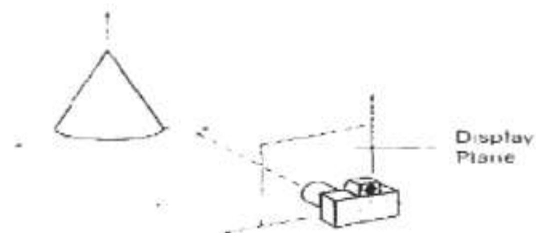
3D viewing

In 2D we specify a window on 2D world and a view port on 2D view surface
objects in world are clipped against window and are then transformed into view port for display.

Added complexity caused by

i. added dimension
ii. display devise are only 2D



Solution is accomplished by introducing projections that transform 3D objects onto 2D plane

In 3D we specify view volume(only those objects within view volume will appear in display on output device others are clipped from display) in world , projection onto projection plane and view port on view surface

So objects in 3D world are clipped against 3D view volume and are then projected
the contents of projection of view volume onto projection plane called window are then transformed (mapped onto) view port for display.

4.5 Projections

The process of transforming points in coordinate system of dimension 'n' into points in a coordinate system of dimension less than 'n' is called **projection**. Projection is required as the display device is a 2D plane and objects are modeled in 3D space (by considering the third dimension or depth or z component).

Projection of 3D object is defined by straight projection rays(projectors) emanating from center of projection, passing thru each point of object and intersecting a projection plane to form projection.

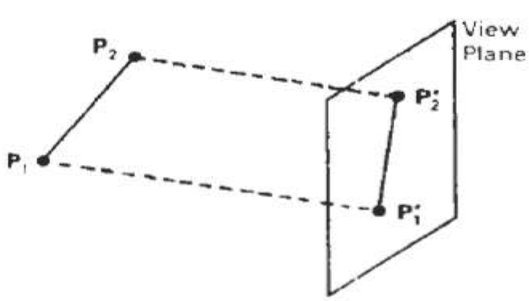
These are planar geometric projection as the projection is onto a plane rather than some curved surface and uses straight rather than curved projectors.

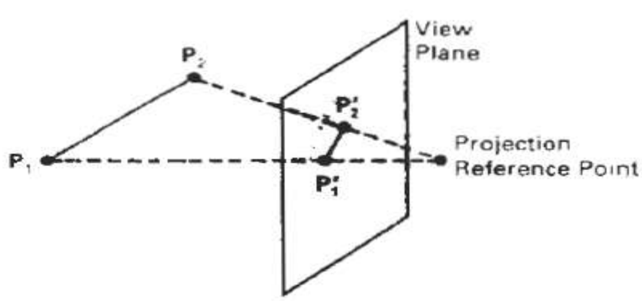
2 Types of Projections

i. distinction is in relation of center of projection to projection plane

ii. if the distance from one to other
is finite then projection is perspective

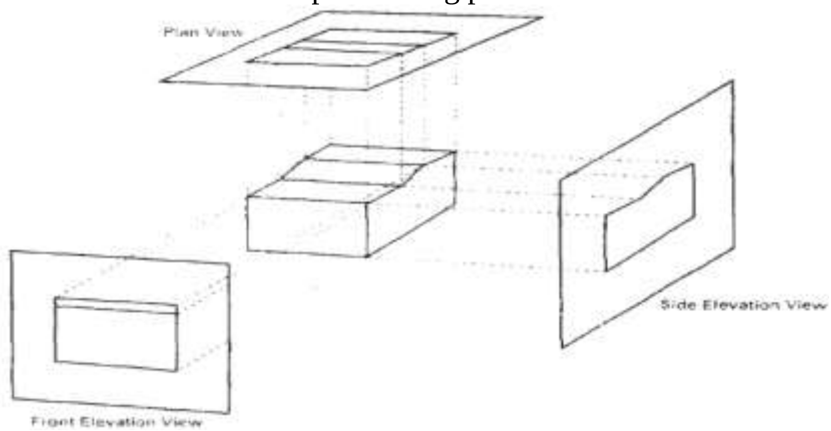
iii. if the distance is infinite then projection is parallel.





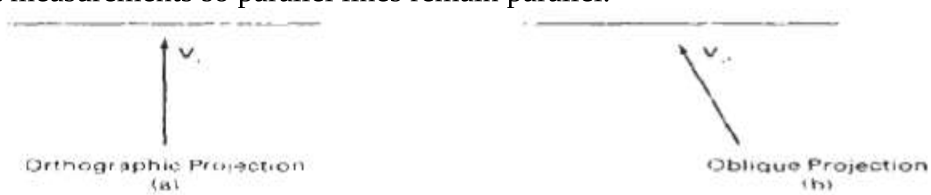
4.5.1 Parallel Projection

Coordinate positions are transformed to the view plane along parallel lines



Preserves relative proportions of objects so that accurate views of various sides of an object are obtained but doesn't give realistic representation of the 3D object.

Can be used for exact measurements so parallel lines remain parallel.



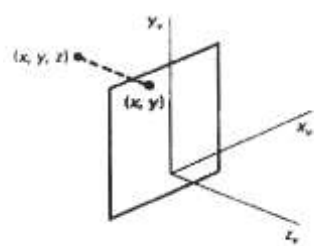
a. Orthographic Parallel Projection

When projection is perpendicular to view plane we have orthographic parallel projection.

Used to produce the front, side and top views of an object. Front, side and rear orthographic projections of an object are called elevations

Top orthographic projection is called a plan view.

Used in Engineering and architectural drawings.



Views that display more than one face of an object are called axonometric orthographic projections. Most commonly used axonometric projection is the isometric projection.

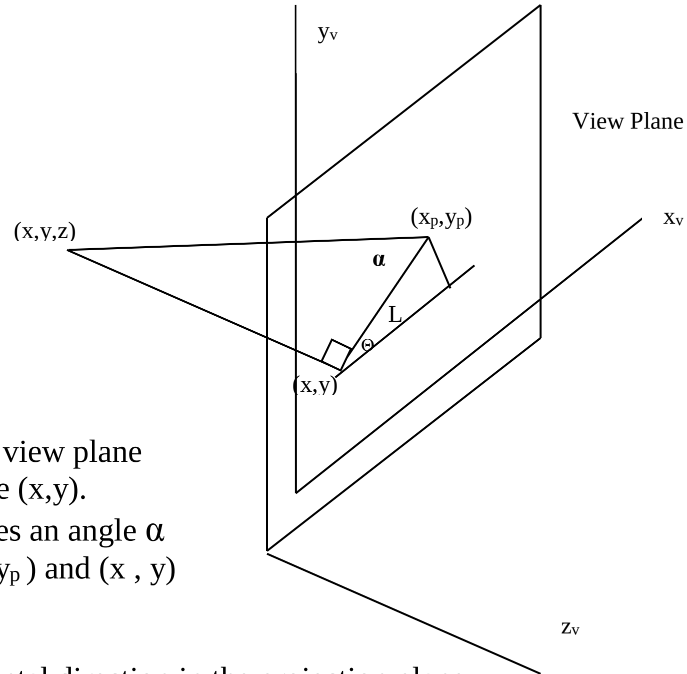
Transformation equations

If view plane is placed at position z_{vp} along z_v axis then any point (x,y,z) is transformed to projection as

$$x_p = x \,,\; y_p = y$$

z value is preserved for depth information needed (visible surface detection).

b. Oblique Parallel Projection



Obtained by projecting points along parallel lines that are not perpendicular to projection plane.

Often specified with two angles Θ and α
Point (x ,y ,z) is projected to position(x_p , y_p) on the view plane
Orthographic projection coordinated on the plane are (x,y).
Oblique projection line from (x,y,z) to (x_p , y_p) makes an angle α with then line on the projection plane that joins (x_p , y_p) and (x , y)

This line of length L is at an angle Θ with the horizontal direction in the projection plane.
Expressing projection coordinates in terms of x, y, L and Θ as

$$x_p = x + L\cos\Theta$$
$$y_p = y + L\sin\Theta$$

L depends on the angle α and z coordinate of point to be projected

$$\tan\alpha = z/L$$

thus,

$$L = z/\tan\alpha$$
$$= zL_1 \qquad L_1 \text{ is the inverse of } \tan\alpha$$

so the oblique projection equations are

$$x_p = x + z(L_1\cos\Theta)$$
$$y_p = y + z(L_1\sin\Theta)$$

the transformation matrix for producing any parallel projection onto the $x_v\,y_v$ plane can be written as

$$M_{parallel} = \begin{pmatrix} 1 & 0 & L_1\cos\Theta & 0 \\ 0 & 1 & L_1\sin\Theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Orthographic projection is obtained when $L_1 = 0$ (occurs at projection angle α of 90*)
Oblique projection is obtained with non zero values for L_1.

4.5.2 Perspective Projection

A perspective projection whose center is a point at infinity becomes a parallel projection

Visual effect of perspective projection is similar to that of photographic system and human visual system.

Size of perspective projection of an object varies inversely with distance of that object from the center of projection

Although objects tend to look realistic, is not particularly useful for recording exact shape and measurements of objects.

Perspective projection of any set of parallel lines that are not parallel to projection plane converge to vanishing point

In 3D parallel lines meet only at infinity

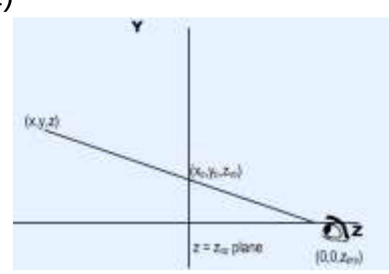
If the set of lines parallel to one of three principal axes then vanishing point is called axis vanishing point

eg. If projection plane cuts only z axis and normal to it , only z axis has principle vanishing point as lines parallel to either y or x axes are also parallel to projection plane and has no vanishing points
In fig lines parallel to x,y do not converge only lines parallel to z axis converge

It is a method for generating a view of a three-dimensional scene by **projecting points to the display plane along converging paths**.
This causes **objects farther from the viewing position to be displayed smaller** than objects of the same size that are nearer to the viewing position.
In a perspective projection, parallel lines in a scene that are not parallel to the display plane are **projected into converging lines.**
Scenes displayed using perspective projections **appear more realistic**, since this is the way that eyes and camera lens form images.
In the perspective projection view, parallel lines appear to converge to a distant point in the background, and distant objects appear smaller than objects closer to the viewing position.
To obtain a perspective projection of a three-dimensional object, transform points along projection lines that meet at the **projection reference point**.
Suppose the perspective reference point is set at position z_{prp} along the z_v, axis, and the view plane is placed at z_{vp} .
Equations describing coordinate positions along this perspective projection line in parametric form as

$$x' = x - xu$$
$$y' = y - yu$$
$$z' = z - (z - z_{prp})u$$

Parameter 'u' takes values from 0 to 1, and coordinate position (x', y', z') represents any point along the projection line.
When u = 0, we are at position P = (x,y, z)



At the other end of the line, u = 1 and we have the projection reference point coordinates $(0,0, z_{prp})$ On the view plane, $z' = z_{vp}$, and we can solve the z' equation for parameter u at this position along the projection line:

$$u = \frac{z_{vp} - z}{z_{prp} - z}$$

Substituting this value of u into the equations for x' and y', we obtain the perspective transformation equations

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) = x\left(\frac{d_p}{z_{prp} - z}\right)$$

$$y_p = y\left(\frac{z_{prp} - z_{vp}}{z_{prp} - z}\right) = y\left(\frac{d_p}{z_{prp} - z}\right)$$

where, $d_p = z_{prp} - z_{vp}$ is the distance of the view plane from the projection reference point.

Using a three-dimensional homogeneous-coordinate representation, we can write the perspective projection transformation in matrix form as

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -z_{vp}/d_p & z_{vp}(z_{prp}/d_p) \\ 0 & 0 & -1/d_p & z_{prp}/d_p \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

In this representation, the homogeneous factor is

$$h = \frac{z_{prp} - z}{d_p}$$

and the projection coordinates on the view plane are calculated from the homogeneous coordinates as

$$x_p = x_h/h, \qquad y_p = y_h/h$$

where the original z-coordinate value would be retained in projection coordinates for visible-surface and other depth processing.

In general, the projection reference point does not have to be along the z_v axis.

There are a number of special cases for the perspective transformation equations.

If the view plane is taken to be the ***XY*** plane, then $z_{vp} = 0$ and the projection coordinates are

$$x_p = x\left(\frac{z_{prp}}{z_{prp} - z}\right) = x\left(\frac{1}{1 - z/z_{prp}}\right)$$

$$y_p = y\left(\frac{z_{prp}}{z_{prp} - z}\right) = y\left(\frac{1}{1 - z/z_{prp}}\right)$$

And, in **some** graphics packages, the projection reference point is always taken to be at the viewing-coordinate origin. In this case, $z_{prp} = 0$ and the projection coordinates on the viewing plane are

$$x_p = x\left(\frac{z_{vp}}{z}\right) = x\left(\frac{1}{z/z_{vp}}\right)$$

$$y_p = y\left(\frac{z_{vp}}{z}\right) = y\left(\frac{1}{z/z_{vp}}\right)$$

When a three-dimensional object is projected onto a view plane using perspective transformation equations, any set of parallel lines in the object that are not parallel to the plane are projected into converging lines.

Parallel Lines that are parallel to the view plane will be projected as parallel lines.

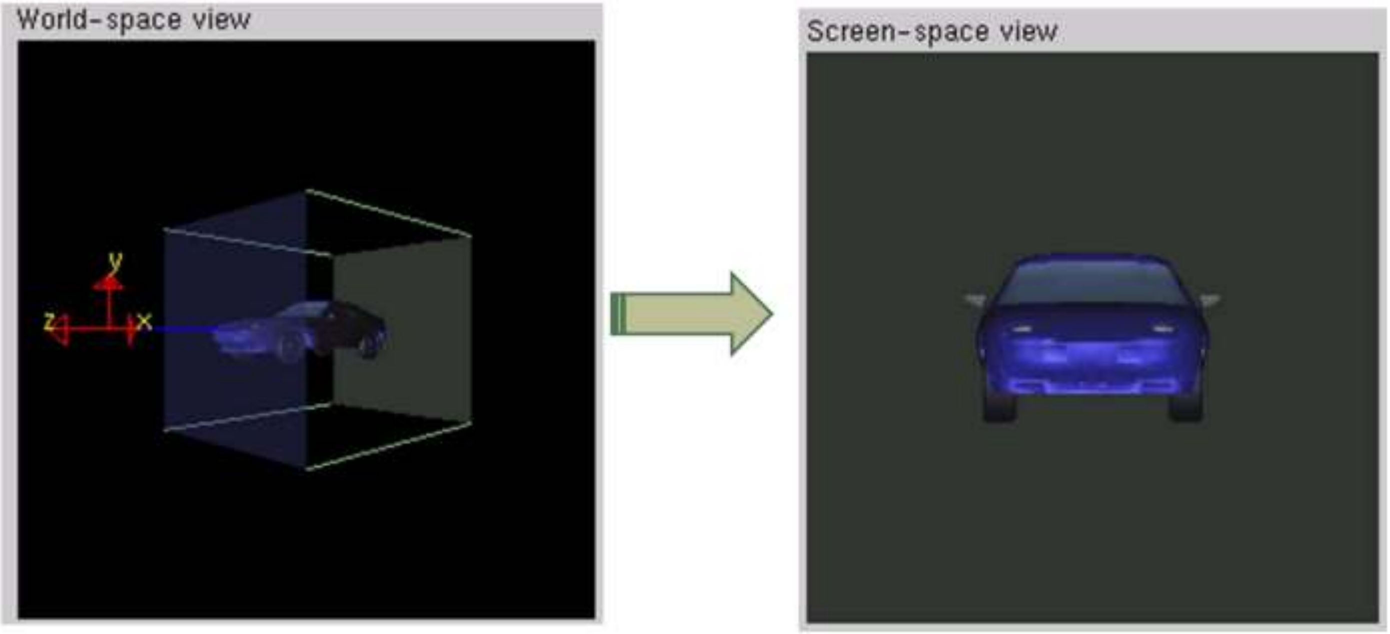
The point at which a set of projected parallel lines appears to converge is called a vanishing point.

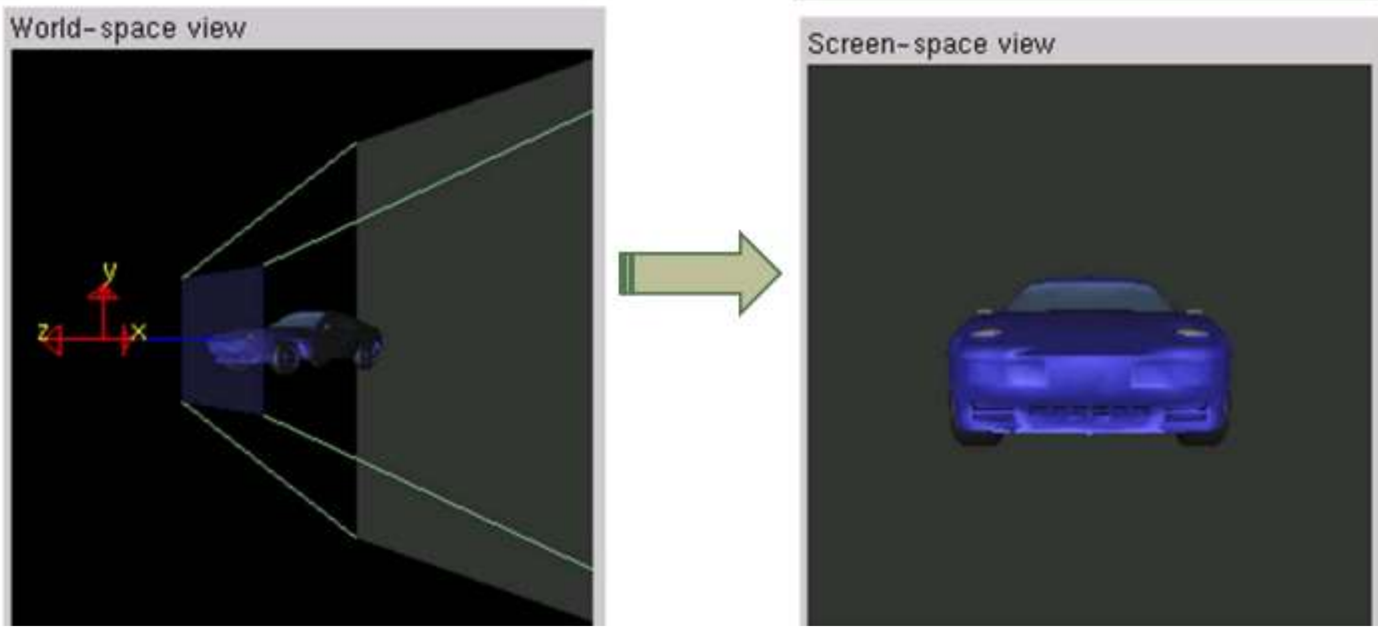
Each such set of projected parallel lines will have a separate vanishing point; and in general, a scene can have any number of vanishing points, depending on how many sets of parallel lines there are in the scene.

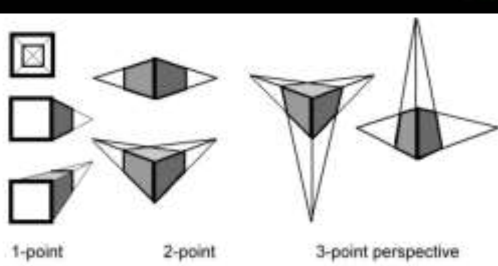
The vanishing point for any set of lines that are parallel to one of the principal axes of an object is referred to as a principal vanishing point.

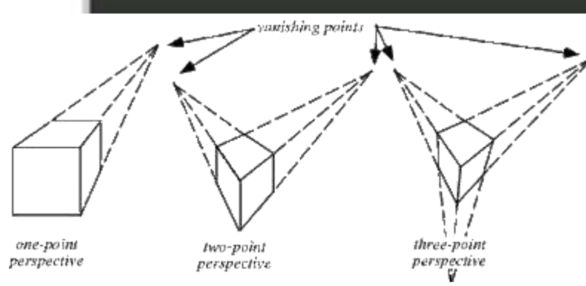
The number of principal vanishing points (one, two, or three) are controlled with the orientation of the projection plane, and perspective projections are accordingly classified as one-point, two-point, or **three-point** projections.

The number of principal vanishing points in a projection is determined by the number of principal axes intersecting the view plane.

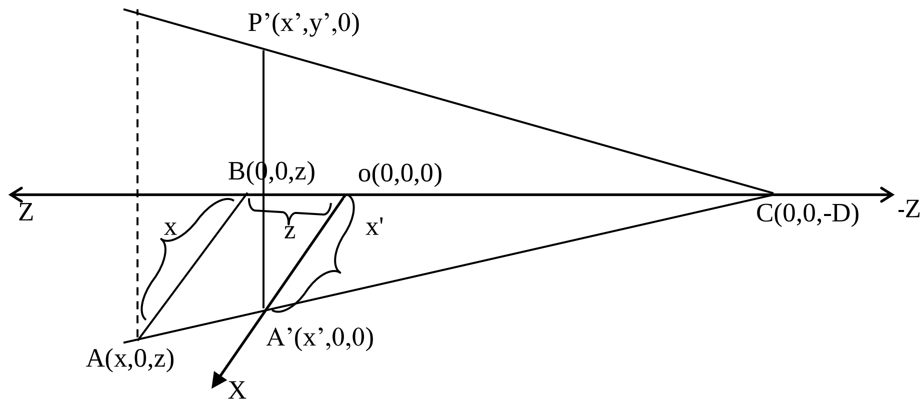








Perspective Projection (Standard)



Here, center of Projection is 'C' (0,0,-D) along the direction of Z axis so the reference point is taken of world coordinate space W_c and the normal vector N is aligned with the y axis.

So now the view plane vp is the xy plane and center of projection is C (0,0,-D) now from similar triangles ABC and A'OC

$$\frac{x}{x'} = \frac{z+D}{D} = \frac{AC}{A'C}$$

$$\text{or } \frac{xD}{z+D} = x' \quad \text{or} \quad x' = \frac{Dx}{z+D}$$

Similarly from triangles APC and A'P'C

$$\frac{y}{y'} = \frac{z+D}{D} = \frac{AC}{A'C}$$

$$\text{or } y' = \frac{Dy}{z+D}$$

and z' =0

now in homogenous coordinates

$$\text{Persv} = \begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix} = \frac{1}{z+D}\begin{pmatrix} Dx \\ Dy \\ 0 \\ z+D \end{pmatrix} = \frac{1}{z+D}\begin{pmatrix} D & 0 & 0 & 0 \\ 0 & D & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & D \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

A unit cube is projected into xy plane . Draw the projected image using standard perspective transformation where center of projection is (0,0,-20)

Here,

Center of projection = (0,0,-20) i.e. d = 20

$$\text{Persp} = \begin{pmatrix} 20 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 20 \end{pmatrix}$$

The cube represented in Homogenous coordinate is

$$V(A,B,C,D,E,F,G,H) = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$V' = \text{Persp} * V \begin{pmatrix} 20 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 20 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= 1/z+D \begin{pmatrix} 0 & 20 & 20 & 0 & 0 & 0 & 20 & 20 \\ 0 & 0 & 20 & 20 & 20 & 0 & 0 & 20 \\ 0 & 0 & 0 & 0 & 20 & 20 & 20 & 20 \\ 20 & 20 & 20 & 20 & 21 & 21 & 21 & 21 \end{pmatrix}$$

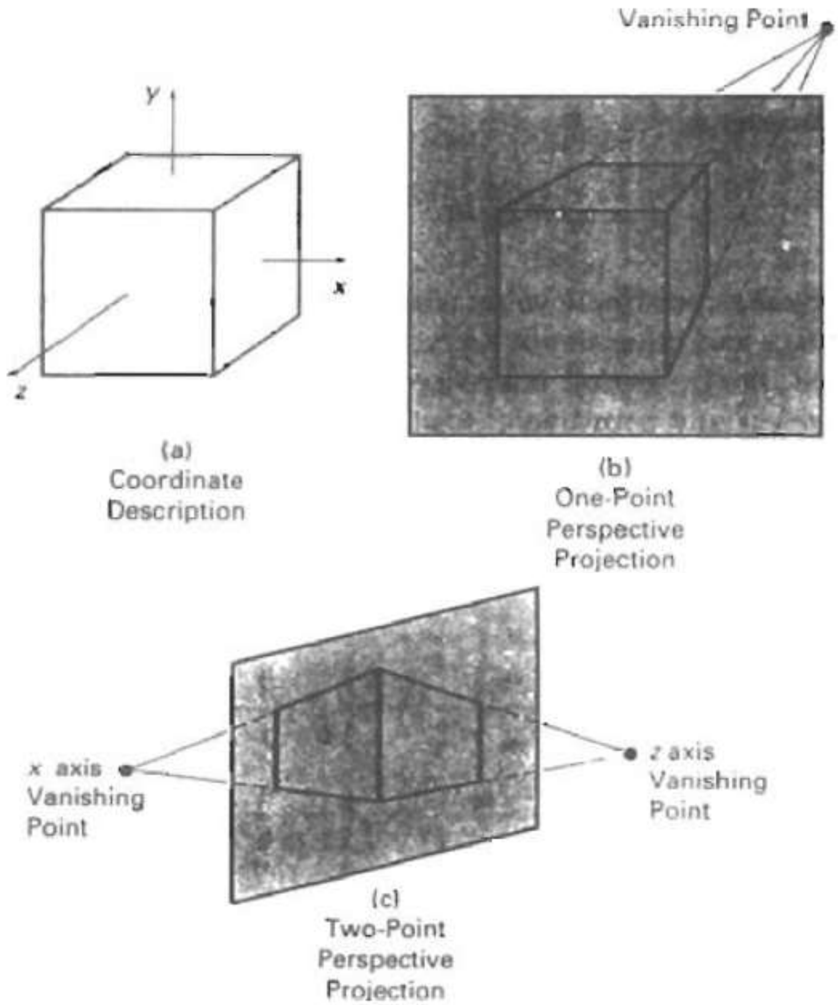
Hence,

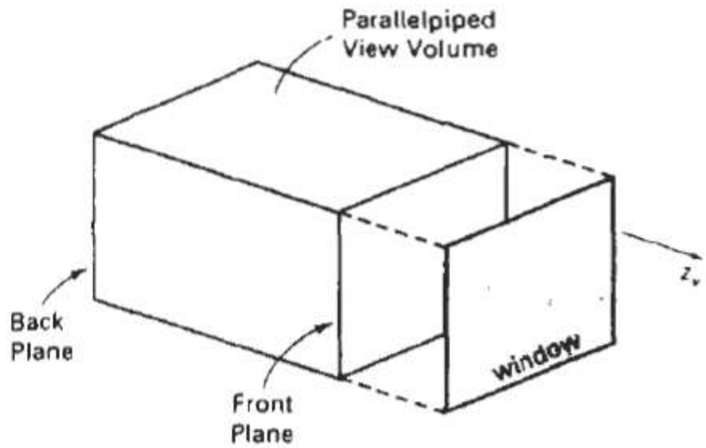
$1/(z+D) = 1/ (0 + 20) = 1/ 20$

A' =(0,0,0) B' =(1,0,0) C' =(1,1,0) D' =(0,1,0)

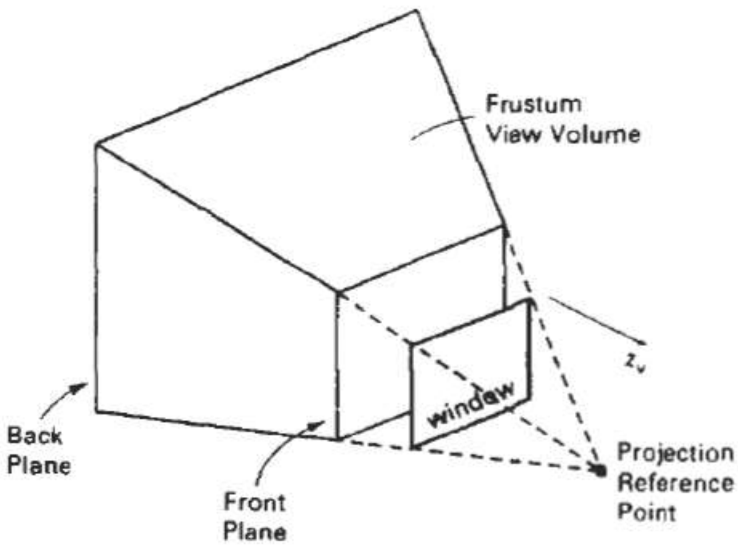
$1/(z+D) = 1/ (1 + 20) = 1/ 21$

E' =(0,20/21,0) F' =(0,0,0) G' =(20/21,0,0) H' =(20/21, 20/21,0)


Vanishing Point
y
x
z
(a)
Coordinate
Description
(b)
One-Point
Perspective
Projection
x axis
Vanishing
Point
z axis
Vanishing
Point
(c)
Two-Point
Perspective
Projection



Parallel Projection



Perspective Projection