) Refrigerator – 1.5 kW, working for 10 hours/day

(v) An electric clock – 10 watts input

If the cost of electricity is 35 paise/unit, what will be the monthly electric charges?

Solution:

Daily energy consumption

$= 60 \times 4 \times 4 + 40 \times 4 \times 6 + 1000 \times 1 \times 2 + 1500 \times 10 + 10 \times 24 = 19160$ Wh

$\therefore$ Monthly consumption $= 19160$ Wh $\times 30$

$= 574800$ Wh $= 574.8$ kWh

Hence, Monthly electric charge $= 574.8 \times$ Re. 0.35

$=$ Rs. 201.18

9. **A 100 W, 250 V bulb is put in series with a 40 W, 250 V bulb across 500V supply. What will be the current drawn? What will be the power consumed by each bulb? Will such a combination work?**

Solution:

Resistance of 100 W bulb,

$$R_{100} = \frac{V^2}{W_1} = \frac{(250)^2}{100} = 625\ \Omega$$

Resistance of 40 W bulb,

$$R_{40} = \frac{V^2}{W_2} = \frac{(250)^2}{40} = 1562.5\ \Omega$$

When both bulbs are connected in series across 500 V supply, current through each bulb,

$$I = \frac{500}{R_{100} + R_{40}} = \frac{500}{625 + 1562.5} = 0.2286\text{ A.}$$

Power consumed by 100 W bulb $= I^2 R_{100} = (0.2286)^2 \times 625 = 32.66$ W

Power consumed by 40 W bulb $= I^2 R_{40}$

$= (0.2286)^2 \times 1562.5$

$= 81.65$ W

Voltage across 100 W bulb $= 0.2286 \times 625$

$= 142.875$ V

Voltage across 40 W bulb $= 0.2286 \times 1562.5 = 357.1875$ V

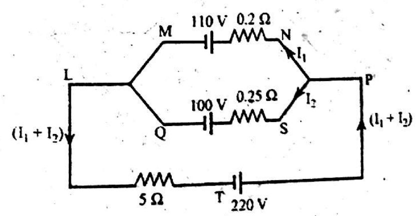
Hence, such a combination will not work because the voltage appearing across 40 W bulb is more than 250V, which is the rated voltage.

10. **A battery having an emf of 110 V and an internal resistance of 0.2 Ω is connected in parallel with another battery with e.m.f. of 100 V and a resistance of 0.25 Ω. The two in parallel are placed in series with regulating resistance of 5 ohms and connected across 220 V mains. Calculate:**

(i) The magnitude and direction of the current in each battery.

(ii) The total current taken from the mains supply.

Solution:



Let the direction of flow of currents I_1 and I_2 be as shown in figure.

Applying KVL to loop LMNPSQL, we get

$$110 + 0.2\,I_1 - 0.25I_2 - 100 = 0$$

Or, $0.2I_1 - 0.25\,I_2 = -10$

Or, $I_1 - 1.25I_2 = -50$ .... (i)

Applying KVL to loop LMNPTL, we get

$$110 + 0.2I_1 - 220 + 5(I_1 + I_2) = 0$$

Or, $5.2I_1 + 5I_2 = 110$

Or, $I_1 + 0.96I_2 = 21.15$ ....(ii)

Subtracting equation (ii) from equation (i), we get,

$$2.2I_2 = -71.15$$

$\therefore$ $I_2 = 32.19$ A

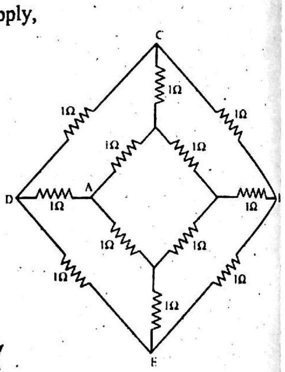
And $I_1 = -9.75$ A

Since I_1 turns out to be negative, its actual direction of flow is opposite to that shown in figure. In other words it is not a charging current but a discharging one. However, I_2 is charging current.

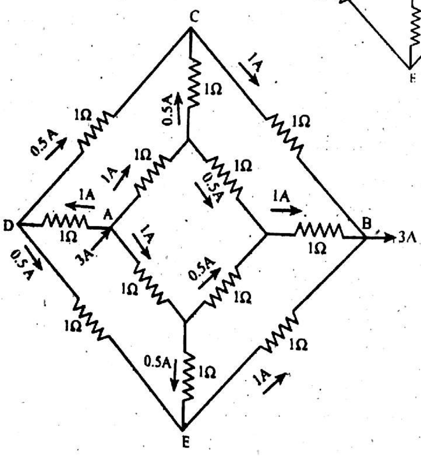
The total current taken from the mains supply,

$$I_1 + I_2 = -9.75 + 32.19$$
$$= 22.44 \text{ A}$$

11. For the network shown below, find the resistance between junction A and B.



Solution:



This problem can easily be solved by introducing a current of 3A at point A, and the current coming out of B will also be 3A. The division of currents in different branches is shown in figure above.

Now,

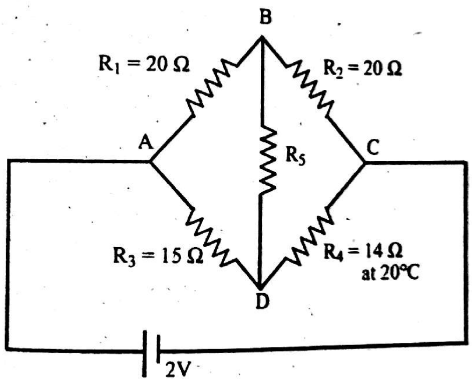
Applying Kirchhoff's law, the voltage between point A and point B,

$$V_{AB} = V_{AD} + V_{DC} + V_{CB}$$
$$= 1 \times 1 + 0.5 \times 1 + 1 \times 1$$
$$= 2.5 \text{ V}$$

$\therefore$ Resistance,

$$R_{AB} = \frac{V_{AB}}{I_{AB}}$$
$$= \frac{2.5}{3}$$
$$= 0.833\ \Omega.$$

12. In the circuit shown in the figure, resistors R_1, R_2, R_3 have negligible temperature coefficient of resistance. If resistor R_4 has a temperature coefficient of resistance of 0.004 per °C, and a resistance of 14 Ohms at 20°C, what rise in temperature is required to reduce the current in resistor R_5 to zero?



Solution:

For no current in R_5, the wheatstone bridge is balanced.

$$\therefore \frac{20}{15} = \frac{20}{R_4}$$

$$\therefore R_4 = 15\Omega$$

Let, t be temperature at which 14Ω resistance becomes 15Ω

$$\therefore \quad \frac{R_t}{R_{20}} = \frac{15}{14}$$

$$= \frac{R_0[1 + \alpha_0 t]}{R_0[1 + \alpha_0 \times 20]}$$

$$= \frac{1 + t \times 0.004}{1 + 20 \times 0.004}$$

or, $$\frac{15}{14} = \frac{1 + 0.04t}{1.08}$$

or, $$1 + 0.004t = 1.08 \times \frac{15}{14}$$

$$\therefore \quad t = \frac{0.15714}{0.004}$$

$$= 39.29°C$$

3

NETWORK THEOREMS

3.1 NETWORK TERMINOLOGY

While discussing network theorems and techniques, one often comes across the following terms.

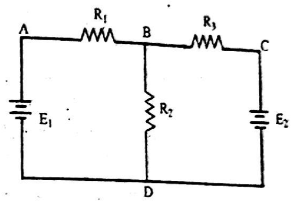


fig. 3.1

i) Active element

An active element is one which supplies electrical energy to the circuit. Thus in figure, E_1 and E_2 are the active elements because they supply energy to the circuit.

ii) Passive element

A passive element is one which receives electrical energy and then either converts it into heat (resistance) or stores in an electric field (capacitance) or magnetic field (inductance). Thus in figure, there are three passive elements namely R_1, R_2 and R_3

iii) Node

A node of a network is an equipotential surface at which two or more circuit elements are joined. Here A, B, C and D are nodes.

iv) Junction

A junction is that point in a network where three or more circuit elements are joined. In figure, there are only two junction points, B and D.

V) Branch

A branch is that part of a network which lies between two junction points. Referring to figure, there are a total of three branches; BAD, BCD and BD.

vi) Loop

A loop is any closed path of a network. Thus in figure ABDA, BCDB and ABCDA are the loops.

vii) Mesh

A mesh is the most elementary form of a loop and cannot be further divided into other loops. In figure, both loops ABDA and BCDB are meshes because they cannot be further divided into other loops.

viii) Unilateral circuit

A unilateral circuit is one whose characteristics are not same in either direction. For example: diode, transistor.

ix) Bialateral circuit

A bilateral circuit is one whose characteristics are same in either direction. For example: resistor, transmission line.

3.2.1 Nodal analysis

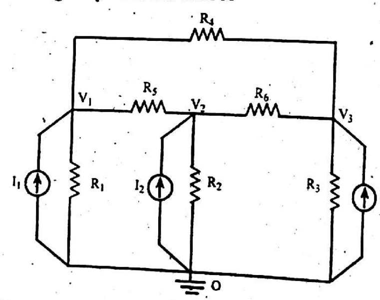
This method of circuit analysis is based on seeking the solution of a network in terms of voltage of nodes measured with respect to reference node. A reference node is usually the point in network called ground. Nodal analysis is the application of KCL.

Steps for nodal analysis

- Find the possible number of nodes.
- Select one node as the ground reference. The choice doesn't affect the result and is just a matter of convention. Choosing the node with most connections can simplify the analysis.
- Assign a variable for each node whose voltage is unknown. If the voltage is already known, it is not necessary to assign a variable.
- For each unknown voltage, form an equation based on Kirchhoff's current law.
- If there are voltage sources between two unknown voltages, join the two nodes as supernode. The currents of the two nodes are combined in a single equation and a new equation for the voltage is formed.
- Solve the system of simultaneous equations for each unknown voltage.

On the basis of source present, the circuit may be classified as follows and can be solved accordingly by Nodal analysis.

I) Circuit containing only current source



Applying KCL at node 1,

$$\frac{V_1-0}{R_1}+\frac{V_1-V_2}{R_5}+\frac{V_1-V_3}{R_4}=I_1$$

or, $$\left(\frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_5}\right)V_1+\left(-\frac{1}{R_5}\right)V_2+\left(-\frac{1}{R_4}\right)V_3=I_1 \text{ ..... (i)}$$

or, $$G_{11}V_1+G_{12}V_2+G_{13}V_3=I_1 \text{ ....(ia)}$$

Applying KCL at node 2,

$$\frac{V_2-0}{R_2}+\frac{V_2-V_1}{R_5}+\frac{V_2-V_3}{R_6}=I_2$$

or, $$\left(-\frac{1}{R_5}\right)V_1+\left(\frac{1}{R_2}+\frac{1}{R_5}+\frac{1}{R_6}\right)V_2+\left(-\frac{1}{R_6}\right)V_3=I_2 \text{ ..... (ii)}$$

or, $$G_{21}V_1+G_{22}V_2+G_{23}V_3=I_2 \text{ ....(iia)}$$

Applying KCL at node 3,

$$\frac{V_3-0}{R_3}+\frac{V_3-V_2}{R_6}+\frac{V_3-V_1}{R_4}=I_3 \text{ ....(iii)}$$

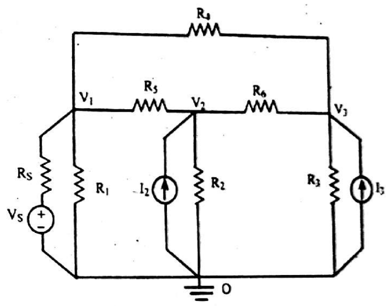
or, $$\left(-\frac{1}{R_4}\right)V_1+\left(-\frac{1}{R_6}\right)V_2+\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_6}\right)V_3=I_3 \text{ ..... (iii)}$$

or, $$G_{31}V_1+G_{32}V_2+G_{33}V_3=I_3 \text{ ....(iiia)}$$

Solving equations (i), (ii) and (iii), we can find node voltages V_1, V_2 and V_3.

II) Circuit containing voltage source in addition to current source.

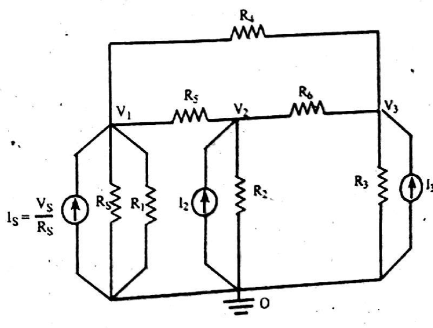
a) Voltage source transformable into current source.



Applying KCL at node 1,

$$\frac{V_1-V_2}{R_5}+\frac{V_1-V_3}{R_4}+\frac{V_1-V_S}{R_S}+\frac{V_1-0}{R_1}=0 \text{ ....(i)}$$

As voltage source is transformable into current source,



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Applying KCL at node 1,

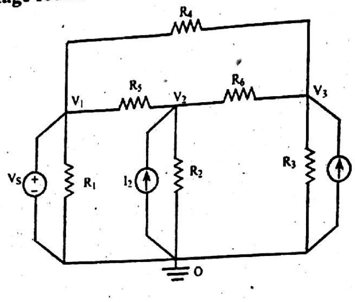
$$\frac{V_1-V_2}{R_5}+\frac{V_1-V_3}{R_4}+\frac{V_1-0}{R_S}+\frac{V_1-0}{R_1}=\frac{V_S}{R_S}$$

or, $$\left(\frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_5}+\frac{1}{R_S}\right)V_1+\left(-\frac{1}{R_5}\right)V_2+\left(-\frac{1}{R_4}\right)V_3=\frac{V_S}{R_S}\text{ ......(i)}$$

Equations (ii) and (iii) are same as above.

b) Voltage source not transformable into current source

i) Voltage source involving reference node

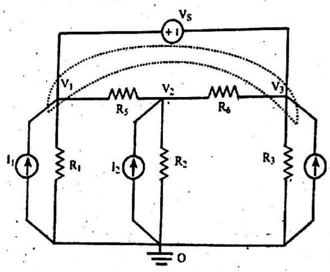


Here, $V_S = V_1 - 0$

or, $V_1 = V_S$ ....(i)

Equation (ii) and (iii) are same as above.

ii) Voltage source not involving reference node



Writing KCL for at the supernode 1 and 3,

$$\frac{V_1-0}{R_1}+\frac{V_1-V_2}{R_5}+\frac{V_3-0}{R_3}+\frac{V_3-V_2}{R_6}=I_1+I_3\text{ ....(i)}$$

Here, $V_1 - V_3 = V_S$..............................(iii)

Equation (ii) is same as above.

3.2.2 Mesh analysis

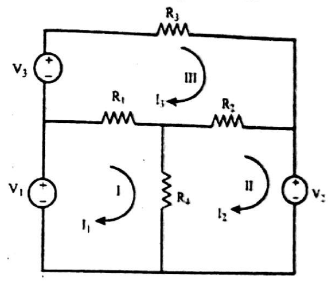
In this method, the solution of a given network is obtained in terms of mesh or loop current which are assigned arbitrary for each mesh. It is the application of Kirchhoff's voltage law.

Steps for Mesh Analysis

- Find the possible number of mesh.
- Assume the smallest number of mesh currents so that at least one mesh current links every element. As a matter of convenience, all mesh currents can be assumed to have a clockwise direction.
- For each mesh, write down Kirchhoff's voltage law equation. When more than one mesh current flows through an element, the algebraic sum of currents should be used. The algebraic sum of mesh currents may be the sum or the difference of the currents flowing through the element depending on the direction of mesh currents.
- Solve the above equations and from the mesh currents find the branch currents.

On the basis of source present the circuit may be classified as follows and can be solved accordingly by mesh analysis.

I) Circuit containing only voltage sources



Here, I_1, I_2 and I_3 are the loop current of loop I, loop II and loop III whose directions are assumed to be clockwise as shown in figure.

Applying KVL to the loop I, we get

$V_1 - (I_1 - I_3)R_1 - (I_1 - I_2)R_4 = 0$

or, $(R_1 + R_4)I_1 + (-R_4)I_2 + (-R_1)I_3 = V_1$ ....(i)

or, $R_{11}I_1 + R_{12}I_2 + R_{13}I_3 = V_1$ ....(ia)

Applying KVL to the loop II, we get

$-V_2 - (I_2 - I_1)R_4 - (I_2 - I_3)R_2 = 0$

or, $(-R_4)I_1 + (R_2 + R_4)I_2 + (-R_2)I_3 = V_2$ ....(ii)

or, $R_{21}I_1 + R_{22}I_2 + R_{23}I_3 = -V_2$ ....(iib)

Applying KVL to the loop III, we get

$V_3 - I_3R_3 - (I_3 - I_2)R_2 - (I_3 - I_1)R_1 = 0$

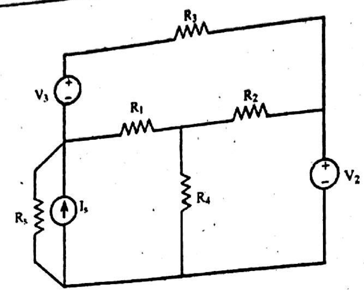
or, $(-R_1)I_1 + (-R_2)I_2 + (R_1 + R_2 + R_3)I_3 = V_3$ ....(iii)

or, $R_{31}I_1 + R_{32}I_2 + R_{33}I_3 = V_3$ ....(iiia)

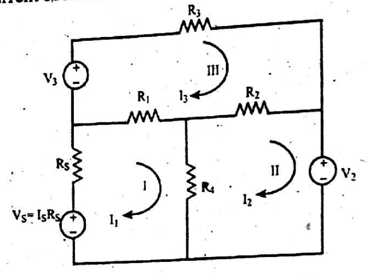
Solving equations (i), (ii) and (iii), we can find I_1, I_2 and I_3.

II) Circuit containing current source in addition to voltage source.

a) Current source transformable into voltage source.



Transforming the current source into voltage source.



Applying KVL to the loop I, we get

$I_S R_S - I_1 R_S - R_1 (I_1 - I_3) - R_4 (I_1 - I_2) = 0$

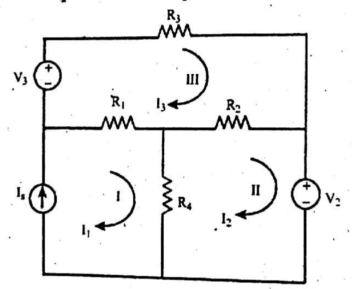
or, $(R_S + R_1 + R_4) I_1 + (-R_4) I_2 + (-R_1) I_3 = I_S R_S$ ....(i)

or, $R_{11} I_1 + R_{12} I_2 + R_{13} I_3 = I_S R_S$ ....(ia)

Equations (ii) and (iii) are same as above

b) Current source not transformable into voltage source

i) Current source present in the perimeter of any individual loop.



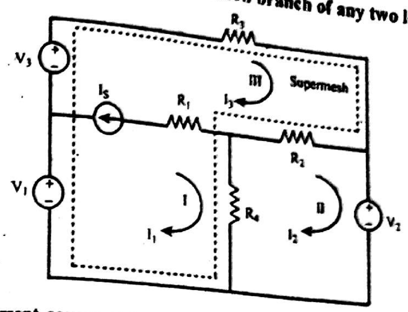
As we are interested to know the loop current and we can see that current source is present in the perimeter of loop I,

So, directly we write

$I_1 = I_S$ ....(i)

Equations (ii) and (iii) are same as above.

ii) Current source present in the common branch of any two loops.



If the current source occurs as a common element between the two meshes then one of the way is to form a supermesh from two meshes that contain current source in their common branch and then write KVL equation for supermesh.

Here, $I_3 - I_1 = I_S$ ....(i)

Applying KVL for supermesh I and III, we get

$V_3 - I_3 R_3 - R_2 (I_3 - I_2) - R_4 (I_1 - I_2) + V_1 = 0$

or, $I_3 R_3 + R_2 (I_3 - I_2) + R_4 (I_1 - I_2) = V_1 + V_3$

Equation (ii) is same is above.

3.3 Star–delta and delta–star transformaiton

In many circuit applicaitons, we encounter components connected together in one of two ways to form a three – terminal network; the 'Delta' or Δ (also known as the pi or π) configuraton and the 'Star' or Y (also known as the T) configuration:

3.3.1 Delta – Star transformation

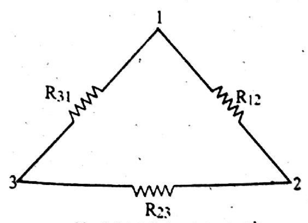


Fig. 3.2 (a) Delta (Δ) network

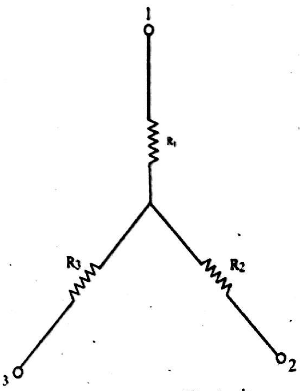


Fig 3.2 (b) Star (Y) network

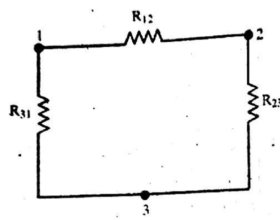


Fig. 3.3 (a) Pi (π) network

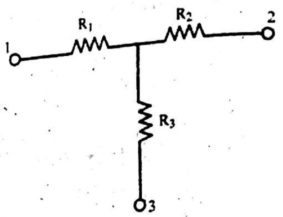


Fig. 3.3 (b) Tee (T) network

Consider the two circuits shown in the figure 3.2 (a) and figure 3.2 (b). They will be equivalent if the resistance measured between any two terminals 1, 2 and 3 is the same in the two cases.

$$[R_{12}]_Y = [R_{12}]_\Delta \text{ ....(i)}$$

or, $$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \text{ ....(ii)}$$

Similarly, $$R_2 + R_3 = \frac{R_{23}(R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \text{ ....(iii)}$$

and, $$R_3 + R_1 = \frac{R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \text{ ....(iv)}$$

Adding equations (ii), (iii) and (iv), we get

$$R_1 + R_2 + R_2 + R_3 + R_3 + R_1 = \frac{R_{12}(R_{23} + R_{31}) + R_{23}(R_{31} + R_{12}) + R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}}$$

or, $$2(R_1 + R_2 + R_3) = \frac{2(R_{12}R_{23} + R_{23}R_{31} + R_{31}R_{12})}{R_{12} + R_{23} + R_{31}}$$

or, $$R_1 + R_2 + R_3 = \frac{R_{12}R_{23} + R_{23}R_{31} + R_{31}R_{12}}{R_{12} + R_{23} + R_{31}} \text{ ....(v)}$$

Subtracting equations (ii), (iii), (iv) from (v), we get

$$R_1 + R_2 + R_3 - (R_1 + R_2) = \frac{R_{12}R_{23} + R_{23}R_{31} + R_{31}R_{12} - R_{12}R_{23} - R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$\therefore\ R_3 = \frac{R_{23}R_{31}}{R_{12} + R_{23} + R_{31}} \text{ .... (vi)}$$

Similarly, $$R_2 = \frac{R_{12}R_{23}}{R_{12} + R_{23} + R_{31}} \text{ .... (vii)}$$

$$R_1 = \frac{R_{31}R_{12}}{R_{12} + R_{23} + R_{31}} \text{ .... (viii)}$$

From above it may be noted that resistance of each arm of the star is given by the product of the resistance of the two delta sides that meet at its end divided by the sum of the three delta resistances.

3.3.2 Star - delta transformation

Now, Multiplying equations (viii) and (vii), (vi) and (vi), (vi) and (viii) and then adding them, we get,

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_{12} R_{23} R_{31}}{R_{12} + R_{23} + R_{31}} \text{ .... (ix)}$$

Dividing equation (ix) by equation (vi), we get

$$R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = R_1 + R_2 + \frac{R_1 R_2}{R_3} \text{ .... (x)}$$

Similarly, $$R_{23} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = R_2 + R_3 + \frac{R_2 R_3}{R_1} \text{ .... (xi)}$$

$$R_{31} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = R_1 + R_3 + \frac{R_3 R_1}{R_2} \text{ .... (xii)}$$

Hence, the equivalent delta resistance between any two terminals is given by the sum of star resistances between those terminals plus the product of these two star resistances divide by the third star resistance.

3.4 Superposition theorem

If a number of voltage or current sources are acting simultaneously in a linear network, the resultant current in any branch is the algebraic sum of the currents that would be produced in it when each source acts alone replacing all other independent sources by their internal resistances.

Steps for solving a network using the principle of superposition

- Take only one independent source of voltage/current and deactive the other independent voltage/ current sources (for voltage sources, remove the source and short circuit the respective circuit terminals and for current sources, just delete the source keeping the respective circuit terminals open). Obtain branch currents.
- Repeat the above step for each of the independent sources.

The total current in any branch of the circuit is the algebraic sum of currents due to each source. When finding total current in any branch, it is necessary to take into account the directions of the currents caused by each individual source, currents flowing in the same direction being additive, currents flowing in opposite direction being subtractive.

Explanation

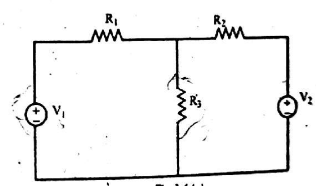


Fig 3.4 (a)

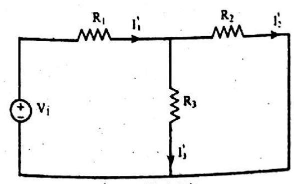


Fig 3.4 (b)

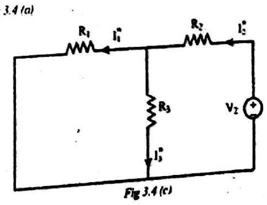


Fig 3.4 (c)

In figure 3.4 (a), to apply Superposition theorem, let us first take the source V_1 alone at first replacing V_2 by short circuit [Fig. 3.4 (b)]

Here, $I_1' = \dfrac{V_1}{\dfrac{R_2R_3}{R_2+R_3}+R_1}$

$I_2' = I_1'\dfrac{R_3}{R_2+R_3}$ and $I_3' = I_1' - I_2'$

Next, replacing V_1 by short circuit, let the circuit be energizd by V_2 only [Fig 3.4c]

Here, $I_2'' = \dfrac{V_2}{\dfrac{R_1R_3}{R_1+R_3}+R_2}$

$I_1'' = I_2''\dfrac{R_3}{R_1+R_3}$ and $I_3'' = I_2'' - I_1''$

As per Superposition theorem,

$I_3 = I_3' + I_3''$

$I_2 = I_2' - I_2''$

$I_1 = I_1' - I_1''$

It may be noted that during application of Superposition, the direction of currents calculated for each source should be taken care of.

3.5 Thevenin's theorem

A linear two terminal network can be replaced by an equivalent circuit composed of a voltage source V_{Th} in series with a resistance R_{Th}. The voltage V_{Th} is the voltage across the open circuited terminals and R_{Th} is the equivalent resistance of the network as seen from the terminals with all independent source suppressed.

To understand the application of this theorem, consider the circuit shown in figure 3.5 (a). It is desired to find the current through the resistance R_L. The steps are;

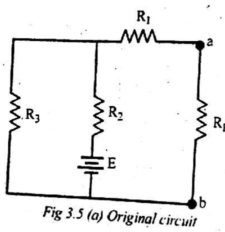


Fig 3.5 (a) Original circuit

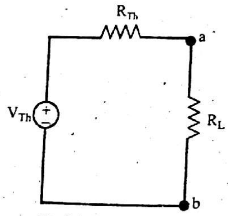


Fig 3.5(b) Thevenin's equivalent Circuit

i) Remove the resistance R_L, thus creating an open circuit at terminals ab.

ii) Find out the voltage between ab. This voltage is V_{Th}.

In the given circuit, $V_{Th} = E - \dfrac{ER_2}{R_2+R_3}$

iii) Short circuit the battery and find the resistance R_{Th} of the network as seen from terminals ab. Actually, voltage source are removed by internal resistance and current source if present is open circuited.

iv) The Thevenin's equivalent circuit for terminals ab is shown in fig 3.5 (b). The current through resistance R_L is;

$$I_{R_L} = \frac{V_{Th}}{R_{Th}+R_L}$$

3.6 Norton's theorem

Any two terminals of a network containing linear passive and active elements may be replaced by an equivalent current source I_N in parallel with a resistance R_N, where I_N is the current flowing through a short circuit placed across the terminals and R_N is the equivalent resistance of the network as seen from the two terminals with all independent sources suppressed.

To understand the application of this theorem consider the circuit shown in figure 3.6 (a). It is desired to find the current through the resistance R_L. The steps are:

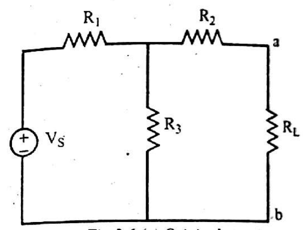


Fig 3.6 (a) Original circuit

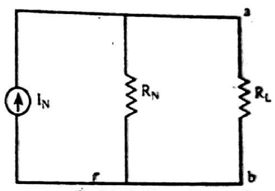


Fig 3.6 (b) Norton's equivalent circuit

i) Short circuit the pair of terminals at which Norton's equivalent is to be determined.

ii) Find the current flowing through the short circuited path. This is I_N.

iii) Redraw the circuit with all the voltage sources short circuited and all the current sources open circuited. Determine the resistance of the network as seen from the terminals ab. This is R_N (R_N is same as R_{Th}).

iv) The Norton's equivalent circuit for terminals ab is shown in figure 3.6 (b). The current through resistance R_L is;

$$I_L = I_N \times \frac{R_N}{R_N + R_L}$$

3.7 Maximum Power transfer theorem

For loads connected directly to a dc voltage supply, maximum power will be delivered to the load when the load resistance is equal to the internal resistance of the source i.e. $R_L = R_S$.

Consider the circuit of figure 3.7 where a battery having source resistance R_S is supplying by a load resistance R_L. The power delivered to the load is

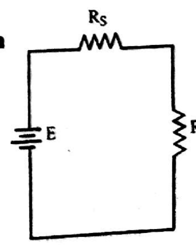


Fig. 3.7

$$P = I^2R_L = \frac{E^2R_L}{(R_L+R_S)^2} \quad \text{............(i)}$$

When R_L is zero, P is zero. When R_L is infinite, P is again zero. For the intermediate value of R_L the power delivered is maximum. This value of R_L can be found by putting $\frac{dP}{dR_L}$ equal to zero.

$$\frac{dP}{dR_L} = 0$$

or, $$\frac{d}{dR_L}\left[\frac{E^2R_L}{(R_L + R_S)^2}\right] = 0$$

or, $$\frac{(R_L + R_S)^2 E^2 - E^2R_L \times 2(R_L + R_S)}{(R_S + R_L)^4} = 0$$

or, $E^2 [(R_S + R_L)^2 - 2R_L (R_S + R_L)] = 0$

or, $E^2 (R_S + R_L) (R_S + R_L - 2R_L) = 0$

or, $E^2 (R_S + R_L) (R_S - R_L) = 0$

or, $E^2 (R_S^2 - R_L^2) = 0$

Since, $E \neq 0$

So, $R_S = R_L$

Thus, the power delivered to the load is maximum when the load resistance equals the source resistance.

Now,

If $R_L = R_S$ then from equation (i)

Power delivered to load;

$$P = I^2R_L = \frac{E^2R_L}{4R_L^2} = \frac{E^2}{4R_L}$$

Steps for soultion of a network utilizing maximum power transfer theorem

i) Remove the load resistance and find Thevenin's resistance (R_{Th}) of the source network looking through the open circuited load terminals.

ii) As per maximum power transfer theorem, this R_{Th} is the load resistance of the network i.e. $R_L = R_{Th}$ that allows maximum power transfer.

iii) Find the Thevenin's voltage (V_{Th}) across the open circuited load terminals.

iv) Maximum power transfer is given by, $\frac{V_{Th}^2}{4R_{Th}}$

Note: We can also use Norton's theorem to find maximum power transfer and is given by, $\frac{I_N^2R_N}{4}$

3.8 Reciprocity theorem

In a linear bilateral network having one independent source and no dependent sources, an important relation exists between a source voltage in one branch and current in some other branch.

The reciprocity theorem states that, "If a source of emf E, located at one point in a network composed of linear bilateral circuit elements, produces a current I at a selected point in the network, the same source of emf E acting at the second point will produce the same current at the first point.

Reciprocity theorem is also applicable to network containing a single current source. In this case the theorem states that, "If a current source I, loacted at one network containing linear bilateral circuit elements produces a voltage V at a second point in the network, the same source of current I acting at second point will produce the same voltage V at the first point.

Steps for Solving a network using Reciprocity theorem

i) The branches between which reciprocity is to be established are to be selected first.

ii) The current in the branch is obtained using conventional network analysis.

iii) The voltage source is interchanged between the branches concerned.

iv) The current in the branch where the voltage source was existing earlier is calculated.

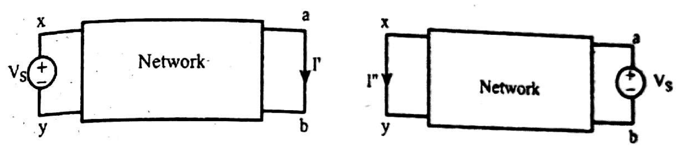


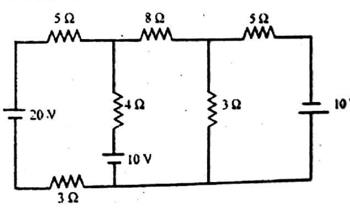
Fig. 3.8

It may be observed that the currents obtained in step-2 and step-4 are identical to each other for the validation of reciprocity theorem.

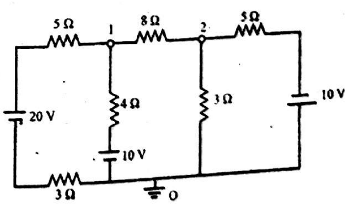
Exam Solutions

(Nodal and Mesh Analysis)

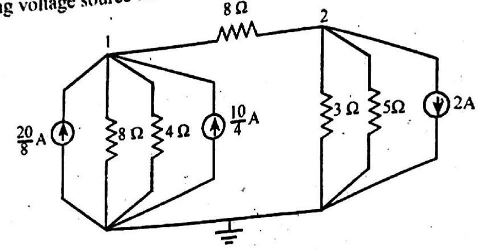
1. **Use nodal analysis method to find the current through 8Ω resistor in the circuit shown below.** **[2064 Shrawan]**



Solution:



Transforming voltage source into current source,



Let O be the reference node and the voltage of nodes 1 and 2 be V_1 and V_2 respectively.

Applying KCL at node 1;

$$\frac{20}{8}+\frac{10}{4}=\frac{V_1-0}{8}+\frac{V_1-0}{4}+\frac{V_1-V_2}{8}$$

or, $5=\frac{V_1}{8}+\frac{V_1}{4}+\frac{V_1-V_2}{8}$

or, $5=\frac{1}{2}V_1-\frac{1}{8}V_2$ ....(i)

Applying KCL at node 2;

$$0 = \frac{V_2-V_1}{8}+\frac{V_2-0}{3}+\frac{V_2-0}{5}+2$$

or, $0 = -\frac{V_1}{8}+\frac{V_2}{8}+\frac{V_2}{3}+\frac{V_2}{5}+2$

or, $-2 = -\frac{V_1}{8}+\frac{79}{120}V_2$ ....(ii)

Solving equations (i) & (ii) we get

$V_1 = 9.7010$ volt

$V_2 = -1.1960$ volt

The negative voltage at node 2 indicates that node 2 is at lower potential with respect to reference node.

Hence,

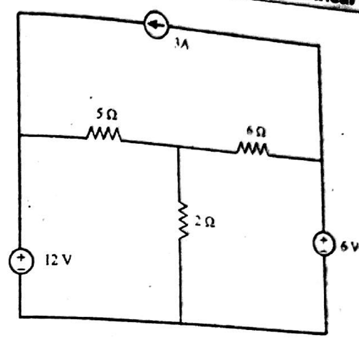
Current flowing through 8Ω resistor

$$= \frac{V_1-V_2}{8}$$

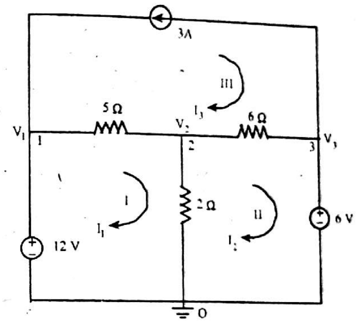
$$= \frac{9.7010-(-1.1960)}{8}$$

$$= 1.3621 \text{ A}$$

2. Use mesh current analysis method to calculate node voltages and current through resistors in the following network. **[2063 Kartik]**



Solution:



The three loop currents are shown in figure. For these loops, applying Kirchhoff's voltage law,

Here, $I_3 = -3A$

Loop I:

$12-5(I_1-I_3)-2(I_1-I_2)=0$

or, $12-5I_1+5I_3-2I_1+2I_2=0$

or, $12-5I_1-15-2I_1+2I_2=0$ [$\because I_3=-3A$]

or, $-7I_1+2I_2=3$ ....(i)

Loop II:

$-6-2(I_2-I_1)-6(I_2-I_3)=0$

or, $-6-2I_2+2I_1-6I_2+6I_3=0$

or, $2I_1-8I_2-18=6$ [$\because I_3=-3A$]

or, $2I_1-8I_2=24$ ....(ii)

Solving (i) & (ii), we get

$I_1=-1.3846A$

$I_2=-3.3462$ A

The negative sign indicates that the current flows in the direction opposite to our assumptions.

Suppose 0 be the reference node

So, $V_1 = 12V \quad V_3 = 6V$

We know,

$V_1 - V_2 = 5 \times (3 - 1.3846) = 8.077V$

or, $V_2 = V_1 - 8.077 = 12 - 8.077$

$\therefore V_2 = 3.923$ V

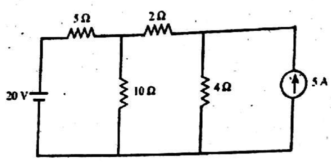
Hence, Current through 5Ω resistor = $3 - 1.3846 = 1.6154A$ (1 to 2)

Current through 6Ω resistor = $3.3462 - 3 = 0.3462A$ (3 to 2)

Current through 2Ω resistor = $3.3462 - 1.3846 = 1.9616A$ (2 to 0)

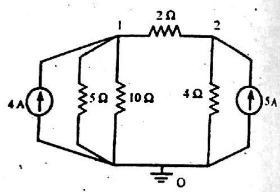
3. Calculate current through 10Ω resistor in the following network.

[2063 Kartik]



Solution:

Converting the voltage source into current source and undergoing nodal analysis.



Let 0 be the reference node and the voltage of nodes 1 and 2 be V_1 and V_2 respectively.

Applying KCL at node 1;

$$4 = \frac{V_1 - 0}{5} + \frac{V_1 - 0}{10} + \frac{V_1 - V_2}{2}$$

or, $$4 = \frac{V_1}{5} + \frac{V_1}{10} + \frac{V_1 - V_2}{2}$$

or, $$\frac{4}{5} V_1 - \frac{V_2}{2} = 4 \quad \text{.............(i)}$$

Applying KCL at node 2;

$$5 = \frac{V_2 - V_1}{2} + \frac{V_2 - 0}{4}$$

or, $$5 = -\frac{V_1}{2} + \frac{V_2}{2} + \frac{V_2}{4}$$

or, $$5 = -\frac{V_1}{2} + \left(\frac{1}{2} + \frac{1}{4}\right) V_2$$

or, $$5 = -\frac{V_1}{2} + \frac{3}{4} V_2$$

Solving equations (i) and (ii), we get

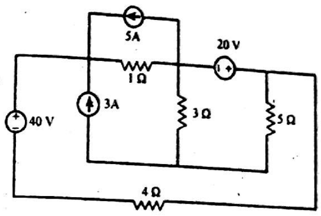
$V_1 = 15.7143V$

$V_2 = 17.1429V$

Now, Current through 10Ω resistor $= \frac{15.7143 - 0}{10} = 1.5714A$

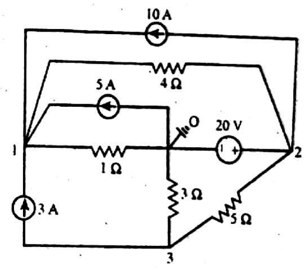
4. For the circuit shown below, determine the power dissipated by 5Ω resistor in the circuit using nodal analysis.

[2066 Kartik]



Solution:

Converting possible voltage source into current source.



Let 0 be the reference node and the voltage of nodes 1, 2 and 3 be V_1, V_2 and V_3 respectively,

Here, $V_2 = 20$ V

Applying KCL at node 3, we get

$$0 = 3 + \frac{V_3 - 0}{3} + \frac{V_3 - V_2}{5}$$

or, $$0 = 3 + \frac{V_3}{3} + \frac{V_3}{5} - \frac{20}{5} \qquad [\because V_2 = 20 V]$$

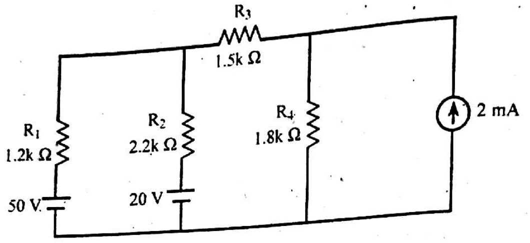
or, $$0 = \frac{8}{15} V_3 - 1$$

$\therefore$ $$V_3 = \frac{15}{8} V$$

Now, Power dissipated by 5Ω resistor

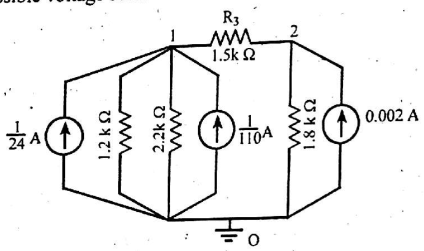
$$= \left(\frac{V_2 - V_3}{5}\right)^2 \times 5 = \left(\frac{20 - \frac{15}{8}}{5}\right)^2 \times 5 = 65.7031 \text{ watt}$$

5. **Using nodal analysis determine the current that flows through resistor R_3. [2066 Magh]**



Solution:

Converting possible voltage source into current source.



Let 0 be the reference node and the voltage of nodes 1 and 2 be V_1 and V_2 respectively.

Applying KCL at node 1, we get

$$\frac{1}{24} + \frac{1}{110} = \frac{V_1 - 0}{1.2 \times 1000} + \frac{V_1 - 0}{2.2 \times 1000} + \frac{V_1 - V_2}{1.5 \times 1000}$$

or, $0.0508 = \frac{43}{22000} V_1 - \frac{1}{500} V_2$ .... (i)

Applying KCL at node 2, we get

$$0.002 = \frac{V_2 - 0}{1.8 \times 1000} + \frac{V_2 - V_1}{1.5 \times 1000}$$

or, $0.002 = -\frac{1}{1500} V_1 + \frac{11}{9000} V_2$ .... (ii)

Solving equations (i) and (ii), we get

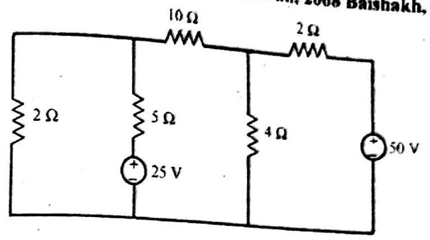
$V_1 = 32.6171$ V

$V_2 = 19.4275$V

∴ Current flowing through R_3 resistor

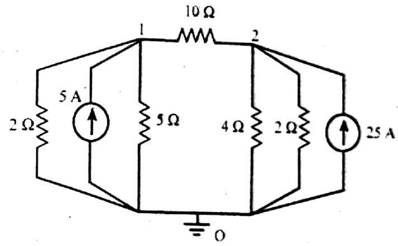
$$= \frac{V_1 - V_2}{R_3} = \frac{32.6171 - 19.4275}{1.5 \times 1000} = 8.79 \times 10^{-3} A$$

6. **Use nodal method to find the current through 10Ω resistor for circuit shown below. [2071 Shrawan, 2068 Baishakh, 2067 Ashad]**



Solution:

Converting the possible voltage source into current source.



Let 0 be the reference node and the voltage of node 1 and 2 be V_1 and V_2 respectively.

Applying KCL at node 1, we get

$$5 = \frac{V_1 - 0}{2} + \frac{V_1 - 0}{5} + \frac{V_1 - V_2}{10}$$

or, $5 = \frac{4}{5} V_1 - \frac{1}{10} V_2$ .... (i)

Applying KCL at node 2, we get

$$25 = \frac{V_2 - 0}{4} + \frac{V_2 - 0}{2} + \frac{V_2 - V_1}{10}$$

or, $25 = -\frac{1}{10} V_1 + \frac{17}{20} V_2$ ....(ii)

Solving equations (i) and (ii), we get

$V_1 = 10.075$ V

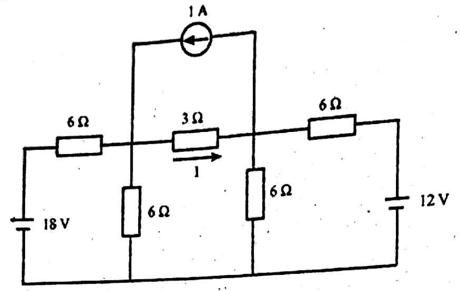
$V_2 = 30.597$ V

∴ Current through 10Ω resistor $= \frac{V_2 - V_1}{10}$

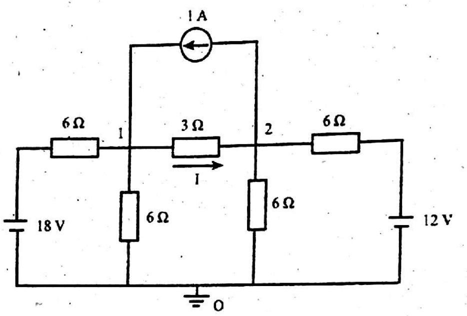
$$= \frac{30.597 - 10.075}{10}$$

$$= 2.052 \text{ A}$$

7. Find the current I in the circuit of figure given below by applying nodal voltage. [2067 Mangsir]



Solution:



Let 0 the reference node and the voltage of node 1 and 2 be V_1 and V_2 respectively.

Applying KCL at node 1, we get

$$1 = \frac{V_1 - 18}{6} + \frac{V_1 - 0}{6} + \frac{V_1 - V_2}{3}$$

or, $1 = \frac{2}{3}V_1 - \frac{1}{3}V_2 - 3$

or, $\frac{2}{3}V_1 - \frac{1}{3}V_2 = 4$ .... (i)

Applying KCL at node 2, we get

$$0 = \frac{V_2 - V_1}{3} + \frac{V_2 - 0}{6} + \frac{V_2 - 12}{6} + 1$$

or, $0 = -\frac{1}{3}V_1 + \frac{2}{3}V_2 - 2 + 1$

or, $\frac{1}{3}V_1 - \frac{2}{3}V_2 = -1$ ....(ii)

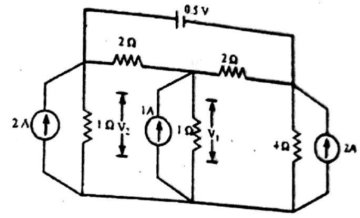
Solving equations (1) and (ii), we get

$V_1 = 9V$, $V_2 = 6V$

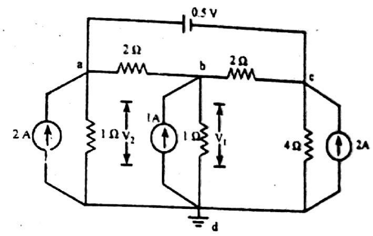
∴ Current flowing through 3Ω resistor

$$I = \frac{V_1 - V_2}{3} = \frac{9-6}{3} = \frac{3}{3} = 1A$$

8. Find the values of V_1, V_2 and the current flowing through the 4Ω resistor. [2070 Bhadra]



Solution:



Let d be the reference node and the voltage of node a, b and c be V_a, V_b and V_C respectively.

Here, node a and node c are supernode.

So, $V_a - V_c = 0.5$ ....(i)

Also, $2 + 2 = \frac{V_a - 0}{1} + \frac{V_a - V_b}{2} + \frac{V_c - 0}{4} + \frac{V_c - V_b}{2}$

or, $4 = V_a + \frac{V_a}{2} - \frac{V_b}{2} + \frac{V_c}{4} + \frac{V_c}{2} - \frac{V_b}{2}$

or, $4 = \frac{3}{2}V_a - V_b + \frac{3}{4}V_c$ ....(ii)

Applying KCL at node b, we get,

$$1 = \frac{V_b - 0}{1} + \frac{V_b - V_c}{2} + \frac{V_b - V_a}{2}$$

or, $1 = V_b + \frac{V_b}{2} - \frac{V_c}{2} + \frac{V_b}{2} - \frac{V_a}{2}$

or, $1 = -\frac{V_a}{2} + 2V_b - \frac{V_c}{2}$ ....(ii)

Solving equations (i), (ii) and (iii), we get

$V_a = 2.714V$

$V_b = 1.732$ V

$V_c = 2.214$ V

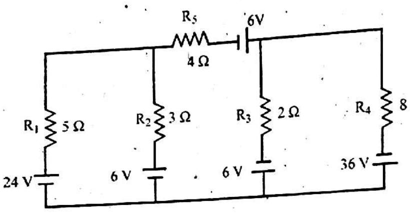
Therefore,

$V_1 = V_b = 1.732$ V

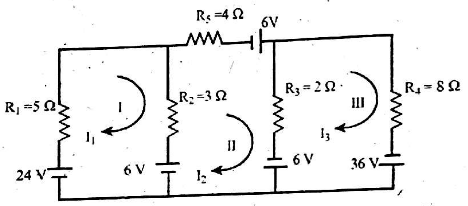
$V_2 = V_a = 2.714$V

Current flowing through 4Ω resistor $= \frac{V_C - 0}{4} = \frac{2.214}{4} = 0.554$A

9. Determine the current flowing through each resistor in the circuit shown below. [2064 Shrawan]



Solution:



The three loop currents are shown in figure. For these loops, applying KVL,

Loop – I:

$24 - 5I_1 - 3(I_1 - I_2) - 6 = 0$

or, $24 - 5I_1 - 3I_1 + 3I_2 - 6 = 0$

or, $-8I_1 + 3I_2 = -18$ ....(i)

Loop II:

$6 - 3(I_2 - I_1) - 4I_2 + 6 - 2(I_2 - I_3) + 6 = 0$

or, $6 - 3I_2 + 3I_1 - 4I_2 + 6 - 2I_2 + 2I_3 + 6 = 0$

or, $3I_1 - 9I_2 + 2I_3 = -18$ .... (ii)

Loop III:

$-6 - 2(I_3 - I_2) - 8I_3 + 36 = 0$

or, $-6 - 2I_3 + 2I_2 - 8I_3 + 36 = 0$

or, $2I_2 - 10I_3 = -30$ .... (iii)

Solving equation (i), (ii) and (iii) we get

$I_1 = 3.7926$ A, $I_2 = 4.1137$ A, $I_3 = 3.8227$ A

Hence,

Current flowing through R_1 resistor = 3.7926 A

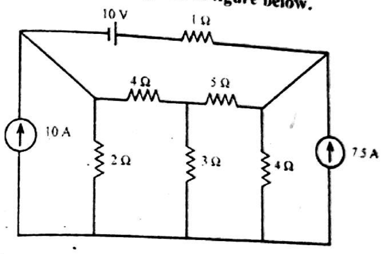
Current flowing through R_2 resistor = 4.1137 – 3.7926 = 0.3211 A

Current flowing through R_3 resistor = 4.1137 – 3.8227 = 0.2910 A

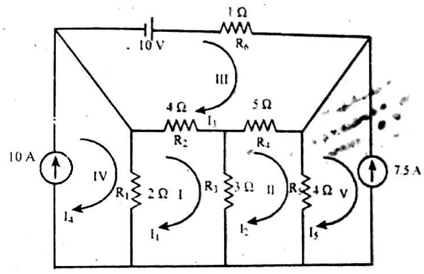
Current flowing through R_4 resistor = 3.8227 A

Current flowing through R_5 resistor = 4.1137 A

10. Use loop - current method to find the currents through each of the resistance for the network given in figure below. [2064 Poush]



Solution:



Consider loop currents be as shown in figure.

Here,

$I_4 = 10$ A, $I_5 = -7.5$ A

Applying KVL on loops I, II and III, we get

Loop I:

$-4(I_1 - I_3) - 3(I_1 - I_2) - 2(I_1 - I_4) = 0$

or, $-9I_1 + 3I_2 + 4I_3 = -20$ .... (i)

Loop II:

$-5(I_2 - I_3) - 4(I_2 - I_5) - 3(I_2 - I_1) = 0$

or, $3I_1 - 12I_2 + 5I_3 = 30$ .... (ii)

Loop III:

$10 - I_3 - 5(I$ [illegible] $) - 4(I_3 - I_1) = 0$

or, $4I_1 + 5I_2 -$ [illegible] $= -10$ .... (iii)

Solving equations (i), ([illegible] and (iii), we get

$I_1 = 2.2737$ A, I_2 [illegible] 1.4349 A, $I_3 = 1.1921$ A

Now,

Current flowing through $R_1 = 2\,\Omega$ resistor $= 10 - 2.2737 = 7.7263$ A

Current flowing through $R_2 = 4\Omega$ resistor $= 2.2737 - 1.1921 = 1.0816$ A

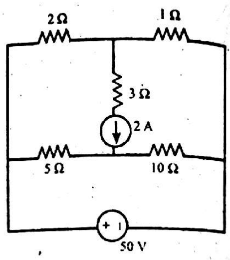
Current flowing through $R_3 = 3\Omega$ resistor $= 2.2737 + 1.4349 = 3.7086$ A

Current flowing through $R_4 = 5\Omega$ resistor $= 1.1921 + 1.4349 = 2.6270$ A

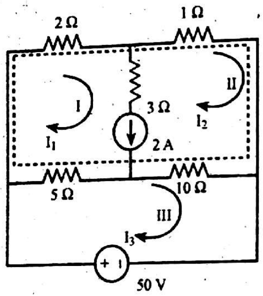
Current flowing through $R_5 = 4\Omega$ resistor $= 7.5 - 1.4349 = 6.0651$ A

Current flowing through $R_6 = 1\Omega$ resistor $= 1.1921$ A

11. Determine the current in the 5Ω resistor in the network shown below, using loop formulation method. [2066 Kartik]



Solution:



Consider loop currents be as shown in figure.

Here,

Mesh I and mesh II are super mesh.

so,

$I_1 - I_2 = 2$ .... (i)

Applying KVL on super mesh, we get

$-2I_1 - I_2 - 10(I_2 - I_3) - 5(I_1 - I_3) = 0$

or, $-2I_1 - I_2 - 10I_2 + 10\,I_3 - 5I_1 + 5I_3 = 0$

or, $-7I_1 - 11I_2 + 15\,I_3 = 0$ .... (ii)

Applying kVL on mesh III, we get

$50 - 5(I_3 - I_1) - 10\,(I_3 - I_2) = 0$

or, $50 - 5I_3 + 5I_1 - 10\,I_3 + 10\,I_2 = 0$

or, $5I_1 + 10\,I_2 - 15I_3 = -50$ .... (iii)

Solving equations (i), (ii) and (iii) we get

$I_1 = 17.3333$ A, $I_2 = 15.3333$ A, $I_3 = 19.3333$A

So,

The current flowing in 5Ω resistor

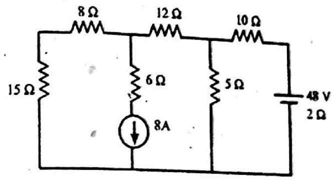
$= I_3 - I_1$

$= 19.3333 - 17.3333$

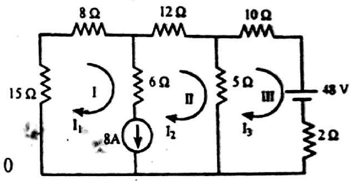
$= 2$ A

12. Calculate the current through 15 Ω resistor in figure given below:

[2066 magh]



Solution:



Consider loop currents be as shown in figure.Here mesh I and mesh II are supermesh

Here, $I_1 - I_2 = 8$ ....(i)

Applying KVL on supermesh, we get

$-15I_1 - 8I_1 - 12\,I_2 - 5(I_2 - I_3) = 0$

or, $-23\,I_1 - 12I_2 - 5I_2 + 5I_3 = 0$

or, $-23\,I_1 - 17\,I_2 + 5I_3 = 0$................ (ii)

Applyign KVL in mesh III we get

$-10I_3 - 48 - 2I_3 - 5(I_3 - I_2) = 0$

or, $-10I_3 - 48 - 2I_3 - 5I_3 + 5I_2 = 0$

or, $5I_2 - 17\,I_3 = 48$ .... (iii)

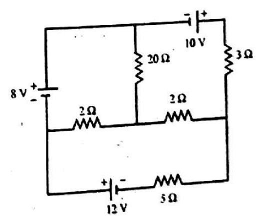
Solving (i), (ii) and (iii) we get

$I_1 = 2.858$ A, $I_2 = -5.142$ A, $I_3 = -4.3359$ A

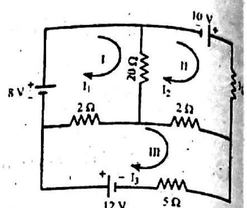
$\therefore$ Current flowing through 15Ω resistor = 2.858 A

13. Determine current in 5Ω resistor by mesh analysis in figure below.

[2068 Chaitra]



Solution:



Consider loop currents be as shown in figure.

Applying KVL on mesh I, we get

$8 - 20(I_1 - I_2) - 2(I_1 - I_3) = 0$

or, $8 - 20I_1 + 20I_2 - 2I_1 + 2I_3 = 0$

or, $-22I_1 + 20I_2 + 2I_3 = -8$ .... (i)

Applying KVL on mesh II, we get

$10 - 3I_2 - 2(I_2 - I_3) - 20(I_2 - I_1) = 0$

or, $10 - 3I_2 - 2I_2 + 2I_3 - 20I_2 + 20I_1 = 0$

or, $20I_1 - 25I_2 + 2I_3 = -10$ .... (ii)

Applying KVL on mesh III, we get

$12 - 2(I_3 - I_1) - 2(I_3 - I_2) - 5I_3 = 0$

or, $12 - 2I_3 + 2I_1 - 2I_3 + 2I_2 - 5I_3 = 0$

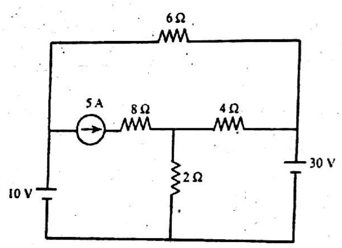
or, $2I_1 + 2I_2 - 9I_3 = -12$ .... (iii)

Solving equations (i), (ii) and (iii) we get

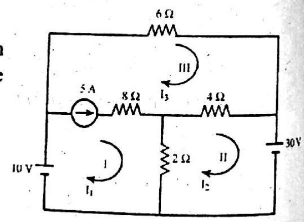
$I_1 = 4.679\text{ A}, I_2 = 4.411\text{ A}, I_3 = 3.353\text{A}$

Hence, Current in 5Ω resistor is 3.353 A

14. Find all branch current using mesh analysis method in the followin circuit. **[2070 Bharda**



Solution:



The three mesh currents are shown in figure. Here mesh I and mesh III are supermesh

So, $I_1 - I_3 = 5\text{A}$ .... (i)

Applying KVL on supermesh I and III, we get

$10 - 6I_3 - 4(I_3 - I_2) - 2(I_1 - I_2) = 0$

or, $10 - 6I_3 - 4I_3 + 4I_2 - 2I_1 + 2I_2 = 0$

or, $-2I_1 + 6I_2 - 10I_3 = -10$ .... (ii)

Applying KVL on mesh II,we get

$-30 - 2(I_2 - I_1) - 4(I_2 - I_3) = 0$

or, $-30 - 2I_2 + 2I_1 - 4I_2 + 4I_3 = 0$

or, $2I_1 - 6I_2 + 4I_3 = 30$ .... (iii)

Solving equations (i), (ii) and (iii), we get

$I_1 = \frac{5}{3}\text{ A} = 1.667\text{ A}$

$I_2 = -\frac{20}{3}\text{ A} = -6.667\text{ A}$

$I_3 = -\frac{10}{3}\text{ A} = -3.333\text{ A}$

Negative current indicates that our direction of assumption of current flow is opposite to its actual flow.

Now, Current in a branch containing 6Ω resistor = 3.333 A

Current in a branch containing 8Ω resistor = 5 A

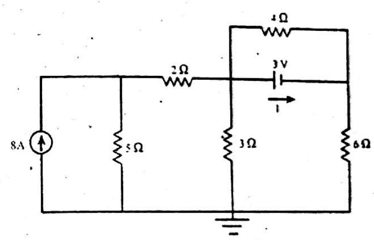
Current in a branch containing 4Ω resistor = 6.667 − 3.333 = 3.334 A

Current in a branch containing 2Ω resistor = 1.667 + 6.667 = 8.334 A

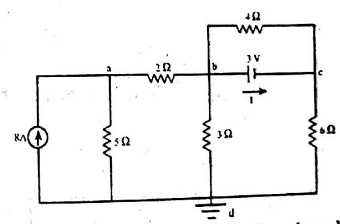
Current in a branch containing 10 V battery = 1.667 A

Current in a branch containing 30 V battery = 6.667 A

15. Find the value of 'I' through the voltage source using Nodal analysis. **[2071 Magh]**



Solution:



Let d be the reference node and the voltage of node a, b and c be V_a, V_b and V_c respectively.

Here, node b and node c are super node.

So, $V_b - V_c = 3$ .... (i)

Also,

$$0 = \frac{V_b - V_a}{2} + \frac{V_b - 0}{3} + \frac{V_c - 0}{6}$$

$$0 = \frac{V_b}{2} - \frac{V_a}{2} + \frac{V_b}{3} + \frac{V_c}{6}$$

or, $\frac{1}{2}V_a - \frac{5}{6}V_b - \frac{1}{6}V_c = 0$ .... (ii)

Applying KCL at node a, we get

$$8 = \frac{V_a - V_b}{2} + \frac{V_a - 0}{5}$$

or, $8 = \frac{V_a}{2} - \frac{V_b}{2} + \frac{V_a}{5}$

or, $8 = \frac{7}{10}V_a - \frac{1}{2}V_b$ .... (iii)

Solving equations (i), (ii) and (iii), we get

$$V_a = \frac{55}{3}\text{V}$$

$$V_b = \frac{29}{3}\text{V}$$

$$V_c = \frac{20}{3}\text{V}$$

We are now clear that with reference to node d, node a is at higher poten compared to node b and node c. Also, node b is at higher potential compared node c.

Now,

applying KCL at node b, in the above circuit.

(Incoming current)$_{\text{nodeb}}$ = (Outgoing current)$_{\text{nodeb}}$

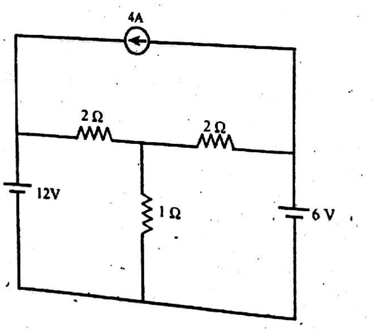
$$\frac{V_a - V_b}{2} = \frac{V_b - V_c}{4} + \frac{V_b - 0}{3} + I$$

or, $\frac{55-29}{3\times 2} = \frac{29-20}{3\times 4} + \frac{29}{3\times 3} + I$

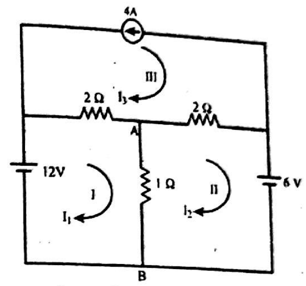
or, $\frac{13}{3} = \frac{3}{4} + \frac{29}{9} + I$

$\therefore\ I = \frac{13}{36} = 0.3611$ A

16. Calculate the current flowing through 1 ohm resistor for network show below using loop current method. **[2071 chaitr**



Solution:



The three loop currents are shown in figure. For these loops, we apply Kirchhoff's Voltage Law.

Here, $I_3 = -4$A

Loop I :

$12 - 2(I_1 - I_3) - 1(I_1 - I_2) = 0$

or, $12 - 2I_1 + 2I_3 - I_1 + I_2 = 0$

or, $-3I_1 + I_2 + 2I_3 = -12$

or, $-3I_1 + I_2 + 2(-4) = -12$

or, $-3I_1 + I_2 = -4$ .... (i)

Loop II :

$-6 - 1(I_2 - I_1) - 2(I_2 - I_3) = 0$

or, $-6 - I_2 + I_1 - 2I_2 + 2I_3 = 0$

or, $I_1 - 3I_2 + 2I_3 = 6$

or, $I_1 - 3I_2 + 2(-4) = 6$

or, $I_1 - 3I_2 = 14$ .... (ii)

Solving equations (i) and (ii), we get

$I_1 = -0.25$A,

$I_2 = -4.75$A, Negative sign indicates that the current flows in the direction opposite to our assumptions.

Now, the current flowing through

1Ω resistor $= I_2 - I_1$

$= 4.75 - 0.25$

$= 4.5$A (top to bottom) (A to B)

Additional questions (Nodal and mesh analysis)

1. In the circuit configuration of figure shown below, determine the number of

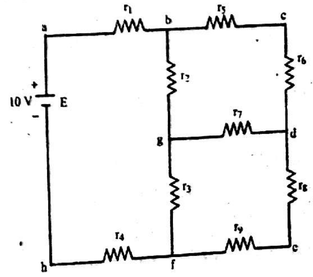
i. circuit elements

ii. nodes

iii. junction points

iv. branches

v. meshes



Solution:

i. No. of circuit elements = 10 [9 resistors + 1 voltage source]

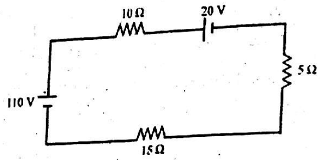
ii. No of nodes = 8 [a, b, c,d,e,f,g, h]

iii. No. of junction points = 4[b, d, g, f]

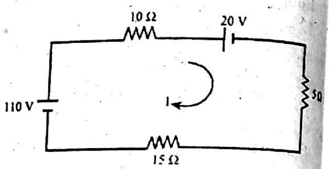
iv. No. of branches = 6[r_2, r_3, r_7, ($r_1 + r_4$), ($r_9 + r_8$), ($r_5 + r_6$)]

v. No. of meshes = 3[abgfha, bcdgb, defgd]

2. Find the current that flow and p.d. across each resistor in the circuit given below



Solution:



Applying KVL to mesh, we get

$110 - 10I - 20 - 5I - 15I = 0$

or, $-30I + 90 = 0$

∴ $I = 3$ A

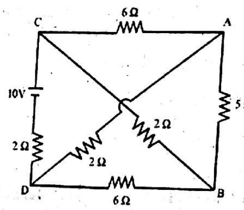
Current through each resistor, I = 3A

p.d. across 10 Ω resistor = 10 × 3 = 30 V

p.d. across 5 Ω resistor = 5 × 3 = 15 V

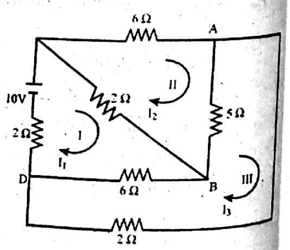
p.d, across 15 Ω resistor = 15 × 3 = 45 V

3. Calculate the voltage across AB in the circuit shown below and indicate the polarity of voltage.



Solution:

The circuit can be redrawn as,



Applying KVL to mesh I,we get

$10 - 2(I_1 - I_2) - 6(I_1 - I_3) - 2I_1 = 0$

or, $10 - 2I_1 + 2I_2 - 6I_1 + 6I_3 - 2I_1 = 0$

or, $-5I_1 + I_2 + 3I_3 = -5$ .... (i)

Applying KVL to mesh II, we get

$-6I_2 - 5(I_2 - I_3) - 2(I_2 - I_1) = 0$

or, $-6I_2 - 5I_2 + 5I_3 - 2I_2 + 2I_1 = 0$

or, $2I_1 - 13I_2 + 5I_3 = 0$ .... (ii)

Applying KVL to mesh III, we get,

$-2I_3 - 6(I_3 - I_1) - 5(I_3 - I_2) = 0$

or, $-2I_3 - 6I_3 + 6I_1 - 5I_3 + 5I_2 = 0$

or, $6I_1 + 5I_2 - 13I_3 = 0$ .... (iii)

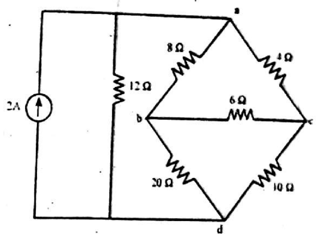
Solving equations (i), (ii) and (iii),we get

$I_1 = 1.8$ A, $I_2 = 0.7$ A, $I_3 = 1.1$ A

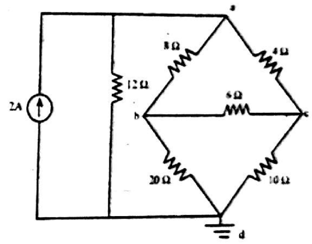
∴ Current in branch AB, $(I_3 - I_2) = 1.1 - 0.7 = 0.4$ A (B to A)

∴ Voltage across AB, $V_{BA} = V_B - V_A = 0.4 \times 5 = 2$ V

4. Use nodal analysis method to find the voltage drop across 10 Ω resistor of the circuit shown below:



Solution:



The circuit can be redrawn as,

Let d be the reference node and the voltage of node a, b, c and d be V_a,V_b, V_c and V_d respectively, then $V_d = 0$

Applying KCL at node a, we get

$$\frac{V_a - V_b}{8} + \frac{V_a - V_c}{4} + \frac{V_a - V_d}{12} = 2$$

or, $$\left(\frac{1}{12} + \frac{1}{8} + \frac{1}{4}\right) V_a - \frac{V_b}{8} - \frac{V_c}{4} = 2$$

or, $$\frac{11}{24} V_a - \frac{V_b}{8} - \frac{V_c}{4} = 2 \text{ .... (i)}$$

Applying KCL at node b, we get

$$\frac{V_b - V_a}{8} + \frac{V_b - V_c}{6} + \frac{V_b - V_d}{20} = 0$$

or, $$-\frac{1}{8} V_a + \left(\frac{1}{8} + \frac{1}{6} + \frac{1}{20}\right) V_b - \frac{V_c}{6} = 0$$

or, $$-\frac{1}{8} V_a + \frac{41}{120} V_b - \frac{V_c}{6} = 0 \text{ .... (ii)}$$

Applying KCL at node c, we get

$$\frac{V_c - V_a}{4} + \frac{V_c - V_b}{6} + \frac{V_c - V_d}{10} = 0$$

or, $-\frac{1}{4}V_a - \frac{1}{6}V_b + \left(\frac{1}{4}+\frac{1}{6}+\frac{1}{10}\right)V_c = 0$

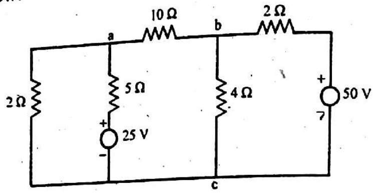
or, $-\frac{1}{4}V_a - \frac{1}{6}V_b + \frac{31}{60}V_c = 0$ .... (iii)

Solving equations (i), (ii) and (iii) we get

$V_a = 10.5$ V, $V_b = 7.5$ V, $V_c = 7.5$ V

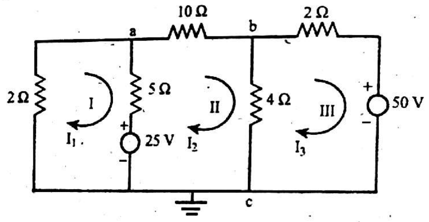
Hence; Voltage drop across 10 Ω resistor = V_c = 7.5 V.

5. Use the node voltage method to solve the mesh currents in the network shown in below:



Solution:

Let, c be the reference node and the voltage of node a and b be V_a and respectively.



Applying KCL at node a;

$$\frac{V_a-V_c}{2}+\frac{V_a-25}{5}+\frac{V_a-V_b}{10}=0$$

or, $\frac{V_a-0}{2}+\frac{V_a-25}{5}+\frac{V_a-V_b}{10}=0$

or, $\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{10}\right)V_a-\frac{1}{10}V_b=\frac{25}{5}$

or, $\frac{4}{5}V_a-\frac{1}{10}V_b=5$ .... (i)

Applying KCL at node b;

$$\frac{V_b-V_c}{4}+\frac{V_b-V_a}{10}+\frac{V_b-50}{2}=0$$

or, $-\frac{1}{10}V_a+\left(\frac{1}{4}+\frac{1}{10}+\frac{1}{2}\right)V_b-\frac{1}{4}V_c=\frac{50}{2}$

or, $-\frac{1}{10}V_a+\frac{17}{20}V_b=25$ .... (ii)

Solving (i) and (ii), we get

$V_a = 10.0746$ V, $V_b = 30.5970$ V

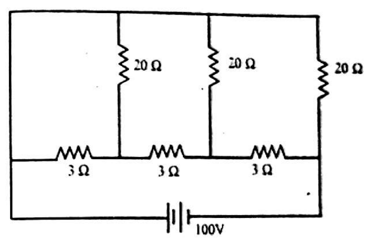
Mesh current I_1 can be determined by finding the current in resistor 2Ω of first mesh. Also mesh current I_2 can be determined by finding the current in resistor 10Ω of second mesh and finally mesh current I_3 can be determined by finding the current in resistor 2Ω of third mesh.

$I_1 = I_{2\Omega} = \frac{V_a-0}{2} = \frac{10.0746}{2} = 5.0373$ A (Clock wise)

$I_2 = I_{10\Omega} = \frac{V_b-V_a}{10} = \frac{30.5970-10.0746}{10} = 2.0522$ A (clockwise)

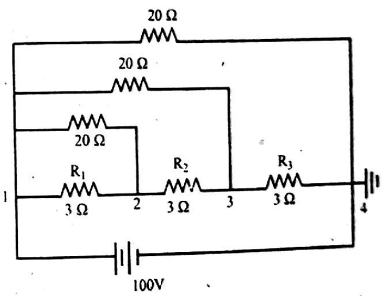
$I_3 = I_{2\Omega} = \frac{V_b-50-0}{2} = \frac{30.5970-50}{2} = -9.7015$ A (counter clockwise)

6. Determine the voltage across the 3 ohms resistors in the circuit shown.



Solution:

Redrawing the figure and solving by using nodal analysis.



Consider node 4 as the reference node and the voltage of nodes 1, 2 and 3 be V_1, V_2 and V_3 respectively then $V_1 = 100$ V

Applying KCL at node 2;

$$\frac{V_2-V_1}{3}+\frac{V_2-V_1}{20}+\frac{V_2-V_3}{3}=0$$

or, $\left(\frac{1}{3}+\frac{1}{20}+\frac{1}{3}\right)V_2-\frac{1}{3}V_3=\frac{100}{3}+\frac{100}{20}$ [$\because V_1 = 100$ V] or, $\frac{43}{60}V_2 - \frac{1}{3}V_3 = \frac{115}{3}$ .... (i)

Applying KCL at node 3;

$$\frac{V_3 - V_1}{20} + \frac{V_3 - V_2}{3} + \frac{V_3 - 0}{3} = 0$$

or, $-\frac{1}{3}V_2 + \left(\frac{1}{20} + \frac{1}{3} + \frac{1}{3}\right)V_3 = \frac{100}{20}$ [∵ $V_1 = 100$ V]

or, $-\frac{1}{3}V_2 + \frac{43}{60}V_3 = 5$ .... (ii)

Solving equations (i) and (ii), we get

$V_2 = 72.3948$ V

$V_3 = 40.6487$ V

Hence, Voltage across 3Ω (R_1) = $V_1 - V_2$

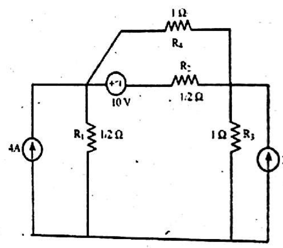
= 100 − 72.3948 = 27.6052 V

Voltage across 3Ω (R_2) = $V_2 - V_3$

= 72.3948 − 40.6487 = 31.74610 V

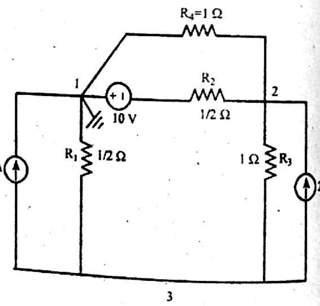
Voltage across 3Ω (R_3) = $V_3 - 0$ = 40.6487 V

7. Find the power loss in the resistor R_1 of the network shown below, using nodal analysis.



Solution:

Considering node 1 as the reference node and the voltage of nodes 2 and 3 be V_2 and V_3 respectively. Then, $V_1 = 0$



Applying KCL at node 2;

$$\frac{V_2 + 10 - 0}{\frac{1}{2}} + \frac{V_2 - 0}{1} + \frac{V_2 - V_3}{1} = 2$$

or, $2V_2 + 20 + 2V_2 - V_3 = 2$

or, $4V_2 - V_3 = -18$ .... (i)

Applying KCL at node 3;

$$4 + 2 + \frac{V_3 - 0}{\frac{1}{2}} + \frac{V_3 - V_2}{1} = 0$$

or, $6 + 2V_2 + V_3 - V_2 = 0$

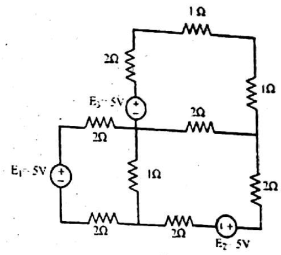
or, $-V_2 + 3V_3 = -6$ .... (ii)

Solving equations (i) and (ii), we get

$V_2 = -5.455$ V, $V_3 = -3.818$ V

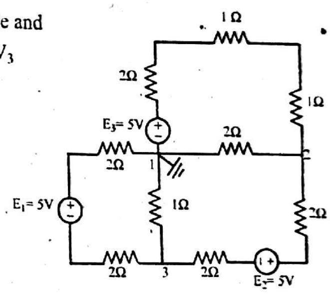
∴ Power loss in $R_1 = \frac{(V_1 - V_3)^2}{R_1} = \frac{(0 + 3.818)^2}{\frac{1}{2}} = 29.154$ watt

8. Using nodal method, find the battery currents in the circuit of fig below:



Solution:

Considering node 1 as the reference node and the voltage of nodes 2 and 3 be V_2 and V_3 respectively. Then, $V_1 = 0$



Applying KCL at node 2;

$$\frac{V_2 - 0}{2} + \frac{V_2 - 5 - 0}{(1 + 1 + 2)} + \frac{V_2 - 5 - V_3}{(2 + 2)} = 0$$

or, $\frac{V_2}{2} + \frac{V_2}{4} - \frac{5}{4} + \frac{V_2}{4} - \frac{5}{4} - \frac{V_3}{4} = 0$

or, $V_2 - \frac{V_3}{4} = 2.5$ .... (i)

Applying KCL at node 3;

$$\frac{V_3 - 0}{1} + \frac{V_3 + 5 - 0}{2 + 2} + \frac{V_3 + 5 - V_2}{2 + 2} = 0$$

or, $\frac{V_3}{1} + \frac{V_3}{4} + \frac{5}{4} + \frac{V_3}{4} + \frac{5}{4} - \frac{V_2}{4} = 0$

or, $-\frac{V_2}{4} + \frac{3}{2}V_3 = -2.5$ .... (ii)

Solving equations (i) and (ii), we get

$V_2 = 2.174$ V, $V_3 = -1.304$ V

Current of E_1 = $\frac{V_1 - E_1 - V_3}{2 + 2} = \frac{0 - 5 - (-1.304)}{4} = -0.924$ A (Discharging)

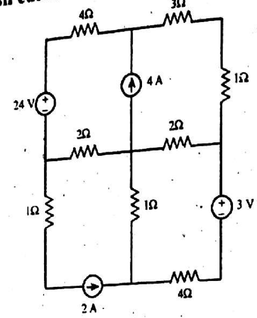
Current of E_2 = $\frac{V_2 - E_2 - V_3}{4} = \frac{2.174 - 5 - (-1.304)}{4}$

= −0.3805 A (Dishcharging)

Current of $E_3 = \dfrac{V_2 - 5 - 0}{4}$

$= \dfrac{2.174 - 5}{4} = -0.707$ A (Dishcharging)

8. Determine the mesh currents in the given circuit using mesh analysis.



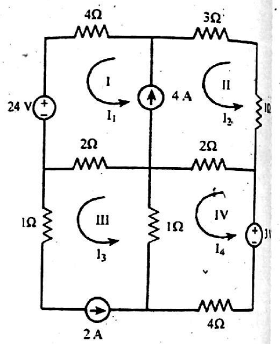
Solution:

Consider loop currents be as shown in figure $I_3 = 2A$

Here, Mesh I and mesh II are supermesh

so,

$I_1 - I_2 = 4A$ .... (i)



Applying KVL on super mesh

$-24 - 2(I_1 - I_3) - 2(I_2 - I_4) - 4I_2 - 4I_1 = 0$

or, $-24 - 2I_1 + 2I_3 - 2I_2 + 2I_4 - 4I_2 - 4I_1 = 0$

or, $-6I_1 - 6I_2 + 2I_4 = 20$ .... (ii) $[\because I_3 = 2A]$

Using equation (i), equation (ii) can be written as,

or, $-6(4 + I_2) - 6I_2 + 2I_4 = 20$

or, $-24 - 6I_2 - 6I_2 + 2I_4 = 20$

or, $-6I_2 + I_4 = 22$ .... (iii)

Applying KVL on mesh IV, we get

$-4I_4 + 3 - 2(I_4 - I_2) - 1(I_4 - I_3) = 0$

or, $-4I_4 + 3 - 2I_4 + 2I_2 - I_4 + I_3 = 0$

or, $2I_2 - 7I_4 = -5$ .... (iv) $[\because I_3 = 2A]$

Solving equation (iii) and equation (iv) we get

$I_2 = -3.725$ A, $I_4 = -0.35$ A

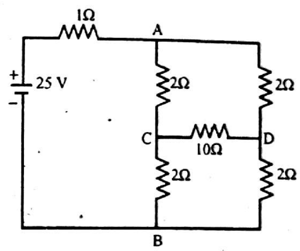
$\therefore$ $I_1 = 4 + I_2 = 4 + (-3.725) = 0.275$ A

The negative current indicates that the direction of current flow is opposite to our assumed direction.

Star – delta and delta – star transformation

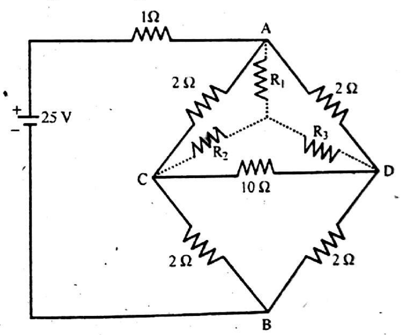
Exam solutions

1. Using Delta - star transformation determine the current drawn from the supply for the network given below. **[2064 Poush]**



Solution:

The network given in figure can be redrawn as,



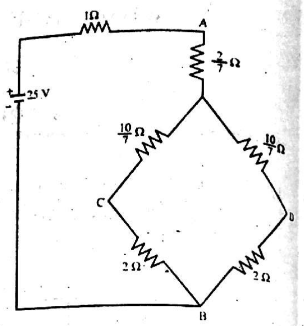
The three delta connected resistances between nodes A, C and D can be converted into equivalent star configuration. The values are,

$$R_1 = \frac{2 \times 2}{2 + 10 + 2} = \frac{2}{7}\ \Omega$$

$$R_2 = \frac{10 \times 2}{2 + 10 + 2} = \frac{10}{7}\ \Omega$$

$$R_3 = \frac{2 \times 10}{2 + 10 + 2} = \frac{10}{7}\ \Omega$$

After this transformation, the circuit is shown in figure below.



$$\text{Total resistance} = 1 + \frac{2}{7} + \left(\frac{10}{7}+2\right) \| \left(\frac{10}{7}+2\right)$$

$$= 1 + \frac{2}{7} + \left(\frac{24}{7} \| \frac{24}{7}\right)$$

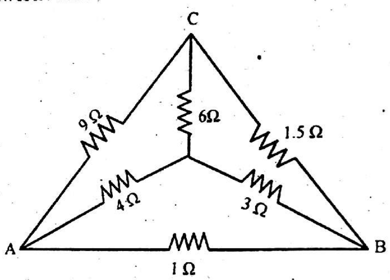
$$= 1 + \frac{2}{7} + \frac{12}{7}$$

$$= 3\ \Omega$$

$\therefore$ Current drawn from the supply $= \frac{25}{3}$

$= 8.333$ A.

2. **In the network shown below, using star/delta transformation, calculate the network resistance between A and B.** **[2066 Kartik]**



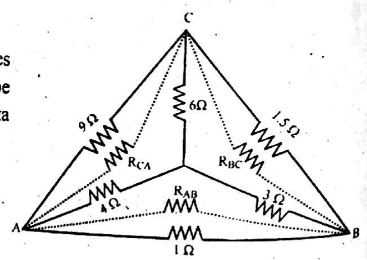
Solution:

Here, the three star connected resistances between nodes A, B and C can be converted into equivalent delta configuration. The values are,

$$R_{AB} = \frac{4\times3+3\times6+6\times4}{6} = 9\ \Omega$$

$$R_{BC} = \frac{4\times3+3\times6+6\times4}{4} = 13.5\ \Omega$$

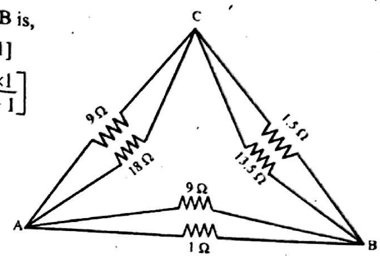
$$R_{CA} = \frac{4\times3+3\times6+6\times4}{3} = 18\ \Omega$$



After this transformation, the circuit is shown in the figure below.

Now,

Equivalent resistance between A and B is,



$$R_{AB} = [(9 \| 18) + (1.5 \| 13.5)] \| [9 \| 1]$$

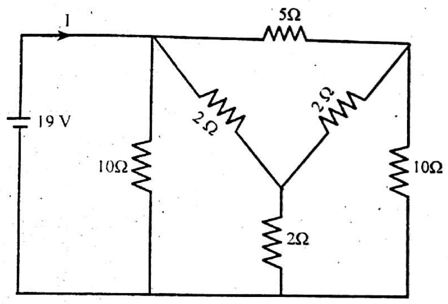
$$= \left[\frac{9\times18}{9+18} + \frac{1.5\times13.5}{1.5+13.5}\right] \| \left[\frac{9\times1}{9+1}\right]$$

$$= \frac{147}{20} \| \frac{9}{10}$$

$$= \frac{\frac{147}{20}\times\frac{9}{10}}{\frac{147}{20}+\frac{9}{10}}$$

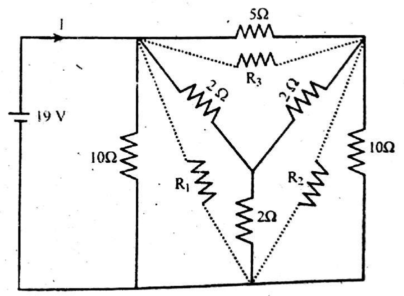
$$= 0.8018\ \Omega$$

3. **Find the current I as shown in figure below using star - delta transformation.** **[2068 Chaitra]**



Solution:

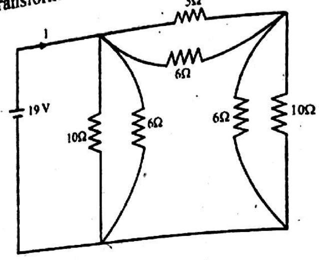
Three resistances 2Ω, 2Ω and 2Ω are star connected. Transforming them into delta with ends at the same points.



$$R_1 = \frac{2\times2+2\times2+2\times2}{2} = 6\ \Omega$$

Similarly, $R_2 = 6\ \Omega$, $R_3 = 6\ \Omega$

After this transformation, the circuit is shown in the figure below.



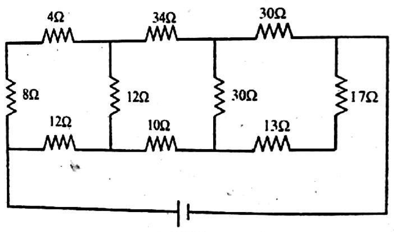
Total resistance = $[(10 \parallel 6) + (5 \parallel 6)] \parallel (10 \parallel 16)$

$$= \left[\frac{10\times 6}{10+6} + \frac{5\times 6}{5+6}\right] \parallel \left[\frac{10\times 6}{10+6}\right]$$

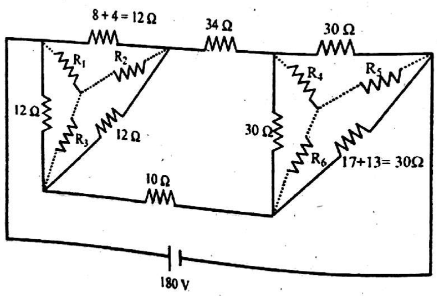
$$= \frac{285}{44} \parallel \frac{60}{16} = \frac{\frac{285}{44}\times\frac{60}{16}}{\frac{285}{44}+\frac{60}{16}} = 2.375\ \Omega$$

$\therefore$ Current supplied by the battery, $I = \dfrac{19}{2.375} = 8\ A$

4. **Determine the value of current in 10 ohm resistor in the network shown in figure below using Star/Delta conversions.** **[2069 Bhadra]**



Solution:



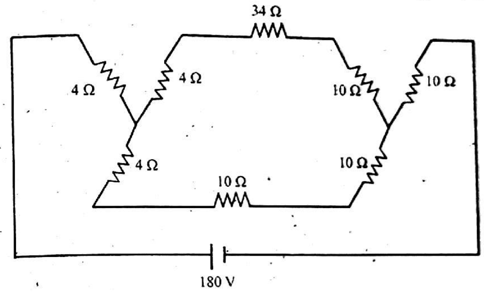
Three resistances at the left in the circuit (12Ω, 12Ω, 12Ω) and three resistances at the right in the circuit (30Ω, 30Ω, 30Ω) are delta connected. So, transforming them into star.

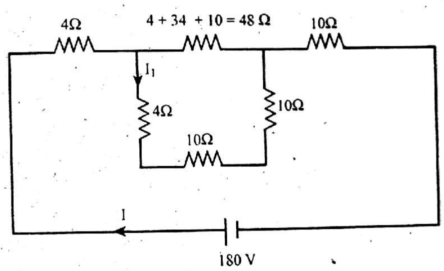
We get,

$$R_1 = R_2 = R_3 = \frac{12\times 12}{12+12+12} = 4\ \Omega$$

$$R_4 = R_5 = R_6 = \frac{30\times 30}{30+30+30} = 10\ \Omega$$

After this transformation, the circuit is shown in the figure below:





Equivalent resistance of the circuit,

$$R_{eq} = 4 + [48 \parallel (4 + 10 + 10)] + 10$$

$$= 4 + [48 \parallel 24] + 10$$

$$= 4 + \frac{48\times 24}{48+24} + 10$$

$$= 30\ \Omega$$

$\therefore$ Current, $I = \dfrac{180}{30} = 6\ A$

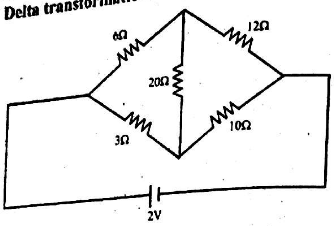
Now,

Current through 24 Ω (4 + 10 + 10 = 24 Ω) branch,

$$I_1 = \frac{I}{24+48}\times 48 = \frac{6}{72}\times 48 = 4\ A$$

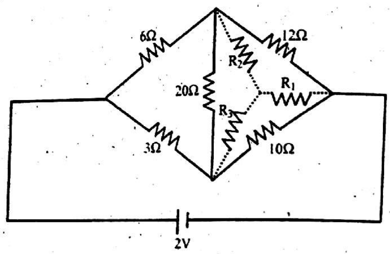
Hence, the current through 10 Ω resistor (which is a part of series branch of 24 ohms) is also 4A.

5. **Determine the current in 20 Ω resistor of the network shown in figure by using Star - Delta transformation.** **[2070 Cha**



Solution:

Resistances 12 Ω, 10 Ω and 20 Ω are delta connected. Transforming them into star with ends at the same points.

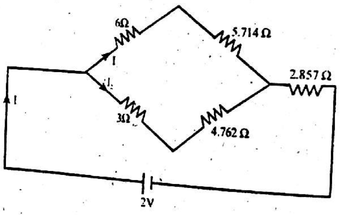


$$R_1 = \frac{12 \times 10}{12+10+20} = 2.857\ \Omega$$

$$R_2 = \frac{20 \times 12}{12+10+20} = 5.714\ \Omega$$

$$R_3 = \frac{20 \times 10}{12+10+20} = 4.762\ \Omega$$

After this transformation, the circuit is shown in figure below.



Here, Current, $I = \dfrac{2}{[(6+5.714) \parallel (3+4.762)] + 2.857}$

$$= \frac{2}{[11.714 \parallel 7.762] + 2.857}$$

$$= \frac{2}{4.669 + 2.857} = 0.266\ \text{A.}$$

Using current division rule,

$$I_1 = \frac{I}{(6+5.714)+(3+4.762)} \times (3+4.762)$$

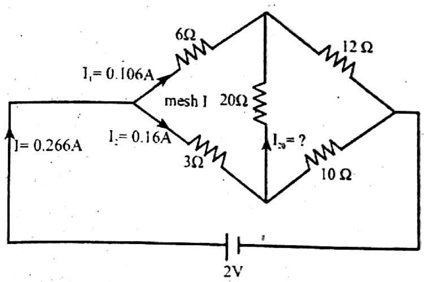
$$= \frac{0.266}{19.476} \times 7.762$$

$$= 0.106\ \text{A}$$

Simply, $I_2 = I - I_1$ (using KCL)

$= 0.266 - 0.106 = 0.160$ A.

Now, to find the current in 20 Ωresistor;



Applying KVL in mesh I considering branch current method, we get

$$-6\,I_1 + 20\,I_{20} + 3\,I_2 = 0$$

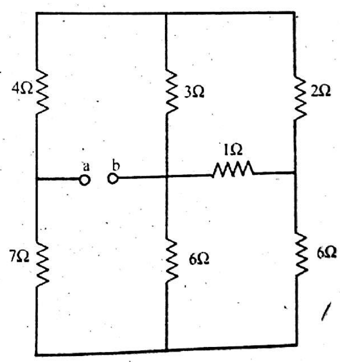
Or, $-6 \times 0.106 + 20 \times I_{20} + 3 \times 0.16 = 0$

Or, $-0.156 + 20 \times I_{20} = 0$

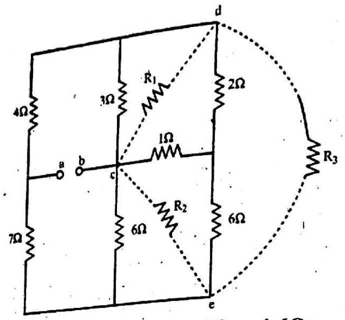
Or, $I_{20} = \dfrac{0.156}{20}$

$\therefore$ $I_{20} = 7.8 \times 10^{-3} = 7.8$ mA.

6. **Using star - delta transformation, find the equivalent resistance between terminals 'a' and 'b'.**



Solution:



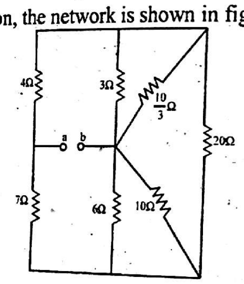
In the right side of the given network, 1Ω, 2Ω and 6Ω are connected in star. Transforming them into delta,

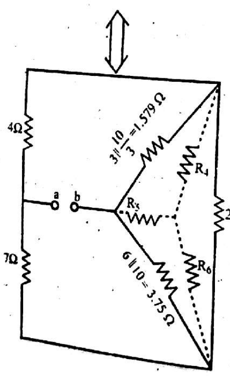
$$R_1 = \frac{1 \times 2 + 2 \times 6 + 6 \times 1}{6} = \frac{10}{3}\,\Omega$$

$$R_2 = \frac{1 \times 2 + 2 \times 6 + 6 \times 1}{2} = 10\,\Omega$$

$$R_3 = \frac{1 \times 2 + 2 \times 6 + 6 \times 1}{1} = 20\,\Omega$$

After this transformation, the network is shown in figure below,





Considering delta to star transformation,
We get

$$R_4 = \frac{1.579 \times 20}{1.579 + 20 + 3.75}$$

$$= 1.247\,\Omega$$

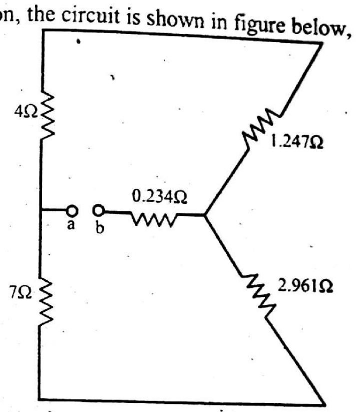
$$R_5 = \frac{3.75 \times 1.579}{1.579 + 20 + 3.75}$$

$$= 0.234\,\Omega$$

$$R_6 = \frac{20 \times 3.75}{1.579 + 20 + 3.75}$$

$$= 2.961\,\Omega$$

After transformation, the circuit is shown in figure below,



Hence,
the equivalent resistance between terminals 'a' and 'b',

$$R_{ab} = [(4 + 1.247) \,//\, (7 + 2.961)] + 0.234$$

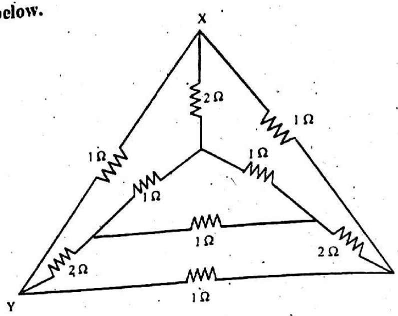
$$= [5.247 \,//\, 9.96] + 0.23$$

$$= 3.437 + 0.234$$

$$= 3.671\,\Omega$$

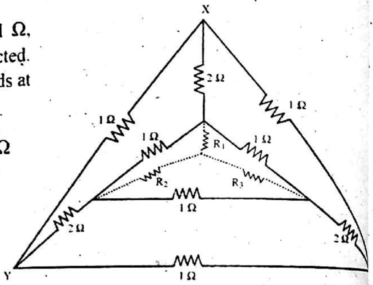
Additional Questions

1. Determine the resistance between the point X and Y for the network given below.



Solution:

Inner three resistances 1 Ω, 1Ω and 1 Ω are delta connected. Transforming them into star with ends at the same points.

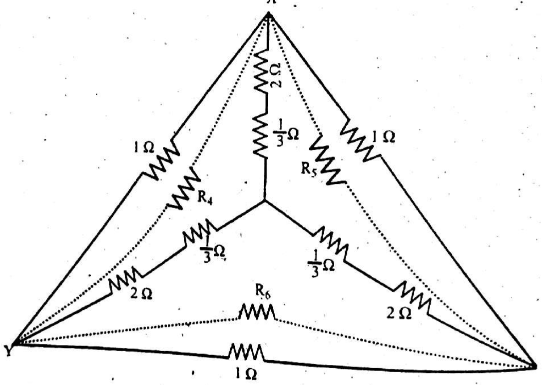


$$R_1 = \frac{1\times1}{1+1+1} = \frac{1}{3}\ \Omega$$

Similarly,

$$R_2 = \frac{1}{3}\ \Omega,\ R_3 = \frac{1}{3}\ \Omega$$

After this transformation, the circuit is shown in the figure below,



Converting inner star connected resistances

$2+\frac{1}{3} = \frac{7}{3}\ \Omega, \frac{7}{3}\ \Omega$ and $\frac{7}{3}\ \Omega$ into delta connection.

$$R_4 = \frac{\frac{7}{3}\times\frac{7}{3}+\frac{7}{3}\times\frac{7}{3}+\frac{7}{3}\times\frac{7}{3}}{\frac{7}{3}} = 7\ \Omega$$

Similarly,

$$R_5 = 7\ \Omega,\ R_6 = 7\ \Omega$$

Now,

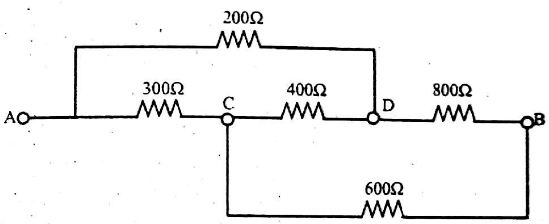
The resistance between the points X and are Y is,

$$R_{xy} = [(R_5 \parallel 1) + (R_6 \parallel 1)] \parallel [R_4 \parallel 1]$$

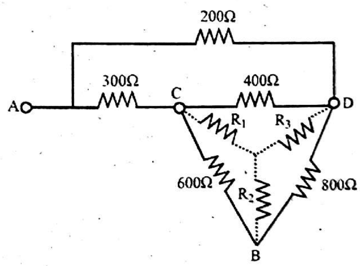
$$= [(7 \parallel 1) + (7 \parallel 1)] \parallel [7 \parallel 1] = \left[\frac{7}{8}+\frac{7}{8}\right] \parallel \left[\frac{7}{8}\right] = \frac{7}{4} \parallel \frac{7}{8}$$

$$\therefore R_{xy} = \frac{7}{12}\ \Omega$$

2. Using Star/delta and delta/ star conversion, find the resistance between the terminals A and B.



Solution:



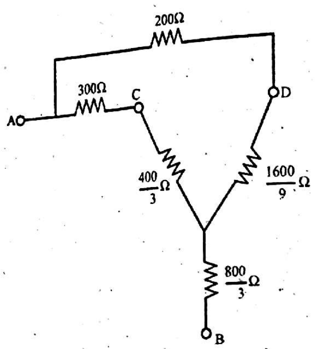
The three delta connected resistances between nodes B, C and D can be connected into equivalent star configuration. The values are,

$$R_1 = \frac{600\times400}{600+400+800} = \frac{400}{3}\ \Omega$$

$$R_2 = \frac{600 \times 800}{600 + 400 + 800} = \frac{800}{3}\ \Omega$$

$$R_3 = \frac{400 \times 800}{600 + 400 + 800} = \frac{1600}{9}\ \Omega$$

After this transformation the circuit is shown in the figure below.



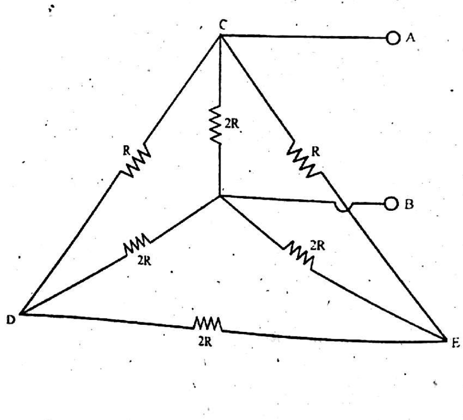
$$\therefore R_{AB} = \left[\left(300 + \frac{400}{3}\right) \| \left(200 + \frac{1600}{9}\right)\right] + \frac{800}{3}$$

$$= \left[\frac{1300}{3} \| \frac{3400}{9}\right] + \frac{800}{3}$$

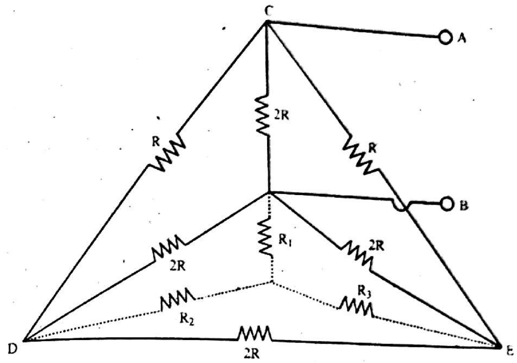
$$= 201.826 + \frac{800}{3}$$

$$= 468.493\ \Omega$$

3. **Making use of star/ delta transformation, determine the resista[...] between terminals A and B.**



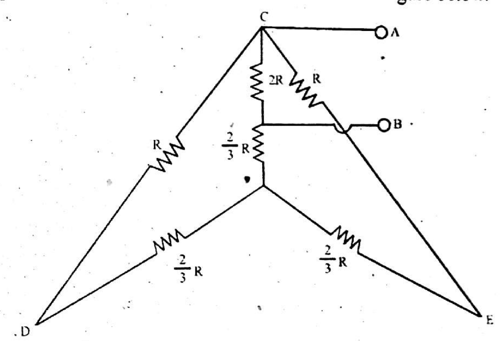
Solution:



The three delta connected resistances between nodes B, D and E can be converted into equivalent star configuration. The values are,

$$R_1 = R_2 = R_3 = \frac{2R \times 2R}{2R + 2R + 2R} = \frac{4R^2}{6R} = \frac{2}{3}R$$

After this transformation the circuit is shown in figure below.



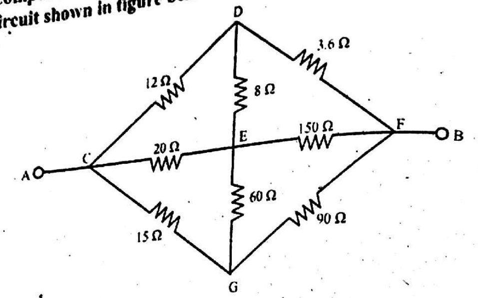
$$R_{AB} = \left[\left\{\left(R + \frac{2}{3}R\right) \| \left(R + \frac{2}{3}R\right)\right\} + \frac{2}{3}R\right] \| [2\,R]$$

$$= \left[\left(\frac{5}{3}R \| \frac{5}{3}R\right) + \frac{2}{3}R\right] \| 2\,R$$

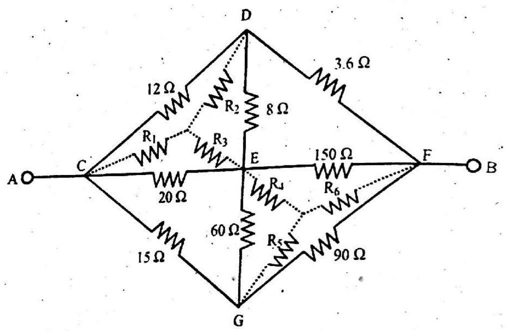
$$= \left(\frac{5}{6}R + \frac{2}{3}R\right) \| 2\,R$$

$$= \frac{3}{2}R \| 2\,R = \frac{\frac{3}{2}R \times 2\,R}{\frac{3}{2}R + 2\,R} = \frac{3R}{\frac{7}{2}} = \frac{6}{7}R$$

4. **Compute the resistance measured between terminals A and B of the circuit shown in figure below.**



Solution:



Considering delta to star transformation between nodes C, D, E and E, F, G.

$$R_1 = \frac{12 \times 20}{12 + 20 + 8} = 6\ \Omega$$

$$R_2 = \frac{12 \times 8}{12 + 20 + 8} = 2.4\ \Omega$$

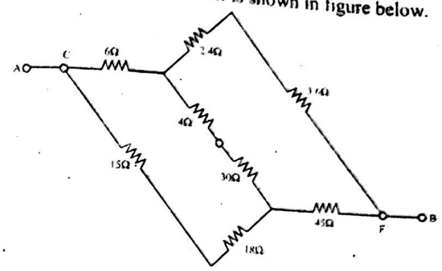
$$R_3 = \frac{20 \times 8}{12 + 20 + 8} = 4\ \Omega$$

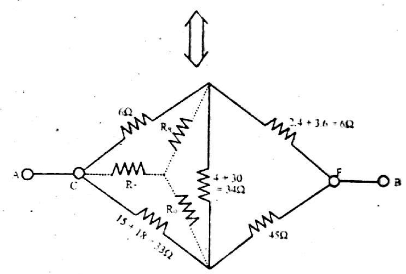
$$R_4 = \frac{60 \times 150}{60 + 150 + 90} = 30\ \Omega$$

$$R_5 = \frac{60 \times 90}{60 + 150 + 90} = 18\ \Omega$$

$$R_6 = \frac{150 \times 90}{60 + 150 + 90} = 45\ \Omega$$

After transformation, the circuit is shown in figure below.





Considering delta to star transformation;

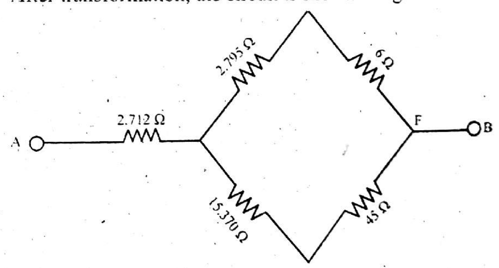
We get

$$R_8 = \frac{6 \times 34}{6 + 34 + 33} = 2.795\ \Omega$$

$$R_7 = \frac{6 \times 33}{6 + 33 + 34} = 2.712\ \Omega$$

$$R_9 = \frac{33 \times 34}{6 + 33 + 34} = 15.370\ \Omega$$

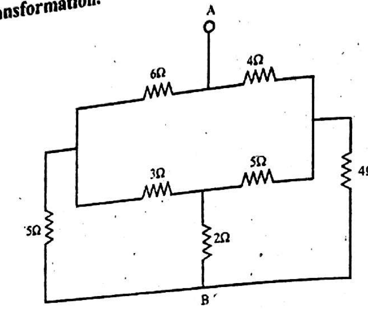
After transformation, the circuit is shown in figure below.



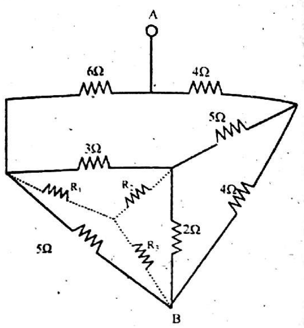
Hence, Resistance between terminals A and B,

$$R_{AB} = 2.712 + [(6 + 2.795) \,||\, (15.370 + 45)]$$
$$= 2.712 + [8.795 \,||\, 60.370]$$
$$= 2.712 + 7.677 = 10.389\ \Omega$$

5. **Find the equivalent resistance between terminal A and B using delta to star transformation.**



Solution:



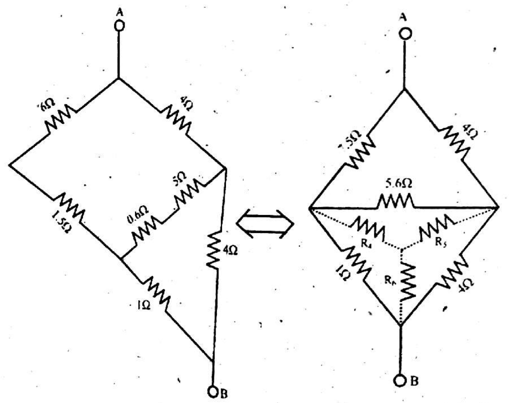
Considering delta to star transformation,

$$R_1 = \frac{5 \times 3}{5+3+2} = 1.5\ \Omega$$

$$R_2 = \frac{3 \times 2}{5+3+2} = 0.6\ \Omega$$

$$R_3 = \frac{5 \times 2}{5+3+2} = 1\ \Omega$$

After transformation;



Again,

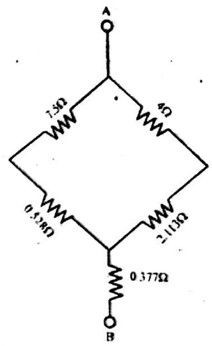
Considering delta to star transformation,

$$R_4 = \frac{1 \times 5.6}{1+5.6+4} = 0.528\ \Omega$$

$$R_5 = \frac{5.6 \times 4}{1+5.6+4} = 2.113\ \Omega$$

$$R_6 = \frac{1 \times 4}{1+5.6+4} = 0.377\ \Omega$$

After transformation,



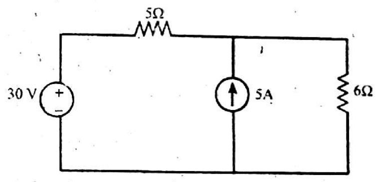
$$R_{AB} = [(7.5 + 0.528) \parallel (4 + 2.113)] + 0.377$$

$$= (8.028 \parallel 6.113) + 0.377 = 3.847\ \Omega$$

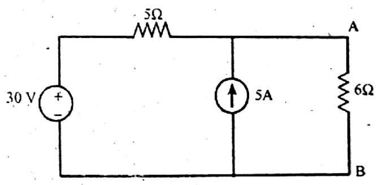
Superposition theorem

Exam solutions:

1. **Use Superposition theorem to calculate current through the 6 Ω resistor in the following network.** **[2063 Kartik]**

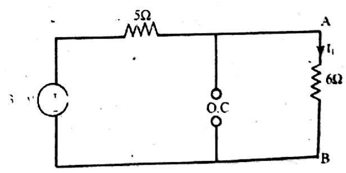


Solution:



Here are two sources in the given circuit. We shall determine the current through the 6 Ω resistor due to each source acting alone.

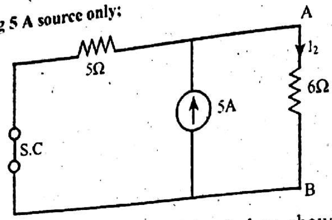
Considering 30 V source only;



Here, The 5 A current source is open - circuited then by Ohm's law

$$I_1 = \frac{30}{(5+6)} = 2.727 \text{ A (A to B)}$$

Considering 5 A source only:



The 30V voltage source is short circuited as shown in figure. The 5A current source is supplying current to 5Ω and 6Ω resistors in parallel.

By current division rule,

$$I_2 = \frac{5}{5+6} \times 5 = 2.273 \text{ A (A to B)}$$

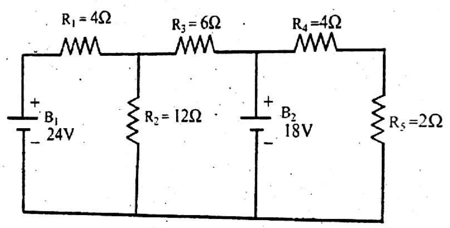
Now,

From the principle of Superposition,

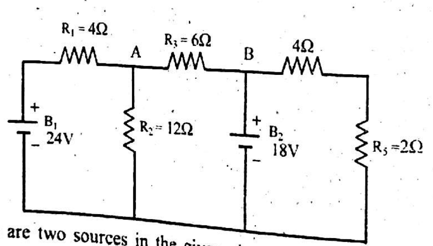
Current through 6 Ω resistor is,

$$I = I_1 + I_2$$
$$= 2.727 + 2.273 = 5 \text{ A (A to B)}$$

2. **Using Superposition theorem, find the current in resistor R_3 in the circuit shown in figure below.** [2065 Kartik]

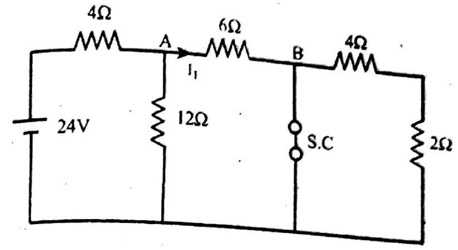


Solution:



Here are two sources in the given circuit. We shall determine the current through the R_3 resistor due to each source acting alone.

Considering B_1 = 24V source only;



Total resistance = $(6 \parallel 12) + 4$

$$= \frac{6 \times 12}{6+12} + 4 = 8\ \Omega$$

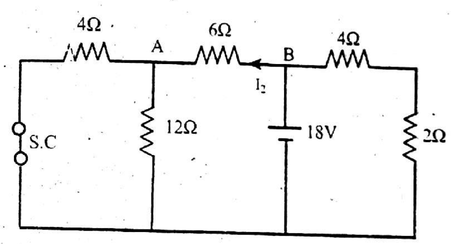
Current supplied by battery $= \frac{24}{8} = 3$ A

Using current division rule,

Current through 6 Ω resistor, $I_1 = \frac{3}{6+12} \times 12$

$$= 2 \text{ A (A to B)}$$

Considering B_2 = 18V source only:



Total resistance $= [(4 \parallel 12) + 6] \parallel [(4+2)]$

$$= \left[\frac{4 \times 12}{4+12} + 6\right] \parallel [6]$$

$$= 9 \parallel 6$$

$$= \frac{9 \times 6}{9+6} = 3.6\ \Omega$$

Current supplied by the battery $= \frac{18}{3.6} = 5$ A

Using current division rule,

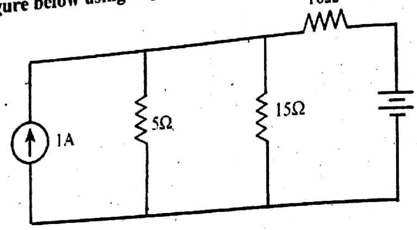
Current through 6 Ω resistor,

$$I_2 = \frac{5}{[(4 \parallel 12) + 6] + (4+2)} \times (4+2)$$

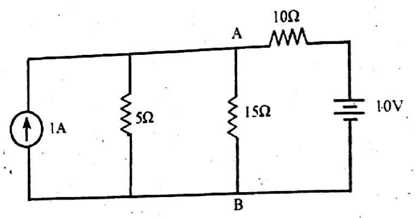
$$= \frac{5}{3+6+6} \times 6 = 2 \text{ A (B to A)}$$

Now,
From principle of Superposition,
Current through 6 Ω resistor $= I_1 - I_2$
$= 0$ A

3. **Calculate the current in the 15 Ω resistor in the network shown in figure below using Superposition theorem.** **[2067 Ashadh]**

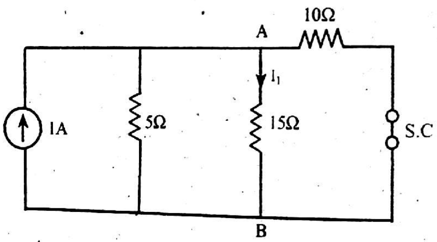


Solution:



Here are two sources in the given circuit. We shall determine the current through the 15 Ω resistor due to each source acting alone.

Considering 1 A current source only;

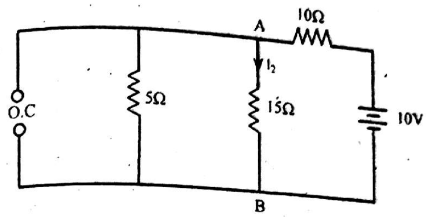


Here, 5 Ω, 15 Ω and 10 Ω resistors are connected in parallel.
Using current division rule,

Current in the 15 Ω resistor, $I_1 = \dfrac{\frac{1}{15}}{\frac{1}{5}+\frac{1}{15}+\frac{1}{10}} \times 1$

$= \dfrac{2}{11}$ A (A to B)

Considering 10V battery source only;



Total resistance $= (5 \parallel 15) + 10 = \dfrac{5 \times 15}{5+15} + 10$

$= \dfrac{55}{4}\ \Omega = 13.75\ \Omega$

Current supplied by the battery

$= \dfrac{10}{13.75}$

$= \dfrac{8}{11}$ A

Current through 15 Ω resistor,

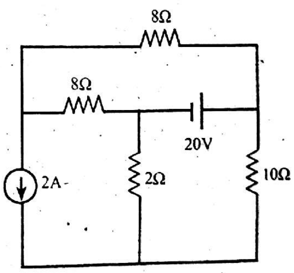
$= \dfrac{\frac{8}{11}}{5+15} \times 5$

$= \dfrac{2}{11}$ A (A to B)

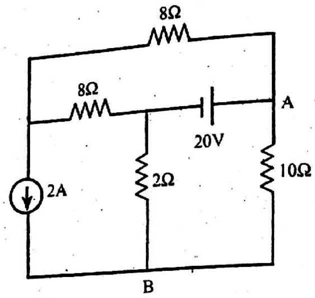
Now,
From the principle of Superposition,
Current through 15 Ω resistor $= I_1 + I_2$

$= \dfrac{2}{11} + \dfrac{2}{11} = \dfrac{4}{11}$ A (A to B).

4. **Use Superposition theorem to find the current flowing through the 10 Ω resistor shown in the figure.** **[2068 Baishakh]**

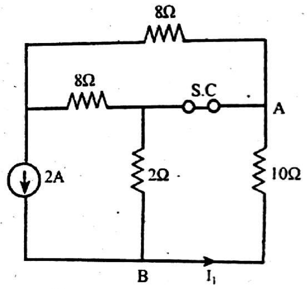


Solution:



Here are two sources in the given circuit. We shall determine the current through the 10 Ω resistor due to each source acting alone.

Considering 2A current source only;

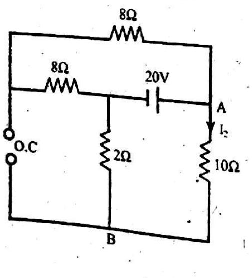


Using current division rule,

Current flowing through 10 Ω resistor, $I_1 = \frac{2}{2+10} \times 2$

$= \frac{4}{12} = \frac{1}{3}$ A (B to A)

Considering 20V battery source only;



Current flowing through 10 Ω resistors,

$I_2 = \frac{20}{2+10}$

$= \frac{5}{3}$ A (A to B)

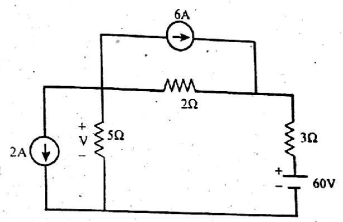
Now, from the principle of Superposition,

Current through the 10 Ω resistor $= I_2 - I_1$

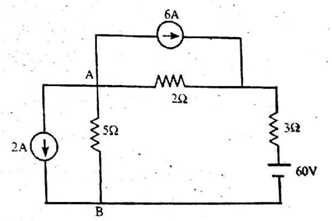
$= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

$= 1.333$ A (A to B)

5. **Apply Superposition theorem to the circuit shown below to find the voltage drop V across the 5 Ω resistor.** **[2068 Bhadra]**

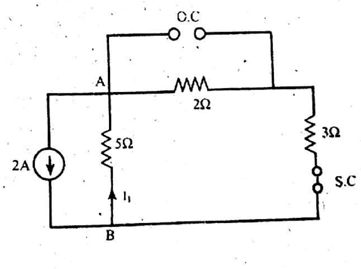


Solution:



Here are three sources in the given circuit. We shall determine the current through the 5 Ω resistor due to each source acting alone.

Considering 2 A current source only;

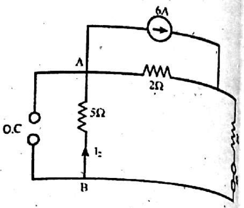


Using current division rules,

Current through 5 Ω resistor,

$$I_1 = \frac{2}{5+(3+2)} \times (3+2) = 1\text{ A (B to A)}$$

Considering 6 A current source only:



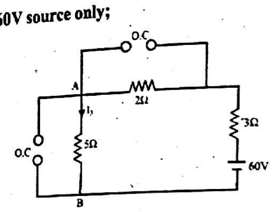
Using current division rule,

Current through 5 Ω resistor,

$$I_2 = \frac{6}{2+(5+3)} \times 2$$

$$= \frac{6}{5}\text{A (B to A)}$$

Considering 60V source only;



Current through 5 Ω resistor, by Ohm's law

$$I_3 = \frac{60}{5+2+3} = 6\text{ A (A to B)}$$

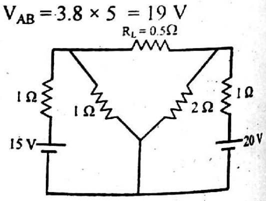
Now, from the principle of Superposition, we get

Current through 5 Ω resistor $= I_3 - I_1 - I_2 = 6 - 1 - \frac{6}{5}$

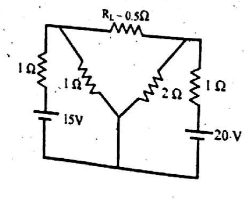
$= 3.8$ A (A to B)

∴ Voltage across 5 Ω resistors, $V = V_{AB} = 3.8 \times 5 = 19$ V

6. **Find the current in 0.5 Ω resistor in the following network shown, by using Superposition theorem.[2069 Ashad]**

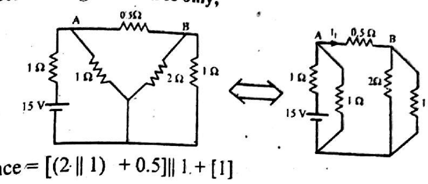


Solution:



Here are two sources in the given circuit. We shall determine the current through $R_L = 0.5\ \Omega$ resistor due to each source acting alone.

Considering 15V source only;



Total resistance $= [(2 \parallel 1) + 0.5] \parallel 1 + [1]$

$$= \left[\frac{2}{3} + 0.5\right] \parallel 1 + [1] = \frac{7}{6} \parallel 1 + [1]$$

$$= \frac{7}{13} + 1$$

$$= \frac{20}{13}\ \Omega$$

Current supplied by the battery $= \dfrac{15}{\left(\frac{20}{13}\right)}$

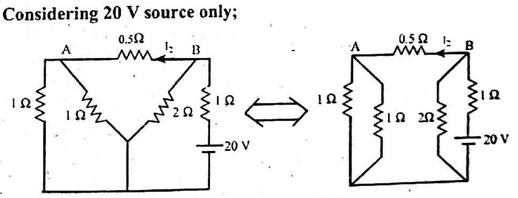
$= 9.75$ A.

Current flowing through 0.5 Ω resistor,

$$I_1 = \frac{9.75}{1 + \left(0.5 + \frac{2}{3}\right)} \times 1$$

$= 4.5$ A (A to B)

Considering 20 V source only;



Total resistance $= [(1 \parallel 1) + 0.5] \parallel 2 + [1]$

$$= (0.5 + 0.5) \parallel 2 + [1] = 1 \parallel 2 + [1] = \frac{2}{3} + 1 = \frac{5}{3}\ \Omega$$

Current supplied by the battery $= \dfrac{20}{\left(\frac{5}{3}\right)} = 12$ A

Current flowing through 0.5 Ω resistor

$$I_2 = \frac{12}{[(1 \parallel 1) + 0.5] + 2} \times 2$$

∴ $I_2 = 8$A (B to A)

Now, from the principle of Superposition,

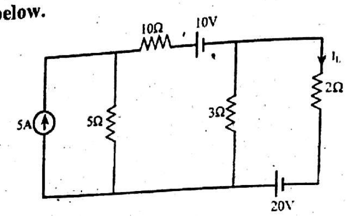
We get,

Current through 0.5 Ω resistor

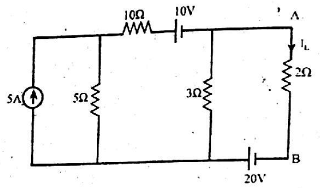
$= I_2 - I_1 = 8 - 4.5$

$= 3.5$ (B to A)

7. **Use Superposition theorem to find the current I_L through 2 Ω resistor in figure below. [2070 Asha]**

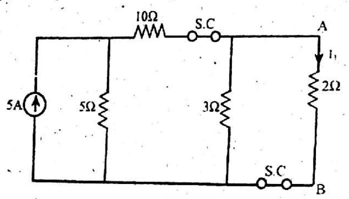


Solution:



Here are three sources in the given circuit. We shall determine the current through the 2 Ω resistor due to each source acting alone.

Considering 5A current source only;



Current flowing through 10 Ω resistor

$$= \frac{5}{5 + \left(10 + \frac{3 \times 2}{3+2}\right)} \times 5$$

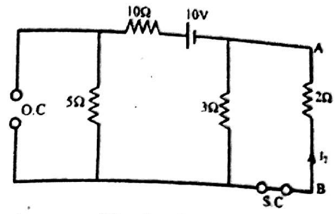
$= 1.543$ A

Current flowing through 2 Ω resistor,

$$I_1 = \frac{1.543}{3+2} \times 3$$

$= 0.926$ A (A to B)

Considering 10V source only;



Total resistance = $10 + 5 + (3 \parallel 2)$

$$= 10 + 5 + \frac{3 \times 2}{3+2}$$

$= 16.2$ Ω

Current supplied by the battery

$$= \frac{10}{16.2}$$

$= 0.617$ A

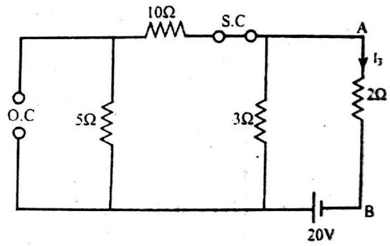
Using current division rule,

Current flowing through 2 Ω resistor,

$$= \frac{0.617}{3+2} \times 3$$

$= 0.37$ A (B to A)

Considering 20 V source only;



Total resistance $= [(5+10) \parallel 3] + 2$

$= [15 \parallel 3] + 2$

$$= \frac{15 \times 3}{15+3} + 2$$

$= 4.5$ Ω.

Current flowing through 2 Ω resistor

$$I_3 = \frac{20}{4.5}$$

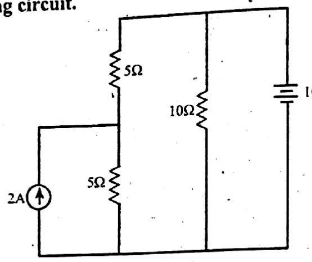
$= 4.444$ A (A to B)

Now,

From the principle of Superposition, we get,
Current flowing through 2 Ω resistor,

$$I_L = I_1 + I_3 - I_2$$
$$= 0.926 + 4.444 - 0.37$$
$$= 5\text{ A} \quad (\text{A to B})$$

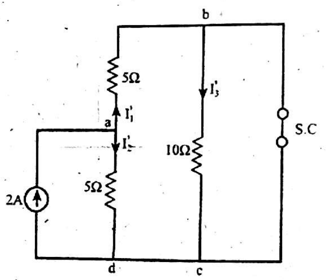
8. **Using Superposition theorem, determine currents in all the resistors of the following circuit.** **[2070 Bhadra]**



Solution:

We have two sources in the given circuit. We shall determine the current through the resistors due to each source acting alone.

Considering 2A source only;



Here,

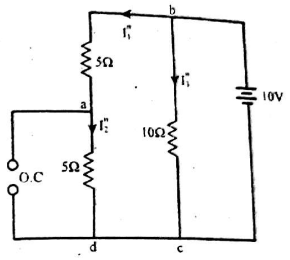
$I_3' = 0$ (current flows through short circuit path)

Using current division rule,

$I_1' = \frac{2}{5+5} \times 5 = 1\text{ A}$ (a to b)

$I_2' = \frac{2}{5+5} \times 5 = 1\text{ A}$ (a to d)

Considering 10V source only;



Here, $I_3'' = \frac{V_{bc}}{10} = \frac{10}{10} = 1\text{ A}$ (b to c)

$I_1'' = \frac{V_{bd}}{5+5} = \frac{10}{10} = 1\text{ A}$ (b to a)

Also, $I_2'' = 1\text{ A}$ (a to d)

Now,

From principle of Superposition,

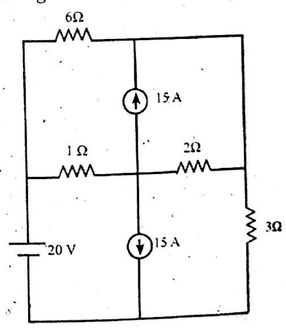
$I_1 = I_1' - I_1'' = 1 - 1 = 0$

$I_2 = I_2' + I_2'' = 1 + 1 = 2\text{ A}$ (a to d)

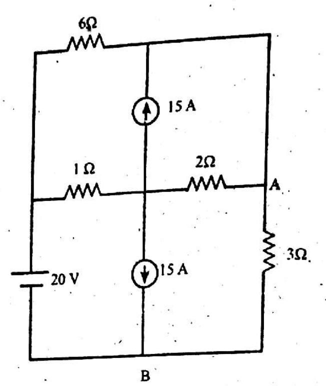
$I_3 = I_3'' + I_3' = 1 - 0 = 1\text{ A}$ (b to c)

Hence, current through 5 Ω resistor (at top) is 0A, current through 5 Ω resistor (at bottom) is 2A and current through 10 Ω resistor is 1 A.

9. **Calculate the voltage drop across 3 Ω resistor using Superposition theorem in the circuit given below.** **[2071 Bhadra]**

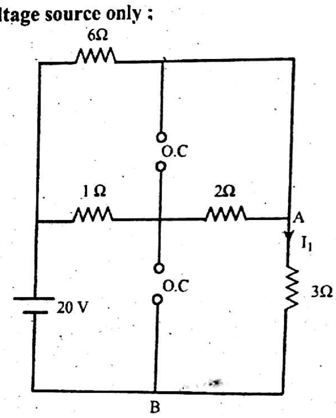


Solution:



Here are three sources in the given circuit. We shall determine the voltage drop across 3 Ω resistor due to each source acting alone.

Considering 20V voltage source only ;



Total resistance = [6 // (1 + 2)] + 3
= [6 // 3] + 3
$= \frac{6 \times 3}{6+3} + 3$
= 5 Ω

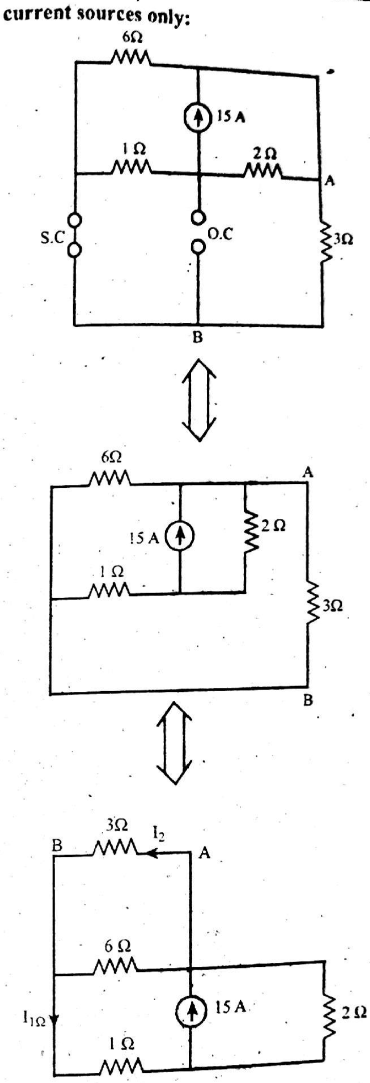
Current flowing through 3 Ω resistor,

$I_1 = \frac{20}{5}$
= 4 A (A to B)

Voltage drop across 3 Ω resistor,

$V'_{AB} = 4 \times 3$
= 12 V

Considering 15A current sources only:



Using current division rule,

$I_{1\Omega} = \frac{15}{[(3 // 6) + 1] + 2} \times 2$
$= \frac{15}{5} \times 2$
= 6 A

Again, Using current division rule,

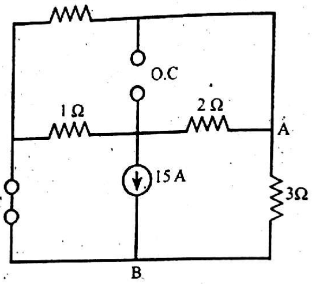
$I_2 = \frac{6}{3+6} \times 6$

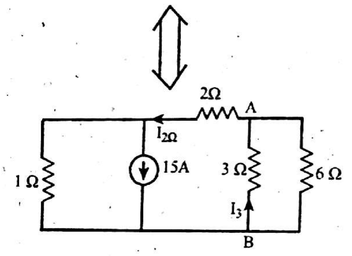
$= 4$ A (A to B)

Voltage drop across 3 Ω resistor,

$V''_{AB} = 4 \times 3 = 12$ V

Considering 15A current source only :





Using current division rule,

$I_{2\Omega} = \frac{15}{1 + [(3 // 6) + 2]} \times 1$

$= \frac{15}{5} \times 1 = 3$ A

Again, using current division rule,

$I_3 = \frac{3}{3+6} \times 6$

$= 2$ A (B to A)

Voltage drop across 3 Ω resistor,

$V'''_{BA} = 2 \times 3 = 6$V

Now,

From principle of Superposition,

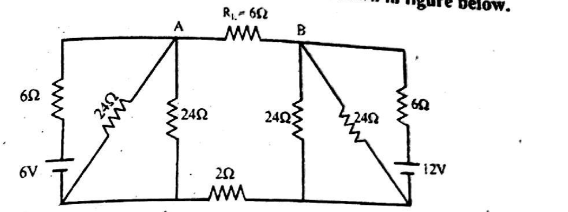
Voltage drop across 3 Ω resistor,

$V_{3\Omega} = V_{AB} = V'_{AB} + V''_{AB} - V'''_{BA}$

$= 12 + 12 - 6 = 18$ V

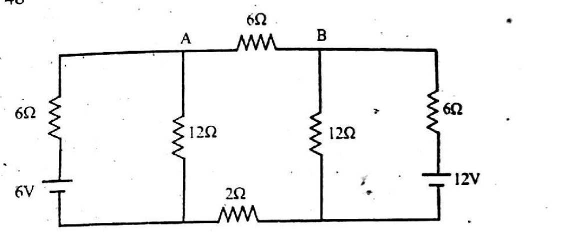
Additional questions

1. Use the Superposition theorem to determine the current in the current in the branch AB of the network shown in figure below.



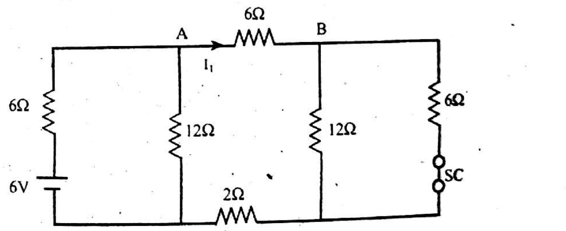
Solution:

Converting both 24 Ω || 24 Ω parallel combinations by equivalent $\frac{24 \times 24}{48} = 12$ Ω resistor, the circuit becomes as shown below.



Here are two sources in the given circuit. We shall determine the current through the 6Ω resistor due to each source acting alone.

Considering 6V source only;



Total resistance = [(12 || 6) + 2 + 6] || 12 + [6]

= [(4 + 2 + 6) || 12] + [6]

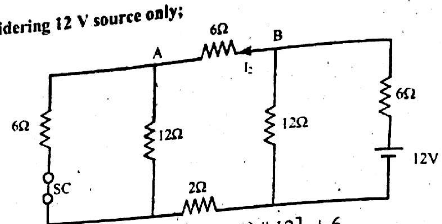
= 6 + 6 = 12Ω

Current supplied by the battery $= \frac{6}{12} = 0.5$ A

Current through 6 Ω resistor, $I_1 = \frac{0.5}{12 + (6 + 4 + 2)} \times 12$

$= 0.25$ A (A to B)

Considering 12 V source only;



Total resistance $= [((6 \parallel 12) + 6 + 2) \parallel 12] + 6$

$= [(4 + 6 + 2) \parallel 12] + 6$

$= 6 + 6 = 12\Omega$

Current supplied by the battery $= \dfrac{12}{12} = 1$ A

Current through 6 Ω resistor, $I_2 = \dfrac{1}{12 + [6 + 2 + (6 \parallel 12)]} \times 12$

$= 0.5$ A (B to A)

Now, From the principle of Superposition,

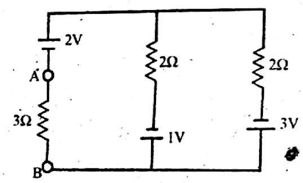
Current through the 6Ω resistor is,

$I = I_2 - I_1$

$= 0.5 - 0.25$

$= 0.25$ A (B to A)

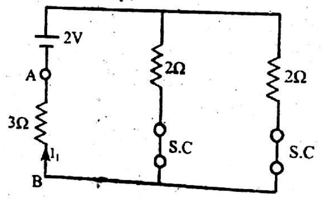
2. **Find the current through 3 ohms resistor of the circuit shown below by Superposition theorem.**



Solution:

There are three sources in the given circuit. We shall determine the current through the 3 Ω resistor due to each source acting alone.

Considering 2V source only;



Total resistance $= (2 \parallel 2) + 3$

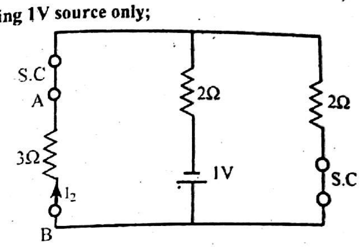
$= 1 + 3$

$= 4\,\Omega$

Current supplied by the battery $= \dfrac{2}{4} =$ 0.5 A

Current through 3 Ω resistor, $I_1 = 0.5$ A (B to A)

Considering 1V source only;



Total resistance $= (3 \parallel 2) + 2$

$= \dfrac{6}{5} + 2$

$= 3.2\,\Omega$

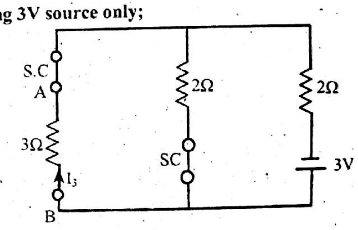
Current supplied by the battery $= \dfrac{1}{3.2}$

$= 0.313$ A

Current through 3 Ω resistor,

$I_2 = \dfrac{0.313}{3 + 2} \times 2 = 0.125$ A (B to A)

Considering 3V source only;



Total resistance $= (3 \parallel 2) + 2$

$= \dfrac{3 \times 2}{3 + 2} + 2$

$= 3.2\,\Omega$

Current supplied by the battery $= \dfrac{3}{3.2} = 0.938$ A

Current through 3 Ω resistor $= \dfrac{0.938}{3 + 2} \times 2$

$= 0.375$ A (B to A)

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Now,

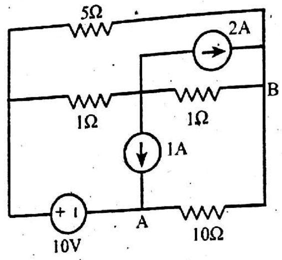
From the principle of Superposition,

Current through the 3 Ω resistor

$= I_1 + I_2 + I_3$

$= 0.5 + 0.125 + 0.375 = 1$ A (B to A)

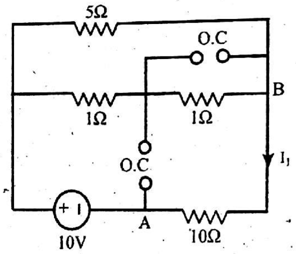
3. **Find the current in the 10Ω resistor in the circuit below using Superposition theorem.**



Solution:

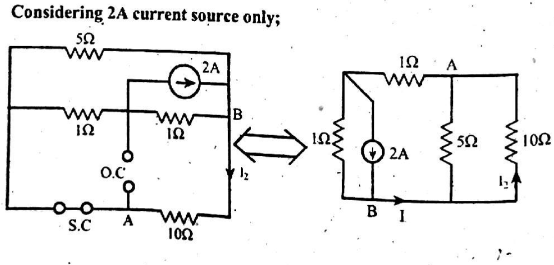
We have three sources in the given circuit. We shall determine the current through the 10 Ω resistor due to each source acting alone.

Considering 10V source only;



$$I_1 = \frac{10}{\frac{5\times(1+1)}{5+(1+1)}+10} = \frac{10}{\frac{10}{7}+10} = \frac{7}{8} = 0.875 \text{ A} \quad \text{(B to A)}$$

Considering 2A current source only;



Using current division rule,

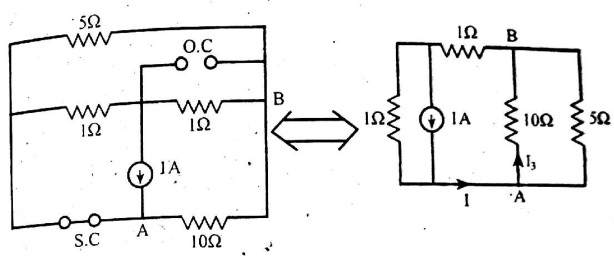
$$I = \frac{2}{1+[1+(5\,\|\,10)]} \times 1$$

$$= \frac{2}{1+\left[1+\frac{10}{3}\right]}$$

$$= 0.375 \text{ A}$$

$$\therefore \quad I_2 = \frac{I}{5+10} \times 5 = \frac{0.375}{15} \times 5 = 0.125 \text{ A} \quad \textbf{(B to A)}$$

Considering 1A current source only;



Using current division rule,

$$I = \frac{1}{1+[1+(10\,\|\,5)]} \times 1$$

$$= \frac{1}{1+\left[1+\frac{50}{15}\right]}$$

$$= 0.1875 \text{ A}$$

$$I_3 = \frac{I}{10+5} \times 5 = \frac{0.1875}{15} \times 5 = 0.0625 \text{ A} \quad \text{(B to A)}$$

Now,

From principle of Superposition,

Current flowing through 10 Ω resistor,

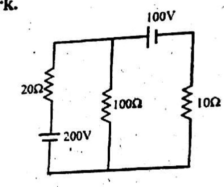
$= I_1 + I_2 - I_3$

$= 0.875 + 0.125 - 0.0625$

$= 0.9375$ A (B to A)

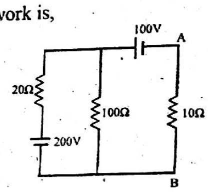
Thevenin's theorem
Exam solutions

1. **Use Thevenin's theorem to calculate current through 10Ω resistor in the following network.** **[2064 Shrawan]**

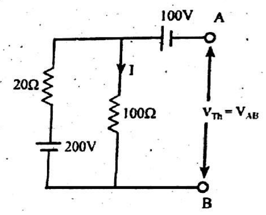


Solution:

Here the given network is,



To find V_{Th} ;



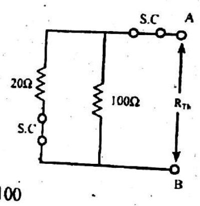
$$V_{Th} = V_{AB} = V_A - V_B$$

$$= 100 \times \frac{200}{20 + 100} - 100 \text{ [write KVL equation; move from B to A]}$$

$$= 66.67 \text{ V}$$

This indicates that A is at a higher potential with respect to B.

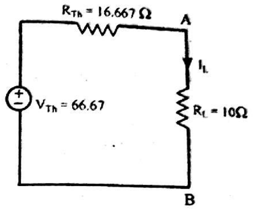
To find R_{Th} ;



$$R_{Th} = R_{AB} = 20 \parallel 100$$

$$= \frac{20 \times 100}{20 + 100} = 16.667 \,\Omega$$

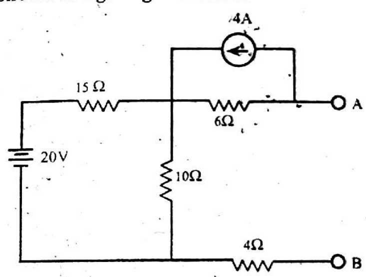
To find current I_L in $R_L = 10\,\Omega$



By Ohm's law,

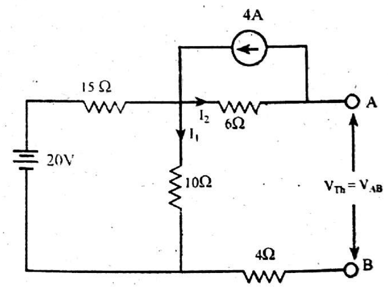
$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{66.67}{16.667 + 10} = 2.5 \text{ A} \quad \text{(A to B)}$$

2. **Find the Thevenin's equivalent circuit for terminal pair AB of the network shown in figure given below.** **[2067 Mangsir]**



Solution:

To find V_{Th} ;



$$V_{Th} = V_{AB}$$

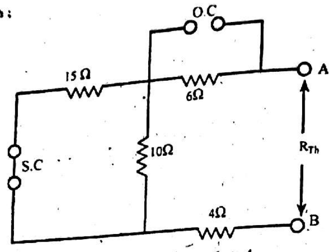
$$= V_A - V_B$$

$$= 10 \times I_1 - 6 \times I_2 \quad \text{[Write KVL equation; move from B to A]}$$

$$= 10 \times \frac{20}{15 + 10} - 6 \times 4 = -16 \text{ V}$$

Which indicates that B is at higher potential with respect to A.

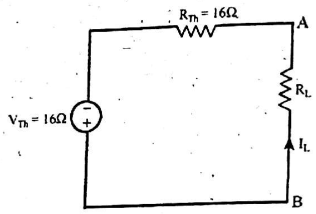
To find R_{Th}:



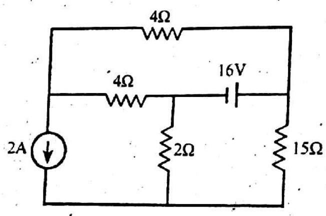
$$R_{Th} = R_{AB} = (15 \| 10) + 6 + 4$$

$$= \frac{15 \times 10}{15 + 10} + 6 + 4 = 16\ \Omega$$

Thevenin's equivalent circuit

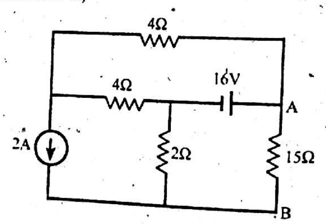


3. **Use Thevenin's theorem to find the current flowing through 15Ω resistor of the network of figure below. [2069 Bhadra]**

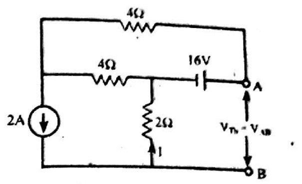


Solution:

Here, the given network is,



To find V_{Th};



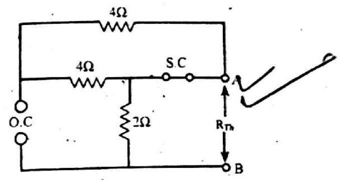
$V_{Th} = V_{AB} = V_A - V_B$

$= -2 \times I + 16$ [Write KVL equation; move from B to A]

$= -2 \times 2 + 16 = 12\ V$

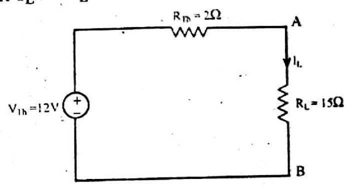
Which indicates A is at higher potential with respect to B.

To find R_{Th};



$R_{Th} = 2\ \Omega$

To find current I_L in $R_L = 15\ \Omega$

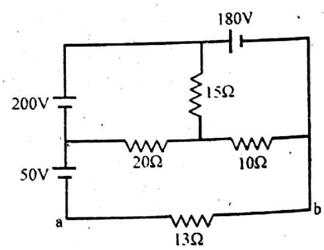


By Ohm's law,

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{12}{2 + 15}$$

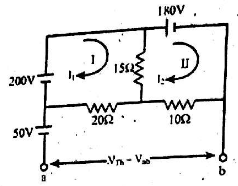
$= 0.706\ A$ (A to B)

4. **Applying Thevenin's theorem, calculate the magnitude and direction of current in the 13Ω resistor in the circuit shown in the following figure. [2069 Ashad]**



Solution:

To find V_{Th} ;



Applying KVL on mesh I and mesh II, we get

Mesh I:

$$200 - 15(I_1 - I_2) - 20\, I_1 = 0$$

Or, $\quad -35\, I_1 + 15\, I_2 = -200$ ....(i)

Mesh II :

$$-180 - 10\, I_2 - 15\,(I_2 - I_1) = 0$$

Or, $\quad 15\, I_1 - 25\, I_2 = 180$ .... (ii)

Solving equations (i) and (ii), we get

$$I_1 = 3.538 \text{ A}$$

$$I_2 = -5.077 \text{ A}$$

Negative current indicates that our direction of assumption of current flow is opposite to the actual flow.

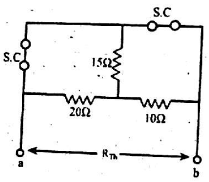
$\therefore \quad V_{Th} = V_{ab} = V_a - V_b$

$= 10 \times I_2 - 20 \times I_1 - 50$[Write KVL equation; move from b to a]

$= 10 \times 5.077 - 20 \times 3.538 - 50 = -70\text{V}$

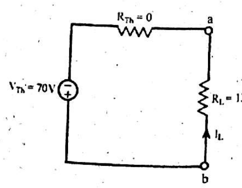
Which indicates b is at higher potential with respect to a.

To find R_{Th} ;



$R_{Th} = 0$

To find current I_L in $R_L = 13\Omega$

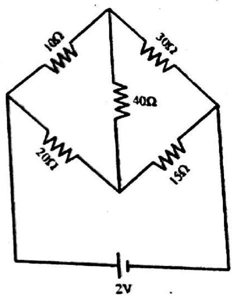


By Ohm's law,

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{70}{0 + 13} = 5.385 \text{ A (b to a)}$$

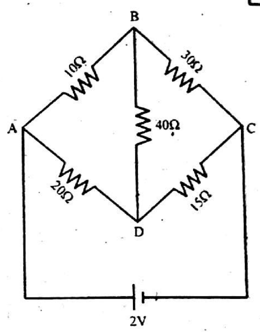
6. **The resistance of the various arms of a Wheatstone bridge are shown in figure below. The battery has an emf of 2V. Using Thevenis's theorem, determine the value and direction of current in the 40Ω resistor.**

[2070 Bhadra]



Solution:

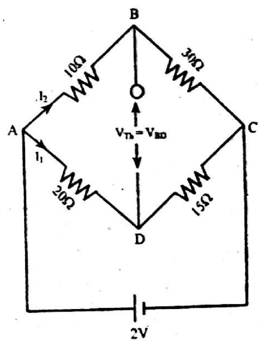
Here, the given circuit is,



To find V_{Th} ;

Here,

$$V_{AB} = V_A - V_B$$
$$= I_2 \times 10$$
$$= \frac{2}{10 + 30} \times 10$$
$$= 0.5 \text{ V}$$

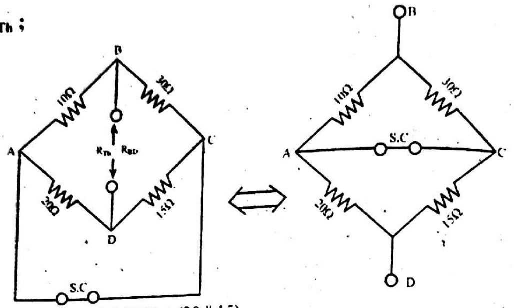


$$V_{AD} = V_A - V_D$$
$$= I_1 \times 20$$
$$= \frac{2}{20 + 15} \times 20$$
$$= 1.143\text{V}$$

Now,

$$V_{Th} = V_{BD} = V_{AD} - V_{AB}$$
$$= 1.143 - 0.5$$
$$= 0.643\text{V}$$

To find R_{Th} ;



$\therefore \quad R_{Th} = R_{BD} = (10 \,||\, 30) + (20 \,||\, 15)$

$= \dfrac{10 \times 30}{10 + 30} + \dfrac{20 \times 15}{20 + 15} = 7.5 + 8.571 = 16.071\ \Omega$

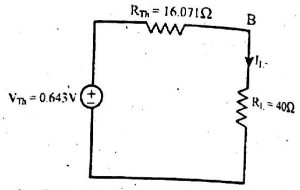
To find current I_L in $R_L = 40\ \Omega$;

By Ohm's law,

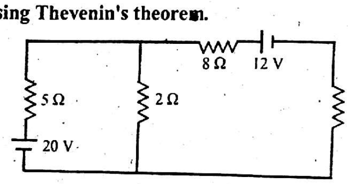
$I_L = \dfrac{V_{Th}}{R_{Th} + R_L}$

$= \dfrac{0.643}{16.071 + 40}$

$= 0.0115$ A (B to D)

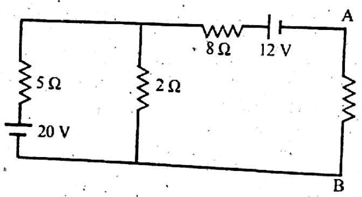


6. For the circuit shown in figure below, calculate the current in the 10 ohm resistance using Thevenin's theorem. **[2071 Shrawan]**

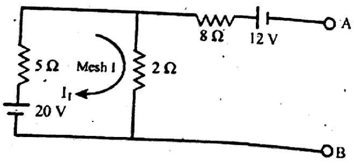


Solution :-

Here, the given circuit is,



To find V_{Th} ;



Applying KVL on mesh I, we get

$20 - 5 I_1 - 2 I_1 = 0$

or, $7I_1 = 20$

$\therefore I_1 = \dfrac{20}{7}$ A

$\therefore V_{Th} = V_{AB} = V_A - V_B$

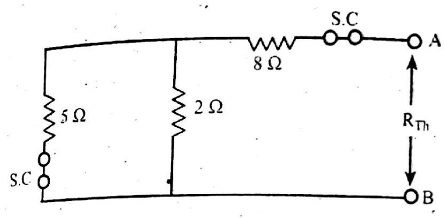
$= 2 I_1 - 8 \times 0 - 12$ [write KVL equation; move from B to A]

$= 2 \times \dfrac{20}{7} - 12$

$= -6.286$ V

Which indicates B is at higher potential with respect to A.

To find R_{Th};

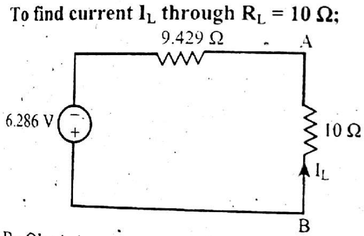


$R_{Th} = (5 \,//\, 2) + 8$

$= \dfrac{5 \times 2}{5 + 2} + 8$

$= 9.429\ \Omega$

To find current I_L through $R_L = 10\ \Omega$;



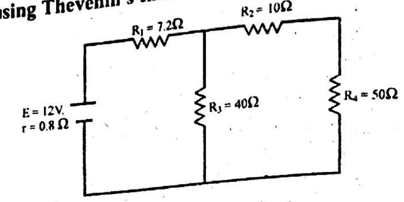
By Ohm's law,

$I_L = \dfrac{V_{Th}}{R_{Th} + R_L} = \dfrac{6.286}{9.429 + 10}$

$= 0.3235$ A (B to A)

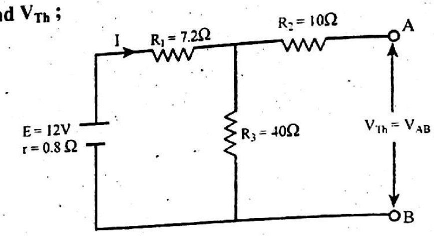
Additional questions

1. **Determine the current through 50Ω resistance in the circuit shown below using Thevenin's theorem.**



Solution:

To find V_{Th} ;



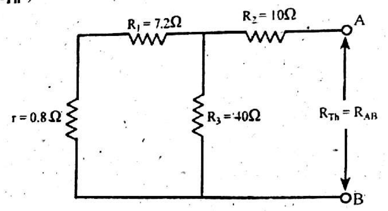
$$V_{Th} = V_{AB} = IR_3$$

$$= \left(\frac{E}{r + R_1 + R_3}\right) \times R_3$$

$$= \left(\frac{12}{0.8 + 7.2 + 40}\right) \times 40$$

$$= 10V$$

To find R_{Th} ;



$$R_{Th} = [(R_1 + r) \parallel R_3] + R_2$$

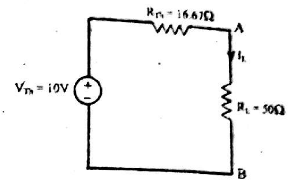
$$= [(7.2 + 0.8) \parallel 40] + 10$$

$$= (8 \parallel 40) + 10$$

$$= \frac{8 \times 40}{8 + 40} + 10$$

$$= 16.67\ \Omega$$

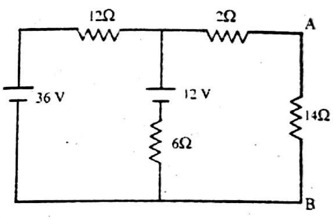
To find current I_L in $R_L = 50\ \Omega$;



By Ohm's law,

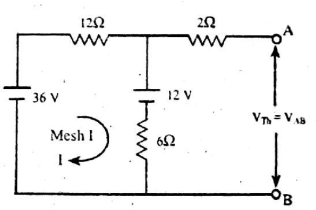
$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{10}{16.67 + 50} = 0.15\text{ A (A to B)}$$

2. **Using Thevenin's theorem, calculate the potential difference across terminals A and B of the given circuit.**



Solution:

To find V_{Th} ;



Applying KVL on Mesh I, we get

$$36 - 12\,I - 12 - 6I = 0$$

or, $-18\,I + 24 = 0$

$$\therefore \quad I = \frac{4}{3}\text{ A}$$

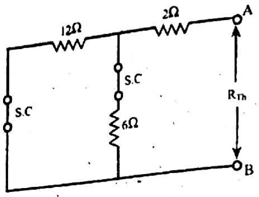
$$\therefore \quad V_{Th} = V_{AB} = V_A - V_B$$

$$= 6\,I + 12 \qquad \text{[Write KVL equation; move from B to A]}$$

$$= 6 \times \frac{4}{3} + 12 = \frac{24}{3} + 12 = 20\text{ V}$$

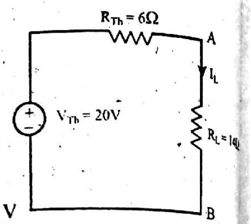
Which indicates A is at higher potential with respect to B.

To find R_{Th} :



$$R_{Th} = (12 \parallel 6) + 2 = \frac{12 \times 6}{12 + 6} + 2 = 6\ \Omega$$

To find p.d across 14 Ω resistance.

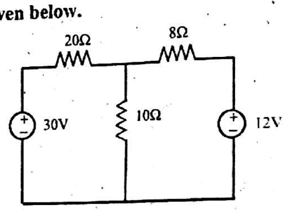


Using voltage divider rule,

$$\text{P.d across 14 }\Omega\text{ resistance} = \frac{V_{Th}}{R_{Th} + R_L} \times R_L$$

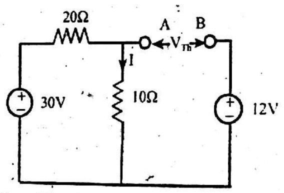
$$= \frac{20}{6 + 14} \times 14 = 14\ V$$

3. **Using Thevenin's theorem, calculate the current through the 8 Ω resistor of the circuit given below.**



Solution:

To find V_{Th};



$$V_{Th} = V_{AB} = V_A - V_B$$

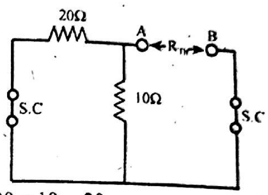
$= -12 + 10 \times I$ [Write KVL eqution; move from B to A]

$$= -12 + 10 \times \frac{30}{(20 + 10)}$$

$$= -2V$$

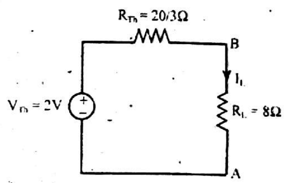
It means that point A is at lower potential with respect to B, or point B is at higher potential than point A.

To find R_{Th};



$$R_{Th} = 20 \parallel 10 = \frac{20 \times 10}{20 + 10} = \frac{20}{3}\ \Omega$$

To find current I_L in $R_L = 8\Omega$;

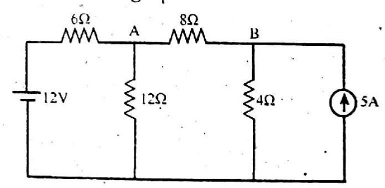


By Ohm's law,

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{2}{\frac{20}{3} + 8}$$

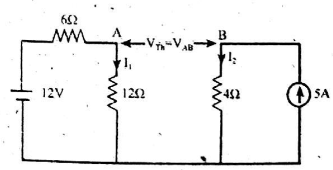
$$= \frac{3}{22} = 0.136\ A\ (B\ to\ A)$$

4. **Using Thevenin's theorem, calculate the current flowing through the 8Ω resistor in the circuit given below.**



Solution:

To find V_{Th};



$$V_{Th} = V_{AB} = V_A - V_B$$

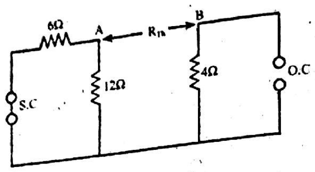
$= -4I_2 + 12\ I_1$ [Write KVL equation; move from B to A]

$$= -4 \times 5 + 12 \times \frac{12}{12 + 16}$$

$$= -12\ V$$

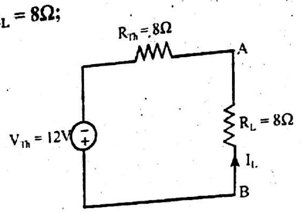
Which indicates that B is at higher potential with respect to A.

To find R_{Th};



$$R_{Th} = (6 \parallel 12) + 4 = \frac{6 \times 12}{6+12} + 4 = 8\Omega$$

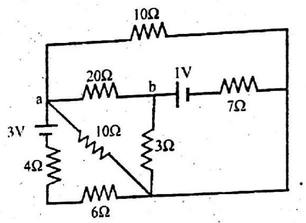
To find current I_L in $R_L = 8\Omega$;



By Ohm's law

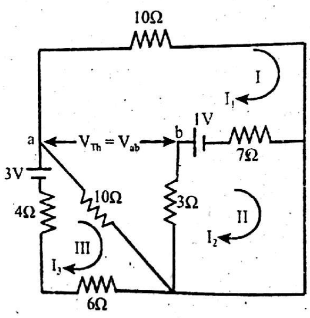
$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{12}{8+8} = 0.75\ A \quad \text{(B to A)}$$

5. **By the use of Thevenin's theorem, find the value of the current in 20 ohms resistor.**



Solution:

To find V_{Th};



Applying KVL to mesh I, we get

$$-10 I_1 - 7(I_1 - I_2) + 1 - 3(I_1 - I_2) - 10(I_1 - I_3) = 0$$

or, $-30 I_1 + 10 I_2 + 10 I_3 = -1$ .... (1)

Applying KVL to mesh II, we get

$$-3(I_2 - I_1) - 1 - 7(I_2 - I_1) = 0$$

or, $10 I_1 - 10 I_2 = 1$ .... (2)

Applying KVL to mesh III, we get

$$-(6+4) I_3 + 3 - 10(I_3 - I_1) = 0$$

or, $10 I_1 - 20 I_3 = -3$ .... (3)

Solving equations (1), (2) and (3) we get

$$I_1 = \frac{1}{10}\ A,\ I_2 = 0,\ I_3 = \frac{1}{5}\ A$$

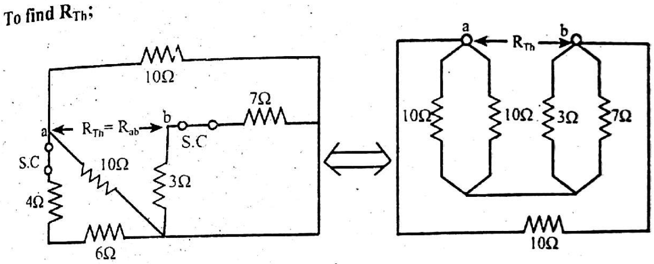
$V_{Th} = V_{ab} = V_a - V_b$

∴ $= -3(I_1 - I_2) + 10(I_3 - I_1)$ [write KVL equation; move from B to A]

$$= -3 \times \left(\frac{1}{10} - 0\right) + 10\left(\frac{1}{5} - \frac{1}{10}\right)$$

$$= -\frac{3}{10} + 10 \times \frac{1}{10} = 0.7V$$

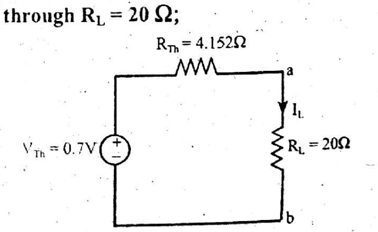
To find R_{Th};



$$R_{Th} = [(10 \parallel 10) + (3 \parallel 7)] \parallel 10 = \left[\frac{10 \times 10}{10+10} + \frac{3 \times 7}{3+7}\right] \parallel 10$$

$$= 7.1 \parallel 10 = 4.152\ \Omega$$

To find current I_L through $R_L = 20\ \Omega$;



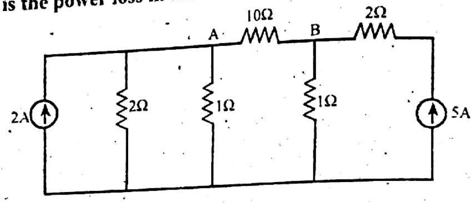
By Ohm's law,

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$= \frac{0.7}{4.152 + 20}$$

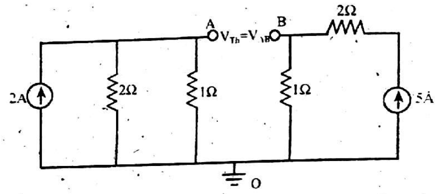
$$= 0.029 \text{ A (a to b)}$$

6. **What is the power loss in the 10 Ω resistor? Use Thevenin's theorem.**



Solution:

To find V_{Th};



Here, we use nodal analysis to find V_{Th}

Applying KCL at node A, we get

$$\frac{V_A - 0}{2} + \frac{V_A - 0}{1} = 2$$

or, $$V_A\left(\frac{1}{2} + 1\right) = 2$$

∴ $$V_A = \frac{4}{3} V$$

Also,

Applying KCL at node B, we get

$$\frac{V_B - 0}{1} = 5$$

∴ $V_B = 5V$

∴ $V_{Th} = V_{AB} = V_A - V_B$

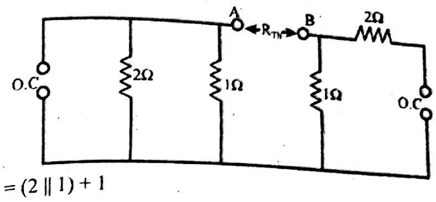
$$= \frac{4}{3} - 5$$

$$= -3.667 \text{ V}$$

Which indicates B is at higher potential with respect to A.

∴ $V_{BA} = 3.667$ V

To find R_{Th};

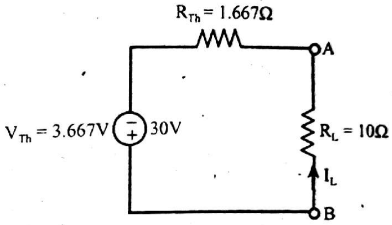


$$R_{Th} = R_{AB} = (2 \parallel 1) + 1$$

$$= \frac{2 \times 1}{2 + 1} + 1$$

$$= 1.667\ \Omega$$

To find current I_L through $R_L = 10\ \Omega$;



By Ohm's law

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

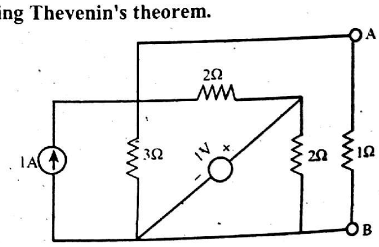
$$= \frac{3.667}{1.667 + 10}$$

$$= 0.314 \text{ A (B to A)}$$

∴ Power loss in 10Ω resistor $= (0.314)^2 \times 10$

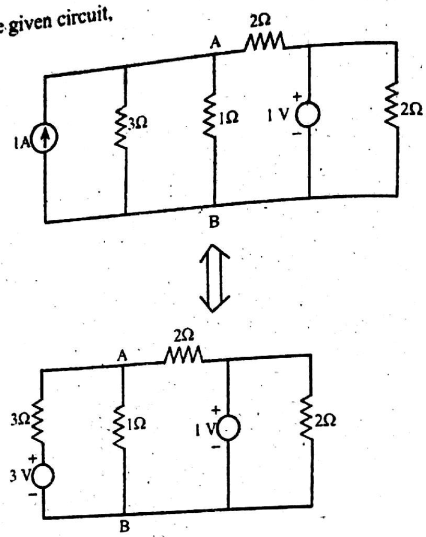
$$= 0.986 \text{ W}$$

7. **Determine the current in the 1 Ω resistor across AB of network shown below using Thevenin's theorem.**

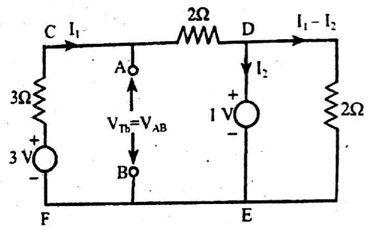


Solution:

Redrawing of the given circuit,



To find V_{Th};



Applying KVL to mesh CDEFC,we get

$3 - 3I_1 - 2I_1 - 1 = 0$

or, $2 - 5I_1 = 0$

$\therefore$ $I_1 = \frac{2}{5} A$

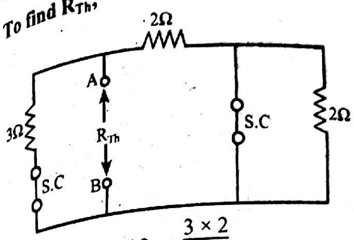
$\therefore$ $V_{Th} = V_{AB} = V_A - V_B$

$= 3 - 3I_1$ [Write KVL equation; move from B to A]

$= 3 - 3 \times \frac{2}{5} = 1.8$ V

which indicates A is at higher potenital with respect to B.

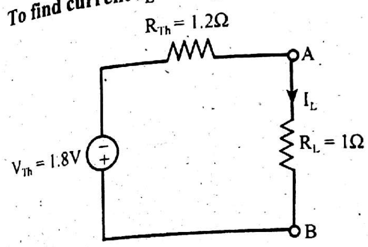
To find R_{Th};



$$R_{Th} = 3 \parallel 2 = \frac{3 \times 2}{3 + 2}$$

$$= 1.2\ \Omega$$

To find current I_L through $R_L = 1\Omega$;



By Ohm's law

$\therefore$ $$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$= \frac{1.8}{1.2 + 1}$$

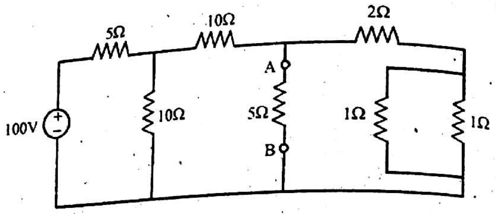
$$= 0.818 \text{ A (A to B)}$$

Norton's Theorem

Exam solutions:

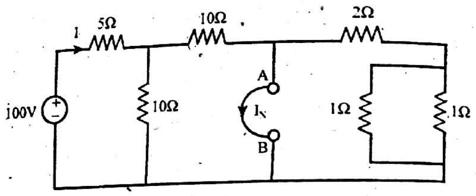
1. Determine the current flowing through the 5 Ω resistor conne... between AB in the circuit shown below using Norton's theorem.

[2066 Kar...



Solution:

To find I_N;



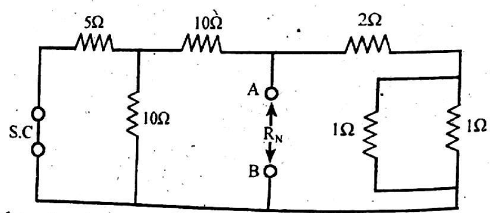
Here,

Current supplied by the battery

$$I = \frac{100}{(10 \parallel 10) + 5} = \frac{100}{\frac{10 \times 10}{10 + 10} + 5} = 10\text{ A}$$

Short circuit current, $I_N = \frac{I}{10 + 10} \times 10 = \frac{10}{10 + 10} \times 10 = 5\text{A (A to B)}$

To find R_N;



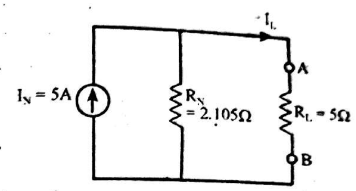
Resistance of the network seen from the terminals A and B

$R_N = R_{AB}$

$= [(5 \parallel 10) + 10] \parallel [2 + (1 \parallel 1)]$

$$= \left(\frac{50}{15} + 10\right) \parallel \left(2 + \frac{1}{2}\right) = \frac{40}{3} \parallel \frac{5}{2} = \frac{\frac{40}{3} \times \frac{5}{2}}{\frac{40}{3} + \frac{5}{2}} = \frac{40}{19} = 2.105\ \Omega$$

To find current I_L in $R_L = 5\ \Omega$;

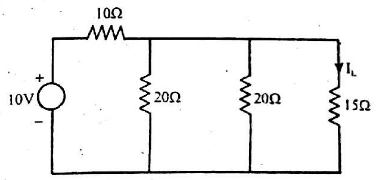


Using current division rule,

Current through 5 Ω resistor,

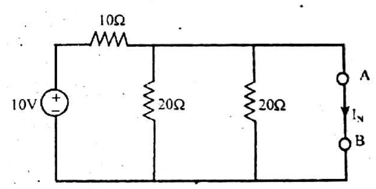
$$I_L = \frac{5}{2.105 + 5} \times 2.105 = 1.48\text{ A (A to B)}$$

2. **Determine the current I_L through 15 Ω resistor in the network by Norton's theorem.** **[2067 Ashadh]**



Solution:

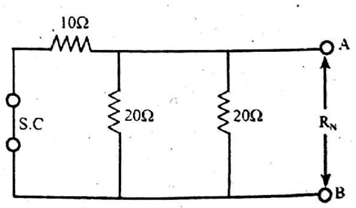
To find I_N;



With 15 Ω resistor removed and terminals AB short circuited,

Short circuit current, $I_N = \frac{10}{10} = 1\text{ A (A to B)}$

To find R_N;

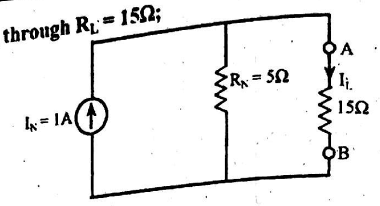


Resistance of the network seen from the terminals A and B is:

$R_N = 10 \parallel 20 \parallel 20$

$$= \frac{10 \times 20 \times 20}{10 \times 20 + 20 \times 20 + 20 \times 10} = \frac{4000}{800} = 5\ \Omega$$

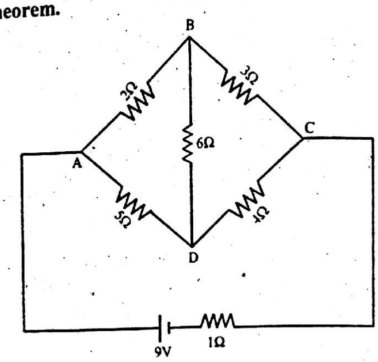
To find current I_L through $R_L = 15\Omega$;



Using current division rule,

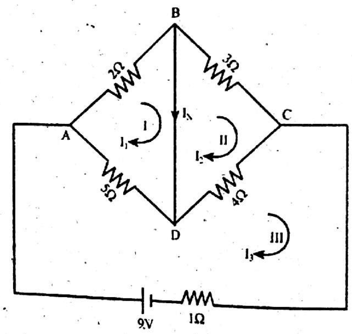
Current through 15Ω resistor $= \frac{1}{5+15} \times 5 = \frac{1}{4} = 0.25$ A (A to B or top to bottom)

3. **Calculate the current in the 6Ω resistor in the network shown below using Norton's theorem.** [2067 Magh]



Solution:

To find I_N;



Using mesh analysis, applying KVL on mesh I, mesh II and mesh III

Mesh I:

$$-2I_1 - 5(I_1 - I_3) = 0$$

or, $-2I_1 - 5I_1 + 5I_3 = 0$

or, $-7I_1 + 5I_3 = 0$ .... (i)

Mesh II:

$$-3I_2 - 4(I_2 - I_3) = 0$$

or, $-7I_2 + 4I_3 = 0$ .... (ii)

Mesh III:

$$9 - 5(I_3 - I_1) - 4(I_3 - I_2) - I_3 = 0$$

or, $9 - 5I_3 + 5I_1 - 4I_3 + 4I_2 - I_3 = 0$

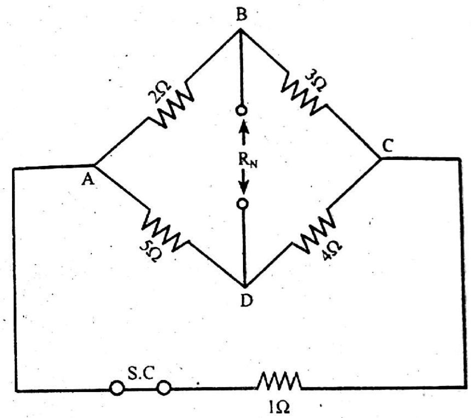
or, $5I_1 + 4I_2 - 10I_3 = -9$ .... (iii)

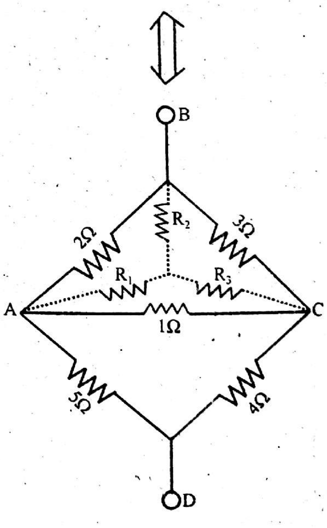
Solving equations (i), (ii) and (iii), we get

$I_1 = 1.552$ A, $I_2 = 1.241$ A, $I_3 = 2.172$ A

$\therefore$ $I_N = I_1 - I_2 = 1.552 - 1.241 = 0.311$ A (B to D)

To find R_N;





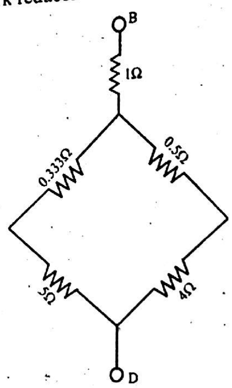
Using Delta – Star transformation;

$$R_1 = \frac{2 \times 1}{2+3+1} = 0.333\ \Omega$$

$$R_2 = \frac{2 \times 3}{2+3+1} = 1\ \Omega$$

$$R_3 = \frac{3 \times 1}{2+3+1} = 0.5\ \Omega$$

After transformation, network reduces into the form,

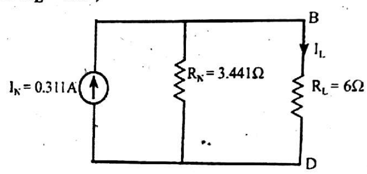


Resistance of the network seen from the terminals B and D is;

$$R_N = 1 + [(5 + 0.333) \parallel (0.5 + 4)]$$

$$= 1 + (5.333 \parallel 4.5) = 1 + \frac{5.333 \times 4.5}{5.333 + 4.5} = 3.441\ \Omega$$

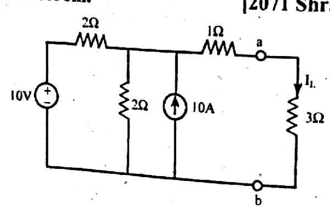
To find current I_L in $R_L = 6\Omega$;



Using current division rule,

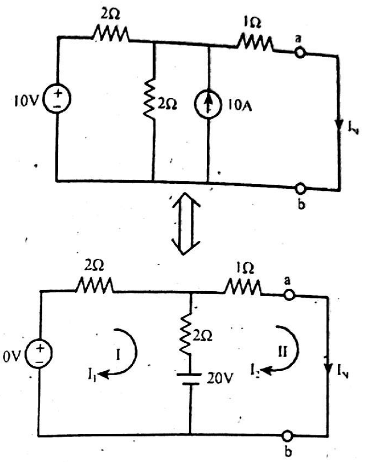
Current in 6Ω resistor, $I_L = \frac{0.311}{3.441 + 6} \times 3.441 = 0.113$ A (B to D)

4. **Determine the power dissipated in 3Ω resistor in the circuit shown below using Norton's theorem.** **[2071 Shrawan, 2066 Baishakh]**



Solution:

To find I_N;



Applying KVL on mesh I and Mesh II, we get

Mesh I:

$$10 - 2I_1 - 2(I_1 - I_2) - 20 = 0$$

or, $-2I_1 - 2I_1 + 2I_2 = 20 - 10$

or, $-4I_1 + 2I_2 = 10$ .... (i)

Mesh II:

$$-I_2 + 20 - 2(I_2 - I_1) = 0$$

or, $-I_2 + 20 - 2I_2 + 2I_1 = 0$

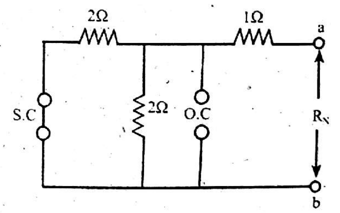
or, $2I_1 - 3I_2 = -20$ .... (ii)

Solving equations (i) and (ii), we get

$I_1 = 1.25$ A, $I_2 = 7.5$A

∴ Short circuit current, $I_N = 7.5$ A (a to b)

To find R_N;

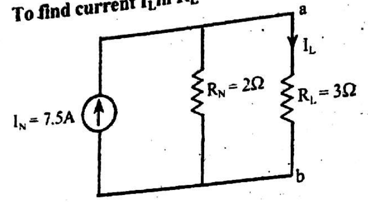


Resistance of the network seen form the terminals a and b is;

$$R_N = R_{ab} = (2 \parallel 2) + 1$$

$$= \frac{2 \times 2}{2+2} + 1 = 1 + 1 = 2\Omega$$

To find current I_L in $R_L = 3\Omega$



Using current division rule,

Current in 3Ω resistor, $I_L = \frac{7.5}{2+3} \times 2$

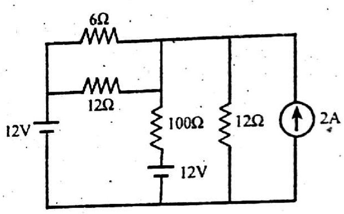
$\therefore$ $I_L = 3$ A (a to b)

Hence,

the power dissipated in 3Ω resistor in the circuit,

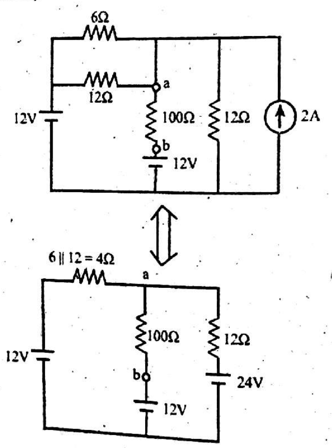
$= I_L^2 \times 3 = 3^2 \times 3 = 9 \times 3 = 27$ W

5. **Use Norton's theorem to find the current through $100\ \Omega$ resistor of the circuit below.** **[2070 Magh]**

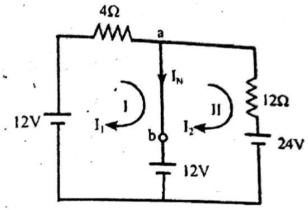


Solution:

The given circuit is;



To find I_N;



Applying KVL to mesh I, we get

$12 - 4I_1 - 12 = 0$

$\therefore$ $I_1 = 0$

Applying KVL to mesh II, we get

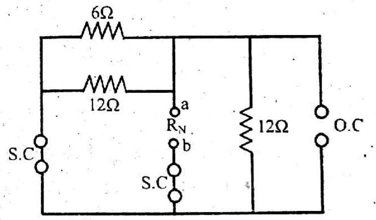
$-12 I_2 - 24 + 12 = 0$

$\therefore$ $I_2 = -1$ A

Negative current indicates that the direction of current is opposite to our assumed direction.

$\therefore$ $I_N = I_2 = 1$ A (a to b)

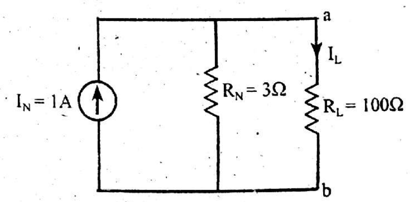
To find R_N;



Resistance of the network seen from the terminals a and b is,

$$R_N = R_{ab} = 6 \,||\, 12 \,||\, 12 = \frac{6 \times 12 \times 12}{6 \times 12 + 12 \times 12 + 12 \times 6} = 3\Omega$$

To find current I_L through $R_L = 100\ \Omega$;



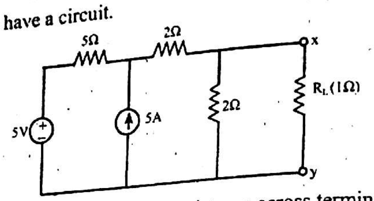
using current division rule,

Current through $100\ \Omega$ resistor, $I_L = \frac{1}{3 + 100} \times 3 = 0.02912$ A (a to b)

6. **Write down the steps to calculate Norton's equivalent resistance in the circuit with a suitable example.** **[2070 Ashad]**

Solution:

Suppose we have a circuit.



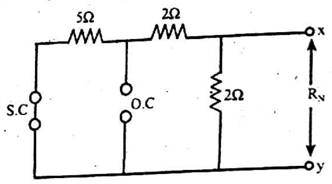
Here, we want to find Norton's equivalent resistance across terminals xy. For this,

i. We remove load resistance (R_L), thus creating an open circuit at terminals xy.

ii. We redraw the circuit with all the voltage sources short circuited and all current sources open circuited.

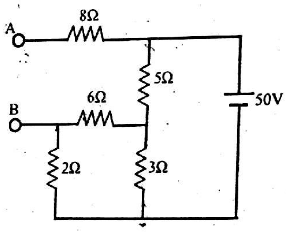
iii. Determining the resistance of the network as seen from the terminals xy as R_N (Norton's equivalent resistance).

To find R_N in above circuit;



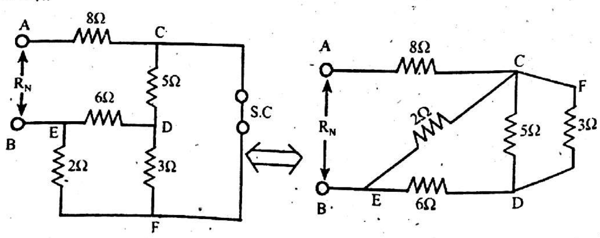
$$\therefore \quad R_N = (5+2) \parallel 2 = 7 \parallel 2 = \frac{7 \times 2}{7+2} = 1.556\ \Omega$$

7. **Find the Norton's equivalent resistance between the terminals A and B in the given circuit.** **[2070 Bhadra]**



Solution:

To find R_N;



Norton's equivalent resistance

$$\therefore \quad R_N = R_{AB} = [[(5 \parallel 3) + 6] \parallel 2] + 8$$

$$= \left[\left(\frac{15}{8} + 6\right) \parallel 2\right] + 8$$

$$= [7.875 \parallel 2] + 8$$

$$= \frac{126}{79} + 8$$

$$= 9.595\ \Omega$$

8. **"Thevenin's theorem and Norton's theorem are dual of each other". Justify the statement with suitable example.** **[2070 Ashad]**

Ans:

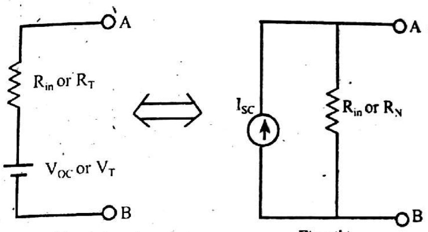


Fig: (a) *Fig: (b)*

Thevenin's equivalent can be converted into its Norton's equivalent and vice- versa. Thevenin's equivalent is shown in fig (a). Norton's current source equals the current Isc which flows through a short across terminals A and B.

Hence,

$$I_{SC} = \frac{V_{OC}}{R_{in}}$$

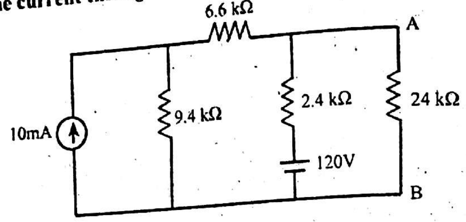
Likewise a Norton's circuit can be converted into its Thevenin's equivalent. The Thevenin's equivalent source V_{OC} or V_T is the voltage on open circuit and is given as

$$V_{OC} \text{ or } V_T = I_{SC} R_{in}$$

Hence, Thevenin's theorem and Norton's theorem are dual of each other.

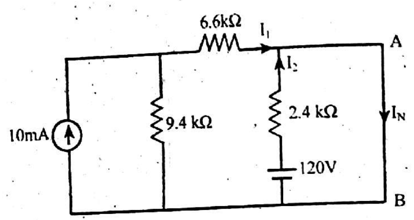
Additional Questions:

1. **For the network shown below derive Norton's equivalent circuit and find the current through 24 kΩ resistance.**



Solution:

To find I_N;

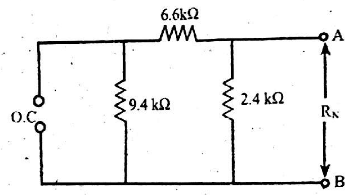


I_N = Current due to 10 mA source + Current due to 120 V source

$= I_1 + I_2$

$= \frac{10}{9.4+6.6} \times 9.4 + \frac{120}{2.4} = 55.875$ mA (A to B)

To find R_N;

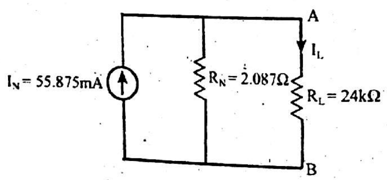


Resistance of the network seen from the terminals A and B is;

$R_N = R_{AB} = (6.6 + 9.4) \parallel 2.4$

$= 16 \parallel 2.4 = 2.087\ \Omega$

To find current I_L through R_L = 24 kΩ;

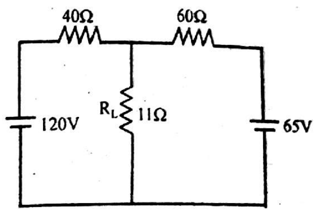


Using current division rule,

Current through 24 kΩ resistor,

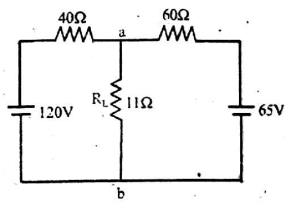
$I_L = \frac{55.875}{2.087 + 24} \times 2.087 = 4.47$ mA

2. **Find the current through R_L in the circuit below using Norton's theorem.**

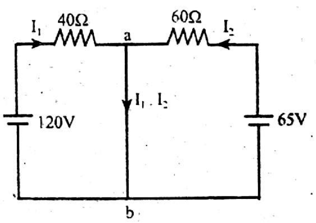


Solution:

The given circuit is;



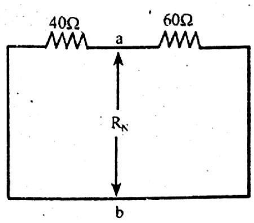
To find I_N;



$I_N = I_1 + I_2$

$= \frac{120}{40} + \frac{65}{60} = 4.083$ A (a to b)

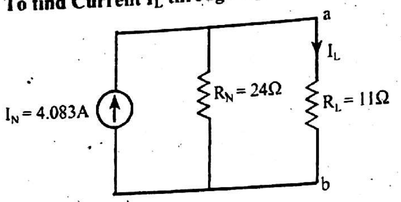
To find R_N;



Resistance of the network seen from the terminals a and b is;

$$R_N = R_{ab} = 40 \parallel 60 = \frac{40 \times 60}{40 + 60} = 24\ \Omega$$

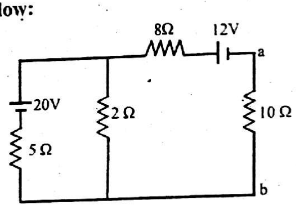
To find Current I_L through $R_L = 11\Omega$;



Using current division rule,

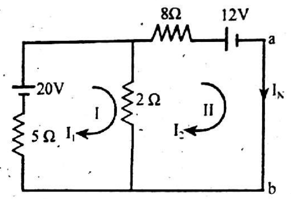
$$I_L = \frac{4.083}{24 + 11} \times 24 = 2.8\ A \text{ (a to b)}$$

3. **Use Norton's theorem to find the current in 10Ω resistance in the circuit given below:**



Solution:

To find I_N;



Applying KVL on mesh I, we get

$20 - 2(I_1 - I_2) - 5I_1 = 0$

or, $-2I_1 + 2I_2 - 5I_1 = -20$

or, $-7I_1 + 2I_2 = -20$ .... (i)

Applying KVL on mesh II, we get

$-8I_2 - 12 - 2(I_2 - I_1) = 0$

or, $-8I_2 - 12 - 2I_2 + 2I_1 = 0$

or, $2I_1 - 10I_2 = 12$ .... (ii)

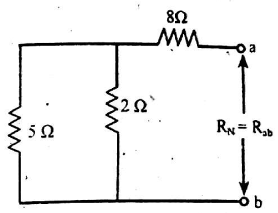
Solving equations (i) and (ii), we get

$$I_1 = \frac{8}{3}A,\ I_2 = -\frac{2}{3}A$$

Negative current indicates that our direction of assumption of flow of current is opposite to actual flow of current

$\therefore\ I_2 = I_N = \frac{2}{3}A$ (b to a)

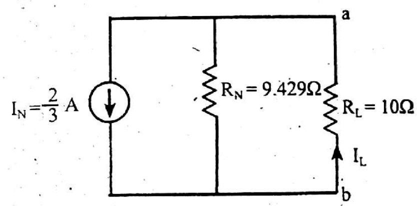
To find R_N;



Resistance of the network seen from the terminals a and b is;

$$R_N = R_{ab} = (5 \parallel 2) + 8 = \frac{5 \times 2}{5 + 2} + 8 = 9.429\ \Omega$$

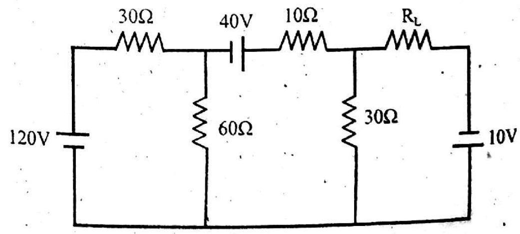
To find Current I_L through $R_L = 10\Omega$;



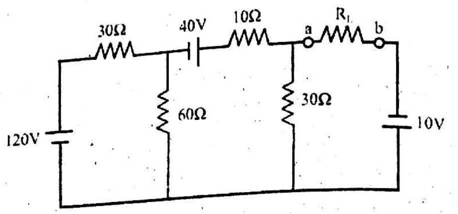
Using current division rule,

$$I_L = \frac{\frac{2}{3}}{9.429 + 10} \times 9.429 = 0.324\ A \text{ (b to a)}$$

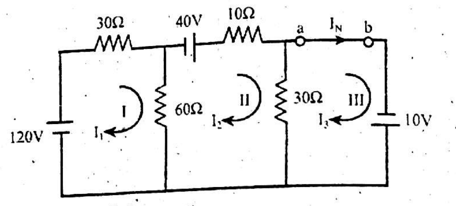
4. **The two terminal battery resistor network shown below is to be connected to the load resistor (R_L) of 24 ohms. Determine the load current and power delivered by using Norton's theorem.**



Solution:



To find I_N:



Applying KVL to mesh I, we get

$120 - 30I_1 - 60(I_1 - I_2) = 0$

or, $\quad -90\, I_1 + 60I_2 = -\,120$ .... (i)

Applying KVL to mesh II, we get

$40 - 10I_2 - 30\,(I_2 - I_3) - 60\,(I_2 - I_1) = 0$

or, $\quad 60I_1 - 100\, I_2 + 30I_3 = -40$ .... (ii)

Applying KVL to mesh III, we get

$10 - 30\,(I_3 - I_2) = 0$

or, $\quad 30I_2 - 30I_3 = -\,10$ .... (iii)

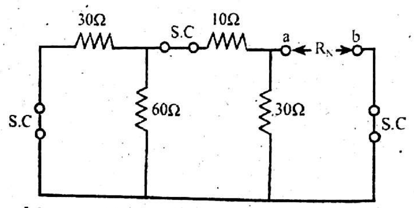
Solving equations (i), (ii) and (iii), we get

$I_1 = 4.22$ A, $I_2 = 4.33$ A, $I_3 = 4.67$ A

∴ Norton's current or short circuit current is,

$I_N = I_3 = 4.67$ A (a to b)

To find R_N;



Resistance of the network seen from the terminals a and b is;

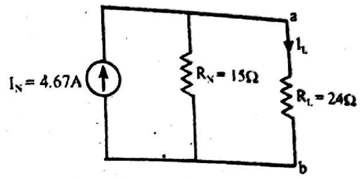
$R_N = R_{ab} = [(30 \,||\, 60) + 10] \,||30$

$= [20 + 10] \,||\, 30$

$= 30 \,||\, 30$

$= 15\,\Omega$

To find current I_L through $R_L = 24\,\Omega$;



Using current division rule,

$$I_L = \frac{I_N}{R_N + R_L} \times R_N$$

$$= \frac{4.67}{15 + 24} \times 15$$

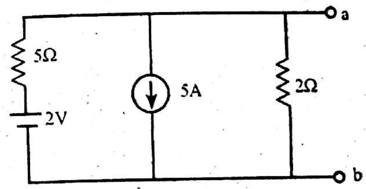
$= 1.796$ A

Power delivered, $P = I_L^2\, R_L$

$= (1.796)^2 \times 24$

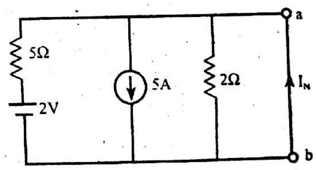
$= 77.415$ W

5. Find the Norton's equivalent circuit across a – b for the network shown in figure below. Use Superposition theorem to find the short circuit current at network terminals.

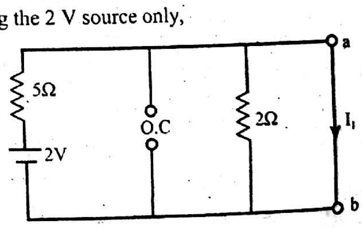


Solution:

To find I_N;

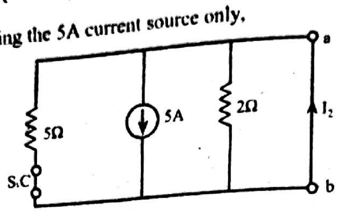


Firstly taking the 2 V source only,



$I_1 = \frac{2}{5} = 0.4$ A (a to b)

Secondly, taking the 5A current source only,



$I_2 = 5$ A (b to a)

Now,

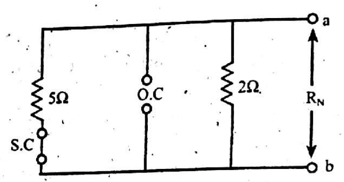
From principle of Superposition, we get

$I_N = I_2 - I_1$

$= 5 - 0.4$

$= 4.6$ A (b to a)

To find R_N;

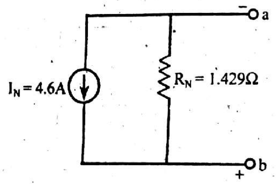


Resistance of the network seen from the terminals a and b is;

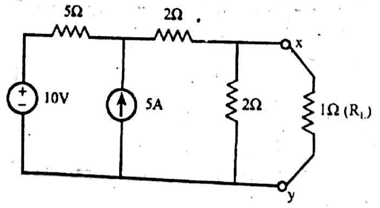
$R_N = R_{ab} = 5 \parallel 2$

$= \frac{5 \times 2}{5 + 2} = 1.429\ \Omega$

Norton's equivalent circuit;

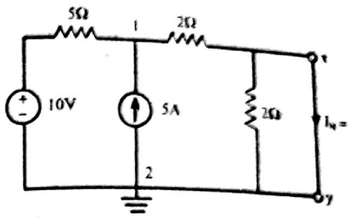


6. Find the power loss in 1Ω resistor (R_L) using Norton's theorem.



Solution:

To find I_N;



Assuming node 1 with potential V and node 2 as the reference node.

Then,

Applying KCL at node 1, we get

$$\frac{V - 10 - 0}{5} + \frac{V - 0}{2} = 5$$

or, $$\frac{V - 10}{5} + \frac{V}{2} = 5$$

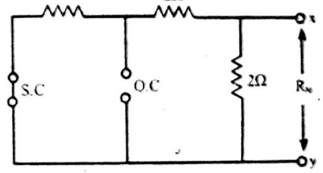
or, $$\frac{V}{5} + \frac{V}{2} = 5 + \frac{10}{5}$$

or, $$\left(\frac{1}{5} + \frac{1}{2}\right) V = 7$$

$\therefore$ V = 10V

$\therefore$ $I_N = \frac{V - 0}{2} = \frac{10}{2} = 5$A (x to y)

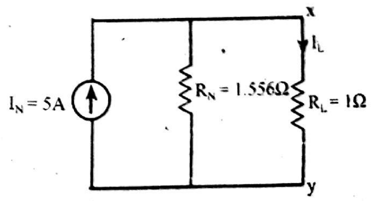
To find R_N;



Resistance of the network seen from the terminals x and y is;

$R_N = R_{xy} = (5 + 2) \parallel 2 = 7 \parallel 2 = 1.556\ \Omega$

To find current I_L through $R_L = 1\ \Omega$;

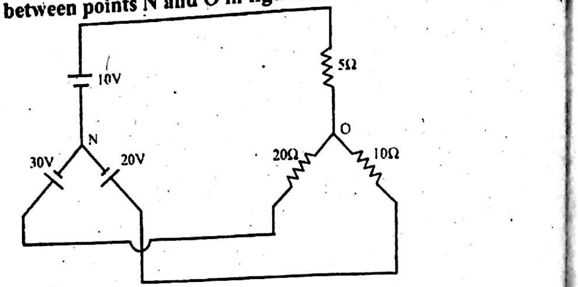


Using current division rule,

$$I_L = \frac{5}{1.556 + 1} \times 1.556 = 3.043 \text{ A (x to y)}$$

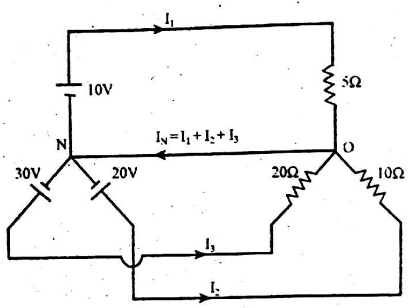
∴ Power loss in 1Ω resistor = $(3.043)^2 \times 1 = 9.26$ W

7. **Using Norton's theorem, find current which would flow in a 25Ω resistor connected between points N and O in figure below:**



Solution:

To find I_N;

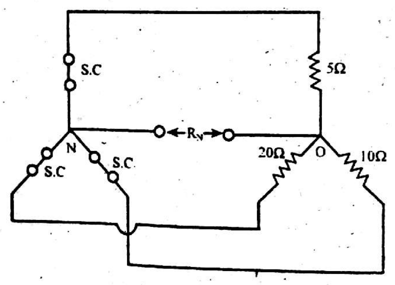


Total current in short - circuit across ON is equal to the sum of currents driven by different batteries through their respective resistances.

$$I_N = I_{SC} = I_1 + I_2 + I_3$$

$$= \frac{10}{5} + \frac{20}{10} + \frac{30}{20} = 5.5 \text{ A}$$

To find R_N;

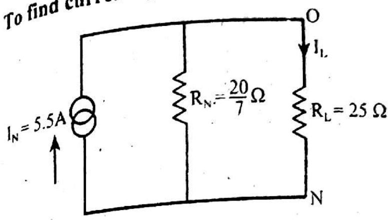


Resistance of the network seen from the terminals N and O is,

$$R_N = R_{NO} = 5 \,||\, 20 \,||\, 10$$

$$= \frac{5 \times 20 \times 10}{5 \times 20 + 20 \times 10 + 10 \times 5} = \frac{20}{7} \, \Omega$$

To find current I_L through $R_L = 25\ \Omega$;



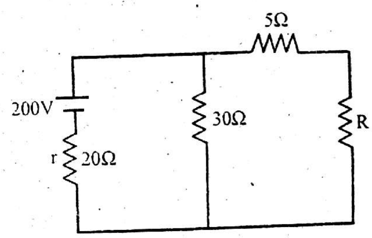
Using current division rule,

$$I_L = \frac{I_N}{R_N + R_L} \times R_N = \frac{5.5}{\frac{20}{7} + 25} \times \frac{20}{7} = 0.564 \text{ A} \qquad \text{(O to N)}$$

Maximum Power Transfer Theorem

Exam solutions

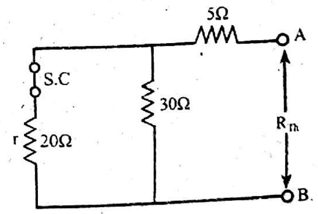
1. **In the network shown below find the resistance R so that maximum power is transferred by 200 V source of internal resistance 20 Ω.** **[2063 Kartik]**



Solution:

In order to determine maximum power transfer, we determine the Thevenin's equivalent network.

To find R_{Th};

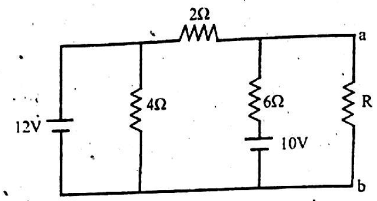


$$R_{Th} = (20 \parallel 30) + 5$$
$$= \frac{20 \times 30}{20 + 30} + 5 = 17\ \Omega$$

Hence, the value of resistance R must be 17 Ω for the maximum power to be drawn by it following maximum power transfer theorem ($R = R_{Th} = 17\Omega$).

2. **Calculate the value of R to receive maximum power and hence the maximum power received by it for the network given in figure.**

[2064 Poush]

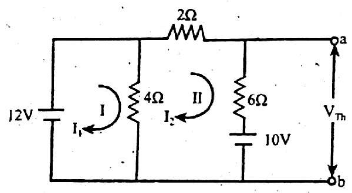


Solution:

Here,

The open circuit voltage V_{OC} (also called Thevenin's voltage V_{Th}) which appears across terminals a and b is calculated and also the resistance of the circuit as looked into the network (also called Thevenin's resistance R_{Th}) from the points a and b is calculated.

To find V_{Th};



Applying KVL on mesh I, we get

$$12 - 4(I_1 - I_2) = 0$$

or, $-4I_1 + 4I_2 = -12$ ....(i)

Applying KVL on mesh II, we get

$$-2I_2 - 6I_2 - 10 - 4(I_2 - I_1) = 0$$

or, $-8I_2 - 10 - 4I_2 + 4I_1 = 0$

or, $4I_1 - 12I_2 = 10$ .... (ii)

Solving equations (i) and (ii), we get

$I_1 = 3.25$ A, $I_2 = 0.25$ A

∴ Open circuit voltage

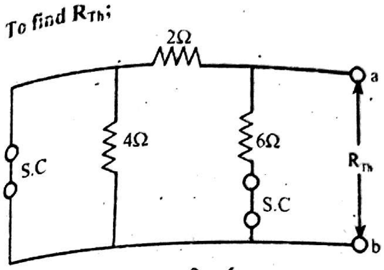
$V_{OC} = V_{Th} = V_{ab}$

$= V_a - V_b$

$= 10 + 6 \times I_2$ [Write KVL equation; move from b to a]

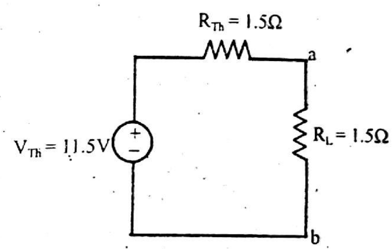
$= 10 + 6 \times 0.25 = 11.5V$

To find R_{Th};



$$R_{Th} = 2 \parallel 6 = \frac{2 \times 6}{2 + 6} = 1.5\ \Omega$$

For maximum power transfer, R_L should be equal to R_{Th} i.e. 1.5 Ω. The whole circuit upto ab can now be replaced by a single source of e.m.f. and single resistance as,



Maximum power drawn by load $= \frac{V_{Th}^2}{4R_L}$

$$= \frac{11.5^2}{4 \times 1.5}$$

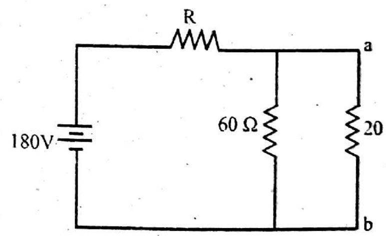
$$= 22.042\ W$$

3. **Consider the circuit shown in the figure: Determine**

i. The value of R so that the load of 20 Ω should draw maximum power.

ii. The value of maximum power drawn by the load.

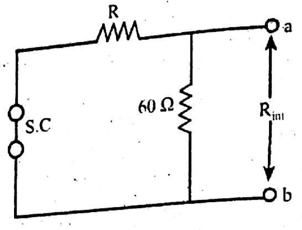
[2065 Kartik]



Solution:

Here, load given is of 20Ω. From maximum power transfer theorem, maximum power will be delivered to the load when the load resistance is equal to the internal resistance of the source ($R_{int} = R_{Th}$). So, $R_{int} = 20\Omega$

i. **To find R_{int};**



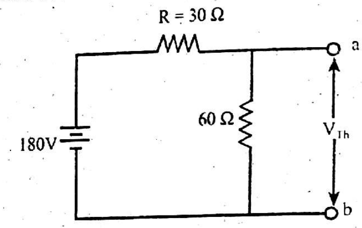
$$R_{int} = R \,||\, 60 = \frac{60\,R}{R+60}$$

or, $\quad 20 = \dfrac{60\,R}{R+60}$

or, $\quad 20R + 1200 = 60\,R$

$\therefore \quad R = 30\Omega$

ii. We find open circuit voltage V_{OC} (also called the Thevenin's voltage V_{Th}) across terminals a and b.



$$V_{Th} = V_{ab} = \frac{180}{30+60} \times 60$$

$\therefore \quad V_{Th} = 120$ V

$\therefore \quad$ Maximum power drawn by the load,

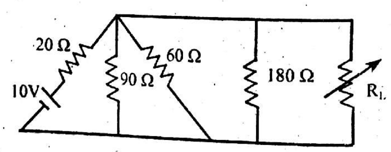
$$P_{max} = \frac{V_{Th}^2}{4R_L}$$

$$= \frac{120^2}{4 \times 20}$$

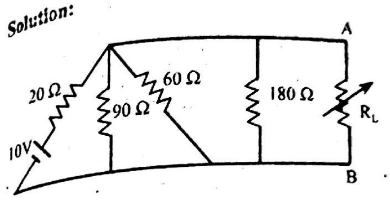
$$= 180 \text{ watt}$$

4. For the circuit shown below, what should be the value of R_L to get maximum power ? What is the maximum power delivered to the load?

[2066 Kartik]



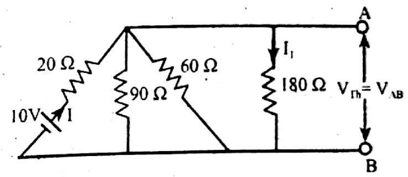
Solution:



Here,

The open circuit voltage V_{OC} (also called Thevenin's voltage V_{Th}) which appears across terminals A and B is calculated and also the resistance of the circuit as looked into the network (also called Thevenin's resistance R_{Th}) from the points A and B is calculated.

To find V_{Th};



Current delivered by battery, $I = \dfrac{10}{20 + (90 \,||\, 60 \,||\, 180)}$

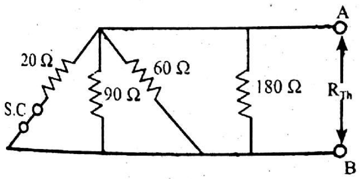
$$= \frac{10}{20 + \dfrac{90 \times 60 \times 180}{(90 \times 60 + 60 \times 180 + 180 \times 90)}} = \frac{10}{20+30} = \frac{1}{5}\text{ A}$$

$\therefore \quad V_{Th} = V_{AB}$

$\quad$ = Source emf – voltage drop in 20Ω resistor

$$= 10 - I \times 20 = 10 - \frac{1}{5} \times 20 = 6\text{ V}$$

To find R_{Th};



$$R_{Th} = R_{AB} = 20 \,||\, 90 \,||\, 60 \,||\, 180$$

$$= \left(\frac{20 \times 90}{20+90}\right) \,||\, \left(\frac{60 \times 180}{60+180}\right)$$

$$= \frac{180}{11} \,||\, 45 = 12\,\Omega$$

As per the maximum power transfer theorem,

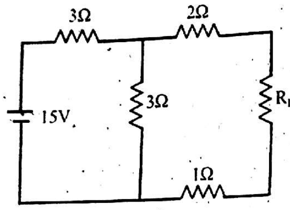
$$R_{Th} = R_L = 12\,\Omega$$

Then,

Amount of maximum power delivered to the load is given by,

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{6^2}{4 \times 12} = 0.75 \text{ W}$$

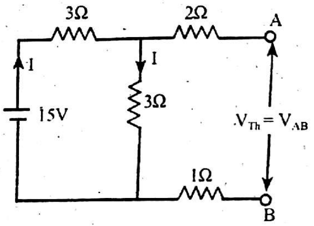
5. **Find the value of R_L such that maximum power will be transferred to. Find the value of the maximum power.** [2068 Bhadra]



Solution:

In order to determine the maximum power transfer, we determine the Thevenin's equivalent network.

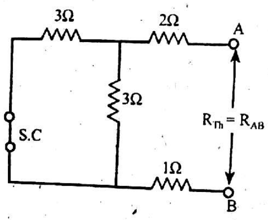
To find V_{Th};



Using voltage divider rule,

$$V_{Th} = V_{AB} = I \times 3 = \frac{15}{3+3} \times 3 = 7.5 \text{ V}$$

To find R_{Th};



$R_{Th} = R_{AB}$

$= (3 \parallel 3) + 2 + 1$

$= \frac{3 \times 3}{3+3} + 3 = 4.5\ \Omega$

As per the maximum power transfer theorem,

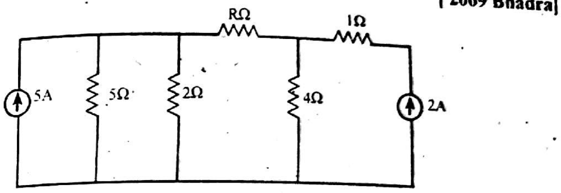
$R_{Th} = R_L = 4.5\ \Omega$

Then,

Amount of maximum power delivered to the load is given by,

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{7.5^2}{4 \times 4.5} = 3.125 \text{ watt}$$

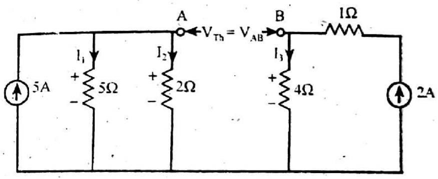
6. **Find the value of R such that maximum power transfer takes place from the current sources to the load R in figure below. Obtain the amount of power transfer.** [2069 Bhadra]



Solution:

In order to determine the maximum power transfer, we determine the Thevenin's equivalent circuit.

To find V_{Th};



Using current division rule,

$$I_2 = \frac{5}{5+2} \times 5 = 3.571 \text{ A}$$

$I_3 = 2\text{A}$

Then,

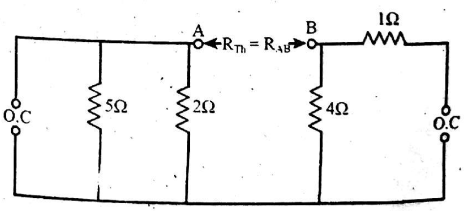
$V_{Th} = V_{AB} = V_A - V_B$

$= -4I_3 + 2I_2$ [Write KVL equation; move from B to A]

$= -4 \times 2 + 2 \times 3.571 = -0.858$ V

Which indicates B is at higher potential with respect to A.

To find R_{Th};



$R_{Th} = R_{AB} = (5 \parallel 2) + 4$

$= \dfrac{5 \times 2}{5 + 2} + 4 = 5.429\ \Omega$

As per maximum power transfer theorem,

$R = R_{Th} = 5.429\ \Omega$

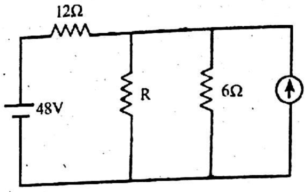
Then,

Amount of maximum power transfer is given by,

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

$$= \frac{0.858^2}{4 \times 5.429} = 0.034 \text{ watt}$$

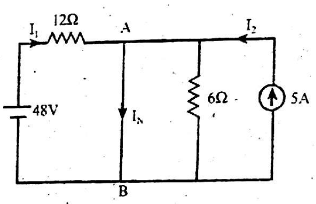
7. **Use Norton's theorem to calculate the value of R that will ab maximum power from the circuit shown in the figure below. Al calculate the maxumum power drawn by it.** **[2069 Chai**



Solution:

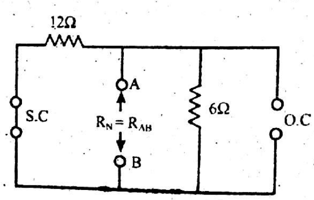
In order to determine the maximum power drawn by R, we first determi Norton's equivalent network.

To find I_N;



$$I_N = I_1 + I_2 = \frac{48}{12} + 5 = 9\text{ A} \quad (\text{A to B})$$

To find R_N;



$R_N = R_{AB} = 12 \parallel 6$

$= \dfrac{12 \times 6}{12 + 6} = 4\Omega$

As per maximum power transfer theorem,

$R_N = R = 4\Omega$

Then,

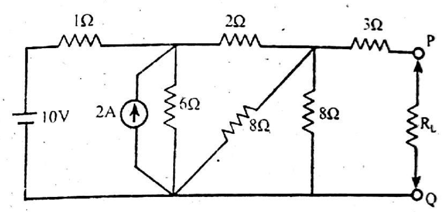
Maximum power drawn by R;

$$P_{max} = \left(\frac{I_N}{2}\right)^2 \times R$$

$$= \frac{I_N^2}{4} \times R = \left(\frac{9}{2}\right)^2 \times 4 = 81 \text{ watt}$$

8. **Using maximum power transfer theorem, find the value of R_L connected between terminals P and Q so that maximum power is developed across R_L. Find the value of maximum power also.**

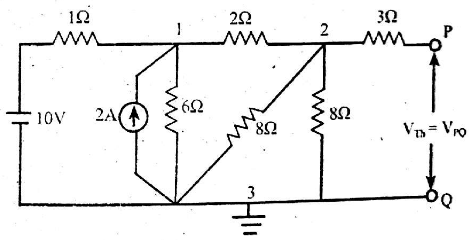
[2068 Magh]



Solution:

In order to determine the maximum power transfer, we determine the Thevenin's equivalent circuit.

To find V_{Th};



In order to find V_{Th}, here we use nodal analysis.

For this, let node 3 be the reference node. The voltage at node 1 and node 2 be V_1 and V_2 respectively.

Then, Applying KCL at node 1:

$$\frac{V_1 - 10 - 0}{1} + \frac{V_1 - 0}{6} + \frac{V_1 - V_2}{2} = 2$$

or, $\quad V_1 - 10 + \dfrac{V_1}{6} + \dfrac{V_1}{2} - \dfrac{V_2}{2} = 2$

or, $\frac{5}{3}V_1 - \frac{V_2}{2} = 12$ .... (i)

Applying KCL at node 2;

$$\frac{V_2 - 0}{8} + \frac{V_2 - 0}{8} + \frac{V_2 - V_1}{2} = 0$$

or, $\frac{V_2}{8} + \frac{V_2}{8} + \frac{V_2}{2} - \frac{V_1}{2} = 0$

or, $-\frac{1}{2}V_1 + \frac{3}{4}V_2 = 0$ .... (ii)

Solving equations (i) and (ii), we get

$V_1 = 9V,\ V_2 = 6V$

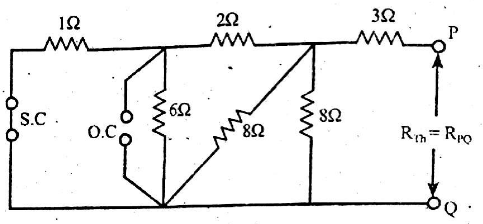
∴ $V_{Th} = V_{PQ}$

= Voltage drop in 8Ω resistor

$= V_2$

$= 6\ V$

To find R_{Th};



$R_{Th} = R_{PQ}$

$= [\{(1 \| 6) + 2\} \| 8 \| 8] + 3$

$$= \left[\left(\frac{6}{7} + 2\right) \| 8 \| 8\right] + 3 = \left[\frac{20}{7} \| 8 \| 8\right] + 3 = \frac{\frac{20}{7} \times 8 \times 8}{\frac{20}{7} \times 8 + 8 \times 8 + 8 \times \frac{20}{7}} + 3$$

$= \frac{5}{3} + 3 = 4.67\ \Omega$

As per maximum power transfer theorem,

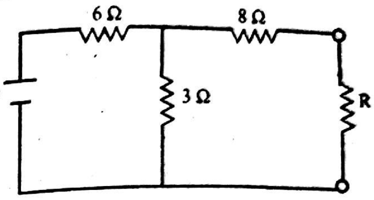
$R_{Th} = R_L = 4.67\ \Omega$

Then,

Maximum power drawn by R_L;

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{6^2}{4 \times 4.67} = 1.93 \text{ watt}$$

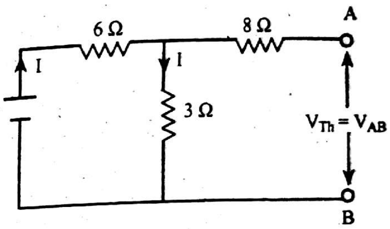
9. Determine the value of R for maximum power to R and calculate the power delivered under this condition. **[2071 Bhadra]**



Solution:

In order to determine maximum power transfer, we determine the Thevenin's equivalent network.

To find V_{Th};

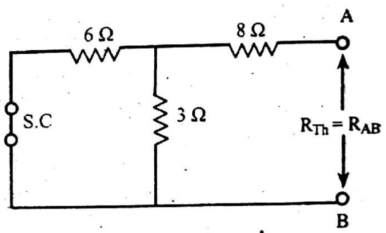


Using voltage divider rule,

$V_{Th} = V_{AB} = I \times 3$

$= \frac{12}{6+3} \times 3 = 4V$

To find R_{Th};



$R_{Th} = R_{AB} = (6 \,//\, 3) + 8$

$= \frac{6 \times 3}{6 + 3} + 8$

$= 10\ \Omega$

As per the maximum power transfer theorem,

$R_{Th} = R_L = 10\ \Omega$

Then,

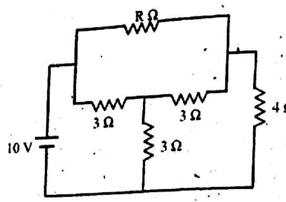
Amount of maximum power delivered to the load is given by,

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{4^2}{4 \times 10} = 0.4 \text{ watt}$$

Hence, the value of R is $10\ \Omega$

For maximum power and power delivered under this condition is 0.4 watt.

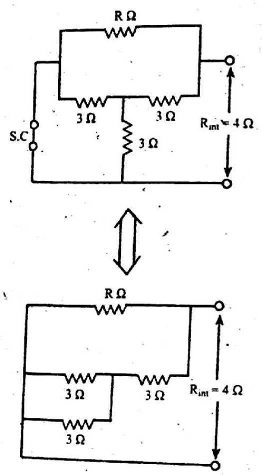
10. Determine the value of R in the given network such that 4 Ω resistor consumes maximum power. [2071 ...]



Solution :–

Maximum power will be delivered to the load of 4 resistance when the load resistance is equal to the internal resistance of the source ($R_{int} = R_{Th}$). So, $R_{int} = 4\Omega$

To Find R_{int};



We know,

$R_{int} = [(3//3) + 3] // R$

or, $4 = 4.5 // R$

or, $4 = \dfrac{4.5R}{4.5 + R}$

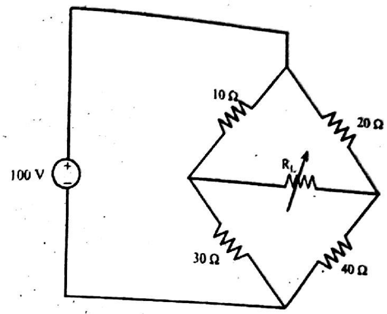
or, $4(4.5 + R) = 4.5\ R$

or, $18 + 4R = 4.5\ R$

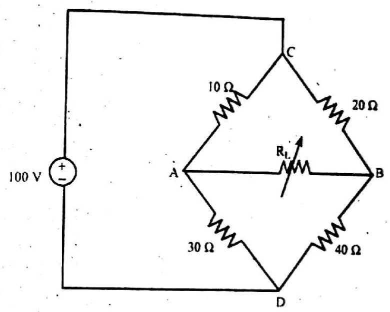
or, $0.5\ R = 18$

$\therefore\ R = 36\ \Omega$

11. Determine the value of load resistance R_L to receive maximum power from the source. Also, find the maximum power delivered to the load in the circuit shown in figure below. [2071 chaitra]

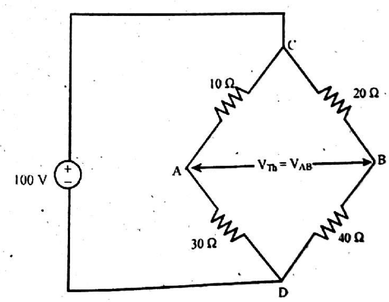


Solution:



Here, the open circuit voltage V_{oc} (also called Thevenin's voltage V_{Th}) which appears across terminals A and B is calculated and also the resistance of the circuit as looked into the network (also called Thevenin's resistance R_{Th}) from the points A and B is calculated.

To find V_{Th} ;



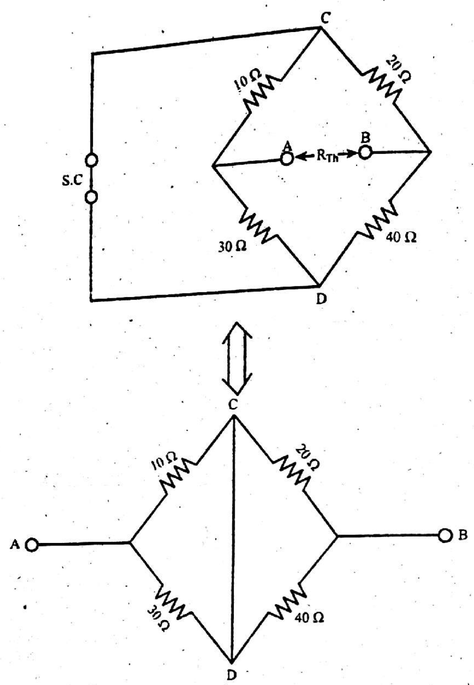
$V_{Th} = V_{AB} = V_{CA} - V_{CB}$

$= \dfrac{100}{10 + 30} \times 10 - \dfrac{100}{20 + 40} \times 20$

$= 25 - \dfrac{100}{\ldots} = -8.33\ V$

The negative sign indicates that B is at higher potential with respect to A.

To find R_{Th};



$\therefore \quad R_{Th} = R_{AB} = (10 // 30) + (20 // 40)$

$= \dfrac{10 \times 30}{10 + 30} + \dfrac{20 \times 40}{20 + 40}$

$= 7.5\ \Omega + 13.33$

$= 20.83\ \Omega$

As per maximum power transfer theorem ,

$R_{Th} = R_{AB} = R_L = 20.83\ \Omega$

Then,

~~Maximum power~~ delivered to the load R_L ;

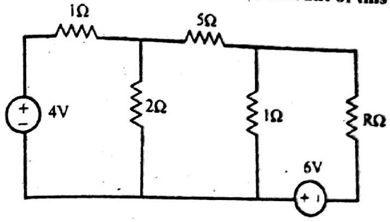
$P_{Max} = \dfrac{V_{Th}^2}{4R_{Th}}$

$= \dfrac{8.33^2}{4 \times 20.83}$

$= 0.883$ watt

Additional questions

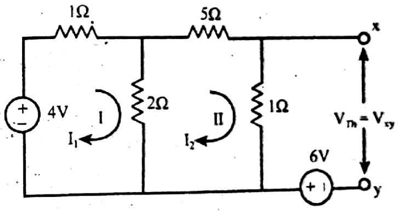
1. **Find the value of R in the circuit of figure below such that maximum power transfer takes place. What is the amount of this power?**



Solution:

In order to determine the maximum power transfer, we determine the Thevenin's equivalent circuit.

To find V_{Th};



Applying KVL to mesh I, we get

$4 - I_1 - 2(I_1 - I_2) = 0$

or, $4 - I_1 - 2I_1 + 2I_2 = 0$

or, $-3I_1 + 2I_2 = -4$ .... (i)

Applying KVL to mesh II, we get

$-5I_2 - I_2 - 2(I_2 - I_1) = 0$

or, $-6I_2 - 2I_2 + 2I_1 = 0$

or, $2I_1 - 8I_2 = 0$ .... (ii)

Solving equations (i) and (ii), we get

$I_1 = 1.6$ A, $I_2 = 0.4$ A

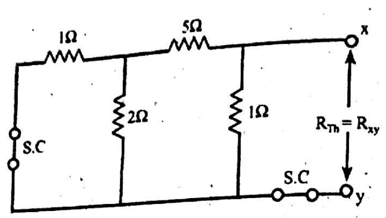
$\therefore \quad V_{Th} = V_{xy} = V_x - V_y$

$= 6 + 1 \times I_2$ [Write KVL equation; move from y to x]

$= 6 + 1 \times 0.4$

$= 6.4$ V

To find R_{Th};



$$R_{Th} = R_{xy}$$

$$= [(1 \parallel 2) + 5] \parallel 1 = \left[\frac{2}{3} + 5\right] \parallel 1 = \frac{17}{3} \parallel 1 = 0.85\ \Omega$$

Now,

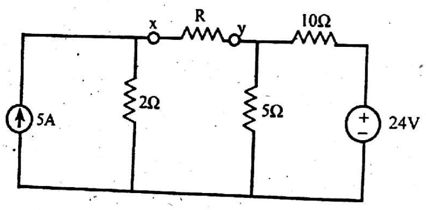
For maximum power, $P_{L(max)}$;

$$R = R_{Th} = 0.85\ \Omega$$

$$P_{L(max\ power)} = \frac{V_{Th}^2}{4R_{Th}}$$

$$= \frac{6.4^2}{4 \times 0.85} = 12.04\ \text{W}$$

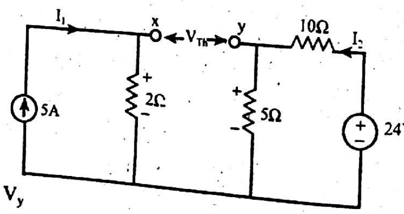
2. **What should be the value of R such that maximum power transfer can take place from the rest of the network to R in figure below. Obtain the amount of this power.**



Solution:

In order to determine the maximum power transfer, we determine the Thevenin's equivalent circuit.

To find V_{Th};

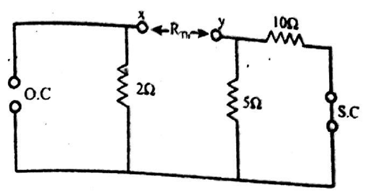


$$V_{Th} = V_{xy} = V_x - V_y$$

$$= -5\,I_2 + 2I_1 \quad \text{[Write KVL equation; move from y to x]}$$

$$= -5 \times \left(\frac{24}{5+10}\right) + 2 \times 5 = 2\ \text{volt}$$

To find R_{Th};



$$R_{Th} = R_{xy}$$

$$= (5 \parallel 10) + 2 = \frac{5 \times 10}{5 + 10} + 2 = \frac{16}{3}\ \Omega = 5.33\ \Omega$$

As per maximum power transfer theorem,

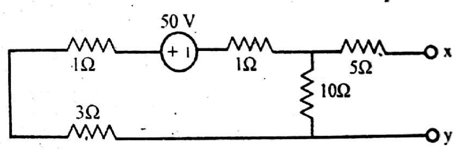
$$R = R_{Th} = \frac{16}{3}\ \Omega = 5.33\ \Omega$$

And,

$$P_{L(max\ power)} = \frac{V_{Th}^2}{4R_{Th}}$$

$$= \frac{2^2}{4 \times 5.33} = 0.188\ \text{watt}$$

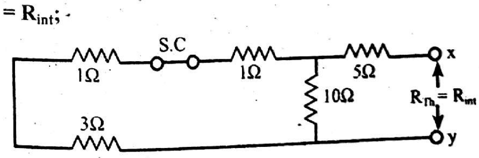
3. **What resistance should be connected across x – y in the circuit shown in figure below such that maximum power is developed across this load resistance? What is the amount of this maximum power?**



Solution:

Let R_L be the resistance that is to be connected across x – y for maximum power transfer from source to load. As per maximum power transfer theorem, R_L should be equal to the internal resistance of the networking looking through x – y.

To find $R_{Th} = R_{int}$;



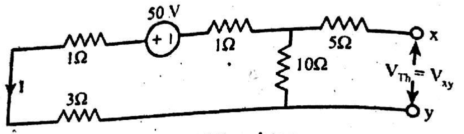
$$R_{int} = [(1 + 1 + 3) \parallel 10] + 5$$

$$= [5 \parallel 10] + 5$$

$$= \frac{5 \times 10}{5 + 10} + 5$$

$$= \frac{50}{15} + 5 = 8.333\ \Omega$$

To find V_{Th};



V_{Th} $= V_{xy}$ = Voltage drop in 10Ω resistor

$$= -10I = -10 \times \frac{50}{1+1+3+10} = -10 \times \frac{50}{15} = -33.33\ V$$

Which indicates y is at higher potential with respect to x.

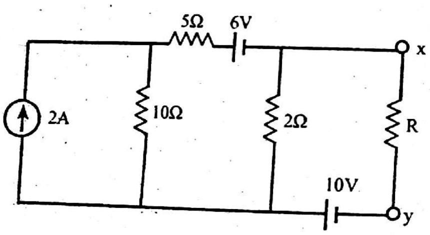
Amount of maximum power transfer is given by,

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

$$= \frac{33.33^2}{4 \times 8.333}$$

$$= 33.33 \text{ watt}$$

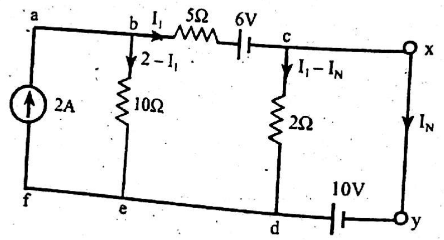
4. **Find R to have maximum power transfer in the circuit given below, obtain the amount of maximum power.**



Solution:

In order to determine the maximum power transfer, we determine Nort equivalent circuit.

To find I_N;



Here,

We use branch current method to find I_N. For this,

Applying KVL in loop bcdeb, we get

$-5I_1 - 6 - 2(I_1 - I_N) + 10(2 - I_1) = 0$

or, $-5I_1 - 6 - 2I_1 + 2I_N + 20 - 10I_1 = 0$

or, $-17I_1 + 2I_N = -14$ ....(i)

Applying KVL in loop cxydc, we get

$10 + 2(I_1 - I_N) = 0$

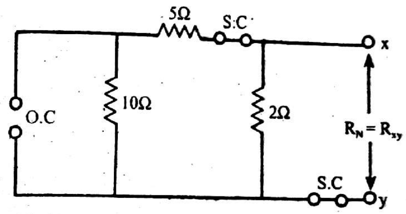
or, $10 + 2I_1 - 2I_N = 0$

or, $2I_1 - 2I_N = -10$ .... (ii)

Solving equations (i) and (ii), we get

$I_1 = 1.6$ A, $I_N = 6.6$ A (x to y)

To find R_N;



$$R_N = R_{xy}$$

$$= (10+5) \parallel 2$$

$$= 15 \parallel 2$$

$$= \frac{15 \times 2}{15 + 2}$$

$$= 1.765\ \Omega$$

As per the maximum power transfer theorem,

$R_{Th} = R = 1.765\ \Omega$

And,

Amount of maximum power,

$$P_{max} = \left(\frac{I_N}{2}\right)^2 \times R$$

$$= \frac{I_N^2 R}{4}$$

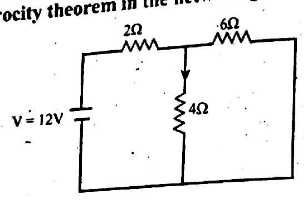
$$= \frac{6.6^2 \times 1.765}{4}$$

$$= 19.22 \text{ watt}$$

Reciprocity Theorem

Exam Solutions

1. Verify the reciprocity theorem in the network given below. [2067 Mangsir]



Solution:

In order to verify the reciprocity theorem, the network is drawn as;

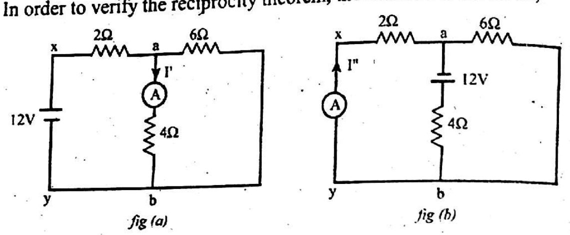


fig (a) *fig (b)*

Considering fig (a),

Equivalent resistance, $R_{eq} = (4 \parallel 6) + 2 = \frac{4 \times 6}{4+6} + 2 = 4.4\ \Omega$

Current supplied by battery $= \frac{12}{4.4} = \frac{30}{11}$ A

Ammeter current, $I' = \frac{\frac{30}{11}}{4+6} \times 6 = \frac{18}{11}$ A

Now, **Considering fig (b),**

Equivalent resistance, $R_{eq} = (2 \parallel 6) + 4 = \frac{2 \times 6}{2+6} + 4 = 5.5\ \Omega$

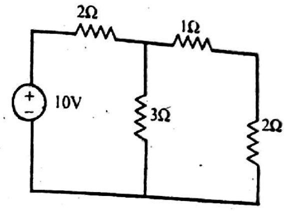
Current supplied by battery $= \frac{12}{5.5} = \frac{24}{11}$ A

Ammeter current, $I'' = \frac{\frac{24}{11}}{2+6} \times 6 = \frac{18}{11}$ A

Hence, we observed that when the source was in branch xy as in fig (a), the ab branch current was $\frac{18}{11}$ A and when the source was in branch ab as in fig (b), the xy branch current was $\frac{18}{11}$ A. This proves the reciprocity theorem.

Additional Questions

1. Show the application of reciprocity theorem in the network given below.



Solution:

Let the circuit be drawn as;

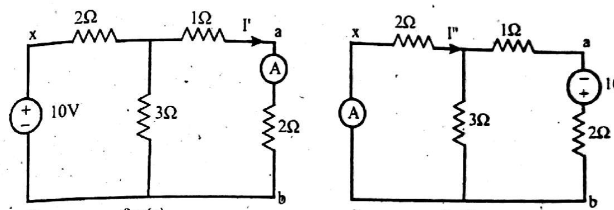


fig (a) *fig (b)*

Considering fig (a),

Equivalent resistance,

$R_{eq} = [(1+2) \parallel 3] + 2$

$= (3 \parallel 3) + 2$

$= 3.5\ \Omega$

Current supplied by battery $= \frac{10}{3.5} = \frac{20}{7}$ A

Ammeter current, $I' = \frac{\frac{20}{7}}{3 + (1+2)} \times 3 = \frac{10}{7}$ A

Now,

Considering fig (b),

Equivalent resistance, $R_{eq} = (2 \parallel 3) + (1+2)$

$= \frac{2 \times 3}{2+3} + 3$

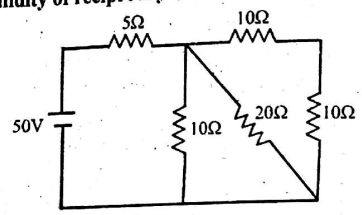
$= 4.2\ \Omega$

Current supplied by battery $= \frac{10}{4.2} = \frac{50}{21}$ A

Ammeter current, $I'' = \frac{\frac{50}{21}}{3+2} \times 3 = \frac{10}{7}$ A

Hence, we observed that when the source was in branch xy as in fig (a), the branch current was $\frac{10}{7}$ A and when the source was in branch ab as in fig (b), the branch current was $\frac{10}{7}$ A. This proves the reciprocity theorem.

2. Show the validity of reciprocity theorem in the circuit given below:



Solution:

Let the circuit be drawn as;

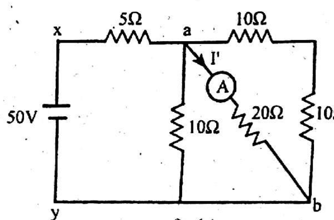


fig (a)

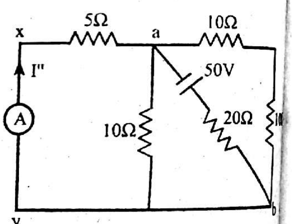


fig (b)

Considering fig (a),

Equivalent resistance, $R_{eq} = (10 \| 20 \| 20) + 5$
$= 5 + 5$
$= 10\,\Omega$

Current supplied by battery $= \frac{50}{10}$
$= 5$A.

Ammeter current, $I' = \frac{\frac{1}{20}}{\frac{1}{10}+\frac{1}{20}+\frac{1}{20}} \times 5 = 1.25$ A

Now,

Considering fig (b),

Equivalent resistance,

$R_{eq} = (5 \| 10 \| 20) + 20$
$= 2.86 + 20$
$= 22.86\,\Omega$

Current supplied by battery $= \frac{50}{22.86}$
$= 2.187$ A

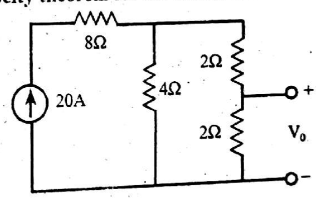
Ammeter current,

$$I'' = \frac{\frac{1}{5}}{\frac{1}{5}+\frac{1}{10}+\frac{1}{20}} \times 2.187$$

$= 1.25$ A

Hence, we observed tnat when the source was in branch xy as in fig (a), the ab branch current is 1.25 A and when the source was in branch ab as in fig (b), the xy branch current becomes 1.25 A. This proves the reciprocity theorem.

3. Verify the reciprocity theorem for the circuit shown in figure.



Solution:

Let the circuit be drawn as;

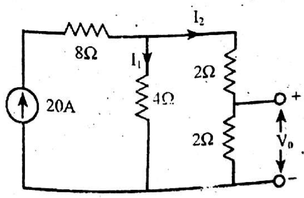


fig (a)

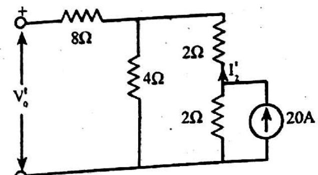


fig (b)

Considering fig (a),

By current division rule

$$I_2 = 20 \times \frac{4}{4+(2+2)}$$

$$= 10\ A$$

Therefore, $V_0 = 2 \times 10$

$$= 20\ V$$

Considering fig (b),

By current division rule,

$$I_2' = 20 \times \frac{2}{2+(4+2)}$$

$$= 5\ A$$

Therefore, $V_0' = 5 \times 4$

$$= 20\ V$$

From the above, it can be seen that $V_0 = V_0'$. Hence, the reciprocity theorem is verified.

❖ ❖ ❖

4

INDUCTANCE AND CAPACITANCE IN ELECTRIC CIRCUITS

4.1 Capacitor

A capacitor is circuit element capable of storing energy in an electrostatic field. In its elementary form, it consists of two insulated parallel metallic plates with air or any other insulating material separating the plates.

4.2 Capacitance

Capacitance is a measure of ability of a capacitor to store an electric charge when a potential difference is applied between the two plates. It is denoted by 'C' and it's unit is farad (F).

A capacitor has a capacitance of one farad when 1 volt charges it with 1 coulomb of electricity.

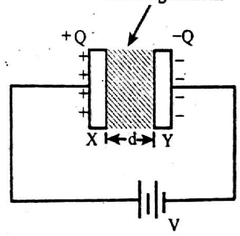


Fig. 4.1

It has been found experimentally that charge Q stored in a capacitor is directly proportional to the p.d. (V) across the plates.

i.e. $Q \propto V$

or, $\frac{Q}{V} = \text{constant} = C$

The constant of proportionality C is called Capacitance of the capacitor.

Electric field strength (E) between two plates,

$E = \frac{V}{d}$ .... (i) [where d is distance between two parallel plates]

If the charge on the plates X and Y is of Q coulombs,

Electric flux density, $D = \frac{Q}{A}$ .... (ii) [where A is overlapping area of parallel plates]

Dividing equation (ii) by equation (i), we get

$$\frac{D}{E}=\frac{Q}{A}\times\frac{d}{V}=\frac{Q}{V}\times\frac{d}{A}=\frac{Cd}{A} \quad \text{.................. (iii)}$$

The ratio of electric flux density in a vacuum (or free space) to the electric field intensity is termed the permittivity of the free space, represented by ε_0.

So,

$$\varepsilon_0 = \frac{D}{E}.$$

Then, equation (iii) becomes

$$\varepsilon_0 = \frac{Cd}{A}$$

$$\therefore \quad C = \frac{\varepsilon_0 A}{d} \text{ farad (with air as medium)}$$

$$\therefore \quad C = \frac{\varepsilon_0 \varepsilon_r A}{d} \text{ farad (in a medium)}$$

4.3 Energy stored in capacitor

Energy stored in capacitor during the interval of raising the charge by small amount dq.

$$dw = Vdq = \frac{q}{C}dq$$

$$\therefore W = \int_0^Q \frac{q}{C}dq$$

$$= \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{C}\frac{Q^2}{2} = \frac{1}{C}\times\frac{C^2V^2}{2} = \frac{1}{2}CV^2$$

$\therefore$ Energy stored in capacitor,

$$W = \frac{1}{2}CV^2$$

4.4.1 Capacitors in Series

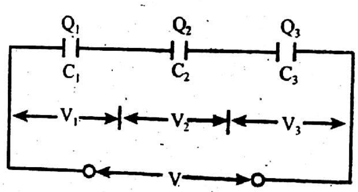


Fig. 4.2

Consider three capacitors connected in series and V be the potential difference across their combination. Since, current will be same through all capacitors, the charge Q across the capacitors will also be same.

$$Q_T = Q_1 = Q_2 = Q_3 = Q$$

As, $V_T = V = V_1 + V_2 + V_3$

And, $C=\frac{Q}{V}$ So, $V=\frac{Q}{C}$

$$\frac{Q_T}{C_T}=\frac{Q_1}{C_1}+\frac{Q_2}{C_2}+\frac{Q_3}{C_3}$$

$$\therefore \quad \frac{1}{C_T}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

The calculation of total series capacitance is similar to the calculation of total resistance of parallel resistors.

4.4.2 Capacitors in Parallel

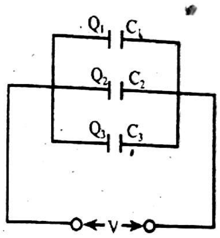


Fig. 4.3

Consider three capacitors connected in parallel and V be the potential difference across the combination. Since potential difference will be same through all capacitors so,

$$V_T = V = V_1 = V_2 = V_3$$

Charge Q is the sum of charge in each capacitor. So,

$$Q = Q_1 + Q_2 + Q_3$$

Since ,

$$Q = C_T V_T = CV$$

Then,

$$C_T V_T = C_1 V_1 + C_2 V_2 + C_3 V_3$$

$$\therefore \quad C_T = C_1 + C_2 + C_3$$

The calculation of total parallel capacitance is similar to the calculation of total resistance of series resistors.

4.5 Inductance

Whenever a conductor is wound around an iron core (or air core) usually in the form of a solenoid, it develops a property known as inductance.

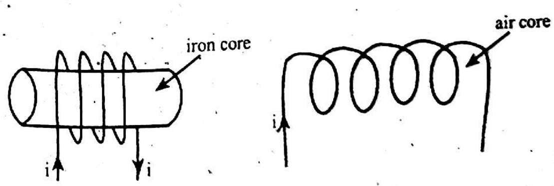


fig. 4.4 Inductors

Inductance is the property of a circuit element that opposes the rise or fall current through it by inducing emf across the circuit element. The circuit element having this property is known as inductor.

4.6 Self inductance

Self inductance is usually just called inductance symbolized by L. Self inductance is a measure of a coils ability to establish an induced voltage as a result of a change in its current. The induced voltage always opposes the change in current which is basically a statement of Lenz's law. The unit of inductance is henry (H).

Let us consider an inductor with copper wire wound on an iron ring as shown.

Self induced emf,

$$e = -N\frac{d\phi}{dt} \quad \text{.........(i)}$$

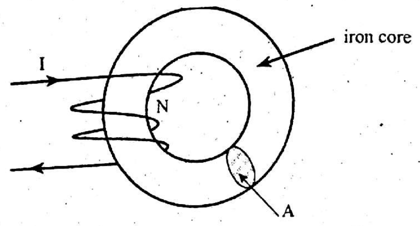


Fig. 4.5

Flux is defined as the ratio of magnetomotive force (mmf) to the reluctance.

i.e $\phi = \frac{\text{MMF(magnetomotive force)}}{\text{R(reluctance)}}$ .... (ii)

We know,

MMF = NI

Since,

$$R \propto \frac{l}{A}, \; R \propto \frac{1}{A}$$

or, $R \propto \frac{l}{A}$

$\therefore$ $R = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A}$

Where,

$\mu = \mu_0 \mu_r$ = permeability of medium

$\mu_0 = 4\pi \times 10^{-7} \frac{H}{m}$

Then,

Equation (ii) can be written as,

$$\phi = \frac{NI}{\frac{l}{A\mu_0\mu_r}} \quad \text{.............(iii)}$$

Now, substituting the value of ϕ from equation (iii) in equation (i), we get,

$$e = -N\frac{d}{dt}\left[\frac{NI}{\frac{l}{A\mu_0\mu_r}}\right]$$

$$= -N\frac{N}{\frac{l}{A\mu_0\mu_r}} \times \frac{dI}{dt}$$

$$= -N^2\frac{\mu_0\mu_r A}{l} \times \frac{dI}{dt}$$

$$\therefore \; e = -L\frac{dI}{dt} \quad \text{.......... (iv)}$$

L Li L
I

$\phi = \frac{MMF}{Reln.}$

$\Rightarrow \frac{NI}{\frac{l}{\mu_0\mu_r A}}$

Where,

$L = \frac{N^2\mu_0\mu_r A}{l} = \frac{N\phi}{I}$ is coefficient of self inductance. $\mu_0\mu_r A$.

A coil is said to have a self inductance of one henry if a current of 1A, when following through it, produces flux linkage of 1 wb-turn in it.

$\phi = \frac{N\mu_0\mu_r A I}{l}$.

4.7 Energy Stored in an inductor

When the current in a circuit, of a coil of inductance L henry, increases from zero to its maximum steady value of I amperes, work has to be done against the opposing induced emf. Let i and e are the respective value of the current and induced emf after a time t seconds. Then the work done in establishing the steady state value of current is given by,

$$W = \int_0^t i\,e\,dt = \int_0^t i\left(L\frac{di}{dt}\right)dt = \int_0^I Lidi$$

$\Rightarrow$ $\mathcal{E} = N\frac{d\phi}{dt}$

$\therefore E = W = \frac{1}{2}LI^2$ joules

$\Rightarrow \frac{N^2\mu_0\mu_r A}{l} \cdot \frac{dI}{dt}$

This is the expression for energy stored in an inductor.

$\mathcal{E} = L\frac{dI}{dt}$.

4.8 Magnetic coupling

Two coils are said to be magnetically coupled, if either full or part of the magnetic flux produced by one links with that of the other. Let L_1 = self inductance of coil '1'; L_2 = self inductance of coil '2' and M = mutual inductances of two coils. Then,

$L = \frac{N^2\mu_0\mu_r A}{l}$.

$$M = k\sqrt{L_1L_2}$$

Where k is coefficient of coupling. If the full flux produced by coil '1' links with the flux produced by coil '2' then k = 1 and $M = \sqrt{L_1L_2}$.

If there is no common flux between the two coils, then they are said to be magnetically isolated.

4.9.1 Inductors in series

i) Series - aiding

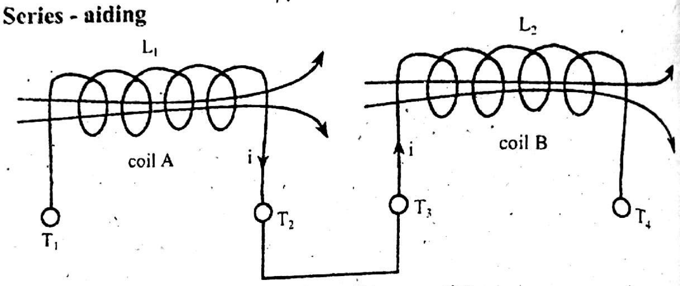


Fig. 4.6 Series aiding connection

Let the two coils be so joined in series that their fluxes (or m.m.fs) are additive i.e. in the same direction.

Let,

M = coefficient of mutual inductance

L_1 = coefficient of self inductance of first coil

L_2 = coefficient of self inductance of second coil

Then, Self induced emf in A = $e_1^L = -L_1 \dfrac{di}{dt}$

Mutually induced emf in A due to change of current in B is $e_1^M = -M\dfrac{di}{dt}$

total emf induced in coil A;

$$e_1 = e_1^L + e_1^M$$

$$= -(L_1 + M)\frac{di}{dt}$$

Similarly,

Total emf induced in coil B;

$$e_2 = -(L_2 + M)\frac{di}{dt}$$

So, the total induced emf in the circuit is given as,

$$e = e_1 + e_2$$

$$= -(L_1 + M)\frac{di}{dt} - (L_2 + M)\frac{di}{dt}$$

$$= -\frac{di}{dt}(L_1 + L_2 + 2M) \text{ .... (i)}$$

If L is the equivalent inductance then total induced emf in that single coil would have been

$$e = -L\frac{di}{dt} \text{ .... (ii)}$$

Equating equations (i) and (ii),we get

$$\therefore \quad L = L_1 + L_2 + 2M$$

ii) Series opposing

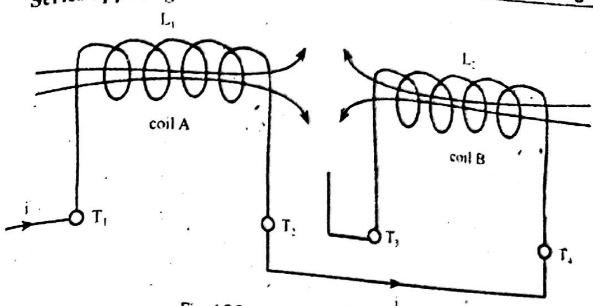


Fig. 4.7 Series opposing connection

For series opposing connections, mutually induced emf opposes the self induced emf.

As before,

Total emf induced in coil A;

$$e_1 = -(L_1 - M)\frac{di}{dt}$$

Similarly,

Total emf induced in coil B;

$$e_2 = -(L_2 - M)\frac{di}{dt}$$

So, the total induced emf in the circuit is given as;

$$e = e_1 + e_2 = -(L_1 + L_2 - 2M)\frac{di}{dt} \text{ .... (iii)}$$

If L is the equivalent inductance then total induced emf in that single coil would have been;

$$e = -L\frac{di}{dt} \text{ .... (iv)}$$

From equations (iii) and (iv), we get

$$\therefore \quad L = L_1 + L_2 - 2M$$

In general

$L = L_1 + L_2 + 2M$ (if m.m.fs are additive)

$L = L_1 + L_2 - 2M$ (if m.m.fs are subtractive)

4.9.2 Inductors in parallel

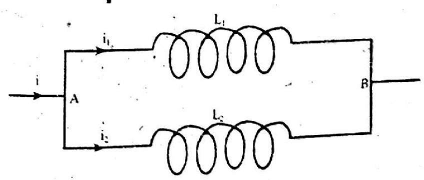


Fig. 4.8

Let the two coils be joined in parallel that the mutual field assists the separate fields.

Let,

M = coefficient of mutual inductance

L_1 = coefficient of self inductance of first coil

L_2 = coefficient of self inductance of second coil

Here,

At node A, by KCL

$i = i_1 + i_2$

or, $\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$ .... (i)

In each coils, both self and mutual induced emf are produced. Since the coils are in parallel, these emfs are equal.

i.e. $e_1^L + e_1^M = e_2^L + e_2^M$

or, $-L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = -L_2\frac{di_2}{dt} - M\frac{di_1}{dt}$

or, $(L_2 - M)\frac{di_2}{dt} = (L_1 - M)\frac{di_1}{dt}$

or, $\frac{di_1}{dt} = \frac{(L_2 - M)}{(L_1 - M)}\frac{di_2}{dt}$ .... (ii)

From equations (i) and (ii)

$\frac{di}{dt} = \frac{L_2 - M}{L_1 - M}\frac{di_2}{dt} + \frac{di_2}{dt}$

$\frac{di}{dt} = \left[\left(\frac{L_2 - M}{L_1 - M}\right) + 1\right]\frac{di_2}{dt}$ .... (iii)

If L is the equivalent inductance of the combination then induced emf

$e = -L\frac{di}{dt}$

= induced emf in parallel combination

= induced emf in any one coil

i.e. $-L\frac{di}{dt} = -L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$

or, $\frac{di}{dt} = \frac{L_1}{L}\frac{di_1}{dt} + \frac{M}{L}\frac{di_2}{dt}$ .... (iv)

or, $\frac{di}{dt} = \frac{1}{L}\left(L_1\frac{di_1}{dt} + M\frac{di_2}{dt}\right)$

Substituting the value of $\frac{di_1}{dt}$ from equation (ii) in equation (iv), we get

$\frac{di}{dt} = \frac{1}{L}\left[L_1\frac{(L_2 - M)}{(L_1 - M)} + M\right]\frac{di_2}{dt}$ .... (v)

From equations (iii) and (v), we have

$\frac{L_2 - M}{L_1 - M} + 1 = \frac{1}{L}\left[L_1\frac{(L_2 - M)}{(L_1 - M)} + M\right]$

or, $\frac{L_2 - M + L_1 - M}{L_1 - M} = \frac{1}{L}\left[\frac{L_1L_2 - L_1M + L_1M - M^2}{L_1 - M}\right]$

$\therefore$ $L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$ [When mutual field assists the separate fields]

Similarly,

$L = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$ [When the two fields oppose each other]

Exam Solutions

1. **A capacitor of 2 μF is charged to 600 V. Find the energy stored in the capacitor and calculate power if the capacitor is discharged uniformly to 400 V in 3 seconds.**
[2064 Shrawan]

Solution:

Here

Capacitance, $C = 2\mu F$

Voltage to which the capacitor has been charged, $V_1 = 600$ V

Voltage to which the capacitor has been discharged, $V_2 = 400$ V

Energy stored in the capacitor, before discharge,

$W_1 = \frac{1}{2}CV_1^2 = \frac{1}{2} \times 2 \times 10^{-6} \times 600^2$

$= 0.36$ J

Energy stored in the capacitor, after discharge

$W_2 = \frac{1}{2}CV_2^2 = \frac{1}{2} \times 2 \times 10^{-6} \times 400^2$

$= 0.16$ J

Energy removed from the capacitor

$= W_1 - W_2$

$= 0.36 - 0.16$

$= 0.2$ J

Duration of discharge, t = 3 seconds

Power, $P = \frac{\text{Energy removed in J or W-s}}{\text{t in seconds}}$

$= \frac{0.2}{3} = \frac{1}{15}$ watt

2. **A solenoid of 1200 turns has iron core of cross - section area 80 cm^2 and mean length 0.4m. Find the self inductance of the coil and calculate induced emf if a current of 0.2 A is switched OFF in 0.01 sec. (Relative permeability of iron is 1000)**
[2063 Kartik]

Solution:

Here,

$\ell = 0.4$ m , N = 1200 turns, A = 80 cm^2,

$\mu_0 = 4\pi \times 10^{-7}$, $\mu_r = 1000$

$$\text{Self - inductance, } L = \frac{N\phi}{I}$$

$$= \frac{N}{I} \times \frac{NI}{\frac{\ell}{A\mu_0\mu_r}}$$

$$= \frac{N^2 A\mu_0 \mu_r I}{I \times \ell}$$

$$= \frac{N^2 A\mu_0 \mu_r}{\ell}$$

$$= \frac{1200^2 \times 80 \times 10^{-4} \times 4\pi \times 10^{-7} \times 1000}{0.4}$$

$$= 36.19 \text{ H}$$

$$\text{Induced emf, } e = L\frac{di}{dt}$$

$$= 36.19 \times \frac{0.2 - 0}{0.01}$$

$$= 723.82 \text{ V}$$

3. **An inductor is to be made with a copper wire wound on an iron core having mean length of 50 cm with a cross sectional area of 60mm^2. If the required value of inductance is 70 mH, calculate the number of turns required. It is given that the relative permeability of the core is 1400.**
[2066 Magh]

Solution:

Here,

ℓ = 50 cm = 0.5 m, A = 60 mm^2, L = 70 mH

$\mu_0 = 4\pi \times 10^{-7}$, $\mu_r = 1400$, N = ?

Since,

$$L = \frac{N^2 A\mu_0 \mu_r}{\ell}$$

or, $$70 \times 10^{-3} = \frac{N^2 \times 60 \times 10^{-6} \times 4\pi \times 10^{-7} \times 1400}{0.5}$$

$\therefore$ N = 575.82 ≈ 576 turns

4. **An air cored coil is 2.5 cm long and has an average cross - sectional area of 2cm^2. Determine the number of turns if the coil has an inductance of 100 μH.**
[2068 Chaitra]

Solution:

Here,

$L = 100\mu H = 100 \times 10^{-6}$ H, ℓ = 2.5 cm = 0.025 m

$A = 2\ cm^2 = 2 \times 10^{-4}\ m^2$, $\mu_0 = 4\pi \times 10^{-7}$, $\mu_r = 1$

Since,

$$L = \frac{N^2 A\mu_0 \mu_r}{\ell}$$

or, $$100 \times 10^{-6} = \frac{N^2 \times 2 \times 10^{-4} \times 4\pi \times 10^{-7} \times 1}{0.025}$$

$\therefore$ N = 99.74 ≈ 100 turns

5. **Two coils of inductance of 4 and 6 henry are connected in parallel. If their mutual inductance is 3 henry, calculate the equivalent inductance of the combination if mutual inductance**

i) Supports the self inductance, and

ii) Oppose the self inductance
[2068 Shrawan]

Solution:

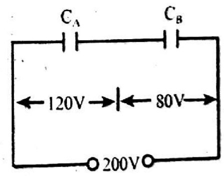
i. $$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{4 \times 6 - 3^2}{4 + 6 - 2 \times 3} = 3.75 \text{ H}$$

ii. $$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \frac{4 \times 6 - 3^2}{4 + 6 + 2 \times 3} = 0.9375 \text{ H}$$

6. **Two capacitors, A and B are connected in series across a 200 V dc supply. The p.d. across A is 120 V. This p.d. is increased to 140V, when a 3μF capacitor is connected in parallel with B. Calculate the capacitances of A and B.**
[2069 Bhadra]

Solution:

Let the capacitances of the capacitors A and B be C_A and C_B respectively and potential difference across them be V_A and V_B respectively.



$V_B = 200 - V_A = 200 - 120 = 80$ V

For series connection, charge is same

i.e. $Q = C_A V_A = C_B V_B$

or, $C_A \times 120 = C_B \times 80$

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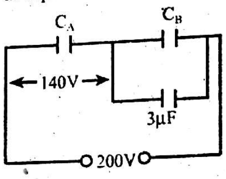
or, $C_A = \frac{80}{120} C_B$ .... (i)

Now,

When a capacitor of 3μF is connected in parallel with capacitor B, the equivalent capacitance of capacitor B and capacitor of 3μF is



$C_B' = C_B + 3$

New charges on C_A and C_B' will be equal.

i.e. $140\ C_A = (200 - 140)(C_B + 3)$

or, $140\ C_A = 60\ (C_B + 3)$

or, $140 \times \frac{80}{120} C_B = 60\ (C_B + 3)$

or, $\frac{100}{3} C_B = 180$

∴ $C_B = 5.4\ \mu F$

From equation (i)

$C_A = \frac{80}{120} \times 5.4 = 3.6\ \mu F$

6. **Calculate the inductance that must be connected in parallel with a 100 mH inductor to give a total inductance of 70 mH. Assume no mutual inductance between the two.** [2070 Ashad]

Solution:

Let L_1 and L_2 be the inductances of inductors 1 and 2 connected in parallel. Also, L_{eq} be the equivalent inductance of the parallel connection. Since, no mutual inductance is between the two

so, M= 0

Since,

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

or, $\frac{1}{70} = \frac{1}{100} + \frac{1}{L_2}$

∴ $L_2 = 233.33$ mH

7. **A capacitor with capacitance of 2μF is connected in series with another capacitor whose capacitance is C_x. If the equivalent capacitance of the combination is 1.5 μF calculate the value of C_x. What would be the equivalent capacitance if they were connected in parallel?** (2071 Magh)

Solution:

Let C_1 and C_2 be the capacitance of capacitor 1 and capacitor 2 respectively. Then,

By question,

$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$ (in series)

or, $C_{eq} = \frac{2C_x}{2 + C_x}$

or, $1.5 = \frac{2C_x}{2 + C_x}$

or, $3 + 1.5\ C_x = 2\ C_x$

∴ $C_x = 6$ F

Now,

$C_{eq} = C_1 + C_2$ (in parallel)

$= 2 + 6$

$= 8\ \mu F$

Hence,

The value of C_x is 6 μF and the equivalent capacitance is 8 μF when capacitors are connected in parallel.

Additional questions

1. **The total capacitance of two capacitors is 0.025μF, when connected in series and 0.15 μF, when connected in parallel. Find the capacitance of each capacitor.**

Solution:

Let C_1 and C_2 be the capacitances of capacitor 1 and capacitor 2 respectively.

Then,

$C_1 + C_2 = 0.15$ (in parallel)

or, $C_2 = 0.15 - C_1$ .... (i)

Also,

$\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{0.025}$ (in series)

or, $\frac{C_1 C_2}{C_1 + C_2} = 0.025$ .... (ii)

From equations (i)and (ii) we get

$$\frac{C_1(0.15 - C_1)}{0.15} = 0.025$$

or, $0.15\ C_1 - C_1^2 = 0.00375$

or, $C_1^2 - 0.15\ C_1 + 0.00375 = 0$

On solving this quadratic equation, we get

$C_1 = 0.1183\mu F$ or $0.0317\ \mu F$

Hence,

$C_2 = (0.15 - 0.1183)\ \mu F$ or $(0.15 - 0.0317)\mu F$

$= 0.0317 \mu F$ or $0.1183 \mu F$

$\therefore$ One capacitor is of capacitance $0.1183\ \mu F$ and other of capacitance $0.0317\ \mu F$.

2. Three capacitors A, B and C are connected in series across 100 V d.c. supply. The p.d. across the capacitors are 20 V, 30 V and 50 V respectively. If the capacitance of A is 10 μF, calculate the capacitances of B and C.

Solution:

Here,

$C_A = 10\ \mu F,\ V_A = 20\ V,\ V_B = 30\ V,\ V_C = 50\ V$

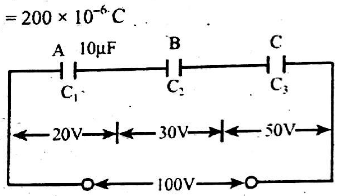
Now,

Charge on capacitor A,

$Q_A = C_A V_A = 10 \times 10^{-6} \times 20 = 200 \times 10^{-6}\ C$

Since, the three capacitors A, B and C are joined in series, so the charge on all of them must be the same i.e.,

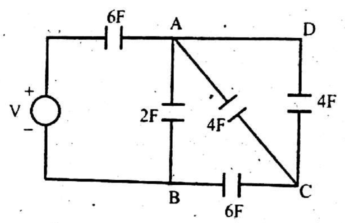
$Q_B = Q_C = Q_A = 200 \times 10^{-6}\ C$



$$\text{Capacitance, } C_B = \frac{Q_B}{V_B} = \frac{200 \times 10^{-6}}{30} = 6.667\ \mu F$$

$$\text{Capacitance, } C_C = \frac{Q_C}{V_C} = \frac{200 \times 10^{-6}}{50} = 4\ \mu F$$

3. Determine the equivalent capacitance of the combination shown in the figure.



Solution:

Between A and C, there are two 4 F capacitors in parallel, so

$C_{AC} = 4 + 4 = 8\ F$

C_{BD} = Equivalent of 8F in series with 6F

$$C_{BD} = \frac{8 \times 6}{8 + 6} = \frac{24}{7}\ F$$

C_{AB} = Equivalent of 2F in parallel with $\frac{24}{7}$ F

$$C_{AB} = \frac{24}{7} + 2 = \frac{38}{7}\ F$$

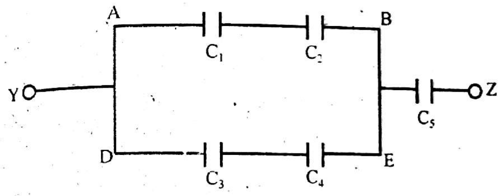
Now,

Equivalent capacitance of combination

= Equivalent of 6 F in series with $\frac{38}{7}$ F

$$\therefore \quad C_{eq} = \frac{\frac{38}{7} \times 6}{\frac{38}{7} + 6} = 2.85\ F$$

4. Find the equivalent capacitance of the following network.



Solution:

$C_P = C_{AB} + C_{DE}$

$$= \frac{C_1 C_2}{C_1 + C_2} + \frac{C_3 C_4}{C_3 + C_4}$$

$\therefore \quad C_{eq} = C_{YZ}$

$$= \frac{C_P C_5}{C_P + C_5} = \frac{\left(\frac{C_1 C_2}{C_1 + C_2} + \frac{C_3 C_4}{C_3 + C_4}\right) C_5}{\left(\frac{C_1 C_2}{C_1 + C_2} + \frac{C_3 C_4}{C_3 + C_4}\right) + C_5}$$

5. Two coils have self inductances of 6.8 mH and 4.5 mH respectively. When they are connected in series, the total inductances in series aiding and series opposing connections are 19.6 mH and 3mH respectively. Find mutual inductance and coefficient of coupling.

Solution:

Here,

$L_{additive} = 19.6$ mH

$L_{subtractive} = 3$ mH

$L_1 = 6.8$ mH

$L_2 = 4.5$ mH

$\because$ $L_{additive} = L_1 + L_2 + 2M$

or, $19.6 = 6.8 + 4.5 + 2M$

$\therefore$ $M = 4.15$ mH

Also

$M = k\sqrt{L_1L_2}$

or, $k = \dfrac{M}{\sqrt{L_1L_2}}$

$= \dfrac{4.15}{\sqrt{6.8 \times 4.5}}$

$\therefore$ $k = 0.7502$

6. **The coefficient of coupling between two coils is 0.85. Coil 1 has 250 turn. When the current in coil 1 is 2A, the total flux of this coil is 3×10^{-4} weber. When i_1 is changed from 2A to zero linearly in 2 milliseconds, the voltage induced in coil 2 is 63.75V. Find L_1, L_2, M and N_2.**

Solution:

$$L_1 = \frac{N_1\phi_1}{I_1} = \frac{250 \times 3 \times 10^{-4}}{2} = 37.5 \times 10^{-3} \text{H}$$

$$v_2 = M\frac{di_1}{dt}$$

or, $63.75 = M \times \dfrac{2}{2 \times 10^{-3}}$

$\therefore$ $M = 63.75 \times 10^{-3}$ H

Also, $M = k\sqrt{L_1L_2}$

or, $63.75 \times 10^{-3} = 0.85\sqrt{37.5 \times 10^{-3} \times L_2}$

$\therefore$ $L_2 = 150 \times 10^{-3}$ H

Since, self inductance $\propto N^2$

$$\frac{L_1}{L_2} = \frac{N_1^2}{N_2^2}$$

or, $\dfrac{37.5 \times 10^{-3}}{150 \times 10^{-3}} = \dfrac{250^2}{N_2^2}$

$\therefore$ $N_2 = 500$ turns

5

ALTERNATING QUANTITIES

BASIC TERM & CONCEPTS

An alternating quantity (voltage or current) is one which changes continuously in magnitude and alternates in direction at regular intervals of time.

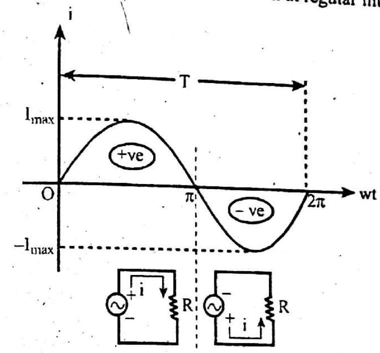


Fig. 5.1

Waveform – The shape of the curve of the voltage or current when plotted against time as base is called wave form. Fig. 5.1 shows the waveform of an alternating current varying sinusoidally.

[Sinusoid refers to the signal that has form of the sine or cosine function]

Time period- The time taken by the alternating quantities (voltage/current) to complete one cycle is called time period and is denoted by T.

Frequency-The number of cycles completed in one second is called frequency and is denoted by f.

$\boxed{f = \frac{1}{T}}$ Unit → hertz (Hz) or cycles/second

Amplitude- The maximum value which an alternating quantity attains during one complete cycle is called its amplitude.

In fig. 5.1, amplitude is denoted by I_{max}.

Phase – The phase of an alternating quantity is defined as the fraction of time period that it has elapsed since its waveform has passed through zero position of the reference line.

In fig. 5.2,

Phase of A = $\frac{T}{8}$

Phase of B = $\frac{T}{4}$

Phase of C = $\frac{T}{2}$

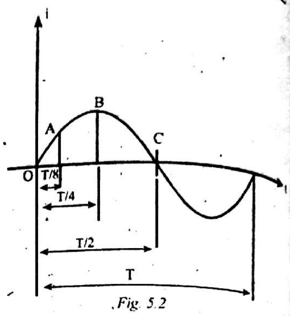


Fig. 5.2

Phase difference

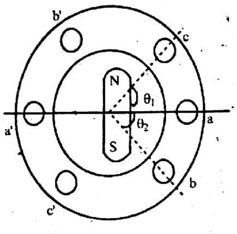


Fig. 5.3

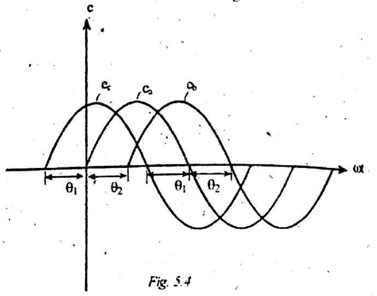


Fig. 5.4

Consider a generator having three coils a – a', b – b' & c – c'. According to Faraday's law of electromagnetic induction emf will induce in all three coils namely e_a, e_b and e_c. But, the instant at which they attend maximum value is different and so is the time at which they pass through zero of reference line.

At time when e_a passes through zero, e_c has already passed through zero i.e. e_c is θ_1 angle ahead of e_a. Similarly, e_b is θ_2 angle behind e_a.

Therefore, the phase difference between e_a and e_c is θ_1 and e_c is leading e_a by a phase angle of θ_1. Similarly, the phase difference between e_a and e_b is θ_2 and e_b is lagging e_a by θ_2 phase angle.

Hence,
$$e_a = E_m \sin\omega t$$
$$e_b = E_m \sin(\omega t - \theta_2)$$
$$e_c = E_m \sin(\omega t + \theta_1)$$

Instantaneous value of alternating quantity

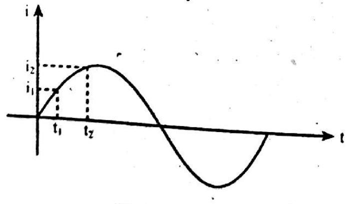


Fig. 5.5

The value of the alternating quantities at any particular instant of time, is called instantaneous value of alternating quantity.

- Instantaneous value of current at time $t_1 = i_1$
- Instantaneous value of current at time $t_2 = i_2$

Average value of alternating quantity

The average value of an alternating current is equal to the value of direct current which transfers across any circuit the same amount of charge during a given time as is transferred by the alternating current for the same time in the same circuit.

Mathematically integral form,

$$\therefore I_{avg} = \frac{1}{T}\int_0^T i\,dt$$

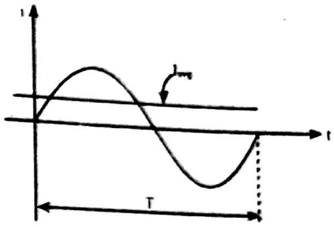


Fig. 5.6

[However, for symmetrical waveform, average value for a complete cycle is zero]

Symmetrical waveforms are those sinusoidal or non-sinusolidal wave forms whose two half cycles (positive and negative) are exactly similar.

Root – mean square (rms)/ Effective/ Virtual value of alternating quantity

The rms value of an alternating current or voltage is equal to that direct current or voltage which when flows or applied to a given resistance for a given time produces the same amount of heat as when the alternating current or voltage is flowing or applied to the same resistance for the same time.

In integral form,

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$$

(Note : rms value of a waveform is greater than the average value of the particular waveform]

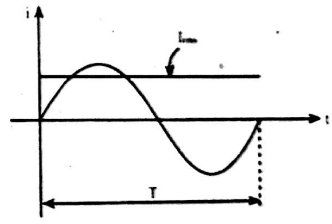


Fig. 5.7

Form factor

$$\text{Form factor} = \frac{\text{Rms value of alternating quantity}}{\text{Average value of alternating quantity}}$$

Peak factor

$$\text{Peak factor} = \frac{\text{Peak value of alternating quantity}}{\text{Rms value of alternating quantity}}$$

Phasor Diagram

An alternating quantities (voltage/current) are represented by straight lines having definite direction and length. Such lines are called phasors and diagrams in which phasors represent currents, voltages and their phase difference are known as phasor diagrams. The phasor diagrams can be drawn either to represent maximum or effective values.

Some of common conventions are–

1. Counter - clockwise direction of rotation of phasors is usually taken as positive direction of rotation of phasors i.e. a phasor rotated in counter - clockwise direction from a given phasor is said to lead the given phasor while a phasor rotated in clockwise direction is said to lag the given phasor.
2. For series circuit, the current phasor is usually taken as reference phasor. For parallel circuit, voltage phasor is usually taken as reference phasor.

Some illustration of phasor diagrams.

$e_a = E_m \sin \omega t$

$e_b = E_m \sin (\omega t - \theta_2)$

$e_c = E_m \sin (\omega t + \theta_1)$

Phasor diagram

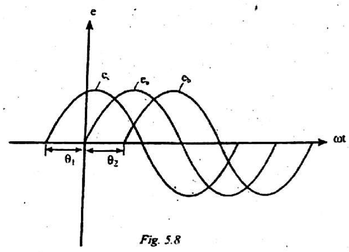


Fig. 5.8

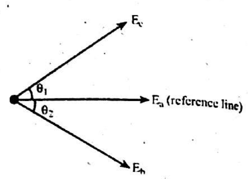


Fig. 5.9

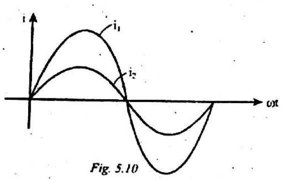


Fig. 5.10

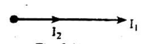


Fig. 5.11

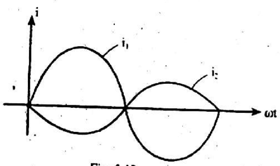


Fig. 5.12

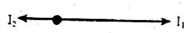


Fig. 5.13

Note:

$\sin n\pi = 0$
$\cos n\pi = (-1)^n$
$\cos 2n\pi = 1$

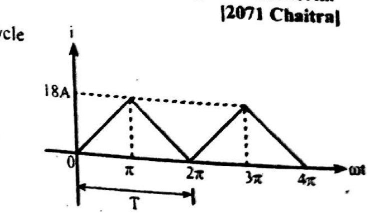
Steps to find out rms and average values

1. Find the time period of the waveform i.e. 'T'
2. Find the required equation of the waveform for a complete cycle.
3. Find the rms and average values using the equations.

Exam Solutions

1. **Calculate the rms value of current of the following triangular wave form.** **[2071 Chaitra]**

Solution:



The waveform completes one cycle from 0 to 2π.

Hence, time period of the waveform is 2π as denoted in fig.

$\therefore$ Time period (T) = 2π

Required equation

For $0 \le \omega t < \pi$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\text{or,} \quad y - 0 = \frac{18 - 0}{\pi - 0}(x - 0)$$

$$\text{or,} \quad y = \frac{18\,x}{\pi}$$

$$\therefore \quad i = \frac{18}{\pi}(\omega t) \qquad [0 \le \omega t < \pi]$$

For $\pi < \omega t < 2\pi$

$$\text{or,} \quad y - 18 = \frac{0 - 18}{2\pi - \pi}(x - \pi)$$

$$\text{or,} \quad y = 18 - \frac{18}{\pi}(x - \pi)$$

$$\text{or,} \quad y = 18 - \frac{18}{\pi}x + 18$$

$$\text{or,} \quad y = 36 - \frac{18}{\pi}x$$

$$\therefore \quad i = 36 - \frac{18}{\pi}(\omega t) \qquad [\pi < \omega t < 2\pi]$$

Rms value of current.

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\, d(\omega t)}$$

$$I^2_{rms} = \frac{1}{2\pi}\left[\int_0^\pi \left\{\frac{18}{\pi}(\omega t)\right\}^2 d(\omega t) + \int_\pi^{2\pi} \left\{36 - \frac{18}{\pi}(\omega t)\right\}^2 d(\omega t)\right]$$

$$= \frac{1}{2\pi}$$

$$\left[\int_0^\pi \frac{324}{\pi^2}\times(\omega t)^2 d(\omega t) + \int_\pi^{2\pi} \left\{(36)^2 - 2\times 36\times\frac{18}{\pi}(\omega t) + \frac{324}{\pi^2}(\omega t)^2 d\omega t\right\}\right]$$

$$= \frac{1}{2\pi}\left[\frac{324(\omega t)^3}{\pi^2\; 3}\Bigg|_0^\pi + 1296(\omega t)\Bigg|_\pi^{2\pi} - \frac{1296}{\pi}\frac{(\omega t)^2}{2}\Bigg|_\pi^{2\pi} + \frac{324}{\pi^2}\frac{(\omega t)^3}{3}\Bigg|_\pi^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[\frac{324}{\pi^2}\frac{\pi^3}{3} + 1296\pi - \frac{1296}{\pi}\frac{3\pi^2}{2} + \frac{324}{\pi^2}\frac{7\pi^3}{3}\right]$$

$$= \frac{1}{2\pi}\left[\frac{324\pi}{3} + 1296\pi - \frac{1296\times3}{2}\pi + \frac{324\times7\pi}{3}\right]$$

$$= \frac{1}{2\pi}\times216\pi$$

$$= 108$$

or, $I_{rms} = \sqrt{108}$

$= 10.39$ A

∴ The rms value of current of the above waveform is 10.39 A.

2. An alternating current of frequency 50 Hz has a maximum value of 120 A. Write down the equation for its instantaneous value. Find also the instantaneous value after 1/360 sec and the time taken to reach 96A for the first time. **[2071 Magh]**

Solution:

Given,

Frequency (f) = 50 Hz

Maximum value of current (I_m) = 120 A

Equation for its instantaneous value

$i = I_m \sin(\omega t)$

∴ $i = 120 \sin(2\pi f t)$

$= 120 \sin(2\pi \times 50t)$

$= 120 \sin(100\pi t)$

∴ $i = 120 \sin(100\pi t)$

∴ Instantaneous value after 1/360 sec;

$i = 120 \sin(100\pi t)$

$$= 120 \sin\left(100\times\pi\times\frac{1}{360}\right)$$

$$= 120 \sin\left(100\times\frac{1}{360}\times180°\right)$$

$= 120 \sin 50° = 91.92$ A

∴ The time taken to reach 96A for the first time,

$i = 120 \sin(100\pi t)$

or, $96 = 120 \sin(100\pi t)$

or, $\frac{96}{120} = \sin(100\pi t)$

or, $\sin^{-1}\left(\frac{96}{120}\right) = 100\pi t$

or, $0.927 = 100\pi t$

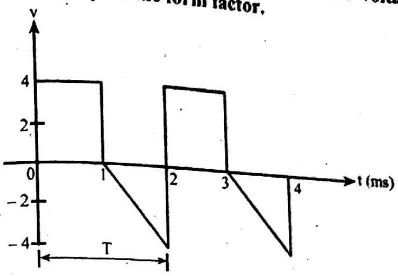
or, $t = 2.95 \times 10^{-3}$

∴ $t = 2.95$ milli second

3. Define cycle, time period, angular velocity, frequency, and average and rms value of an alternating quantity. **[2071 Shawan]**

[Please refer to the theory]

4. Calculate the Rms value and Average value of the voltage wave given below and hence compute the form factor. **[2071 Bhadra]**



Solution:

Time period (T) = 2 ms

Required equation:

For $0 < t < 1$

∴ $v = 4$

For $1 < t < 2$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

or, $y - 0 = \frac{-4-0}{2-1}(x-1)$

or, $y = -4x + 4$

or, $y = 4 - 4x$

∴ $v = 4 - 4t$

Rms value of voltage

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 dt}$$

$$V^2_{rms} = \frac{1}{2}\left[\int_0^1 4^2\, dt + \int_1^2 (4 - 40t)^2\, dt\right]$$

$$V^2_{rms} = \frac{1}{2}\left[16t\,\Big|_0^1 + \int_1^2 (16 - 32t + 16t^2)\, dt\right]$$

or, $$V^2_{rms} = \frac{1}{2}\left[16 + 16t\,\Big|_1^2 - 32\frac{t^2}{2}\Big|_1^2 + \frac{16t^3}{3}\Big|_1^2\right]$$

or, $$V^2_{rms} = \frac{1}{2}\left[16 + 16 - 48 + \frac{112}{3}\right]$$

or, $V^2_{rms} = \frac{1}{2}\times\frac{64}{3}$

or, $V^2_{rms} = \frac{32}{3}$

$$\therefore \quad V_{rms} = \sqrt{\frac{32}{3}} = 3.266 \text{ volt}$$

Average value of voltage,

$$V_{avg} = \frac{1}{T}\int_0^T v\,dt$$

$$= \frac{1}{2}\left[\int_0^1 4dt + \int_1^2 (4-4t)\,dt\right]$$

$$= \frac{1}{2}\left[4t\Big|_0^1 + 4t\Big|_1^2 - \frac{4t^2}{2}\Big|_1^2\right]$$

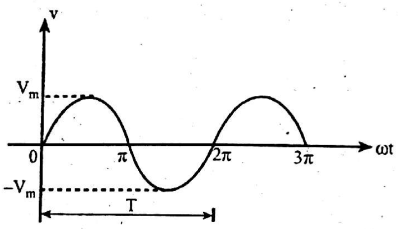
$$= \frac{1}{2}[4+4-6]$$

$$= \frac{1}{2}\times 2$$

$$= 1 \text{ volt}$$

$$\text{Form factor} = \frac{V_{rms}}{V_{avg}} = \frac{3.266}{1} = 3.266$$

5. **Find the rms and average values of the given figure.**



Solution:

The waveform completes one cycle from 0 to 2π

Hence, time period of the waveform is 2π as denoted in fig.

$\therefore$ Time period (T) = 2π

Required equations:

$$v = V_m \sin\omega t \qquad \text{for } [0 \le \omega t \le 2\pi]$$

$$\therefore V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 d\omega t} \qquad \text{for } [0 \le \omega t \le 2\pi]$$

or, $$V_{rms}^2 = \frac{1}{2\pi}\int_0^{2\pi} (V_m \sin \omega t)^2\, d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\omega t\, d\omega t$$

$$= \frac{1}{2\pi} V_m^2 \int_0^{2\pi} \frac{1-\cos 2\omega t}{2}\, d\omega t$$

$$= \frac{V_m^2}{2\pi}\times\frac{1}{2}\left[\omega t\Big|_0^{2\pi} - \frac{\sin 2\omega t}{2}\Big|_0^{2\pi}\right]$$

$$= \frac{V_m^2}{4\pi}\times\left[2\pi - 0 - \frac{\sin 4\pi - \sin 0}{2}\right]$$

$$= \frac{V_m^2}{4\pi}\times 2\pi = \frac{V_m^2}{2}$$

$$\boxed{\therefore V_{rms} = \frac{V_m}{\sqrt{2}}}$$

Similarly,

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}} \text{ for sinusoidal current waveform}$$

Average value in complete cycle,

$$V_{average} = \frac{1}{T}\int_0^T v\,d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t\, d\omega t$$

$$= \frac{1}{2\pi}\times V_m[-\cos\omega t]_0^{2\pi}$$

$$= \frac{V_m}{2\pi}[-\cos 2\pi + \cos 0]$$

$$= \frac{V_m}{2\pi}[-1+1] \qquad [\because \cos 2\pi = (-1)^2 = 1]$$

$$= 0$$

Thus, for symmetrical waveforms, average value in half cycle is calculated.

Average value in half cycle

$$V_{average} = \frac{1}{\pi}\int_0^{\pi} v\,d\omega t$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t\, d\omega t$$

$$= \frac{1}{\pi}\times V_m[-\cos\omega t]_0^{\pi}$$

$$= \frac{V_m}{\pi}[-\cos\pi + \cos 0] \qquad [\because \cos\pi = (-1)^1 = (-1)]$$

$$= \frac{V_m}{\pi}[1+1] = \frac{2V_m}{\pi}$$

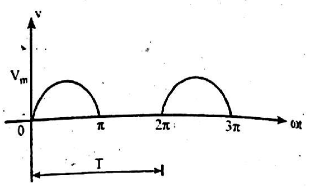
Scanned by CamScanner

$$\boxed{\therefore V_{average} = \frac{2V_m}{\pi}}$$

Similarly,

$$\boxed{I_{average} = \frac{2I_m}{\pi}}$$ for sinusoidal current waveform

6. **Calculate the (i) average value and (ii) rms value of voltage wave shown in fig.** **[2065 Kartik]**



$T = 2\pi$

$v = V_m \sin \omega t [0 \le \omega t \le \pi]$

$v = 0 \quad [\pi \le \omega t < 2\pi]$

(i) $V_{average} = \frac{1}{T}\int_0^T v d\omega t$

$$= \frac{1}{2\pi}\left[\int_0^\pi V_m \sin \omega t\, d\omega t + \int_\pi^{2\pi} 0\, d\omega t\right] = \frac{1}{2\pi} V_m [-\cos\omega t]_0^\pi$$

$$= \frac{V_m}{2\pi}[-\cos\pi + \cos 0] = \frac{V_m}{2\pi}[1+1] = \frac{V_m}{2\pi}\times 2$$

$$\boxed{\therefore V_{average} = \frac{V_m}{\pi}}$$

(ii) $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 d\omega t}$

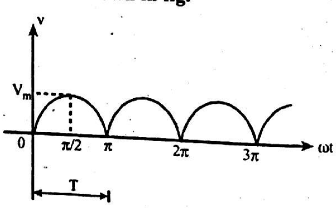
or, $V_{rms}^2 = \frac{1}{2\pi}\left[\int_0^\pi (V_m \sin \omega t)^2 d\omega t + \int_\pi^{2\pi} 0^2 d\omega t\right]$

or, $V_{rms}^2 = \frac{1}{2\pi} V_m^2 \int_0^\pi \frac{1-\cos 2\omega t}{2} d\omega t$

$$= \frac{1}{4\pi} V_m^2 \left[\omega t \Big|_0^\pi - \frac{\sin 2\omega t}{2}\Big|_0^\pi\right] = \frac{1}{4\pi} V_m^2 \times \pi = \frac{V_m^2}{4}$$

$$\boxed{\therefore V_{rms} = \frac{V_m}{2}}$$

7. **Find the (i) average value (ii) rms value and (iii) form factor of the full wave rectified sine wave shown in fig.** **[2064 Poush]**



Solution:

$\therefore T = \pi$

$v = V_m \sin \omega t \; [0 \le \omega t \le \pi]$

(i) $V_{average} = \frac{1}{T}\int_0^T v d\omega t$

$$= \frac{1}{\pi}\int_0^\pi V_m \sin \omega t\, d\omega t$$

$$= \frac{1}{\pi} V_m [-\cos\omega t]_0^\pi = \frac{V_m}{\pi}[-\cos\pi + \cos 0] = \frac{V_m}{\pi}[1+1]$$

$$\boxed{\therefore V_{average} = \frac{2V_m}{\pi}}$$

(ii) $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 d\omega t}$

or, $V_{rms}^2 = \frac{1}{\pi}\int_0^\pi V_m^2 \sin^2\omega t\, d\omega t$

$$= \frac{V_m^2}{\pi}\int_0^\pi \frac{1-\cos 2\omega t}{2} d\omega t$$

$$= \frac{V_m^2}{2\pi}\left[\omega t \Big|_0^\pi - \frac{\sin 2\omega t}{2}\Big|_0^\pi\right] = \frac{V_m^2}{2\pi}\times\pi = \frac{V_m^2}{2}$$

$$\boxed{\therefore V_{rms} = \frac{V_m}{\sqrt{2}}}$$

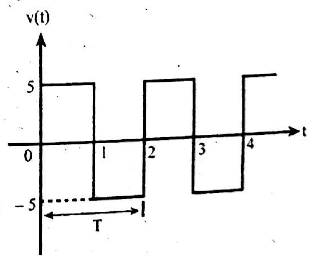
(iii) Form factor $= \frac{V_{rms}}{V_{average}} = \frac{\frac{V_m}{\sqrt{2}}}{\frac{2V_m}{\pi}}$

$$= \frac{V_m}{\sqrt{2}}\times\frac{\pi}{2V_m} = \frac{\pi}{2\sqrt{2}} = 1.11$$

8. **Calculate the average and rms values for the figure shown below:**

$v(t) = 5 \quad [0 \le t < 1]$

$v(t) = -5 \quad [1 < t < 2]$



Solution:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$$

or, $$V_{rms}^2 = \frac{1}{2}\left[\int_0^1 (5)^2 dt + \int_1^2 (-5)^2 dt\right]$$

$$= \frac{1}{2}\left[25t\Big|_0^1 + 25t\Big|_1^2\right]$$

$$= \frac{1}{2}[25 \times 1 + 25 \times 1] = \frac{50}{2} = 25$$

$$\therefore V_{rms} = \sqrt{25} = 5V$$

Average value in complete cycle

$$V_{average} = \frac{1}{T}\int_0^T v\,dt = \frac{1}{2}\left[\int_0^1 5dt + \int_1^2 (-5)dt\right]$$

$$= \frac{1}{2}\left[5t\Big|_0^1 + (-5)t\Big|_1^2\right] = \frac{1}{2}[5 - 5] = 0$$

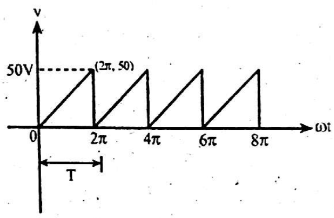
Since, symmetrical waveform's average value in complete cycle is zero.

Average value in half cycle

$$V_{average} = \frac{1}{2}\int_0^1 5dt$$

$$= \frac{5}{2}t\Big|_0^1 = \frac{5}{2}(1 - 0) = 2.5V$$

9. Calculate the average, rms value, form factor and peak factor of the saw tooth waveform as shown in fig. **[2068 Chaitra]**



Solution:

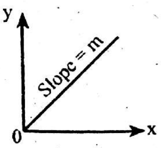
We know

The equation of line is y = mx

i.e. $v = m(\omega t)$

In fig. Slope (m) $= \frac{50}{2\pi}$

$$\therefore v = \frac{50}{2\pi}(\omega t) \quad [0 \le \omega t \le 2\pi]$$



OR Using two point formula;

$x_1 = 0 \qquad y_1 = 0$

$x_2 = 2\pi \qquad y_2 = 50$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$(y - 0) = \frac{50 - 0}{2\pi - 0}(x - 0)$$

$$y = \frac{50}{2\pi}x$$

$$\therefore \quad v = \frac{50}{2\pi}(\omega t) \quad [0 \le \omega t \le 2\pi]$$

Now, $$V_{average} = \frac{1}{T}\int_0^T v\,d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\frac{50}{2\pi}(\omega t)\,d\omega t = \frac{1}{2\pi}\times\frac{50}{2\pi}\frac{(\omega t)^2}{2}\Big|_0^{2\pi}$$

$$= \frac{50}{(2\pi)^2}\times\frac{(2\pi)^2}{2} = 25\text{ V}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 d\omega t}$$

$$V^2_{rms} = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{50}{2\pi}\omega t\right)^2 d\omega t$$

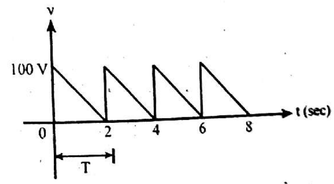
$$= \frac{1}{2\pi}\times\frac{(50)^2}{(2\pi)^2}\frac{(\omega t)^3}{3}\Big|_0^{2\pi} = \frac{(50)^2}{(2\pi)^3}\times\frac{(2\pi)^3}{3} = \frac{(50)^2}{3}$$

$$\therefore V_{rms} = \frac{50}{\sqrt{3}} = 28.8675\text{ V}$$

$$\text{Form factor} = \frac{V_{rms}}{V_{average}} = \frac{28.8675}{25} = 1.15$$

$$\text{Peak factor} = \frac{V_{peak}}{V_{rms}} = \frac{V_m}{V_{rms}} = \frac{50}{28.8675} = 1.73$$

10. Find the average value, rms value, form factor and peak factor of the voltage waveform given below. **[2070 Asad]**



Solution:

$\therefore$ T = 2 sec.

Using two point formula,

$(0,100) \Rightarrow x_1 = 0 \qquad y_1 = 100$

$(2, 0) \Rightarrow x_2 = 2 \qquad y_2 = 0$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 100 = \frac{0 - 100}{2 - 0}(x - 0)$$

$$y = 100 - \frac{100}{2}x$$

$$\therefore \quad v = 100 - 50t \qquad [0 \le t \le 2]$$

$$V_{average} = \frac{1}{T}\int_0^T v\,dt$$

$$= \frac{1}{2}\int_0^2 (100 - 50t)dt = \frac{1}{2}\left[100\,t\Big|_0^2 - 50\frac{t^2}{2}\Big|_0^2\right]$$

$$= \frac{1}{2}\left[100 \times 2 - 50 \times \frac{4}{2}\right] = \frac{1}{2}[200 - 100]$$

$$\therefore V_{average} = 50\text{ V}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 d\omega t}$$

$$\text{or, } V_{rms}^2 = \frac{1}{2}\int_0^2 (100 - 50t)^2 dt$$

$$= \frac{1}{2}\int_0^2 [(100)^2 - 2 \times 100 \times 50t + (50t)^2]dt$$

$$= \frac{1}{2}\left[(100)^2 t\Big|_0^2 - 10000\frac{t^2}{2}\Big|_0^2 + (50)^2\frac{t^3}{3}\Big|_0^2\right]$$

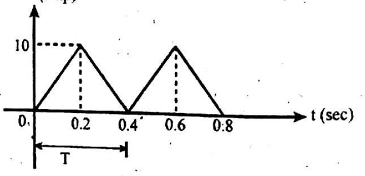
$$= \frac{1}{2}\left[(100)^2 \times 2 - (100)^2 \times \frac{4}{2} + (50)^2 \times \frac{8}{3}\right] = \frac{1}{2}(50)^2 \times \frac{8}{3} = (50)^2 \times \frac{4}{3}$$

$$\therefore V_{rms} = 50 \times \frac{2}{\sqrt{3}} = 57.735$$

$$\text{Form factor} = \frac{V_{rms}}{V_{average}} = \frac{57.735}{50} = 1.154$$

$$\text{Peak factor} = \frac{V_{peak}}{V_{rms}} = \frac{100}{57.735} = 1.732$$

11. Find the average and rms values of the traingular waveform of current shown in fig below. Also calculate the form factor and peak amplitude factor for the triangular wave form **[2063 Kartik]**



Solution:

For $0 \le t < 0.2$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\text{or,} \quad y - 0 = \frac{10 - 0}{0.2 - 0}(x - 0)$$

$$\text{or,} \quad y = \frac{10}{0.2}x$$

$$i = 50\,t \qquad [0 \le t < 0.2]$$

For $0.2 < t < 0.4$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\text{or,} \quad y - 10 = \frac{0 - 10}{0.4 - 0.2}(x - 0.2)$$

$$\text{or,} \quad y = 10 - \frac{10}{0.2}(x - 0.2)$$

$$\text{or,} \quad y = 10 - 50x + 10$$

$$\text{or,} \quad y = 20 - 50x$$

$$\therefore \quad i = 20 - 50t \quad [0.2 < t < 0.4]$$

$$I_{average} = \frac{1}{0.4}\left[\int_0^{0.2} 50\,t\,dt + \int_{0.2}^{0.4}(20 - 50t)\,dt\right]$$

$$= \frac{1}{0.4}\left[50\frac{t^2}{2}\Big|_0^{0.2} + 20t\Big|_{0.2}^{0.4} - 50\frac{t^2}{2}\Big|_{0.2}^{0.4}\right]$$

$$= \frac{1}{0.4}\left[50 \times \frac{(0.2)^2}{2} + 20 \times (0.2) - 50 \times \frac{(0.4)^2}{2} + 50 \times \frac{(0.2)^2}{2}\right] = \frac{1}{0.4} \times 2 = 5A$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

$$I_{rms}^2 = \frac{1}{0.4}\left[\int_0^{0.2}(50t)^2 dt + \int_{0.2}^{0.4}(20 - 50t)^2 dt\right]$$

$$= \frac{1}{0.4}\left[(50)^2\frac{t^3}{3}\Big|_0^{0.2} + \frac{(20 - 50t)^{2+1}}{(2+1)(-50)}\Big|_{0.2}^{0.4}\right]$$

$$\left[\because \int(ax + b)^n dx = \frac{(ax + b)^{n+1}}{(n+1)a}\right]$$

OR Expanding the square term & intergrating individually

$$= \frac{1}{0.4}\left[(50)^2 \times \frac{(0.2)^3}{3} + \frac{(20 - 50 \times 0.4)^3}{3(-50)} - \frac{(20 - 50 \times 0.2)^3}{3(-50)}\right]$$

$$= \frac{1}{0.4}\left[(50)^2 \times \frac{(0.2)^3}{3} - \frac{(10)^3}{3(-50)}\right]$$

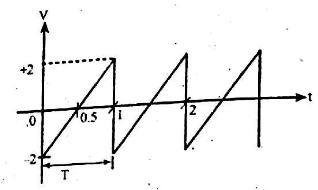
$$\therefore I_{rms} = \sqrt{\frac{100}{3}} = 5.7735A$$

Form factor $= \frac{I_{rms}}{I_{average}} = \frac{5.7735}{5} = 1.154$

Peak amplitude factor $= \frac{I_{peak}}{I_{rms}} = \frac{10}{5.7735} = 1.732$

12. Find the rms and average values of the waveform given in figure below.

[2070 Magh]



Solution:

Using two-point formula for equation throughout 0 to 1

i.e. (0, – 2) and (1, 2)

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

or, $y - (-2) = \frac{2-(-2)}{1-0}(x-0)$

or, $y + 2 = \frac{2+2}{1}(x-0)$

or, $y = 4x - 2$

$\therefore$ $v = 4t - 2 \quad [0 \le t < 1]$

$$V_{rms} = \sqrt{\frac{1}{1}\int_0^1 (4t-2)^2 dt}$$

or, $V_{rms}^2 = \int_0^1 [(4t)^2 - 2 \times 4t \times 2 + (2)^2] dt$

$$= (4)^2 \frac{t^3}{3}\Big|_0^1 - 16\frac{t^2}{2}\Big|_0^1 + 4t\Big|_0^1 = \frac{16}{3} - \frac{16}{2} + 4 = \frac{4}{3}$$

$\therefore$ $V_{rms} = 1.154$ V

Average value in complete cycle

$$V_{average} = \frac{1}{1}\int_0^1 (4t-2)\,dt = 4\frac{t^2}{2}\Big|_0^1 - 2t\Big|_0^1 = \frac{4}{2} - 2 = 0$$

Average value in half cycle

$$V_{average} = \frac{1}{0.5}\int_0^{0.5} (4t-2)\,dt$$

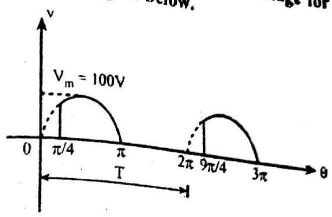
$$= \frac{1}{0.5}\left[4\frac{t^2}{2}\Big|_0^{0.5} - 2t\Big|_0^{0.5}\right] = \frac{1}{0.5}\left[4\frac{(0.5)^2}{2} - 2\,(0.5)\right]$$

$\therefore$ $V_{average} = -1$V

i.e. – ve sign indicate the Average value for lower half cycle, hence lies below the reference line.

13. Determine the average and rms values of voltage for sinusoidal voltage waveform as shown in figure below.

[2067 Mangsir]



Solution:

$v = 0 \quad [0 \le \theta < \frac{\pi}{4}]$

$v = 100 \sin\theta \quad \left[\frac{\pi}{4} < \theta < \pi\right]$

$v = 0 \quad [\pi < \theta < 2\pi]$

$$V_{average} = \frac{1}{2\pi}\int_{\pi/4}^{\pi} 100 \sin\theta\, d\theta$$

$$= \frac{1}{2\pi} 100\,[-\cos\theta]_{\pi/4}^{\pi}$$

$$= \frac{1}{2\pi} 100\left[-\cos\pi + \cos\frac{\pi}{4}\right]$$

$$= \frac{1}{2\pi} 100\left[1 + \frac{1}{\sqrt{2}}\right] = \frac{1}{2\pi} \times 100\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right) = 27.169 \text{ V}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_{\pi/4}^{\pi} (100 \sin\theta)^2 d\theta}$$

or, $V_{rms}^2 = \frac{1}{2\pi}(100)^2 \int_{\pi/4}^{\pi} \frac{1-\cos 2\theta}{2}\, d\theta$

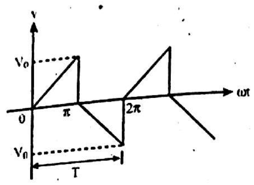
$$= \frac{1}{2\pi}(100)^2 \times \frac{1}{2}\left[\theta\Big|_{\pi/4}^{\pi} - \frac{\sin 2\theta}{2}\Big|_{\pi/4}^{\pi}\right]$$

$$= \frac{1}{4\pi}(100)^2\left[\left(\pi - \frac{\pi}{4}\right) - \frac{\sin 2\pi - \sin 2\frac{\pi}{4}}{2}\right]$$

$$= \frac{1}{4\pi}(100)^2\left[\pi - \frac{\pi}{4} + \frac{\sin\frac{\pi}{2}}{2}\right] = \frac{1}{4\pi}(100)^2\left[\frac{3\pi}{4} + \frac{1}{2}\right] = 2272.88$$

$\therefore$ $V_{rms} = 47.67$ V

14. Determine the average and rms values of voltage for voltage waveform as shown in figure below.



For $0 \le \omega t < \pi$

$(0, 0)$ & (π, V_0)

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

or, $$y - 0 = \frac{V_0 - 0}{\pi - 0}(x - 0)$$

or, $$y = \frac{V_0}{\pi}x$$

$\therefore$ $$v = \frac{V_0}{\pi}(\omega t) \quad [0 \le \omega t < \pi]$$

For $\pi < \omega t < 2\pi$

$(\pi, 0)$ & $(2\pi, -V_0)$

$$y - 0 = \frac{-V_0 - 0}{2\pi - \pi}(x - \pi)$$

or, $$y = \frac{-V_0}{\pi}(x - \pi)$$

or, $$y = V_0 - \frac{V_0}{\pi}x$$

$\therefore$ $$v = V_0 - \frac{V_0}{\pi}(\omega t) \quad [\pi < \omega t < 2\pi]$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\, d\omega t}$$

or, $$V_{rms}^2 = \frac{1}{2\pi}\left[\int_0^{\pi}\left(\frac{V_0}{\pi}\omega t\right)^2 d\omega t + \int_{\pi}^{2\pi}\left(V_0 - \frac{V_0}{\pi}\omega t\right)^2 d\omega t\right]$$

$$= \frac{1}{2\pi}\left[\left(\frac{V_0}{\pi}\right)^2 \times \frac{(\omega t)^3}{3}\Big|_0^{\pi} + \int_{\pi}^{2\pi}\left\{V_0^2 - 2 \times V_0\frac{V_0}{\pi}(\omega t) + \left(\frac{V_0}{\pi}\right)^2(\omega t)^2\right\}d\omega t\right]$$

$$= \frac{1}{2\pi}\left[\left(\frac{V_0}{\pi}\right)^2 \times \frac{\pi^3}{3} + V_0^2(\omega t)\Big|_{\pi}^{2\pi} - \frac{2V_0^2}{\pi}\frac{(\omega t)^2}{2}\Big|_{\pi}^{2\pi} + \left(\frac{V_0}{\pi}\right)^2\frac{(\omega t)^3}{3}\Big|_{\pi}^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[\left(\frac{V_0}{\pi}\right)^2\frac{\pi^3}{3} + V_0^2\pi - \frac{2V_0^2}{\pi}\frac{(2\pi)^2}{2} + \frac{2V_0^2}{\pi}\frac{\pi^2}{2} + \left(\frac{V_0}{\pi}\right)^2\frac{(2\pi)^3}{3} - \left(\frac{V_0}{\pi}\right)^2\frac{(\pi)^3}{3}\right]$$

$$= \frac{1}{2\pi}\left[\frac{V_0^2\pi}{3} + V_0^2\pi - 4V_0^2\pi + V_0^2\pi + \frac{8V_0^2}{3}\pi - \frac{V_0^2\pi}{3}\right] = \frac{1}{2\pi} \times \frac{2}{3}V_0^2\pi = \frac{V_0^2}{3}$$

$$\boxed{\therefore V_{rms} = \frac{V_0}{\sqrt{3}}}$$

Average value in complete cycle

$$V_{average} = \frac{1}{T}\int_0^T v\, d\omega t$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi}\frac{V_0}{\pi}(\omega t)d\omega t + \int_{\pi}^{2\pi}\left(V_0 - \frac{V_0}{\pi}(\omega t)\right)d\omega t\right]$$

$$= \frac{1}{2\pi}\left[\frac{V_0}{\pi}\frac{(\omega t)^2}{2}\Big|_0^{\pi} + V_0(\omega t)\Big|_{\pi}^{2\pi} - \frac{V_0}{\pi}\frac{(\omega t)^2}{2}\Big|_{\pi}^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[\frac{V_0}{\pi}\frac{\pi^2}{2} + V_0\pi - \frac{V_0}{\pi}\frac{(2\pi)^2}{2} + \frac{V_0}{\pi}\frac{(\pi)^2}{2}\right]$$

$$= \frac{1}{2\pi}\left[\frac{V_0\pi}{2} + V_0\pi - 2V_0\pi + \frac{1}{2}V_0\pi\right] = 0$$

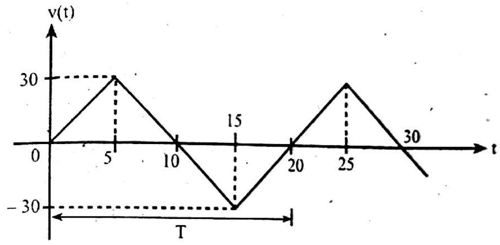
Average value in half cycle

$$V_{average} = \frac{1}{\pi}\int_0^{\pi}\frac{V_0}{\pi}(\omega t)d\omega t = \frac{1}{\pi} \times \frac{V_0}{\pi}\frac{(\omega t)^2}{2}\Big|_0^{\pi} = \frac{1}{\pi} \times \frac{V_0}{\pi} \times \frac{\pi^2}{2}$$

$$\therefore V_{average} = \frac{V_0}{2}$$

Additional Questions

1. **Determine the average and rms values of voltage for current waveform as shown in figure below.**



Solution:

Since the waveform is symmetrical, average value in a complete cycle is zero.

Therefore, average and rms values is calculated in half - cycle.

For $0 \le t < 5$

$$y = mx$$

or, $$v = \frac{30}{5}t$$

$\therefore$ $$v = 6t \quad [0 \le t < 5]$$

For $5 < t < 10$

$(5, 30)$ & $(10, 0)$

$$y - 30 = \frac{0 - 30}{10 - 5}(x - 5)$$

or, $$y = 30 - 6x + 30$$

$\therefore$ $$v = 60 - 6t \quad [5 < t < 10]$$

$$V_{rms} = \sqrt{\frac{1}{T/2}\int_0^{T/2} v^2\, dt}$$

$$\text{or, } V_{rms}^2 = \frac{1}{10}\left[\int_0^5 (6t)^2\, dt + \int_5^{10} (60-6t)^2 dt\right]$$

$$= \frac{1}{10}\left[36\frac{t^3}{3}\Big|_0^5 + \int_5^{10} [(60)^2 - 2\times 60\times 6\,t + (6t)^2]dt\right]$$

$$= \frac{1}{10}\left[36\times\frac{5^3}{3} + (60)^2 t\Big|_5^{10} - 720\frac{t^2}{2}\Big|_5^{10} + (6)^2\frac{t^3}{3}\Big|_5^{10}\right]$$

$$= \frac{1}{10}\left[36\times\frac{125}{3} + (60)^2(10-5) - 720\frac{(10)^2}{2} + 720\frac{(5)^2}{2} + (6)^2\frac{(10)^3}{3} - (6)^2\frac{(5)^3}{3}\right]$$

$$= \frac{1}{10}[1500 + 1800 - 36000 + 9000 + 12000 - 1500] = 300$$

$$\therefore \quad V_{rms} = 17.32\text{ V}$$

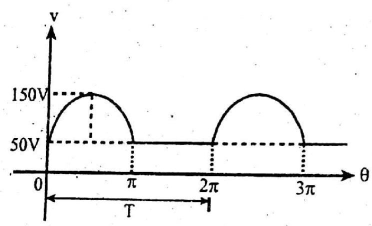
$$V_{average} = \frac{1}{T/2}\int_0^{T/2} v\,dt$$

$$= \frac{1}{10}\left[\int_0^5 6\,t\,dt + \int_5^{10}(60-6t)dt\right]$$

$$= \frac{1}{10}\left[6\frac{t^2}{2}\Big|_0^5 + 60\,t\Big|_5^{10} - 6\frac{t^2}{2}\Big|_5^{10}\right]$$

$$= \frac{1}{10}\left[6\times\frac{(5)^2}{2} + 60\times 5 - 6\frac{(10)^2}{2} + 6\frac{(5)^2}{2}\right] = 15\text{ V}$$

2. **A transmission line carries a dc voltage of 50V and half-wave rectified sinusodal voltage as shown in figure. Calculate (i) average value (ii) rms value.**



$v = 50 + 100\sin\theta$ for $[0 \le \theta < \pi]$

[Note: $V_m \sin\theta \Rightarrow V_m = 150 - 50 = 100$]

$v = 50$ for $[\pi < \theta < 2\pi]$

$$V_{average} = \frac{1}{2\pi}\left[\int_0^\pi (50 + 100\sin\theta)d\theta + \int_\pi^{2\pi} 50\, d\theta\right]$$

$$= \frac{1}{2\pi}\left[50\theta\Big|_0^\pi + 100(-\cos\theta)\Big|_0^\pi + 50\theta\Big|_\pi^{2\pi}\right]$$

$$= \frac{1}{2\pi}[50\pi + 100(-\cos\pi + \cos 0) + 50\pi]$$

$$= \frac{1}{2\pi}[100\pi + 100(1+1)] = 81.83\text{V}$$

$$V_{rms}^2 = \frac{1}{2\pi}\left[\int_0^\pi (50 + 100\sin\theta)^2 d\theta + \int_\pi^{2\pi}(50)^2\, d\theta\right]$$

$$= \frac{1}{2\pi}\left[\int_0^\pi \{(50)^2 + 10000\sin\theta + (100)^2\sin^2\theta\}d\theta + (50)^2\theta\Big|_\pi^{2\pi}\right]$$

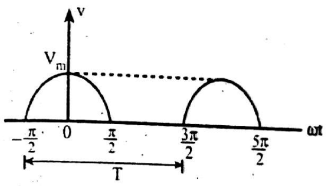
$$= \frac{1}{2\pi}\left[(50)^2\theta\Big|_0^\pi + 10000(-\cos\theta)\Big|_0^\pi + (100)^2\int_0^\pi \frac{1-\cos 2\theta}{2}d\theta + (50)^2\pi\right]$$

$$= \frac{1}{2\pi}\left[(50)^2\pi + 10000\times 2 + (100)^2\left\{\frac{1}{2}\theta\Big|_0^\pi - \frac{1}{2}\frac{\sin 2\theta}{2}\Big|_0^\pi\right\} + (50)^2\pi\right]$$

$$= \frac{1}{2\pi}\left[(50)^2 2\pi + 10000\times 2 + \frac{(100)^2}{2}\pi\right]$$

$$\therefore V_{rms} = 90.46\text{ V}$$

3. **Determine the average and rms values of voltage for sinusoidal voltage waveform as shown in figure below.**



Solution:

$v = V_m\cos\omega t \quad \left[-\frac{\pi}{2} < \omega t < \frac{\pi}{2}\right]$

$v = 0 \quad \left[\frac{\pi}{2} < \omega t < \frac{3\pi}{2}\right]$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 d\omega t}$$

$$V_{rms}^2 = \frac{1}{\left[\frac{3\pi}{2} - \left(-\frac{\pi}{2}\right)\right]}\left[\int_{-\pi/2}^{\pi/2}(V_m\cos\omega t)^2 d\omega t\right]$$

$$= \frac{1}{2\pi}V_m^2\int_{-\pi/2}^{\pi/2}\frac{1+\cos 2\,\omega t}{2}d\omega t$$

$$= \frac{1}{2\pi}V_m^2\frac{1}{2}\left[\omega t\Big|_{-\pi/2}^{\pi/2} + \frac{\sin 2\omega t}{2}\Big|_{-\pi/2}^{\pi/2}\right]$$

$$= \frac{1}{2\pi}V_m^2\frac{1}{2}\left[\frac{\pi}{2} + \frac{\pi}{2} + \frac{\sin\pi - \sin(-\pi)}{2}\right] = \frac{V_m^2}{4\pi}[\pi] = \frac{V_m^2}{4}$$

$$\boxed{\therefore V_{rms} = \frac{V_m}{2}}$$

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$$V_{average} = \frac{1}{T}\int_0^T v\,d\omega t$$

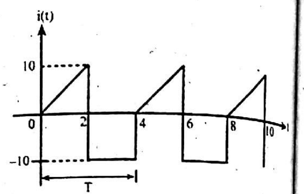
$$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} V_m \cos\omega t\, d\omega t$$

$$= \frac{1}{2\pi} V_m \sin\omega t \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2\pi} V_m\left[\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right] = \frac{V_m}{2\pi}[1-(-1)]$$

$$\boxed{\therefore V_{average} = \frac{V_m}{\pi}}$$

4. **Determine the average and rms values of current for current waveform as shown in figure below.**



Solution:

$$i(t) = \begin{cases} 5t & ; \ 0<t<2 \\ -10 & ; \ 2<t<4 \end{cases}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$$

or, $$I_{rms}^2 = \frac{1}{4}\left[\int_0^2 (5t)^2 dt + \int_2^4 (-10)^2 dt\right]$$

$$= \frac{1}{4}\left[25\frac{t^3}{3}\Big|_0^2 + 100\,t\Big|_2^4\right] = \frac{1}{4}\left[25\frac{(2)^3}{3} + 100\,(4-2)\right]$$

$$= \frac{1}{4}\left[25 \times \frac{8}{3} + 100 \times 2\right] = \frac{200}{3}$$

$\therefore I_{rms} = 8.165$ A

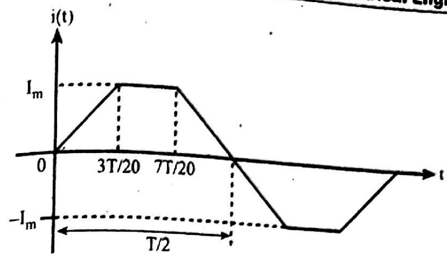
Now, $$I_{average} = \frac{1}{4}\left[\int_0^2 5t\,dt + \int_2^4 (-10)\,dt\right]$$

$$= \frac{1}{4}\left[5\frac{t^2}{2}\Big|_0^2 + (-10)\,t\Big|_2^4\right] = \frac{1}{4}\left[5\frac{(2)^2}{2} + (-10)\,(4-2)\right]$$

$$= \frac{1}{4}[10-20] = -\frac{5}{2} = -2.5 \text{ A}$$

– ve sign indicates the average value lies below the reference line.

5. **Determine the average and rms values of current for current waveform as shown in figure below.**



For $0 < t < \frac{3T}{20}$

$y = mx$

or, $i = \frac{I_m}{\frac{3T}{20}}\,t$

$\therefore$ $i = \frac{20}{3T} I_m t$

For $\frac{3T}{20} < t < \frac{7T}{20}$

$y = I_m$

$\therefore i = I_m$

For $\frac{7T}{20} < t < \frac{T}{2}$

$\left(\frac{7T}{20}, I_m\right)$ & $\left(\frac{T}{2}, 0\right)$

$$y - I_m = \frac{0 - I_m}{\frac{T}{2} - \frac{7T}{20}}\left(x - \frac{7T}{20}\right)$$

or, $$y = I_m - \frac{Im}{\frac{3T}{20}}\left(x - \frac{7T}{20}\right)$$

or, $$y = I_m - \frac{20\,I_m}{3T}\,x + \frac{7}{3} I_m$$

or, $$y = \frac{10}{3} I_m - \frac{20}{3T} I_m\,x$$

$\therefore$ $$i = \frac{10}{3} I_m - \frac{20}{3T} I_m\,t$$

$$i(t) = \begin{cases} \frac{20}{3T} I_m\,t & ; \quad 0<t<\frac{3T}{20} \\ I_m & ; \quad \frac{3T}{20} < t < \frac{7T}{20} \\ \frac{10}{3} I_m - \frac{20}{3T} I_m\,t & ; \quad \frac{7T}{20} < t < \frac{T}{2} \end{cases}$$

$$I_{rms}^2 = \frac{1}{\frac{T}{2}}\left\{\int_0^{3T/20}\left(\frac{20}{3T}I_m t\right)^2 dt + \int_{3T/20}^{7T/20}(I_m)^2\, dt + \int_{7T/20}^{T/2}\left(\frac{10}{3}I_m - \frac{20}{3T}I_m t\right)^2 dt\right\}$$

$$= \frac{2}{T}\left[\left(\frac{20}{3T}\right)^2 I_m^2 \frac{t^3}{3}\Big|_0^{3T/20} + I_m^2 t\Big|_{3T/20}^{7T/20} + \int_{7T/20}^{T/2}\left\{\left(\frac{10}{3}I_m\right)^2 - \frac{400}{9T}I_m^2 t + \left(\frac{20}{3T}I_m\right)^2 t^2\right\}dt\right]$$

$$= \frac{2}{T}\left[\left(\frac{20}{3T}\right)^2 I_m^2\left(\frac{3T}{20}\right)^3\frac{1}{3} + I_m^2\frac{4T}{20} + \left(\frac{10}{3}\right)^2 I_m^2\, t\Big|_{7T/20}^{T/2} - \frac{400}{9T}I_m^2\frac{t^2}{2}\Big|_{7T/20}^{T/2} + \left(\frac{20}{3T}\right)^2 I_m^2\frac{t^3}{3}\Big|_{7T/20}^{T/2}\right]$$

$$= \frac{2}{T}\left[\frac{3T}{20}I_m^2\frac{1}{3} + I_m^2\frac{T}{5} + \left(\frac{10}{3}\right)^2 I_m^2\frac{3T}{20} - \frac{400}{9T}I_m^2\frac{51T^2}{800} + \left(\frac{20}{3T}\right)^2 I_m^2\frac{657T^3}{24000}\right]$$

$$= \frac{2}{T}\left[I_m^2\frac{T}{20} + I_m^2\frac{T}{5} + I_m^2\frac{5T}{3} - \frac{17}{6}I_m^2 T + \frac{73}{60}I_m^2 T\right]$$

$$= \frac{2 I_m^2 T}{T}\left[\frac{1}{20}+\frac{1}{5}+\frac{5}{3}-\frac{17}{6}+\frac{73}{60}\right] = 2I_m^2\times\frac{3}{10}$$

$$\therefore I_{rms} = \sqrt{\frac{3}{5}I_m^2}$$

$$= 0.7745\, I_m$$

$$I_{average} = \frac{1}{\frac{T}{2}}\left[\int_0^{3T/20}\frac{20}{3T}I_m t\, dt + \int_{3T/20}^{7T/20} I_m\, dt + \int_{7T/20}^{T/2}\left(\frac{10}{3}I_m - \frac{20}{3T}I_m t\right)dt\right]$$

$$= \frac{2}{T}\left[\frac{20}{3T}I_m\frac{t^2}{2}\Big|_0^{3T/20} + I_m t\Big|_{3T/20}^{7T/20} + \frac{10}{3}I_m\, t\Big|_{7T/20}^{T/2} - \frac{20}{3T}I_m\frac{t^2}{2}\Big|_{7T/20}^{T/2}\right]$$

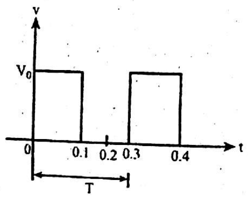
$$= \frac{2}{T}\left[\frac{20}{3T}I_m\frac{1}{2}\left(\frac{3T}{20}\right)^2 + I_m\frac{4T}{20} + \frac{10}{3}I_m\frac{3T}{20} - \frac{20}{3T}I_m\frac{51T^2}{800}\right]$$

$$= \frac{2}{T}\left[\frac{3T}{40}I_m + \frac{T}{5}I_m + \frac{T}{2}I_m - \frac{17}{40}T\, I_m\right]$$

$$= \frac{2}{T}\times T I_m\left[\frac{3}{40}+\frac{1}{5}+\frac{1}{20}-\frac{17}{40}\right]$$

$$= 2I_m\times\frac{7}{20} = \frac{7}{10}I_m$$

6. **Calculate the average and rms values and form factor and peak factor of the following figure.**



$$v = \begin{cases} V_0 \;; & 0 < t < 0.1 \\ 0 \;; & 0.1 < t < 0.3 \end{cases}$$

$$V_{rms}^2 = \frac{1}{0.3}\int_0^{0.1} V_0^2\, dt = \frac{1}{0.3}V_0^2\, t\Big|_0^{0.1} = \frac{1}{0.3}V_0^2\times 0.1 = \frac{V_0^2}{3}$$

$$\therefore V_{rms} = \frac{V_0}{\sqrt{3}}$$

$$V_{average} = \frac{1}{0.3}\int_0^{0.1} V_0\, dt$$

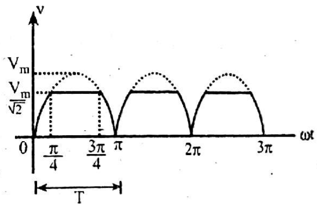
$$= \frac{1}{0.3}V_0\, t\Big|_0^{0.1} = \frac{1}{0.3}V_0\times 0.1$$

$$\therefore V_{average} = \frac{V_0}{3}$$

$$\therefore \text{Form factor} = \frac{V_{rms}}{V_{average}} = \frac{\frac{V_0}{\sqrt{3}}}{\frac{V_0}{3}} = 1.73$$

$$\therefore \text{Peak factor} = \frac{V_{peak}}{V_{rms}} = \frac{V_0}{\frac{V_0}{\sqrt{3}}} = 1.73$$

7. **A full wave - rectified sinusoidal voltage is clipped at $\frac{1}{\sqrt{2}}$ of its max value. Find its rms and average value.**



$$v = \begin{cases} V_m \sin\omega t \;; & 0 < \omega t < \frac{\pi}{4} \\ \frac{V_m}{\sqrt{2}} \;; & \frac{\pi}{4} < \omega t < \frac{3\pi}{4} \\ V_m \sin\omega t \;; & \frac{3\pi}{4} < \omega t < \pi \end{cases}$$

$$V_{average} = \frac{1}{T}\int_0^T v\, d\omega t$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/4} V_m \sin\omega t\, d\omega t + \int_{\pi/4}^{3\pi/4}\frac{V_m}{\sqrt{2}}\, d\omega t + \int_{3\pi/4}^{\pi} V_m \sin\omega t\, d\omega t\right]$$

$$= \frac{1}{\pi}\left[V_m[-\cos\omega t]_0^{\pi/4} + \frac{V_m}{\sqrt{2}}\,\omega t\Big|_{\pi/4}^{3\pi/4} + V_m[-\cos\omega t]_{3\pi/4}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[V_m\left[-\cos\frac{\pi}{4} + \cos 0\right] + \frac{V_m}{\sqrt{2}}\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) + V_m\left[-\cos\pi + \cos\frac{3\pi}{4}\right]\right]$$

$$= \frac{1}{\pi}\left[V_m\left[-\frac{1}{\sqrt{2}}+1\right]+\frac{V_m}{\sqrt{2}}\frac{\pi}{2}+V_m\left(1+\left(\frac{-1}{\sqrt{2}}\right)\right)\right]$$

$$= \frac{1}{\pi}\left[V_m\left(1-\frac{1}{\sqrt{2}}\right)+\frac{V_m\pi}{2\sqrt{2}}+V_m\left(1-\frac{1}{\sqrt{2}}\right)\right] = 0.54\ V_m$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\, d\omega t}$$

$$\text{or, } V_{rms}^2 = \frac{1}{\pi}\left[\int_0^{\pi/4} V_m^2 \sin^2\omega t\, d\omega t + \int_{\pi/4}^{3\pi/4}\left(\frac{V_m}{\sqrt{2}}\right)^2 d\omega t + \int_{3\pi/4}^{\pi} V_m^2 \sin^2\omega t\, d\omega t\right]$$

$$= \frac{1}{\pi}V_m^2\left[\int_0^{\pi/4}\frac{1-\cos 2\omega t}{2}d\omega t + \int_{\pi/4}^{3\pi/4}\frac{1}{2}d\omega t + \int_{3\pi/4}^{\pi}\frac{1-\cos 2\omega t}{2}d\omega t\right]$$

$$= \frac{1}{\pi}\frac{V_m^2}{2}\left[\omega t\Big|_0^{\pi/4} - \frac{\sin 2\omega t}{2}\Big|_0^{\pi/4} + \omega t\Big|_{\pi/4}^{3\pi/4} + \omega t\Big|_{3\pi/4}^{\pi} - \frac{\sin 2\omega t}{2}\Big|_{3\pi/4}^{\pi}\right]$$

$$= \frac{1}{\pi}\frac{V_m^2}{2}\left[\frac{\pi}{4} - \frac{\sin\frac{\pi}{2} - \sin 0}{2} + \frac{\pi}{2} + \frac{\pi}{4} - \frac{\sin 2\pi - \sin\frac{3\pi}{2}}{2}\right]$$

$$= \frac{1}{\pi}\frac{V_m^2}{2}\left[\pi - \frac{1}{2} - \frac{1}{2}\right] = \frac{1}{\pi}\times\frac{V_m^2}{2}[\pi - 1]$$

$$= 0.3408\ V_m^2$$

$\therefore V_{rms} = 0.584\ V_m$

Adding two alternating quantities

Exam Solutions

1. Two currents i_1 and i_2 are given as $i_1 = 10 \sin\left(314t + \frac{\pi}{14}\right)$ A and $i_2 = 8\sin\left(314t - \frac{\pi}{3}\right)$ A. Find (i) $i_1 + i_2$ and (ii) $i_1 - i_2$

Write answer in sinusoidal form. Also draw phasor diagram.

[2070 Bhadra]

Solution:

$i_1 = 10 \sin\left(314t + \frac{\pi}{14}\right)$ A

$i_2 = 8 \sin\left(314t - \frac{\pi}{3}\right)$ A

$i = i_1 + i_2 = 10 \sin\left(314t + \frac{\pi}{14}\right) + 8 \sin\left(314t - \frac{\pi}{3}\right)$

$= 10 \sin 314t \cos\frac{\pi}{14} + 10 \cos 314t \sin\frac{\pi}{14} + 8 \sin 314t \cos\frac{\pi}{3} - 8 \cos 314t \sin\frac{\pi}{3}$

$= 9.75 \sin 314t + 2.22 \cos 314t + 4 \sin 314t - 6.93 \cos 314t$

$= 13.75 \sin 314t - 4.71 \cos 314t$ .... (1)

$i = I_m \sin(\omega t + \alpha)$

$= I_m \sin\omega t \cos\alpha + I_m \cos\omega t \sin\alpha$ ....(2)

Comparing (1) and (2)

$I_m \cos\alpha = 13.75$ ....(3)

$I_m \sin\alpha = -4.71$ ....(4)

Squaring and adding

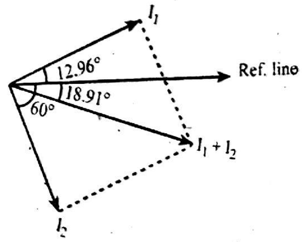
$I_m^2 = (13.75)^2 + (-4.71)^2$

$\therefore I_m = 14.53$ A

Dividing (4) by (3)

$\frac{I_m \sin\alpha}{I_m \cos\alpha} = \frac{-4.71}{13.75}$

$\therefore \alpha = -18.91°$



$\therefore i_1 + i_2 = 14.53 \sin(314t - 18.91°)$A

In phasor notation,

$\bar{I}_1 + \bar{I}_2 = 14.53 \angle -18.91°$A

Again,

$i_1 - i_2 = 10 \sin\left(314t + \frac{\pi}{14}\right) - 8\sin\left(314t - \frac{\pi}{3}\right)$

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$= 10\sin 314t \cos\frac{\pi}{14} + 10\cos 314t \sin\frac{\pi}{14} - 8\sin 314t \cos\frac{\pi}{3}$

$+ 8\cos 314t \sin\frac{\pi}{3}$

$= 9.75 \sin 314t + 2.22 \cos 314 - 4 \sin 314t + 6.93 \cos 314t$

$= 5.75 \sin 314t + 9.15 \cos 314t$ ....(5)

$i_1 - i_2 = I_m \sin(314t + \beta)$

$= I_m \sin 314t \cos\beta + I_m \cos 314t \sin\beta$ ....(6)

Comparing (5) and (6)

$I_m \cos\beta = 5.75$ .... (7)

$I_m \sin\beta = 9.15$ ....(8)

Squaring and adding (3) and (4)

$I_m^2 = (5.75)^2 + (9.15)^2$

$I_m = 10.81$ A

Dividing (8) by (7)

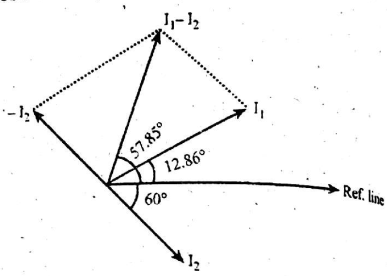
$\frac{I_m \sin\beta}{I_m \cos\beta} = \frac{9.15}{5.75}$

$\therefore \beta = 57.85°$

$\therefore i_1 - i_2 = 10.81 \sin(314t + 57.85°)$

In phasor notation

$\tilde{I}_1 - \tilde{I}_2 = 10.81 \angle 57.85°$A



Alternate Method

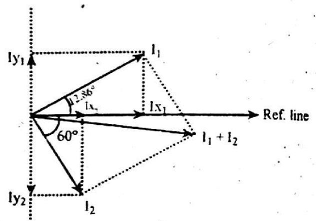
Resolving the phasors I_1 and I_2 into x – component and y - component

$Ix_1 = I_1 \cos 12.86° = 0.975\, I_1$

$Iy_1 = I_1 \sin 12.86° = 0.222\, I_1$

$Ix_2 = I_2 \cos 60° = \frac{1}{2} I_2$

$Iy_2 = I_2 \sin 60° = \frac{\sqrt{3}}{2} I_2$



x – component $I_x = Ix_1 + Ix_2 = 0.975\, I_1 + \frac{1}{2} I_2 = 0.975 \times 10 + \frac{1}{2} \times 8 = 13.75$

y-component $I_y = Iy_1 - Iy_2$

$= 0.222 \times 10 - \frac{\sqrt{3}}{2} \times 8 = -4.708$

Now, Magnitude of current $= \sqrt{I_x^2 + I_y^2}$

$= \sqrt{(13.75)^2 + (-4.708)^2}$

$= 14.53$A

Phase of current $= \tan^{-1}\left(\frac{y - comp}{x - comp}\right)$

$= \tan^{-1}\left(\frac{-4.708}{13.76}\right) = -18.91°$

$i_1 + i_2 = 14.53 \sin(314t - 18.91°)$ A

In phasor notation,

$\tilde{I}_1 + \tilde{I}_2 = 14.53 \angle -18.91°$ A

For $i_1 - i_2$;

x – component $Ix = Ix_1 - Ix_2$

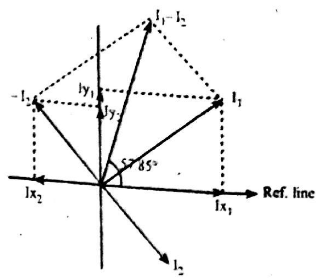
$= 0.975 \times 10 - \frac{1}{2} \times 8$

$= 5.75$

y - component $Iy = Iy_1 + Iy_2$

$= 0.222 \times 10 + \frac{\sqrt{3}}{2} \times 8$

$= 9.148$



Now, mag. of current $= \sqrt{5.75^2 + 9.148^2}$

$= 10.81$A

Phase of current $= \tan^{-1}\left(\frac{9.148}{5.75}\right) = 57.85°$

$i_1 - i_2 = 10.81 \sin(314t + 57.85°)$A

In phasor notation,

$\tilde{I}_1 - \tilde{I}_2 = 10.81 \angle 57.85°$ A

2. **A sinusoidal voltage is applied to three parallel branches yielding branch currents, $i_1 = 14.14 \sin(\omega t - 45°)$, $i_2 = 28.3 \cos(\omega t - 60°)$ and $i_3 = 7.07 \sin(\omega t + 60°)$ (i) Find complete time expression for the source current (ii) Draw the phasor diagram in terms of effective values. Use the voltage as reference. [2069 Chaitra]**

Solution:

Given, $i_1 = 14.14 \sin(\omega t - 45°)$

$i_2 = 28.3 \cos(\omega t - 60°)$

$= 28.3 \sin(90° + \omega t - 60°)$

$= 28.3 \sin(\omega t + 30°)$

$i_3 = 7.07 \sin(\omega t + 60°)$

(i) Let the source current be i,

$i = i_1 + i_2 + i_3 = 14.14 \sin(\omega t - 45°) + 28.3 \sin(\omega t + 30°) + 7.07 \sin(\omega t + 60°)$

$= 14.14 (\sin\omega t \cos 45° - \sin 45° \cos\omega t) + 28.3 (\sin\omega t \cos 30° + \cos\omega t \sin 30°) + 7.07 (\sin\omega t \cos 60° + \sin 60° \cos\omega t)$

$= 9.999 \sin\omega t - 9.99 \cos\omega t + 24.51 \sin\omega t + 14.15 \cos\omega t + 3.53 \sin\omega t + 6.12 \cos\omega t$

$\therefore i = 38.03 \sin\omega t + 10.28 \cos\omega t$ ....(i)

Now, $i = I_m \sin(\omega t + \alpha)$

$= I_m \sin\omega t \cos\alpha + I_m \cos\omega t \sin\alpha$ ....(ii)

Comparing (i) and (ii)

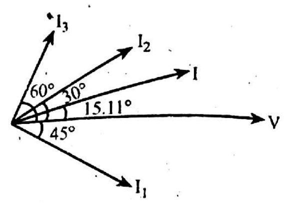
$I_m \cos\alpha = 38.03$ ....(iii)

$I_m \sin\alpha = 10.28$ ....(iv)

Squaring & adding

$$I_m^2 = (38.03)^2 + (10.28)^2$$

$$\therefore \quad I_m = 39.40 \text{ A}$$



Dividing (iv) by (iii)

$$\frac{I_m \sin\alpha}{I_m \cos\alpha} = \frac{10.28}{38.03}$$

or, $$\alpha = \tan^{-1}\left(\frac{10.28}{38.03}\right)$$

$$\therefore \alpha = 15.11°$$

Hence, $\bar{I}$ = $39.40 \angle 15.11°$A (in phasor notation)

$i(t)$ = $39.40 \sin(\omega t + 15.11°)$ (in time domain)

Drawing phasor diagram,

Alternately; Resolving into x – component and y – component

$$i_1 = 14.14 \sin(\omega t - 45°)$$

$$i_2 = 28.3 \cos(\omega t - 60°)$$

$$= 28.3 \sin(90° + \omega t - 60°) = 28.3 \sin(\omega t + 30°)$$

$$i_3 = 7.07 \sin(\omega t + 60°)$$

Resultant x – component

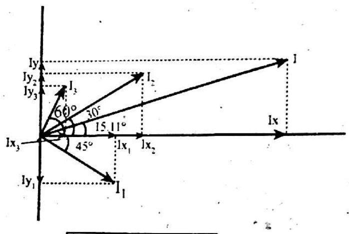
$$I_x = I_{x_1} + I_{x_2} + I_{x_3}$$

$$= 14.14 \cos 45° + 28.3 \cos 30° + 7.07 \cos 60° = 38.04$$

Resultant y - componant

$$I_y = -I_{y_1} + I_{y_2} + I_{y_3}$$

$$= -14.14 \sin 45° + 28.3 \sin 30° + 7.07 \sin 60° = 10.27$$



Magnitude of total current $= \sqrt{(38.04)^2 + (10.27)^2} = 39.402$A

Phase of total current $= \tan^{-1}(10.27/38.04) = 15.11°$

$\therefore i(t) = 39.402 \sin(\omega t + 15.11°)$A

3. **Describe phasor representation and additioin of two sinusoids $i_3 = i_1 + i_2$. Illustrate**
   (i) Position of the phasors for t = 0
   (ii) Sinusoidal waveform for increasing time. **[2069 Bhadra]**

Solution:

Let us consider two sinusoidal currents whose equation is given as

$$i_1 = Im_1 \sin\omega t \text{ and}$$

$$i_2 = Im_2 \sin(\omega t + \theta_2)$$

Addition of these sinusoidal currents is give n by

$$i_3 = i_1 + i_2$$

$$= Im_1 \sin\omega t + Im_2 \sin(\omega t + \theta_2)$$

$$= Im_1 \sin\omega t + Im_2 \sin\omega t \cos\theta_2 + Im_2 \cos\omega t \sin\theta_2$$

$$i_3 = (Im_1 + Im_2 \cos\theta_2) \sin\omega t + (Im_2 \sin\theta_2) \cos\omega t \text{ ....(i)}$$

The addition of two sinusoids of the same frequency results in another sinusoid. Hence i_3 can be represented as;

$$i_3 = Im_3 \sin(\omega t + \theta_3)$$

$$= Im_3 \sin\omega t \cos\theta_3 + Im_3 \sin\theta_3 \cos\omega t$$

$$i_3 = (Im_3 \cos\theta_3) \sin\omega t + (Im_3 \sin\theta_3) \cos\omega t \text{ ....(ii)}$$

From (i) and (ii) we get

$$Im_3 \cos\theta_3 = Im_1 + Im_2 \cos\theta_2 \text{ ....(iii)}$$

$$Im_3 \sin\theta_3 = Im_2 \sin\theta_2 \text{ ....(iv)}$$

From (iii) and (iv) equations we can determine the amplitude Im_3 and phase angle θ_3

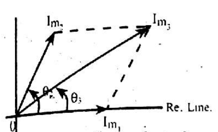


Fig. Phasor diagram for t = 0

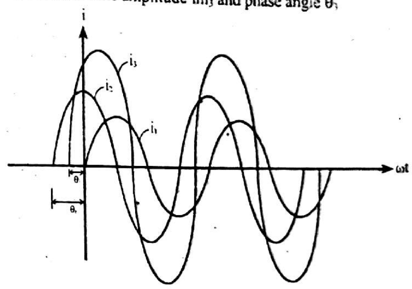


Fig:Sinusoid waveform for increasing time

4. **In a parallel ac circuit consisting of two branches, the branch current are as follows: $i_1(t) = 10 \sin\left(314t - \frac{\pi}{12}\right)$ and $i_2(t) = 5 \sin\left(314t - \frac{\pi}{3}\right)$. Find the expression of total instantaneous current being drawn from the supply by these two branches.** **[2062 Bhadra]**

Solution:

Given, $$i_1(t) = 10 \sin\left(314t - \frac{\pi}{12}\right)$$

$$i_2(t) = 5 \sin\left(314t - \frac{\pi}{3}\right)$$

Total instantaneous current drawn from supply

$$i(t) = i_1(t) + i_2(t)$$

$$= 10 \sin\left(314t - \frac{\pi}{12}\right) + 5 \sin\left(314t - \frac{\pi}{3}\right)$$

$$= 10 \sin 314t \cos\frac{\pi}{12} - 10 \cos 314t \sin\frac{\pi}{12} + 5 \sin 314t \cos\frac{\pi}{3}$$

$- 5\cos 314t \sin\frac{\pi}{3}$

$= 9.66 \sin 314t - 2.59 \cos 314t + 2.5 \sin 314t - 4.33 \cos 314t$

$= 12.16 \sin 314t - 6.92 \cos 314t$

$i(t) = 12.16 \sin 314t - 6.92 \cos 314t$ .... (1)

We need to express i(t) in equation from as

$i(t) = I_m \sin(314t + \alpha) = I_m \sin 314t \cos\alpha + I_m \cos 314t \sin\alpha$ ....(2)

Comparing equation (1) and (2) we get

$I_m \cos\alpha = 12.16$ ....(3)

$I_m \sin\alpha = -6.92$ ....(4)

Squaring and adding (3) and (4)

$I_m^2 \cos^2\alpha + I_m^2 \sin^2\alpha = (12.16)^2 + (-6.92)^2$

or, $I_m^2 = 195.752$

$\therefore I_m = 13.99 \approx 14$ A

Dividing (4) by (3);

$$\frac{I_m \sin\alpha}{I_m \cos\alpha} = \frac{-6.92}{12.16}$$

or, $\tan\alpha = \dfrac{-6.92}{12.16}$

or, $\alpha = \tan^{-1}\left(\dfrac{-6.92}{12.16}\right)$

$\therefore \quad \alpha = -29.64°$

$\therefore \quad i(t) = 14 \sin(314t - 29.64°)$A

Alternately;

Resolving into x – components and y components.

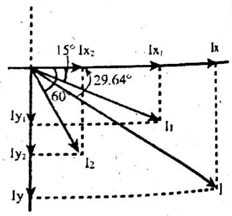
Resultant x – component

$Ix = Ix_1 + Ix_2 = 10\cos 15° + 5\cos 60° = 12.16$

Resultant y - component

$Iy = -Iy_1 - Iy_2$

$= -10\sin 15° - 5\sin 60° = -6.92$



Magnitude of total current $= \sqrt{Ix^2 + Iy^2}$

$= \sqrt{(12.16)^2 + (-6.92)^2}$

$= 13.99 \approx 14$ A

Phase of total current $= \tan^{-1}\left(\dfrac{Iy}{Ix}\right) = \tan^{-1}\left(\dfrac{-6.92}{12.16}\right) = -29.64°$

$\therefore \quad i(t) = 14\sin(314t - 29.64°)$

❁ ❁ ❁

6

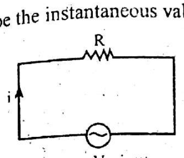
SINGLE PHASE AC CIRCUITS & POWER IN AC CIRCUITS

AC through pure resistor:

Consider an AC circuit with a pure resistance 'R' excited by sinusoidal AC voltage given by

$v = V_m \sin\omega t$

If 'i' be the instantaneous value of current through the circuit, applying Ohm's law



$v = iR$

$i = \dfrac{v}{R} = \dfrac{V_m}{R}\sin\omega t$

$\boxed{\therefore i = I_m \sin\omega t}$ where, $I_m = \dfrac{V_m}{R}$

'i' is also ac in nature and has no phase difference with v'

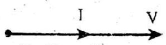


Fig: Phasor diagram

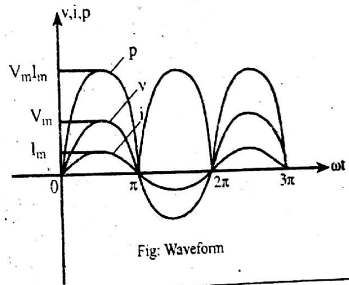


Fig: Waveform

Hence, it is concluded that in purely resistive circuit, voltage and current are in same phase.

Now, instantaneous power is given by

$p = v \times i$

$= V_m \sin\omega t \times I_m \sin\omega t$

$\boxed{p = V_m I_m \sin^2\omega t}$

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The average power in a complete cycle is given by

$$P_{average} = \int_0^{\pi} \frac{p}{\pi - 0}\, d\omega t$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m I_m \sin^2\omega t\, d\omega t$$

$$= \frac{V_m I_m}{\pi}\int_0^{\pi} \frac{1-\cos 2\omega t}{2}\, d\omega t$$

$$= \frac{V_m I_m}{2\pi}\left[\omega t\Big|_0^{\pi} - \frac{\sin 2\omega t}{2}\Big|_0^{\pi}\right]$$

$$= \frac{V_m I_m}{2\pi}(\pi - 0)$$

$$= \frac{V_m I_m}{2}$$

$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

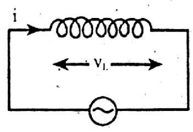
$$= V_{rms}\, I_{rms}$$

$$\boxed{P_{average} = V_{rms}\, I_{rms}}$$

Hence, power in resistive circuit is always positive though fluctuating. This means that voltage source constantly delivers power to the circuit and the circuit consumes it.

AC through pure inductor

Consider an AC circuit with a pure inductance 'L' excited by an ac sinusoidal voltage described by the equation



$v = V_m\sin\omega t$

$v = V_m \sin\omega t$ ....(1)

Let 'i' be the instantaneous value of current through the inductance which is also time VARying in nature.

Hence, according to Faraday's law of electromagnetic induction, emf is induced across the inductance given by equation,

$v_L = L\dfrac{di}{dt}$ ....(2)

By KVL, $v = v_L$

or, $V_m \sin\omega t = L\dfrac{di}{dt}$

or, $di = \dfrac{V_m}{L}\sin\omega t\, dt$

Integrating both sides,

$$\int di = \int \frac{V_m}{L}\sin\omega t\, dt$$

$$\Rightarrow \quad i = \frac{V_m}{L}\left(\frac{-\cos\omega t}{\omega}\right)$$

$$\Rightarrow \quad i = \frac{V_m}{\omega L}(-\cos\omega t)$$

$$\Rightarrow \quad i = \frac{V_m}{\omega L}\sin(\omega t - 90°)$$

$$\Rightarrow \quad i = \frac{V_m}{X_L}\sin(\omega t - 90°)$$

Where, $X_L = \omega L = 2\pi f L$

X_L is known as inductive reactance of L.

Thus, X_L opposes flow of current (functions same as resistor). Its unit is Ohm (Ω).

$\Rightarrow \boxed{i = I_m \sin(\omega t - 90°)}$ where, $I_m = \dfrac{V_m}{X_L}$

Here, 'i' is also ac in nature and lags by 90° with respect to v.

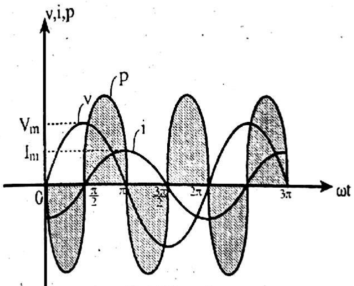


Fig: Waveform

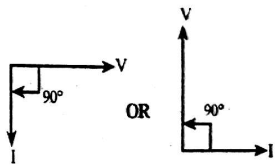


Fig: Phasor diagram

Hence, it is concluded that in purely inductive coil, voltage and current are in different phase; current lags behind applied voltage by $\frac{\pi}{2}$.

Now, Instantaneous power $p = v \times i$

$$= V_m \sin\omega t \times I_m \sin(\omega t - 90°)$$

$$= V_m \sin\omega t \times -I_m \cos\omega t = \frac{-V_m I_m}{2}\sin 2\omega t$$

The power waveform may be plotted by multiplying at every instant the values of voltage and current obtained from their waveforms.

Hence, pure capacitor does not consume real power and produces useful work in the circuit. However, it draws a 90° leading current, which is used in establishing the electric field.

Explanation from waveform

The power curve is a sine wave of twice the frequency of the voltage or current wave. During first quarter cycle the power curve is positive and the circuit draws energy from source and capacitor is charged. During second quarter cycle the power curve is negative, the energy stored in the capacitor is returned to the source i.e. the capacitor is discharged. Thus, the total active energy during each cycle of the current is zero. Thus, in purely capacitive circuit the active power over a complete cycle is zero.

Impedance and Admittance

Resistive circuit	Purely inductive circuit	Purely capacitive circuit
$i = \frac{V_m}{R}\sin\omega t$ In phasor form, $\tilde{I} = \frac{V_m\angle 0°}{R}$ $R = \frac{V_m\angle 0°}{\tilde{I}}$ $R = \frac{\tilde{V}}{\tilde{I}}$ $\boxed{\therefore Z = R}$	$i = \frac{V_m}{X_L}\sin(\omega t - 90°)$ In phasor form, $\tilde{I} = \frac{V_m\angle -90°}{X_L}$ $\tilde{I} = \frac{V_m\angle 0°}{X_L\angle 90°} = \frac{V_m\angle 0°}{jX_L}$ $jX_L = \frac{V_m\angle 0°}{\tilde{I}}$ $jX_L = \frac{\tilde{V}}{\tilde{I}}$ $\boxed{\therefore Z = jX_L}$	$i = \frac{V_m}{X_C}\sin(\omega t + 90°)$ In phasor form, $\tilde{I} = \frac{V_m\angle 90°}{X_C}$ $\tilde{I} = \frac{V_m\angle 0°}{X_C\angle -90°} = \frac{V_m\angle 0°}{-jX_C}$ $\tilde{I} = \frac{j\times V_m\angle 0°}{X_C}$ $[\because j^2 = -1]$ $\frac{X_C}{j} = \frac{V_m\angle 0°}{\tilde{I}}$ $-jX_C = \frac{\tilde{V}}{\tilde{I}}$ $\boxed{\therefore Z = -jX_C}$

Thus, $Z = \frac{\tilde{V}}{\tilde{I}}$

Where, Z is a frequency - dependent quantity known as impedance.

The impedance (Z) of a circuit is the ratio of the phasor voltage (V) to the phasor current (I), measured in ohms (Ω).

As a complex quantity, Z can be expressed in rectangular from as

$Z = R + jX$

Where, R = Re (Z) is the resistance

X = Imag (Z) is the reactance

If X is positive $Z = R + jX$

$Z \rightarrow$ inductive impedance

$X \rightarrow$ inductive reactance

Current lags Voltage

If X is negative $Z = R - jX$

$Z \rightarrow$ Capacitive impendance

$X \rightarrow$ Capacitive reactance

Current leads Voltage

In polar form -

$$Z = |Z|\angle\theta$$

Where, $|Z| = \sqrt{R^2 + X^2}$, $\theta = \tan^{-1}\left(\frac{X}{R}\right)$

and, $R = |Z|\cos\theta$

$X = |Z|\sin\theta$

For Inductive circuit: For Capacitive circuit:

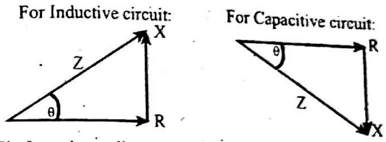


Fig:Impedance diagram Fig:Impedance diagram

Admittance (Y) of a circuit is the reciprocal of impedance (Z), measured in **siemens** (s) or mho (℧).

$$Y = \frac{1}{Z} = \frac{I}{V}$$

$$Y = G + jB$$

Where, G = Re (Y) is the conductance

B = Imag (Y) is the susceptance

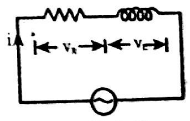
$$G + jB = \frac{1}{R \pm jX} = \frac{1}{R \pm jX} \times \frac{R \mp jX}{R \mp jX}$$

$$= \frac{R \mp jX}{R^2 + X^2}$$

Thus, $G = \frac{R}{R^2 + X^2}$; $B = \mp\frac{X}{R^2 + X^2}$

AC through R– L series circuit

Consider an ac circuit with a resistance connected in series with an inductance and excited by ac voltage.



Now,

Voltage drop across resistance

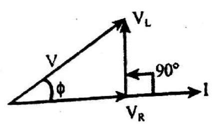
$\tilde{V}_R = \tilde{I}\,R$ [no phase difference between v and i]

Voltage drop across inductance,

$\tilde{V}_L = \tilde{I}\,(jX_L)$ [i lags v_L by 90°]

By KVL, $\tilde{V} = \tilde{V}_R + \tilde{V}_L$ ....(i)

Now, drawing phasor diagram from equation (i)



From diagram

$$V = \sqrt{V_R^2 + V_L^2}$$

or, $V = \sqrt{I^2R^2 + I^2X_L^2}$

or, $V = I\sqrt{R^2 + X_L^2}$

or, $\frac{V}{I} = \sqrt{R^2 + X_L^2}$

or, $\frac{V}{I} = |Z|$

Where, $\boxed{|Z| = \sqrt{R^2 + X_L^2}}$ → impedance of RL circuit.

From phasor diagram, it shows that current lags voltage by a phase angle ϕ which is less than 90°

Now, $\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}$

$\therefore \quad \phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

Also, $\tilde{I} = \frac{\tilde{V}}{Z}$ (in phasor)

$\tilde{I} = \frac{V\angle 0°}{|Z|\angle\phi} = \frac{V}{|Z|}\angle(-\phi)$

$\boxed{\therefore \tilde{I} = I\angle -\phi}$ .... (2) where $I = \frac{V}{|Z|}$

Thus in an inductive circuit, current lags applied voltage by an angle ϕ.

If the instantaneous voltage is represented by

$v = V_m \sin \omega t$

then instantaneous value of current is

$i = I_m \sin(\omega t - \phi)$ where, $I_m = \frac{V_m}{|Z|}$

Now, Instantaneous power $p = v \times i$

$= V_m \sin\omega t \times I_m \sin(\omega t - \phi)$

$= V_m I_m \times \frac{1}{2}[2 \sin\omega t \sin(\omega t - \phi)]$

$= \frac{V_m I_m}{2}[\cos\phi - \cos(2\omega t - \phi)]$

$= \frac{1}{2} V_m I_m \cos\phi - \frac{1}{2} V_m I_m \cos(2\omega t - \phi)$

The power waveform can be plotted by multiplying the values of current and voltage at each instant obtained from their waveforms.

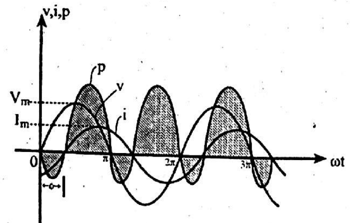


Fig: Waveform

This power consists of two parts

1. Constant part $\rightarrow \frac{1}{2} V_m I_m \cos\phi$

2. Pulsating component $\rightarrow \frac{1}{2} V_m I_m \cos(2\omega t - \phi)$

This component has frequency twice of voltage or current.
The average power of the this pulsating part is zero.

Hence, only constant part $\frac{1}{2} V_m I_m \cos\phi$ contributes power to the circuit.

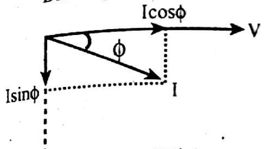
$\therefore P_{average} = \frac{1}{2} V_m I_m \cos\phi = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} \cos\phi$

$\boxed{\therefore P_{average} = VI\cos\phi}$

[Note: V, I refers to rms values of voltage & current respectively]

Power in RL series circuit

Drawing phasor diagram of voltage and current in RL circuit:



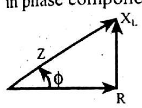
Here, current 'I' has two components.

i. $I\cos\phi \rightarrow$ component of 'I' in phase with 'V'

ii. $I\sin\phi \rightarrow$ component of 'I' in quadrature to 'V'

Accordingly, two types of power can be defined:

1. Active Power (P): It is the product of rms value of the voltage in the circuit and in phase component of rms value of current. Its unit is watt.



Active power, P = V × component of 'I' in phase with 'V'

$\boxed{P = VI\cos\phi}$

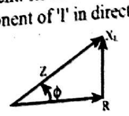
$P = IZ \times I\frac{R}{Z}$

$\boxed{\therefore P = I^2R}$ where, $\frac{R}{Z} = \cos\phi$

Thus, active power is only consumed in resistance. It is the actual power which is responsible for production of heat. Thus, the power which is actually consumed or utilized in an ac circuit is called true or real or active power of the circuit.

2. Reactive power (Q): It is the product of rms value of voltage and quadrature components of rms value of current. Its unit is VAR (volt – ampere reactive).

Reactive power, Q = V × component of 'I' in direction perpendicular to 'V'



$$\boxed{\therefore Q = VI\sin\phi}$$

$$Q = IZ \times I\frac{X_L}{Z}$$

$$\boxed{\therefore Q = I^2X_L}$$

A pure inductor and a pure capacitor doesn't consume any power, as in power, in quarter cycle, whatever power is drawn from the supply source by these components, the same is returned to the supply source in the other quarter cycle. This power which flows, back and forth (i.e. in both direction in the circuit) is called reactive power. It is the power which is responsible for production of flux (electric in capacitor / magnetic in inductor) in the circuit.

Apparent power (S) – It is the total power in the circuit. It is the production of rms value of voltage and rms value of current. Its unit is VA (Volt-Ampere).

$$\boxed{S = VI}$$

$$S = I \times ZI$$

$$\boxed{\therefore S = I^2Z}$$

Also,

$$\begin{aligned} S &= VI \\ S^2 &= (VI)^2\ (\cos^2\phi + \sin^2\phi) \\ &= (VI\cos\phi)^2 + (VI\sin\phi)^2 \\ &= P^2 + Q^2 \end{aligned}$$

$$\boxed{\therefore S = \sqrt{P^2 + Q^2}}$$

Drawing power diagram

$$\therefore \phi = \tan^{-1}\left(\frac{Q}{P}\right)$$

$$\therefore P = VI\cos\phi = S\cos\phi$$

$$\therefore Q = VI\sin\phi = S\sin\phi$$

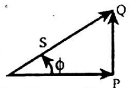


Fig: Power diagram

So, apparent power has two components P and Q

Power factor (p.f) : Cosine of angle of lead or lag is known as power factor.

i.e. $pf = \cos\phi$

Where ϕ is the phase angle and also called as power factor angle.

A circuit in which current lags voltage i.e. an inductive circuit is said to have a lagging power factor.

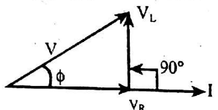


Fig: Phasor diagram

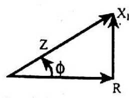


Fig:Impedance triangle

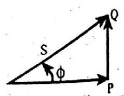


Fig: Power diagram

$$\boxed{p.f. = \cos\phi = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}}$$

i) If $R = X_L$, then $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = 45°$, p.f. $= \cos\phi = \frac{1}{\sqrt{2}} = 0.707$

$\therefore P = Q$

ii) If $R > X_L$, then $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) < 45°$, p.f. > 0.707

$\therefore P > Q$

iii) If $R < X_L$, then $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) > 45°$, p.f. < 0.707

$\therefore P < Q$

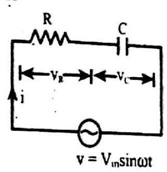
iv) If $X_L = 0$, then $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = 0$, p.f. $= 1$ (maximum value)

$\therefore Q = 0$ [Resistive circuit]

v) If $R = 0$, then $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = 90°$, p.f. $= 0$ (minimum value)

$\therefore P = 0$ [Inductive circuit]

AC through R–C series circuit



Consider an circuit with a resistance 'R' connected in series with a capacitor 'C' and supplied by ac voltage source.

Now, Voltage drop across resistance

$\tilde{V}_R = \tilde{I}\,R$ [no phase difference between v and i]

Voltage drop in capcitance

$\tilde{V}_R = \tilde{I}\ (-jX_C)$ [i leads v by 90°]

By KVL, $\tilde{V} = \tilde{V}_R + \tilde{V}_C$ .... (i)

Representing equation (i) in phasor form

$$V = \sqrt{V_R^2 + V_C^2}$$

or, $V = \sqrt{I^2R^2 + I^2X_C^2}$

or, $V = I\sqrt{R^2 + X_C^2}$

or, $\frac{V}{I} = \sqrt{R^2 + X_C^2}$

$\therefore \frac{V}{I} = Z$

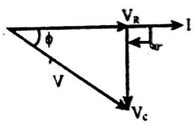


Fig: Phasor diagram

Where, $Z = \sqrt{R^2 + X_C^2}$ = impedance of RC circuit

From phasor diagram, it shows that current leads voltage by an angle ϕ which is less than 90°

From impedance triangle,

$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R}$$

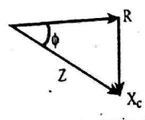


Fig:Impedance triangle

$\therefore \phi = \tan^{-1}\frac{X_C}{R}$

Also, $\tilde{I} = \frac{\tilde{V}}{Z} = \frac{V\angle 0°}{|Z|\angle -\phi}$

$\boxed{\therefore\ \tilde{I} = I\angle\phi}$ where, $I = \frac{V}{|Z|}$

Thus in a capacitive circuit, current leads voltage by an angle ϕ.

The instantaneous voltage and current are described as:

$v = V_m \sin\omega t$

$i = I_m \sin(\omega t + \phi)$ where $I_m = \frac{V_m}{Z}$

Instantaneous power p
$$= v \times i$$
$$= V_m \sin\omega t \times I_m \sin(\omega t + \phi)$$
$$= \frac{V_m I_m}{2}[2\sin\omega t \sin(\omega t + \phi)]$$
$$= \frac{V_m I_m}{2}[\cos\phi - \cos(2\omega t + \phi)]$$
$$= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t + \phi)$$

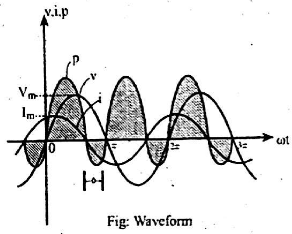


Fig: Waveform

Power in RC circuit

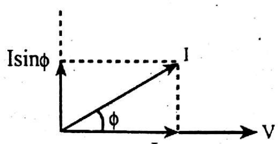


Fig: Phasor diagram

Here, current has two components

⇒ I cosϕ – component in phase with voltage

⇒ I sin ϕ – component perpendicular with voltage

Accordingly two powers can be defined

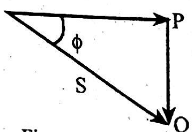


Fig: power diagram

1. Active power, $P = VI\cos\phi$

$$= IZ \times I\frac{R}{Z} = I^2R$$

2. Reactive power, $Q = VI\sin\phi$

$$= IZ \times I\frac{X_C}{Z} = I^2X_C$$

3. Apparent power, $S = VI = \sqrt{P^2 + Q^2}$

Power factor (p.f.) $= \cos\phi = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$

i) If $R = X_C$ then $\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = 45°$, p.f. = 0.707

$\therefore P = Q$

ii) If $R > X_C$ then $\phi = \tan^{-1}\left(\frac{X_C}{R}\right) < 45°$, p.f. > 0.707

$\therefore P > Q$

iii) If $R < X_C$ then $\phi = \tan^{-1}\left(\frac{X_C}{R}\right) > 45°$, p.f. < 0.707

$\therefore P < Q$

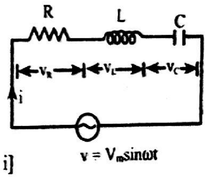
iv) If $X_C = 0$, then $\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = 0$, p.f = 1 (max)

$\therefore Q = 0$ [Resistive circuit]

v) If $R = 0$, then $\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = 90°$ p.f = 0 (min)

$\therefore P = 0$ [Capacitive circuit]

AC through RLC series circuit



Here, Inductive reactance $(X_L) = \omega L = 2\pi fL$

Capacitive reactance $(X_C) = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

Let,

V_R = Voltage drop across R [V_R is in phase with i]

V_L = voltage across inductor [i lags v by 90°]

V_C = voltage across capacitor [i leads v by 90°]

By KVL, $\tilde{V} = \tilde{V}_R + \tilde{V}_L + \tilde{V}_C$

$$= \tilde{I}R + \tilde{I}jX_L + \tilde{I}(-jX_C)$$

$\tilde{V} = \tilde{I}(R + jX_L - jX_C)$ ....(i)

Case I: If $X_L > X_C$ i.e. $V_L > V_C$ i.e. the circuit will inductive

$\tilde{V} = \tilde{I}[R + j(X_L - X_C)]$

or, $\tilde{V} = \tilde{I}Z$

Where, $Z = R + j\,(X_L - X_C)$

= impedance of the circuit

$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$, $\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right)$

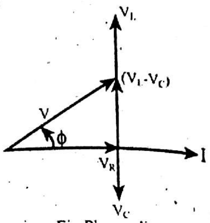


Fig:Phasor diagram

Also, $\tilde{I} = \dfrac{\tilde{V}}{Z}$

$= \dfrac{V\angle 0°}{Z\angle\phi}$

$= \dfrac{V}{|Z|}\angle -\phi$

$\boxed{\tilde{I} = I\angle -\phi}$

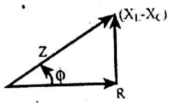


Fig: Impedance triangle

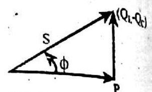


Fig:Power diagram

⇒ i lags v by φ angle and P.F. will be lagging

Thus, if $X_L > X_C$ circuit will be inductive

Case II: If $X_L < X_C$, $V_L < V_C$ i.e. circuit will be capactive

$\tilde{V} = \tilde{I}\,[R - j\,(X_C - X_L)]$

or, $\tilde{V} = \tilde{I}\,Z$

Where, $Z = R - j(X_C - X_L)$

= impedance of the circuit

$|Z| = \sqrt{R^2 + (X_C - X_L)^2}$, $\phi = \tan^{-1}\left[\dfrac{-(X_C - X_L)}{R}\right]$

Also, $\tilde{I} = \dfrac{\tilde{V}}{Z} = \dfrac{V\angle 0°}{|Z|\angle -\phi} = \dfrac{V}{|Z|}\angle\phi$

$\boxed{\therefore \tilde{I} = I\angle\phi}$ where $I = \dfrac{V}{|Z|}$

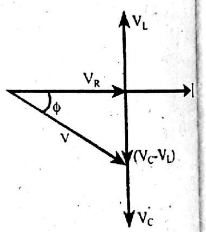


Fig: Phasor diagram

⇒ i leads v by φ angle & p.f. will be leading.

Thus, if $X_C > X_L$ the circuit will be capacitive.

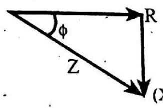


Fig: Impendance triangle

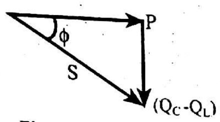
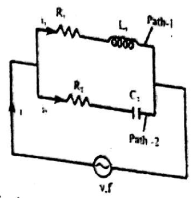


Fig: Phasor diagram

AC parallel circuit



Let, V = Rms value of applied voltage

I_1 = Rms value of current through path - 1

I_2 = Rms value of current through path - 2

Now, $\tilde{I}_1 = \dfrac{\tilde{V}}{Z_1} = \dfrac{\tilde{V}}{R_1 + j\,X_L} = \dfrac{V\angle 0°}{|Z_1|\angle\phi_1} = I_1\angle -\phi_1$

Where, $Z_1 = \sqrt{R_1^2 + (X_L)^2}$, $\phi_1 = \tan^{-1}\left(\dfrac{X_L}{R_1}\right) \Rightarrow i_1$ lags v by ϕ_1

$\tilde{I}_2 = \dfrac{\tilde{V}}{Z_2} = \dfrac{\tilde{V}}{R_2 - jX_C} = \dfrac{V\angle 0°}{|Z_2|\angle -\phi_2} = I_2\angle\phi_2$

Where, $Z_2 = \sqrt{R_2^2 + X_C^2}$, $\phi_2 = \tan^{-1}\left(\dfrac{-X_C}{R_2}\right) \Rightarrow i_2$ leads v by ϕ_2

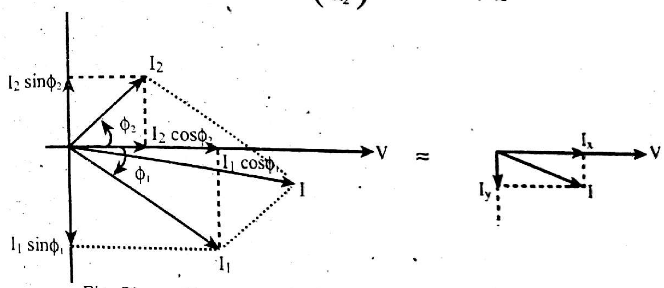


Fig: Phasor diagram

$I_1\cos\phi_1$ = Active component of I_1

$I_1\sin\phi_1$ = Reactive component of I_1

$I_2\cos\phi_2$ = Active component of I_2

$I_2\sin\phi_2$ = Reactive component of I_2

Net active component $I_x = I_1\cos\phi_1 + I_2\cos\phi_2$

Net reactive component $I_y = I_1\sin\phi_1 - I_2\sin\phi_2$

$\therefore I = \sqrt{I_x^2 + I_y^2}$

$\therefore \phi = \tan^{-1}\left(\dfrac{I_y}{I_x}\right)$

$\Rightarrow$ Active power in path 1 = $VI_1 \cos\phi_1$

Reactive power in path 1 = $VI_1 \sin \phi_1$

$\Rightarrow$ Active power in path 2 = $VI_2 \cos\phi_2$

Reactive power in path 2 = $VI_2 \sin \phi_2$

$\therefore$ Total active power $P_{total} = VI_1 \cos\phi_1 + VI_2 \cos\phi_2$

$= VI \cos\phi$

$\therefore$ Total reactive power $Q_{total} = VI_1 \sin\phi_1 - VI_2 \sin\phi_2$

$= VI\sin\phi$

Complex Power

Consider a single phase ac circuit.

Let $\tilde{V} = |V| \angle \theta_1$ and $\tilde{I} = |I| \angle \theta_2$

Then complex power S flowing into the circuit is given by

$$S = \tilde{V}\tilde{I}^* = (|V| \angle \theta_1) \times (|I| \angle -\theta_2)$$

$$= |V|\,|I| \angle (\theta_1 - \theta_2)$$

$$= |V|\,|I| \cos(\theta_1 - \theta_2) + j\,|V|\,|I| \sin(\theta_1 - \theta_2)$$

$$= P + jQ$$

[Convention: Power is taken as positive when following into the circuit or load or out a generator]

$\Rightarrow$ P is positive as $(-90° < (\theta_1 - \theta_2) < 90°)$

$\Rightarrow$ When $(\theta_1 - \theta_2)$ is positive, Q is positive. Thus current lags voltage i.e. the circuit consists of inductive load. So, inductive load absorbs/consumes reactive power.

$\Rightarrow$ When $(\theta_1 - \theta_2)$ is negative, Q is negative. Thus current leads voltage i.e. the circuit consists of capacitive load. So, capacitive load supplies reactive power.)

Consider a circuit in which voltage leads current by angle ϕ [Inductive circuit]

Let,

$$V = V \angle \theta + \phi \,;\, I = I \angle \theta$$

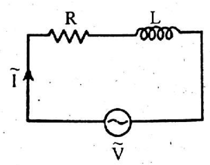
$$S = \tilde{V}\tilde{I}^*$$

$$= (V \angle \theta + \phi) \times (I \angle -\theta)$$

$$= VI \angle \phi$$

$$= VI \cos \phi + j\, VI \sin \phi$$

$$= P + jQ$$



Hence, $P = VI \cos \phi = S \cos \phi$

$Q = VI \sin \phi = S \sin \phi$

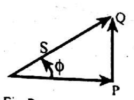


Fig:Power triangle

Power factor and its correction

Power factor is defined as cosine of the phase angle ϕ between voltage and current.

$$\text{p.f.} = \cos \phi$$

Also, $P = VI \cos \phi$

$$\therefore \cos \phi = \frac{P}{VI}$$

$$\therefore I = \frac{P}{V\cos \phi} \quad \text{.................... (1)}$$

Equation (1) shows that if the supply voltage is kept constant, for a given power P required by load, the current I taken by load VARies inversely with the power factor cos ϕ. Hence, more current is drawn at low power factor than it does at high power factor.

Disadvantages of low power factor

As explained above low power factor causes larger current drawn, which has following disadvantages.

1. Higher amount of currents lead to higher copper loses in the system thus the efficiency of system is reduced. i.e. $P_{loss} = I^2R$.
2. Higher currents required larger conductor, transformers, switchgears, etc. Thus capital cost of equipment is increased.
3. Higher currents produce larger voltage drop in cables and other apparatus causing poor voltage regulation.

Power factor improvement

To overcome these disadvantages the power factor of the circuit can be improved by placing a capacitor in parallel to the load.

Consider an inductive circuit supplied by an ac voltage.

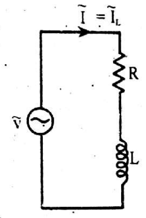


Fig:Circuit diagram1

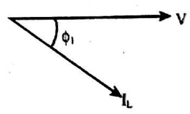


Fig:Phasor diagram1

Let

V = Supply voltage

I_L = load current

ϕ_1 = phase angle by which current I_L lags behind voltage X

$\cos\phi_1$ = original power factor

Let a capacitor C is placed in parallel with the load. It will draw leading current I_C from the supply. Hence total current I is drawn from the supply.

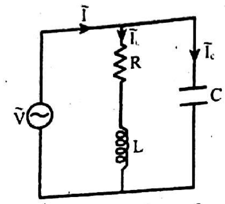


Fig:Circuit diagram2

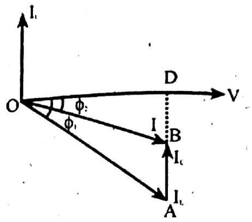


Fig:phasor diagram2

From phasor diagram.

$\phi_2 < \phi_1$

$\cos\phi_2 > \cos\phi_1$

Power factor of circuit 2 > power factor of circuit 1.

1. **Thus, by connecting capacitor power factor is improved from $\cos\phi_1$ to $\cos\phi_2$.**

In Δ OBD of phasor diagram 2,

$\cos\phi_2 = \frac{OD}{OB}$ ⇒ $OD = OB \cos\phi_2 = I \cos\phi_2$ ....(1)

In Δ OAD of phasor diagram 1,

$\cos\phi_1 = \frac{OD}{OA}$ ⇒ $OD = OA\cos\phi_1$

$= I_L \cos\phi_1$..............(2)

From (1) & (2)

$I_L \cos\phi_1 = I \cos\phi_2$ ....(3)

Since, $\cos\phi_2 > \cos\phi_1$

2. **Thus the new current drawn from the supply is less than the load current I_L.**

Multiplying V on both sides of (3)

$VI_L \cos\phi_1 = VI \cos\phi_2$

$\therefore P_1 = P_2$

3. **Thus the power taken from the supply remains same.**

Calculation for rating of capacitor to improve power factor.

Let us draw phasor diagrom for the above circuit.

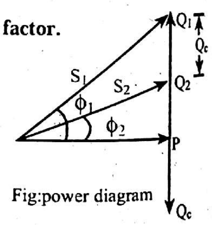


Fig:power diagram

Here,

P = Active power drawn from the supply.

Q_1 = Reactive power taken by load.

Q_2 = Reactive power taken from the supply.

Q_C = leading reactive power drawn by the capacitor from the supply.

From power diagram,

$\tan\phi_1 = \frac{Q_1}{P}$

or, $Q_1 = P\tan\phi_1$ ....(1)

$\tan\phi_2 = \frac{Q_2}{P}$

or, $Q_2 = P\tan\phi_2$................(2)

Now,

$Q_C = Q_1 - Q_2$

$= P\tan\phi_1 - P\tan\phi_2$

$\boxed{\therefore Q_C = P(\tan\phi_1 - \tan\phi_2)}$ VAR

Thus, the above equation gives the VAR rating of the capacitors to improve the power factor from $\cos\phi_1$ to $\cos\phi_2$

The value of capacitance can be obtained

Since,

$Q_C = VI_C \sin 90°$ [as I_c leads V by 90°]

$= VI_C$

Also, $I_C = \frac{V}{X_c} = \frac{V}{\frac{1}{\omega C}} = \omega CV$

$\therefore Q_C = V \times \omega CV$

$= \omega CV^2$

$\boxed{\therefore C = \frac{Q_C}{\omega V^2}}$ F, where V is rms value of supply voltage.

Thus, the above equation gives the capacitance value in farad for power factor correction.

Step - by - step calculation procedure for series AC circuits.

1. **Determine the impedance in rectangular form**

$Z = (R + jX_L - jX_C)\Omega$

$= R + j(X_L - X_C)\,\Omega$

Where, $X_L = \omega L = 2\pi fL\,\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}\,\Omega$

2. **Express the given supply voltage (i.e. as reference phasor) in phasor form.**

Given, $v = V_m \sin\omega t$

$\tilde{V} = V\angle 0°$ where, $V = \frac{V_m}{\sqrt{2}}$; $V \rightarrow$ rms voltage

3. **Determine circuit current by Ohm's law**

$\tilde{I} = \frac{\tilde{V}}{Z} = \frac{V\angle 0°}{R + j(X_L - X_C)} = \frac{V\angle 0°}{Z\angle\phi} = \frac{V}{Z}\angle -\phi$ A

To express $\tilde{I}$ in equation from

$i(t) = I_m \sin(\omega t - \phi)$

Where $I_m = \sqrt{2} \times I = \sqrt{2} \times \frac{V}{Z}$

4. **Determine phase difference between supply voltage and circuit current I_L**

$\phi = 0° - (-\phi) = \phi$

5. **Determine the power factor cos ϕ. Specify whether the power factor is lagging or leading. If current lags voltage the power factor is lagging, if current leads voltage the power factor is leading.**

6. **Determine power in the circuit.**

Active power (P) = VI cos ϕ watt

Reactive power (Q) = VI sin ϕ VAR

Apparent power (S) = VI VA

Alternatively,

$S = \tilde{V}\tilde{I}^*$ Where $\tilde{I}^*$ is complex conjugate of $\tilde{I}$

$= P + jQ$

$\therefore \quad P \quad = \quad Re|\tilde{V}\tilde{I}^*|$ watt

$Q \quad = \quad Imag|\tilde{V}\tilde{I}^*|$ VAR

$S \quad = \quad |\tilde{V}\tilde{I}^*|$ VA

Note: Lagging current gives a positive Q i.e. reactive power is consumed by the circuit.

Leading current gives a negative Q i.e. reactive power is supplied by the circuit.

7. **Determine the voltage drop at each element.**

$\tilde{V}_R = \tilde{I}R$

$\tilde{V}_L = \tilde{I}(jX_L)$

$\tilde{V}_C = \tilde{I}(-jX_C)$

8. **Draw the phasor diagram**

Step - by - step calculation procedure for single -phase AC parallel circuits.

1. Solve each branch by the procedure given for single -phase series circuits.
2. Determine the current through each branch.
3. Add the branch current to determine the total supply current $\tilde{I}$.
4. Determine the phase difference ϕ between $\tilde{V}$ & $\tilde{I}$ and determine the overall power factor cos ϕ.
5. Determine total impedance Z by Ohm's law

$Z = \frac{\tilde{V}}{\tilde{I}}$

Alternately, $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$

$\therefore Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$

Exam Solutions

1. **Derive the equation in for instantaneous current flowing through a pure capacitor when excited by AC sinusoidal voltage. $V = V_m$ sinwt. Draw the waveform of voltage and current and phase or diagram of the circuit. Show analytically and graphically that it does not consume real power.**

[Please refer to the theory] **[2071 Chaitra]**

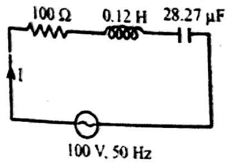
2. **A series R-L-C circuit having R = 100Ω, L = 0.12 H and C = 28.27 μF is fed from a 100 V, 50 Hz supply. Find the current flowing, active power, reactive power, power factor, rms values of voltage across each elements. Also draw phase diagram.**

[2071 Chaitra]

Solution:

Given,

100V, 50 Hz, 1ϕ Supply



Now,

$X_L = 2\pi fL$

$= 2\pi \times 50 \times 0.12$

$= 37.7\ \Omega$

$X_C = \frac{1}{2\pi fC}$

$= \frac{1}{2\pi \times 50 \times 28.27 \times 10^{-6}}$

$= 112.6\ \Omega$

$\therefore$ Total impedance of the circuit

$Z = R + jX_L - jX_C$

$= 100 + j37.7 - j112.6$

$= 100 - j74.9\ \Omega$

$\tilde{V} = 100\angle 0^0$ V

$\therefore \tilde{V} = \tilde{I}Z$

$\tilde{I} = \frac{\tilde{V}}{Z} = \frac{100\angle 0°}{100 - j74.9} = 0.8\angle 36.83°$ A

Phase difference (ϕ) = 0° − 36.83° = −36.83°

Active Power (P) = VI Cos ϕ

$= 100 \times 0.8 \times \text{Cos}(-36.83°)$

$= 64.03$ watt

Reactive power (Q) = VI Sin ϕ

$= 100 \times 0.8 \times \text{Sin}(-36.83°)$

$= -47.95$ VAR

(Negative sign indicates that the circuit supplies reactive power).

Power factor = Cosϕ = Cos(−36.83°)

$= 0.8$ (lead)

Voltage across resistor

$\bar{V}_R = \bar{I} R$

$= (0.8 \angle 36.83°) \times (100)$

$= 80 \angle 36.83°$ V

$\therefore$ Rms value of voltage across resistor = 80 V

Voltage across inductor

$\bar{V}_L = \bar{I} \times (j X_L)$

$= (0.8 \angle 36.83°) \times j\, 37.7$

$= 30.16 \angle 126.83°$ V

$\therefore$ Rms value of voltage across inductor = 30.16 V

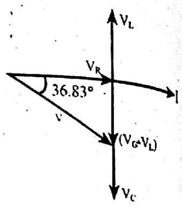
Voltage across capacitor

$\bar{V}_C = \bar{I} \times (-j X_c)$

$= (0.8 \angle 36.83°) \times (-j\, 112.6)$

$= 90.08 \angle -53.17°$ V

$\therefore$ Rms value of voltage across capacitor = 90.08 V

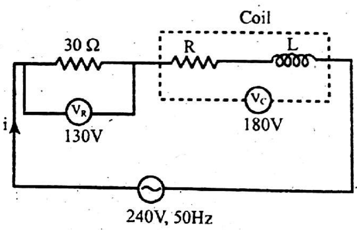


Drawing phasor diagram

Since $X_C > X_L$, $V_C > V_L$

The circuit is capacitive

3. **A coil is connected in series with a resistance of 30Ω across of 240V, 50 Hz power supply. The reading of a voltmeter across coil is 180V and across resistor is 130V. Calculate resistance and reactance of coil. Also find power factor of whole circuit.** [2071 Magh]



Solution:

Given,

Voltage across coil (V_C) = 180V

Voltage across resistor (V_R = 130 V

240 V, 50 Hz power supply.

Total current of the circuit

$$I = \frac{\text{Voltage across resistor } (V_R)}{\text{Resistance}}$$

$$= \frac{130}{30} = 4.33 \text{ A}$$

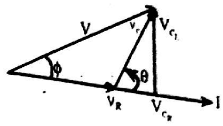
Now,

Impendence of the coil.

$$Z_C = \frac{\text{Voltage across coil}}{\text{total current of the circuit}}$$

$$= \frac{180}{4.33}$$

$$= 41.57\ \Omega$$



Where,

V_{C_R} → Voltage across resistance R of coil.

V_{C_L} → voltage across inductor L of coil.

From phasor diagram,

$V^2 = V_R^2 + V_C^2 + 2 \times V_R \times V_C \times \text{Cos}\theta$ [∵ using vector law of addition]

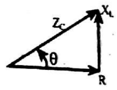
$(240)^2 = (130)^2 + (180)^2 + 2 \times 130 \times 180 \times \text{Cos}\theta$

or, $\text{Cos}\theta = \dfrac{(240)^2 - (130)^2 - (180)^2}{2 \times 130 \times 180}$

$= 0.177$

$\therefore \theta = \text{Cos}^{-1}(0.177) = 79.8°$

Now, impedance diagram of the coil.



$\therefore$ Resistance of the coil, $R = Z_C \text{ Cos}\theta$

$= 41.57 \times 0.177$

$= 7.36\ \Omega$

$\therefore$ Reactance of the coil, $X_L = \sqrt{Z_c^2 - R^2}$

$= \sqrt{(41.57)^2 - (7.36)^2}$

$= 40.91\ \Omega$

Now,

Total resistance of the circuit = 30 + 7.36

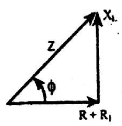
$= 37.36\ \Omega$

Total impedance of whole circuit

$Z = \sqrt{(37.36)^2 + (40.91)^2}$

$= 55.4\ \Omega$

Again, Impedance diagram of whole circuit.



$\therefore R + R_1 = Z \text{ Cos}\phi$

or, $\text{Cos}\phi = \dfrac{R + R_1}{Z}$

$= \dfrac{37.36}{55.4}$

$= 0.674$ (lagging)

$\therefore$ The power of whole circuit = 0.674 (lagging)

4. **Construct a phasor diagram of currents and voltages in a R-L-C series circuit. Assume R = $|X_L|$ = $|0.8\ X_C|$** [2071]

Solution:

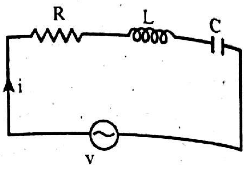
Given,

R = $|X_L| = |0.8X_C|$

$|X_L| = |0.8X_C|$

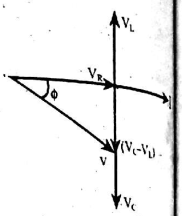
$\therefore \quad X_C > X_L$

Thus the circuit is capacitive.



Drawing phasor diagram

From phasor diagram,



$$\phi = \tan^{-1}\frac{V_C - V_L}{V_R}$$

$$= \tan^{-1}\frac{(IX_C - IX_L)}{IR}$$

$$= \tan^{-1}\frac{I\,(X_C - X_L)}{IR}$$

$$= \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

$[\because R = 0.8\ X_L = X_C]$

$$= \tan^{-1}\left(\frac{R - R/0.8}{R}\right)$$

$$= \tan^{-1}\left(1 - \frac{1}{0.8}\right)$$

$$= \tan^{-1}\left(\frac{-1}{4}\right)$$

$$= -\,14.036^\circ$$

Since, ϕ is negative applied voltage lags current i.e. current leads applied voltage the circuit is capacitive.

5. **A series combination resistor R and inductance L is driven by 25V, 50 Hz supply. The power delivered to R and L are 100 W and 75 VAR. Determine the value of capacitance of a capacitor to be connected in parallel with source to improve its power factor to 0.9 (lagging).** [2071 Magh]

Solution:

Given,

25V, 50 Hz Supply,

Active Power (P) = 100 W

Reactive Power (Q_1) = 75 VAR

Improve power factor (pf_2) = 0.9 (lagging)

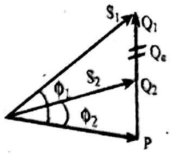
$pf_2 = \cos\phi_2$

or, $0.9 = \cos\phi_2$

$\therefore \quad \phi_2 = 25.84^\circ$

Drawing phasor diagram

Q_1 = 75 VAR



From figure,

Q_2 = P tan ϕ_2

= 100×tan 25.84°

= 48.43 VAR

Reactive power supplied

$Q_C = Q_1 - Q_2$

= 75 − 48.43

= 26.57 VAR

$Q_C = \omega CV^2$

$\therefore \quad C = \dfrac{Q_C}{\omega V^2}$

$= \dfrac{26.57}{2\times\pi\times50\times(25)^2}$

$= 1.353\times10^{-4}$ F

= 135.3 μF

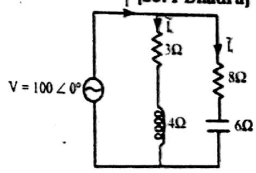
$\therefore$ The value of required capacitance is 135.3 μF.

6. **Explain the operation of purely capacitive circuit excited by a sinusoidal source and hence prove that average power consumed by such circuit is zero. Draw necessary waveforms.** [2071 Bhadra]

[Please refer to the theory]

7. **For the circuit given below, calculate the current I. Draw the phasor diagram of the circuit.** [2071 Bhadra]

Solution:



$\tilde{V}$ = 100 ∠0°

Z_1 = 3 + j 4 Ω

Z_2 = 8 − j 6 Ω

$\tilde{I}_1 = \dfrac{\tilde{V}}{Z_1} = \dfrac{100\angle0^\circ}{3+j4} = 20\angle{-53.13^\circ}$ A

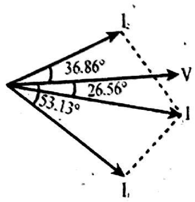
$\tilde{I}_2 = \dfrac{\tilde{V}}{Z_2} = \dfrac{100\angle0^\circ}{8-j6} = 10\angle36.86^\circ$ A

$\therefore \tilde{I} = \tilde{I}_1 + \tilde{I}_2$

$= (20\angle{-53.13^\circ}) + (10\angle36.86^\circ)$

$= 22.36\ \angle{-26.56^\circ}$ A

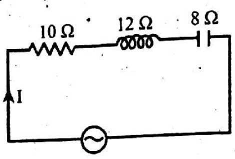
Drawing phasor diagram



8. Derive the equation for the instantaneous current when A.C. voltage is supplied to a series R-L circuit. Draw phasor diagrams and analyze power in the circuit. **[2071 Bhadra]**

[Please refer to the theory]

9. An electric circuit is being supplied by an a.c. source of 100 Vrms. The circuit has a resistance of 10Ω, inductor of 12Ω reactance and capacitance of 8Ω reactance connected in series. Compute the active power and power factor of the circuit. **[2071 Bhadra]**



100 V

Solution:

$\tilde{V} = 100\angle 0°$

$Z = R + jX_L - jX_C$

$= 10 + j12 - j8$

$= 10 + j4$

$\tilde{V} = \tilde{I}\,Z$

$\therefore \tilde{I} = \dfrac{\tilde{V}}{Z}$

$= \dfrac{100\angle 0°}{10 + j4}$

$= 9.28\angle -21.8°$ A

Phase difference $(\phi) = 0° - (-21.8°)$

$= 21.8°$

Active power (P) = VI Cos ϕ

$= 100 \times 9.28 \times \text{Cos } 21.8°$

$= 861.63$ Watt.

Power factor (pf) $= \text{Cos}\phi$

$= \text{Cos } 21.8°$

$= 0.928$ (lagging)

10. Derive the equation for instantaneous current following through a pure capacitor when excited by AC sinusoidal voltage $v = V_m \sin \omega t$. Draw the wave form of voltage and current and phasor diagram of the circuit. Show analytically and graphically that it does not consume real power. **[2070 Chaitra, 2071 Chaitra]**

[Please refer to the theory]

11. A coil takes 1.3 kVA and 1.2 kVAR when connected to a 240V, 50 Hz sinusoidal supply. Calculate (i) Power dissipated (ii) current and (iii) inductance of the coil. **[2070 Chaitra]**

Solution:

Given, 240V, 50Hz supply

Reactive power (Q) = 1.2 kVAR

Apparent power (S) = 1.3 kVA

(i) Power dissipated (P)

$P^2 = S^2 - Q^2$

$P = \sqrt{S^2 - Q^2}$

$= \sqrt{(1.3)^2 - (1.2)^2}$

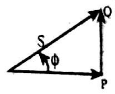
$= 0.5$ kW

$= 500$ W

(ii) Current (I)

Power - diagram

$\tan\phi = \dfrac{Q}{P}$



$\tan\phi = \dfrac{1.2}{0.5}$

$\therefore \phi = 67.38°$

$P = VI\cos\phi$

or, $500 = 240 \times I \times \cos 67.38°$

$\therefore I = \dfrac{500}{240 \times \cos 67.38°} = 5.4166$ A

(iii) Inductance of the coil (L)

$\tilde{V} = \tilde{I}Z$

$240\angle 0° = (5.4166\angle -67.38°)Z$

$\therefore Z = \dfrac{240\angle 0°}{5.4166\angle -67.38°} = 44.308\angle 67.38°$

$\therefore Z = 17.0417 + j\,40.8998\Omega$

Comparing with $Z = R + jX_L$

$\therefore X_L = 40.8998$

or, $2\pi f L = 40.8998$

or, $L = \dfrac{40.8998}{2\pi \times 50}$

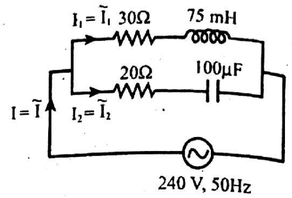
$\therefore L = 0.13018$ H

12. A circuit consisting of a resistance of 30Ω in series with an inductance of 75mH is connected in parallel with a circuit consisting of a resistance of 20Ω in series with a capacitance of 100μF. if the parallel combination is connected to a 240V, 50Hz, single phase supply. Calculate (i) The total current (ii) Power factor (iii) Active and reactive power. Also draw a neat phasor diagram. **[2070 Chaitra]**

Solution:

Given,

240 V, 50Hz, 1ϕ supply



Now,

$$X_L = 2\pi fL = 2\pi \times 50 \times 75 \times 10^{-3} = 23.56\Omega$$

$$\therefore Z_1 = R_1 + jX_L = 30 + j\,23.56\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\Omega$$

$$\therefore Z_2 = R_2 - jX_C = 20 - j31.83\Omega$$

Now,

Total impedance (Z) = $Z_1 \parallel Z_2$

So, $$\tilde{I}_1 = \frac{\tilde{V}}{Z_1} = \frac{240 \angle 0°}{30 + j\,23.56} = 6.29 \angle -38.14°A$$

$$\tilde{I}_2 = \frac{\tilde{V}}{Z_2} = \frac{240 \angle 0°}{20 - j\,31.83} = 6.38 \angle 57.85°A$$

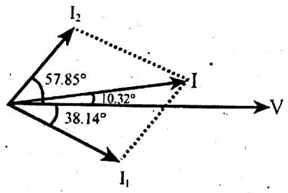
Hence,

(i) Total current ($\tilde{I}$) $= \tilde{I}_1 + \tilde{I}_2$
$= (6.29 \angle -38.14°) + (6.38 \angle 57.85°)$
$= 8.478 \angle 10.32°A$

(ii) Power factor
$\cos\phi = \cos 10.32° = 0.9838$ (lead)

(iii) Active power (P) $= VI\cos\phi$
$= 240 \times 8.478 \times 0.9838 = 2002.229$ W

Reactive power (Q) $= VI \sin\phi$
$= 240 \times 8.478 \times \sin(10.32°) = 364.597$ VAR

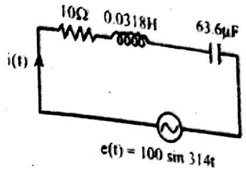


13. Define power factor, active, reactive and apparent power in ac circuit. Also draw the phasor diagram. **[2070 Magh]**

[Please refer to the theory]

14. A voltage e(t) = 100 sin 314t is applied across series circuit consisting of 10Ω resistance, 0.0318 H inductance and a capacitor of 63.6 μF. Calculate expression for i(t), phase difference between voltage and current, power factor, apparent power and active power. **[2070 Magh]**

Solution:



$$R = 10\Omega$$

$$X_L = 2\pi fL = \omega L = 314 \times 0.0318 = 9.9852\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 63.6 \times 10^{-6}} = 50.074\Omega$$

$$Z = R + jX_L - jX_c = 10 + j\,9.9852 - j\,50.074 = 10 - j\,40.088\,\Omega$$

Now, $e(t) = 100 \sin 314t$

Comparing with $e(t) = E_m \sin(\omega t + \phi)$

$\therefore \omega = 314$ $\phi = 0°$

$E_m = 100$

$$\therefore E_{rms} = \frac{Em}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.71 \text{ V}$$

$$\therefore \tilde{E} = 70.71 \angle 0°$$

$$\therefore \tilde{E} = \tilde{I}Z$$

$$\tilde{I} = \frac{\tilde{E}}{Z} = \frac{70.17 \angle 0°}{10 - j\,40.088} = 1.71 \angle 75.99° = 1.71 \angle 76°A$$

76°? I V

$\therefore I_{rms} = 1.71$ A

$I_m = \sqrt{2}\, I_{rms} = \sqrt{2} \times 1.71 = 2.42$ A

$\therefore i(t) = 2.42 \sin(314t + 76°)$A

Phase difference between voltage and current

$\phi = 0° - 76° = -76°$

ϕ 2 -76°

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Power factor = $\cos\phi = \cos(-76°) = 0.2419$ (lead)

Apparent power (S) $= \tilde{V}\,\tilde{I}^*$

$= (70.71\angle 0°) \times (1.71\angle -76°)$

$= 120.91\angle -76°$ VA

$= (29.25 - j117.322)$VA

$\therefore$ Apparent power (S) = 120.91 VA

Comparing with $S = P + jQ$

Active power (P) = 29.25 W

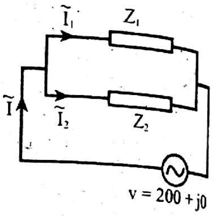
Reactive Power (Q) = – 117.33 VAR (negative sign indicates the circuit supplies reactive power)

15. Two impedances $Z_1 = (10 + j5)$ and $Z_2 = (8 + j6)$ are joined in parallel across a voltage of $V = 200 + j0$. Calculate magnitudes and phases of circuit current and branch currents. Draw phasor diagram. [2070 Bhadra]

Given, $\tilde{V} = 200 + j0$

$= 200\angle 0°$

$Z_1 = 10 + j5\Omega$



Now, $\tilde{I}_1 = \dfrac{\tilde{V}}{Z_1} = \dfrac{200\angle 0°}{10 + j5}$

$= 17.88\angle -26.56°$ A

$\tilde{I}_2 = \dfrac{\tilde{V}}{Z_2} = \dfrac{200\angle 0°}{8 + j6}$

$= 20\angle -36.87°$ A

$\tilde{I} = \tilde{I}_1 + \tilde{I}_2$

$= (17.88\angle -26.55°) + (20\angle -36.87°)$

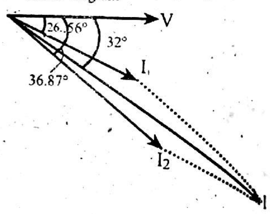
$= 37.73\angle -32°$A

$\therefore$ Circuit current $(\tilde{I}) = 37.73\angle -32°$ A

Branch current $\tilde{I}_1 = 17.88\angle -26.56°$A

$\tilde{I}_2 = 20\angle -36.87°$A

Phasor diagram



16. An inductive load of 4kW at a lagging power factor of 0.8 is connected across a 220V, 50Hz supply. Calculate the value of the capacitance to be connected in parallel with the load to bring the resultant power factor to 0.95 lagging. [2070 Bhadra]

Solution:

Given, 220V, 50Hz supply

Power (P) = 4 kW

$p.f_1$ = 0.8 (lag)

$p.f_2$ = 0.95 (lag)

Now, $p.f_1 = 0.8$

$\cos\phi_1 = 0.8$

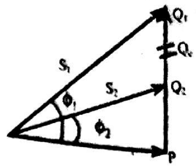
$\therefore \phi_1 = 36.87°$

$p.f_2 = 0.95$

$\cos\phi_2 = 0.95$

$\therefore \phi_2 = 18.19°$

Drawing power diagram



From figure,

$Q_1 = P\tan\phi_1$

$= 4000 \times \tan 36.87°$

$= 3000.01$ VAR

$Q_2 = P\tan\phi_2$

$= 4000 \times \tan 18.19°$

$= 1314.359$ VAR

Reactive Power supplied $(Q_C) = Q_1 - Q_2$

$= 3000.01 - 1314.359$

$= 1685.65$ VAR

$\because Q_C = \omega CV^2$

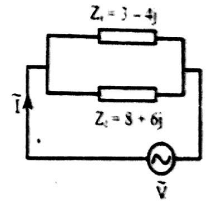
$\therefore C = \dfrac{Q_C}{\omega V^2} = \dfrac{1685.65}{2\times\pi\times 50(220)^2} = 110.86\,\mu F$

$\therefore$ Required value of capacitance (C) = 110.86μF

17. Two impedances (3 – 4j) and (8 + 6j) are connected in parallel across an ac voltage source. If the total current drawn from the source is 25A, find the total active power consumed by the impedances. [2070 Ashad]

Solution:

Given,



$Z_1 = 3 - j4$

$Z_2 = 8 + j6$

Total current drawn (I) = 25A

Let, $\tilde{I} = 25\angle 0°$A

Now,

Total impedance $Z = Z_1 \parallel Z_2 = (3 - j4) \parallel (8 + j6) = 4 - j2\ \Omega$

$\tilde{V} = \tilde{I}Z$

$= (25\angle 0°) \times (4 - j2)$

$= 111.803\angle -26.565°$V

Now, phase different between voltage and current

$\phi = -26.565° - 0°$

$= -26.565°$

Active power consumed (P) = $VI\cos\phi$

$= 111.803 \times 25\cos(-26.565)$

$= 2449.9 = 2500$ watts

18. **An industrial load consists of the following:**

i) A load of 200 kVA @ 0.8 power factor lagging

ii) A load of 50 kW@ unity power factor.

iii) A load of 48 kW@ 0.6 power factor leading

Calculate the total kW, total kVAR, total kVA and the overall power factor. **[2070 Ashad]**

Solution:

Load 1: Since P.f lagging (inductive load)

$S_1 = 200$ kVA

p.$f_1 = 0.8$ (lag)

$\cos\phi_1 = 0.8$

$\therefore \phi_1 = 36.87°$

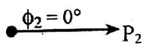
$\therefore P_1 = S_1 \cos\phi_1 = 200 \times 0.8 = 160$ kW

$\therefore Q_1 = S_1 \sin\phi_1 = 200 \times \sin 36.87° = 120$ kVAR

Load 2: Since p.f. = 1 (Resistive load)

p.$f_2 = 1 \Rightarrow \phi_2 = 0°$ $P_2 = 50$ kW

$Q_2 = 0$



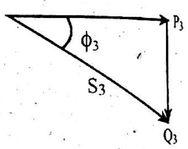
$\therefore S_2 = \sqrt{P_2^2 + Q_2^2} = \sqrt{P_2^2} = P_2 = 50$ kVA

Load 3: Since p.f. leading (Capactive load)

$P_3 = 48$ kW

p.$f_3 = 0.6$

$\phi_3 = 53.13°$



$\therefore Q_3 = P_3 \tan\phi_3 = 48 \tan 53.13° = 64$ kVAR

$\therefore S_3 = \sqrt{P_3^2 + Q_3^2} = \sqrt{48^2 + 64^2} = 80$ kVA

$\therefore$ Total kW(P) $= P_1 + P_2 + P_3$

$= 160 + 50 + 48$

$= 258$ kW

$\therefore$ Total kVAR (Q) $= Q_1 + Q_2 - Q_3$ [phasor sum]

$= 120 + 0 - 64$

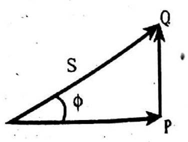
$= 56$ kVAR

$\therefore$ Total kVA(S) $= \sqrt{P^2 + Q^2}$

$= \sqrt{258^2 + 56^2} = 264$ kVA

From fig.

$\cos\phi = \frac{P}{S} = \frac{258}{264} = 0.977$



$\therefore$ Total power factor (P.f) = 0.977 (lag)

19. **A 100 kW load at 0.8 lagging power factor is being supplied by a 220V, 50 Hz source. Calculate the reactive power drawn from the source. If a capacitor connected parallel to the load improves its power factor to 0.9. Find the capacitance of the capacitor. Also calculate the current drawn from the source before and after connecting the capacitor.** **[2070 Ashad]**

Solution:

Given,

220 V, 50 Hz source.

Power (P) = 100 kW

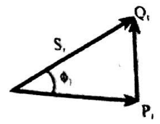
p.$f_1 = 0.8$ (lag)

p.$f_2 = 0.9$ (lag)

p.$f_1 = 0.8$

$\cos\phi_1 = 0.8$

$\phi_1 = 36.87°$



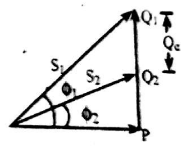
From figure,

$\therefore Q_1 = P\tan\phi_1$

$= 100 \times \tan 36.87°$

$= 75$ kVAR

$\therefore$ Reactive power drawn from the source = 75 kVAR



$P = VI_1 \cos\phi_1$

$100 \times 10^3 = 220 \times I_1 \times 0.8$

$\therefore I_1 = 568.18$A

Current drawn from the source before connecting the capacitor = 568.18 A

p.$f_2 = 0.9$

$\cos\phi_2 = 0.9$

$\therefore \phi_2 = 25.84°$

$\therefore Q_2 = P \tan\phi_2 = 100 \times \tan 25.84° = 48.428$ kVAR

Reactive power compensated by the capacitor

$\therefore Q_C = Q_1 - Q_2 = 75 - 48.428 = 26.572$ kVAR

$\therefore Q_C = \omega CV^2$

$\therefore C = \frac{Q_C}{\omega V^2} = \frac{26.572 \times 10^3}{2\pi \times 50 \times (220)^2}$

$= 1747.547$ μF

$\therefore$ Capacitance of the capacitor = 1747.547 μF

Now, $P = VI_2 \cos\phi_2$

$100 \times 10^3 = 220 \times I_2 \times 0.9$

$\therefore I_2 = 505.05$

$\therefore$ Current drawn from the source after connecting the capacitor $I_2 = 505.05$A

Thus, this shows power factor improvement causes less current to be drawn from the circuit for same amount of active power consumption.

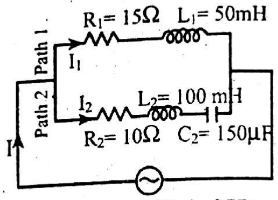
20. For the parallel circuit shown below, calculate

(i) Rms value for current, power factors and active power of path 1.

(ii) Rms value of current, power factor and reactive power of path 2.

(iii) Rms value of current and power factor of the whole circuit.

[2069 Chaitra]



$v = 325 \sin 377t$

Solution:

Given,

$v = 325 \sin 377t$

Comparing with $v = V_m \sin(\omega t + \phi)$

$V_m = 325 \qquad \omega = 377 \qquad \phi = 0°$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{325}{\sqrt{2}} = 229.8 \text{ V}$$

$\therefore \tilde{V} = 229.8 \angle 0° \text{ V}$

$$Z_1 = R_1 + jX_1$$
$$= 15 + j\omega L_1$$
$$= 15 + j\,377 \times 50 \times 10^{-3}$$
$$= 15 + j\,18.85\ \Omega$$

$$Z_2 = R_2 + j\,X_{L_2} - jX_C$$
$$= R_2 + j(X_{L_2} - X_C)$$
$$= 10 + j\left(\omega L_2 - \frac{1}{\omega C}\right)$$
$$= 10 + j\left(377 \times 100 \times 10^{-3} - \frac{1}{377 \times 150 \times 10^{-6}}\right)$$
$$= 10 + j(37.7 - 17.68) = 10 + j20\Omega$$

(i) For Path 1

$\tilde{V} = \tilde{I}_1 Z_1$

$$\tilde{I}_1 = \frac{\tilde{V}}{Z_1} = \frac{229.8 \angle 0°}{15 + j\,18.85} = 9.54 \angle -51.49° \text{A}$$

$\therefore$ Rms value of current of path 1 = 9.54 A

$\therefore \quad p.f_1 = \cos\phi_1 = \cos(0° - (-51.49°))$

$= \cos 51.49°$

$= 0.622$ (lag)

Active power $(P_1) = VI_1 \cos\phi_1$

$= 229.8 \times 9.54 \times 0.622$

$= 1363.605$ watts

(ii) For path 2

$\tilde{V} = \tilde{I}_2 Z_2$

$$\tilde{I}_2 = \frac{\tilde{V}}{Z_2} = \frac{229.8 \angle 0°}{10 + j\,20} = 10.28 \angle -63.43° \text{ A}$$

$\therefore$ Rms value of current of path 2 = 10.28 A

$\therefore$ $p.f_2 = \cos\phi_2 = \cos(0° - (-63.43°)) = \cos 63.43° = 0.447$ (lag)

Reactive power $(Q_2) = VI_2 \sin\phi_2$

$= 229.8 \times 10.28 \sin 63.43°$

$= 2112.85$ VAR

(iii) Since parallel circuit,

$\tilde{I} = \tilde{I}_1 + \tilde{I}_2 = (9.54 \angle -51.49°) + (10.28 \angle -63.43°)$

$= 19.71 \angle -57.68°\text{A}$

Rms value of current of the whole circuit = 19.71 A

$\therefore$ p.f. $= \cos\phi = \cos(0° - (-57.68°))$

$= \cos 57.68°$

$= 0.534$ (lag)

21. In a network shown in figure below, determine:

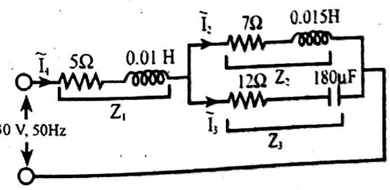
i) Total impedance

ii) Total current

iii) The current in each branch

iv) The overall power factor.

v) Volt amperes, active power & reactive power. **[2069 Chaitra]**



Solution:

Given,

230V, 50Hz supply

$Z_1 = R_1 + jX_{L_1}$

$= 5 + j\,2\pi f L_1$

$= 5 + j\,2\pi \times 50 \times 0.01$

$= 5 + j\,3.14\,\Omega$

$Z_2 = R_2 + jX_{L_2} = 7 + j2\pi \times 50 \times 0.015 = 7 + j\,4.71\,\Omega$

$Z_3 = R_3 - jX_c = 12 - j\dfrac{1}{2\pi \times 50 \times 180 \times 10^{-6}} = 12 - j\,17.68\Omega$

Now,

(i) $Z = Z_1 + (Z_2 \parallel Z_3)$

$= Z_1 + \dfrac{Z_2 Z_3}{Z_2 + Z_3}$

$= (5 + j\,3.14) + \dfrac{(7 + j\,4.71)(12 - j\,17.68)}{(7 + j\,4.71) + (12 - j\,17.68)}$

$= (5 + j\,3.14) + (7.65 + j\,1.68)$

$= (12.65 + j\,4.82)\Omega$

$\therefore$ Total impedance $(Z) = 12.65 + j\,4.82\,\Omega$

Now,

(ii) $\tilde{V} = 230\angle 0°$

$\tilde{I}_1 = \dfrac{\tilde{V}}{Z} = \dfrac{230\angle 0°}{12.65 + j\,4.82} = 16.99\angle -20.86°A \approx 17\angle -20.86°A$

$\therefore$ Total current $= 17\angle -20.86°A$

(iii) Using current division rule,

$\tilde{I}_2 = \dfrac{\frac{1}{Z_2}}{\frac{1}{Z_2} + \frac{1}{Z_3}} \times \tilde{I}_1$

$= \dfrac{\frac{1}{(7 + j\,4.71)}}{\frac{1}{(7 + j\,4.71)} + \frac{1}{(12 - j\,17.68)}} \times (17\angle -20.86)$

$= 15.79\angle -42.37°A$

Using KCL,

$\tilde{I}_3 = \tilde{I}_1 - \tilde{I}_2$

$= (17\angle -20.86°) - (15.79\angle -42.37°)$

$= 6.23\angle 47.39°$

(iv) Phase difference between voltage and current

$\phi = 0° - (-20.86°)$

$= 20.86°$

$p.f = \cos\phi = \cos 20.86° = 0.934$ (lag)

Current lags voltage in phase angle

(v) Volt amperes = Apparent power (S) = VI = 230 × 17 = 3910 VA

Active power (P) = VI cos φ = 230 × 17 × cos 20.86° = 3653.71 watts

Reactive power (Q) = VI sinφ = 230 × 17 × sin 20.86 = 1392.29 VAR

22. **What is power factor in ac circuit? Explain the disadvantages of low power factor.**

[please refer to the theory]

23. **When a voltage v = 10 sin (500 t - 60°) V is applied to a series ac circuit, the current is i = 6 sin (500 t - 10°). Find (i) Power factor (ii) Circuit parameters (iii) Apparent, active and reactive power. (iv) Also indicate whether the circuit is capacitive or inductive and why? [2069 Ashd]**

Solution:

Given,

$v = 10\sin(500t - 60°)V$

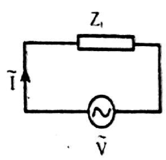
Comparing with $v = V_m \sin(\omega t + \phi)$

$V_m = 10$

$\omega = 500$

$\phi = -60°$

$\therefore V_{rms} = \dfrac{V_m}{\sqrt{2}} = \dfrac{10}{\sqrt{2}} = 7.07\text{ V}$



$\therefore \tilde{V} = 7.07\angle -60°\text{ V}$

$i = 6\sin(500t - 10°)\text{ A}$

Comparing with $i = I_m \sin(500t + \phi)$

$\because I_m = 6 \quad \phi = -10°$

$\therefore I_{rms} = \dfrac{6}{\sqrt{2}} = 4.24\text{ A}$

$\therefore \tilde{I} = 4.24\angle -10°A$

Phase difference between voltage & current $\phi = -60° - (-10°) = -50°$

(i) $\therefore$ p.f = cosφ = cos (– 50°) = 0.6427 (lead)

Lead as current leads voltage in phase angle

(ii) Circuit parameters

$\tilde{V} = \tilde{I}Z$

$\therefore Z = \dfrac{\tilde{V}}{\tilde{I}} = \dfrac{7.07\angle -60°}{4.24\angle -10°}$

$= 1.07 - j\,1.27\,\Omega$

Comparing with $Z = R - jX_C$

$\therefore \quad R = 1.07\Omega$

$\therefore \quad X_C = 1.27\Omega$

Now, $X_C = \dfrac{1}{\omega C} \Rightarrow C = \dfrac{1}{\omega X_C} = \dfrac{1}{500 \times 1.27} = 1574.8\mu F$

$\therefore$ Circuit parameters Resistance (R) = 1.07Ω

Capacitance (C) = $1574.8\mu F$

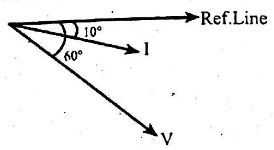
(iii) Apparent power (S) = VI = $7.07 \times 4.24 = 29.976$ VA

Active power (P) = VI $\cos\phi = 7.07 \times 4.24 \times \cos(-50°) = 19.27$ W

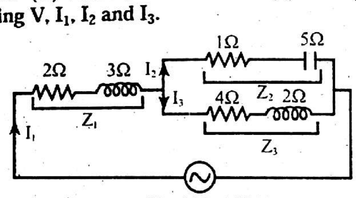
Reactive power (Q) = VI $\sin\phi = 7.07 \times 4.24 \times \sin(-50°) = -22.96$VAR

(– ve sign indicates reactive power is being supplied by the capacitive circuit)

iv) The circuit is capacitive as current leads voltage by 50° phase angle and also impedance $Z = R - jX_C$ which indicates capacitive circuit. The reactive power is being supplied by the circuit which also shows the circuit to be capacitive.



24. Following figure shows a series parallel circuit. Find:
(i) Total impedance (ii) current drawn from the circuit (iii) Voltage across the parallel branches (iv) current following through each parallel branch (v) power factor (vi) Active, reactive and apparent power. Draw phasor diagram showing V, I_1, I_2 and I_3. **[2069 Ashad]**



Solution:

Given,

$\tilde{V} = 100 \angle 0°$V

$Z_1 = 2 + j3\ \Omega$

$Z_2 = 1 - j5\ \Omega$

$Z_3 = 4 + j2\ \Omega$

(i) $Z = Z_1 + (Z_2 \parallel Z_3) = Z_1 + \dfrac{Z_2 Z_3}{Z_2 + Z_3}$

$= (2 + j3) + \dfrac{(1 - j5)(4 + j2)}{(1 - j5) + (4 + j2)}$

$= (2 + j3) + (3.64 - j\,1.41) = 5.64 + j\,1.58\ \Omega$

$\therefore$ Total impedance of the circuit (Z) = $5.64 + j\,1.58\ \Omega = 5.86 \angle 15.71°\Omega$

(ii) $\tilde{V} = \tilde{I}_1 Z$

$\tilde{I}_1 = \dfrac{\tilde{V}}{Z}$

$= \dfrac{100 \angle 0°}{5.64 + j\,1.58}$

$= 17.05 \angle -15.71°$A

$\therefore$ Current drawn from the circuit = 17.05 A

(iii) Voltage across the parallel branches

$\tilde{V}_{parallel} = ?$

$\tilde{V}_{z_1} = \tilde{I}_1 Z_1 = (17.05 \angle -15.71°)(2 + j3)$

$= 61.46 \angle 40.60°$ V

Now, using KVL,

$\tilde{V}_{parallel} = \tilde{V} - \tilde{V}_{z_1}$

$= (100 \angle 0°) - (61.46 \angle 40.60°)$

$= 66.67 \angle -36.87°$ V

$\therefore$ Voltage across the parallel branches = $66.67 \angle -36.87°$ V.

(iv) $\tilde{I}_2 = \dfrac{\tilde{V}_{parallel}}{Z_2}$

$= \dfrac{66.67 \angle -36.87°}{(1 - j5)}$

$= 13.07 \angle 41.82°$A

$\tilde{I}_3 = \dfrac{\tilde{V}_{parallel}}{Z_3} = \dfrac{66.67 \angle -36.87°}{4 + j2} = 14.91 \angle -63.43°$A

(v) Power factor of the circuit

Phase difference between voltage & current $\phi = 0 - (-15.71°) = 15.71°$

$\therefore$ p.f. = $\cos\phi = \cos 15.71° = 0.9626$ (lag)

(vi) Active power (P) = VI$\cos\phi$

$= 100 \times 17.05 \times 0.9626$

$= 1641.23$ W

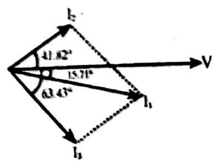
Reactive power (Q) = VI $\sin\phi$

$= 100 \times 17.05 \times \sin 15.71°$

$= 461.66$ VAR

Apparent power (S) = VI

$= 100 \times 17.05 = 1705$ VA



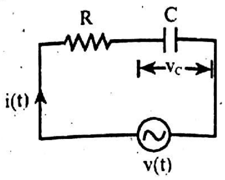
Drawing phasor diagram showing V, I_1, I_2 and I_3

25. What do you mean by complex power? Explain it with the help of an R-L series circuit and power triangle. **[2069 Ashad]**

[Please refer to the theory.]

26. An adjustable resistor R in series with a capacitive of 25μF drawn a current of 0.8 A when connected across 50 Hz supply. Calculate
i) The value of resistor so that the voltage across the capacitor is half of the supply voltage.
ii) Power consumed and
iii) The power factor.

[2069 Ashadh]



Solution:

Given,

Capacitance (C) = 25μF

Current (I) = 0.8 A

50 Hz supply

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 25 \times 10^{-6}} = 127.32\ \Omega$$

(i) Voltage across capacitor $= \frac{1}{2} \times$ supply voltage

$$\Rightarrow \quad V_C = \frac{1}{2} \times V$$

$$\Rightarrow \quad IX_C = \frac{1}{2} IZ$$

$$\Rightarrow \quad Z = 2X_C = 2 \times 127.32 = 254.64\Omega$$

Now, $|Z| = \sqrt{R^2 + X_C^2}$

or, $254.64 = \sqrt{R^2 + (127.32)^2}$

or, $R^2 = (254.64)^2 - (127.32)^2$

$\therefore R = 220.52\ \Omega$

(ii) Power consumed (P) $= I^2R = (0.8)^2 \times 220.52 = 141.14$ watts

(iii) $P = VI \cos\phi$

$141.14 = V \times 0.8 \times \cos\phi$ ....(1)

Now, $V = IZ$

$= 0.8 \times 254.64$

$= 203.712$ V

From (1)

$141.14 = 203.712 \times 0.8 \times \cos\phi$

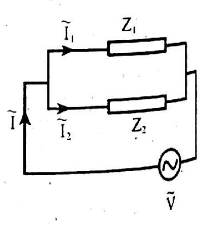
$\therefore \cos\phi = 0.866$

$\therefore$ Power factor $= \cos\phi = 0.866$ (lead)

OR

$$p.f = \frac{R}{Z} = \frac{220.52}{254.64} = 0.866 \text{ (lead)}$$

27. **A voltage of 200 ∠ 53.8° is applied across two impedances in parallel. The values of impedances are (12 + j16) ohm and (10 – j20) ohm. Determine: (i) Total impedances (ii) Total current drawn from the circuit. (iii) Current following through each parallel branch (iv) power factor of the whole circuit. (v) Active, reactive and apparent power. Draw the phasor diagram.** **[2069 Bhadra]**



Solution:

Given,

$\tilde{V} = 200 \angle 53.8°$V

$Z_1 = 12 + j\,16\Omega$

$Z_2 = 10 - j\,20\,\Omega$

(i) $Z = Z_1 \parallel Z_2$

$$= \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \frac{(12 + j16)(10 - j20)}{(12 + j\,16) + (10 - j20)} = 20\,\Omega$$

$\therefore$ Total impedance of the circuit $= 20\Omega$

(ii) $\tilde{V} = \tilde{I}Z$

$$\tilde{I} = \frac{\tilde{V}}{Z} = \frac{200 \angle 53.8°}{20} = 10 \angle 53.8°A$$

(iii) $$\tilde{I}_1 = \frac{\tilde{V}}{Z_1} = \frac{200 \angle 53.8°}{12 + j16} = 10 \angle 0.669°A$$

$$\tilde{I}_2 = \frac{\tilde{V}}{Z_2} = \frac{200 \angle 53.8°}{10 - j20} = 8.94 \angle 117.234°A$$

(iv) Phase difference between voltage and current

$\phi = 53.8° - 53.8° = 0$

$\therefore$ p.f. $= \cos\phi = \cos 0° = 1$

$\therefore$ Power factor of the circuit = unity (1)

(v) Active power (P) $= VI\cos\phi$

$= 200 \times 10 \times \cos 0° = 2000$ Watts

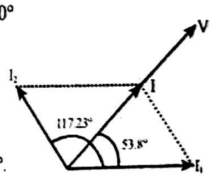
Reactive power (Q) $= VI\sin\phi = 200 \times 10 \times \sin 0°$

$= 0$

Apparent power (S) $= VI$

$= 200 \times 10$

$= 2000$ VA

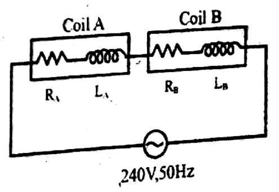


Taking I_1 as reference phasor as its phase angle is nearly 0°.

28. **Two coils A and B are connected in series across a 240V, 50 Hz supply. The resistance of A is 5Ω and the inductance of B is 0.015 H. If the input from the supply is 3 kW and 2 kVAR, find the inductance of A and the resistance of B. Calculate the voltage across each coil.** **[2068 Bhadra]**

Solution:

Given,



$R_A = 5\Omega$

$L_B = 0.015$ H

Active power (P) = 3 kW

Reactive power (Q) = 2 kVAR

$L_A = ?$
$R_B = ?$
$V_A = ?$ $V_B = ?$

We know,

$$P = VI\cos\phi$$
$$3\times10^3 = 240I\cos\phi \text{ ....(i)}$$

Also,

$$Q = VI\sin\phi$$
$$2\times10^3 = 240I\sin\phi \text{ ....(ii)}$$

Dividing (ii) by (i)

$$\frac{2\times10^3}{3\times10^3} = \frac{240I\sin\phi}{240I\cos\phi}$$

or, $\frac{2}{3} = \tan\phi$

$$\therefore \phi = \tan^{-1}\left(\frac{2}{3}\right) = 33.69°$$

Now, $P = VI\cos\phi$

$$3\times10^3 = 240I\cos 33.69°$$
$$\therefore I = 15.02A$$

Now, $\tilde{V} = 240\angle 0°\text{ V}$

$$\tilde{I} = 15.02\angle -33.69°\text{A}$$

Since inductive load i lags v by angle ϕ

Hence, $\tilde{V} = \tilde{I}\,Z$

$$Z = \frac{\tilde{V}}{\tilde{I}} = \frac{240\angle 0°}{15.02\angle -33.69°} = 13.295 + j\,8.863\Omega \text{ ....(i)}$$

Also, Total impedance $Z = Z_A + Z_B$

$$= R_A + jX_{LA} + R_B + j\,X_{LB}$$
$$\therefore Z = (R_A + R_B) + j\,(X_{LA} + X_{LB}) \text{ ....(ii)}$$

Comparing (i) & (ii)

$$R_A + R_B = 13.295 \text{ .... (iii)}$$
$$X_{L_A} + X_{L_B} = 8.863 \text{ .... (iv)}$$

From (iii)

$$R_A + R_B = 13.295$$
$$\Rightarrow 5 + R_B = 13.295$$
$$\Rightarrow R_B = 13.295 - 5$$
$$\therefore R_B = 8.295\ \Omega$$

From (iv)

$$X_{L_A} + X_{L_B} = 8.863$$

or, $\omega L_A + \omega L_B = 8.863$

or, $\omega(L_A + L_B) = 8.863$

or, $2\pi\times50\,(L_A + 0.015) = 8.863$

or, $L_A + 0.015 = 0.0282$

$\therefore L_A = 0.013$ H

Voltage across coil A

$$\tilde{V}_A = \tilde{I}Z_A$$
$$= (15.02\angle -33.69°)\times(R_A + jX_{L_A})$$
$$= (15.02\angle -33.69°)\times(5 + j\,2\pi\times50\times L_A)$$
$$= (15.02\angle -33.69°)\times(5 + j\,2\pi\times50\times0.013)$$
$$= (15.02\angle -33.69°)\times(5 + j\,4.08) = 96.93\angle 5.52°\text{V}$$

Voltage across coil B

$$\tilde{V}_B = \tilde{I}Z_B$$
$$= (15.02\angle -33.69°)\times(R_B + jX_{L_B})$$
$$= (15.02\angle -33.69°)\times(8.295 + j2\pi\times50\times0.015)$$
$$= (15.02\angle -33.69°)\times(8.295 + j4.712) = 143.29\angle -4.09°\text{V}$$

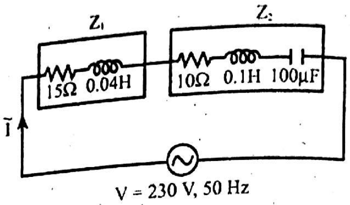
29. **Two impedances consists of (resistance of 15 Ω and series connected inductance of 0.04H) and (resistance of 10Ω, inductance of 0.1 H and a capacitance of 100μF, all in series) are connected in series and are connected to a 230 V, 50 Hz ac source. Find: (i) current drawn (ii) voltage across each impedance (iii) total power factor and (iv) draw the phasor diagram.** **[2068 Bhadra]**

Solution:

Given, 230V, 50Hz ac source

$$Z_1 = R_1 + jX_{L_1}$$
$$= 15 + j\omega L_1$$
$$= 15 + j2\pi\times50\times0.04$$
$$= 15 + j\,12.56\,\Omega$$
$$Z_2 = R_2 + jX_{L_2} - jX_C$$
$$= R_2 + j\,\omega L_2 - j\frac{1}{\omega C}$$
$$= 10 + j2\pi\times50\times0.1 - j\frac{1}{2\pi\times50\times100\times10^{-6}}$$
$$= 10 + j\,31.41 - j\,31.83 = 10 - j\,0.42\,\Omega$$

Total impedance



$$Z = Z_1 + Z_2$$
$$= (15 + j\,12.56) + (10 - j\,0.42)$$
$$= (25 + j\,12.14)\Omega$$

(i) $\tilde{I} = \dfrac{\tilde{V}}{Z} = \dfrac{230 \angle 0^\circ}{25 + j\,12.14} = 8.275 \angle -25.9°A$

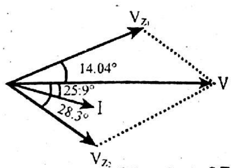
(ii) Voltage across Z_1 $(\tilde{V}_{Z1}) = \tilde{I}Z_1$
$$= (8.275 \angle -25.9°)(15 + j\,12.56)$$
$$= 161.91 \angle 14.04° \text{ V}$$

Voltage across Z_2 $(\tilde{V}_{Z2}) = \tilde{I}Z_2$
$$= (8.275 \angle -25.9°)(10 - j\,0.42)$$
$$= 82.82 \angle -28.30° \text{ V}$$

(iii) Total p.f. $= \cos\phi$
$$= \cos(0 - (-25.9°))$$
$$= \cos 25.9°$$
$$= 0.899 \text{ (lag)}$$

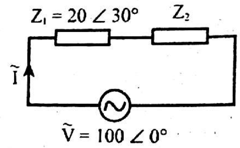
Phasor diagram

Taking voltage as ref. phasor



30. A rms voltage of 100 ∠ 0° is applied to the series combination of Z_1 and Z_2 where Z_1= 20 ∠ 30°. The effective voltage drop across Z_2 is known to be 40 ∠ - 30°. Find the reactive component of Z_2. **[2068 Baisakh]**

Solution:



Given, $\tilde{V}_{Z_2} = 40 \angle -30°$ V

By KVL

$$\tilde{V} = \tilde{V}_{Z_1} + \tilde{V}_{Z_2}$$

or, $100 \angle 0° = \tilde{V}_{Z_1} + 40 \angle -30°$

$\therefore \tilde{V}_{Z_1} = (100 \angle 0°) - (40 \angle -30°)$
$$= 68.351 \angle 17.014° \text{V}$$

$$\tilde{V}_{Z_1} = \tilde{I}Z_1$$

$\therefore \tilde{I} = \dfrac{\tilde{V}_{Z_1}}{Z_1} = \dfrac{68.351 \angle 17.014°}{20 \angle 30°} = 3.418 \angle -12.98° \text{ A}$

$$\tilde{V}_{Z_2} = \tilde{I}Z_2$$

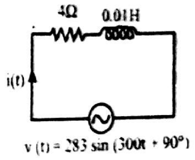
$\therefore Z_2 = \dfrac{\tilde{V}_{Z_2}}{\tilde{I}} = \dfrac{40 \angle -30°}{3.418 \angle -12.98°} = 11.192 - j\,3.424\Omega$

$\therefore$ The reactive component of Z_2 is $-3.424\ \Omega$

31. A series circuit consists of resistance equal to 4Ω and inductance of 0.01H. The applied voltage is 283 sin(300t + 90°) V. Calculate the followings:

(i) Power factor
(ii) Expression for i (t)
(iii) The power dissipated in the circuit
(iv) Voltage drop across each elements and
(v) Draw a phasor diagram **[2068 Baisakh, 2071 Shawan]**

Solution:



Given,

$v = 283\sin(300t + 90°)$

$V_{rms} = \dfrac{283}{\sqrt{2}} = 200.11\text{V}$

$\tilde{V} = 200.11 \angle 90°$ V

Impedance of the circuit (Z) $= R + jX_L$
$$= R + j\omega L$$
$$= 4 + j\,300 \times 0.01 = 4 + j3\Omega$$

Now, $\tilde{I} = \dfrac{\tilde{V}}{Z} = \dfrac{200.11 \angle 90°}{4 + j3} = 40.02 \angle 53.13°A$

$\therefore I_m = \sqrt{2}\, I_{rms} = \sqrt{2} \times 40.02 = 56.6A$

$\therefore i(t) = 56.6 \sin(300t + 53.13°)A$

P.f $= \cos\phi = \cos[90° - 53.13°]$
$$= \cos 36.87° = 0.8 \text{ (lag)}$$

Power dissipated in the circuit (P) $= VI\cos\phi$
$$= 200.11 \times 40.02 \times 0.8$$
$$= 6406.71 \text{ watts}$$

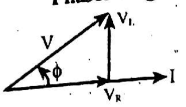
Voltage drop across resistor $\tilde{V}_R = \tilde{I}R$
$$= (40.02 \angle 53.13°) \times 4$$
$$= 160.08 \angle 53.13° \text{ V}$$

Voltage drop across inductor $\tilde{V}_L = \tilde{I}\,(jX_L)$

$= (40.02 \angle 53.13°)\,(j3)$

$= 120.06 \angle 143.13°$ V

Phasor diagram



32. **Define cycle, time period, angular velocity, frequency, average and rms value of an alternating quantity.** **[2068 Baisakh]**

[Please refer to the theory]

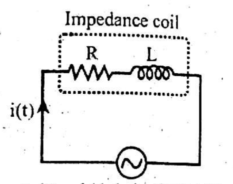
33. **In a purely inductive circuit when excited by a sinusoidal voltage, show mathematically and graphically, that the current lags the applied voltage by 90° and also show the average power consumed in the inductor is zero.** **[2068 Baishakh]**

[Please refer to the theory]

34. **An emf $e_0 = 141.4 \sin(377t + 30°)$ is impressed on the impedance coil having a resistance of 4Ω and an inductive reactance of 1.25Ω measured at 25 Hz. What is the equation of the current? Also find the equation for the resistive drop e_R and inductive drop e_L.** **[2067 Mangsir]**

Solution:

Given,



$e_0(t) = 141.4 \sin(377t + 30°)$

$e_0 = 141.4 \sin(377t + 30°)$

$\omega = 377$

$2\pi f = 377$

$\therefore f = 60$ Hz

At 25 Hz, $X_L = 1.25\,\Omega$

At 60 Hz, $X_L = \dfrac{1.25}{25} \times 60$

$= 3\Omega$

$\therefore Z = R + jX_L = 4 + j3\Omega$

$E_{rms} = \dfrac{E_m}{\sqrt{2}} = \dfrac{141.4}{\sqrt{2}} = 99.98\ \text{V} \approx 100\text{V}$

$\therefore \tilde{E}_0 = 100 \angle 30°$ V

$\tilde{I} = \dfrac{\tilde{E}_0}{Z} = \dfrac{100 \angle 30°}{4 + j3}$

$= 20 \angle -6.86°$A

$I_m = 20\sqrt{2} = 28.28$ A

$\therefore i(t) = 28.28 \sin(377t - 6.86°)$A

$\tilde{E}_R = \tilde{I}R$

$= (20 \angle -6.86°)4$

$= 80 \angle -6.86°$ V

$e_R(t) = 80 \times \sqrt{2} \sin(377t - 6.86°)$

$= 113.14 \sin(377t - 6.86°)$ V

$\tilde{E}_L = \tilde{I}\,(jX_L)$

$= (20 \angle -6.86°)\,(j3)$

$= 60 \angle 83.14°$ V

$e_L(t) = 60 \times \sqrt{2} \sin(377t + 83.14°)$

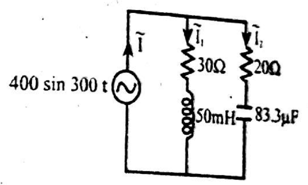
$= 84.85 \sin(377t + 83.14°)$ V

35. **For the circuit below, calculate**

i) Magnitude and phase angles of current in each of the branches.

ii) Active, reactive, apparent power and power factor of the circuit, and

iii) Draw the vector diagram indicating branch currents and supply voltage. **[2067 Ashadh]**



Solution:

Given,

$v = 400 \sin 300t$

$\tilde{V} = \dfrac{400}{\sqrt{2}} \angle 0° = 282.84 \angle 0°$ V

$Z_1 = R_1 + jX_L = 30 + j\omega L = 30 + j\,300 \times 50 \times 10^{-3} = 30 + j15\Omega$

$Z_2 = R_2 - jX_C = 20 - j\dfrac{1}{\omega C} = 20 - j\dfrac{1}{300 \times 83.3 \times 10^{-6}} = 20 - j40\,\Omega$

(i) $\tilde{I}_1 = \dfrac{\tilde{V}}{Z_1} = \dfrac{282.84 \angle 0°}{30 + j15} = 8.43 \angle -26.56°$A

$\tilde{I}_2 = \dfrac{\tilde{V}}{Z_2} = \dfrac{282.84 \angle 0°}{20 - j40} = 6.32 \angle 63.43°$A

By KCL;

$\tilde{I} = \tilde{I}_1 + \tilde{I}_2 = (8.43 \angle -26.56°) + (6.32 \angle 63.43°)$

$= 10.54 \angle 10.32°$ A

Phase difference between voltage and current

$\phi = 0° - 10.32° = -10.32°$

(ii) Active power of the circuit (P) = VI cosϕ

$= 282.84 \times 10.54 \cos(-10.32°)$

$= 2932.9$ watts

Reactive power of the circuit (Q) = VI sinϕ

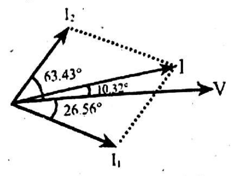
$= 282.84 \times 10.54 \sin(-10.32°)$

$= -534.05$ VAR

(– ve sign indicates reactive power is being supplied by the circuit)

Apparent power of the circuit $(S) = \sqrt{P^2 + Q^2} = 2981.12$ VA

Power factor of the circuit p.f = $\cos\phi = \cos(-10.32°) = 0.9838$ (lead)



36. A circuit consists of the following in parallel

(i) a resistor of 500Ω

(ii) an inductance of 2H

(iii) a capacitor of 10μF

In this circuit a 200V, 50Hz source is applied calculate

(A) the total current drawn from the supply

(B) Complex power

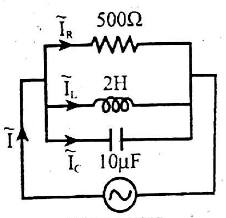
(C) Active power

(D) Reactive power

(E) Power factor of the circuit [2065 Kartik]

Solution:

Given,



$$\tilde{V} = 200\angle 0° \text{ V}$$

$$X_L = 2\pi fL = 2\pi \times 50 \times 2 = 628.32\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}} = 318.31\ \Omega$$

(A) $$\tilde{I}_R = \frac{\tilde{V}}{R} = \frac{200\angle 0°}{500} = 0.4\angle 0° \text{A}$$

$$\tilde{I}_L = \frac{\tilde{V}}{jX_L} = \frac{200\angle 0°}{j\,628.32} = 0.318\angle -90° \text{A}$$

$$\tilde{I}_C = \frac{\tilde{V}}{-jX_C} = \frac{200\angle 0°}{-j\,318.31} = 0.628\angle 90° \text{A}$$

∴ Total current drawn from the supply

$$\tilde{I} = \tilde{I}_R + \tilde{I}_L + \tilde{I}_C = (0.4\angle 0°) + (0.318\angle -90°) + (0.628\angle 90°)$$
$$= 0.506\angle 37.77° \text{A}$$

(B) Complex power is defined as

$$S = \tilde{V}\tilde{I}^*$$
$$= (200\angle 0°) \times (0.506\angle -37.77°)$$
$$= 79.99 - j\,61.98$$
$$= 101.2\angle -37.77° \text{ VA}$$

[Note: $I = 0.506\angle 37.77°$A

$I^* = 0.506\angle -37.77°$A]

(C) Comparing $S = P + jQ$ with $S = 79.99 - j\,61.98$

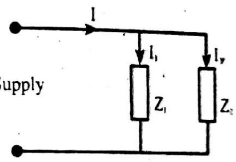
∴ Active power (P) = 79.99 watts

(D) Reactive power (Q) = – 61.98 VAR

(– ve sign indicates capacitive circuit)

(E) P.f = $\cos\phi = \cos(0° - 37.77°) = \cos(-37.77°) = 0.79$ (lead)

37. Two impedances in the circuit shown in figure below are Z_1 = (1k + j2.7k)Ω and Z_2 = (790 – j 1.6k)Ω. The total current taken from the supply is 15 mA. Calculate the two branch currents. [2066 Magh]



Solution:

Given, $\tilde{I} = 15 \text{ mA}\angle 0° = 15 \times 10^{-3}\angle 0°$A

$$Z_1 = 1000 + j\,2.7 \times 1000 = 1000 + j\,2700\ \Omega$$

$$Z_2 = 790 - j\,1.6 \times 1000 = 790 - j\,1600\ \Omega$$

Total impedance $Z = Z_1 \parallel Z_2$

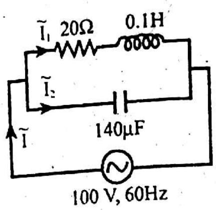
$$= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(1000 + j\,2700)(790 - j\,1600)}{(1000 + j\,2700) + (790 - j\,1600)}$$
$$= 2205.02 - j1057.28\Omega$$

$$\therefore \tilde{V} = \tilde{I}Z = (15 \times 10^{-3}\angle 0°)(2205.02 - j\,1057.28) = 36.68\angle -25.62° \text{ V}$$

$$\therefore \tilde{I}_1 = \frac{\tilde{V}}{Z_1} = \frac{36.68\angle -25.62°}{1000 + j\,2700} = 0.0127\angle -95.29° = 12.74\angle -95.29° \text{ mA}$$

$$\therefore \tilde{I}_2 = \frac{\tilde{V}}{Z_2} = \frac{36.68 \angle -25.62^\circ}{790 - j\,1600} = 0.0205 \angle 38.102^\circ = 20.55 \angle 38.102^\circ \text{mA}$$

38. A coil having resistance of 20Ω and inductance of 0.1H, connected in parallel with a 140 μF capacitor is supplied by 100 V, 60 Hz sinusoidal source. Find active power, reactive power, apparent power and power factor of the circuit and draw the phasor diagram. [2065 Shrawan]



Solution:

Given, $\tilde{V} = 100 \angle 0^\circ$ V

$Z_1 = R + j\,X_L = R + j2\pi fL = 20 + j\,2\pi \times 60 \times 0.1 = 20 + j37.699\Omega$

$Z_2 = -jX_C = -j\dfrac{1}{\omega C} = -j\dfrac{1}{2\pi fC}$

$= -j\dfrac{1}{2\pi \times 60 \times 140 \times 10^{-6}} = -j\,18.95\ \Omega$

Now, $\tilde{I}_1 = \dfrac{\tilde{V}}{Z_1} = \dfrac{100 \angle 0^\circ}{20 + j\,37.699} = 2.34 \angle -62.05^\circ$A

$\tilde{I}_2 = \dfrac{\tilde{V}}{Z_2} = \dfrac{100 \angle 0^\circ}{-j\,18.95} = 5.28 \angle 90^\circ$A

$\therefore \tilde{I} = \tilde{I}_1 + \tilde{I}_2 = (2.34 \angle -62.05) + (5.28 \angle 90^\circ)$

$= 3.39 \angle 71.15^\circ$ A

Power factor $=$ p.f. $= \cos\phi$

$= \cos(0 - 71.15^\circ)$

$= \cos(-71.15^\circ)$

$= 0.323$ (lead).

Active power (P) $= VI\cos\phi$

$= 100 \times 3.39 \times 0.323 = 109.5$ W

Reactive power (Q) $= VI\sin\phi$

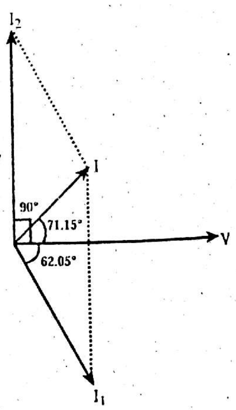
$= 100 \times 3.39 \times \sin(-71.15^\circ)$

$= -320.82$ VAR

Apparent power (S) $= VI$

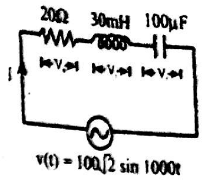
$= 100 \times 3.39$

$= 339$ VA



39. In a R - L - C series circuit has 20Ω resistor, 30 mH inductor and 100μF capacitor. The supply voltage is given as v(t) = 100 √2 sin 1000t. Calculate the phase and magnitude of current and voltages across all the elements in the circuit.

Solution:



Given, $v(t) = 100\sqrt{2}\sin 1000t$

$\omega = 1000$

$V_m = 100\sqrt{2}$

$\therefore V_{rms} = \dfrac{V_m}{\sqrt{2}} = 100$V

$\therefore \tilde{V} = 100 \angle 0^\circ$ V

Impedance $Z = R + jX_L - jX_C$

$X_L = \omega L = 1000 \times 30 \times 10^{-3} = 30\Omega$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{1000 \times 100 \times 10^{-6}} = 10\Omega$

$\therefore Z = R + jX_L - jX_C = 20 + j\,30 - j\,10 = 20 + j\,20\Omega$

as $Z = R + jX_L$ hence the circuit is inductive,

also, $(X_L > X_C)$

$\therefore \tilde{I} = \dfrac{\tilde{V}}{Z} = \dfrac{100 \angle 0^\circ}{20 + j\,20} = 3.53 \angle -45^\circ$ A

$\therefore \tilde{V}_R = \tilde{I}R = (3.53 \angle -45^\circ) \times 20 = 70.6 \angle -45^\circ$ V

$\therefore \tilde{V}_L = \tilde{I}\,(jX_L) = (3.53 \angle -45^\circ) \times (j30) = 105.9 \angle 45^\circ$ V

$\therefore \tilde{V}_C = \tilde{I}\,(-jX_C) = (3.53 \angle -45^\circ) \times (-j10) = 35.3 \angle -135^\circ$ V

Additional Problems

1. The maximum values of the alternating voltage and current are 400V and 20A respectively in a circuit connected to 50 Hz supply and these quantities are sinusoidal. The instantaneous values of voltage and current are 283V and 10 A respectively at t =0 both increasing positively.

(i) Write down the expressions for voltage and current at time t.

(ii) Determine the power consumed in the circuit.

Solution,

Given, maximum voltage $V_{max} = 400$V

maximum current $I_{max} = 20$A

Frequency f = 50 Hz

Instantaneous voltage v= 283V at t = 0

Instantaneous current i = 10 A at t = 0

Let ϕ be the phase difference w.r.t the point corresponding to t = 0.

We have expression of voltage as

$v = V_{max} \sin(\omega t + \phi)$

or, $283 = 400 \sin(\omega \times 0 + \phi)$

or, $283 = 400 \sin\phi$

[2062 Bhadra] $\therefore \phi = \sin^{-1}\left(\frac{283}{400}\right) = 45.03°$

So, expression of voltage at time t

$v = 400 \sin(\omega t + \phi)$

$= 400 \sin(2\pi f t + \phi)$

$= 400 \sin(2\pi \times 50\,t + 45.03°)$

$\therefore v = 400 \sin(314\,t + 45.03°)$

Again, expression for current

$i = I_{max} \sin(\omega t + \phi^1)$

or, $10 = 20 \sin(\omega \times 0 + \phi^1)$

or, $10 = 20 \sin\phi^1$

$\therefore \phi^1 = \sin^{-1}\left(\frac{10}{20}\right) = 30°$

So, expression of current at time t

$i = I_{max} \sin(2\pi f t + \phi^1)$

$= 20 \sin(2\pi \times 50t + 30°)$

$\therefore i = 20 \sin(314\,t + 30°)$

Power consumed in the circuit,

$P = V_{rms} I_{rms} \cos\phi$

$\therefore V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{400}{\sqrt{2}} = 282.84$ V

$\therefore I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 14.14$ A

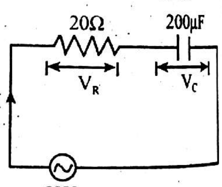
ϕ = phase difference between voltage and current

$= 45.03° - 30° = 15.03°$

$P = 282.84 \times 14.14 \times \cos 15.03° = 3862.54$ W

2. **A circuit of 20 Ω resistance in series with capacitance of 200 μF, connected across 50 Hz supply. The current through the circuit is 10.8 sin 314 t A. Determine the voltage across each component and across the circuit.**

Solution:



Let v be the voltage across the circuit.

$i = 10.8 \sin 314t$

$\tilde{i} = \frac{10.8}{\sqrt{2}} \angle 0° = 7.64 \angle 0°$ A

$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f \times C} = \frac{1}{2\pi \times 50 \times 200 \times 10^{-6}}$

$= 15.91\ \Omega$

impedance of the circuit

$Z = R - jX_c = 20 - j15.91\ \Omega$

$\tilde{V} = \tilde{I}\,Z = (7.64\angle 0°) \times (20 - j15.91) = 195.25 \angle -38.502°$ V

Now, Voltage across resistance

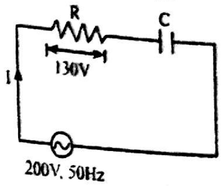
$\tilde{V}_R = \tilde{I}\,R = (7.64 \angle 0°) \times (20) = 152.8 \angle 0°$ V

Voltage across capacitance

$\tilde{V}_c = \tilde{I}\,(-jX_c) = (7.64\angle 0°) \times (-j\,15.91) = 121.55 \angle -90°$ V

3. **A series R – C circuit takes a power of 7,000 W when connected to 200V, 50 Hz supply. The voltage across the resistor is 130 V. Calculate (i) the resistance and impedance (ii) Also write equations for v(t) and i(t).**

Solution:

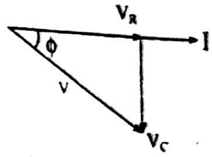


Given, Power (P) = 7,000 W

Voltage (V) = 200 V

Voltage across resistor (V_R) = 130 V

From phasor diagram,



$\cos\phi = \frac{V_R}{V} = \frac{130}{200} = 0.65$ (leading).

$\therefore \phi = \cos^{-1}(0.65) = 49.46°$

We know, $P = VI\cos\phi$

$7{,}000 = 200 \times I \times 0.65$

$\therefore I = \frac{7000}{200 \times 0.65} = 53.84$ A

Resistance $R = \frac{V_R}{I} = \frac{130}{53.84} = 2.41\ \Omega$

Impedance $Z = \frac{V}{I} = \frac{200}{53.84} = 3.71\ \Omega$

Capacitive reactance

$X_c = \sqrt{Z^2 - R^2} = \sqrt{(3.71)^2 - (2.41)^2} = 2.82\ \Omega$

We know, $X_c = \frac{1}{\omega C}$

or, $2.82 = \frac{1}{2\pi \times 50 \times C}$

or, $C = \frac{1}{2\pi \times 50 \times 2.82}$

$\therefore$ Capacitance (C) = 1128.75 μF

Equation for voltage,

$v(t) = V_{max} \sin(\omega t + \phi) = 200 \times \sqrt{2} \sin(2\pi \times 50\,t + 0°)$

[Taking voltage as reference thus $\phi = 0$ at time t]

$= 200 \times \sqrt{2}\ \sin 314\,t$

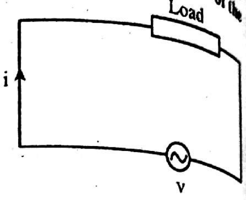
$\therefore v(t) = 282.84 \sin 314\,t$

Equation for current,

$i(t) = I_{max} \times \sin(\omega t + \phi) = 53.84 \times \sqrt{2} \sin(2\pi \times 50t + 49.46°)$

$= 76.14 \sin(314\,t + 49.46°)$

4. An alternating current of 1.5 A flows in a circuit when applied voltage is 300V. The power consumed is 225 W. Find resistance and reactance of the circuit.



Solution:

Current (I) = 1.5 A

Voltage (V) = 300 V

Power consumed (P) = 225 W

$P = VI \cos\phi$

or, $225 = 300 \times 1.5 \times \cos\phi$

or, $\cos\phi = \frac{225}{300 \times 1.5} = 0.5$

$\therefore \phi = \cos^{-1}(0.5) = 60°$

Impedance of the circuit

$Z = \frac{V}{I} = \frac{300}{1.5} = 200\,\Omega$

Resistance (R) = $Z\cos\phi = 200 \times 0.5 = 100\,\Omega$

Reactance (X) = $\sqrt{Z^2 - R^2} = \sqrt{(200)^2 - (100)^2} = 173.2\,\Omega$

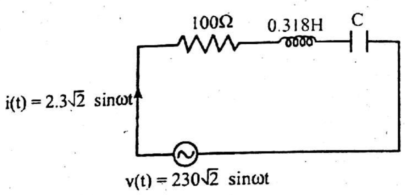
5. A series RLC circuit consists of a 100Ω resistor, an inductor of 0.318 H and capacitor of unknown value. When the circuit is energised by $230\sqrt{2}$ sin ωt volt supply, the current was found to be i = $2.3\sqrt{2}$ sin ωt amperes. Find.

(i) the value of the capacitance

(ii) the voltage across the inductor

(iii) the total power consumed, assume ω = 314.5

Solution:



$v(t) = 230\sqrt{2}\sin\omega t$

$\tilde{V} = \frac{230\sqrt{2}}{\sqrt{2}}\angle 0° = 230\angle 0°\text{ V}$

$i(t) = 2.3\sqrt{2}\sin\omega t$

$\tilde{I} = \frac{2.3\sqrt{2}}{\sqrt{2}}\angle 0° = 2.30\angle 0°\text{ A}$

Impedance of the circuit

$\tilde{Z} = \frac{\tilde{V}}{\tilde{I}} = \frac{230\angle 0°}{2.3\angle 0°} = 100\,\Omega$

We have, Resistance (R) = 100 Ω

Inductive reactance (X_L) = $\omega L = 314.5 \times 0.318 = 100.01\Omega \approx 100\,\Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

or, $100 = \sqrt{(100)^2 + (100 - X_c)^2}$

or, $(100)^2 = (100)^2 + (100 - X_c)^2$

$\therefore X_c = 100\,\Omega$

Capacitive reactance $X_c = \frac{1}{\omega C}$

or, $C = \frac{1}{\omega X_c} = \frac{1}{314.5 \times 100}$

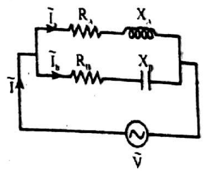
$\therefore$ Capacitance (C) = 31.8 μF

Voltage across inductor,

$\tilde{V}_L = \tilde{I}\,jX_L = (2.3\angle 0°) \times (j100) = 230\angle 90°\text{ V}$

Power consumed (P) = $VI\cos\phi = 230 \times 2.3 \times \cos 0° = 529$ W

6. The currents in each branch of a two branched parallel circuit are given by expressions $i_a = 7.07\sin(314t - \frac{\pi}{4})$ and $i_b = 21.2\sin(314t + \frac{\pi}{3})$. The supply voltage is given by v = 354 sin 314 t. Calculate supply current and the ohmic values of components assuming two pure components in each branch. State whether the reactive components are inductive or capacitive.



Solution:

Given,

v = 354 Sin 314t

$\tilde{V} = \frac{354}{\sqrt{2}}\angle 0° = 250.3\angle 0°\text{ V}$

$i_a = 7.07\sin(314t - \frac{\pi}{4})$

$\tilde{I}_a = \frac{7.07}{\sqrt{2}}\angle -\frac{\pi}{4} = 4.99\angle -45°$

$\approx 5\angle -45°\text{ A}$

$i_b = 21.2\sin(314t + \frac{\pi}{3})$

$\tilde{I}_b = \frac{21.2}{\sqrt{2}}\angle\frac{\pi}{3} = 14.99\angle 60° = 15\angle 60°\text{ A}$

Supply current,

$\tilde{I} = \tilde{I}_a + \tilde{I}_b = (5\angle -45°) + (15\angle 60°) = 14.53\angle 40.59°\text{ A}$

Expression for supply current,

$i = 14.53 \times \sqrt{2}\sin(314t + 40.59°) = 20.54\sin(314t + 40.59°)\text{ A}$

Impedance of branch A,

$$Z_A = \frac{\tilde{V}}{\tilde{I}_a} = \frac{250.3\angle 0^\circ}{5\angle -45^\circ} = 35.39 + j\,35.39\,\Omega$$

Since impedance is in form R + j X;

Resistance of branch A

$R_A = 35.39\,\Omega$

Reactance of branch A,

$X_A = 35.39\,\Omega$

Thus, above reactance is inductive in nature as $Z = R + jX_L$

Impedance of branch B,

$$Z_B = \frac{\tilde{V}}{\tilde{I}_b} = \frac{250.3\angle 0^\circ}{15\angle 60^\circ} = 8.34 - j\,14.45\,\Omega$$

Since impedance is in form R – j X;

Resistance of brance B

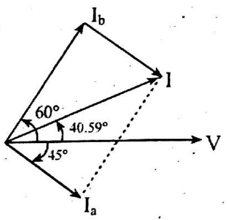
$R_B = 8.34\,\Omega$

Reactance of branch B

$X_B = 14.45\,\Omega$

Thus, above reactance is capacitive in nature as $Z = R - jX_c$

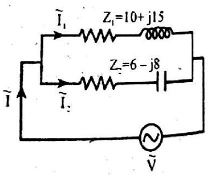
Phasor diagram.



7. **The circuits with impedances $Z_1 = (10 + j15)\ \Omega$ and $Z_2 = (6 - j8)\ \Omega$ are connected in parallel. If the supply current is 20A, what is the power dissipated in each branch.**

Solution:

Given,



$Z_1 = 10 + j15\,\Omega$

$Z_2 = 6 - j8\,\Omega$

Let,

$\tilde{I} = 20\angle 0^\circ$ A

Total impedance of the circuit

$$Z = Z_1 \parallel Z_2 = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$$

$$= \frac{(10 + j15) \times (6 - j8)}{(10 + j15) + (6 - j8)} = 9.67 - j3.61\,\Omega$$

Now, $\tilde{V} = \tilde{I}Z$

$= (20\angle 0^\circ) \times (9.67 - j3.61) = 206.44\angle -20.47^\circ$ V

Branch currents,

$$\tilde{I}_1 = \frac{\tilde{V}}{Z_1} = \frac{206.44\angle -20.47^\circ}{10 + j15} = 11.45\angle -76.78^\circ \text{ A}$$

$$\tilde{I}_2 = \frac{\tilde{V}}{Z_2} = \frac{206.44\angle -20.47^\circ}{6 - j8} = 20.64\angle 32.66^\circ \text{ A}$$

Power dissipated in branch 1,

$P_1 = VI_1 \cos\phi_1$

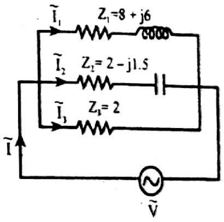
$= 206.44 \times 11.45 \times \cos(-20.47^\circ - (-76.78^\circ)) = 1311.16$ W

Power dissipated in branch 2,

$P_2 = VI_2 \cos\phi_2$

$= 206.44 \times 20.64 \times \cos(-20.47^\circ - 32.66^\circ) = 2556.56$ W

8. **Three impedances $Z_1 = (8 + j6)\Omega$, $Z_2 = (2 - j1.5)\Omega$ and $Z_3 = 2\Omega$ are connected in parallel across a 50 Hz supply. If the current through Z_1 is $(3 + j4)$ A, calculate the current through the other impedances and also the power absorbed by this parallel circuit.**



Given, $I_1 = 3 + j4$ A $= 5\angle 53.13^\circ$ A

$Z_1 = 8 + j6\,\Omega$

$Z_2 = 2 - j\,1.5\,\Omega$

$Z_3 = 2\,\Omega$

Now,

$\tilde{V} = \tilde{I}_1 Z_1 = (3 + j4) \times (8 + j6) = j50 = 50\angle 90^\circ$ V

Current through Z_2

$$\tilde{I}_2 = \frac{\tilde{V}}{Z_2} = \frac{50\angle 90^\circ}{2 - j1.5} = (-12 + j16) = 20\angle 126.87^\circ \text{ A}$$

Current through Z_3

$$\tilde{I}_3 = \frac{\tilde{V}}{Z_3} = \frac{50\angle 90^\circ}{2} = j25 = 25\angle 90^\circ \text{ A}$$

Supply current

$\tilde{I} = \tilde{I}_1 + \tilde{I}_2 + \tilde{I}_3$

$= (3+j4) + (-12+j16) + (j25) = -9 + j45 = 45.89 \angle 101.31° \text{ A}$

Power absorbed by the circuit

$P = VI \cos\phi = 50 \times 45.89 \times \cos(90° - 101.31°) = 2249.94 \text{ W}.$

9. In the circuit shown in figure calculate

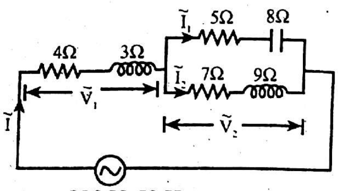
(i) the impedance of the entire circuit

(ii) total current

(iii) overall power factor

(iv) current in each parallel branch

Also draw the phasor diagram showing applied voltage and VARious currents.



220 V, 50 Hz

Solution:

$\tilde{V} = 220 \angle 0° \text{ V}$

impedance of the entire circuit.

$Z = (4+j3) + [(5-j8) \,||\, (7+j9)]$

$= (4+j3) + \left[\dfrac{(5-j8) \times (7+j9)}{(5-j8)+(7+j9)}\right]$

$= (4+j3) + (8.78 - j1.65) = 12.78 + j\,1.35 \ \Omega$

Total current

$\tilde{I} = \dfrac{\tilde{V}}{Z}$

$= \dfrac{220\angle 0°}{12.78 + j1.35} = 17.12 \angle -6.03° \text{ A}$

Power factor,

$pf = \cos\phi = \cos(0° - (-6.03°)) = 0.994 \text{ (lag)}$

Voltage drop across $(4 + j3)\ \Omega$,

$\tilde{V}_1 = \tilde{I}\,(4+j3)$

$= (17.12 \angle -6.03°) \times (4+j3) = 85.6 \angle 30.84° \text{ V}$

Voltage drop across parallel branch.

$\tilde{V}_2 = \tilde{V} - \tilde{V}_1 = (220\angle 0°) - (85.6 \angle 30.84°)$

$= 152.93 \angle -16.67° \text{ V}$

Current in branch $Z_1 = (5 - j8)\ \Omega$

$\tilde{I}_1 = \dfrac{\tilde{V}_2}{Z_1} = \dfrac{152.93 \angle -16.67°}{(5-j8)}$

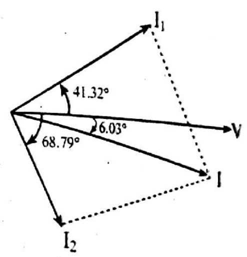
$= 16.21 \angle 41.32° \text{ A}$

Current in branch $Z_2 = (7 + j9)\ \Omega$

$\tilde{I}_2 = \dfrac{\tilde{V}_2}{Z_2} = \dfrac{152.93 \angle -16.67°}{(7+j9)}$

$= 13.41 \angle -68.79° \text{ A}$

Phasor diagram



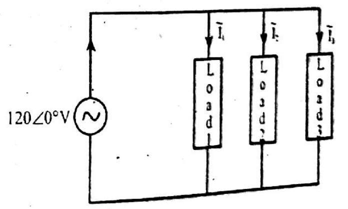
10. **Three loads are connected in parallel to a 120∠0° Vrms source. Load 1 absorbs 60 k VAR at pf = 0.85 lagging, load 2 absorbs 90 kW and 50 k VAR leading, and load 3 absorbs 100kW at pf = 1.**

(a) **Find the equivalent impendance.**

(b) **Calculate the power factor of the parallel combination.**

(c) **Determine the current supplied by the source.**

Solution:



Given, Load 1 absorbs 60 kVAR at pf = 0.85 lagging

Load 2 absorbs 90 kW nd 50kVAR leading

Load 3 absorbs 100 kW at pf = 1

Load 1:

$Q_1 = 60 \text{ kVAR}, \ pf_1 = 0.85 \text{ (lag)}$

$\cos\phi_1 = 0.85$

$\therefore \quad \phi_1 = Cos^{-1}(0.85) = 31.79°$

$Q_1 = VI_1 \, Sin\phi_1$

or, $60 \text{ kVAR} = 120 \times I_1 \times Sin(31.79°)$

or, $I_1 = \dfrac{60 \times 1000}{120 \times Sin(31.79°)}$

$\therefore \quad I_1 = 949.11 \text{ A}$

$Z_1 = \dfrac{\tilde{V}}{\tilde{I}} = \dfrac{120\angle 0°}{949.11\angle -31.79°} = 0.107 + j0.066\ \Omega$

Load 2

$P_2 = 90 \text{ kW}\ Q_2 = 50 \text{ kVAR leading.}$

$P_2 = VI_2 Cos\phi_2$

or, $90 \times 1000 = 120 \times I_2 \times Cos\phi_2$ .... (i)

$Q_2 = VI_2 Sin\phi_2$

or, $50 \times 1000 = 120 \times I_2 \times Sin\phi_2$ .... (ii)

Dividing (ii) by (i).

$\dfrac{50 \times 1000}{90 \times 1000} = \dfrac{120 \times I_2 \times Sin\phi_2}{120 \times I_2 \times Cos\phi_2}$

or, $\dfrac{5}{9} = tan\phi_2$

or, $\phi_2 = tan^{-1}(5/9)$

$\therefore \quad \phi_2 = 29.054°$

$\therefore \quad P_2 = VI_2 Cos\phi_2$

or, $90 \times 1000 = 120 \times I_2 \times Cos(29.054°)$

or, $I_2 = \dfrac{90 \times 1000}{120 \times Cos(29.054°)}$

or, $I_2 = 857.96 \text{ A}$

$Z_2 = \dfrac{\tilde{V}}{\tilde{I_2}} = \dfrac{120\angle 0°}{857.96\angle 29.054°} = 0.122 - j0.067\ \Omega$

Load 3

$P_3 = 100\text{kW},\ pf_3 = 1$

$Cos\phi_3 = 1$

$\therefore \quad \phi_3 = Cos^{-1}(1) = 0°$

$P_3 = 100 \text{ kW}$

or, $VI_3 Cos\phi_3 = 100 \times 1000$

or, $120 \times I_3 \times 1 = 100 \times 1000$

or, $I_3 = \dfrac{100 \times 1000}{120}$

$\therefore \quad I_3 = 833.33 \text{ A}$

$Z_3 = \dfrac{\tilde{V}}{\tilde{I_3}} = \dfrac{120\angle 0°}{833.33\angle 0°} = 0.144\ \Omega$

a) Total impendance = ?

total supply current $\tilde{I} = \tilde{I_1} + \tilde{I_2} + \tilde{I_3}$

$= (949.11\angle -31.79°) + (857.96\angle 29.054°) + (833.33\angle 0°)$

$= 2391.51\angle -1.997° \text{ A}$

total impendance $Z = \dfrac{\tilde{V}}{\tilde{I}}$

$= \dfrac{120\angle 0°}{2391.51\angle -1.997°} = 0.05014 + j1.7485 \times 10^{-3}\ \Omega$

$= 50.14 + j1.7485 \text{ m}\Omega$

(b) Power factor of parallel combination

$pf = Cos\phi$

$= Cos(0° - (-1.997°))$

$= 0.9994 \text{ (lagging)}$

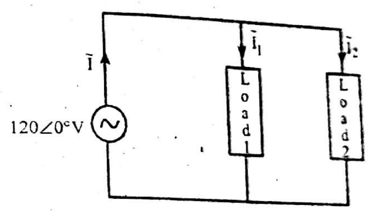
(c) Current supplied by source

$\tilde{I} = 2391.51\angle -1.997° \text{A}$

$= 2.392\angle -1.997° \text{ kA}$

11. **Two loads connected in parallel draw a total of 2.4 kW at 0.8 pf lagging from a 120–V rms, 60-Hz line. One load absorbs 1.5 kW at a 0.707 pf lagging. Determine:**
 (a) **The pf of the second load.**
 (b) **The parallel element required to correct the pf to 0.9 lagging for the two loads.**

Solution:



Given,

120 Vrms, 60 Hz line

Total power $P = 2.4 \text{ kW at } pf = 0.8 \text{ lagging}$

Load 1 $P_1 = 1.5 \text{ kW},\ pf_1 = 0.707 \text{ lagging}$

$pf = Cos\phi$

or, $0.8 = Cos\phi$

or, $\phi = Cos^{-1}(0.8)$

$\therefore \quad \phi = 36.86°$

$P = VI\, Cos\phi$

or, $2.4 \times 1000 = 120 \times I \times Cos(36.86°)$

or, $I = \dfrac{2.4 \times 1000}{120 \times Cos(36.86°)}$

$\therefore \quad I = 24.99 \approx 25 \text{ A}$

$\therefore \quad \tilde{I} = 25\angle -36.86° \text{ A}$

$pf_1 = 0.707$

or, $Cos\phi_1 = 0.707$

or, $\phi_1 = Cos^{-1}(0.707)$

$\therefore \quad \phi_1 = 45.01°$

$P_1 = VI_1 \cos\phi_1$

or, $1.5\times1000 = 120\times I_1\times \cos(45.01°)$

or, $I_1 = \dfrac{1.5\times1000}{120\times\cos(45.01°)}$

$\therefore \quad I_1 = 17.68$ A

$\therefore \quad \tilde{I}_1 = 17.68\angle{-45.01°}$ A

Using KCL,

$\tilde{I} = \tilde{I}_1 + \tilde{I}_2$

or, $\tilde{I}_2 = \tilde{I} - \tilde{I}_1$

$= (25\angle{-36.86°}) - (17.68\angle{-45.01°})$

$\therefore \quad \tilde{I}_2 = 7.9\angle{-18.37°}$ A

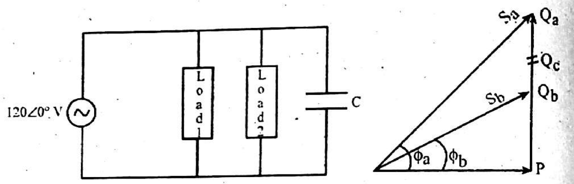
$\therefore \quad pf_2 = \cos\phi_2 = \cos(0°-(-18.37°))$

$= 0.949$ (lagging)

$\therefore$ The pf of the second load = 0.949 (lagging)

Now,

To correct the pf to 0.9 lagging for the two loads capacitor must be connected.



Reactive power supplied by capacitor

$Q_2 = Q_a - Q_b$

$= P\tan\phi_a - P\tan\phi_b$

$= P(\tan\phi_a - \tan\phi_b)$

$[\phi_a = 36.86°$ & pf = 0.9

$\phi_b = \cos^{-1}(0.9) = 25.84°]$

$Q_2 = 2.4\times1000\,(\tan(36.86°) - \tan(25.84°))$

$= 637.08$ VAR.

We know,

$C = \dfrac{Q_c}{\omega V^2} = \dfrac{637.08}{2\pi\times f\times V^2}$

$= \dfrac{637.08}{2\pi\times60\times(120)^2} = 1.1735\times10^{-4}$ F

$= 117.35\ \mu$F

$\therefore$ The required capacitance is 177.35 μF.

❁ ❁ ❁

7

THREE PHASE CIRCUIT ANALYSIS

Generation of a Three Phase Supply

When three identical coils are placed with their axes at 120° apart from each other in the presence of uniformly rotating magnetic field, a sinusoidal voltage is generated across each coil according to Faraday's law of electromagnetism. This is the working principle of a three - phase generator. It has three sets of coils RR', YY' and BB' kept 120° apart from each other mounted on a rotor. When the rotor (magnet) is rotated in clockwise direction with a constant angular velocity of ω rad/s, the flux linkage associated with the coil changes with respect to time.

Hence, according to Faraday's law of electromagnetism, emf will induce in all three coils.

Since, rotor is rotating with constant angular velocity ω, the generated voltages have same frequency. Also, since the coils are identical, the generated voltages have the same magnitudes, but there is a phase difference of 120° between these voltages.

The generated voltage (emfs) in the coils are given by

$e_R = E_m \sin\omega t$

$e_Y = E_m \sin(\omega t - 120°)$

$e_B = E_m \sin(\omega t - 240°)$

$= E_m \sin(\omega t + 120°)$ [Note: phase angle of −240° is same as + 120°]

In polar form

$\tilde{E}_R = E\angle 0°$

$\tilde{E}_Y = E\angle -120°$

$\tilde{E}_B = E\angle -240° = E\angle +120°$

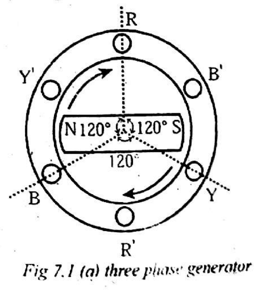


Fig 7.1 (a) three phase generator

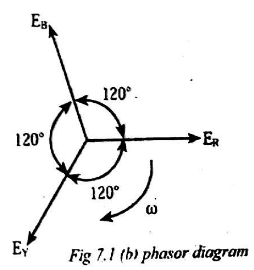


Fig 7.1 (b) phasor diagram

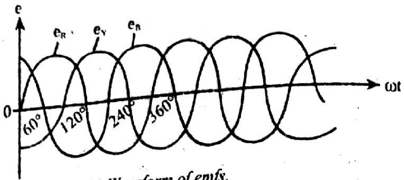


Fig 7.1 (c) Waveform of emfs.

Here, E_R leads E_Y by 120° and E_Y leads E_B by 120°. Also, emfs attain their maximum value in order $E_{RR'}$, $E_{YY'}$ and $E_{BB'}$.

Phase sequence /Phase order: The sequence in which the emf induced in the three - phase attain their peaks is known as phase sequence. For the arrangement shown in figure in which the coils are rotating in clockwise direction, the phase sequence is RYB. If the rotor is rotated in anticlockwise direction, the emfs attain their peaks in order $E_{BB'}$, $E_{YY'}$, $E_{RR'}$. In this case phase sequence is BYR or RBY. Thus phase sequence determines direction of rotation.

Advantages of three phase system over single phase system.

The main advantages of three - phase system over single-phase system are –

(1) A three – phase machine has a smaller size than a single – phase machine of the same power output.

(2) The conductor material required to transmit a given power at a given voltage over a given distance by a three - phase system is less than that by an equivalent single phase system.

(3) In single phase system the power delivered is pulsating and becomes zero at certain intervals. In three phase system power delivered to the load is constant though power of one phase may be negative.

Methods of Connections of Three – Phase system

There are two methods of interconnecting the three phases. They are called star and delta connections.

1. Star or Wye (Y) Connection

If three similar ends are connected together then they form star connected 3ϕ system. The free ends are connected to the external circuit through three conductors called lines. The point N is called neutral point where the three similar ends are connected and wire brought out from neutral point is called neutral wire. The three line conductors and a neutral wire provide a three-phase, four-wire supply. The neutral point is connected to ground.

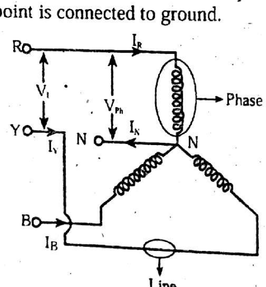


Fig 7.2 (a) 3ϕ 4 wire star connected system

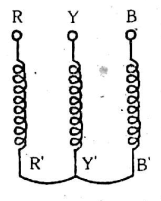


Fig 7.2 (b) Star conn... on

Phase voltage– It is the voltage across a phase coil. In other words, it is the voltage between any line to neutral. It is denoted by V_{ph} or V_P.

In star connection,

Phase Voltage– V_{RN}, V_{BN}, V_{YN}

Line voltage– It is the voltage between two lines. It is denoted by V_L.

In Y connection,

Line Voltage– V_{RY}, V_{YB}, V_{BR}

Phase current – It is the current through phase coil. It is denoted by I_{ph} or I_P.

In Y connection,

Phase Current - I_R, I_Y, I_B

Line current – It is the current through the external wire/line connecting the source and load. It is denoted by I_L.

In Y connection,

Line current – I_R, I_Y, I_B

In star connection,

$$\boxed{\begin{matrix} I_L = I_{Ph} \\ V_L \neq V_{Ph} \end{matrix}}$$

2. Delta or Mesh (Δ) Connection

If the dissimilar ends are connected in such a way that they form a loop, the system is said to be 3ϕ delta connected system.

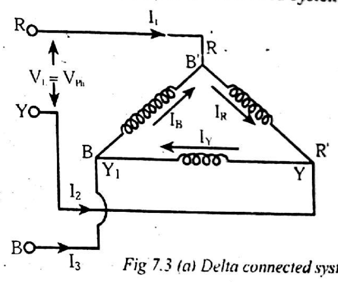


Fig 7.3 (a) Delta connected system

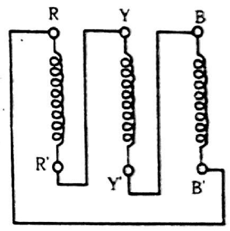


Fig 7.3 (b) Delta connection

In Δ system,

Phase voltages – V_{RY}, V_{YB}, V_{BR}

Phase currents – I_R, I_Y, I_B

Line voltages – V_{RY}, V_{YB}, V_{BR}

Line current – I_1, I_2, I_3

In Δ connection,

$$\boxed{\begin{matrix} V_L = V_{Ph} \\ I_L \neq I_{Ph} \end{matrix}}$$

In Δ system,

Phase voltag... - V_{RY}, V_{YB}, V_{BR}

Phase currents – I... I_Y, I_B

Line voltages – $V_{..}$, V_{YB}, V_{BR}

Line current – I_1, I_2, I_3

In Δ connection,

$$\boxed{\begin{array}{l} V_L = V_{Ph} \\ I_L \neq I_{Ph} \end{array}}$$

Balanced System

A 3ϕ balanced system is one in which

1. all phase voltages are equal in magnitude and displaced from one another by 120°.
2. all phase currents are equal in – magnitude and displaced from one another by 120°.

A 3ϕ balanced load is that in which the loads connected across three phases are identical.

Voltage, Current and Power relation in 3ϕ Y – connected balanced system

From figure:

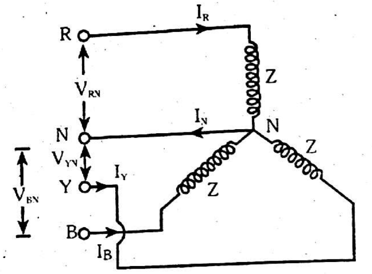


Fig 7.4 (a) 3ϕ Y- connected balanced system

$$I_L = I_{Ph}$$
$$I_R = I_Y = I_B = I_L$$

Let the phase sequence be RYB

$$\tilde{V}_{RN} = V_{Ph} \angle\ 0°$$
$$\tilde{V}_{YN} = V_{Ph} \angle -120°$$
$$\tilde{V}_{BN} = V_{Ph} \angle\ 120°$$

Therefore,

$$\tilde{I}_R = \frac{\tilde{V}_{RN}}{z} = \frac{V_{Ph} \angle\ 0°}{Z \angle\ \phi} = \frac{V_{Ph}}{Z} \angle -\phi$$

$$\tilde{I}_Y = \frac{\tilde{V}_{YN}}{z} = \frac{V_{Ph} \angle -120°}{Z \angle\ \phi} = \frac{V_{Ph}}{Z} \angle -120° - \phi$$

$$\tilde{I}_B = \frac{\tilde{V}_{BN}}{z} = \frac{V_{Ph} \angle\ 120°}{Z \angle\ \phi} = \frac{V_{Ph}}{Z} \angle\ 120° - \phi$$

It is seen that the magnitude of each line current is $\frac{V_{Ph}}{Z}$ and line current are displaced by 120° phase angle from each other.

KCL at point N, $\tilde{I}_N = \tilde{I}_R + \tilde{I}_Y + \tilde{I}_B$

$$= \frac{V_{ph}}{Z} \angle (-\phi) - \frac{V_{ph}}{Z} \angle (-120° - \phi) + \frac{V_{ph}}{Z} \angle (120° - \phi) = 0$$

Hence, phasor sum of line currents must be zero.

Now drawing the phasor diagram.

Also, in a balanced three phase star-connected load the current in the neutral wire is zero.

Note: Steps to draw phasor diagram

1. First draw phasor voltage & phasor current.
2. Then draw line quantities.

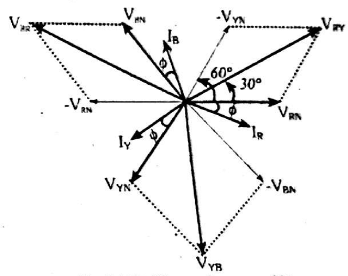


Fig 7.4 (b) Phasor diagram of 3ϕ Y- connected balanced system

From figure,

$V_{RY} = V_{RN} + V_{NY}$	$V_{YB} = V_{YN} + V_{NB}$	$V_{BR} = V_{BN} + V_{NR}$
$= V_{RN} - V_{YN}$	$= V_{YN} - V_{BN}$	$= V_{BN} - V_{RN}$

From figure,

$$V_{RY} = V_{RN} + V_{NY}$$

or, $V_{RY} = V_{RN} - V_{YN}$

or, $V_{RY} = \sqrt{(V_{RN})^2 + (V_{YN})^2 + 2V_{RN} V_{YN} \cos 60°}$

or, $V_L = \sqrt{V_{Ph}^2 + V_{Ph}^2 + 2V_{Ph} V_{Ph} \left(\frac{1}{2}\right)}$

or, $V_L = \sqrt{3V_{Ph}^2}$

∴ $V_L = \sqrt{3}V_{Ph}$

∴ In star connected balanced system

$$\boxed{V_L = \sqrt{3}\ V_{Ph}}$$

Active power (P) = $P_R + P_Y + P_B$

$$= V_{RN}\, I_R \cos\phi + V_{YN} I_Y \cos\phi + V_{BN} I_B \cos\phi$$
$$= V_{Ph}\, I_{Ph} \cos\phi + V_{Ph} I_{Ph} \cos\phi + V_{Ph} I_{Ph} \cos\phi$$

$$\therefore P = 3\,V_{Ph}\,I_{Ph}\cos\phi$$
$$= 3 \times \frac{V_L}{\sqrt{3}} \times I_L \cos\phi$$
$$= \sqrt{3}\,V_L\,I_L\cos\phi$$

$\therefore P = 3V_{Ph}\,I_{Ph}\cos\phi \quad = \sqrt{3}\,V_L\,I_L\cos\phi$

Reactive power (Q) $= Q_R + Q_Y + Q_B = V_{RN}\,I_R\sin\phi + V_{YN}\,I_Y\sin\phi + V_{BN}\,I_B\sin\phi$

$\therefore Q \quad = 3V_{Ph}\,I_{Ph}\sin\phi = 3 \times \frac{V_L}{\sqrt{3}} \times I_L\sin\phi = \sqrt{3}\,V_L I_L\sin\phi$

$\therefore Q = 3V_{Ph}\,I_{Ph}\sin\phi = \sqrt{3}\,V_L I_L\sin\phi$

Thus, in Y connection, we conclude that

(i) $I_L = I_{Ph}$

$V_L = \sqrt{3}\,V_{Ph}$

(ii) Line voltages are 120° apart and also are the phase voltages.

(iii) Line voltage leads the corresponding phase voltage by 30°.

(iv) the phase difference between line currents and corresponding line voltages is (30° + ϕ) with current lagging.

Voltage, Current and Power relation in 3ϕ Δ – connected system

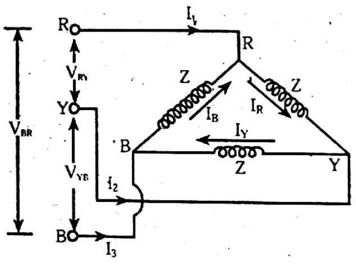


Fig 7.5 (a) 3ϕ Delta connected balanced system

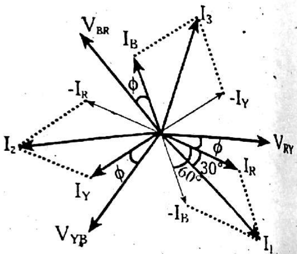


Fig 7.5 (b) Phasor diagram of 3ϕ Delta connected balanced system

Form figure,

$V_L = V_{Ph}$

Let the phase sequence by RYB

Now, drawing the phasor diagram

Note:

1. First draw phase quantities.
2. Then draw line quantities.

Applying KCL at node R

$I_1 + I_B \;=\; I_R$

$I_1 \;=\; I_R - I_B$

at node Y

$I_2 + I_R = I_Y$

$I_2 = I_Y - I_R$

at node B

$I_3 + I_Y = I_B$

$I_3 = I_B - I_Y$

or, $\quad I_L \;=\; \sqrt{I_R^2 + I_B^2 + 2\,I_R I_B\cos 60°}$

or, $\quad I_L \;=\; \sqrt{I_{Ph}^2 + I_{Ph}^2 + 2I_{Ph}I_{Ph}\left(\frac{1}{2}\right)}$

or, $\quad I_L \;=\; \sqrt{3}\,I_{Ph}$

In Δ– connected system: $\boxed{I_L = \sqrt{3}\,I_{Ph}}$

Active power (P) $= P_R + P_Y + P_B$

$= V_{RY}\,I_R\cos\phi + V_{YB}\,I_Y\cos\phi + V_{BR}\,I_B\cos\phi$

$= 3\,V_{Ph}\,I_{Ph}\cos\phi = 3 \times V_L \times \frac{I_L}{\sqrt{3}}\cos\phi$

$= \sqrt{3}\,V_L I_L\cos\phi$

$\therefore P = 3\,V_{Ph}\,I_{Ph}\cos\phi = \sqrt{3}\,V_L\,I_L\cos\phi$

Similarly, Reactive power (Q) $= 3V_{Ph}\,I_{Ph}\sin\phi = \sqrt{3}V_L\,I_L\sin\phi$

Thus in Δ– connected system, we conclude that

i) $V_L = V_{Ph}$

$I_L = \sqrt{3}\,I_{Ph}$

(ii) Line currents are 120° apart and also are the phase currents.

(iii) Line currents are 30° behind the respective phase currents.

(iv) The phase difference between line currents and corresponding line

Dynamometer Wattmeter

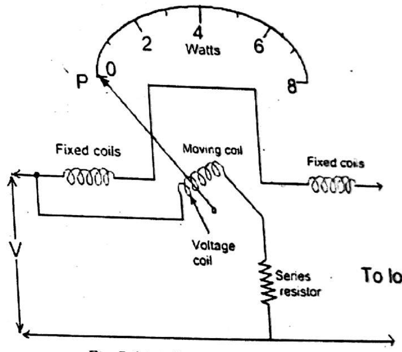


Fig 7.6 (a) Dynamometer Wattmeter

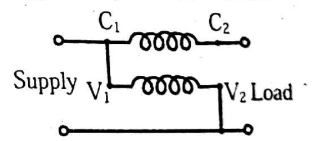


Fig 7.6 (b)Schematic diagram of Dynamometer Wattmeter

A dynamometer wattmeter is a moving coil instrument in which the magnetic field is produced not by a permanent magnet but by fixed coils. The fixed coils are usually arranged in two equal sections F and F placed together and parallel to each other.

The deflecting torque produced in the moving coil is proportional to the product of the magnetic flux density produced by the fixed coil and the current passing through the moving coil.

$\therefore \quad T_d \propto B \times I_2$

But flux produced by the fixed coil is proportional to the current flowing through fixed coil.

$B \propto I_1$

$\boxed{\therefore T_d \propto I_1 \times I_2}$

For dc circuit,

$T_d \propto I_1 I_2$ But $I_2 \propto V$

$\therefore T_d \propto VI_1$

$\therefore T_d \propto$ Power

For ac circuit

$T_d \propto$ average value of $v \times i$

$$T_d \propto \frac{1}{2\pi}\int_0^{2\pi} v \times i \, d\omega t$$

$$T_d \propto \frac{V_m I_m}{2\pi}\int_0^{2\pi} \sin\omega t \sin(\omega t - \phi) \, d\omega t$$

voltages is (30° + ϕ) which is lagging

$$T_d \propto \frac{V_m I_m}{2\pi}\int_0^{2\pi} \frac{\cos\phi - \cos(2\omega t - \phi)}{2} \, d\omega t$$

$$T_d \propto \frac{V_m I_m}{4\pi}\left[\cos\phi \times \omega t - \frac{\sin(2\omega t - \phi)}{2}\right]_0^{2\pi}$$

$$T_d \propto \frac{V_m I_m}{4\pi}\cos\phi \times 2\pi$$

$$T_d \propto \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}\cos\phi$$

$$T_d \propto V_{rms} \times I_{rms}\cos\phi$$

$\therefore T_d \propto$ Active power

Hence, deflection is proportional to active power in ac circuit.

Power Measurement in 1ϕ system by wattmeter

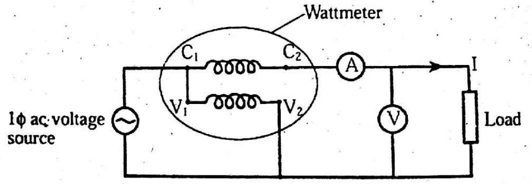


Fig 7.7 Power measurement in single phase circuit

Wattmeter reading

$W = VI\cos\phi$

where,

$\cos\phi$ = Power factor of the load

Now, $\cos\phi = \frac{W}{VI}$ can be calculated

Then, reactive power can be calculated as

$Q = VI \sin\phi$

Power measurement in 3ϕ system by two wattmeter method.

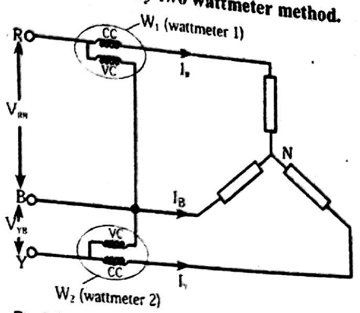


Fig 7.8 (a) Power measurement in 3 phase system

The total power in 3ϕ balanced as well as unbalanced load can be measured by using two-wattmeter method.

As shown in figure, the current coil of W_1 measures I_R, while its potential coil measures line voltage V_{RB}. Similarly, the current coil of W_2 measures I_Y, while its potential coil measures line voltage V_{YB}.

Let an inductive load having power factor cos ϕ is connected to each phase, then the phase current lags the corresponding phase voltage by an angle ϕ as shown in the phasor.

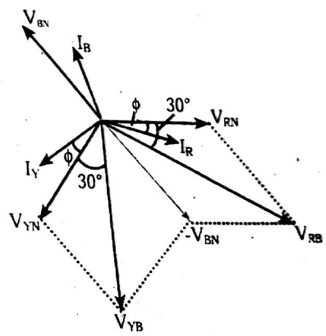


Fig 7.8 (b) Phaser diagram

For W_1,

$V_{RB} = V_{RN} + V_{NB} = V_{RN} - V_{BN}$

For W_2,

$V_{YB} = V_{YN} + V_{NB} = V_{YN} - V_{BN}$

From phasor diagram,

I_R leads V_{RB} by an angle $(30° - \phi)$

Thus, the reading of wattmeter $W_1 = V_{RB} I_R \cos(30° - \phi)$

Similarly,

I_Y lags V_{YB} by angle $(30° + \phi)$

Thus, the reading of wattmeter $W_2 = V_{YB} I_Y \cos(30° + \phi)$

Now, Total active power of the circuit:

$$W = W_1 + W_2$$
$$= V_{RB}\, I_R \cos(30^\circ - \phi) + V_{YB}\, I_Y \cos(30^\circ + \phi)$$
$$= V_L I_L [\cos(30^\circ - \phi) + \cos(30^\circ + \phi)]$$
$$= V_L I_L [2 \cos 30^\circ \times \cos\phi]$$
$$= V_L I_L \times 2 \times \frac{\sqrt{3}}{2} \cos\phi = \sqrt{3}\, V_L I_L \cos\phi$$

$$\therefore \boxed{\begin{array}{c} W = W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\phi \\ \text{i.e. Total active power of } 3\phi \text{ circuit} \end{array}} \quad \text{.......... (1)}$$

Similarly,

$$W_1 - W_2 = V_L I_L \cos(30^\circ - \phi) - V_L I_L \cos(30^\circ + \phi)$$
$$= V_L I_L [\cos(30^\circ - \phi) - \cos(30^\circ + \phi)]$$
$$= V_L I_L \times 2 \times \sin 30^\circ \sin\phi$$
$$= V_L I_L \times 2 \times \frac{1}{2} \times \sin\phi = V_L I_L \sin\phi$$

or, $\sqrt{3}\,(W_1 - W_2) = \sqrt{3}\, V_L I_L \sin\phi$

$$\therefore \boxed{\begin{array}{c} Q = \sqrt{3}(W_1 - W_2) = \sqrt{3}\, V_L I_L \sin\phi \\ \text{i.e. Total Reactive power of } 3\phi \text{ circuit} \end{array}} \quad \text{.......... (2)}$$

From (1) and (2),

Dividing (ii) by (i)

$$\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)} = \frac{\sqrt{3}\, V_L I_L \sin\phi}{\sqrt{3}\, V_L I_L \cos\phi}$$

or, $$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

$$\therefore \boxed{\begin{array}{c} \phi = \tan^{-1}\left[\dfrac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right] \\ \text{i.e. Power factor angle of the circuit} \end{array}} \quad \text{.............(3)}$$

Effect of power factor on Wattmeter Reading

$W_1 = V_L I_L \cos(30^\circ - \phi)$

$W_2 = V_L I_L \cos(30^\circ + \phi)$

ϕ = P.f angle of the load

Case 1: When $\phi = 0^\circ$; $\cos\phi = 1$ (unity p.f.)

$W_1 = V_L I_L \cos 30^\circ$

$W_L = V_L I_L \cos 30^\circ$

$\therefore W_1 = W_2$ = equal reading on both wattmeters

$\Rightarrow \frac{W_2}{W_1} = 1$

This is the case for resistive load

Case 2: When $\phi = 60^\circ$; $\cos 60^\circ = 0.5$ (lagging)

$$W_1 = V_L I_L \cos(30^\circ - 60^\circ) = \frac{\sqrt{3}}{2} V_L I_L$$

$$W_2 = V_L I_L \cos(30^\circ + 60^\circ) = 0$$

$$\therefore W = W_1 + W_2 = \frac{\sqrt{3}}{2} V_L I_L$$

$$\Rightarrow \frac{W_2}{W_1} = 0$$

Case 3: When $60^\circ < \phi < 90^\circ \Rightarrow 0.5 > \cos\phi > 0^\circ$

- W_1 reading is positive.

  W_2 reading is negative.

  To obtain reading on W_2, the connection VC/CC (Voltage Coil or Current coil) must be reversed and thus reading obtained will be taken as – ve reading

  $\Rightarrow \frac{W_2}{W_1}$ = (– ve value)

Case 4: When $\phi = 90^\circ$; $\cos 90^\circ = 0$

$W_1 = V_L I_L \cos(30^\circ - 90^\circ)$

$= V_L I_L \sin 30^\circ$

$W_2 = V_L I_L \cos(30^\circ + 90^\circ)$

$= - V_L I_L \sin 30^\circ$

$\therefore W_1 + W_2 = 0$

$\therefore W_1$ and W_2 readings are equal and opposite

$\Rightarrow \frac{W_2}{W_1} = -1$

This is the case for purely Inductive or Capacitive load.

The above analysis is plotted on curve known as watt – ratio curve.

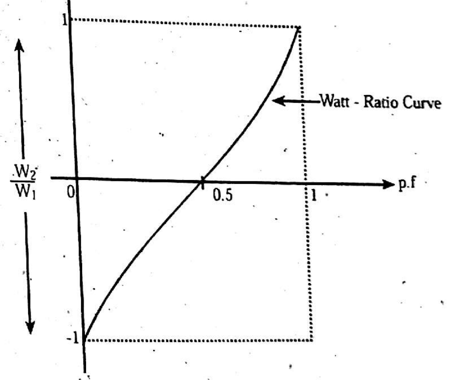


Fig 7.9 Watt Ratio Curve

Hence, if pf of load is less than 0.5 then it shows that one of the wattmeter shows negative reading.

Exam Solutions

1. **Each phase of a 3-Phase, delta-connected load consists of an impendence, Z = 20∠60° ohm. The line voltage is 440 V at 50 Hz. Compute the impendance and the total power. What will be the power consumed by each phase reading of the two wattmeters connected?** **[2071 Chaitra]**

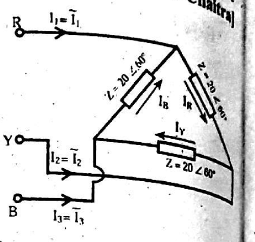
Solution:

Given,

Line voltage (V_L) = 440 V

Frequency (f) = 50 Hz

From figure as Δ-connected,



Line voltage = phase voltage

∴ $V_L = V_{Ph} = 440$ V

Phase Voltages

$$\tilde{V}_{RY} = 440\angle 0° \text{ V}$$

$$\tilde{V}_{YB} = 440\angle -120° \text{V}$$

$$\tilde{V}_{BR} = 440\angle 120° \text{ V}$$

Phase Currents

$$\tilde{I}_R = \frac{\tilde{V}_{RY}}{Z} = \frac{440\angle 0°}{20\angle 60°} = 22\angle -60° \text{ A}$$

$$\tilde{I}_Y = \frac{\tilde{V}_{YB}}{Z} = \frac{440\angle -120°}{20\angle 60°} = 22\angle 180° \text{ A}$$

$$\tilde{I}_B = \frac{\tilde{V}_{BR}}{Z} = \frac{440\angle 120°}{20\angle 60°} = 22\angle 60° \text{ A}$$

OR,

$$\tilde{I}_Y = 22\angle(-60°-120°) = 22\angle -180° = 22\angle 180° \text{ A}$$

$$\tilde{I}_B = 22\angle(-60°+120°) = 22\angle 60° \text{A}$$

Power Consumed by each phase impendance.

∴ Active power per phase (P_R) = $V_{Ph}\, I_{Ph} \cos\phi$

Phase difference $\phi = 0° - (-60°)$

= 60°

∴ P_R = 440×22×Cos 60°

= 4840W

Power consumed by each phase impendance = 4840 W

∴ Total Active power (P) = 3 P_R

= 3×4840

= 14520 W

OR,

Total active power (P) = 3 $V_{Ph}\, I_{Ph} \cos\phi$

= 3×440×22×Cos 60°

= 14520W

Total Reactive Power (Q) = 3 $V_{Ph}\, I_{Ph} \sin\phi$

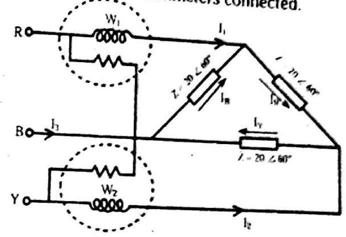
= 3×440×22×Sin60°

= 25149.37 VAR

Apparent Power (S) = $\sqrt{P^2 + Q^2}$

= $\sqrt{(14520)^2 + (25149.37)^2}$ = 29039.99 ≈ 29040 VA

Now, To find the reading of the two wattmeters connected.



Let, W_1 and W_2 be the readings of the two wattmeters connected.

Total Active Power (P) = $W_1 + W_2$.

Or, 14520 = $W_1 + W_2$ ....(i)

Total Reactive Power (Q) = $\sqrt{3}\,(W_1 - W_2)$

or, 25149.37 = $\sqrt{3}\,(W_1 - W_2)$

or, $\frac{25149.37}{\sqrt{3}} = W_1 - W_2$ ....(ii)

Adding equations (i) and (2), we get

$14520 + \frac{25149.37}{\sqrt{3}} = 2W_1$

or, 29039.99 = $2W_1$

∴ W_1 = 14519.99 W ≈ 14520 W

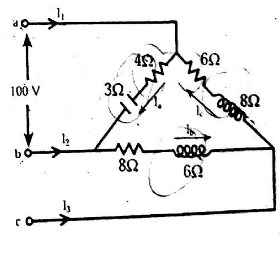
Using equation (i)

$W_1 + W_2 = 14520$

∴ W_2 = 14520 − 14520 = 0

∴ The readings of two wattmeters connected are 14520 W and 0.

2. **Discuss the advantages of three phase ac system over single phase ac system. For the given unbalanced delta connected load, find the phase currents, line currents and total power consumed by the load when phase sequence is abc. Construct the phasor diagram of currents and voltages in the load.** **[2071 Magh]**



Solution:
[Please refer to the theory for first part of the question]
For Δ-connected system

Line voltage = Phase voltage
$V_L = V_{Ph} = 100$ V

Phase Voltages

$\widetilde{V}_{ab} = 100\angle 0°$ V

$\widetilde{V}_{bc} = 100\angle -120°$V

$\widetilde{V}_{ca} = 100\angle 120°$ V

$Z_a = 4 - j3$

$Z_b = 8 + j6$

$Z_c = 6 + j8$

Phase Currents

$$\widetilde{I}_a = \frac{\widetilde{V}_{ab}}{Z_a} = \frac{100\angle 0°}{4-j3} = 20\angle 36.86° \text{ A}$$

$$\widetilde{I}_b = \frac{\widetilde{V}_{bc}}{Z_b} = \frac{100\angle -120°}{8+j6} = 10\angle -156.86° \text{ A}$$

$$\widetilde{I}_c = \frac{\widetilde{V}_{ca}}{Z_c} = \frac{100\angle 120°}{6+j8} = 10\angle 66.86° \text{ A}$$

Line Currents

$\widetilde{I}_1 + \widetilde{I}_c = \widetilde{I}_a$

$\therefore\ \widetilde{I}_1 = \widetilde{I}_a - \widetilde{I}_c$
$= (20\angle 36.86°) - (10\angle 66.86°) = 12.39\angle 13.06°$ A

$\widetilde{I}_2 + \widetilde{I}_a = \widetilde{I}_b$

$\therefore\ \widetilde{I}_2 = \widetilde{I}_b - \widetilde{I}_a$
$= (10\angle -156.86°) - (20\angle 36.86°) = 29.81\angle -147.70°$ A

$\widetilde{I}_3 + \widetilde{I}_b = \widetilde{I}_c$

$\therefore\ \widetilde{I}_3 = \widetilde{I}_c - \widetilde{I}_b$
$= (10\angle 66.86°) - (10\angle -156.86°)$
$= 18.56\angle 45°$ A

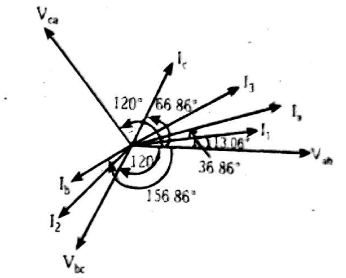
Total Power consumed by load (P) = ?
Active Power in Phase a $(P_a) = V_{ph} I_{ph} \cos\phi$
$= 100 \times 20 \times \cos(0° - 36.86°)$
$= 1600.21$ W

Active Power in Phase b (P_b)
$= 100 \times 10 \times \cos(-120° - (-156.86°))$
$= 800.10$ W

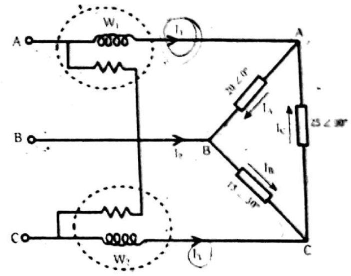
Active Power in Phase c (P_C)
$= 100 \times 10 \times \cos(120° - 66.86°) = 599.86$ W

Total Power consumed by load
(P) $= P_a + P_b + P_c$
$= 1600.21 + 800.10 + 599.86 = 3000.17$ W

Phasor diagram



3. **The supply system is 230 V, 3-Phase, 50 Hz. Determine the reading of wattmeters W_1 and W_2. Phase sequence is AB-BC-CA.** **[2071 Bhadra]**



Solution:
Given, 230V, 3φ, 50 Hz Supply
As Δ-connected system,
$V_L = V_{Ph} = 230$ V

Phase Voltages

$\widetilde{V}_{AB} = 230\angle 0°$ V

$\widetilde{V}_{BC} = 230\angle -120°$V

$\widetilde{V}_{CA} = 230\angle 120°$ V

$Z_A = 20\angle 0°$

$Z_B = 15\angle 30°$

$Z_C = 25\angle 90°$

Phase Currents:

$$\widetilde{I}_A = \frac{\widetilde{V}_{AB}}{Z_A} = \frac{230\angle 0°}{20\angle 0°} = 11.5\angle 0° \text{ A}$$

$$\widetilde{I}_B = \frac{\widetilde{V}_{BC}}{Z_B} = \frac{230\angle -120°}{15\angle 30°} = 15.33\angle -150° \text{ A}$$

$$\overline{I}_C = \frac{\widetilde{V}_{CA}}{Z_C} = \frac{230\angle 120^\circ}{25\angle 90^\circ} = 9.2\angle 30^\circ \text{ A}$$

Line Currents:

KCL at node A.

$$\overline{I}_1 + \overline{I}_C = \overline{I}_A$$

$\therefore$ $\overline{I}_1 = \overline{I}_A - \overline{I}_C = (11.5\angle 0^\circ) - (9.2\angle 30^\circ) = 5.79\angle -52.47^\circ$ A

KCL at node C.

$$\overline{I}_3 + \overline{I}_B = \overline{I}_C$$

$\therefore$ $\overline{I}_3 = \overline{I}_C - \overline{I}_B$
$= (9.2\angle 30^\circ) - (15.33\angle -150^\circ) = 24.53\angle 30^\circ$ A

As shown in figure, the current coil of W_1 measure I_1 and potential coil of W_1 measures V_{AB}.

$\therefore$ Reading of Wattmeter

$W_1 = V_{AB} \times I_1 \times \cos\phi_1$

Where ϕ_1 is phase difference between V_{AB} & I_1

$\therefore$ $W_1 = 230 \times 5.79 \times \cos(0^\circ - (-52.47)) = 811.24$ W

Similarly, the current coil of W_2 measures I_3 and its potential coil measures V_{CB}.

$$\widetilde{V}_{CB} = -\widetilde{V}_{BC} = -(230\angle -120^\circ) = 230\angle 60^\circ \text{ V}$$

$\therefore$ Reading of wattmeter

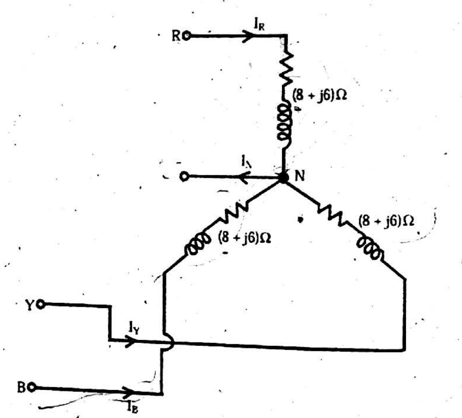
$W_2 = V_{CB} \times I_3 \times \cos\phi_2$

Where ϕ_2 is phase difference between V_{CB} & I_3.

$\therefore$ $W_2 = 230 \times 24.53 \times \cos(60^\circ - 30^\circ) = 4886.02$ W

$\therefore$ The readings of two wattmeter W_1 and W_2 are 811.24 W and 4886.02 W respectively.

4. **Calculate the amount of current through the neutral of a balanced 3–phase star connected circuit. Also verify with the phasor diagram.**
Let a balanced 3–phase star connected load of (8+j6) Ω phase is connected to 3ϕ, 230 V, 50 Hz supply.



Solution:

From figure Y–connected load

Line current = phase current

$\therefore$ $I_L = I_{Ph}$

$$V_{Ph} = \frac{230}{\sqrt{3}} = 132.8 \text{ V}$$

Impendance per phase (Z) = (8 + j6) Ω

$\widetilde{V}_{RN} = 132.8\angle 0^\circ$ V

$\widetilde{V}_{YN} = 132.8\angle -120^\circ$V

$\widetilde{V}_{BN} = 132.8\angle 120^\circ$ V

$$\overline{I}_R = \frac{\widetilde{V}_{RN}}{Z} = \frac{132.8\angle 0^\circ}{8+j6} = 13.28\angle -36.87^\circ \text{ A}$$

$$\overline{I}_Y = \frac{\widetilde{V}_{YN}}{Z} = \frac{132.8\angle -120^\circ}{8+j6} = 13.28\angle -156.87^\circ \text{ A}$$

$$\overline{I}_B = \frac{\widetilde{V}_{BN}}{Z} = \frac{132.8\angle 120^\circ}{8+j6} = 13.28\angle 83.13^\circ \text{ A.}$$

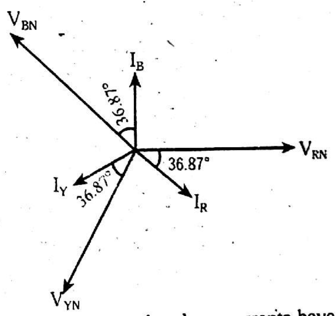
Applying KCL at N,

$\overline{I}_N = \overline{I}_R + \overline{I}_Y + \overline{I}_B$
$= (13.28\angle -36.87^\circ) + (13.28\angle -156.87^\circ) + (13.28\angle 83.13^\circ) = 0$

$\therefore$ $\overline{I}_N = 0$

Thus the current through neutral of a balanced 3 phase star connected circuit is zero.

Phasor diagram



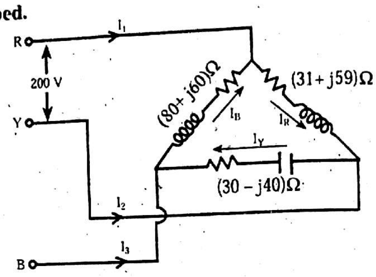
From phasor diagram, it is clear that the phase currents have equal magnitude and are 120° apart from each other thus, their algebraic sum is zero.

5. **A 415 V, 3 Phase, 50 Hz induction motor takes 50 kW power from supply mains at 0.72 power factor lagging. A bank of capacitors is connected in delta across the line to improve the overall power factor. Calculate the capacitance per phase in order to raise power factor to 0.9 lagging.** **[2071 Shawan]**

Solution:

[Please refer to solution of 2068 Baisakh]

6. **Three loads (31+j59) Ω, (30 – j40)Ω and (80+j60)Ω are connected in delta to a 3-phase, 200 V supply. Find the phase currents, line currents and total power absorbed.** **[2071 Shawan]**



Solution:

Let the phase sequence be RYB.

From figure Δ-connected system,

Line voltage = Phase voltage

∴ $V_L = V_{Ph} = 220$ V

Phase voltage:

$$\tilde{V}_{RY} = 200\angle 0° \text{ V}$$

$$\tilde{V}_{YB} = 200\angle -120° \text{V}$$

$$\tilde{V}_{BR} = 200\angle 120° \text{ V}$$

$$Z_R = 31 + j59\Omega$$

$$Z_Y = 30 - j40\Omega$$

$$Z_B = 80 + j60\Omega$$

Phase Currents

$$\tilde{I}_R = \frac{\tilde{V}_{RY}}{Z_R} = \frac{200\angle 0°}{31 + j59} = 3\angle -62.28° \text{ A}$$

$$\tilde{I}_Y = \frac{\tilde{V}_{YB}}{Z_Y} = \frac{200\angle -120°}{30 - j40} = 4\angle -66.87° \text{ A}$$

$$\tilde{I}_B = \frac{\tilde{V}_{BR}}{Z_B} = \frac{200\angle 120°}{80 + j60} = 2\angle 83.13° \text{ A}$$

Line Currents:

$$\tilde{I}_1 + \tilde{I}_B = \tilde{I}_R$$

$$\therefore \tilde{I}_1 = \tilde{I}_R - \tilde{I}_B$$
$$= (3\angle -62.28°) - (2\angle 83.13°)$$
$$= 4.78\angle -76.01° \text{ A}$$

$$\tilde{I}_2 + \tilde{I}_R = \tilde{I}_Y$$

$$\therefore \tilde{I}_2 = \tilde{I}_Y - \tilde{I}_R = (4\angle -66.87°) - (3\angle -62.28°) = 1.037\angle -80.24° \text{ A}$$

$$\tilde{I}_3 + \tilde{I}_Y = \tilde{I}_B$$

$$\therefore \tilde{I}_3 = \tilde{I}_B - \tilde{I}_Y$$
$$= (2\angle 83.13°) - (4\angle -66.87°) = 5.82\angle 103.23° \text{ A}$$

Active Power in phase R

$$P_R = V_{RY} \times I_R \times \cos\phi_R$$
$$= 200 \times 3 \times \cos(0° - (-62.28)) = 279.09 \text{ W}$$

Active Power in phase Y

$$P_Y = V_{YB} \times I_Y \times \cos\phi_Y$$
$$= 200 \times 4 \times \cos(-120 - (-66.87)) = 480 \text{ W}$$

Active Power in phase B

$$P_B = V_{BR} \times I_B \times \cos\phi_B$$
$$= 200 \times 2 \times \cos(120 - 83.13) = 320 \text{ W}$$

∴ Total power absorbed (P) = $P_R + P_Y + P_B$

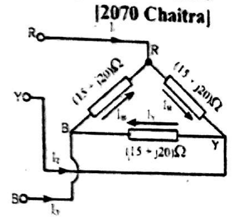
$= 279.09 + 480 + 320 = 1079.09$ W

7. **What are the two ways of connecting a 3 - phase system? Draw their phasor diagrams and write down the relationship between phase and line voltages and phase and line current for these system. [2070 Chaitra]**

[Please refer to the theory]

8. **A 220V, 3 – phase voltage is applied to a balanced delta connected 3–phase load of phase impedance (15 + j20) Ω calculate:**
   (i) **The phase voltages**
   (ii) **The phasor current in each line**
   (iii) **The power consumed per phase**
   (iv) **Draw the phasor diagram**
   (v) **What is the phasor sum of three line currents? Why does it have this value?** **[2070 Chaitra]**

Solution:



Given, Line voltage (V_L) = 220V

[Remember the quantities given in the question is always assumed to be line quantities unless stated to be the other quantities]

From figure as Δ- connected system

Line voltage = phase voltage

∴ $V_L = V_{Ph} = 220$ V

Since balanced load condition

All phase impedances are equal i.e. Z = (15 + j 20) Ω

[It is convenient to do the calculation in terms of phase and then use to find the required solution]

(i) The phase voltages

$$\tilde{V}_{RY} = 220\angle 0° \text{V}$$

$$\tilde{V}_{YB} = 220\angle -120° \text{V}$$

$$\tilde{V}_{BR} = 220\angle 120° \text{ V}$$

All phase voltages are 120° apart

(ii) Phasor current in each line

$$\bar{I}_R = \frac{\bar{V}_{RY}}{Z} = \frac{220\angle 0°}{15 + j\,20} = 8.8\angle -53.13°A$$

Now, in balanced Δ-connected system all line currents and phase currents are 120° apart. Using this fact, we can write

$\bar{I}_Y = 8.8\angle(-53.13° - 120°)$ [Magnitude is same as it is balanced system]

$= 8.8\angle -173.13°$ A

$\bar{I}_B = 8.8\angle(-53.13° + 120°)$

$= 8.8\angle 66.87°$ A

OR $$\bar{I}_Y = \frac{\bar{V}_{YB}}{Z} = \frac{220\angle -120°}{15 + j\,20} = 8.8\angle -173.13°A$$

$$\bar{I}_B = \frac{\bar{V}_{YB}}{Z} = \frac{220\angle 120°}{15 + j\,20} = 8.8\angle 66.87°A$$

Line currents

KCL at node R,

$\bar{I}_1 + \bar{I}_B = \bar{I}_R$

$\bar{I}_1 = \bar{I}_R - \bar{I}_B$

$= (8.8\angle -53.13°) - (8.8\angle 66.87°) = 15.24\angle -83.13°A$

Similarly,

KCL at node Y,

$\bar{I}_2 + \bar{I}_R = \bar{I}_Y$

$\bar{I}_2 = \bar{I}_Y - \bar{I}_R$

$= (8.8\angle -173.13°) - (8.8\angle -53.13°)$

$= 15.24\angle 156.87°A$

KCL at node B,

$\bar{I}_3 + \bar{I}_Y = \bar{I}_B$

$\bar{I}_3 = \bar{I}_B - \bar{I}_Y$

$= (8.8\angle 66.87°) - (8.8\angle -173.13°) = 15.24\angle 36.87°A$

OR

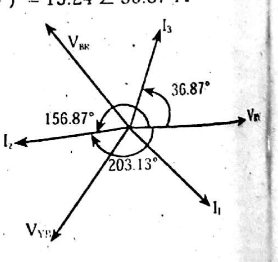
$\bar{I}_2 = 15.24\angle(-83.13° - 120°)$

$= 15.24\angle -203.13°$ A

$= 15.24\angle 156.87°$ A

$\bar{I}_3 = 15.24\angle(-83.13° + 120°)$

$= 15.24\angle 36.87°$ A



(iii) Power consumed per phase

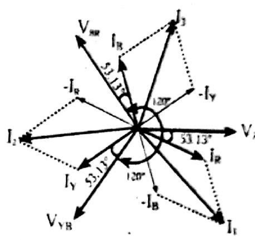
∴ Active power per phase $(P_R) = V_{Ph}\, I_{Ph} \cos\phi$

The phase difference $\phi = 0° - (-53.13°) = 53.13°$

∴ $P_R = 220 \times 8.8 \times \cos 53.13° = 1161.6027$ watts

Power consumed per phase = 1161.6027 watts

(iv) Phasor diagram



(v) Phasor sum of three line currents

$= \bar{I}_1 + \bar{I}_2 + \bar{I}_3$

$= (15.24\angle -83.13°) + (15.24\angle 156.87°) + (15.24\angle 36.87)$

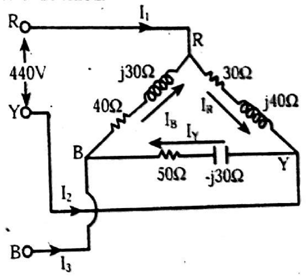
$= 0$

The phasor sum of three line currents is zero because the line currents are equal in magnitude as in a balanced system and have a phase difference of 120° amongst themselves.

9. Explain two – wattmeter method for the measurement of power in a balanced three phase load. [2070 Chaitra]

[Please refer to the theory]

10. For the delta connected load, find the phase currents, line currents, power (active, reactive and apparent) in each phases. Also determine the total active power consumed.



Solution:

Given, $V_L = 440$ V

For Δ – connected load

$V_L = V_{Ph} = 440$ V

$Z_1 = (30 + j\,40)\Omega$

$Z_2 = (50 - j30)\Omega$

$Z_3 = (40 + j30)\Omega$

$\tilde{V}_{RY} = 440 \angle 0°$ V

$\tilde{V}_{YB} = 440 \angle -120°$ V

$\tilde{V}_{BR} = 440 \angle 120°$ V

Phase Currents

$$\tilde{I}_R = \frac{\tilde{V}_{RY}}{Z_1} = \frac{440 \angle 0°}{30 + j40} = 8.8 \angle -53.13°\text{A}$$

$$\tilde{I}_Y = \frac{\tilde{V}_{YB}}{Z_2} = \frac{440 \angle -120°}{50 - j30} = 7.54 \angle -89.03°\text{A}$$

$$\tilde{I}_B = \frac{\tilde{V}_{BR}}{Z_3} = \frac{440 \angle 120°}{40 + j30} = 8.8 \angle 83.13°\text{A}$$

Line currents:

At node R, $\tilde{I}_1 + \tilde{I}_B = \tilde{I}_R$

$\tilde{I}_1 = \tilde{I}_R - \tilde{I}_B = (8.8 \angle -53.13°) - (8.8 \angle 83.13°) = 16.33 \angle -75°\text{A}$

At node Y, $\tilde{I}_2 + \tilde{I}_R = \tilde{I}_Y$

$\tilde{I}_2 = \tilde{I}_Y - \tilde{I}_R = (7.54 \angle -89.03°) - (8.8 \angle -53.13°)$

$= 5.176 \angle -174.47°\text{A}$

At node B, $\tilde{I}_3 + \tilde{I}_Y = \tilde{I}_B$

$\tilde{I}_3 = \tilde{I}_B - \tilde{I}_Y = (8.8 \angle 83.13°) - (7.54 \angle -89.03°) = 16.301 \angle 86.75°\text{A}$

Powers in each phases

Power in R Phase

$\phi_R = 0° - (-53.13°) = 53.13°$

Active power $(P_R) = V_{RY} I_R \cdot \cos \phi_R$

$= 440 \times 8.8 \times \cos (53.13°)$

$= 2323.205$ W

Reactive power $(Q_R) = V_{RY} I_R \sin \phi_R$

$= 440 \times 8.8 \times \sin (53.13°)$

$= 3097.595$ VAR

Apparent power $(S_R) = V_{RY} I_R = 440 \times 8.8 = 3872$ VA

Power in Y phase

$\phi_Y = -120° - (-89.03°) = -30.97°$

$P_Y = V_{YB} I_Y \cos \phi_Y = 440 \times 7.54 \times \cos (-30.97°)$

$= 2844.63$ W

$Q_Y = V_{YB} I_Y \sin \phi_Y = 440 \times 7.54 \times \sin (-30.97°) = -1707.2$ VAR

(– ve sign indicates this phase supplies reactive power)

$S_Y = V_{YB} I_Y = 440 \times 7.54 = 3317.6$ VA

Power in B phase

$\phi_B = 120° - 83.13° = 36.87°$

$P_B = 440 \times 8.8 \cos (36.87°) = 3097.595$ W

$Q_B = 440 \times 8.8 \sin (36.87°) = 2323.205$ VAR

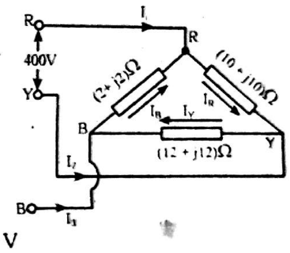
$S_B = 440 \times 8.8 = 3872$ VA

Total active power consumed (P) $= P_R + P_Y + P_B$

$= 2323.205 + 2844.63 + 3097.595 = 8265.43$

Watts

11. **Three impedances of (10 + j10) Ω, (12 + j 12)Ω and (2 + j2)Ω are connected in delta to a 3– phase system with line voltage 400 V. Calculate all the phase currents, line currents, active powers, reactive powers and apparent power.**



Given, $V_L = 400$ V

In Δ– connected system, $V_L = V_{ph} = 400$ V

$\tilde{V}_{RY} = 400 \angle 0°$ V

$\tilde{V}_{YB} = 400 \angle -120°$ V

$\tilde{V}_{BR} = 400 \angle 120°$ V

$Z_1 = 10 + j\,10\,\Omega$

$Z_2 = 12 + j\,12\,\Omega$

$Z_3 = 2 + j\,2\,\Omega$

Phase Currents

$$\tilde{I}_R = \frac{\tilde{V}_{RY}}{Z_1} = \frac{400 \angle 0°}{10 + j10} = 28.28 \angle -45°\text{A}$$

$$\tilde{I}_Y = \frac{\tilde{V}_{YB}}{Z_2} = \frac{400 \angle -120°}{12 + j12} = 23.57 \angle -165°\text{A}$$

$$\tilde{I}_B = \frac{\tilde{V}_{BR}}{Z_3} = \frac{400 \angle 120°}{2 + j\,2} = 141.42 \angle 75°\text{A}$$

Line Currents

$\tilde{I}_1 = \tilde{I}_R - \tilde{I}_B = (28.28 \angle -45°) - (141.42 \angle 75°)$

$= 157.47 \angle -96.05°\text{A}$

$\tilde{I}_2 = \tilde{I}_Y - \tilde{I}_R = (23.57 \angle -165°) - (28.28 \angle -45°)$

$= 44.96 \angle 161.997°\text{A} = 44.96 \angle 162°\text{A}$

$\tilde{I}_3 = \tilde{I}_B - \tilde{I}_Y = (141.42 \angle 75°) - (23.57 \angle -165°) = 154.56 \angle 67.41°\text{A}$

Power in R phase

$\phi_R = 0 - (-45°) = 45°$

Active power $(P_R) = V_{RY}I_R\cos\phi_R = 400 \times 28.28 \times \cos 45° = 7998.79W$

Reactive power $(Q_R) = V_{RY}I_R\sin\phi_R = 400 \times 28.28 \times \sin 45° = 7998.79VAR$

Apparent power $(S_R) = V_{RY}I_R = 400 \times 28.28 = 11312$ VA

Power in Y phase

$\phi_Y = -120° - (-165°) = 45°$

$P_Y = V_{YB}I_Y\cos\phi_Y = 400 \times 23.57 \times \cos 45° = 6666.602$ W

[2070 Bhadra] $Q_Y = V_{YB}I_Y\sin\phi_Y = 400 \times 23.57 \times \sin 45° = 6666.602$ VAR

$S_Y = V_{YB}I_Y = 400 \times 23.57 = 9428VA$

Powers in B Phase

$\phi_B = 120° - 75° = 45°$

$P_B = 400 \times 141.42 \times \cos 45° = 39999.61$ W

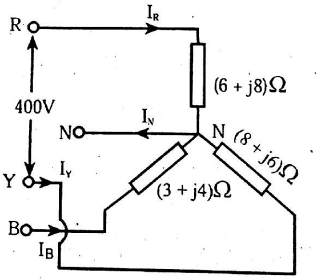
$Q_B = 400 \times 141.42 \times \sin 45° = 39999.61$ VAR

$S_B = 400 \times 141.42 = 56568$ VA

12. **With the help of necessary phasor diagram and circuit diagram, explain the two wattmeter method of Active Power Measurement in three Phase AC system? What is the VAriation of wattmeter readings with load Power Factor?** **[2070 Ashad]**

[Please refer to the theory]

13. **Three impedances of (6 + j8) Ω, (8 + j6) Ω, (3 + j4) Ω, are connected in star to a 3– phase, 4 wire system for which the line voltage is 400 V. Find the line currents, and active and reactive and apparent power per phase. Also find the current through neutral wire.** **[2069 Ashad]**



Solution,

Given,

$V_L = 400$ V

$$V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

In Y – connected system,

Line current = phase current

$I_L = I_{Ph}$

$Z_1 = (6 + j8)\Omega \quad Z_2 = (8 + j6)\Omega \quad Z_3 = (3 + j4)\Omega$

Phase currents/line currents

$\bar{V}_{RN} = 230.94 \angle 0°$ V

$\bar{V}_{YN} = 230.94 \angle -120°$ V

$\bar{V}_{BN} = 230.94 \angle 120°$ V

$$\bar{I}_R = \frac{\bar{V}_{RN}}{Z_1} = \frac{230.94 \angle 0°}{(6 + j8)} = 23.094 \angle -53.13° \text{ A}$$

$$\bar{I}_Y = \frac{\bar{V}_{YN}}{Z_1} = \frac{230.94 \angle -120°}{(8 + j6)} = 23.094 \angle -156.87° \text{ A}$$

$$\bar{I}_B = \frac{\bar{V}_{BN}}{Z_3} = \frac{230.94 \angle 120°}{(3 + j4)} = 46.188 \angle 66.87° \text{ A}$$

Current through neutral wire $\bar{I}_N = \bar{I}_R + \bar{I}_Y + \bar{I}_B = 18.403 \angle 54.21°$A

Power in R phase

$\phi_R = 0° - (-53.13°) = 53.13°$

Active power $(P_R) = V_{RN}I_R\cos\phi_R = 230.94 \times 23.094 \times \cos 53.13° = 3200$ W

Reactive power $(Q_R) = V_{RN}I_R\sin\phi_R = 230.94 \times 23.094 \times \sin 53.13° = 4266.65$ VAR

Apparent power $(S_R) = V_{RN}I_R = 230.94 \times 23.094 = 5333.33$ VA

Power in Y Phase

$\phi_Y = -120° - (-156.87°) = 36.87°$

$P_Y = 230.94 \times 23.094 \times \cos 36.87° = 4266.65$ W

$Q_Y = 230.94 \times 23.094 \times \sin 36.87° = 3200$ VAR

$S_Y = 230.94 \times 23.094 = 5333.33$ VA

Power in B phase

$\phi_B = 120° - (66.87°) = 53.13°$

$P_B = 230.94 \times 46.188 \times \cos 53.13° = 6400$ W

$Q_B = 230.94 \times 46.188 \times \sin 53.13° = 8533.31$ VAR

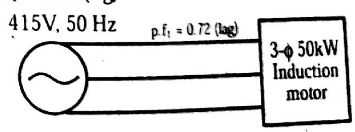
$S_B = 230.94 \times 46.188 = 10666.66$ VA

14. **A three phase induction motor takes 50 kW at 415V, 50Hz and a power factor of 0.72 lagging. Determine the kVAR rating of capacitor bank to improve the power factor to 0.9 lagging. What capacitance per phase is required if the capacitor bank is connected in star connection? What is the advantage of power factor correction from the source point of view of motor itself?** **[2069 Chaitra]**

Solution,

Given,

415 V, 50Hz $P.f_1 = 0.72$ (lag)



Power factor $(P.f_1) = 0.72$ (lag)

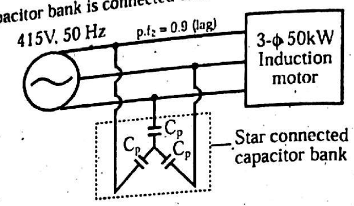
$\Rightarrow \phi_1 = \cos^{-1}(0.72) = 43.94°$

kVAR rating of capacitor bank = ?

$P.f_2 = 0.9$ (lag)

$\Rightarrow \phi_2 = \cos^{-1}(0.9) = 25.84°$

Now, given that the capacitor bank is connected to star connection



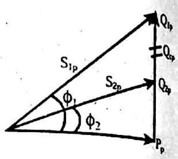
Power per phase (P_P) $= \frac{50}{3}$ kW = 16.67 kW

Now, Reactive power supplied when P.f₁ = 0.72 per phase

$$Q_{1p} = P_p \tan \phi_1$$

Reactive power supplied when P.f₂ = 0.9 per phase

$$Q_{2p} = P_p \tan \phi_2$$



From figure

kVAR rating per phase $Q_{CP} = Q_{1P} - Q_{2P}$

$$= P_P \tan\phi_1 - P_p \tan \phi_2$$
$$= P_P (\tan \phi_1 - \tan\phi_2)$$
$$= 16.67 (\tan 43.94° - \tan 25.84°)$$
$$= 7.99 \text{ kVAR} \approx 8 \text{ kVAR}$$

∴ Required kVAR rating of capacitor bank

$$Q_C = 3 \times Q_{CP} = 3 \times 8 = 24 \text{ kVAR}$$

We know, $Q_{CP} = \omega V_{Ph}^2 C_P$

$$C_P = \frac{Q_{CP}}{\omega V_{Ph}^2}$$

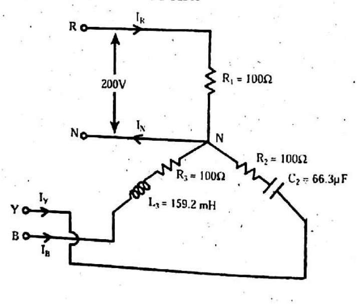
Now, since star - connection

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6 \text{ V}$$

$$\therefore C_P = \frac{8 \times 10^3}{2\pi \times 50 \times (239.6)^2} = 4.435 \times 10^{-4} \text{F} = 443.57 \ \mu\text{F}$$

∴ Required capacitance per phase = 443.57 μF

15. In a 3 – phase, 4 wire Wye connected system the phase voltage V_{Ph} = 200V and its frequency is 60 Hz. The load impedance components are R_1 = 100Ω, R_2 = 100Ω, C_2 = 66.3 μF, R_3 = 100Ω, L_3 = 159.2 mH. Calculate the three line currents and the neutral current. **[2069 Chaitra]**



Solution,

Given, 3ϕ, 4 wire Y – connected system.

V_{Ph} = 200 V, 60 Hz

$$Z_1 = 100 \ \Omega$$

$$Z_2 = 100 - j\frac{1}{2 \times \pi \times 60 \times 66.3 \times 10^{-6}}$$

$$= 100 - j\,40 \ \Omega$$

$$Z_3 = 100 + j\,2\pi \times 60 \times 159.2 \times 10^{-3} = 100 + j\,60\Omega$$

In Y – connected system

$$\therefore \tilde{I}_R = \frac{\tilde{V}_{RN}}{Z_1} = \frac{200 \angle 0°}{100} = 2 \angle 0° \text{ A}$$

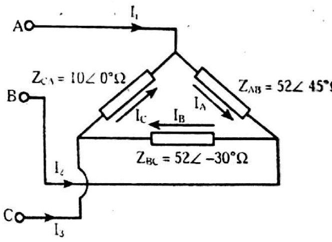
$$\therefore \tilde{I}_Y = \frac{\tilde{V}_{YN}}{Z_2} = \frac{200 \angle -120°}{100 - j\,40} = 1.85 \angle -98.2° \text{ A}$$

$$\therefore \tilde{I}_B = \frac{\tilde{V}_{BN}}{Z_3} = \frac{200 \angle 120°}{100 + j60} = 1.71 \angle 89.04° \text{ A}$$

Neutral current $\tilde{I}_N = \tilde{I}_R + \tilde{I}_Y + \tilde{I}_B = 2 \angle 0° + 1.85 \angle -98.2° + 1.71 \angle 89.04°$

$$= 1.77 \angle -3.93° \text{ A}$$

16. A delta connected load of $Z_{AB} = 52 \angle 45°$ Ω, $Z_{BC} = 52 \angle -30°$ Ω and $Z_{CA} = 10 \angle 0°$ Ω are connected to a 380 V,3 phase ac source. Find the magnitude of the line currents and total power absorbed by loads, when phase sequence is ABC. **[2068 Chaitra]**



Solution:

380V, 3ϕ ac source

$$Z_{AB} = 52 \angle 45° \ \Omega$$
$$Z_{BC} = 52 \angle -30° \ \Omega$$
$$Z_{CA} = 10 \angle 0° \ \Omega$$

In Δ – connected system

$$V_L = V_{Ph} = 380 \text{ V}$$

$$\tilde{V}_{AB} = 380 \angle 0° \text{ V}$$

$\tilde{V}_{BC} = 380 \angle -12$[illegible] V

$\tilde{V}_{CA} = 380 \angle 1$[illegible] V

Phase Currents

$$\bar{I}_A = \frac{\bar{V}_{AB}}{Z_{AB}} = \frac{380\angle 0^\circ}{52\angle 45^\circ} = 7.3\angle -45^\circ A$$

$$\bar{I}_B = \frac{\bar{V}_{BC}}{Z_{BC}} = \frac{380\angle -120^\circ}{52\angle -30^\circ} = 7.3\angle -90^\circ A$$

$$\bar{I}_B = \frac{\bar{V}_{CA}}{Z_{CA}} = \frac{380\angle 120^\circ}{10\angle 0^\circ} = 3.8\angle 120^\circ A$$

Line Currents

$$\bar{I}_1 = \bar{I}_A - \bar{I}_C = (7.3\angle -45^\circ) - (3.8\angle 120^\circ) = 11.01\angle -50.12^\circ A$$

$$\bar{I}_2 = \bar{I}_B - \bar{I}_A = (7.3\angle -90^\circ) - (7.3\angle -45^\circ) = 5.58\angle -157.5^\circ A$$

$$\bar{I}_3 = \bar{I}_C - \bar{I}_B = (3.8\angle 120^\circ) - (7.3\angle -90^\circ) = 10.76\angle 100.17^\circ A$$

Active power in A phase (P_A) $= V_{AB}I_A \cos\phi_A$
$= 380 \times 7.3 \cos[0^\circ - (-45^\circ)]$
$= 380 \times 7.3 \cos 45^\circ = 1961.514$ W

Active power in B phase (P_B) $= V_{BC}I_B \cos\phi_B$
$= 380 \times 7.3 \cos[-120^\circ - (-90^\circ)]$
$= 380 \times 7.3 \cos(-30^\circ)$
$= 2402.35$ W

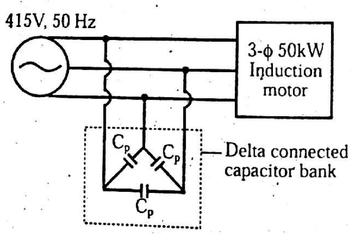
Active power in C phase (P_C) $= V_{CA}I_C \cos\phi_C$
$= 380 \times 3.8 \cos[120^\circ - 120^\circ] = 1444$ W

Total power absorbed by loads $= P_A + P_B + P_C$
$= 1961.514 + 2402.35 + 1444$
$= 5807.864$ W

17. **What are the advantages of three phase AC system over single phase ac system? [2068 Chaitra]**

[Please refer to the theory]

18. **A 415V, 3 phase, 50 Hz induction motor takes 50 kW power from supply mains at 0.72 power factor lagging. Capacitors are connected in delta across the line to improve the overall power factor. Calculate the capacitance per phase in order to raise the power factor to 0.9 lagging. [2068 Baisakh]**



Solution

Now, Power per phase $(P_p) = \frac{50}{3}$ kW = 16.67 kW

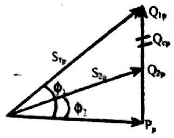
P.f$_1$ = 0.72 (lag)

$\Rightarrow \phi_1 = \cos^{-1}(0.72) = 43.94^\circ$

P.f$_2$ = 0.9 (lag)

$\Rightarrow \phi_2 = \cos^{-1}(0.9) = 25.84^\circ$

From figure,



$Q_{1P} = P_P \tan\phi_1 = 16.67 \times \tan 43.94^\circ = 16.06$ kVAR

$Q_{2P} = P_P \tan\phi_2 = 16.67 \times \tan 25.84^\circ = 8.07$ kVAR

$Q_{CP} = Q_{1P} - Q_{2P} = 16.06 - 8.07 \approx 8$ kVAR

Since Δ– connected capacitor bank

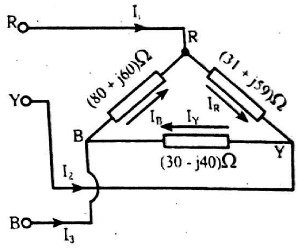
$V_L = V_{Ph} = 415$ V

$Q_{CP} = \omega V_{Ph}^2 C_P$

$$C_P = \frac{Q_{CP}}{\omega V_{Ph}^2} = \frac{8\times 10^3}{2\pi \times 50 \times (415)^2} = 147.86\ \mu F$$

∴ Required capacitance per phase = 147.86μF

19. **Three phase loads (31 + j 59)Ω, (30 − j40)Ω and (80 + j60)Ω are connected in delta to a 3 phase, 200V supply. Find the phase currents. Line currents and total power absorbed. [2068 Baisakh]**



Solution:

In Δ – connected system

$V_L = V_{Ph} = 200$V

Phase Currents

$$\bar{I}_R = \frac{220\angle 0^\circ}{31 + j59} = 3.3\angle -62.28^\circ A$$

$$\bar{I}_Y = \frac{220\angle -120^\circ}{30 - j40} = 4.4\angle -66.87^\circ A$$

$$\bar{I}_B = \frac{220\angle 120^\circ}{80 + j60} = 2.2\angle 83.13^\circ A$$

Line currents

$$\bar{I}_1 = \bar{I}_R - \bar{I}_B = (3.3\angle -62.28^\circ) - (2.2\angle 83.13^\circ) = 5.26\angle -76.01^\circ A$$

$$\bar{I}_2 = \bar{I}_Y - \bar{I}_R = (4.4\angle -66.87^\circ) - (3.3\angle -62.28^\circ) = 1.14\angle -80.24^\circ A$$

$$\bar{I}_3 = \bar{I}_B - \bar{I}_Y = (2.2\angle 83.13^\circ) - (4.4\angle -66.87^\circ) = 6.4\angle 103.23^\circ A$$

$P_R = V_{RY}I_R \cos\phi_R = 200 \times 3.3 \times \cos(0^\circ - (-62.28^\circ))$
$= 306.997\ W \approx 307$ W

$P_Y = V_{YB}I_Y \cos\phi_Y = 200 \times 4.4 \times \cos(-120^\circ - (-66.87^\circ))$
$= 528$ W

$P_B = V_{BR}I_B \cos\phi_B = 200 \times 2.2 \times \cos(120° - 83.13°)$

$= 351.9995 \text{ W} \approx 352 \text{ W}$

Total power absorbed (P) $= P_R + P_Y + P_B$

$= 307 + 528 + 352 = 1187 \text{ W}$

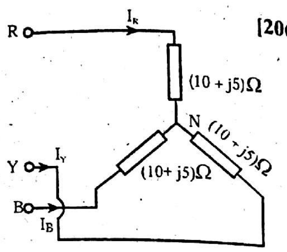
20. A balanced star connected load with impedance (10 + j3)Ω per phase is fed from a balanced 3 phase 400 volt supply. Calculate:

i) The phase voltages

ii) The line currents

iii) The power absorbed and

iv) Draw the phasor diagram. **[2067 Mangsir]**



Solution: $Z = (10 + j5)\Omega$

$V_L = 400V$

(i) Phase voltages

$$V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$\tilde{V}_{RN} = 230.94 \angle 0°V$

$\tilde{V}_{YN} = 230.94 \angle -120°V$

$\tilde{V}_{BN} = 230.94 \angle 120°V$

(ii) Line currents

In Y – connected system,

Line currents = phase currents

$$\tilde{I}_R = \frac{\tilde{V}_{RN}}{Z} = \frac{230.94 \angle 0°}{(10 + j5)} = 20.65 \angle -26.56°A$$

$\tilde{I}_Y = 20.65 \angle (-26.56° - 120°) = 20.65 \angle -146.56° \text{ A}$

$\tilde{I}_B = 20.65 \angle (-26.56° + 120°) = 20.65 \angle 93.44°A$

(iii) Power absorbed (P) $= 3V_{Ph}I_{Ph} \cos\phi$

$= 3 \times 230.94 \times 20.65 \times \cos(26.56°) = 12796.895$ watts

OR $P = \sqrt{3}V_L I_L \cos\phi$

$= \sqrt{3} \times 400 \times 20.65 \times \cos(26.56°)$

$= 12796.9$ watts

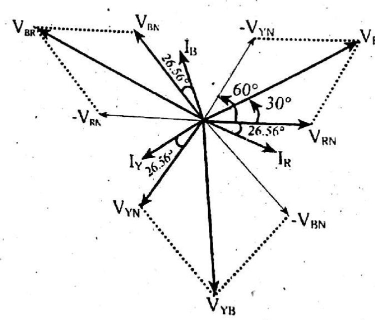
(iv) Phasor diagram

$\tilde{V}_{RY} = \tilde{V}_{RN} + \tilde{V}_{NY}$

$= \tilde{V}_{RN} - \tilde{V}_{YN}$

$\tilde{V}_{YB} = \tilde{V}_{YN} - \tilde{V}_{BN}$

$\tilde{V}_{BR} = \tilde{V}_{BN} - \tilde{V}_{RN}$



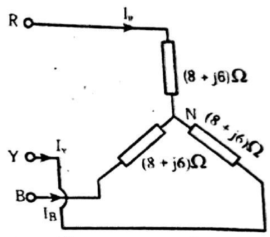
21. A balanced star connected load of (8 + j 6) Ω per phase is connected to a three - phase, 50Hz, 380 V supply. Find the line current, power factor, active power, reactive power and total volt – amps. Also draw the phasor diagram. **[2065 Kartik]**

Solution:

$V_L = 380 \text{ V}$

$$V_{Ph} = \frac{380}{\sqrt{3}} \text{ V} = 219.39 \text{ V}$$

$Z = (8 + j6)\Omega$



In Y – connected system, line current = phase current

$$\tilde{I}_R = \frac{\tilde{V}_{RN}}{Z} = \frac{219.39 \angle 0°}{8 + j6} = 21.94 \angle -36.87° \text{ A}$$

(i) Line current $(I_L) = 21.94$ A

(ii) P.f. $= \cos\phi = \cos(0 - (-36.87)) = \cos 36.87 = 0.8$ (lag)

(iii) Active power $(P) = \sqrt{3}V_L I_L \cos\phi$

$= \sqrt{3} \times 380 \times 21.94 \times 0.8$

$= 11552.36$ W

(v) Reactive power $(Q) = \sqrt{3}V_L I_L \sin\phi$

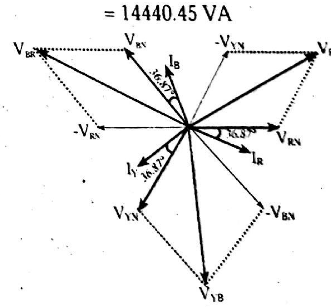
$= \sqrt{3} \times 380 \times 21.94 \times \sin(36.87°)$

$= 8664.293$ VAR

(v) Total volt-amp $(S) = \sqrt{3}V_L I_L$

$= \sqrt{3} \times 380 \times 21.94$

$= 14440.45$ VA



22. A 380 V balanced three – phase voltage source is supplying a delta connected load bank with each of the loads equal to 10 – j5 Ohm. Determine the phase current, line current and magnitude of voltage across each of the loads. Also calculate total active (real) power consumption of the load and power factor. Construct the phasor diagram of currents and voltages in the circuit. **[2062 Bhadra]**

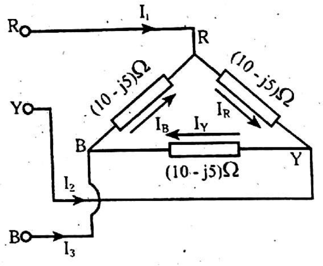
Solution:

In Δ- connected system

$V_L = 380$ V , $V_L = V_{Ph} = 380$ V

$Z = (10 - j5)\Omega$

Phase current



$$\tilde{I}_R = \frac{\tilde{V}_{RY}}{Z} = \frac{380\angle 0°}{10-j5} = 33.99\angle 26.56° \approx 34\angle 26.56°\text{A}$$

∴ Phase current (I_{Ph}) = 34 A

∴ Line current (I_L) $= \sqrt{3}\, I_{Ph} = \sqrt{3} \times 34 = 58.89$A

Magnitude of voltage across each of the loads = 380 V

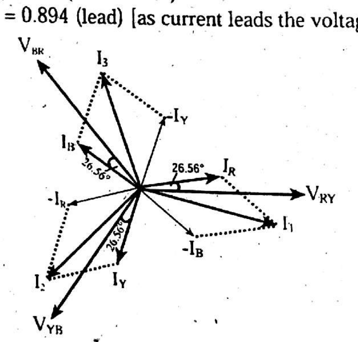
[As Δ connected $V_L = V_{Ph}$]

Total active power consumption of the load

$$\begin{aligned} P &= \sqrt{3}\, V_L I_L \cos\phi \\ &= \sqrt{3} \times 380 \times 58.89 \cos(0° - 26.56°) \\ &= \sqrt{3} \times 380 \times 58.89 \cos(-26.56°) \\ &= 34669.68 \text{ W} \end{aligned}$$

Power factor (P.f) = cos (0° – 26.56°)

= 0.894 (lead) [as current leads the voltage]



23. The power input to a motor is measured by two wattmeters, which indicate 40 kW and 50 kW respectively. If the power factor of the motor be changed to 0.8 leading, determine the reading of two wattmeters. The total input power remains the same. Draw vector diagram for the second condition of load. **[2066 Magh]**

Given,

Total input power = 40 kW + 50 kW

= 90 kW

Total power factor (P.f) = 0.8 (lead)

Let W_1 and W_2 be the required wattmeter readings.

As total input power remains the same

$W_1 + W_2 = 90$ kW ....(i)

We know,

$W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\phi$ ....(ii)

$W_1 - W_2 = V_L I_L \sin\phi$ ....(iii)

Dividing (iii) by (ii)

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\phi}{\sqrt{3}\, V_L I_L \cos\phi}$$

or, $$\frac{W_1 - W_2}{W_1 + W_2} = \frac{\tan\phi}{\sqrt{3}} \text{ .... (iv)}$$

We have,

P.f = 0.8

cosϕ = 0.8

∴ ϕ = 36.87°

Putting in equation (iv)

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{\tan 36.87°}{\sqrt{3}}$$

⇒ $W_1 - W_2 = 0.43\,(W_1 + W_2)$

⇒ $W_1 - W_2 - 0.43\,W_1 - 0.43\,W_2 = 0$

⇒ $0.57\,W_1 - 1.43\,W_2 = 0$ ....(v)

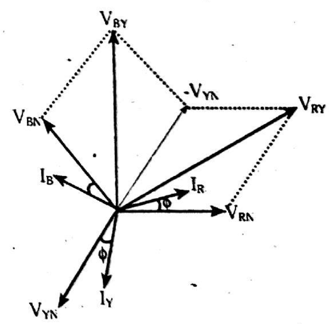
Solving (i) & (v)

∴ $W_1 = 64.35$ kW

∴ $W_2 = 25.65$ kW

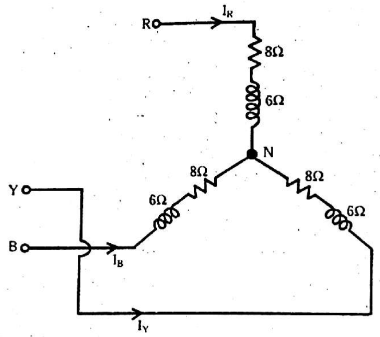
∴ The required readings of two wattmeters are 64.35 kW and 25.65 kW

Considering star-connected load



Additional Problems

1. A star - connected three phase load has a resistance of 8 Ω and an inductive reactance of 6 Ω in each phase. It is fed from a 400V, 3 -phase balanced supply. Determine the line current power factor, active and reactive power. Draw phasor diagram showing phase and line voltages and currents. If power measurements is made using two wattmeter method , what will be the readings of both wattmeters ?



Solution:

Given,

400 V, 3 – ϕ balanced supply

$Z = 8 + j6\ \Omega$

For star - connected system,

line current = phase current

$I_L = I_{ph}$

$V_L = 400V$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94\ V$$

Line currents

$$\tilde{I}_R = \frac{\tilde{V}_{RN}}{Z}$$

$$= \frac{230.94\angle 0°}{8+j6} = 23.094\angle -36.86°\ A$$

$$\tilde{I}_Y = 23.094\angle(-36.86° - 120°)$$
$$= 23.094\angle -156.86°\ A$$

$$\tilde{I}_B = 23.094\angle(-36.86° + 120°)$$
$$= 23.094\angle 83.14°\ A$$

Power factor (pf) = Cos ϕ

= Cos (0° – (–36.86°))

= 0.8 (lagging)

Active power (P) = $3V_{ph}\,I_{ph}\,\text{Cos}\,\phi$

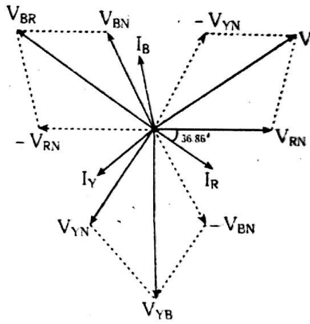
= 3 × 230.94 × 23.094 × 0.8

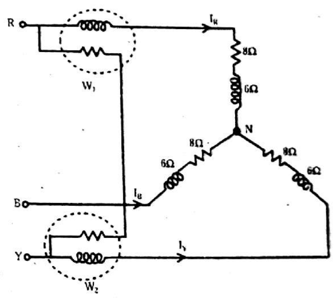
= 12.799.99 W

Reactive power (Q) = $3\,V_{ph}\,I_{ph}\,\text{Sin}\,\phi$

= 3 × 230.94 × 23.094 × Sin(36.86°)

= 9597.78 VAR





Wattmeter readings.

$W_1 = V_{RB}\,I_R\cos(30° - \phi)$

= 400 × 23.094 cos (30° – 36.86°)

= 9171.47 W

$W_2 = V_{YB}\,I_Y\cos(30° + \phi)$

= 400 × 23.094 cos (30° + 36.86°) = 3630.18 W

2. **A star - connected balanced load is supplied from a 3ϕ balanced supply with a line voltage of 416 V at a frequency of 50 Hz. Each phase of the load consists of a resistance and a capacitor joined in series and the**

readings of two wattmeters connected to measure the load power supplied are 782 W and 1,980 W, both positive. Calculate

(i) power factor of the circuit

(ii) Line current

(iii) the capacitance of each capacitor

Solution:

Given,

3ϕ, 416 V, 50Hz supply

$W_1 = 782$ W

$W_2 = 1980$ W

Total active power $P = W_1 + W_2$

$= 782 + 1980 = 2762$ W

we have,

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

$$\text{or, } \tan\phi = \frac{\sqrt{3}(782 - 1980)}{(782 + 1980)}$$

or, $\tan\phi = -0.75$

$\therefore \phi = -36.869°$

$\therefore$ power factor (Pf) $= \cos\phi$

$= \cos(-36.869°)$

$= 0.8$ (lead)

We have,

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

$$\text{or, } 2762 = \sqrt{3} \times 416 \times I_L \times 0.8$$

$$\text{or, } I_L = \frac{2762}{\sqrt{3} \times 416 \times 0.8}$$

$\therefore$ line current $(I_L) = 4.79$ A

Now, Impedance of the circuit per phase

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{V_L/\sqrt{3}}{I_L}$$

$$= \frac{\frac{416}{\sqrt{3}}}{4.79}$$

$= 50.14\ \Omega$

Reactance per phase,

$X_{ph} = Z_{ph} \sin\phi$

$= 50.14 \times \sin(36.869°)$

$= 30.08\ \Omega$

$$X_{ph} = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$\text{or, } C = \frac{1}{2\pi f X_{ph}}$$

$$= \frac{1}{2\pi \times 50 \times 30.08} = 105.82\ \mu F$$

$\therefore$ Capacitance of each

capacitor $= 105.82\ \mu F$

3. **The power input to a synchronous motor is measured by two wattmeters both of which indicate 50 kW. If the pf of the motor be changed to 0.866 leading determine the readings of the wattmeters, the total input power remaining the same.**

Solution:

Let the readings of the wattmeters for second condition be W_1 and W_2

Total input power of load $= 2 \times 50$kW

$W_1 + W_2 = 100$ kW .....(1)

pf = 0.866 (leading)

$\cos\phi = 0.866$

$\phi = \cos^{-1}(0.866) = 30°$ (lead)

We have,

$$\tan\phi = \sqrt{3}\frac{(W_1 - W_2)}{(W_1 + W_2)}$$

$$\text{or, } \tan 30° = \sqrt{3}\frac{(W_1 - W_2)}{(W_1 + W_2)}$$

$$\text{or, } 0.577 = \sqrt{3}\frac{(W_1 - W_2)}{(W_1 + W_2)}$$

$$\text{or, } 0.577 = \sqrt{3}\frac{(W_1 - W_2)}{100}$$

$$\text{or, } W_1 - W_2 = \frac{100 \times 0.577}{\sqrt{3}} \text{ kW}$$

$\therefore W_1 - W_2 = 33.313$ kW ....(2)

From eqns (1) and (2)

$W_1 = 66.656$ kW

$W_2 = 33.343$ kW

4. **Two wattmeter method is used to measure the power taken by a 3-ϕ inductive motor on no-load. The wattmeter readings are 375W and -50W.**

Calculate.

i) **pf of the motor at no load.**

ii) **Phase difference of voltage and current in two wattmeters.**

iii) **Reactive power taken by the load.**

Solution:

Given, $W_1 = 375$ W

$W_2 = -50$W

i) We have,

$$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

or, $\tan\phi = \dfrac{\sqrt{3}(375-(-50))}{(375+(-50))}$

or, $\tan\phi = 2.2649$

or, $\phi = \tan^{-1}(2.2649)$

$\therefore\ \phi = 66.18°$

Power factor (pf) = $\cos\phi = \cos(66.18°) = 0.403$

(ii) Phase angle difference in W_1 Wattmeter

$= 30° - \phi$

$= 30° - 66.18°$

$= -36.18°$

Phase angle difference in W_2 Wattmeter

$= 30° + \phi$

$= 30° + 66.18°$

$= 96.18°$

(iii) Reactive Power (Q)

$= \sqrt{3}\,(W_1 - W_2)$

$= \sqrt{3}\,(375 - (-50))$

$= 736.12$ VAR

5. **A 3ϕ motor load has pf of 0.397 lagging. Two wattmeter connected to measure power so that the input is 30 kW. Find reading on each wattmeters.**

Solution:

Let W_1 and W_2 be readings on two wattmeters.

pf = 0.397

$\cos\phi = 0.397$

$\therefore\ \phi = \cos^{-1}(0.397) = 66.61°$

Given,

Total input power = 30 kW

$\therefore\ W_1 + W_2 = 30$ kW .... (i)

$$\tan\phi = \sqrt{3}\,\frac{(W_1 - W_2)}{(W_1 + W_2)}$$

Or, $\tan(66.61°) = \sqrt{3}\,\dfrac{(W_1 - W_2)}{30\text{ kW}}$

Or, $\dfrac{2.31 \times 30\text{ kW}}{\sqrt{3}} = (W_1 - W_2)$

$\therefore\ W_1 - W_2 = 40.1$ kW .... (ii)

Solving (i) & (ii) We get,

$W_1 = 35.01$ kW

$W_2 = -5.01$ kW

6. **A three-phase source delivers 4.8 kVA to a wye-connected load with a phase voltage of 208 V and a power factor of 0.9 lagging. Calculate the source line current and the source line voltage.**

Solution:

Apparent power delivered (S) = 4.8 kVA

Phase voltage (V_{Ph}) = 208V

Power factor pf = 0.9 (lagging)

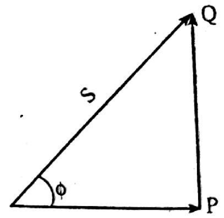
pf = $\cos\phi = 0.9$

$\therefore\ \phi = \cos^{-1}(0.9) = 25.84°$

Total Active Power (P) = $S\cos\phi$

$= 4.8 \times 1000 \times \cos(25.84°)$

$= 4320.07$ W



Total Reactive Power (Q) = $S\sin\phi$

$= 4.8 \times 1000 \times \sin(25.84°)$

$= 2092.12$ VAR

Total active Power (P) = $3V_{Ph}I_{Ph}\cos\phi$

Or, $4320.07 = 3 \times 208 \times I_{Ph} \times \cos(25.84°)$

Or, $I_{Ph} = \dfrac{4320.07}{3 \times 208 \times \cos(25.84°)}$

$\therefore\ I_{Ph} = 7.69$ A

Since load is wye-connected,

Phase current = line current

$I_{Ph} = I_L = 7.69$ A

$\therefore$ The Source line current is 7.69 A

Line voltage = $\sqrt{3} \times$ Phase voltage

$= \sqrt{3} \times V_{Ph}$

$= \sqrt{3} \times 208$

$= 360.3$V

$\therefore$ The source line voltage is 360.3V.

7. **Each phase load consists of a 20Ω resistor and a 10Ω inductive reactance. With a line voltage of 220V rms, calculate the average power taken by the load if**
   (a) the three–phase loads are delta–connected.
   (b) the loads are wye-connected.

Solution:

$Z = 20 + j10\ \Omega$

Line voltage $V_L = 220$V rms

(a) Loads are delta-connected,

In Δ-connection,

Line voltage = Phase voltage

$\therefore\ V_L = V_{Ph} = 220$V

$\tilde{V}_R = 220\angle 0°$V

$\therefore\ \tilde{I}_R = \dfrac{\tilde{V}_R}{Z} = \dfrac{220\angle 0°}{20 + j10} = 9.84\angle -26.56°$ A

$\therefore$ Average power (P) = $3V_{Ph}I_{Ph}\cos\phi$

$= 3 \times 220 \times 9.84 \times \cos(0° - (-26.56°))$

$= 5809.02$W

(b) Loads are wye–connected,

Phase voltage = $\frac{\text{Line voltage}}{\sqrt{3}}$

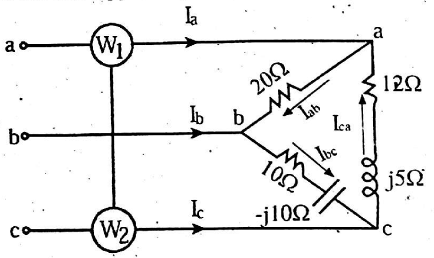
$V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{220}{\sqrt{3}} = 127.02V$

$\therefore \quad \tilde{V}_R = 127.02\angle 0°V$

$\tilde{I}_R = \frac{\tilde{V}_R}{Z} = \frac{127.02\angle 0°}{20+j10} = 5.68\angle -26.56°A$

$\therefore$ Average power (P) = $3V_{Ph}\, I_{Ph}\, Cos\phi$
$= 3 \times 127.02 \times 5.68 \times Cos(0° - (-26.56°))$
$= 1936W$

8. In figure, two wattmeters are properly connected to the unbalanced load supplied by a balanced source such that $V_{ab} = 208\angle 0°V$ with positive phase sequence.
a) Determine the reading of each wattmeter.
b) Calculate the total apparent power absorbed by the load.



Phase voltages:

$\tilde{V}_{ab} = 208\angle 0°V$

$\tilde{V}_{bc} = 208\angle -120°V$

$\tilde{V}_{ca} = 208\angle 120°V$

Phase currents:

$\tilde{I}_{ab} = \frac{\tilde{V}_{ab}}{20} = \frac{208\angle 0°}{20} = 10.4\angle 0°A$

$\tilde{I}_{bc} = \frac{208\angle -120°}{10-j10} = 14.71\angle -75°A$

$\tilde{I}_{ca} = \frac{208\angle 120°}{12+j5} = 16\angle 97.38°A$

Line Currents.

$\tilde{I}_a + \tilde{I}_{ca} = \tilde{I}_{ab}$

$\therefore \quad \tilde{I}_a = \tilde{I}_{ab} - \tilde{I}_{ca} = (10.4\angle 0°) - (16\angle 97.38°) = 20.17\angle -51.86°\ A$

$\tilde{I}_b + \tilde{I}_{ab} = \tilde{I}_{bc}$

$\therefore \quad \tilde{I}_b = \tilde{I}_{bc} - \tilde{I}_{ab} = (14.71\angle -75°) - (10.4\angle 0°) = 15.66\angle -114.89°\ A$

$\tilde{I}_c + \tilde{I}_{bc} = \tilde{I}_{ca}$

$\therefore \quad \tilde{I}_c = \tilde{I}_{ca} - \tilde{I}_{bc} = (16\angle 97.38°) - (14.71\angle -75°) = 30.64\angle 101.03°\ A$

W_1 measure V_{ab} voltage and I_a line current.

$\therefore \quad W_1 = V_{ab}\, I_a\, Cos\phi$
$= 208 \times 20.17 \times Cos\ (0° - (-51.86°))$
$= 2590.99\ W$

W_2 measures V_{cb} voltage and I_c line current.

$\tilde{V}_{cb} = -\tilde{V}_{bc} = -(208\angle -120°) = 208\angle 60°\ V$

$\therefore \quad W_2 = V_{cb}\, I_c\, Cos\phi'$
$= 208 \times 30.64 \times Cos(60° - 101.03°)$
$= 4807.66W$

Now,

Power absorbed by Phase A.

Active power $(P_A) = V_{ab}\, I_{ab}\, Cos\phi_a = 208 \times 10.4 \times Cos(0° - 0°)$
$= 2163.2W$

Reactive power $(Q_A) = V_{ab}\, I_{ab}\, Sin\phi_a = 208 \times 10.4 \times sin0° = 0$

Power absorbed by phase B.

Active power $(P_B) = V_{bc}\, I_{bc}\, Cos\phi_b = 208 \times 14.71 \times Cos(-120° - (-75°)$
$= 2163.5W$

Reactive power $(Q_B) = V_{bc}\, I_{bc}\, Sin\phi_b = 208 \times 14.71 \times Sin(-120° - (-75°))$
$= -2163.5VAR$

Power absorbed by phase C.

Active power $(P_C) = V_{ca}\, I_{ca}\, Cos\phi_c = 208 \times 16 \times Cos(120° - 97.38°)$
$= 3071.99 \approx 3072W$

Reactive power $(Q_C) = V_{ca}\, I_{ca}\, Sin\phi_c = 208 \times 16 \times Sin(120° - 97.38°)$
$= 1280VAR$

Total Active Power (P) $= P_A + P_B + P_C$
$= 2163.2 + 2163.5 + 3072$
$= 7398.7\ W$

Total Reactive Power (Q) $= Q_A + Q_B + Q_C$
$= 0 + (-2163.5) + 1280$
$= -883.5\ VAR$

Total apparent Power absorbed by load

$S = \sqrt{P^2 + Q^2} = \sqrt{(7398.7)^2 + (-883.5)^2}$
$= 7451.26\ VA$

Some Theoretical Questions

1. Explain emf, potential difference and current with a circuit diagram.
2. Define electric circuit. Explain the constituent parts of an electric system.
3. State Ohm's law and write down its limitations.
4. On what factor does the resistance offered by a conductor depends upon?
5. Define the temperature coefficient of resistance and explain the effect of temperature on resistance of a substance.
6. What are ideal and practical voltage and current sources? Explain.
7. Explain how can we convert a voltage source into a current source and a current source into a voltage source?
8. Explain the following:
   a. Series circuit
   b. Parallel circuit
9. Compute equivalent resistance of three resistors connected in
   i. Series
   ii. Parallel
10. What is current divider rule ? Explain with example.
11. What is voltage divider rule? Explain with example.
12. What do you understand by duality between series and parallel circuits?
13. State and explain Kirchhoff's laws.
14. Explain power and energy in dc circuit.
15. Explain the nodal method of solving a network.
16. Explain the loop current method of solving a network.
17. Explain and write the equations for delta-star conversion and for star-delta conversion.
18. State and explain Superposition theorem with a suitable example. Also mention its limitations.
19. State Norton's theorem with an example and list out the steps for nortonizing of a given circuit.
20. State Thevenin's theorem with an example and list out the steps for thevenizing of a given circuit. Also list out the major advantages offered by the use of this theorem.
21. Show that Thevenin's and Norton's theorems are dual to each other.
22. State and explain Maximum power transfer theorem.
23. State and explain reciprocity theorem with a suitable example.
24. Define the following terms
    a. Resistance
    b. Inductance
    c. Capacitance
25. Describe Capacitance form circuit view point, and geometric view point.
26. State the definition of the capacitance and from it write an equation for the charge stored in a capacitor.
27. Derive the expression of energy stored in an inductive coil.
28. Derive the expression of energy stored in a capacitor.
29. Derive an expression for the equivalent inductance of two inductors when they are connectd in parallel (i) Aiding combination (ii) Opposing combination.
30. Derive an expression for the equivalent inductance of two inductors when they are connected in series (i) Aiding combination (ii) Opposing Ccombination.
31. Explain about series and parallel combination of capacitors.

32. Derive an equation for the capacitance of a parallel plate capacitor.
33. Explain the process of charging and discharging of capacitor with neat sketch.
34. List out the difference between ac and dc system.
35. Explain generation of sinusoidal emf with diagram.
36. Define the following terms for an alternating quantity.
    a. Frequency b. Cycle
    c. Phase d. Phase difference
    e. Time period f. Average value
    g. RMS value h. Form factor
    i. Peak factor j. Angular velocity
    k. Instantaneous value
37. Define the following terms with phasor and waveform (i) lagging (ii) leading (iii) in phase. How would you calculate the RMS value of a waveform?
38. Why do we express an ac voltage or current by its RMS value? Discuss.
39. Derive the equation for instantaneous current flowing through a pure resistor when excitd by ac sinusoidal voltage $v = v_m \sin\omega t$. Draw the wave form of voltage and current and phasor diagram of the circuit. Show analytically and graphically that is consumes active power.
40. Derive the equation for instantaneous current flowing through a pure capacitor when excited by ac sinusoidal voltage $v = v_m \sin\omega t$. Draw the waveform of voltage and current and phasor diagram of the circuit. Show analytically and graphically that it does not consume any active power.
41. Derive th relationship between the voltage and current for a purely inductive circuit excited by ac voltage source. Also show that the average power consumed by a purely inductive circuit is zero.
42. Draw wave diagram and phasor diagram to illustrate clearly the relation between voltage and current in the case of
    i. R – L series circuit.
    ii. R – C series circuit.
    iii. R – L – C series circuit.
43. What are active, reactive and apparent power? Draw the power triangle.
44. Derive the expression for instantaneous power in RL and RC series circuit.
45. Derive expressions for impedance and power factor for an R–L– C series circuit when applied with ac voltage. Draw also the phasor diagram.
46. What do you mean by complex power? Explain it with the help of R–L series circuit and power triangle.
47. What is power factor in ac circuit? Explain the disadvantages of low power factor.
48. Explain the requirement of power factor and the method of its correction.
49. What are the advantages of three phase ac system over single phase ac system.
50. What are the two ways of connection in a 3 - phase system? Draw their phasor diagrams and write down the relationship betwen phase and line voltages and currents for these systems.
51. Compare the star and delta 3 - phase connection.
52. Explain two wattmeter method for the measurement of power in a balanced three phase load. What is the variation of wattmeter readings with load power factor.
53. Describe the measurement of power in a unbalanced three phase load by two wattmeter method with neat sketch of circuit diagram.

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