NEPAL COLLEGE OF INFORMATION TECHNOLOGY

Balkumari, Lalitpur

Affiliated to Pokhara University

ASSIGNMENT FOR COMPUTER GRAPHICS

ASSIGNMENT 3

Submitted by:
Name: Arpan Adhikari
Roll No.: 231309
BE-CE (3^{rd} SEM)

Submitted to:
Saroj Shakya

Arpan Adhikari
Roll No. 231309
Computer Graphics
Assignment 3

1) A point P(20,20) in a circular window with its center at (100,100) and a radius of 90 pixels is required to be mapped to a point in circular viewport having its at (50,50) and radius of 40 pixels. Where will the point P be placed after the transformation?

Soln

Radius in window $(w_r) = 90$ $\quad (x_{wc}, y_{wc}) = (100,100)$
Radius in viewport $(v_r) = 40$ $\quad (x_{vc}, y_{vc}) = (50,50)$

So, $S_x = \frac{v_r}{w_r} = \frac{40}{90} = 0.44$ $\quad S_y = \frac{v_r}{w_r} = 0.44$

Composite Matrix $(C_m) = T_{(x_{vc}, y_{vc})} \cdot S_{wv} \cdot T_{(-x_{wc}, -y_{wc})}$

$$= \begin{bmatrix} 1 & 0 & 50 \\ 0 & 1 & 50 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.44 & 0 & 0 \\ 0 & 0.44 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -100 \\ 0 & 1 & -100 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.44 & 0 & 6 \\ 0 & 0.44 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

So, $P' = C_m \cdot P$

$$= \begin{bmatrix} 0.44 & 0 & 6 \\ 0 & 0.44 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 20 \\ 1 \end{bmatrix} = \begin{bmatrix} 14.8 \\ 14.8 \\ 1 \end{bmatrix}$$

Hence, the point P(20,20) after being mapped to viewport will be P'(14.8, 14.8) ≈ P'(15,15) Ans

Q2) A triangle with vertices A(5,2), B(4,1), C(6,1) is required to be rotated in a clockwise direction by 45 degrees about any arbitany point (4,4). Find out the final coordinate positions of the triangle after performing the desired transformation.

Soln

Here, $\theta = -45°$ (cw)

Now,

composite matrix $(C_m) = T_{(4,4)} \cdot R_{\theta_{-45°}}^{cw} \cdot T_{(-4,-4)}$

$$= \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-45) & -\sin(-45) & 0 \\ \sin(-45) & \cos(-45) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.707 & 0.707 & -1.66 \\ -0.707 & 0.707 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, $P' = C_m \cdot P$

$$= \begin{bmatrix} 0.707 & 0.707 & -1.66 \\ -0.707 & 0.707 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 4 & 6 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3.29 & 1.87 & 3.29 \\ 1.87 & 1.87 & 0.46 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\approx \begin{bmatrix} 3 & 2 & 3 \\ 2 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Hence, the vertices of triangle after transformation are $A'(3,2)$, $B'(2,2)$, $C'(3,0)$.

Q3) Reflect a triangle $A(1,0)$, $B(3,1)$, $C(1,2)$ about line $y=-2x+5$ then scale it about a fixed point $P(10,10)$.

Sol^n:

Here, $y=-2x+5$

So, $m=-2$, $c=5$

or, $\theta = \tan^{-1}(-2)$

$\therefore \theta = -63.43^\circ$

Lets assume scaling factor as '2' in both directions.

$$C_m = S_{S_x=2, S_y=2} \cdot T_{(0,5)} \cdot R\theta^{-63.43^\circ}_{cw} \cdot R_{fx} \cdot R\theta^{63.43^\circ}_{cw} \cdot T_{(0,-5)}$$

$$= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 63.43^\circ & \sin 63.43^\circ & 0 \\ -\sin 63.43^\circ & \cos 63.43^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 63.43^\circ & -\sin 63.43^\circ & 0 \\ \sin 63.43^\circ & \cos 63.45 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -0.599 & -0.80 & 4 \\ -0.80 & 0.599 & -2.99 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1.19 & -1.60 & 8.00 \\ -1.60 & 1.19 & 4.00 \\ 0 & 0 & 1 \end{pmatrix}$$

So, $P' = C_m . P$

$$= \begin{pmatrix} -1.19 & -1.60 & 8 \\ -1.60 & 1.19 & 4 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 6.81 & 2.83 & 3.61 \\ 2.4 & 0.39 & 4.78 \\ 1 & 1 & 1 \end{pmatrix}$$

Hence, Final coordinates are
(7,2), (3,0), ([illegible].4, 5) Ans

Q4) A triangle with vertices A(5,2), B(4,1), C(6,1) is required to be reflected about an arbitary line $y = 2x + 1$. Find out the final coordinate positions of the triangle after performing the desired transformation.

Soln

Given line, $y = 2x + 1$

So, $m = 2$

or, $\theta = \tan^{-1}(2)$

$\therefore \theta = 63.43°$

Composite Matrix $(C_m) = R_{\theta_{ccw}}^{63.43} \; R_{fx} \; R_{\theta_{cw}}^{-63.43}$

$$= \begin{bmatrix} \cos 63.43° & -\sin 63.43° & 0 \\ \sin 63.43° & \cos 63.43° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.59 & 0.81 & 0 \\ 0.81 & 0.59 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, $P' = C_m \times P$

$$= \begin{bmatrix} -0.59 & 0.81 & 0 \\ 0.81 & 0.59 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 4 & 6 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1.33 & -1.55 & -2.73 \\ 5.23 & 3.83 & 5.45 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\approx \begin{bmatrix} -1 & -2 & -3 \\ 5 & 4 & 5 \\ 1 & 1 & 1 \end{bmatrix}$$

Hence, vertices after transformation are $A'(-1,5)$, $B'(-2,4)$, $C'(-3,5)$.

Q5) A triangle with vertices $A(5,2)$, $B(4,1)$, $C(6,1)$ is required to be rotated by 45 degrees in CC direction about (i) origin and (ii) line $y=5$.

soln

i) origin

Step 1: Rotate the object by 45° in CCW.

$$R_{\theta=45^\circ, CCW} = \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The required component matrin is as,

$$= \begin{bmatrix} 0.7071 & -0.7071 & 0 \\ 0.7071 & 0.7071 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, $\begin{bmatrix} 0.7071 & -0.7071 & 0 \\ 0.7071 & 0.7071 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 4 & 6 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2.1213 & 2.1213 & 3.5355 \\ 4.9497 & 3.5355 & 4.9497 \\ 1 & 1 & 1 \end{bmatrix}$$

(ii) $y=5$ line :

step1 : Translate the line $y=5$ to x-axis.

step2 : Rotate the object by 45° in ccw.

step3 : Translate back the line along with rotated object to original position

The composite matrix is as;

$$C_m = T(0,5) \cdot R_{\theta \, ccw}^{45°} \cdot T(0,-5)$$

$$C_m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7071 & -0.707 & 0 \\ 0.7071 & 0.7071 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7071 & -0.7071 & 3.53 \\ 0.7071 & 0.7071 & 1.46 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, The matrix coordinates after rotation is,

$$p' = \begin{bmatrix} 0.7071 & -0.7071 & 3.53 \\ 0.7071 & 0.7071 & 1.46 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 4 & 6 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5.6513 & 5.6513 & 7.0655 \\ 6.4097 & 4.9955 & 6.4097 \\ 1 & 1 & 1 \end{bmatrix}$$

Hence, the points are $A'(6,6)$, $B'(6,5)$, $C'(7,6)$.

Q6) Clip a line with end point coordinates $A(-1,6)$, $B(5,-8)$ against a clip window with its lower left corner at $(-2,-5)$ and upper right corner at $(4,8)$ using cohen-sutherland algorithm.

Sol^n

Step 1 : Establish region codes for line end points.

	$A(-1,6)$				$B(5,-8)$		
L :	$-1 < -2$	F	'0'	L :	$5 < -2$	F	0
R :	$-1 > 4$	F	'0'	R :	$5 > 4$	T	1
B :	$6 < -5$	F	'0'	B :	$-8 < -5$	T	1
T :	$6 > 8$	F	'0'	T :	$-8 \not> 8$	F	0

So, Bits = 0000   Bits for B = 0110.

So, this is not the case of full visibility.

step 2: Perform AND operation of region codes.

$$\begin{array}{r} 0000 \\ 0110 \\ \hline 0000 \end{array}$$

so, this is the case of partial visibility.

~~For P~~

Here, slope(m) $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8-6}{5-(-1)} = \frac{-14}{6}$

Step 3: Determine intersection points of line with window boundaries.

For B(5, -8),

For bit 2,

$x = x_{wmax} = 4$

and, $y = y_1 + m(x - x_1)$

$= -8 + \frac{-14}{6}(4 - 5)$

$\therefore y = \frac{-17}{3}$

$(x', y') = (4, \frac{-17}{3})$

For bit 3, $y = y_{wmin} = -5$

and,

$x = x_1 + \frac{1}{m}(y - y_1)$

$= 4 + \frac{6}{-14}\left(-5 - \left(\frac{-17}{3}\right)\right)$

$\text{or}, x = \cancel{1.2} 3.71$

$\therefore x \approx \cancel{1} 4$

Hence, ~~B is~~ $(x'', y'') = (\cancel{2}4, -5)$

The coordinates of the end points of visible line segment after clipping are $A'(-1, 6)$ and $B'(\cancel{2}4, -5)$. Ans

Q7) Derive the composite transformation matrix that reflects an object about line 'L' with necessary figures.

Soln

Let the line 'L' be considered as $y = mx + c$.

step1: The line is translated through factor $(0, -c)$ along with the object to be reflected, such that the line passes through origin.

step2: Now, the line along with the object is rotated until it aligns with one of the major coordinate axes (consider X-axis) by angle θ.

where, $\theta = \tan^{-1}(m)$ → Derived from eqn. value

step 3: ~~Tak~~ The reflection of the object is taken along x-axis.

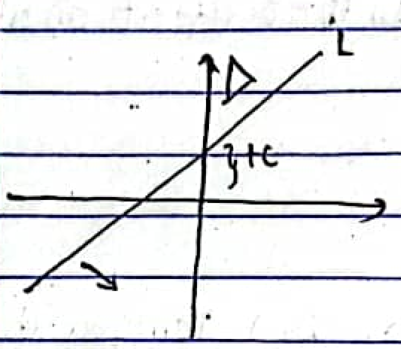
step 4: After reflection, the line along with the object is rotated back to the same slope with angle 'θ' but in opposite direction.

step 5: Now, we perform inverse translation to bring the line back to its initial location along with the reflected object.

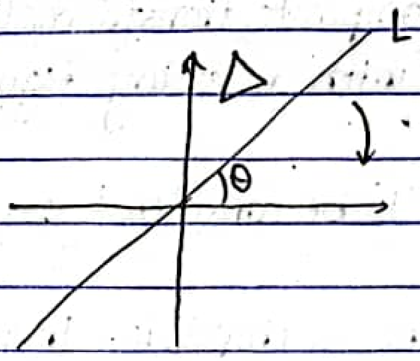
The composite matrix is,

$$C_m = T_{(0,+c)} \cdot R_{\theta_{ccw}} R_{fx} \cdot R_{\theta_{cw}} \cdot T_{(0,-c)}$$

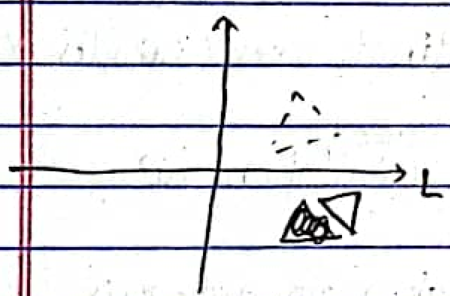
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ +\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{pmatrix}$$



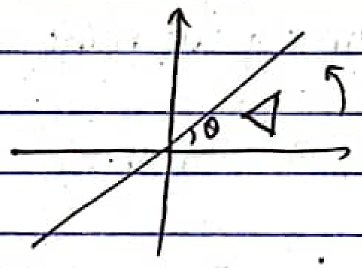
Step : 1 (Translation)



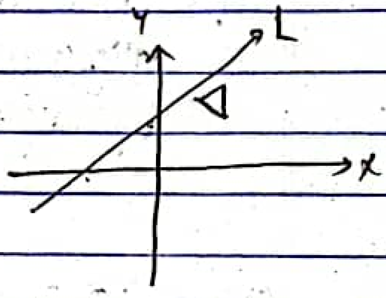
Step 2 : Rotation



step 3 : Reflection



Step 4 : Inverse Rotation



step 5 : Inverse translation

Q8) Find scaling transformation matrix to scale s_x, s_y units with respect to fixed point $P(x,y)$.

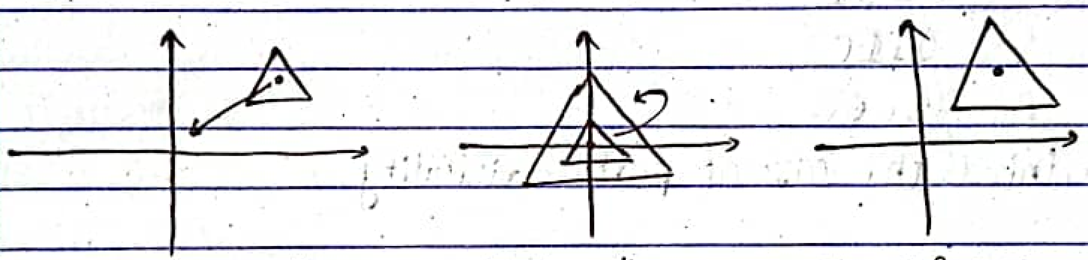
sol^n

Step 1 : The fixed point $P(x,y)$ along with the object to be rotated about it, is translated to origin.
Step 2 : Now scale the object about origin by the desired scaling factor.
Step 3 : Translate back the fixed point which is at the coordinate origin along with the scaled object to its original location.

The composite matrix for these transformations is as follows:

$$Cm = T_{(x,y)} \cdot S_{s_x,s_y} \cdot T_{(-x,-y)}$$

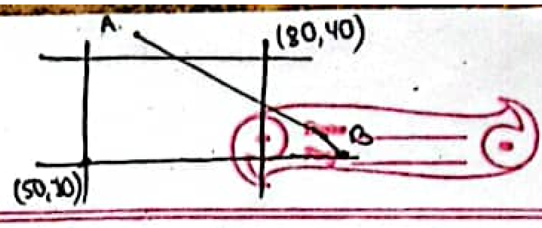
$$= \begin{pmatrix} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -x \\ 0 & 1 & -y \\ 0 & 0 & 1 \end{pmatrix}$$



Step: 1 (Translate) Step2: scaling Step 3: Reverse Translation

Q9) Use cohen Sutherland's algorithm to clip line (60,50) (100,10) against window (50,10) (80,40).

Sol^n



Given points, A(60,50) B(100,10)

$(x_{wmin}, y_{wmin}) = (50,10)$ $(x_{wmax}, y_{wmax}) = (80,40)$

Step 1 : Establish region codes for line end points.

A(60,50) :			B(100,10)		
L : 60 < 50	F	0	L : 100 < 50	F	0
R : 60 > 80	F	0	R : 100 > 80	T	1
B : 50 < 10	F	0	B : 10 < 10	F	0
T : 50 > 40	T	1	T : 10 > 40	F	0
Region code ⇒ ~~000~~1000			Region code ⇒ ~~0100~~ 0010		

Hence, this isn't case of full visibility.

Step 2 : Perform logical AND operation of region codes.

0001
0100
————
0000

Hence, this is the case of partial visibility.

Step 3 : Determine intersection point of line with window boundaries.

For A, $(x_1, y_1) = (60,50)$

$y = y_{wmax} = 40$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-50}{100-60} = \frac{-40}{40} = -1$

$x = \frac{1}{m}(y - y_1) + x_1$

$= \frac{1}{-1}(40-50) + 60$

$\therefore x = 70$

For B, $(x_1, y_1) = (100, 10)$

$x = x_{wmax} = 80$

$y = m(x - x_1) + y_1$

$= -1(80 - 100) + 10$

$\therefore y = 30$

Hence, the coordinates of the end points of visible line segment after clipping are ~~A'(~~, A'(70,40) and B'(80,30). Ans

Q10) show that 2D reflection through x-axis followed by 2D reflection through line $y = -x$ is equivalent to a pure rotation (90 degrees) about origin.

Soln

To prove, lets deduce the composite matrix C_{m1} and C_{m2} for two cases.

For first case,

~~C_{m1}~~ R_{fx} Given line, $y = -x + 0$

so, $m = -1$

$\therefore \theta = -45°$

composite matrix for line reflection $= R_{\theta_{cw}}^{-45°} \cdot R_{fx} \cdot R_{\theta_{ccw}}^{45°}$

Then, Final C_m with reflection through x-axis is,

$C_{m1} = R_{\theta_{cw}}^{-45°} \cdot R_{fx} \cdot R_{\theta_{ccw}}^{45°} \cdot R_{fx}$

$$= \begin{pmatrix} \cos 45° & \sin 45° & 0 \\ -\sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 45° & -\sin 45° & 0 \\ \sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

So, $C_{m1} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Now, For the second case,

$C_{m2} = R_{\theta\,cw}^{-90^\circ}$

$$= \begin{pmatrix} \cos 90^\circ & \sin 90^\circ & 0 \\ -\sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Hence, we can observe $C_{m1} = C_{m2}$

So, 2D reflection through x-axis followed by 2D reflection through line $y = -x$ is equivalent to a pure rotation (90°) about origin.

Q11) Prove that scaling followed by rotation is equivalent to shearing.

Sol^n

Q12) Triangle with vertices A(1,1), B(7,1), C(4,3) is required to be rotated about any arbitrary fixed point (4,2) in a counter clock wise direction by 90 degrees. What will be the final coordinates of the triangle?

Soln

Composite Matrix $(C_m) = T_{(4,2)} \, R_{\theta\,ccw}^{90^\circ} \, T_{(-4,-2)}$

$$= \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 90^\circ & -\sin 90^\circ & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 & 6 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

So, Final coordinates, $P' = C_m . P$

$$= \begin{pmatrix} 0 & -1 & 6 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 7 & 4 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 5 & 3 \\ -1 & 5 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

Hence, final coordinates triangle after rotation are
A(5,-1), B(5,5), C(3,2). Ans

Q13) A point P(20,20) in a window with its lower most left corner at (10,10) and uppermost right corner at (100,100) is required to be mapped to a point in viewport having its lower most left corner at (30,30) and uppermost right corner at (90,80). Where will the point, P be placed after the transformation?

Soln

Given, $P(20,20)$

$(x_{wmin}, y_{wmin}) = (10,10)$ $(x_{vmin}, y_{vmin}) = (30,30)$

$(x_{wmax}, y_{wmax}) = (100,100)$ $(x_{vmax}, y_{vmax}) = (90,80)$

For this,

composite matrix $= T_{(x_{vmin}, y_{vmin})} \cdot S_{s_x, s_y} \cdot T_{-(x_{wmin}, y_{wmin})}$ —— ①

Here, $S_x = \dfrac{x_{vmax} - x_{vmin}}{x_{wmax} - x_{wmin}}$

$= \dfrac{90-30}{100-10} = \dfrac{60}{90} = 0.67$

$S_y = \dfrac{y_{vmax} - y_{vmin}}{y_{wmax} - y_{wmin}}$

$= \dfrac{80-30}{100-10} = \dfrac{50}{90} = 0.56$

So, $C_m = \begin{pmatrix} 1 & 0 & 30 \\ 0 & 1 & 30 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.67 & 0 & 0 \\ 0 & 0.56 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -10 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 0.67 & 0 & 23.3 \\ 0 & 0.56 & 24.4 \\ 0 & 0 & 1 \end{pmatrix}$

So, $P' = C_m \cdot P$

$= \begin{pmatrix} 0.67 & 0 & 23.3 \\ 0 & 0.56 & 24.4 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 20 \\ 20 \\ 1 \end{pmatrix} = \begin{pmatrix} 36.7 \\ 35.6 \\ 1 \end{pmatrix} \approx \begin{pmatrix} 37 \\ 36 \\ 1 \end{pmatrix}$

Hence, the point P will be placed at (37,36) Ans

Q14) Derive a composite transformation matrix for mapping a point P ~~for~~ in a circular window with a radius of 50 pixels and having its center at (50,50) to a circular view port with a radius of 20 pixels and having its center at (10,10).

Sol^n

Here, $(x_{wc}, y_{wc}) = (50,50)$

$(x_{vc}, y_{vc}) = (10,10)$

Then, $w_r = 50$ pixels $\quad v_r = 20$ pixels

So, $s_x = s_y = \frac{v_r}{w_r} = \frac{2\cancel{0}}{5\cancel{0}} = 0.4$

$C_m = T_{(x_{vc}, y_{vc})} \cdot S_{s_x, s_y} \cdot T_{(-x_{wc}, -y_{wc})}$

$$= \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.4 & 0 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -50 \\ 0 & 1 & -50 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.4 & 0 & -10 \\ 0 & 0.4 & -10 \\ 0 & 0 & 1 \end{pmatrix} \text{Ans}$$

Q15) A window has its lower most left corner at (-5,10) and its upper most right corner at (35,40), clip a line segment that has two end points (-20,5) and (60,20). If a viewport is defined by (6,17) and (90,80). Find the view port coordinates for the intersection points.

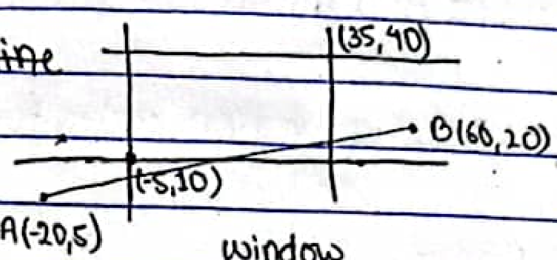
Sol^n

$(x_{wmin}, y_{wmin}) = (-5, 10)$ $\qquad$ $(x_{vmin}, y_{vmin}) = (6, 17)$

$(x_{wmax}, y_{wmax}) = (35, 40)$ $\qquad$ $(x_{vmax}, y_{vmax}) = (90, 80)$

End points of line : A (-20,5) and B (60,20)

~~Step 1 : Establish region codes for line~~



window

First, lets ~~translate~~ transform points to viewport.

$$S_x = \frac{x_{vmax} - x_{vmin}}{x_{wmax} - x_{wmin}} = \frac{90-6}{35-(-5)} = 2.1 \qquad S_y = \frac{y_{vmax} - y_{vmin}}{y_{wmax} - y_{wmin}} = \frac{80-17}{40-10} = 2.1$$

$$C_m = T_{(x_{vmin}, y_{vmin})} \cdot S_{s_x, s_y} \cdot T_{(-x_{wmin}, -y_{wmin})}$$

$$= \begin{pmatrix} 1 & 0 & 6 \\ 0 & 1 & 17 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2.1 & 0 & 0 \\ 0 & 2.1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2.1 & 0 & 16.5 \\ 0 & 2.1 & -4 \\ 0 & 0 & 1 \end{pmatrix}$$

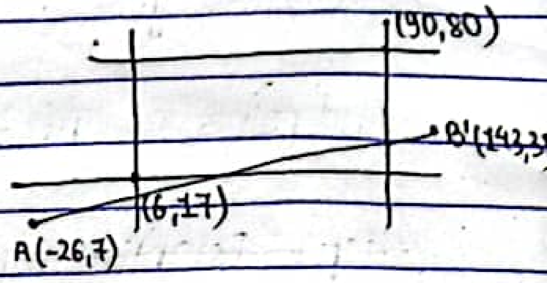
The points mapped to viewport will be A' and B'.

$$P' = C_m \cdot P$$

$$= \begin{pmatrix} 2.1 & 0 & 16.5 \\ 0 & 2.1 & -4 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -20 & 60 \\ 5 & 20 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} -25.5 & 142.5 \\ 6.5 & 38 \\ 1 & 1 \end{pmatrix}$$

so, $A'(-25.5, 6.5)$ and $B'(142.5, 38)$

$\approx A'(-26, 7)$ and $B'(143, 38)$



Now, Lets find intersection points.

step1: Develop region codes for line end points.

$A'(-26,7)$			$B'(143,38)$		
L : $-26 < 6$	T	1	L : $143 < 6$	F	0
R : $-26 > 90$	F	0	R : $143 > 90$	T	1
B : $7 < 17$	T	1	B : $38 < 17$	F	0
T : $7 > 80$	F	0	T : $38 > 80$	F	0
~~Regi~~		(0101 ~~1010~~)			(0010 ~~0100~~)

So, this is not the case of full visibility.

Step 2: Perform AND operation of region code of end points.

$$\begin{array}{r} 1010 \\ 0100 \\ \hline 0000 \end{array}$$

So, this is the case of partial visibility.

Step 3: Find intersection points for end points.

~~For B'(1~~ slope(m) $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{38-7}{143+26} = \dfrac{31}{169}$

For $B'(143, 38)$,

$x = x_{wmax} = 90$

$y = m(x - x_1) + y_1$

$= \dfrac{31}{169}(90 - 143) + 38$ $\therefore y = 28.27 \approx 28$ $(90, 28)$

For $A'(-26,7)$,

For bit 1,

$x = x_{wmin} = 6$

$y = m(x - x_1) + y_1$

$= \frac{31}{169}(6+26) + 7$

$\therefore y = 12.86$ ~~≈13~~ So, $(x', y') = (6, 12.86)$

For bit 3,

~~$y = y_{wmin} = 17$~~ $y = y_{wmin} = 17$

$x = \frac{1}{m}(y - y_1') + x_1'$

$= \frac{169}{31}(17 - 12.86) + 6$

$\therefore x = 28.57 \approx 29$ So, $(29, 17)$

Hence, final coordinates of visible line end points are
~~A(90,~~ A(29,17) and B(90,28) Ans

Q16) Reduce a triangle with coordinates A(0,0), B(0,4), C(8,4), D(8,0) to twice its original about a fixed point P(4,2) also derive a transformation matrix to convert this rectangle into a square.

Sol^n

$C_m = T_{(4,2)} \cdot S_{S_x = \frac{1}{2}, S_y = \frac{1}{2}} \cdot T(-4,-2)$

$$= \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\therefore C_m = \begin{pmatrix} 0.5 & 0 & -2 \\ 0 & 0.5 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P' = C_m \cdot P$$

$$= \begin{pmatrix} 0.5 & 0 & -2 \\ 0 & 0.5 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 8 & 8 \\ 0 & 4 & 4 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$= \cancel{\begin{pmatrix} -4 & -4 & 12 & 12 \\ -2 & 6 & 6 & -2 \\ 1 & 1 & 1 & 1 \end{pmatrix}} = \begin{pmatrix} 2 & 2 & 6 & 6 \\ 1 & 3 & 3 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

So, So, rectangle coordinates after reduction are
(2,1), (2,3), (6,3), (6,1). Ans

To convert rectangle to square, Lets calculate 'l' & 'b' for rectangle,

$$l = 8 - 0 = 8$$
$$b = 4 - 0 = 4$$

So, Lets shorten the length from 8 to 4 in x-axis to be square.

$$\text{so, } S_x = \frac{4}{8} = 0.5$$

$$S_y = 1$$

So, using fixed point, we first translate the fixed point to center (0,0), scale by $S_x = 0.5$, $S_y = 1$ and translate back.

The transformation matrix will be,

$$C_m = T_{(+x_f, y_f)} \cdot S_{S_x = 0.5, S_y = 1} \cdot T_{(-x_f, -y_f)}$$

$$= \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.5 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ Ans}$$

Q17) A triangle with vertices A(0,10), B(10,0), C(-10,0) is required to be shifted down by 5 units, then rotate in anticlockwise direction by 30° and scale by twice its original size. What will be the final location?

Soln

Step 1: shift down by 5 units i.e. $T_{(-5,-5)}$

Step 2: Rotate ccw by 30° i.e. $R_{\theta^{30°}_{ccw}}$

Step 3: scale by $S_x = 2, S_y = 2$.

So, $C_m = S_{S_x=2, S_y=2} \cdot R_{\theta^{30°}_{ccw}} \cdot T_{(-5,-5)}$

$$= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 30° & -\sin 30° & 0 \\ \sin 30° & \cos 30° & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -5 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1.73 & -1 & -3.66 \\ 1 & 1.73 & -13.66 \\ 0 & 0 & 1 \end{pmatrix}$$

So, $P' = C_m \cdot P$

$$= \begin{pmatrix} 1.73 & -1 & -3.66 \\ 1 & 1.73 & -13.66 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 10 & -10 \\ 10 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -13.66 & 13.66 & -20.98 \\ 3.66 & -3.66 & -23.66 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\approx \begin{pmatrix} 14 & 14 & -21 \\ 4 & -4 & -24 \\ 1 & 1 & 1 \end{pmatrix}$$

Hence, final location will be A(14,4), B(14,-4), C(-21,-24). Ans

Q18) Rotate triangle A(0,0), B(1,1), C(5,2) about origin and about point P(-1,-1) by 45° in a cc direction.

Sol:

$$\text{Composite matrix } (C_m) = T_{(-1,-1)} \cdot R_{\theta_{ccw}^{45°}} \cdot T_{(1,1)} \cdot R_{\theta_{ccw}^{45°}}$$

$$= \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 45° & -\sin 45° & 0 \\ \sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 45° & -\sin 45° & 0 \\ \sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & 0.414 \\ 0 & 0 & 1 \end{pmatrix}$$

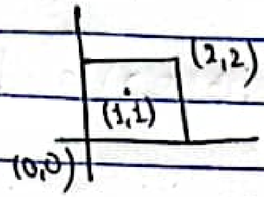
$$P' = C_m \cdot P$$

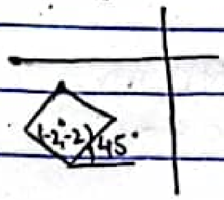
$$= \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & 0.414 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -2 & -3 \\ 0.414 & 1.414 & 5.414 \\ 1 & 1 & 1 \end{pmatrix}$$

Final points : A'(-1,0), B'(-2,1), C'(-3,5) Ans

Q19) Write the series of transformation in matrix that are needed to place the square shown in the figure A reduced to one third of its original size, into the position shown in fig. B where the center of square is at (-2,-2).





soln

$$C_m = T_{(-2,-2)} \cdot R_{\theta\,ccw}^{45°} \cdot S_{s_x=\frac{1}{3},\, s_y=\frac{1}{3}} \cdot T_{(-1,-1)}$$

$$= \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 45° & -\sin 45° & 0 \\ \sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.236 & -0.236 & -2 \\ 0.236 & 0.236 & -2.47 \\ 0 & 0 & 1 \end{pmatrix}$$

To prove, $P' = C_m \cdot P$

$$= \begin{pmatrix} 0.236 & -0.236 & -2 \\ 0.236 & 0.236 & -2.47 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix} //$$

Hence, series is as:

step 1: Translation by factor (-1, -1) to bring object to origin. (fixed point)

step 2: scaling by factor 1/3 in both axis.

Step 3: Rotation by 45° in counter clockwise direction.
Step 4: Translate ~~to~~ by factor (-2, -2) to move fixed point to req^d point as in fig.

Q20) Reflect a ~~point~~ Triangle with vertices A(4,1), B(5,2) and C(4,3) about line $3x-4y+8=0$.

Sol^n

Given line, $-4y = -3x-8$

or, $y = \frac{3}{4}x + 2$

Here, $m = \frac{3}{4}$; $c = 2$

so, $\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87°$

For reflection,

$$C_m = T_{(0,+2)} \cdot R_{\theta_{ccw}}^{36.87°} \cdot R_{fx} \cdot R_{\theta_{cw}}^{-36.87°} \cdot T_{(0,-2)}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 36.87 & -\sin 36.87 & 0 \\ \sin 36.87 & \cos 36.87 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 36.87 & \sin 36.87 & 0 \\ -\sin 36.87 & \cos 36.87 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.279 & 0.960 & -1.92 \\ 0.960 & -0.279 & 0.559 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.279 & 0.960 & -1.92 \\ 0.960 & -0.279 & 2.559 \\ 0 & 0 & 1 \end{pmatrix}$$

So, $P' = C_m . P$

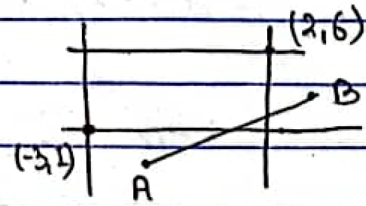
$$= \begin{pmatrix} 0.279 & 0.960 & -1.92 \\ 0.960 & -0.279 & 2.559 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 & 5 & 4 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.159 & 1.39 & 2.079 \\ 6.12 & 6.80 & 5.560 \\ 1 & 1 & 1 \end{pmatrix}$$

Hence, Final triangle vertices are approximately,
A(0,6), B(1,7), C(2,6) Ans

Q21) Clip a line with end points A(1,-2) and B(3,3) against a clip window with its lower left corner at (-3,1) and upper right corner at (2,6).

Soln



Line end points: A(1,-2), B(3,3)

Step 1: Develop region codes for line end points.

A(1,-2):			B(3,3)		
L: 1<-3	F	0	L: 3<-3	F	0
R: 1>2	F	0	R: 3>2	T	1
B: -2<1	T	1	B: 3<1	F	0
T: -2>6	F	0	T: 3>6	F	0
code: (0100)			Code: (0010)		

So, this is not the case of full visibility.

Step 2: Perform AND operation of region codes of end points.

$$\begin{array}{r} 0100 \\ 0010 \\ \hline 0000 \end{array}$$

So, this is the case of partial visibility.

Step 3: Find intersection points with clip window.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{3-1} = \frac{5}{2}$$

For A, (1, -2)

$$y = y_{wmin} = 1$$

$$x = \frac{1}{m}(y - y_1) + x_1$$

$$= \frac{2}{5}(1+2) + 1$$

$$= 2.2 \qquad (2.2, 1) \approx (2, 1)$$

For B, (3, 3)

$$x = x_{wmax} = 2$$

$$y = m(x - x_1) + y_1$$

$$= \frac{5}{2}(2-3) + 3$$

$$= \qquad (2, 0.5) \approx (2, 1)$$

Hence, the visible end points of line are A(2,1) and B(2,1)
i.e. line passes through intersection point of clip window.

Q22) Prove that reflection about line $y=-x$ is equivalent to reflection about y-axis followed by a counter clockwise rotation by 90°.

Sol^n

Case I : Reflection about line $y=-x$.

Here, $m=-1$

or, $\theta = \tan^{-1}(-1)$

$\therefore \theta = -45°$

$$C_m = R_{\theta\,CW}^{-45°} \cdot R_{fx} \cdot R_{\theta\,CCW}^{+45°}$$

$$= \begin{pmatrix} \cos 45° & \sin 45° & 0 \\ -\sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 45° & -\sin 45° & 0 \\ \sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\therefore C_{m_1} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Case II : Counter clockwise rotation by 90° after R_{fy}.

$$C_m = R_{\theta\,CCW}^{90°} \cdot R_{fy}$$

$$= \begin{pmatrix} \cos 90° & -\sin 90° & 0 \\ \sin 90° & \cos 90° & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\therefore C_{m_2} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

As the composite matrix obtained in both cases are same, reflection about $y = -x$ is equivalent to reflection about y-axis followed by a counter clockwise rotation by 90 degrees.