Data Encoding and Modulation

Chapter 9
Data Communication

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Outline

1. Data Encoding
   - Line Coding Schemes: NRZ, RZ, Manchester, AMI
2. Modulation
   - Analog Modulation
     - Amplitude Modulation (AM)
     - Frequency Modulation (FM)
     - Phase Modulation (PM)
   - Analog-to-Digital Conversion
     - Pulse Code Modulation (PCM)
     - Delta Modulation
   - Digital Modulation

Modulation

- process by which data/information is converted into electrical/digital signals for transferring that signal over a medium
  - process of imposing a low-frequency **modulating signal** onto a high-frequency carrier signal by changing its characteristics
    - amplitude, frequency and phase
- increases strength for maximum reach of the signals
- Carrier is needed to facilitate the transportation of the modulated signal across a band-pass channel from the transmitter to the receiver
- Message/ modulating signals also known as **baseband signals** are the band of low frequencies representing the original signal
  - baseband signal may be analog signal or digital signal
- Frequency of carrier signals is almost always higher than that of the baseband signal
  - carrier signals are usually sinusoidal wave

Modulation

General sinusoidal $A_c \cos(2\pi f_c t + \phi(t))$

- A_c : Amplitude
- f_c : Carrier frequency
- ϕ : Phase
- t : time

The reverse process of modulation: **demodulation**

Need for Modulation

- Baseband signals are incompatible for **direct transmission**
- For such a signal, to **travel longer distances**, its strength has to be increased by modulating with a high-frequency carrier wave, which doesn't affect the parameters of the modulating signal.

Advantages of Modulation

1. Reception quality improves: modulation techniques reduced the noise to a great extent. This improves the quality of reception
2. Communication range increases: the frequency of baseband signals is low and the low-frequency signal can not travel a long distance when they are transmitted
    - The attenuation reduces with an increase in the frequency of the transmitted signals, and travel longer distance
    - The modulation process increases the frequency of the signal to be transmitted
        - ★ Modulation increases the range of communication
3. Avoids mixing of Signals: If the baseband signals are transmitted without using the modulation by more than one transmitter, then all the signals will be in the same frequency range
    - all the signals get mixed together and a receiver cannot separate them from each other
    - if each baseband signal is used to modulate a different carrier then they will occupy different slots in the frequency domain
    - modulation avoids mixing of signals

Advantages of Modulation

4. Multiplexing of signals occurs: The multiplexing allows the same channel to be used by many signals, without getting mixed with each other or different frequency signals can be transmitted at the same time
5. Antenna size gets reduced: The antenna height must be multiple of $\lambda/4 \rightarrow \lambda = \frac{c}{f}$, f is the frequency of the signal to be transmitted
   - The minimum antenna height required to transmit a baseband signal of $f = 10kHz$ is
     $\frac{\lambda}{4} = \frac{c}{4f} = \frac{3\times10^8}{4\times10\times10^3} = 7.5km$
   - The antenna of the height is practically impossible
   - Let us consider modulated signal at $f = 100MHz$, the minimum antenna height is given by
     $\frac{\lambda}{4} = \frac{c}{4f} = \frac{3\times10^8}{4\times100\times10^6} = 0.75m$
   - The antenna of the height can easily be installed
6. Adjustments in the bandwidth are allowed: baseband signal is used to modulate a desired carrier frequency

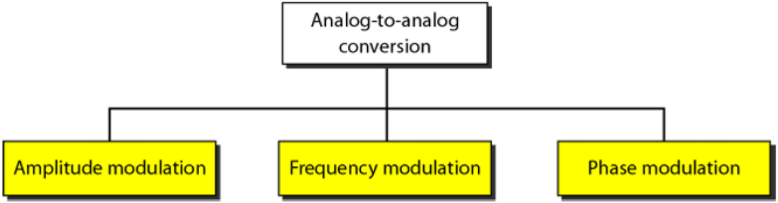
Types of Modulation

1. Analog modulation
   a. Amplitude Modulation (AM)
   b. Angle Modulation
      i. Frequency Modulation (FM)
      ii. Phase Modulation (PM)
2. Digital modulation
   a. Pulse Code Modulation (PCM)
   b. Delta Modulation (DM)
   c. Shift Keying
      i. Amplitude Shift Keying (ASK)
      ii. Frequency Shift Keying (FSK)
      iii. Phase Shift Keying (PSK)

Analog Modulation

- Analog modulation refers to the process of transferring an analog baseband (low frequency) signal, like an audio or TV signal over a higher frequency signal such as a radio frequency band
  - Analog (low frequency) to Analog (higher frequency) conversion
- Types of modulation:
  1. **Amplitude Modulation:** amplitude of the high-frequency carrier wave is varied by the instantaneous amplitude of the modulating signal
  2. **Angle Modulation:** angle of the carrier wave is varied, in accordance with the instantaneous value of the modulating signal.
     a. **Frequency Modulation:** frequency of the carrier wave is varied, in accordance with the instantaneous value of the modulating signal
     b. **Phase Modulation:** phase of the high-frequency carrier wave is varied in accordance with the instantaneous value of the modulating signal

Analog Modulation

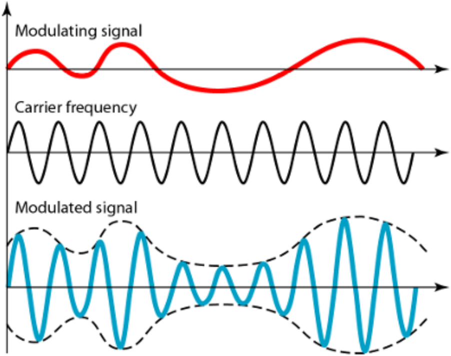


Amplitude Modulation (AM)

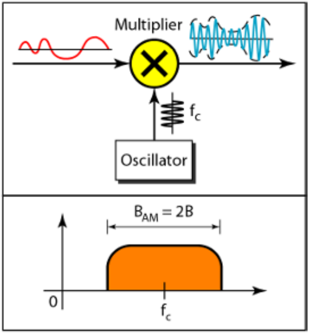
- The amplitude of the high-frequency carrier wave i.e., a sinusoidal wave is varied in accordance with the instantaneous amplitude of the modulating signal
- The frequency of the carrier remains constant
- The information is contained in its amplitude variation
- Let the modulating signal $m(t) = A_m \cos(2\pi f_m t)$ and carrier signal $c(t) = A_c \cos(2\pi f_c t)$
  - A_m and A_c are the amplitude of the modulating signal and the carrier signal respectively
  - f_m and f_c are the frequency of the modulating signal and the carrier signal respectively
- The equation of Amplitude Modulated wave will be

$$\begin{aligned} s(t) &= [A_c + A_m \cos(2\pi f_m t)] \cos(2\pi f_c t) \\ &= A_c[(1 + \mu) \cos(2\pi f_m t)] \cos(2\pi f_c t) \end{aligned} \tag{1}$$

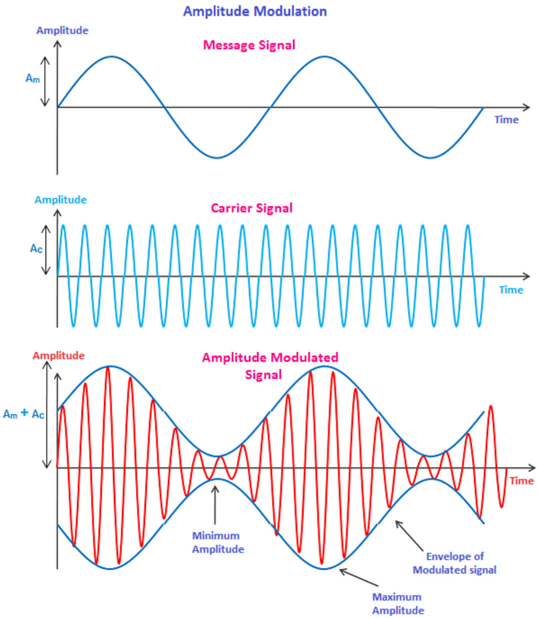
- $\mu = \frac{A_m}{A_c}$ modulation index



Modulating signal
Carrier frequency
Modulated signal



Multiplier
f_c
Oscillator
$B_{AM} = 2B$
0
f_c


Amplitude Modulation
Amplitude
Message Signal
Am
Time
Amplitude
Carrier Signal
Ac
Time
Amplitude
Amplitude Modulated
Signal
Am + Ac
Time
Minimum
Amplitude
Envelope of
Modulated signal
Maximum
Amplitude

Modulation index of AM wave

- range of modulation index (μ) should be $0 < \mu < 1$
- $mu > 1$: over-modulated, and distortion will take place in the modulated signal
- Let A_{max} and A_{min} be the maximum and minimum amplitudes of the modulated wave
- Maximum amplitude of the modulated wave, when $\cos(2\pi f_m t) = 1$

$$A_{max} = A_c + A_m \tag{2}$$

- Minimum amplitude of the modulated wave, when $\cos(2\pi f_m t) = -1$

$$A_{min} = A_c - A_m \tag{3}$$

- Adding eq. 2 and eq. 3

$$A_{max} + A_{min} = A_c + A_m + A_c - A_m$$

$$A_c = \frac{A_{max} + A_{min}}{2} \tag{4}$$

Modulation index of AM wave

- Subtracting eq. 2 and eq. 3

$$A_{max} - A_{min} = A_c + A_m - A_c + A_m$$
$$A_m = \frac{A_{max} - A_{min}}{2} \tag{5}$$

- Modulation index μ

$$\mu = \frac{A_m}{A_c} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} \tag{6}$$

Bandwidth of AM wave

Bandwidth (B) is the difference between the highest and lowest frequencies of the signal $B = f_{max} - f_{min}$ Consider the amplitude-modulated wave

$$\begin{aligned} s(t) &= A_c[(1+\mu)\cos(2\pi f_m t)]\cos(2\pi f_c t) \\ &= A_c \cos(2\pi f_c t) + A_c\mu \cos(2\pi f_c t)\cos(2\pi f_m t) \\ &= \underbrace{A_c \cos(2\pi f_c t)}_{carrier} + \underbrace{\frac{A_c\mu}{2}\cos[2\pi(f_c + f_m)t]}_{upper\ \ sideband} + \underbrace{\frac{A_c\mu}{2}\cos[2\pi(f_c - f_m)t]}_{lower\ \ sideband} \end{aligned}$$

- amplitude modulated wave has three frequencies
- carrier frequency f_c, upper sideband frequency $f_c + f_m$ and lower sideband frequency $f_c - f_m$

$$f_{max} = f_c + f_m \text{ and } f_{min} = f_c - f_m$$
$$B = f_{max} - f_{min} = 2f_m$$

Power of AM wave

- Power of AM wave is equal to the sum of powers of a carrier, upper sideband, and lower sideband frequency components

$$P_t = P_c + P_{USB} + P_{LSB}$$

- Standard formula for the power of cos signal is

$$P = \frac{v_{rms}^2}{R} = \frac{(v_m/\sqrt{2})^2}{2}$$

- Carrier power $P_c = \frac{(A_c/\sqrt{2})^2}{R} = \frac{A_c^2}{2R}$
- Upper sideband power and Lower sideband power
  $P_{USB} = P_{LSB} = \frac{(A_c\mu/2\sqrt{2})^2}{R} = \frac{A_c^2\mu^2}{8R}$

$$\begin{aligned} P_t &= \frac{A_c^2}{2R} + \frac{A_c^2\mu^2}{8R} + \frac{A_c^2\mu^2}{8R} \\ &= \left(\frac{A_c^2}{2R}\right)\left(1 + \frac{\mu^2}{4} + \frac{\mu^2}{4}\right) = P_c\left(1 + \frac{\mu^2}{2}\right) \end{aligned} \tag{7}$$

AM

Advantages

- **Few components needed:** At the receiver side, the original signal is extracted (demodulated) using a circuit consisting of very few components.
- **Low cost:** The components used in amplitude modulation is very cheap. So the AM transmitter and AM receiver build at low cost.
- It is simple to implement.
- **Long distance communication:** Amplitude-modulated waves can travel a longer distance.

Disadvantages

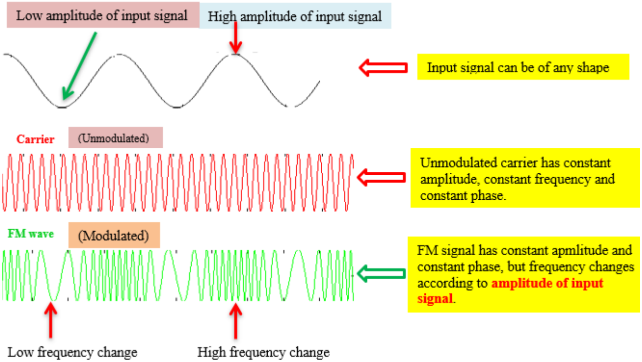
- not efficient in terms of its power usage
- not efficient in terms of its use of bandwidth, requiring a bandwidth equal to twice that of the highest audio frequency
- prone to high levels of noise because most noise is amplitude based and obviously AM detectors are sensitive to it.

Numerical

1. A carrier wave of frequency $f = 1MHz$ with a pack voltage of $20V$ is used to modulate a signal of frequency $1kHz$ with a pack voltage of $10v$. Find out the modulation index, bandwidth and transmit Power.
2. Find the modulation index (μ), bandwidth and transmit Power of the given wave $y = 10\cos(1800\pi t) + 20\cos(2000\pi t) + 10\cos(2200\pi t)$.
3. A modulating signal $m(t) = 10\cos(2\pi times 10^3 t)$ is amplitude modulated with a carrier signal $c(t) = 50\cos(2\pi \times 10^5 t)$. Find the modulation index, the carrier power, and the power required for transmitting AM wave.
4. The equation of amplitude wave is given by $s(t) = 20[1 + 0.8\cos(2\pi \times 10^3 t)]\cos(4\pi \times 10^5 t)$. Find the carrier power, the total sideband power, and the bandwidth of AM wave.

Frequency Modulation (FM)

- The frequency of the carrier signal is modulated to follow the changing voltage level (amplitude) of the modulating signal
- The peak amplitude and phase of the carrier signal remain constant



FM

- The frequency of the modulated wave increases, when the amplitude of the modulating or message signal increases
- The frequency of the modulated wave decreases, when the amplitude of the modulating signal decreases
- The frequency of the modulated wave remains constant and it is equal to the frequency of the carrier signal when the amplitude of the modulating signal is zero.
- The equation of frequency modulated wave will be

$$
\begin{aligned}
s(t) &= A_c + A_m \cos(2\pi f_m t)] \cos(2\pi f_c t) \\
&= A_c[(1+\mu)\cos(2\pi f_m t)] \cos(2\pi f_c t)
\end{aligned}
\tag{8}
$$

- $\mu = \frac{A_m}{A_c}$ modulation index

Phase Modulation (PM)

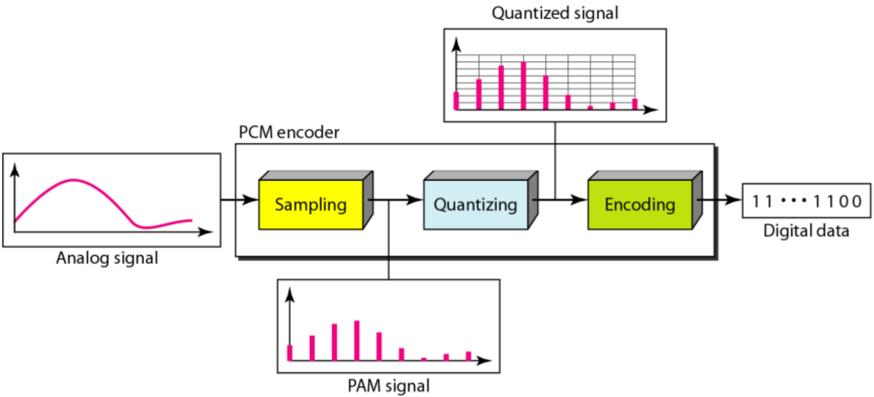
Analog-to-Digital Conversion

- To handle the transmission of analog message signals such as voice, video by digital means
  - signal has to undergo an analog-to-digital conversion
- The carrier is no longer a continuous signal but consists of a pulse train
- Encoding Analog Data as Digital Signal
- Techniques used in analog-to-digital conversion
  1. Pulse Code Modulation (PCM)
  2. Delta Modulation (DM)

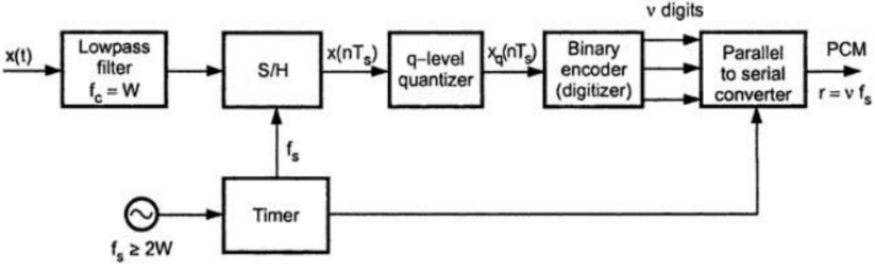
Pulse Code Modulation (PCM)

- A common technique to change an analog signal to digital data (digitization) is called pulse code modulation (PCM)
- PCM encoder has three processes:
  1. The analog signal is sampled
     - ★ analog signal is sampled every T_s s
       where T_s: the sample interval or period
     - ★ sampling rate or sampling frequency $f_s = \frac{1}{T_s}$
     - ★ Sampling frequency f_s is selected sufficiently above Nyquist rate to reproduce the original analog signal, i.e., $f_s \geq 2f_m$
     - ★ Output of sample $x(nT_s)$ is discrete in time and continuous in amplitude
  2. The sampled signal is quantized
     - ★ sample is rounded off to the nearest one of a finite set of discrete levels
  3. The quantized values are encoded as streams of bits.

Components of PCM encoder



Components of PCM encoder



Quantization

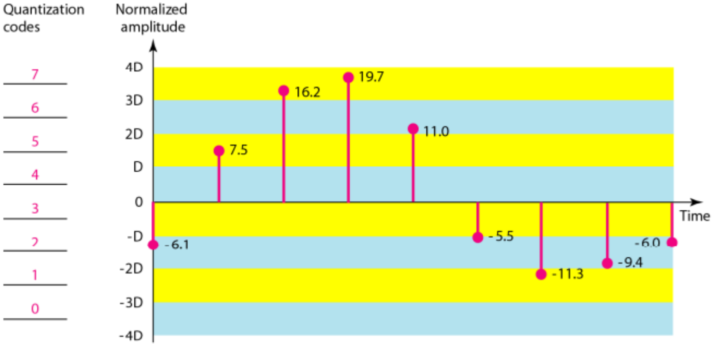
- A q-level quantizer compares input $x(nT_s)$ with its fixed digital levels
- The number of levels, depends on the range of the amplitudes of the analog signal and how accurately we need to recover the signal
- Steps in quantization
  1. Original analog signal has instantaneous amplitudes between $V_{\min}$ and $V_{\max}$
  2. Divied the range into q zones, each of height Δ

$$\Delta = \frac{V_{\max} - V_{\min}}{q}$$

  3. Assign quantized values of 0 to $q - 1$ to the midpoint of each zone
  4. Approximate the value of the sample amplitude to the quantized values
- **Quantization Error:** error created in the quantization process and given as

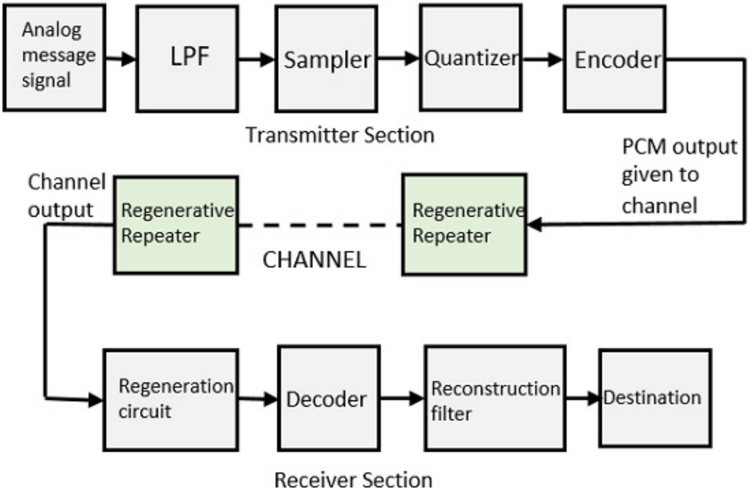
$$e(nT_s) = x_q(nT_s) - x(nT_s)$$

Quantization



Normalized PAM values	-1.22	1.50	3.24	3.94	2.20	-1.10	-2.26	-1.88	-1.20
Normalized quantized values	-1.50	1.50	3.50	3.50	2.50	-1.50	-2.50	-1.50	-1.50
Normalized error	-0.38	0	+0.26	-0.44	+0.30	-0.40	-0.24	+0.38	-0.30
Quantization code	2	5	7	7	6	2	1	2	2
Encoded words	010	101	111	111	110	010	001	010	010

Elements of PCM system



PCM Decoder/Receiver

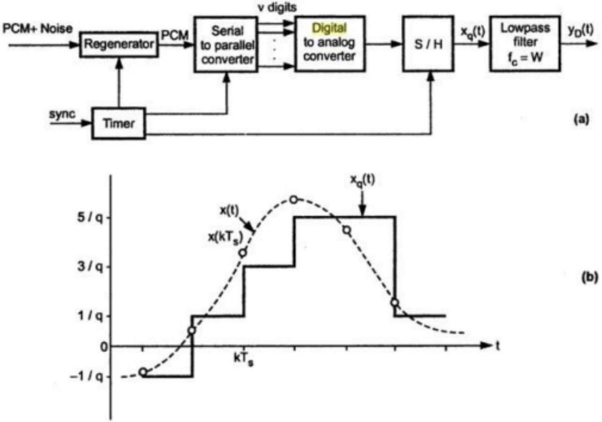


Fig. 9 (a) PCM receiver
(b) Reconstructed waveform

Advantage of PCM

- Immunity to transmission noise and interference
- Possible to regenerate the coded signal along the transmission path
- Communication can kept private and secured by the use of an encryption technique
- Possible to store the signal and process it whenever required
- Possible to use uniform format for different kinds of message or baseband signals

Disadvantage of PCM

- Increased transmission bandwidth
- Increased system complexity

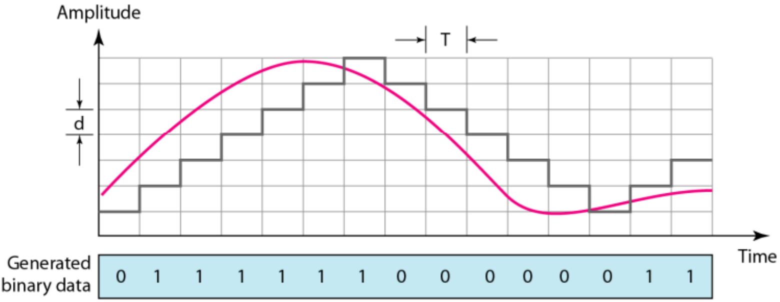
Delta Modulation

- PCM is a very complex technique and requires large bandwidth
  - to reduce the complexity and bandwidth of PCM: **delta modulation**
- Delta modulation transmits only one bit per sample
  - present sample value is compared with the previous sample value and result whether
    - the amplitude is increased or decreased
- Input signal $x(t)$ is approximated to step signal $\hat{x}(t)$ by the delta modulator
  - step sized is kept fixed
- Difference between the input signal $x(t)$ and staircase approximated signal $\hat{x}(t)$ is confined to two levels, i.e., $+\delta$ and $-\delta$

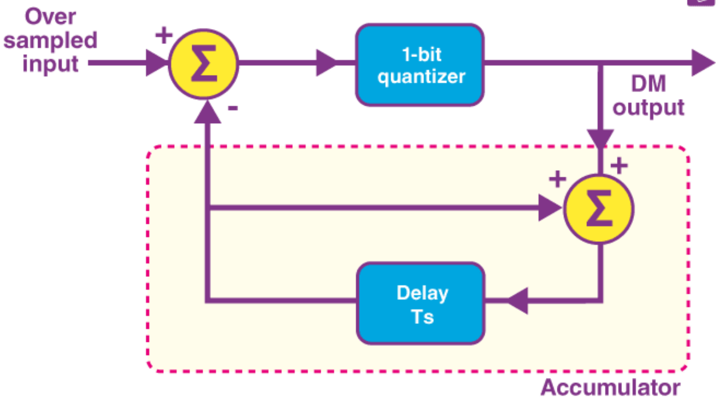
$$x(t) > \hat{x}(t) \rightarrow \text{Increase } \hat{x}(t) \text{ by } \delta$$
$$x(t) < \hat{x}(t) \rightarrow \text{Decrease } \hat{x}(t) \text{ by } \delta$$

- If the δ is positive, the process records a 1; if it is negative, the process records a 0

Delta Modulation



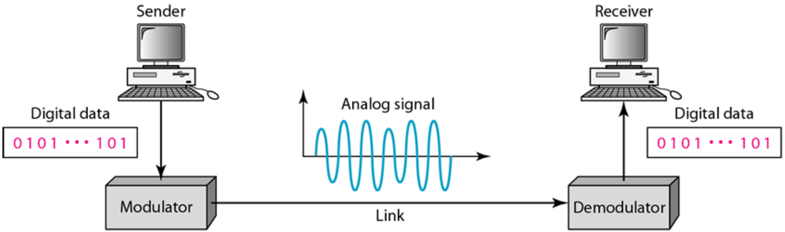
Delta Modulation



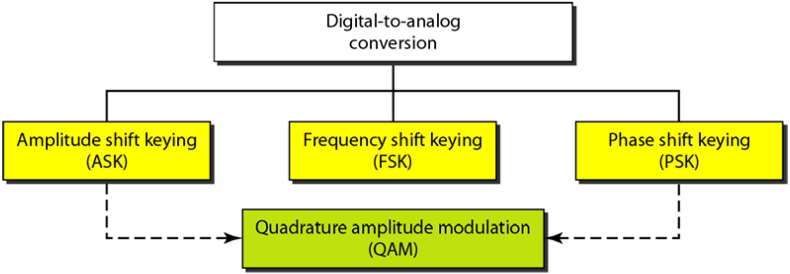
Digital Modulations

- Process of changing one of the characteristics of an analog signal based on the information in digital data
- In digital Modulation: an analog carrier signal is modulated by a discrete signal
  - Digital modulation: digital to analog
  - Demodulation: analog to digital
- If the information is digital changing parameters is called "keying"
- The modulating signal represents as time sequence of symbols or pulses
  - Each symbol has k finite states
  - Each symbol represents n bits of information where $n = \log_2 k$ bits/symbol
- The source information is normally represented as a baseband (low-pass) signal $m(t)$

Digital Modulation



Type of Digital Modulation



Amplitude Shift Keying (ASK)

- The amplitude of the signal is changed in response to information and all else is kept fixed
- Two binary values are represented by two different amplitudes of the carrier frequency
- Carrier signal: $A_c \sin(2\pi f_c t)$
- The modulated signal can be written as

$$
\begin{array}{lc}
s(t) = m(t) A_c \sin(2\pi f_c t) & \text{Symbol} \\
s(t) = A_1 \sin(2\pi f_c t) & 1 \\
s(t) = A_2 \sin(2\pi f_c t) & 0
\end{array}
$$

$A_2 < A_1$

Amplitude Shift Keying (ASK)

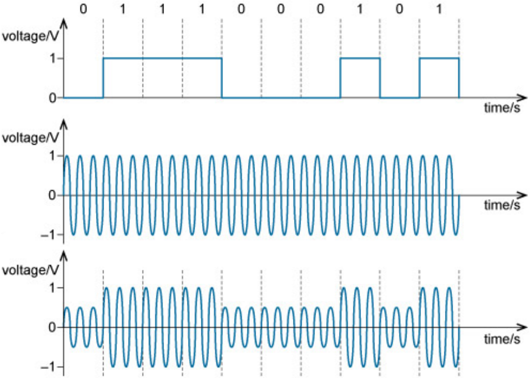


Figure: Amplitude Shift Keying

On/OFF Keying (OOK)

Special Case of Amplitude Shift Keying (ASK)

- Bit 1: amplitude of message signal $A_m = 1$
- Bit 0: amplitude of message signal $A_m = 0$
-
- The modulated signal is

$$s(t) = \begin{cases} A_c \sin(2\pi f_c t) & \text{symbol "1"} \\ 0 & \text{symbol "0"} \end{cases}$$

- OOK is also used in optical communication systems

On/OFF Keying (OOK)

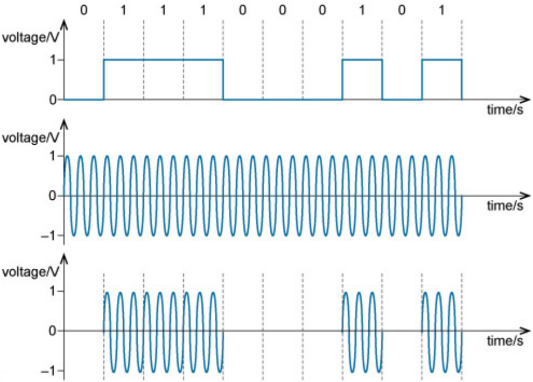


Figure: On/OFF Keying

ASK

Baud Rate

- For ASK, One bit (0 or 1) to represent one symbol
- Rate of symbol transmission i.e., baud rate (S) will be same as bit rate R, i.e., $S = R$

The bandwidth (B) of ASK in terms of Baud Rate $B = (1 + d)S$

- d is a modulation process and its value lies between 0 and 1
- The middle of the bandwidth is where f_c, carrier frequency, is located

We have an available bandwidth of 100 kHz which spans from 200 to 300 kHz. What are the carrier frequency and the bit rate if we modulated our data by using ASK with $d = 1$?

The middle of the bandwidth $(B) = 100kHz$ is located at 250 kHz which is carrier frequency f_c and $d = 1$. The bandwidth of the ASK signal is given by

$$B = (1 + d)S \quad [\because S = R] \rightarrow R = \frac{100}{2} = 50 \ kbps$$

ASK

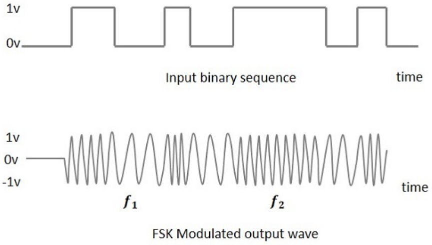
- Advantage: simple and easy to generate and detect
- Disadvantage: very sensitive to noise
- Application: used for data transmission which required a low bit rate (up to 100 bps)

Frequency Shift Keying (FSK)

- Frequency f_c of carrier is changed in response to the information
- Carrier signal: $A_c \cos(\omega_c t)$ $[\because \omega = 2\pi f]$
- The modulated signal is

$$s(t) = \begin{cases} A_c \cos((\omega_c + \delta\omega)t) & \text{symbol "1"} \\ A_c \cos((\omega_c - \delta\omega)t) & \text{symbol "0"} \end{cases}$$

- FSK modulated wave is used high frequency for a high or 1
- FSK modulated wave is used low frequency for a low or 0



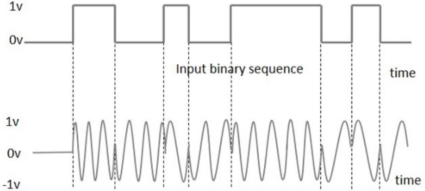
Frequency Shift Keying (FSK)

- Advantage: easy to implement
  - It has better noise immunity than ASK
- Disadvantage: required high bandwidth
- Application: used in low-speed modems having bit rates below 1200*bps*

Phase Shift Keying (PSK)

- Phase ϕ of carrier is changed in response to the information
  - One phase change encodes 0 while another phase change encodes 1
- Carrier signal: $A_c \cos(\omega_c t + \phi(t))$ $[\because \omega = 2\pi f]$
- The modulated signal is

$$s(t) = \begin{cases} A_c \cos(\omega_c t + 0) = A_c \cos(\omega_c t) & \text{symbol "1"} \\ A_c \cos(\omega_c t + \pi) = -A_c \cos(\omega_c t) & \text{symbol "0"} \end{cases}$$



BPSK Modulated output wave

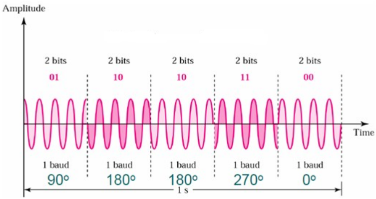
Phase Shift Keying (PSK)

- Advantage: required lower bandwidth than a BFSK signal
  - very good noise immunity
- Disadvantage: Generation and detection of BPSK is quite complicated
- Application: preferred BPSK modems over FSK modems due to low bandwidth

Quadrature Phase Shift Keying (QPSK)

- Variation of BPSK
  - Multilevel Modulation Technique: 2 bits per symbol
- Modulating signals onto carrier signal using four phase $0^0, 90^0, 180^0, 270^0$ states to code two digital bits
- The modulated signal is $S(t) = A_c \cos(2\pi f_c t + \phi(t))$

Dibit	Phase $\phi(t)$
00	0
01	90
10	180
11	270



M-PSK

- The idea can be extended to 8-PSK, 16-PSK, 32- PSK,. . . .
- The limitation is the ability of equipment to distinguish small differences in signal's phase

Tribit	Phase $\phi(t)$
000	0
001	45
010	90
011	135
100	180
101	225
110	270
111	315

Quadrature Amplitude Modulation (QAM)

- If both the amplitude and the phase are varied proportional to the information signal
  - Combination of phase shifting and amplitude shifting

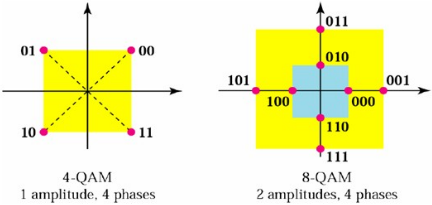
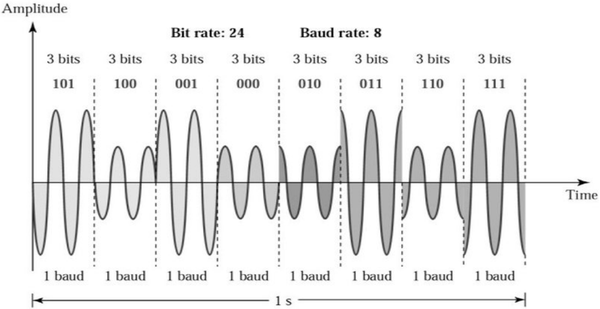


Figure: Constellation diagram

- More bit symbol can transmit simultaneously
  - Increase data rate
- Bandwidth efficient

8-QAM



Amplitude
Bit rate: 24
Baud rate: 8
3 bits
3 bits
3 bits
3 bits
3 bits
3 bits
3 bits
3 bits
101
100
001
000
010
011
110
111
Time
1 baud
1 baud
1 baud
1 baud
1 baud
1 baud
1 baud
1 baud
1 s

Calculate the baud rate of a 16-QAM signal if the bit rate is 64000 bps.

Given: Bite rate $R = 64000$ *bps* and System $=$ 16-QAM
For a 16-QAM system, i.e., $M = 4$, the number of bits per symbol
$r = \log_2 M = \log_2 16 = 4$

$$\begin{aligned} \text{Baud rate} S &= \frac{R}{r} \\ &= \frac{64000}{4} \\ &= 16000 \; baud/s \end{aligned}$$