Pre-University Examination subject wise paper collection

Algebra And Geometry

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POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Algebra and Geometry

Semester : Spring

Year : 2023
Full Marks: 100
Pass Marks: 45
Time : 3hrs

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Check consistency and solve by Gauss elimination method $x + y + z = 6, x - y + z = 2, 2x + y - z = 1.$ 7

   b) Define eigen value and vector. Find the eigen value, eigenvector and diagonalize of the matrix $A = \begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}$ 8

2. a) Solve the linear programming problem by simplex method.[constructing duality] 7
   Minimize $Z= 2x_1+ 9x_2+ x_3$ subject to $x_1+ 4x_2+ 2x_3 \geq 5$, $3x_1+x_2+2x_3\geq 4$, $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$.

   OR

   Using simplex method maximize $Z = 150x_1+300x_2$ subject to $2x_1+x_2\leq 16$, $x_1+x_2\leq 8$ *and* $x_2\leq 3.5$, $x_1\geq 0$, $x_2\geq 0$.

   b) Solve (Big M- method) Maximize $z = -3x_1 +7x_2$ subject to $2x_1 + 3x_2\leq 5$, $5x_1 +2x_2\geq 3$, $x_2 \leq 1$ 8

3. a) State D' Alembert's Ratio test show that 7

   i. $\sum\frac{1}{n}$ is divergent.

   ii. $\sum\frac{1}{n^2}$ is convergent.

   iii. $\sum\frac{(-1)^n}{n}$ is conditional convergent.

   b) Find the center, radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x)^{2n+1}}{(-4)^n}$. 8

4. a) Define eccentricity of a conic section, and derive the equation of the ellipse in its standard form. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

OR

Show that the line $lx + my + n = 0$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $a^2l^2 - b^2m^2 = n^2$. Also find the point of contact.

b) Show that the equations: $16x^2 - 24xy + 9y^2 - 104x - 172y + 44 = 0$ represents a parabola. Also find the equation of axis and vertex. 8

5. a) Define scalar triple product and reciprocal vectors. Find the reciprocal vector of 7

$\overrightarrow{2i} + \overrightarrow{3j} - \vec{k}$, $\vec{i} - \vec{j} - \overrightarrow{2k}$, $-\vec{i} + \overrightarrow{2j} + \overrightarrow{2k}$.

b) Find the distance of the point (3,-4,5) from the plane $2x+5y+6z=16$ measured along a line with direction cosines proportional to 2,1,-2

6. a) Find the shortest distance between the lines $\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2}$ and $\frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}$ Also Find the equation of shortest distance 7

b) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle. 8

7. Attempt all the questions 2.5×4

a) Show that $[\vec{a} \times \vec{b}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$

b) Find the centre the hyperbola:

$5x^2-4y^2+20x+8y=4$.

c) Show that the mapping T: $R^2 \rightarrow R^2$ define by

$T(x, y) = (x, x+y)$ is linear.

d) Plane through the OX and OY include an angle α. Show that their lines of intersection lie on the cone is $(z^2 + x^2)(y^2 + z^2)\cos^2\alpha = x^2y^2$.

POKHARA UNIVERSITY
FACULTY OF SCIENCE AND TECHNOLOGY
SCHOOL OF ENGINEERING

Exam	Final Internal Examination 2024		
Level	B.E.	F M	100
Program	BoCE	PM	45
Year/ Part	I/II	Time	3 Hrs

Subject: Algebra and Geometry

Candidates are required to give answers in their own words as far as practicable. The figure in the margin indicates full marks. Assume suitable data if necessary. Attempt all the questions.

1.	a)	Classify the system of linear equations on the basis of rank. Test the consistency and solve $x + 2y - 3z = 9, 2x - y + 2z = -8, 3x - y - 4z = 3$.	8
	b)	Define basis of a vector space. Show that the vectors $\{(1, 2, 0), (1, 0, 3), (0, 1, 1)\}$ forms a basis of R^3.	7
2.	a)	Find the eigen value and corresponding eigen vectors of the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.	7
	b)	Construct the dual problem and solve by using simplex method of the given primal: Minimize $Z = 20x_1 + 30x_2$ subject to $x_1 + 4x_2 \geq 8,\ x_1 + x_2 \geq 5,\ 2x_1 + x_2 \geq 7,\ x_1, x_2 \geq 0$.	8
3.	a)	Prove that if the vectors $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, then $\vec{a} \times \vec{b},\ \vec{b} \times \vec{c},\ \vec{c} \times \vec{a}$ are also non-coplanar.	7
	b)	Is the series $\sum_{n=1}^{\infty} \frac{(n+3)!}{3!\, n!\, 3^n}$ convergent ?	8
4.	a)	Find the interval of convergence, center of convergence and radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(n+1)(x-4)^n}{10^n}$	7
	b)	Find the centre, vertex, eccentricity, foci, length of major axis of the ellipse $9x^2 + 4y^2 - 18x - 16y - 11 = 0$. **OR** Derive the equation of ellipse in standard form.	8
5.	a)	Show that the equation $9x^2 - 24xy + 16y^2 - 50x - 100y + 225 = 0$ represents a parabola and find its (i) axis (ii) vertex (iii) directrix (iv) focus (v) equation and length of latus rectum.	8

	b) Find the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$. Also find the equation of the line of shortest distance. **OR** Find the image of the point P(1, 3, 4) in the plane $2x-y+z+3=0$.	7
6.	**a)** Find the centre and radius of the circle $x^2+y^2+z^2+12x-13y-16z+111=0,\ 2x+2y+z=17.$	8
	b) Define right circular cone. Find the equation to a right circular cone whose vertex is origin O, axis OX and semi-vertical angle α.	7
7.	***Attempt all the questions.*** **a)** Verify Cayley Hamilton theorem for the matrix $A=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}$. **b)** Find the volume of the parallelepiped whose concurrent edges are given by $\vec{a}=\vec{i}+2\vec{j}+3\vec{k},\ \vec{b}=3\vec{i}+4\vec{j}-5\vec{k},\ \vec{c}=\vec{i}-2\vec{j}+3\vec{k}.$ **c)** Find the equation of the hyperbola whose focus is at (6, 0), directrix is the line $4x-3y=6$ and eccentricity is $\frac{5}{4}$. **d)** Prove that the line $\frac{x+1}{-2}=\frac{y+2}{3}=\frac{z+5}{4}$ lies in the plane $x+2y-z=0$.	10

*** *Best of Luck* ***

NEPAL COLLEGE OF INFORMATION TECHNOLOGY

Assessment: Spring

Level: Bachelor
Programme: BE IT 2 M1 ,M2 CE, Day (II)
Course: Algebra and Geometry

Year: 2024
Full Marks: 100
Pass Marks: 45
Time: 3 Hrs

Candidates are required to give their answers in their own words as far practicable. The figure in the margin indicates full marks.

Attempt all the Questions.

1. a) Define Eigen value and Eigen vectors. Find Eigen value, Eigen(8) vector and diagonalize of the matrix $A=\begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}$

   b) Check the consistency, if consistence .Solve by using Gauss elimination method (7)

$$2x - y + 3z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2$$

OR

Show that {(1,0,2), (2,-1,1), (0,1,1)} forms a basis for R^3 space.

2. a) State p-test. (i) Test the convergence of the given series: $\sum[\sqrt{n^3+1} - \sqrt{n^3-1}\,]$ (ii)show that $\sum \frac{(-1)^n}{n}$ is conditionally convergent.(7)

   b) Find the radius of convergence and interval of convergence of the series $\sum_{x=0}^{\infty} \frac{(x-2)^n}{n10^n}$ (8)

3. a) Find the distance of the point (3,-4,5) from the plane 2x+5y-6z=16 measured along the line with direction cosines proportional to 2,1,-2 (8)

   b) Find the image of the point (1,3,4) in the plane 2x-y+z+3= 0 (7)

4. a) Find equation of the cone with vertex at (0,0,3) and guiding curve is $x^2 + y^2 = 4, z = 0$. (7)

OR

Find the equation of right circular cylinder whose guiding curve is a circle $x^2 + y^2 + z^2 = 9, x - y + z = 3$.

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26 july 2024

9 b) Define skew lines and shortest distance between them. Find magnitude and equation of shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$. (8)

5. (a) Obtained the equation of sphere having the circle (7)
$x^2 + y^2 + z^2 + 7y - 2z + 2 = 0, \quad x + y + z = 3$ as a great circle.

(b) Show that

i) $(\boldsymbol{b} \times \boldsymbol{c}) \times (\boldsymbol{c} \times \boldsymbol{a}) = [\boldsymbol{a}\ \boldsymbol{b}\ \boldsymbol{c}]\boldsymbol{c}$ (8)

ii) $[\boldsymbol{a}+\boldsymbol{b} \quad \boldsymbol{b}+\boldsymbol{c} \quad \boldsymbol{c}+\boldsymbol{a}] = 2[\boldsymbol{a}\ \ \boldsymbol{b}\ \ \boldsymbol{c}]$

6 (a) Solve linear programming problem by using the Big M method (8)
Maximize Z= $5x_1 - 2x_2$ subject to $3x_1 - 4x_2 \leq 2,\ x_1 + 2x_2 \geq 4,\ x_2 \leq 4 \quad x_1, x_2 \geq 0$

(b) Find the dual of the primal problem and solve using simplex method: (8)
minimize $z = 8x_1 + 9x_2$ subject to $x_1 + 3x_2 \geq 4, 2x_1 + x_2 \geq 5, x_1; x_2 \geq 0$.

7. Answer all : (4*2.5=10)

a) Show that the transformation T:$R^2 \rightarrow R^2$ defined by T(x,y) = (x+y, x-y) is linear.

b) Find the direction cosine of normal of a plane $x + 2y - 3z = 5$.

c) Find the rank of the given matrix $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix}$

d) Define scalar triple product of three vectors and write its geometrical interpretation.

4M - (M-2)4 + 5

(M-2)2

POKHARA ENGINEERING COLLEGE

Internal Assessment Examination

Level: Bachelor
Program: BE
Course: Algebra and Geometry

Year: 2024
Full Marks: 100
Pass Marks: 45
Time: 3 hrs

Candidates are required to give their answer in their own words as far as practicable The figure in the margin indicate full marks.

Attempt all the question

1 a) Check the given system of linear equations is consistent or not, if consistent then solve by Gauss elimination method. 7

$5x-2y+3z=2 \qquad -3x-5z=-3 \qquad x+y-4z=4$

b) Find Eigen value and corresponding Eigen vectors of the matrix. 8

$$A=\begin{pmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{pmatrix}$$

OR

Define Rank of matrix Find Rank of following matrix by using Echelon form.

$$\begin{pmatrix}2 & 1 & 3\\ 4 & 7 & 13\\ 4 & -3 & -1\end{pmatrix}$$

2 a) Define basis for a vector space. 7
Show that the vectors (1,2,1), (2,1,0), (1,-1,2) form a basis of R^3.

b) Using simplex method, solve the following LPP 8
Maximize $Z=2x_1-x_2+2x_3$, subject to constraints $x_1+2x_2-2x_3\le 20$, $2x_1+x_2\le 10$, $x_2+2x_3\le 5$ $x_1,x_2,x_3\ge 0$

OR

Construct the dual problem and solve by simplex method.
Minimize $Z=8x_1+9x_2$, subject to constraints $x_1+x_2\ge 6$, $3x_1+x_2\ge 21$ $x_1,x_2\ge 0$

3 a) Find interval of convergence, centre of convergence and radius of convergence of the series. $\sum_{n=1}^{\infty}\frac{(n+1)(x-4)^n}{10^n}$ 7

b) Show that the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}$ is conditionally convergent. 4

c) Test the convergence and divergence of the following series by using D'Alembert ratio test $\frac{1}{2}+\frac{4}{2^2}+\frac{9}{2^3}+\dots$ 4

4 a) Define scalar and vector product three vectors. Find the volume of a parallelepiped whose concurrent edges are represented by the vectors. $\bar{i}+\bar{j}+\bar{k}$; $\bar{i}-\bar{j}+\bar{k}$ and $\bar{i}+2\bar{j}-\bar{k}$ 7

5 a) Define conic section and derive the standard equation of Hyperbola. 7

b) Determine the vertex, equation of axes , focus and directrix of the conic $16x^2+9y^2-24xy-104x-172y+44=0$ 8

OR

Obtain the condition that the line lx+my+n=0 may touch the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$..

6 a) Show that the lines $\frac{x+4}{3}=\frac{y+6}{5}=\frac{1-z}{2}$ and 3x-2y+z+5=0=2x+3y+4z are coplanar. Find their point of intersection and the plane in which they lie. 7

b) Obtain the equation of the sphere which passes through the three points(1,0,0),(0,1,0),(0,0,1) and has its radius as small as possible 8

10

7 Attempt all:

a) Find centre and radius of the sphere $x^2+y^2+z^2$-2x+4y-6z=11.

Show that the vectors $\bar{a}\times(\bar{b}\times\bar{c})$, $\bar{b}\times(\bar{c}\times\bar{a})$, $\bar{c}\times(\bar{a}\times\bar{b})$ are coplanar.

b) Show that the mapping $T:R^2\rightarrow R^2$ defined by T(x,y)=(x,x+y) is

c) linear

d) Verify Cayley Hamilton theorem $A=\begin{pmatrix}1 & 2\\ 2 & 4\end{pmatrix}$

LUMBINI ENGINEERING COLLEGE

Second Internal Exam (2023)

Level: Bachelor | Semester: Spring | Full Marks : 100

Programme : B.E. | | Pass Marks :45

Course : Algebra and Geometry | | Time :3hrs.

Attempt all the questions.

1. a) Check the consistency of the given system of equations and solve it: (7)

   $5x + 3y + 7z = 4; \quad 3x + 26y + 2z = 9; \quad 7x + 2y + 10z = 5$

   b) Define basis of a vector space over the field. Show that the vectors $(1,2,1)$, $(2,1,0)$, $(1,-1,2)$ form a basis of R^3. (8)

2. a) Find the eigen value and eigen vector of the matrix: $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. (8)

   b) Use simplex method to find the minimize value of $Z = -x_1 + x_2 - 3x_3$ subject to the constraints:

   $x_1 + x_2 + x_3 \le 10, \; -2x_1 + x_3 \ge -2, \quad 2x_1 - 2x_2 + 3x_3 \le 0, \quad x_1, x_2, x_3 \ge 0$. (7)

3. a) Show that the four points with position vectors $(4,5,1), (0,-1,-1), (3,9,4)$ and $(-4,4,4)$ are coplanar. (7)

   b) If $\vec{a'}, \vec{b'}, \vec{c'}$ is a reciprocal system to $\vec{a}, \vec{b}, \vec{c}$ then show that $\vec{a}, \vec{b}, \vec{c}$ is also reciprocal system to $\vec{a'}, \vec{b'}, \vec{c'}$ (8)

   OR

   If $\vec{a}, \vec{b}, \vec{c}$ are coplanar and $\vec{a}$ is not parallel to $\vec{b}$, prove that

   $$\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix} \vec{c} = \begin{vmatrix} \vec{c}.\vec{a} & \vec{a}.\vec{b} \\ \vec{c}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix} \vec{a} + \begin{vmatrix} \vec{a}.\vec{a} & \vec{c}.\vec{a} \\ \vec{a}.\vec{b} & \vec{c}.\vec{b} \end{vmatrix} \vec{b}.$$

4. a) Find the condition that the line $lx + my + n = 0$ may be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Also find the point of contact. (8)

   b) Find the equation of the tangents to the hyperbola $3x^2 - 4y^2 = 12$ which are perpendicular to the line $y = x + 2$. Also find the point of contact. (7)

5. a) Find the magnitude and equation of the shortest distance between the lines $\frac{x-5}{3} = \frac{7-y}{16} = \frac{z-3}{7}$ and $\frac{x-9}{3} = \frac{y-13}{8} = \frac{15-z}{5}$. (8)

   b) Find the equation of the tangent planes to the sphere $x^2 + y^2 + z^2 + 6x - 2z + 1 = 0$ which passes through $\frac{16-x}{1} = z = \frac{2y+30}{3}$. (7)

   OR

   Show that the equation to a right circular cone whose vertex is O, axis OX and semi - vertical angle α is $y^2 + z^2 = x^2 tan^2\alpha$

6. a) Test the convergence or divergence of series : $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots\ldots\ldots$ (7)

   b) Prove that for infinite series $\sum a_n$ to be convergent it is necessary that $\lim_{n\to\infty}(a_n) = 0$. By taking suitable example show that the converse may not be true. (8)

7. Attempt all the questions. $(4 \times 2.5 = 1($

   a) Find the rank of matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{bmatrix}$.

   b) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$.

   c) Show that the mapping $T: R^3 \to R^2$ defined by $T(x,y,z) = (x + y - z, \; 0)$ is linear.

   d) Find the equation of hyperbola with vertex at $(\pm 2, \; 0)$ and foci at $(\pm 5, \; 0)$.

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Lumbini Engineering, Management & Science College

Final Internal Assessment Exam

Year: 2024

Level: Bachelor

Full Mark: 100

Program: Computer/ Electrical 2nd sem.

Pass Mark: 45

Course: Algebra and Geometry

Attempt all questions

1.a) Define consistency of a system of linear equations. Test the consistency and solve $x-y+2z+1=0,\ 2x+y+z=1,\ x+2y-z=2.$ (7)

b) Find the eigen values and corresponding eigen vectors of the matrix $A=\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ (8)

OR

Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{pmatrix}$. Also find the inverse of the matrix by using this theorem.

2.a) Find the dual of the primal and solve by using simplex method. (7)

Minimize $Z= 21x_1+50x_2$ subject to constraints $2x_1+5x_2\geq12,\ 3x_1+7x_2\geq17,\ x_1, x_2\geq0.$

b) Maximize the given LPP by Big M method. Maximize $Z= -3x_1+7x_2$ subject to the constraints $2x_1+3x_2\leq5,\ 5x_1+2x_2\geq3,\ x_2\leq1,\ x_1, x_2\geq0.$ (8)

3.a) Find the set of reciprocal system of vectors for (7)
$\hat{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k},\quad \hat{b} = \hat{\imath} - \hat{\jmath} - 2\hat{k},\quad \hat{c} = -\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

b) Reduce the equation of the line $x+2y+3z-6=0=3x+4y+5z-2$ in the symmetrical form. (8)

4.a) Prove that the lines (8)

$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ *and* $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar.

Also, find the equation of the plane containing them.

OR

Find the shortest distance between the lines

$\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}$ *and* $\frac{x-2}{3}=\frac{1-y}{5}=\frac{z+2}{2}$. Find also the equation of shortest distance.

b) Find the equations of the tangent planes to the sphere $x^2+y^2+z^2+2x-4y+6z-7=0$ which intersect the line $6x-3y-23=0=3z+2$. (7)

5.a) Find the condition that the line $lx+my+n=0$ touches the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Also find the point of contact.

OR

Define conic section. Derive the equation of the ellipse in standard form. (7)

b) Show that the equation $16x^2-24xy+9y^2-104x-172y+44=0$ represents of parabola. Also find the equation of axis and vertex. (8)

6.a) State Cauchy Root Test. Test the convergence or divergence of the series $\sum\left(\frac{n-\ln n}{2n}\right)^n$. (7)

b) Find the centre, radius and interval of convergence of power series $\sum_{n=1}^{\infty}\frac{(x)^{2n+1}}{(-4)^n}$ (8)

7. Attempt all questions: **(4*2.5=10)**

a. Show that $[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}]=[\vec{a} \quad \vec{b} \quad \vec{c}]^2$

b. Show that the mapping T: $R^2\to R^2$ defined by $T(x,y)=(x+y,\ x-y)$ is linear.

c. Show that the vectors (1, -1, 1), (1, 1, 1) and (2, 0, 3) generate R^3.

d. Find the equation of the cone with vertex (α, β, γ) and base $y^2=4ax$, $z=0$.

NEPAL ENGINEERING COLLEGE
[Set 2]

Level: Bachelor Assessment Year : 2023
Programme: BE (Computer/CRE) Full Marks: 100
Course: **Algebra and Geometry** Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt any ALL questions.

1. a) Define consistency of the system of linear equations. Classify the system of linear equations according to the nature of its solution. Test the consistency of the system of equations. If consistent, solve by Gauss elimination method 1+2+5

   $x - 5y + 2z = -1,\ 2x + y + z = 1,\ x + 2y - z = 2.$

   b) i. Define linearly dependent and independent set of vectors. Are the vectors (1, 1, 1), (1, 2, 3), (– 1, 1, 3) linearly dependent? 4+4

   ii. Define linear transformation. Is the transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by $T(x, y) = |x + y|$ a linear transformation?

2. a) State Cayley-Hamilton theorem. Verify the theorem for the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ and also find the inverse of the matrix. 1+3+3

   OR

   Find the eigen values and eigenvectors of the matrix $\begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ 3+4

   b) Constructing duality, solve the LPP: Minimize $z = 8x_1 + 9x_2$, subject to $x_1 + 3x_2 \geq 4,\ 2x_1 + x_2 \geq 5,\ x_1 \geq 0,\ x_2 \geq 0$ 8

3. a) Find the reciprocal system of the set of vectors: 7

   $2\vec{i} + 3\vec{j} - \vec{k},\ \vec{i} - \vec{j} - 2\vec{k},\ -\vec{i} + 2\vec{j} + 2\vec{k}$

   b) i) If $\vec{a}, \vec{b}, \vec{c}$ are three coplanar vectors, prove that 4+4

   $\vec{b} + \vec{c},\ \vec{c} + \vec{a}$ *and* $\vec{a} + \vec{b}$ are also coplanar

   ii) If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, prove that $\vec{b} \times \vec{c},\ \vec{c} \times \vec{a}$ *and* $\vec{a} \times \vec{b}$ are also non- coplanar.

4. a) State the p-test for the convergence of an infinite series. Also test the convergence of the series $\sum \frac{n}{1+n\sqrt{n}}$ 7

b) Find the interval centre and radius of convergence of the series $\sum \frac{(x-5)^n}{n5^n}$ 8

5. a) Find the centre, vertices, foci, eccentricity and length of latus rectum of the ellipse $9x^2 - 16y^2 - 72x + 96y - 144 = 0$. 7

b) Find the condition of a line $x + ny + n = 0$ to be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Also find the point of contact. 8

6. a) Prove that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{2} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect at a point. Also find the point of intersection and the plane containing them. 8

OR

Find the length and the equation of the line of shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

b) Find the centre and the radius of the circle of intersection of the plane $x - 2y + 3z = 3$ and the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$. 7

OR

Prove that the second degree equation

$$2x^2 - 4yz - 8xy - 4x - 2y + 6z + 35 = 0$$

Represents a cone. Also find its vertex.

7. **Attempt ALL questions (2.5 × 4)** 10

a) Is the set {(1, 1,0), (1, 0, 1), (3, 1, 2)} forms a basis of R^3 ?

b) Find the equation of a line through the point (3, 2, – 6) and parallel to the line $\frac{x-2}{2} = \frac{y-4}{3} = \frac{z-6}{5}$.

c) Find the centre and the radius of the sphere $x^2 + y^2 + z^2 - 6x + 4y - 2z + 5 = 0$.

d) Find the radius of the sphere having (1, – 2, 4) and (3, 2. 2) as two ends of a diameters.

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National Academy of Science and Technology

(Affiliated to Pokhara University)

Dhangadhi, Kailali

Pre University Examinations

Level : Bachelor | Semester: II_Spring | Year : 2024

Programme: B.E.(Civil/Computer) | F.M. : 100

Course: Algebra and Geometry | P.M. : 45

Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt all the questions.

1.a) Define consistency of the system of linear equations. Investigate for what values of p and q , the system of the equations $x+y+z=6$, $x+2y+5z = 10$, $2x+3y+pz=q$ has

i. No solution

ii. A unique solution

iii. an infinite number of solutions. [7]

b) Define Eigen values and vectors of a square matrix with its characteristics equation. If the Eigen values and the corresponding eigenvectors of the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. [8]

2.a) Verify the Cayley Hamilton theorem by matrix A, and using it, find the inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$ [7]

b) Using simplex method maximize $Z = 30x_1+20x_2$ subject to $-x_1+x_2 \le 5$, $2x_1+x_2 \le 10$, $x_1 \ge 0$, $x_2 \ge 0$ [8]

3.a) Find the dual of given Lpp and solve by using simple method Minimize $Z=8x_1+9x_2$ subject to $x_1+x_2 \ge 5$, $3x_1+x_2 \ge 21$, $x_1 \ge 0$, $x_2 \ge 0$. [7]

b) Define vector triple product. If

$\vec{a} = \vec{i} - 2\vec{j} - \vec{k}$, $\vec{b} = 2\vec{i} + \vec{j} - 2\vec{k}$ and $\vec{c} = -2\vec{i} + 3\vec{j} + 2\vec{k}$

find $\vec{a} \times (\vec{b} \times \vec{c})$. Also verify that

$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$. [8]

4.a) Find the set of reciprocal vectors of

$\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = \vec{i} - \vec{j} - 2\vec{k}$ and $\vec{c} = -\vec{i} + 2\vec{j} + 2\vec{k}$. [7]

b) Define eccentricity of a conic section, and derive the equation of a ellipse in its standard form. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ [8]

5.a) Find the condition that the line lx+my+n=0 touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, Find the point of contact. [7]

OR

Find the center, eccentricity, foci and directrix of the hyperbola

$9x^2 - 16y^2 - 72x + 96y - 144 = 0$

b) Find the center, radius of convergence and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(n+1)(x-4)^n}{10^n}$ [8]

6.a) Define shortest distance between two skew lines is space. Find the length and equation of shortest distance between the lines.

$\frac{x-1}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ and $\frac{x-4}{3} = \frac{y-5}{4} = \frac{z-6}{5}$. [7]

b) Find the equation of the sphere for which the Circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great Circle. Determine its center and radius. [8]

OR

Find the equation of cone with vertex (α, β, γ) and base $y^2 = 4ax$, z = 0.

7. Attempt all the questions. [4*2.5=10]

a) Solve system of linear equation by Cramer's rule:
5x-3y=37 and -2x+7y= -38

b) Test the convergence of series: $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$

c) Find the angle between following pair of planes: 2x + 3y + 5z = 0 and x- 2y+z =20

d) Find the centre and radius of the sphere
$x^2 + y^2 + z^2 + 2x - 4y - 6z + 5 = 0$

COSMOS COLLEGE
OF MANAGEMENT AND TECHNOLOGY

Term Test I

Date: 2081/02/27		Full Marks	50
Level	BE	Time	1.5 hrs
Programme	BE IT, BCE		
Semester	II		

Subject: - *Algebra and geometry*

- *Candidates are required to give their answers in their own words as far as practicable*
- *Attempt All questions.*
- *The figures in the margin indicate Full Marks.*
- *Assume suitable data if necessary.*

1. a) Define an ellipse. Derive the standard equation of an ellipse. [8]

   b) If $\vec{a} = \hat{i} - 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - 2\hat{k}$, find $\vec{a} \times (\vec{b} \times \vec{c})$, $(\vec{a} \times \vec{b}) \times \vec{c}$ and $|\vec{a} \times (\vec{b} \times \vec{c})|$. Also show that $\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$. [7]

2. a) Define *Skew lines* and *line of shortest distance* between two skew lines. Find the shortest distance between the skew lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$. Also find the equation of the line of *S.D.* [8]

   b) Define consistency of a system of equations. Solve $x + 2y - z = 3, 3x - y + z = 1, 2x - 2y + 3z = 2$. [7]

3. a) State Cayley-Hamilton theorem. Find A^{-1} by using it if $A = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$ [8]

   b) Find the distance from the point $(-1, -5, -10)$ to the point where the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ meets the plane $x - y + z = 5$. [7]

4. Attempt any two. [2*2.5=5]

   a) Show that the vectors $\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{a} \times \vec{b})$ are coplanar.

   b) Define *Linear Transformation*. Show that $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x + 2y - z, 0)$ is linear.

   c) Define *basis* of a vector space. Check whether the vectors $(1,2,2), (2,5,4), (2,7,4)$ form a basis of $\mathbb{R}^3$ or not?

COSMOS COLLEGE
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Term Test II

Date: 2081/03/06			
Level	BE	Full Marks	50
Programme	BEIT, BEX, BCE, BCV	Time	
Semester	II	1.5 hrs	

Subject: - Algebra and Geometry

- ✓ *Candidates are required to give their answers in their own words as far as practicable.*
- ✓ *Attempt All questions.*
- ✓ *The figures in the margin indicate Full Marks.*
- ✓ *Assume suitable data if necessary.*

1. a) Fid the dual f the primal and solve by using Simplex method,
Mininimize $z = 8x_1 + 9x_2$ Subject to the constraints; [7]
$x_1 + 3x_2 \geq 4, 2x_1 + x_2 \geq 5,\quad x_1, x_2 \geq 0.$

   b) Solve the Linnear Programming Problem by Big-M method;
Maxi nize $Z = 4x_1 + 2x_2$ Subject to the constraints; [8]
$3x_1 + x_2 \leq 27, x_1 + x_2 \geq 21,\quad x_1, x_2 \geq 0.$

2. a) Find the interval of convergence, Centre of convergence and radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-5)^n}{n\,5^n}$. [7]

   b) Show that the equation $9x^2 - 24xy + 16y^2 - 50x - 100y + 225 = 0$ represents a parabola and find its axis, vertex, directrix, focus and equation and length of latus rectum. [8]

3. a) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9, x - 2y + 2z = 5$ as a great Circle, determine its centre and radius. [7]

   b) Find the equation of right circular cylinder whose guiding curve is $x^2 + y^2 + z^2 - x - y - z = 0, x + y + z = 1.$ [8]

[2.5*2 = 5]

4. Attempt *any two* questions

   a) Find the equation of the tangent plane to the sphere
$x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$
at the point $(-1, 4, -2)$.

   b) Test the convergence or divergence of the series; $1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \ldots\ldots$

   c) Test the convergence or divergence of the series; $\sum [\sqrt{n^4 - 1} - n^2]$.

   d) Find the equation of the hyperbola with focus at (6, 0) and vertex at (4, 0).