

Chapter - 2.

Py - I

Wave motion - 4 hrs.

1. Wave:

A wave is a continuous transfer of energy and momentum in the form of disturbance with definite velocity from one part to another part of a region. Actually, wave is due to repeated simple harmonic motion of particle carrying energy. But, it is not due to actual linear transformation of particle carrying energy.

2. Wave motion: When a disturbance produced at a point is handed over from particle to particle without any actual linear transformation of disturbed particles but by their repeated simple harmonic motion about the mean position, the motion is called wave motion. During wave motion, there is transfer of energy and linear momentum from one part of region to another part.

3. Some important terms of a wave:

(i) Displacement (y): When a wave propagates through material medium, it produces simple harmonic motion to the particle of that medium. The distance of a particle from its mean position at given instant is called the displacement of particle (y). For a particle executing SHM with angular velocity (ω), displacement is given by

$$y = a \sin \omega t$$

(ii) Amplitude (a): It is defined as the maximum displacement of the particle from its mean position. The quantity ' a ' in equation $y = a \sin \omega t$ is called amplitude.

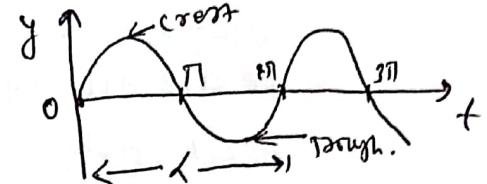
(iii) Crest: It is the point of maximum positive displacement in transverse wave.

- (iv) **Dish**: It is the point of maximum negative displacement in transverse wave.
- (v) **Compression**: It is the region of greater density or greater pressure in the medium during propagation of longitudinal wave.
- (vi) **Rarefaction**: It is the region of smaller density or smaller pressure in the medium during propagation of longitudinal wave.
- (vii) **Frequency (f)**: The number of complete oscillation made by the oscillating particle in one second is called frequency. It is also defined as the reciprocal of time period T . That is $f = \frac{1}{T}$.
- (viii) **Time period (T)**: Time taken by a vibrating particle in the medium to complete one oscillation is called time period T .
- (ix) **wavelength (λ)**: It is the distance between two neighbouring crests or distance between two neighbouring troughs or distance between two neighbouring points vibrating together in exactly the same way. In other word, wavelength is the distance travelled by the wave in one complete oscillation of particle. It is denoted by λ .
- (x) **Phase and phase angle**: The information that indicates position and direction of a particle at particular time is called phase of wave. The angular displacement ' θ ' of the given particle gives its phase and is called phase angle. The phase angle is expressed as $\theta = \omega t$.

(xiii) Wave velocity (v): It is the distance travelled by a wave in one second. If we consider time for one complete oscillation of the particle then wave velocity v is given by

$$v = \frac{\text{distance travelled by wave} (\lambda)}{\text{Time taken} (T)} = \frac{\lambda}{T} = \lambda \cdot \frac{1}{T} = 1 \cdot f$$

$$\therefore v = f \times \lambda.$$



4. Characteristics of wave motion
 $\underbrace{\text{wavy}}_{\text{shape}} \times \underbrace{\text{wavy}}_{\text{shape}} \times \underbrace{\text{wavy}}_{\text{shape}} \times \underbrace{\text{wavy}}_{\text{shape}}$

wave motion has following characteristics.

- ① The wave motion is the disturbance set up in a medium due to repeated periodic motion of the particles.
- ② As the wave travels forward, particles of the medium vibrate about their mean position.
- ③ As the wave advances, there is successive time difference or phase difference between particles of medium.
- ④ In each vibration, particle of medium transfers energy which it got from previous particle to the next particle.
- ⑤ particle velocity and wave velocity are different. i.e. wave velocity $v_w = f \times \lambda$ (const. for a medium)
 particle's velocity $v_p = \frac{dy}{dt} = \frac{d}{dt} (A \sin \omega t) = A \omega \cos \omega t$.
- ⑥ Wave velocity is uniform while particle velocity is different at different position. It is maximum at mean position and minimum at extreme position.
- ⑦ Each particle in the medium starts vibrating a little later than the preceding one.

(5) Types of wave motion

Wave motion are classified based on the modes of vibration, medium, of propagation, and state of motion of the medium.

(1) On the modes of vibration:

There are two types of wave motion on the basis of mode of vibration. They are

(a) Transverse wave motion: in transverse wave motion, particle of the medium vibrate about their mean position but the direction of vibration is perpendicular to the direction of wave propagation. Transverse wave motions can be setup only in solid and over stagnant surface of liquid. Transverse wave travels in the form of alternate crest and trough but does not produce vibration in density and pressure in the medium.

Examples of transverse wave:

- (i) vibration of string and rods.
- (ii) mechanical wave setup over the stagnant surface of water
- (iii) wave produced in the prongs of vibration tuning fork.

(Note: all electromagnetic waves like light, heat etc are transverse wave).

(b) longitudinal wave motion:

In longitudinal wave motion, particles of the medium vibrate simple harmonically about their mean position and along the direction of wave propagation. Longitudinal wave motions can be setup in all media (solid, liquid,

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the successive vibration of the particle of the medium is called progressive wave.

(b) Stationary waves: When two progressive waves of the same amplitude and frequency travelling through a medium with the same speed but in opposite direction superimpose on each other, stationary or standing wave is formed. Stationary wave does not travel in either direction and all the particles of the medium are permanently at rest. There is no flow of energy along the wave.

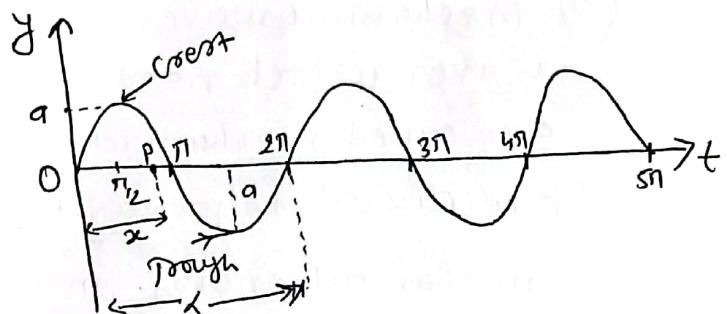
6. Equation of progressive wave: (iml)

A wave, whose crest and trough or compression and rarefaction travel forward continuously with the speed of wave is called progressive wave.

In other word, a wave travels from one region of medium to another is called progressive wave. In progressive wave, all particles vibrate with same amplitude, frequency, and period but there is gradual phase difference between successive particles.

Consider a wave travelling from left to right along x -axis as shown in figure above. If a particle at origin 0 has amplitude 'a' then displacement of that particles at any instant of time 't' is given by,

$$y = a \sin \omega t \quad \text{--- (1)}$$



[Fy: progressive wave motion]

Let a particle at a point 'P' at a 'x' distance from O, so that disturbance reaches to 'P' later than at 'O'. Therefore, particle at 'P' lags the phase to that at point 'O'. Let the phase lag is ϕ . Then its displacement is given by

$$y = a \sin(\omega t - \phi) \quad \text{--- (2)}$$

From above figure,

$$y = a \sin\left(\omega t - \frac{2\pi}{\lambda} \cdot x\right) \text{ since phase } \phi = \frac{2\pi}{\lambda} \cdot x. \quad \text{--- (3)}$$

$$\text{or } y = a \sin(\omega t - kx) \quad \text{--- (4)} \text{ where } k = \frac{2\pi}{\lambda} \text{ is wave number or propagation constant.}$$

From eqn (3) we get,

$$y = a \sin\left(2\pi f \cdot t - \frac{2\pi}{\lambda} \cdot x\right)$$

$$\text{or } y = a \sin\left(2\pi \frac{v}{\lambda} \cdot t - \frac{2\pi}{\lambda} \cdot x\right)$$

$$\Rightarrow y = a \sin \frac{2\pi}{\lambda} (vt - x) \quad \text{--- (5)}$$

$$v = f \times \lambda$$

$$f = \frac{v}{\lambda}$$

These eqn. (2), (4) and (5) are called equation of progressive wave.

Note: If wave travels from right to left then particle at 'P' vibrates earlier than particle at O. Therefore, particle at P leads phase to the particle at O by phase angle $\phi = \frac{2\pi}{\lambda} \cdot x$.

and eqn are;

$$\begin{aligned} y &= a \sin(\omega t + \phi) \\ y &= a \sin(\omega t + kx) \\ y &= a \sin \frac{2\pi}{\lambda} (vt + x) \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \quad \text{--- (6)}$$

are called eqn of progressive waves.

Differential form of wave equation.

We know that, equation of plane progressive wave,

$$y = a \sin \frac{2\pi}{\lambda} (vt - x) \quad \text{--- (1)}$$

Differentiating eqn (1) w.r.t. 't' we get

$$\frac{dy}{dt} = a \cos \frac{2\pi}{\lambda} (vt - x) \cdot \left(\frac{2\pi}{\lambda} \cdot v\right)$$

$$\text{or } \frac{dy}{dt} = \frac{2\pi}{\lambda} v a \cos \frac{2\pi}{\lambda} (vt - x) \quad \text{--- (2)}$$

Again differentiating eqn (2) w.r.t t we get

$$\frac{d^2y}{dt^2} = -\frac{2\pi}{\lambda} v a \sin \frac{2\pi}{\lambda} (vt - x) (\frac{2\pi}{\lambda} \cdot v)$$

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2}{\lambda^2} v^2 a \sin \frac{2\pi}{\lambda} (vt - x) \quad \text{--- (3)}$$

Similarly, differentiating eqn (1) w.r.t x, we get

$$\frac{dy}{dx} = +a \cos \frac{2\pi}{\lambda} (vt - x) (\frac{2\pi}{\lambda}) \quad \text{--- (4)}$$

again, differentiating eqn (4) w.r.t x, we get

$$\frac{d^2y}{dx^2} = +a \sin \frac{2\pi}{\lambda} (vt - x) (\frac{2\pi}{\lambda}) (\frac{2\pi}{\lambda}) (-1)$$

$$\frac{d^2y}{dx^2} = -\frac{4\pi^2}{\lambda^2} a \sin \frac{2\pi}{\lambda} (vt - x) \quad \text{--- (5)}$$

Comparing eqn (3) and (5) we get,

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \quad \text{--- (6)}$$

This eqn (6) is differential form of wave equation.

(#) Relation between wave velocity and particle velocity :

We know that, equation of plane progressive wave,

$$y = a \sin(\omega t - kx) \quad \text{--- (1)}$$

Differentiating eqn (1) w.r.t t, we get

$$\frac{dy}{dt} = aw \cos(\omega t - kx) \quad \text{--- (2)}$$

Differentiating eqn (1) w.r.t x, we get

$$\frac{dy}{dx} = ac(-k) \cos(\omega t - kx) \quad \text{--- (3)}$$

Dividing eqn ② by eqn ③, we get.

(M-9)

$$\frac{dy/dt}{dy/dx} = -\frac{\omega}{k} \Rightarrow \frac{dy}{dt} = -\frac{\omega}{k} \frac{dy}{dx}$$

or $\frac{dy}{dt} = -\frac{2\pi f}{\lambda} \frac{dy}{dx}$ since $\omega = 2\pi f$, $k = \frac{2\pi}{\lambda}$.

or $\frac{dy}{dt} = -v \times k \frac{dy}{dx}$

or $\frac{dy}{dt} = -v \frac{dy}{dx}$ — ④

Therefore, particle velocity = $-(\text{wave velocity}) \times \text{slope of displacement curve.}$

⑤ particle acceleration:

We have an eqn. of plane progressive wave,

$$y = a \sin(\omega t - kx) \quad \text{--- ①}$$

Differentiating eqn ①, w.r.t. t, we get

$$\frac{dy}{dt} = v_p = a\omega \cos(\omega t - kx) = \text{particle velocity}$$

$$\frac{d^2y}{dt^2} = a_p = -a\omega^2 \sin(\omega t - kx)$$

$$\therefore a_p = -\omega^2 y \quad \text{where } y = a \sin(\omega t - kx).$$

For maximum, particle velocity,

$$(v_p)_{\max} = a\omega \text{ since } \cos(\omega t - kx) = 1.$$

For maximum particle acceleration,

$$(a_p)_{\max} = -\omega^2 a. \text{ since } \sin(\omega t - kx) = 1.$$

7. Energy, power, intensity of plane progressive wave (i_{pp})

In progressive wave, there is no transfer of the medium, but there is always transfer of energy in the direction of propagation of the wave. Energy transfer in wave is done by vibrating particles of the medium.

The potential energy is given by

$$P.E = - \int_0^L F \cdot dy = - \int_0^L -k y dy \Rightarrow \frac{1}{2} k y^2 \Rightarrow \frac{1}{2} k y_L^2 \quad \text{--- (1)}$$

Hence, k is called force constant, $K = m\omega^2$ since $y_L = \int_0^m \frac{dx}{K}$.

Therefore, eqn (1) can be written as,

$$P.E = \frac{1}{2} m\omega^2 R^2$$

$$\text{or } P.E = \frac{1}{2} m\omega^2 a^2 \sin^2(\omega t - \omega x) \quad \text{--- (2)}$$

Potential energy per unit volume is given by
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$$(P.E)_V = \frac{1}{2} \frac{m}{V} a^2 \omega^2 \sin^2(\omega t - \omega x) \quad \text{--- (3)}$$

$$(P.E)_V = \frac{1}{2} \frac{\rho}{V} \rho a^2 \omega^2 \sin^2(\omega t - \omega x) \quad \text{--- (3) where } \frac{\rho}{V} = \rho.$$

$$\text{Again, kinetic energy } (K.E) = \frac{1}{2} m v^2 \\ (K.E)_V = \frac{1}{2} \frac{m}{V} a^2 \omega^2 \cos^2(\omega t - \omega x) \quad \text{--- (4)}$$

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or $K.E = \frac{1}{2} m a^2 \omega^2 \cos^2(\omega t - \omega x) \quad \text{--- (4)}$
Kinetic energy per unit volume is given by

$$(K.E)_V = \frac{1}{2} \frac{m}{V} a^2 \omega^2 \cos^2(\omega t - \omega x) \quad \text{--- (5)}$$

$$(K.E)_V = \frac{1}{2} \frac{\rho}{V} \rho a^2 \omega^2 \cos^2(\omega t - \omega x) \quad \text{--- (5)} \\ (E)_V = (K.E)_V + (P.E)_V. \\ \text{Total energy per unit volume } = (K.E)_V + (P.E)_V.$$

Using eqn (3) and eqn (5), we get

$$(E)_V = \frac{1}{2} \rho a^2 \omega^2 \left\{ \cos^2(\omega t - \omega x) + \sin^2(\omega t - \omega x) \right\} \quad \text{--- (6)}$$

$$\therefore (E)_V = \frac{1}{2} \rho a^2 \omega^2 \quad \boxed{\text{--- (6)}}$$

We find the average K.E. over a complete wavelet thus
we find the average K.E. over a complete wavelet

$$(K.E)_{av} = \frac{\int (K.E)_V dx}{\lambda} = \frac{1}{\lambda} \int_0^\lambda \frac{1}{2} \rho a^2 \omega^2 \cos^2(\omega t - \omega x) dx \\ = \frac{1}{\lambda} \left(\frac{1}{2} \rho a^2 \omega^2 \right) \int_0^\lambda \left[\frac{1}{2} (1 + \cos 2(\omega t - \omega x)) \right] dx \\ = \frac{1}{\lambda} \left(\frac{1}{2} \rho a^2 \omega^2 \right) \left[\frac{1}{2} (t + \frac{1}{\omega} \sin 2(\omega t - \omega x)) \right]_0^\lambda \\ = \frac{1}{\lambda} \left(\frac{1}{2} \rho a^2 \omega^2 \right) \left[\frac{1}{2} (t + \frac{1}{\omega} \sin 2(\omega t - \omega \lambda)) \right]$$

(P.T.O)

$$\therefore (K.E)_{av} = \frac{1}{4} \frac{\rho a^2 w^2}{\lambda} \left[\int_0^\lambda dx + \int_0^\lambda \{ \cos \{ 2(wt - kx) \} \} dx \right]$$

$$= \frac{1}{4} \frac{\rho a^2 w^2}{\lambda} \times \lambda$$

$$(K.E)_{av} = \frac{1}{4} \rho a^2 w^2 \quad \dots \textcircled{7}$$

Comparing eqn \textcircled{7} with eqn \textcircled{6}, we get

$$(K.E)_{av} = \frac{1}{2} \text{ total energy} \underset{\text{constant}}{=} \text{constant} \quad \textcircled{8}$$

Again, average $(P.E)_{av}$ over a complete wave length

$$(P.E)_{av} = \frac{1}{2} \int_0^\lambda (P.E)_v = \frac{1}{2} \int_0^\lambda \frac{1}{2} \rho a^2 w^2 \sin^2(wt - kx) dx$$

$$(P.E)_{av} = \frac{1}{2} \frac{\rho a^2 w^2}{\lambda} \int_0^\lambda \left[\frac{1}{2} (1 - \cos 2(wt - kx)) \right] dx$$

$$(P.E)_{av} = \frac{1}{4} \frac{\rho a^2 w^2}{\lambda} \cdot \lambda$$

$$\therefore (P.E)_{av} = \frac{1}{2} \times \text{Total energy} \underset{\text{constant}}{=} \text{constant.} \quad \textcircled{9}$$

Therefore, this shows that average K.E. per unit volume are equal to P.E. per unit volume and each equal to half the total energy per unit volume. It is seen that both K.E. and P.E. depend upon the values of x and t . But K.E. per unit volume, P.E. per unit volume and total energy density are independent of either i.e. constants.

⑤ Power of progressive wave: (imp)

Total energy transported by wave per unit time is called power. we have the relation, Total energy per unit volume is

$$\frac{E}{V} = (E)_v = \frac{1}{2} \rho a^2 w^2 \quad (\text{since from eqn } \textcircled{6})$$

$$\therefore E = \frac{1}{2} \rho V a^2 w^2 \quad \dots \textcircled{1}$$

where, $V = A \times l$ since A = cross-section area through which wave travels.

l = distance travelled by wave.

also, $V = \frac{d}{t} \Rightarrow d = V t$. where V = speed of wave (Pg-132)

Therefore, $V = A \times V t$. —②

Using eqⁿ ② in eqⁿ ①, we get,

$$E = \frac{1}{2} A V t g a^2 w^2 —③$$

Rate of heat transfer or power $P = \frac{E}{t} = \frac{1}{2} A V g a^2 w^2$ —④

(ii) Intensity of progressive wave (I). (imp)

The rate of flow of energy per unit area of cross-section through which wave travels is called intensity; which is given by $I = \frac{P}{A} = \frac{1}{A} [\frac{1}{2} A V g a^2 w^2]$

$$\left[\therefore I = \frac{1}{2} V g a^2 w^2 \right] —⑤$$

Hence, both power and intensity of travelling wave is directly proportional to square of amplitude.

So, eqⁿ ① will be $I \propto a^2$,

(iii) Standing or stationary wave: (imp)

When two progressive wave of same amplitude and frequency travelling in opposite direction with same speed superpose each other, the results gives stationary or standing wave.

Stationary in the sense that wave seems to be moving but there is no net flow of energy along the wave.

Equation of stationary wave: (imp)

Suppose two progressive wave of same amplitude 'a' and frequency f are travelling in opposite direction with same speed along a string. let one wave is travelling along +ve x-axis whose eqⁿ is given by $y_1 = a \sin (\omega t - kx)$ —①