$\omega_0 = \sqrt{\frac{k}{m}}$, ωx .

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Mechanical Oscillation:

Oscillation is a very general term, it indicates any kind of to and fro motion.

Types of oscillation:

- Free oscillation
- Damped oscillation
- Forced oscillation

- Free oscillation:
  - Ideal case
  - No external force is applied.
  - Natural oscillation with no resistive force
  - constant amplitude, energy & time period

- Damped oscillation:
  - Damping nature agent.
  - Resistive force also known as damping force.
  - No external force is applied.
  - Amplitude varies with the time & energy decreases.

- Forced oscillation:
  - Resistive force is there.
  - ~~is~~ External periodic force is applied to make forced oscillation into free oscillation.
  - Amplitude is constant.

∴ Free

Numericals: A particle oscillates simple harmonically takes time to move from one extreme point to mean position is 0.25sec. Find the period and frequency of oscillation.

Sol^n Time period to move from one extreme point to mean position $= \frac{\text{Total time for 1 oscillation}}{4}$

or, $0.25 = \frac{T}{4}$

$\therefore T = 1sec.$

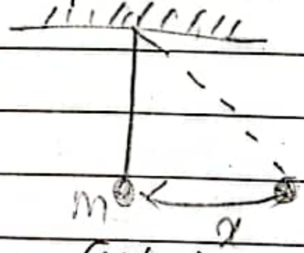
Now,

$$\text{frequency} = \frac{1}{\text{Total time}}$$

$$= \frac{1}{1} = 1rps.$$

Differential equation of free oscillation
OR
Force equation of simple harmonic motion.

Consider a particle of mass 'm' is moving freely. let x be the displacement from mean position.



Then,

the restoring force is given by,

$$F = -kx \quad \text{———} \quad (1)$$

Where k is force constant. Unit is (N/m)

$$P.E = -\int F.dx$$

$$= -\int -kx.dx$$

$$= \frac{1}{2}kx^2 \quad \text{———} \quad (2)$$

$$K.E = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 \quad \left[\because v = \frac{dx}{dt}\right]$$

$$\therefore K.E = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 \quad \text{—— (3)}$$

Now,

$$T.E = K.E + P.E$$

$$= \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 + \frac{1}{2} k x^2 \quad \text{—— (4)}$$

For free oscillation, energy is constant i.e.

$$\frac{dE}{dt} = 0$$

$$\text{or, } \frac{d}{dt}\left[\frac{1}{2} m \left(\frac{dx}{dt}\right)^2 + \frac{1}{2} k x^2\right] = 0$$

$$\text{or, } \frac{1}{\cancel{2}} m \times \cancel{2} \cdot \frac{dx}{dt} \cdot \frac{d}{dt}\left(\frac{dx}{dt}\right) + \frac{1}{\cancel{2}} \times k \times \cancel{2} x \cdot \frac{dx}{dt} = 0$$

$$\text{or, } m\left(\frac{d^2x}{dt^2}\right) + kx = 0$$

$$\text{or, } \frac{d^2x}{dt^2} + \frac{k}{m} x = 0 \quad [\because \text{Dividing both sides by } m]$$

$$\therefore \frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \text{—— (5)} \quad \left(\because \omega = \sqrt{\frac{k}{m}}\right).$$

Hence, Eq^n (5) is the ~~equation~~ required differential eq^n of free osillation where;

$$\omega = \sqrt{\frac{k}{m}}$$

$$\text{or, } 2\pi f = \sqrt{\frac{k}{m}}$$

$$\therefore \boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

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The sol^n of eq^n (5) can be written as;
$x = A\sin(\omega t + \phi)$

(Q) An oscillatory particle is represented by; $y = Ae^{i\omega t}$. where y is displacement from mean position & ω is angular frequency. Show that the particle is in S.H.M.

$\psi = Ae^{-i\omega t \cdot (kx)}$

Sol^n We have;

$$y = Ae^{i\omega t}$$

$$\frac{dy}{dt} = \frac{d}{dt}(Ae^{i\omega t})$$

$$= A\frac{de^{i\omega t}}{dt}$$

$$= Ae^{i\omega t} \cdot \frac{d}{dt}(i\omega t)$$

$$\Rightarrow Ae^{i\omega t} \cdot i\omega$$

Again, $$\frac{d^2y}{dt^2} = \frac{d}{dt}(Ae^{i\omega t} \cdot i\omega)$$

$$\Rightarrow Ai\omega \frac{d}{dt}(e^{i\omega t})$$

$$= Ai\omega e^{i\omega t} \cdot \frac{d}{dt}(i\omega t)$$

$$= Ai\omega e^{i\omega t} \cdot i\omega$$

$$= A(i\omega)^2 e^{i\omega t}$$

$$= y(i\omega)^2$$

$$= yi^2\omega^2$$

$$= -\omega^2 y$$

$$\therefore \frac{d^2y}{dt^2} + \omega^2 y = 0 \quad \text{—— (1)}$$

Eq^n (1) shows the eq^n of S.H.M. Hence, the motion is in S.H.M.

- **Damped oscillation:**

A free oscillation is an ideal concept. There exists a resistive force in all oscillation. As a result, the amplitude of oscillation continuously decrease, this oscillation is known as damped oscillation. Forces acting on the system are given by; Damping Force & Restoring force:

(i) Restoring force;

$$F_r = -kx \quad \text{—— (i)}$$

where k is force constant.

(ii) Damping force:

$$F_d = -b\frac{dx}{dt} \quad \text{—— (ii)}$$

where b is damping constant or factor. Unit is (kg/s)

Total force $F = F_r + F_d$

$$ma = -kx - b\frac{dx}{dt} \qquad \left[\because F = ma = m\frac{d^2x}{dt^2}\right]$$

$$\text{or, } m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

$$\text{or, } m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \quad \text{—— (iii)}$$

$$\text{or, } \boxed{\frac{d^2x}{dt^2} + \frac{b}{2m}\frac{2dx}{dt} + \omega_0^2 x = 0} \qquad \left[\because \omega_0 = \sqrt{\frac{k}{m}}\right]$$

Eqⁿ (III) is the required differential equation of damped oscillation.

Solⁿ of eqⁿ (III) can be written as;

$$x = \underbrace{A\, e^{-bt/2m}}_{\text{Amplitude}} \sin(\omega t + \phi).$$

Amplitude exponentially decrease.

where; $\omega_d = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$.

$$\boxed{f_d = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}}$$

Case: 1 If $\frac{k}{m} > \left(\frac{b}{2m}\right)^2$, there is oscillation and the condⁿ is underdamped.

Case: 2 : If $\frac{k}{m} = \frac{b^2}{4m^2}$ $\rightarrow \boxed{b^2 = 4mk}$, there is no oscillation & the condⁿ is critically damped.

Case: 3 If $\frac{k}{m} < \frac{b^2}{4m^2}$, there is no oscillation the condⁿ is over damped.

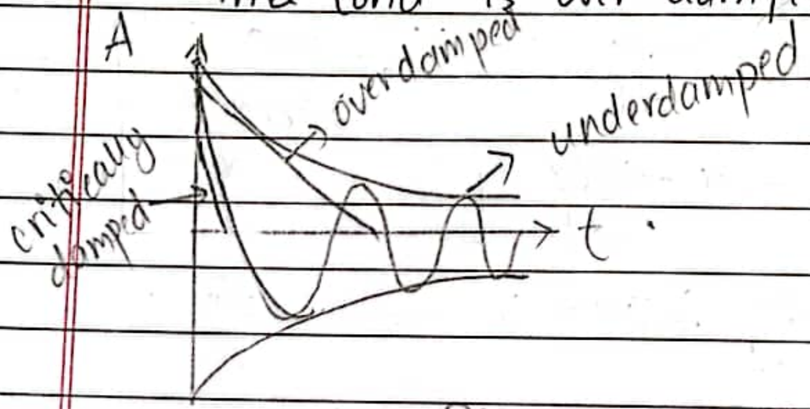


Fig. damped oscillation.

- Forced oscillation:

→ When periodic external force is applied on a damped oscillation to behave as a free oscillation. Such oscillation is known as forced oscillation.
The forces acting on the system is given by; restoring force, damping force & external force.

(1) Restoring force: $F_r = -kx$ —— (i)
(2) Damping force: $F_d = -b\frac{dx}{dt}$ —— (ii).

(3) External force;
$$F_e = F_0 \sin\omega t \quad \text{—— (iii)}$$
↳ This shows external force is applied as the amplitude decrea

Total force (F) = $F_r + F_d + F_e$
$$= -kx - b\frac{dx}{dt} + F_0\sin\omega t$$

or, $m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = F_0\sin\omega t$ ~~= 0~~

$$\therefore F_0\sin\omega t = m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx \quad \text{—— (iv)}.$$

Eqⁿ (iv) is the required differential equation of forced oscillation.
Solⁿ of eqⁿ (iv) can be written as;
~~Fr x = A₀sin(ω~~
$$x = A\sin(\omega t + \phi) \quad \text{—— (v)}$$

where,
$$A = \frac{\frac{F_0}{m}}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{b\omega}{m}\right)^2}} \;;\; \omega_0 = \sqrt{\frac{k}{m}}.$$

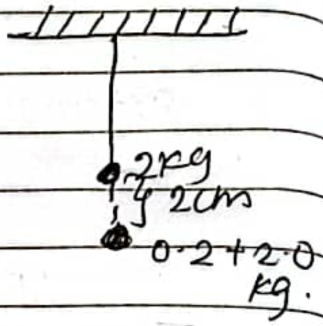
when driving frequency matches with the natural angular frequency; the amplitude of oscillation is maximum. This condⁿ is known as resonance & the frequency is called resonant frequency.

☆☆ Phase constant (ϕ) = $\cancel{\tan^{-1}}$

$$= \tan^{-1}\left[\frac{\omega_0^2 - \omega^2}{b\omega/m}\right] \quad \text{——— (vi)}$$

P.g no. 27

① Sol^n



mass (m_1) = 2kg.

external mass (m_2) = 200gm = 0.2kg

Total mass (M) = 2.2kg

Total displacement ($\cancel{x}$) = $\cancel{(x+2)}$

= $(x + 0.02)$m

$$f = \frac{1}{2\pi}\sqrt{\frac{\cancel{k}}{m}} = \frac{1}{2\pi}\sqrt{\frac{\cancel{0.02}}{2.2}}$$

=

$F = mg$

$= (2+0.2)\times 10 = 22N$

or, $F = +kx$

$22 = +k \times 0.02$

$\therefore k = \frac{22}{0.02}$

= 1100 N/m

Total time (T)

$= \cancel{1}\ 2\pi\sqrt{\frac{m}{k}}$

$= 2\pi\sqrt{\frac{2.2}{1100}}$

$= 0.28$ sec =

$= 2\pi\sqrt{\frac{2}{1078}}$

$\Rightarrow 0.27$ sec.

//

(2) ⇒

Sol: $m = 500gm = 0.5kg$.

$k = 50N/m$.

$b = 0.15 kg/s$

~~frequency = $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$~~ $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

~~Time pe~~ $= 1.59 N/m$

Restoring force $(F_r) = -kx$

or, $0.5 \times 9.8 = 50 \times x$

$\therefore x = 0.098m$.

Damping force $(F_d) = -b\frac{dx}{dt}$

$= \cancel{0.15 \times 0.098}$

~~Time period = $2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.5}{50}}$~~

~~$= 0.62 sec$~~

$$f_d = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

$$= \frac{1}{2\pi}\sqrt{\frac{50}{0.5} - \left(\frac{0.15}{2 \times 0.5}\right)^2}$$

$= 1.591$ ~~N/m~~ rps

Now, Time period $(T) = \frac{1}{f_d} = \frac{1}{1.591} = 0.628 sec$ //

③

$m = 1kg$

$K = 25 N/m.$

$\cancel{fd} =$

$f(\text{undamped}) = \frac{2}{\sqrt{3}} \times (f\text{damped})$

Now,

$$fd = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} \quad \text{—— ①}$$

or, $\cancel{\frac{f}{2/\sqrt{3}}} =$

$$fn = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{25}{1}} = \frac{5}{2\pi}$$

or, $\frac{5}{2\pi} = \frac{2}{\sqrt{3}} \times fd$

$\therefore fd = \frac{5}{2\pi} \times \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{4\pi}$

Now, In eqn ①

$$\frac{5\sqrt{3}}{4\cancel{\pi}2} = \frac{1}{\cancel{2\pi}}\sqrt{\frac{25}{1} - \left(\frac{b}{2}\right)^2}$$

or, $\left(\frac{5\sqrt{3}}{2}\right)^2 = 25 - \frac{b^2}{4}$

or, $\frac{75}{4} = 25 - \frac{b^2}{4}$

or, $\frac{b^2}{4} = 25 - \frac{75}{4}$

or, $b^2 = 25$

$\therefore b = 5 kg/sec.$

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(4) $m = 0.5kg$

$k = 12.5 N/m$.

$$Fn = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{12.5}{0.5}} = \cancel{0.796}\ 0.7957$$

$Fd = Fn - 0.2\%$ of Fn

$$= 0.7957 - \frac{0.2}{100} \times 0.7957$$

$$= \cancel{0.78}.\quad 0.7941$$

Now,

(a) ⇒ $$Fd = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

$$\text{or, } (0.78 \times 2\pi)^2 = \frac{12.5}{0.5} - \frac{b^2}{(2 \times 0.5)^2}$$

or, $24.898 = 25 - \frac{b^2}{1}$

orr, $99.072 = 100 - b^2$

$\therefore b = \cancel{1.98 kg/sec}\ 0.96 kg/sec$.

$$\text{or, } \cancel{24.898}\ \frac{b^2}{4} = 25 - 24.898$$

or, $b^2 = 0.102 \cancel{\times 4}$

$\therefore \cancel{b = 0.63 kg/sec}$.

$\therefore b = 0.319 kg/sec$.

(b) ⇒

$b = 0.319 kg/sec$

(c) ⇒ $b^2 = 4mk$

or, ~~$0.102 = 4 \times 0.5$~~ or, $0.102 = 4 \times 0.5 \times k$

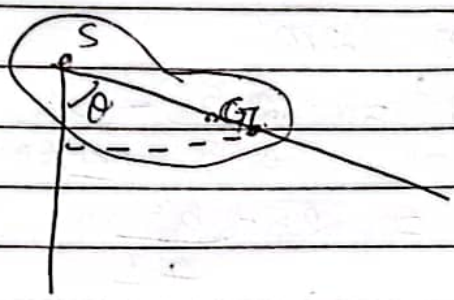
$\therefore b = \sqrt{4 \times 0.5 \times 12.5}$ $\therefore k =$

=

or, $b = \sqrt{4 \times 0.5 \times 12.5}$

$= 5 kg/sec$ //.

Compound / Physical Pendulum :



~~For rig~~

Compound pendulum / physical pendulum:

A rigid body of whatever shape and size is capable of oscillating about the horizontal axis passing through it is called compound pendulum. All real objects are physical pendulums.

Consider a rigid body of mass m is suspended at point 'S' as shown in figure. 'G' represents the position of centre of gravity. Then, distance betn 'S' & 'G' is called length of pendulum (l).

$GN = l\sin\theta$ [$\because \theta$ be the angular displacement from mean position then,

The restoring torque is given by,

$$\tau = -mgl\sin\theta \quad \text{--- (1)}$$

[$\because$ -ve sign indicates that $\tau \propto \theta$]

If I & α are the moment of inertia & angular accleration;

$$\tau = I\alpha \quad \text{--- (11)}$$

or, $-mgl\sin\theta = I\dfrac{d^2\theta}{dt^2}$ $\left[\because \alpha = \dfrac{d^2\theta}{dt^2}\right]$.

or, $I\dfrac{d^2\theta}{dt^2} + mgl\sin\theta = 0$.

or, $\dfrac{d^2\theta}{dt^2} + \dfrac{mgl}{I}\sin\theta = 0$

or, Now, let us suppose θ is very small. So, $\sin\theta \approx \theta$

$\therefore \dfrac{d^2\theta}{dt^2} + \dfrac{mgl}{I}\theta = 0$

$$\therefore \frac{d^2\theta}{dt^2} + \left(\frac{mgl}{I}\right)\theta = 0$$

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \quad \text{——— (III)}$$

where $\omega = \sqrt{\frac{mgl}{I}}$.

Eqn (III) is similar to the differential equation of S.H.M. So, the motion of compound pendulum is also in S.H.M.

If 'k' be the radius of gyration, then the total moment of inertia according to parallel axis theorem is given by:

$$I = I_c + ml^2$$
$$= mk^2 + ml^2$$
$$\therefore I = m(k^2 + l^2) \quad \text{——— (IV)}$$

Then;

$$T = \frac{2\pi}{\omega}$$
$$= 2\pi\sqrt{\frac{I}{mgl}}$$
$$= 2\pi\sqrt{\frac{\cancel{m}(k^2+l^2)}{\cancel{m}gl}} = 2\pi\sqrt{\frac{k^2+l^2}{gl}}$$
$$= 2\pi\sqrt{\frac{\frac{k^2}{l}+l}{g}} \quad \text{——— (V)}$$
$$= 2\pi\sqrt{\frac{L}{g}} \quad \text{——— (VI)}$$

where,

$L = \frac{k^2}{l} + l =$ Effective length of compound pendulum.

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we have;

Time period of simple ~~pendu~~ pendulum is:

$T = 2\pi\sqrt{\frac{L}{g}}$ — (vii) Hence L is the distance

betn point of suspension & pt. of oscillation.

Hen from eqn (6) & eqn (7) & (8) — the effective length must be represented by the direction betn points of suspension & point of oscillation in compound pendulum.

Hence,

the point of ~~sus~~ oscillation of pendulum always like beyond the CG.

Interchangeability of pt. suspension and pt of ~~osc~~ ~~oscillation~~ oscillation:

We have,

Time period of compound pendulum is

$$T = 2\pi\sqrt{\frac{\frac{k^2}{l}+l}{g}} \quad \text{—— (1)}$$

where l = distance betn pt. of suspension & C.G.

$\frac{k^2}{l}$ = distance betn CG and pt. of oscillation

Let $\frac{k^2}{l} = l' \Rightarrow k^2 = ll'$

Then,

$$T = 2\pi\sqrt{\frac{\frac{ll'}{l}+l}{g}}$$

$$= 2\pi\sqrt{\frac{l'+l}{g}} \quad \text{—— (2)}$$

If we invert the pendulum and is suspended at point O such that there will be new length of pendulum. 'l'' and new time T'.

$$T' = 2\pi\sqrt{\frac{\frac{k^2}{l'} + l'}{g}}$$

Hence, the point of oscillation of compound pendulum always lies beyond the C.G.

$$\therefore T' = 2\pi\sqrt{\frac{\frac{ll'}{l'} + l'}{g}}$$

$$= 2\pi\sqrt{\frac{l+l'}{g}} \quad \text{——— (3)}$$

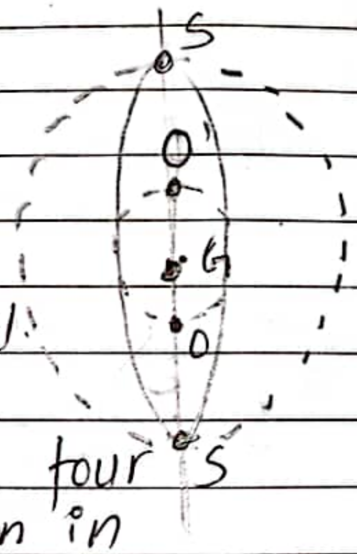
From eq^n (2) & (3), ~~the~~ T = T'

So, the pt. of suspension & pt. of oscillation are interchangeable / reciprocal to each other.

N.F.E

Existence of 4 collinear points

Let us draw two circle centre at CG having radii 'L' and $\frac{k^2}{L}$, the circle cuts the vertical axis passing through CG at four points; S, O', S', O as shown in figure.



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The time period of the pendulum suspended at S, O', O & S' is same. Hence, these 4 points are known as collinear points.

Mathematically,

$$T = 2\pi\sqrt{\frac{\frac{K^2}{l}+l}{g}} \quad \text{——— (1)}$$

Squaring b.s, we get,

$$T^2 = \frac{4\pi^2}{g}\left(\frac{K^2}{l}+l\right)$$

$$\text{or, } T^2 = \frac{4\pi^2}{g}\left(\frac{K^2+l^2}{l}\right)$$

$$\text{or, } \frac{gT^2 l}{4\pi^2} = K^2+l^2$$

$$\text{or, } K^2+l^2-\frac{gT^2}{4\pi^2}\times l = 0 \quad \text{——— (2)}$$

$$\text{or, } l^2-\frac{gT^2 l}{4\pi^2}+K^2 = 0$$

eq^n (2) is quadratic in 'l'. So, eq^n must have two roots to satisfy the equation. i.e. l_1 & l_2 having same time period.

Sum of roots $= l_1 + l_2$

$$= \frac{gT^2}{4\pi^2}$$

$$\text{or, } L = \frac{gT^2}{4\pi^2}$$

$$\therefore \cancel{4\pi^2 L} \quad \boxed{g = \frac{4\pi^2 L}{T^2}}$$

Product of roots $= l_1 \cdot l_2 = k^2$

$\therefore k = \sqrt{l_1 \cdot l_2}$

$= \sqrt{L}$.

If $l_1 = l$, then $\boxed{l_2 = \frac{k^2}{l}}$ //

Here, two distinctly different length of pendulum 'l' and $\frac{k^2}{l}$ ~~an~~ has same time period. Since, there is two point of oscillation on other side of C.G. As we know, point of suspension and point of oscillation are interchangeable. So, there are distinctly different four points having same time period. These 4 points known as collinear points.

Most important for Pailan

Minimum and Maximum Time Period of Compound Pendulum.

(Q) Show that time period of compound pendulum is minimum. If the length of pendulum is equals to radius of ($l=k$) gyration.

(Q2) Show that the time period of compound pendulum is minimum when the point of suspension & point of oscillation are equidistant from C.G.

l & $\frac{k^2}{l}$ equidistant,

From above, $\boxed{l = k}$

Ans: Given, length of pendulum = radius of gyration.

We have, Time period of compound pendulum is.

$$T = 2\pi\sqrt{\frac{k^2/l + l}{g}} \quad \text{——— (1)}.$$

Squaring b.s. we get,

$$T^2 = \frac{4\pi^2}{g}\left(\frac{k^2}{l} + l\right) \quad .... (2).$$

differentiating eqⁿ (2) w.r.t. l on b.s. we get,

$$2T \cdot \frac{dT}{dL} = \frac{4\pi^2}{g}\left[\frac{-k^2}{l^2} + 1\right] \quad \text{——— (3)}$$

For maximum & minimum, time period $\frac{dT}{dl} = 0$.

$$\text{or, } 0 = \frac{4\pi^2}{g}\left(\frac{-k^2}{l^2} + 1\right)$$

$$\text{or, } 0 = \frac{-k^2}{l^2} + 1$$

$$\text{or, } \frac{k^2}{l^2} = 1$$

$$\text{or, } k^2 = l^2$$

$$\therefore \cancel{k = \pm l} \quad l = \pm k$$

Since, length can't be negative, $\therefore \boxed{l = k}$

Again, differentiating eqⁿ (3) w.r.t. l on both sides, we ge:

$$2T\frac{d^2T}{dL^2} + 2\left(\frac{dT}{dL}\right)^2 = \frac{4\pi^2}{g}\left(\frac{2k^2}{l^3}\right) \quad .... (4).$$

For $l = k$ and $\frac{dT}{dl} = 0$

$$\text{or, } 2T\left(\frac{d^2T}{dL^2}\right) = \frac{4\pi^2}{g}\left(\frac{2k^2}{k^3}\right)$$

or, $\frac{d^2T}{dl^2} = \frac{4\pi^2}{gTk} > 0$

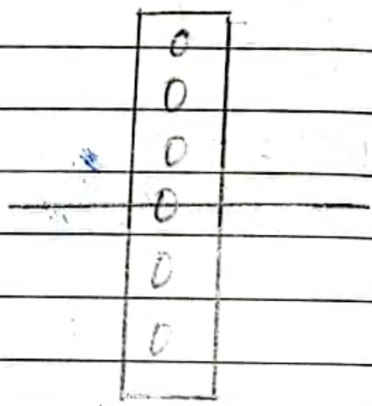
Hence at $l = k$, the time period is minimum and $l = 0$, $T = \infty$ (maximum) //.

$$T = 2\pi\sqrt{\frac{\frac{k^2}{k} + k}{g}}$$

$$\boxed{T = 2\pi\sqrt{\frac{2k}{g}}}$$ —— ⑤ is the required equation for ~~tim~~ minimum time period of oscillation.

* Since, at $l = k$, the time period is minimum.
Here,
distance betⁿ pt. of suspension & C.G = $l = k$.
" " C.G & pt. of oscillation $= \frac{k^2}{l} = k$.

Bar-Pendulum:

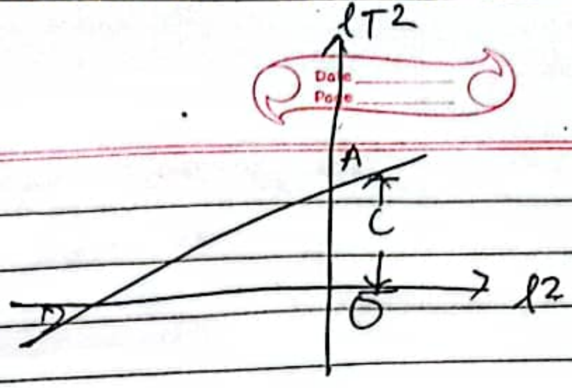


$$T = 2\pi\sqrt{\frac{\frac{k^2}{l} + l}{g}}$$

or, $T^2 = \frac{4\pi^2}{g}\left(\frac{k^2 + l^2}{l}\right)$

or, $\underset{y}{lT^2} = \underset{c}{\frac{4\pi^2 k^2}{g}} + \frac{4\pi^2}{g}\underset{x}{l^2}$

or, comparing it with $y = mx + c$,



$$\text{Slope} = \frac{4\pi^2}{g} = \frac{OA}{OD}$$

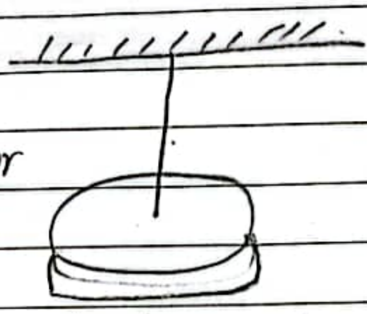
$$g = \frac{4\pi^2}{\text{slope}}.$$

$$\text{Intercept} = OA = \frac{4\pi^2}{g}k^2$$

$$= \frac{OA}{OD}k^2$$

$$\therefore k = \sqrt{OD}.$$

Torsion Pendulum:



A rigid body like a cylinder or disc is suspended at its midpoint by a long and thin wire to the rigid support, constitutes a torsional pendulum. It is so called because when it is twisted ~~at~~ & then released, it executes ~~S.H.M.~~ torsion vibration.

When the disc is rotated by a certain angle θ, the wire too get twisted by same angle 'θ'. then the restoring torque is given by,

$$\tau = -c\theta \quad \ldots (1)$$

where, $c = \frac{\pi \eta r^4}{2l}$ [Torsion].

η = modulus rigidity of the wire
r = radius of suspension wire
l = length of " " .

5-5-5

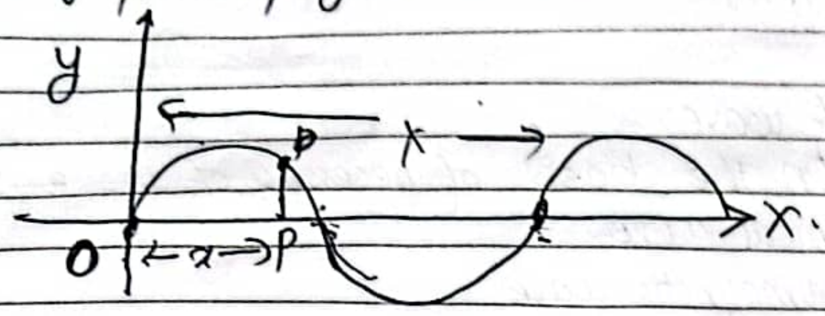
Chapter 2:

Wave Motion:

Types of wave:
(A) On the basis of necessity of the ~~wave~~ medium.
(i) mechanical wave
(ii) electromagnetic wave:

(B) On the basis of disturbance, there are following Type
(i) Transverse wave
(ii) Longitudinal wave

Imp: Eqⁿ of plane progressive wave:



At point O, the displacement of vibrating particle is given by;

$$y = a\sin\omega t \quad \text{——— (1)}.$$

At point P, the displacement of vibrating particle can be written as;

$$y = a\sin(\omega t - \phi) \ldots\ldots (2)$$

For λ path difference, the corresponding phase difference is 2π.

For 1 path difference " " " is $\frac{2\pi}{\lambda}$

For x " " " " is $\frac{2\pi}{\lambda}x = \phi$.

per(m)

Then;

$$y = a\sin\left(\omega t - \frac{2\pi}{\lambda}x\right)$$

$$= a\sin(\omega t - kx) \quad \text{——— (3)}.$$

$k = \frac{2\pi}{\lambda} \rightarrow$ wave number.

$\omega = 2\pi f$.

$$y = a\sin\left(2\pi f t - \frac{2\pi}{\lambda}x\right)$$

$$= a\sin\frac{2\pi}{\lambda}(f\lambda t - x)$$

$$= a\sin\frac{2\pi}{\lambda}(vt - x) \quad [\because v = f \times \lambda] \quad \text{—— (4)}$$

eqⁿ (3) & (4) are the required expressions for plane progressive wave.

2nd case,

If the wave is travelling along the -ve x-direction then,

$$\left.\begin{aligned} y &= a\sin(\omega t + kx) \\ y &= a\sin(vt - x) \end{aligned}\right\} \text{—— (5)}$$

Particle velocity:

$$V_p = \frac{dy}{dt}$$

$$= \frac{d}{dt}\{a\sin(\omega t - kx)\}$$

$$\Rightarrow a\frac{d}{dt}\sin(\omega t - kx)$$

$$= a\cos(\omega t - kx)\cdot\omega$$

$$= a\omega\cos(\omega t - kx)$$

$(V_p)_{max} = a\omega \quad [\because \cos(\omega t - kx) = 1]$ maxm

CamScanner

Particle accl^n:

$$a_p = \frac{d^2y}{dt^2}$$

$$= \frac{d}{dt}(v_p)$$

$$= \frac{d}{dt}[a\omega\cos(\omega t - kx)]$$

$$= a\omega[-\sin(\omega t - kx)\cdot\omega]$$

$$= -a\omega^2\sin(\omega t - kx)$$

$\boxed{(a_p)_{max} = -a\omega^2}$ $[\because \sin(\omega t - kx) = 1]$.

$= a\omega^2$ Numerically;

$\therefore (a_{p\,max}) = a\omega\cdot\omega$

$= (v_p)_{max}\cdot\omega$ ##

Relation between particle velocity & wave velocity.

Show that the particle velocity at any point is equals to the product of wave velocity and slope of displacement curve at that point.

Sol^n The eq^n of plane progressive wave is

$y = a\sin(\omega t - kx)$ —— (1).

Now, particle velocity;

$$V_p = \frac{dy}{dt}$$

$$= \frac{d}{dt}\{a\sin(\omega t - kx)\}$$

$$V = \frac{\omega}{K}$$

$= a\omega\cos(\omega t - kx) \ldots \ldots (2)$

Again, differentiating eqⁿ (1) w.r.t 'x' on both sides, we ge

$$\frac{dy}{dx} = \frac{d}{dx}\{a\sin(\omega t - kx)\}$$

$$= -ak\cos(\omega t - kx) \quad \text{--- (3)}$$

Dividing eqⁿ (2) by eqⁿ (3), we get:

$$\frac{V_P}{\left(\frac{dy}{dx}\right)} = \frac{a\omega\cos(\omega t - kx)}{-ak\cos(\omega t - kx)} = \frac{-2\pi f}{\frac{2\pi}{\lambda}} = -f\lambda = v$$

$$\therefore \boxed{V_P = -v \times \frac{dy}{dx}}$$

which shows that the particle velocity at any point is equals to the product of wave velocity & slope of displacement curve at that point.

Differential eqⁿ of plane progressive wave / General wave eqⁿ

The eqⁿ of plane progressive wave is:

$$y = a\sin(\omega t - kx) \quad \text{--- (1)}$$

differentiating eqⁿ (1) wrt 't' on both sides, we get,

$$\frac{dy}{dt} = \frac{d}{dt}(a\sin(\omega t - kx))$$

$$= a\cos(\omega t - kx) \cdot \omega$$

Again, differentiating w.r.t 't' on both sides, we get,

$$\frac{d^2y}{dt^2} = \frac{d}{dt}\left[a\omega\cos(\omega t - kx)\right]$$

$$= -a\omega^2 \sin(\omega t - kx) \quad \text{——— (2)}$$

Also, differentiating eqⁿ (1) w.r.t. 'x' on b.s we get.

$$\frac{dy}{dx} = \frac{d}{dx}\left[a\sin(\omega t - kx)\right]$$

$$= a\cos(\omega t - kx)\cdot(-k)$$

$$= -ak\cos(\omega t - kx)$$

Again,

differentiating w.r.t. 'x' on b.s. we get,

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\left[a\cos(\omega t - kx)\right](-k)\right]$$

$$= -ak\sin(\omega t - kx)](-k)$$

$$= -ak^2\sin(\omega t - kx) \quad \text{——— (3)}$$

dividing eqⁿ (3) by eqⁿ (2), we get,

$$\frac{\frac{d^2y}{dx^2}}{\frac{d^2y}{dt^2}} = \frac{-a\omega^2\sin(\omega t - kx)}{-ak^2\sin(\omega t - kx)}$$

$$= \frac{\omega^2}{\cancel{\phi}}$$

$$= \frac{\omega^2}{k^2}$$

$$= \left(\frac{\omega}{k}\right)^2$$

$$= \frac{-ak^2\sin(\omega t - kx)}{-a\omega^2\sin(\omega t - kx)}$$

$$= \frac{k^2}{\omega^2} = \left(\frac{k}{\omega}\right)^2 = \left(\frac{\frac{2\pi}{\lambda}}{2\pi f}\right)^2$$

$$= \left(\frac{2\pi}{\lambda} \times \frac{1}{2\pi f}\right)^2$$

$$= \frac{1}{v^2}$$

or, $\frac{d^2y}{dx^2} \times \frac{dt^2}{d^2y} = \frac{1}{v^2}$

$$\therefore \boxed{\frac{d^2y}{dx^2} = \frac{1}{v^2} \times \frac{d^2y}{dt^2}}$$

This is the required differential equation of plane progressive wave.

Imp Long qn

Energy, Power and Intensity of Plane progressive wave.

All

* Show that energy/power/intensity of a plane progressive wave is directly proportional to the square of amplitude.

Soln:

The expression of plane progressive wave is

$$y = a\sin(\omega t - kx) \quad \text{———} \quad (1)$$

The restoring force is given by;

$$F = -ky \quad \text{———} \quad (2)$$

where k is force constant.

It is assumed that the potential energy ~~is~~ at mean position is zero:

$$P.E = \int -F\,dy$$

ϑ = volume
v = velocity.

$= \int -ky\,dy$

$= \frac{1}{2}ky^2$ $\left[\because \omega = \sqrt{\frac{k}{m}}\right]$

$= \frac{1}{2}m\omega^2 a^2 \sin^2(\omega t - kx)$

$= \frac{1}{2}\rho\vartheta\omega^2 a^2 \sin^2(\omega t - kx)$ $\left[\because \rho = \frac{m}{v}\right]$

$\therefore$ ~~pe~~ P.E per unit volume $= \frac{1}{2}\rho\omega^2 a^2 \sin^2(\omega t - kx)$ — (3)

$\left[\because \frac{P.E}{\rho\vartheta}\right]$

$K.E = \frac{1}{2}m\left[\frac{dy}{dt}\right]^2$ $\left[\because v = \frac{dy}{dt}\right]$

$= \frac{1}{2}m\left[a\omega\cos(\omega t - kx)\right]^2$

$= \frac{1}{2}m(a\omega)^2\cos^2(\omega t - kx)$

$= \cancel{\frac{1}{2}ma\omega(1}$ -

$= \frac{1}{2}m(a\omega)^2\cos^2(\omega t - kx)$

$\frac{K.E}{\vartheta} = \frac{1}{2}\rho\omega^2\cos^2(\omega t - kx)a^2$ — (4)

Total energy per unit volume $= \frac{K.E}{vol} + \frac{P.E}{Vol}$.

Joule

$\frac{E}{Vol} =$ $= \cancel{\frac{1}{2}\rho\omega^2 a^2}$

$= \frac{1}{2}\rho\omega^2 a^2\cos^2(\omega t - kx) + \frac{1}{2}\rho\omega^2 a^2\sin^2(\omega t - kx)$

$= \frac{1}{2}\rho\omega^2 a^2\left[\cos^2(\omega t - kx) + \sin^2(\omega t - kx)\right]$

$$\left[\begin{array}{l} I = \frac{P}{A} \\ P = \frac{\mathcal{E}}{t} \end{array}\right.$$

$= \frac{1}{2}\rho\omega^2 a^2 \quad [\because \sin^2\theta + \cos^2\theta = 1]$

$\frac{\mathcal{E}}{V}$ & $\boxed{\frac{\mathcal{E}}{\vartheta} = \frac{1}{2}\rho\omega^2 a^2} \longrightarrow (5)$

avg. K.E = half

Total energy of plane progressive wave is always a constant quantity.

Power (P) = ~~Energy~~ It is the rate of flow of energy.

watt

i.e. $P = \frac{\mathcal{E}}{t}$

$= \frac{\frac{1}{2}\rho\omega^2 v a^2}{t} \quad \left[\because v = \text{Area} \times d \atop = A \cdot \text{distance}\right]$

$= \frac{\frac{1}{2}\rho\omega^2 A d a^2}{t}$

$= \frac{\frac{1}{2}\rho\omega^2 A v \cancel{t} a^2}{\cancel{t}} \quad \left[\because d \quad v = \frac{d}{t}\right]$

$= \frac{1}{2}\rho\omega^2 A v a^2$

$\boxed{P = \frac{1}{2}\rho A v a^2 \omega^2} \longrightarrow (6)$

This is the required equation of power of differential equation.

Intensity: It is the rate of flow of energy per unit area of cross section.

watt/m^2

i.e. $I = \frac{P}{A}$

$\boxed{I = \frac{1}{2}\rho v a^2 \omega^2} \longleftarrow (7)$

* Show that average P.E per unit volume and average K.E per unit volume are equals to half of total energy per unit volume over a wavelength.

* Average $\frac{K.E}{V}$ = Average $\frac{P.E}{V}$

Sol^n Continue upto $\frac{E}{Vol.}$ which is in eq^n 5.

$$\frac{E}{Vol} = \frac{1}{2}\rho\omega^2 a^2$$

Average P.E per unit volume over a wavelength

$$= \frac{1}{\lambda}\int_0^\lambda \frac{1}{2}\rho\omega^2 a^2 \sin^2(\omega t - kx)\,dx$$

$$= \frac{1}{\lambda}\,\frac{1}{2}\rho\omega^2 a^2 \int_0^\lambda \sin^2(\omega t - kx)\,dx$$

$$= \frac{1}{\lambda} \times \frac{1}{2}\rho\omega^2 a^2 \int_0^\lambda \frac{1-\cos 2(\omega t - kx)}{2}\,dx$$

$$= \frac{1}{4\lambda}\rho\omega^2 a^2 \int_0^\lambda [1 - \cos 2(\omega t - kx)]\,dx$$

$$= \frac{1}{4\lambda}\rho\omega^2 a^2 \int_0^\lambda dx - \underbrace{\int_0^\lambda \cos 2(\omega t - kx)\,dx}_{0}$$

$$= \frac{1}{4\cancel{\lambda}}\rho\omega^2 a^2 x\Big|_0^\lambda$$

$$= \frac{1}{4\cancel{\lambda}}\rho\omega^2 a^2 \cancel{\lambda} = \frac{1}{4}\rho\omega^2 a^2$$

~~Ans~~

Average K.E per unit volume over a wavelength

$$= \frac{1}{\lambda}\int_0^\lambda \frac{1}{2}\rho\omega^2a^2\cos^2(\omega t-kx)\,dx$$

$$= \frac{1}{\lambda}\times\frac{1}{2}\rho\omega^2a^2\int_0^\lambda \cos^2(\omega t-kx)\,dx$$

$$= \frac{1}{2}\times\frac{1}{\lambda}\rho\omega^2a^2\int_0^\lambda \underbrace{1-\sin^2(\omega t-kx)}_{\cos^2\theta}\,dx$$

$$= \frac{1}{2\lambda}\rho\omega^2a^2\int_0^\lambda dx - \int_0^\lambda \sin^2(\omega t-kx)\,dx$$

$$= \frac{1}{2\lambda}\rho\omega^2a^2\times\lambda -$$

$$= \frac{1}{2\lambda}\rho\omega^2a^2\int_0^\lambda \frac{1+\cos 2(\omega t-kx)}{2}\,dx$$

$$= \frac{1}{4\lambda}\rho\omega^2a^2\int_0^\lambda dx + \int_0^\lambda \cancel{\cos 2(\omega t-kx)\,dx} \rightarrow 0$$

$$= \frac{1}{4\cancel{\lambda}}\rho\omega^2a^2\times\cancel{\lambda}$$

$$= \frac{1}{4}\rho\omega^2a^2$$

$$\therefore \boxed{\frac{K.E}{V}(avg) = \frac{1}{4}\rho\omega^2a^2}$$

$$\int_0^\lambda \cos 2(\omega t-kx)\,dx$$

$$= \left.\frac{\sin 2(\omega t-kx)}{-2k}\right|_0^\lambda$$

$$= -\frac{1}{2k}\left[\sin 2\left(\frac{2\pi}{T}\cdot\frac{T}{\,}-k\lambda\right)\right] - \frac{1}{2k}\left[\sin 2(0)\right]$$

$$= \frac{-1}{2k}\left[\sin 2\left[2\pi-2\pi\right]\right]$$

$$= -\frac{1}{2k}\,0 = 0$$

~~It show~~

The required eq^n shows that avg. $\frac{P.E}{V}$ and avg. $\frac{K.E}{V}$ is equal to the half of $\frac{T.E}{V}$ over a wavelength.

1.

General equation of plane progressive wave is;

$y = a\sin(\omega t - kx)$ —— (1)

The equation is;

$y = \cancel{0.03(} \ 0.03\sin(60\pi t - 0.03\pi x)$ —— (2)

Comparing eq^n (1) with (2), we get

$\therefore a = 0.03\,cm = 3\times10^{-4}m$

$\omega = 60\pi$

or, $\sqrt{\frac{K}{m}} = 60\pi$

or, $2\pi f = 60\pi$

$\therefore f = \frac{60\pi}{2\pi} = 30\,Hz$

$$\boxed{\begin{aligned} k &= \frac{2\pi}{\lambda} \\ \text{or, } 0.03\pi &= \frac{2\pi}{\lambda} \\ \therefore \lambda &= \frac{2}{0.03} = 66.67m \end{aligned}}$$

particle

$(v_p)_{max} = a\omega$

$= 0.03 \times 60\pi$

$= 5.65\,cm/s$

Q.n ②

$$y = 0.03\sin(3\pi t - 0.03\pi x) \quad \text{—— ①}$$

Sol^n: $y = a\sin(\omega t - kx)$ —— ②.

$\therefore a = 0.03\,cm$.

$\omega = 3\pi$

$or,\ 2\pi f = 3\pi$

$\therefore f = \frac{3}{2} = 1.5Hz$.

$k = 0.03\pi$.

$\frac{2\pi}{\lambda} = 0.03\pi$

$\therefore \lambda = \frac{2}{0.03} = \cancel{6.667cm} \cdot 66.67$

Velocity of the wave $(v) = f\lambda$

$= \cancel{1.5 \times 6.667}$

$= \cancel{10cm/s}$.

$= 1.5 \times 66.67$

$= 100m/s$.

phase difference $(\phi) = \frac{2\pi}{\lambda}x$

$= \frac{2\pi}{\lambda} \times 0.05$

$= 4.71 \times 10^{-3}$ ans.

* A progressive ~~wave~~ & stationary simple harmonic wave having frequency 256 Hz & each having same velocity 20m/s.

① Determine the phase difference bet^n 2 vibrating point in a progressive wave at a distance of 5cm apart.

② Write the appropriate progressive wave eqⁿ if $a = 2cm$
③ Distance between nodes & stationary wave.

$f = 256\,Hz$
$v = 20\,m/s$.

Here, $\cancel{k = 2\pi}$ $\omega = 2\pi f$
$= 2\pi \times 256$
$= 512\pi$

$v = f\lambda$
or, $\frac{20}{256} = \lambda$ $\therefore \lambda = 0.078\,m.$

① ⇒ $\phi = \frac{2\pi}{\lambda} x$ $x = 5cm$
$= 5\times10^{-2}\,m$
$= \cancel{4}\ \frac{32}{25}\pi$.

$\therefore k = \frac{2\pi}{\lambda} = \cancel{8}\ \frac{1000}{39}\pi$.

② ⇒ The appropriate progressive wave eqⁿ of.

$y = a\sin(\omega t - kx)$ $a = 2cm$
$= 0.02\sin\left(512\pi t - \frac{1000}{39}\pi x\right) = 0.02\,m$. //

③ → distance betⁿ node & antinode $= \lambda/2 =$

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* Calculate the ratio of intensity of following two waves ~~eq~~:

$y_1 = 4\sin(\pi t - 10x)$ cm and $y_2 = 2\sin(2\pi t - 100x)$ cm

from y_1;

$a = 4$ cm

$\omega = \pi$

$k = 10$

$\text{or}, \frac{2\pi}{\lambda} = 10 \quad \therefore \lambda = \frac{2\pi}{10} = 0.62$cm

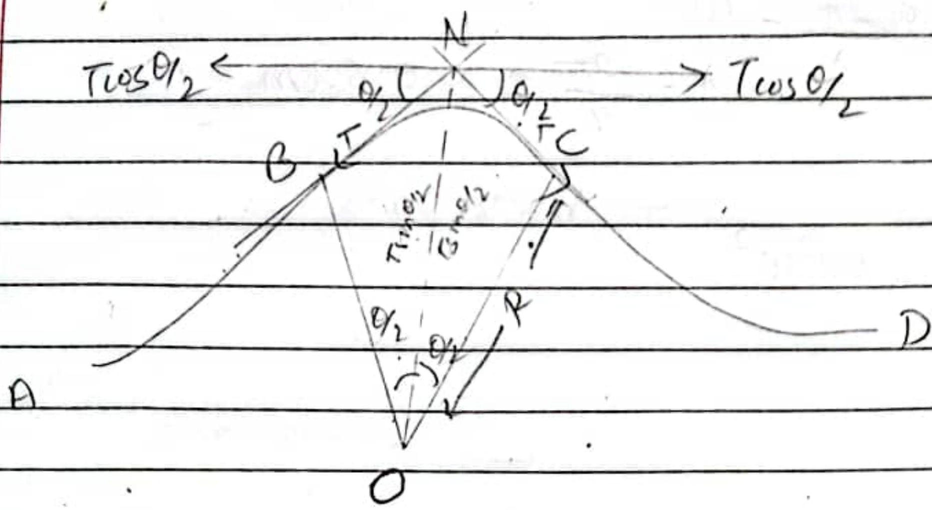
$I_1 = \frac{1}{2} \rho v a^2 \omega^2$

* Speed of transverse wave along a stretched string:

(7) (a) Show that speed of transverse wave along a stretched string doesnot depends on frequency & wavelength but depends upon linear density & tension on the string.

Also show that rate of energy transport is

$$v = \sqrt{\frac{T}{\mu}}$$



consider a ABCD a symmetrical pulse moving from left to right with speed 'v'. Let us choose a frame of reference such that the pulse remains stationery and string element moves from right to left with same speed 'v'.

Let 'T' be the tension at point B & C. The direction of tension is tangentially and the Tension can be resolved into two components. The horizontal components are equal & positive. So, they are cancelled but vertical components acting towards the centre are added to give resultant tension i.e.

$$T = T\sin\theta/_2 + T\sin\theta/_2$$
$$= 2T\sin\theta/_2$$

For very small angle θ, $\sin\theta/2 \approx \theta/2$.

i.e. $T = 2T\theta/2$

or, $T = T\theta$ ——— (1)

If m and l are the mass and arc length of the string element BC. Then,

linear density $(\mu) = \frac{m}{l}$ ——— (2)

The centripetal force acting towards the centre is given by;

$$F = \frac{mv^2}{R} \quad \text{——— (3)}$$

~~Eq.~~ From (1) & (3), we get,

$F = T$

or, $\frac{mv^2}{R} = T\theta$

or, $\frac{mv^2}{R} = T \times \frac{l}{R}$ $\left(\because \theta = \frac{l}{r}\right)$

or, $T = \frac{mv^2}{l}$

or, $T = \mu v^2$

$$\boxed{\therefore v = \sqrt{\frac{T}{\mu}}} \quad \text{——— (4)}$$

which is the required equation for the ~~veloc~~ speed of transverse wave along stretched string.

Hence, speed of transverse wave along a stretched string doesnot depends on frequency & wavelength but depends upon linear density & tension on the string

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(8) We have,

Rate of energy transport $= \frac{\varepsilon}{vol}$

$= \frac{1}{2} \rho \omega^2 a^2$

$\therefore$ Energy $= \frac{1}{2} \times \rho \times V \times \omega^2 \times a^2$ $\quad [\because \rho = \frac{m}{V}]$

$= \frac{1}{2} m \omega^2 a^2$

~~$\therefore \varepsilon$~~

Now,

$$\frac{\varepsilon}{t} = \frac{1}{2} \frac{m}{l} \cdot l \omega^2 a^2 / t$$

$$= \frac{\frac{1}{2} \mu \times v \times t \times \omega^2 \times a^2}{t} \quad [\because v = \frac{l}{t}]$$

$$\boxed{P = \frac{1}{2} \mu v \omega^2 a^2}$$

$$\text{Intensity} = \frac{P}{A} = \frac{\frac{1}{2} \mu v \omega^2 a^2}{A}$$

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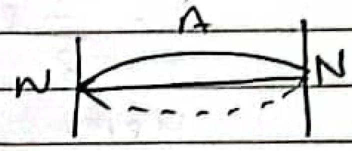
Sonometer: (5) (Fundamental wave).

$$\therefore \boxed{f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}}$$

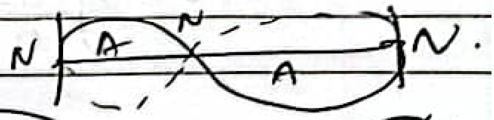
Standing wave & Resonance:

Harmonics & Overtone

$\lambda_1 = \frac{2l}{1} \longrightarrow$ 1st harmonics



$\lambda_2 = \frac{2l}{2} \longrightarrow$ 2nd



$\lambda_3 = \frac{2l}{3} \longrightarrow$ 3rd.

⋮

$\lambda_n = \frac{2l}{n} \longrightarrow$ nth harmonics

where $n = 1, 2, 3, \ldots$

$$f = \frac{v}{\lambda} = \frac{v}{\frac{2l}{n}}$$

$$= \frac{n}{2l} \times v$$

$$\boxed{f = \frac{n}{2l}\sqrt{\frac{T}{\mu}}} \quad \left[\because v = \sqrt{\frac{T}{\mu}}\right]$$

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(3) $a = 1cm = 0.01m$
~~freq~~ $f = 2Hz$
$\mu = 1gm/m = 10^{-3}kg/m$
$T = 20N$.

$$P = \frac{1}{2}\mu v \omega^2 a^2$$

Here,
$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{20}{10^{-3}}} = 141.42 m/s.$$

$$\omega = 2\pi f = 2\pi \times 2 = 4\pi$$

$$P = \frac{1}{2} \times 10^{-3} \times 141.42 \times (4\pi)^2 \times (0.01)^2$$
$$= 116 \times 10^{-3} \text{ Watt}$$

(4) $l = 2.72m$
$m = 263gm = 0.263kg$
$T = 36.1 N$
$a = 2.5mm = 2.5 \times 10^{-3}m$
$P = 85.5W$.
$f = ?$

$$\mu = \frac{m}{l} = \frac{0.263}{2.72} = 0.096 kg/m$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{36.1}{0.096}} = 19.32 m/s.$$

or, $P = \frac{1}{2} \mu v \omega^2 a^2$

or, $85.5 = \frac{1}{2} \times 0.096 \times 19.32 \times \omega^2 \times (2.5 \times 10^{-3})^2$.

or, $\omega^2 = \sqrt{\frac{85.5 \times 2}{0.096 \times 19.32 \times (2.5 \times 10^{-3})^2}}$ ~~1000m~~.

or, $2\pi f = \cancel{3840.775}$ $\quad 2\pi f = 3827.022$

$\therefore f = \cancel{611.27} Hz$. $\quad \therefore f = 609.08 Hz$.

⑤

$f = 1000Hz$. $\quad , \omega = 2\pi f = 2\pi \times 1000$

$= 2000\pi$

$a = 10^{-9} cm = 10^{-11} m$

$v = 340 m/s$

$\rho = 0.0013 g/cc = \frac{0.0013 \times 10^{+3}}{10} kg/m^3$.

$A = \frac{v}{d}$ $\quad = 1.3 kg/m^3$.

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(6) $m = 3gm = 3\times10^{-3}\ kg$

$l = 80cm = 0.8m$

$T = 25N$

$f = 120Hz$

$a = 1.6mm = 1.6\times10^{-3}m$

$\mu = \frac{3\times10^{-3}}{0.8} = 3.75\times10^{-3}$

$$avg.p = \frac{1}{2}\mu v\omega^2 a^2$$
$$= \frac{1}{2}\times\frac{m}{l}\times\sqrt{\frac{T}{\mu}}\times(2\pi f)^2\times a^2$$
$$= \frac{m\times 4\pi^2 f^2\times a^2}{2l}\times\sqrt{\frac{T}{\mu}}$$
$$= \frac{3\times10^{-3}\times 2\times\pi\times 120^2\times(1.6\times10^{-3})^2}{0.8}\times\sqrt{\frac{25}{3.75\times10^{-3}}}$$
$$=$$

$$\omega = 2\pi f = 2\pi\times120 = 240\pi$$
$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{25}{3.75\times10^{-3}}} = 81.64m/s.$$

$$avg.P = \frac{1}{2}\mu v\omega^2 a^2$$
$$= \frac{1}{2}\times 81.64\times 3.75\times10^{-3}\times(240\pi)^2\times(1.6\times10^{-3})^2$$
$$= 0.22\ Watt.$$

If the wave amplitude is halved;

$$P' = \frac{1}{2}\mu\omega^2\left(\frac{1}{2}a\right)^2 v$$
$$= \frac{1}{2}\mu\omega^2\frac{a^2}{4} = \frac{1}{8}\mu\omega^2 a^2 v$$
$$= 0.055\ W.$$

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$V = f\lambda$

Acoustics:- ~~of~~

Sound wave - longitudinal wave:

$v = 332\,m/s$ at NTP.

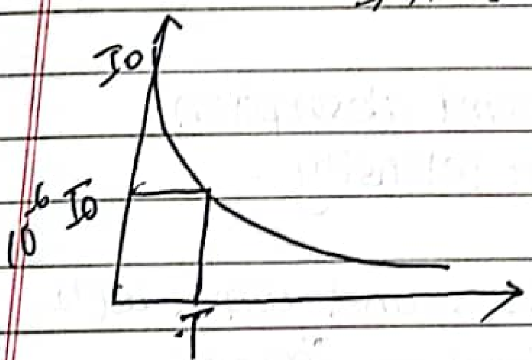
On the basis of frequency
- Infrasonic waves $< 20 Hz$
- Audible waves $20 Hz - 20 kHz$.
- Ultrasonic waves $> 20 kHz$

$$\lambda = \frac{v}{f} = \frac{332}{20 kHz} = \frac{332}{20 \times 10^3 Hz} =$$

Acoustic of building:

Reverberation of sound:- repetition / persistent of ~~sound~~ sound when the sound source is stopped / ceased.

$\Rightarrow$ time taken by sound intensity to fall



T is the reverberation of time:

Absorption of sound

$\alpha \rightarrow$ coefficient of sound absorption.

$\alpha = 1$ (perfect absorber) Open space.

~~1 Sabine~~ $\alpha = \dfrac{\text{sound energy absorbed by the } \cancel{\text{surface}}}{\text{~~dist~~ sound energy '' '' the open space of same area}}$

α cannot exceed 1.

$\alpha = \dfrac{\text{Sound energy absorbed by surface.}}{\text{~~sound~~ strike on the openspace of same area.}}$

$A = \alpha S$

$\alpha = \dfrac{\alpha_1 S_1 + \alpha_2 S_2 + \cdots + \alpha_n S_n}{S_1 + S_2 + \cdots + S_n} = \dfrac{\sum \alpha_i S_i}{S_i}$

$\alpha = \dfrac{\sum \alpha_i S_i}{S}$

Sabine's formula for the Reverberation time T

$= \dfrac{0.158\, V}{\alpha S}$ in S.I. [$\because V$ = volume]
[S = Surface Area]
(S_i)

The fall of intensity of the sound depends upon no. of reflections per second (n), initial intensity of sound 'i' & time interval (δt) i.e;

$\delta I = -\alpha n I \delta t$

where α is the coeff. of sound absorption.
(-ve) sign indicates the fall of intensity.

The average distance travelled by the sound wave bet^n 2 successive reflections is given by Jaegar as-

$d = \dfrac{4V}{S}$ [$\because V$ = Volume].

If 'v' be the velocity of sound in air. then.

$\delta t = \dfrac{d}{v}$

$\therefore d = v\delta t$

Then the no. of reflections per second is represented by

$$'n' = \frac{1}{\bar{t}}$$

$$= \frac{S\upsilon}{4V}$$

$$\therefore \delta I = -\frac{\alpha S\upsilon}{4V} \cdot I \cdot \delta t$$

$$\therefore \frac{\delta I}{I} = -\frac{\alpha S\upsilon}{4V} \delta t$$

$\therefore$ In any limit, the increment can be replaced by derivative;

$$\frac{dI}{I} = -\frac{\alpha S\upsilon}{4V} dt$$

At $t=0$, $I = I_0$ (maximum intensity)

At any instant of time, $I = I_t$

$$\int_{I_0}^{I_t} \frac{dI}{I} = \int_{0}^{t} -\frac{\alpha S\upsilon}{4V} dt$$

Integrating,

$$\int_{I_0}^{I} \frac{dI}{I} = \int_0^t -\frac{\alpha S\upsilon}{4V} dt$$

$$\therefore \ln I \Big|_{I_0}^{I} = -\frac{\alpha S\upsilon}{4V} \times t \Big|_0^t$$

$$\therefore \ln(I - I_0) = -\frac{\alpha S\upsilon}{4V} t$$

$$\therefore \ln\left(\frac{I}{I_0}\right) = -\frac{\alpha S\upsilon}{4V} t$$

For Reverberation Time, T; $I = 10^{-6} I_0$.

Then,

$$\ln\left(\frac{10^{-6}\cancel{I_0}}{\cancel{I_0}}\right) = -\frac{\alpha S v}{4V} \cdot T$$

$$\text{or, } \cancel{-}6\ln 10 = \cancel{-}\frac{\alpha S v}{4V} T$$

$$\text{or, } \frac{6\ln 10 \times 4V}{\alpha S v} = T$$

$$\text{or, } T = \frac{6 \times 2.30 \times 4V}{\alpha S v}$$

$$= \frac{13.81 \times 4V}{\alpha S v}$$

$$\therefore \boxed{T = \frac{55.26\,V}{\alpha S v}}$$

for $v = 350 m/s$

$$T = \frac{0.158V}{\alpha S}$$

For $v = 1100 m/s$, $T = \frac{0.05V}{\alpha S}$

Noise Pollution:

Any undesired or irritating sound that has an impact on the health and well-being of people and other living things is refered to as noise pollution.

Sound is measured in decibels (dB). A decibel value above 80 is considered to be noise pollution.

Causes of Noise Pollution:

① Domestic sources: People utilise devices heavily in their daily lives and are constantly surrounded by them.

$T \propto \frac{1}{\text{no. of person}}$

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(Q) The time of reverberation of an empty hall and with ~~cofdth~~ 600 audiences in the hall is 1.6 sec and 1.45 sec respectively. Find the reverberation time with 700 audience in the hall.

⇒ According to question,

$$T_1 = 1.6\,sec = \frac{0.158V}{\alpha s} \text{ —— (I)}$$

$$T_2 = 1.45\,sec = \frac{0.158V}{\alpha s + 600} \text{ —— (II)}.$$

$$\frac{T_1}{T_2} = \frac{(0.158V)}{\alpha s} \times \frac{\alpha s + 600}{(0.158V)}$$

$$= \frac{\alpha s + 600}{\alpha s}$$

or, $\frac{1.6}{1.45} = \frac{\alpha s + 600}{\alpha s}$

or, $\cancel{\alpha s =}$ $1.6\alpha s - 1.45\alpha s = 870$

$\therefore \alpha s = 5800\,m^2$

From eqn (I),

$$T_1 = \frac{0.158V}{\alpha s}$$

or, $\frac{1.6 \times 5800}{0.158} = V \quad \therefore V = 58734.17\,m^3$

$\therefore T_3 = \frac{0.158V}{\alpha s + 700}$ [∵ with 700 audiences]

$= 1.42\,sec$ //.

(Q) A classroom has dimensions $(10 \times 8 \times 5) m^3$. The reverberation time is 1.4 sec. Calculate the total absorption of its surface and average absorption coefficient.

Sol^n $V = l \times b \times h$

$= 10 \times 8 \times 5 = 400 m^3$.

$T_1 = 1.4 sec$.

$$T = \frac{0.158V}{\alpha S}$$

or, $$1.4 = \frac{0.158 \times 400}{\alpha S}$$

$\therefore \alpha S = 45.14 m^2$

$\therefore$ The total absorption of its surface $(\alpha s) = 45.14 m^2$

Surface area $(A) = 2(lb + lh + bh)$

$= 2(10 \times 8 + 10 \times 5 + 8 \times 5)$

$= 2(80 + 50 + 40)$

$= 340 m^2$.

Average absorption coefficient $= \frac{\Sigma \alpha S}{\Sigma S}$

$= \frac{45.14}{340}$

$\therefore \alpha = \quad = 0.13$ ~~sec~~

$\therefore \alpha = 0.132$ //.

Numerical

P no 1.

⇒ Sol^n, $l = 40ft$
$b = 100ft$
$h = 20ft$
$V = l \times b \times h = 40 \times 100 \times 20$
$= 80{,}000$ ~~m³~~ cubic.ft.

(i) Area of plaster $= 7500$ sq.ft
$\alpha_1 = 0.03$.
$\therefore$ Absorption coefficient $= \alpha_1 S_1$
$= 0.03 \times 7500$
$= 225 ft^2$

$\alpha_2 S_2 = 0.06 \times 600$
$= 36\ ft^2$
$\alpha_3 S_3 = 400 \times 0.025$
$= 10\ ft^2$
$\alpha_4 S_4 = 0.3 \times 600 = 180\ ft^2$
$\alpha_5 S_5 = 500 \times 0.4 = 200\ ft^2$

Total absorption $(\Sigma \alpha S) = 225 + 36 + 10 + 180 + 200$
$= 651\ ft^2$

$$T = \frac{0.05 V}{\alpha S}$$
$$= \frac{0.05 \times 80000}{651}$$
$= \cancel{0.615ec}$. 6.14sec //.

Q.no. 2

Soln

Volume (V) = $20 \times 15 \times 10\ m^3$

$= 3000 m^3$.

$T_1 = 3.5 sec$.

Now, $T = \frac{0.158V}{\alpha S}$

or, $3.5 = \frac{0.158 \times 3000}{\alpha S}$

$\therefore \alpha S = 135.42 m^2$.

or, $\alpha = \frac{135.42}{2(lb+bh+lh)} = 0.104$ //.

$T_2 = 2.5 sec$.

$\alpha = 0.5$.

~~Now, $T_2 = \frac{0.158V}{\alpha S}$~~

~~or, $2.5 = \frac{0.158 \times 3000}{0.5 \times S + \alpha s}$~~

~~$\therefore S = 379.2 m^2$.~~

~~Area of the wall =~~

$T_2 = \frac{0.158V}{\alpha S + \alpha_c S_c}$

or $2.5 = \frac{0.158 \times 3000}{135.42 + 0.5 \times S_c}$

or, ~~$0.5S_c = 1.400$~~

or, $135.42 + 0.5S_c = \frac{0.158 \times 3000}{2.5}$

or, $0.5S_c = 54.18$.

$\therefore S_c = 108.36 m^2$ //.

(3)

$V = 45{,}000$ cu.ft.

$T_1 = 1.5$ sec.

Total absorbing power of all the surface in the hall (αs) = ?.

Now,

$$T_1 = \frac{0.05V}{\alpha S}$$

or, $$1.5 = \frac{0.05 \times 45{,}000}{\alpha S}$$

$\therefore \alpha s = 1500\ ft^2$

Now,

~~At for~~ $\cancel{T_2 = \frac{0.05V}{\alpha S + \alpha.}}$

Average absorption coefficient $= \frac{\Sigma \alpha s}{S}$

$$= \frac{1500}{8000}$$

$$= 0.1875$$ //.

(4).

$V = 80{,}000$ ~~sq~~ cub.ft.

Absorption coefficient (αs) ≈ 1000 sq.ft.

Now,

$$T = \frac{0.05V}{\alpha S}$$

$$= \frac{80{,}000 \times 0.05}{1000}$$

$$= 4\ \text{sec}$$ //.

(5) Volume $(V) = 980m^3$.

$S_1 = 150m^2$, $\alpha_1 = 0.03$

$S_2 = 95m^2$ $\alpha_2 = 0.80$

$S_3 = 90m^2$ $\alpha_3 = 0.06$.

Now,

$\Sigma\alpha s = \alpha_1 s_1 + \alpha_2 s_2 + \alpha_3 s_3$

$= 0.03 \times 150 + 0.80 \times 95 + 0.06 \times 90$

$= 85.9m^2$

Average sound absorption coefficient $= \frac{\Sigma\alpha s}{S}$

$= \frac{85.9}{335}$

$= 0.256$ //.

$T = \frac{0.158V}{\alpha S}$

$= \frac{0.158 \times 980}{85.9}$

$= 1.80 sec$ //.

(4):

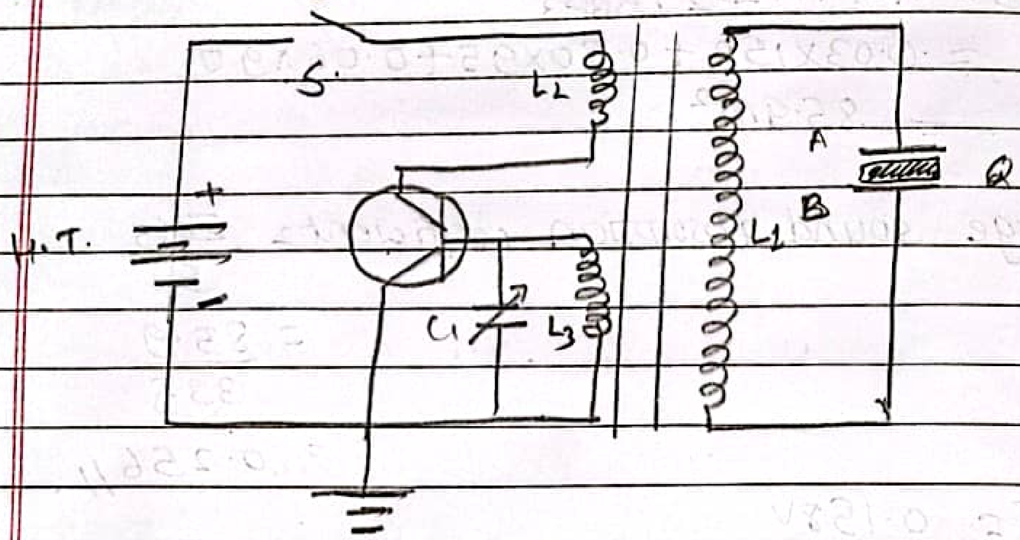
$X =$

Ultrasonic Waves:

(1) Piezoelectric generation

(2) Magnetostriction generator.

(1) Piezoelectric generation (Quartz, Tourmaline, Rochelle salt)



The frequency of vibration is;

$$f = \frac{n}{2l}\sqrt{\frac{Y}{\rho}}$$ where $n = 1, 2, 3, \ldots$ (modes of vibration)

Y = Young's modulus of elasticity

ρ = density of the crystal

l = length of crystal.

(2) Magnetostriction generator. (Fe, Co, alloys of...)

- ferromagnetic material is used.

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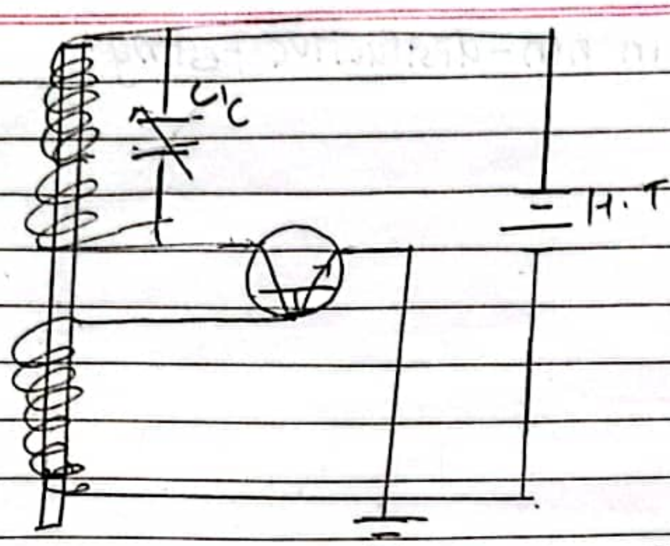


frequency is given by, $f = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}$

where, l = length of rod
ρ = density of rod
Y = Young's modulus of elasticity

Applications of Ultrasonic waves.

① They are highly directed. widely used in various ways:

(a) ~~length~~ depth of sea.

(b) used in SONAR - Sound Navigation and Ranging identification and detection of underwater objects like ships, submarines, iceberg etc in the ocean.

(c) investigation in ~~water~~ matter properties.

(d) weilding of metals.

In Medical field.

(a) locate the internal organs and defect inside the human body.

(b) kill bacteria, yeasts decrease,

(c) lamed the small ~~insec~~ animals.

(d) used in bloodless brain surgery.

(NDT)

Ultrasonic method in non-destructive testing:

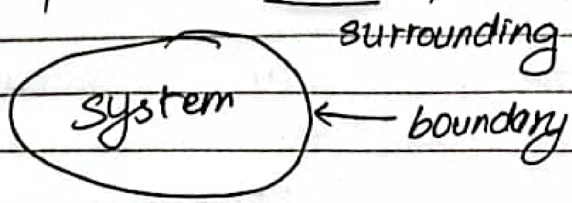
(3 questions min)

11/07

Chapter- 8

Thermodynamics and Heat Transfer

System: Space and volume for investigation.



(a) Open system (ctrl volume(Energy & mass) .
(b) closed system of control mass)
(c) Isolated system

- Closed system ⇒ mass remains constant, exchange of energy.

- open system ⇒ volume remains constant, exchange of mass & energy.

- Isolated system ⇒ Neither energy transfers nor mass exchanges.
  Ideal case) ⇒ Tea kept in thermos flask ⇒ like to be isolated.

- Macroscopic and Microscopic Points of view.
  ⇓ classical thermodynamics - ⇓ statistical thermodynamics.

Thermodynamic property [Intensive / Extensive

Intensive property :- independent of mass.
(T, P) ⇒ exception e.g. temperature, pressure
Represented by lowercase letter.

(ii) Extensive property → dependent of size, matter & mass.
E.g. Total volume, Total momentum. Represented by uppercase letter. (m) → exception.

$\frac{\text{density}}{\text{mass}}$ = specific density, divide by mass

specific energy = $\frac{\text{energy}}{\text{mass}}$

Extensive properties per unit mass are called specific properties.

Thermodynamic state & equilibrium.

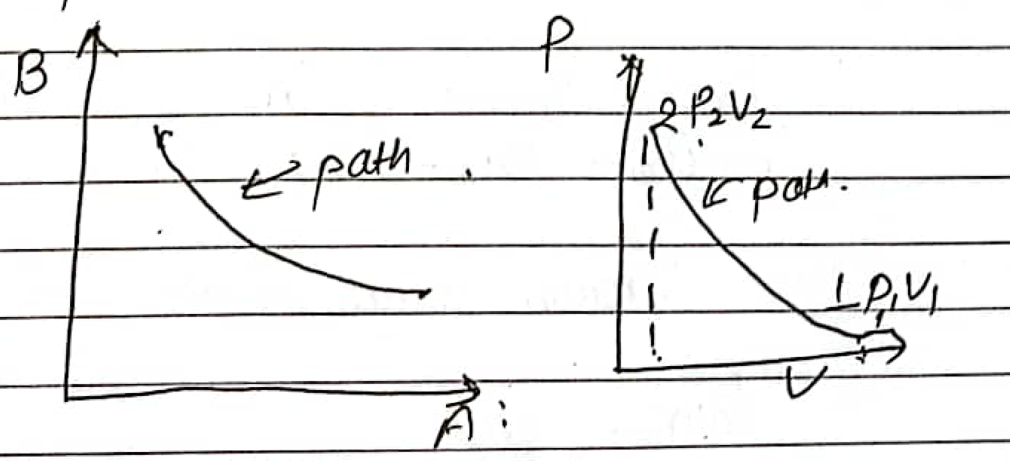
Stable condⁿ for thermodynamic

1. Mechanical Equilibrium: No change in pressure
2. Thermal equilibrium → temp same throughout the entire system.
3. Chemical equilibrium → No change in chemical composition.

State ⇒ distinct property of system

Thermodynamic process (Reversible and Irreversible).

Property:



Thermodynamic process 1 2

Reversible: quasi-static or quasi-equilibrium
- ideal case
- slow phenomena.

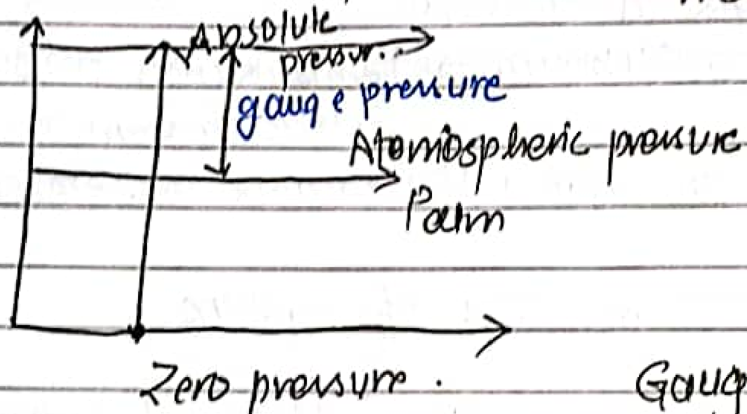
Irreversible :

Some common properties
① Volume (V)

Sp. volume $(v) = \dfrac{\text{volume}}{\text{mass}}$.

② Pressure (P)

Specific pressure $(P) = \dfrac{\text{pressure}}{\text{mass}}$.



Gauge pressure = -ve
Vacuum gauge.

difference between atm pressure & absolute pr

$P_{abs} = P_{atm} + P_{gauge}$

$\therefore P_{vac} = P_{atm} - P_{abs}$

Vacuum gauge = (-ve)

$P_{atm} = \rho h g$

Energy:

(1) stored energy

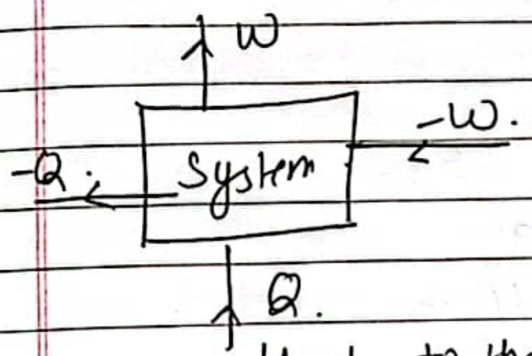
Internal (U) = $u = \frac{U}{M}$.

(b) Potential.

(c) Kinetic

Total energy = U + P.E.

Transition energy: (1) Work
(2) Heat.



Work done by the system = +ve
Work done on the system = -ve

Q Heat to the system = +ve
Heat from the system = -ve

Numericals:

Q no.1

gauge pressure (P_{gauge}) = -15 kPa

ρ_{hg} = 13600 kg/m³

P_{atm} = 13600 kg/m³.

$\therefore P_{abs} = P_{atm} + P_{gauge}$

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Q.no-1

$P_{gauge} = -15kPa = -15\times10^3 Pa$.

$h = 760mm$
$= 0.76m$.

$P_{atm} = \rho g h$
$= 13,600 \times 9.79 \times 760\times10^{-3}$
$= \cancel{10189.44}$
$= 101189.44\ Pa$.
$= 10.11\times10^4\ Pa$.

Now,
$P_{abs} = P_{atm} + P_{gauge}$
$= 10.11\times10^4 - 15\times10^3$
$= 86189.44 Pa$.
$= 86.189 kPa$ //.

Q.no.2

In Inlet,
$P_{abs} = 6000kPa$.
$h = 76cm$ of Hg
$= \cancel{7.6m}\ 0.76m$
$\rho = 13,600 kg/m^3$ and $g = 9.79 m/s^2$.

$\therefore \cancel{P_{abs}}\ P_{\cancel{abs}}^{atm} = \rho h g$
$= 13,600 \times 0.7\cancel{6} \times 9.79$
$= 1.018\times10^{6} Pa$ $\qquad 10.11\times10^4$.
$= 1.0118\times10^3\ \cancel{Pa}\ kPa$

Now, $P_{gauge} = P_{abs} - P_{atm}$
$= \cancel{6000 kPa - 1.0118\times10^3}$
$= \cancel{-1.005\times10^6 Pa}$
$= \cancel{-1.005\times10^3}$

$P_{gauge} = P_{abs} - P_{atm}$
$= (6000 - 10.11 \times 10)$ kPa
$= 5898.9$ kPa.

Now,
For outlet,
$P_{abs} = 6$ kPa
$P_{atm} = 10.11 \times 10^4$
$= \cancel{101.1 \text{ kPa}} (.10.11 \times 10)$ kPa

$\therefore P_{gauge} = P_{abs} - P_{atm}$
$\Rightarrow (6 - 10.11 \times 10)$
$= -95.1$ kPa //.

Q. no. 3.

Sol^n: $P_{abs} = 50$ kPa
Pressure of surrounding $(P_{atm}) = 100$ kPa.

$P_{gauge} = P_{abs} - P_{atm}$
$= 50 - 100$
$= (-50$ kPa$)$ //.

Q. no. 4.

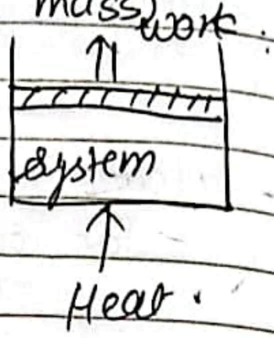
Sol^n:

Control Mass:

* First law of thermodynamics: (control mass)

Energy input − Energy output = E(residue)

$\downarrow$ Heat $\quad\quad \downarrow$ work



⇒ Statement:

It states that, "Energy neither can be created nor be destroyed but transform into various forms of energy and the total energy remains same." Using "law of conservation of energy", "Energy entering to the system & energy releasing from the system is equal to ~~zero~~ "change in ~~total~~ energy of system."

$$E_{in} - E_{out} = \Delta E_s$$

$$Q - W = \Delta E_{cm} \quad [\because \text{change in energy for control mass}]$$

$$= \Delta KE + \Delta PE + \Delta U$$

~~Energy~~ In a closed system, ΔKE & ΔPE are negligible.

So, $\Delta E_{cm} = \Delta U$

i.e., $Q - W = \Delta U$

or, $dQ - dW = dU$

(1) For cyclic process: $dU = 0$

$$dQ - dW = 0$$

or, $dQ = dW$

integrating,

$$\boxed{Q_{net} = W_{net}}$$

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49 ~~73~~
63 77

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(2) For process 1-2

$$Q_{12} - W_{12} = U_2 - U_1$$

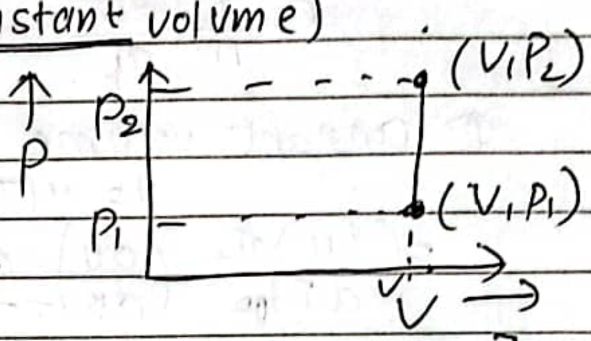
or, $$\boxed{Q_{12} = W_{12} + U_2 - U_1 \\ = W_{12} + \Delta U}$$

P isobaric
V isochoric
T isothermal
— adiabatic

PV curve Applications of 1^{st} law of thermodynamics: (C.M)

(W_{12})

1) Isochoric process (constant volume)



We have,

$$Q_{12} = W_{12} + U_2 - U_1$$
$$= \int_1^2 PdV + U_2 - U_1$$
$$\Rightarrow 0 + U_2 - U_1$$

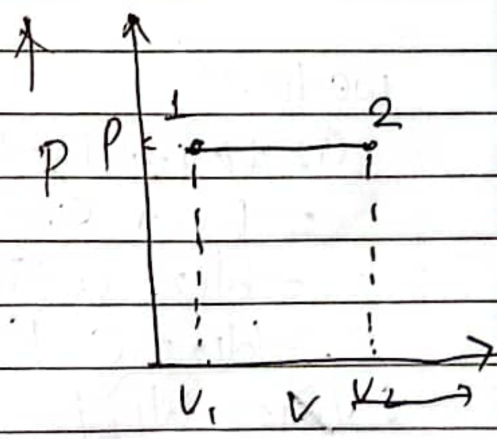
$W_{12} = \int_{P_1}^{P_2} PdV = 0$

[$\because$ Volume is constant
$\therefore \Delta V = 0$]

$\therefore$ $\boxed{Q_{12} = U_2 - U_1} = \Delta U$ is the required equation for isochoric process.

Meaning: Hence, the heat supplied to the system is used for the change in internal energy.

(2) Isobaric process (constant pressure)



We have,

$W_{12} = \int_{V_1}^{V_2} PdV$

$$Q_{12} = W_{12} + U_2 - U_1$$
$$= \int_{V_1}^{V_2} PdV + U_2 - U_1$$
$$= \int_1^2 P(\cancel{V_2 - V_1}) + U_2 - U_1 \quad dV$$
$$= P(V_2 - V_1) + U_2 - U_1$$

$\therefore P(V_2 - V_1)$

or, $\cancel{PV_2 - PV_1 =}$

or, $Q_{12} = PV_2 - PV_1 + U_2 - U_1$

$= (PU_2 + U_2) - (PV_1 + U_1)$

$= H_2 - H_1 = \Delta H$

Supplied Heat is required to change the enthalpy of system.

(3) Specific heat capacity :

$$C = \frac{1}{m}\frac{dQ}{dT}$$

(i) Constant volume

$u = u(T, v)$

$$= \left(\frac{du}{dT}\right)_v dT + \left(\frac{du}{dv}\right)_T dv$$

$$= \left(\frac{du}{dT}\right)_v dT$$

$du = C_v dT$

$$\therefore C_v = \left(\frac{du}{dT}\right)_v$$

(ii) constant pressure.

$h = h(T, P)$

$$= \left(\frac{dh}{dT}\right)_P dT + \left(\frac{dh}{dP}\right)_T dP.$$

$$= \left(\frac{dh}{dT}\right)_P . dT$$

$dh = C_p dT$

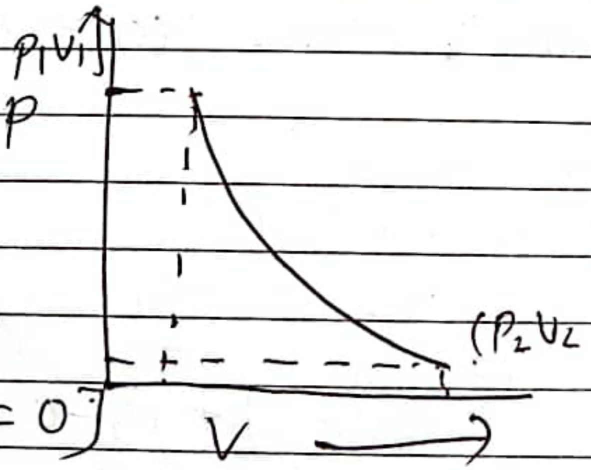
$$\therefore C_p = \left(\frac{dh}{dT}\right)_P$$

(4) Isothermal process

~~(iii)~~ (constant temperature)

$PV = \text{constant}$ (Ideal gas eqn).

$Q_{12} = W_{12} + U_2 - U_1$



we have;

$Q_{12} = W_{12} + U_2 - U_1$

$= W_{12} + C_v dT$

$= W_{12} + C_v(T_2 - T_1)$

$= W_{12} + 0 \quad [\because \Delta T = 0]$

$$\therefore \boxed{Q_{12} = W_{12}}$$

$\cancel{PV = C}$

$\cancel{\int PV = \int C}$

$W_{1-2} = \int_1^2 P\,dV$ $\left[\because PV = C \quad \therefore P = \frac{C}{V}\right]$; $P_1V_1 = P_2V_2$

$W_{1-2} = \int_{V_1}^{V_2} \frac{C}{V} dV$

or $W_{12} = C\int_{V_1}^{V_2} \frac{dV}{V}$

$= C\ln (V)_{V_1}^{V_2}$

$= C\ln(V_2 - V_1) \cancel{+C}$

$= C\ln\left(\frac{V_2}{V_1}\right)$

Hence,

$W_{12} = P_1V_1 \ln\left(\frac{V_2}{V_1}\right)$

$\therefore Q_{12} = P_2V_2 \ln\left(\frac{V_2}{V_1}\right)$

required equation for the isothermal process.

η of carnot engine :)

For Numericals;

(5) Polytropic process: . (Adiabatic) ; $PV^{\gamma} = C$

$PV^n = C$

$$W_{12} = \frac{P_2V_2 - P_1V_1}{1-n} \quad (n \neq 1).$$

$$Q_{12} = W_{12} + U_2 - U_1$$
$$= W_{12} + mC_v dT$$

9 marks

Second Law of Thermodynamics: (Most Imp)

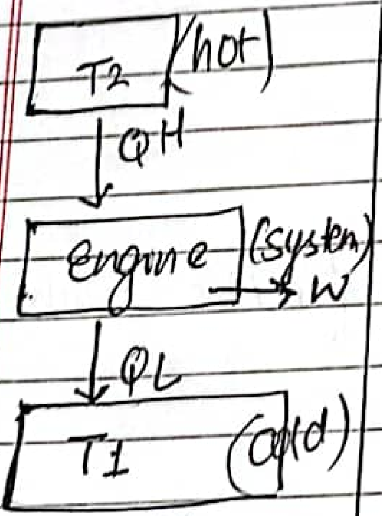
~~(1) Heat pump~~

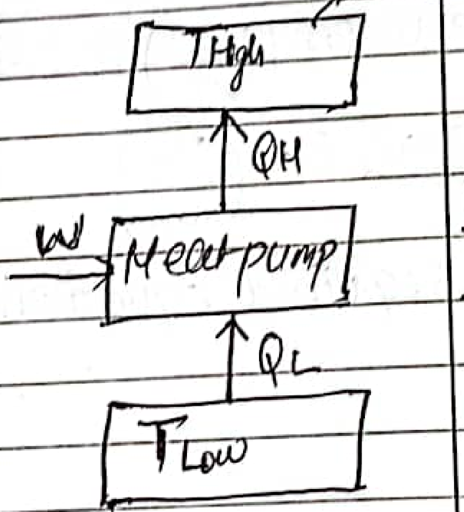
cyclic Heat engine $\boxed{\oint dQ = \oint dW}$

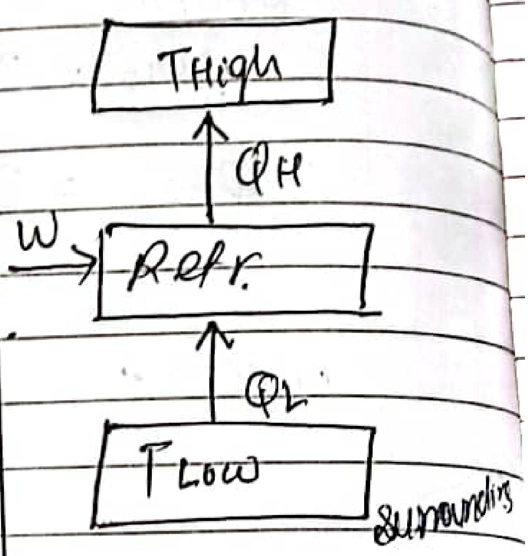
(1) Heat engine (η)

Heat pump (COP)

Refrigerator. (COP)







$T_2 > T_1$

$$\boxed{\eta = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H}}$$

$\oint dQ = \oint dW$

$\therefore W = Q_H - Q_L$

$$\eta = \left(1 - \frac{Q_L}{Q_H}\right) = \left(1 - \frac{T_L}{T_H}\right)$$

$$(COP)_{HP} = \frac{Q_H}{W}$$

$\oint dQ = \oint dW$

$W = (Q_L - Q_H)$

or, $-W = -Q_H + Q_L$

$\therefore W = Q_H - Q_L$

Date ______
Page ______

Reservoir

→ atmosphere, sea, surroundings. no change even after the input or output from them is called reservoir.

For heat pump ;-

$$(COP)_{HP} = \frac{Q_H}{W}$$

$$= \frac{Q_H}{Q_H - Q_L}$$

$$= \frac{1}{1 - \left(\frac{Q_L}{Q_H}\right)}$$

At temp

Reversible pump (ideal) $\frac{1}{\left(1 - \frac{T_L}{T_H}\right)}$

For refrigerator,

$$(COP)_{HP} = \frac{Q_L}{W}.$$

$$= \frac{Q_L}{Q_H - Q_L}$$

$$= \frac{1}{\left(\frac{Q_H}{Q_L} - 1\right)}$$

Reversible ~~eff~~ (Ideal) $\frac{1}{\left(\frac{T_H}{T_L} - 1\right)}$

Components of Refrigerator: (Cyclic)

① Compressor ⇒ 1st step;
high temp ~~low temp~~ vapours enter; it is compressed into liquid.

② Condenser ⇒ cool down ~~the~~ & changes its state.

③ Expansion valve

④ Evaporator:

Statements of Second Law Of thermodynamics:

① Clausius - Statement → Heat Low → High (X) self acting

② Kelvin - Planck statement. → efficiency (00%) impossible

Solved example:

$\frac{X_L}{R}$

Heat Transfer:
Modes
- Conduction
- convection
- Radiation:

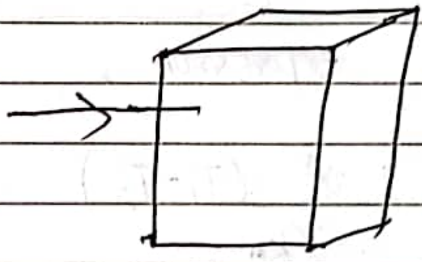
① conduction :- solid
- attached

② convection :- liquid (fluid ⇒ to the surroundings.
No attachment.

③ Radiation :- No necessity of medium

Fourier law of mode of heat transfer :-

One dimensional Steady state Heat Conduction through plane wall:-



$$\frac{dQ}{dT} \propto A\frac{dT}{dx}$$

or, $\frac{dQ}{dT} = \frac{-kA\,dT}{dx}$.

towards the decrease in temperature
k is thermal conductivity of the material.

or, $\frac{J/s}{m^2 \times \frac{K}{m}} = -k$

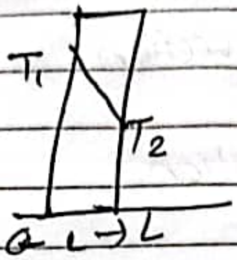
The SI unit of K is;
$k = W/mK$
watt per metre kelvin.

High temp $= T_1$

Low temp $= T_2$

Heat transfer through 1-D steady wall:

Using Fourier's law;



$T_1 > T_2$

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

or, $$\frac{dQ}{dt}dx = -kA\,dT$$

Integrating,

or, $$\int_0^L \frac{dQ}{dt}dx = -kA\int_{T_1}^{T_2} dT$$

$$= -kA(T_2 - T_1)$$

$$= +kA(T_1 - T_2) \quad (\because T_1 > T_2)$$

or, $$\frac{dQ}{dt}\cdot L = kA(T_1 - T_2)$$

or, $$\boxed{\frac{dQ}{dt} = \frac{kA(T_1 - T_2)}{L}}$$ is the required expression:

or, $$\frac{dQ}{dt} = \frac{(T_1 - T_2)}{L/kA} \qquad \left[\because V = IR,\ \frac{V}{I} = R\right.$$

$$= \frac{(T_1 - T_2)}{R_{th}}$$

$\therefore R_{Th} = L/kA$ ($\because$ Thermal resistance)

Convection:

$$\frac{dQ}{dt} = hA(T_S - T_f)$$

$\therefore h = W/m^2K$

$\therefore W = hm^2 \times K$

$T_S > T_f$

T_S = space temp.

T_f = fluid temp.

h = coefficient of conductive heat transfer.

A = exposed Area.

Radiation:

(Stefan's law of black body radiation)

$$\frac{dQ}{dt} = \epsilon\sigma A(T_1^4 - T_2^4) \cdot \begin{pmatrix} \epsilon < 1 \\ T_1 > T_2 \end{pmatrix}.$$

$\Rightarrow \sigma \Rightarrow$ Stefan's coefficient.

$\epsilon \Rightarrow$ emissivity.

$W = \sigma m^2 K^4$

$\therefore \sigma = W/m^2K^4$

Emmissive power.

$\frac{dQ}{dt} \Rightarrow \sigma T^4$

Rate of heat transfer;

Numerical Problems:

Q.no.1.

(a) $dT = 24°K;\ T_1 - T_2 = 24°$.

$l = 3cm = 0.03m$

~~h=80~~

$\frac{1}{A}\frac{dQ}{dt} = 80 W/m^2$

So,

$$\frac{dQ}{dt} = KA\frac{dT}{dl}$$

or, $$\frac{dQ}{dt} = \frac{KA(T_1 - T_2)}{L}$$

or, $$\frac{dQ}{dt} \cdot \frac{1}{A} = K\left(\frac{24}{0.03}\right)$$

or, $$\frac{80}{0.03} \cancel{\times 24} = \cancel{K}$$

$\therefore$ ~~K=~~

or, $$\frac{80}{800} = K$$

$\therefore K = 0.1 W/m^2K$ //

(b) ⇒ $A = 0.8m^2$

$l = 5cm = 0.05m$

$K = 0.25 W/mK$

$\frac{dQ}{dt} = 1600 W$

$T_1 - T_2 = ?$

(5) ⇒

$d = 2mm = 2\times10^{-3}m$; $A = \frac{\pi d^2}{4} =$

$l = \cancel{800}\ 8cm = 0.08m$

$h = 4500\ W/m^2K$.

Area of the wire (A) = $2\pi r l$

$= 2\pi\times10^{-3}\times0.08$

$= 5.02\times10^{-4}$.

$T_s = 120°C = (120+273)K$

$T_f = 100°C = (100+273)K$

$T_s - T_f = 20°K$

$\frac{dQ}{dt} = ?$

We know,

$\frac{dQ}{dt} = hA(T_s - T_f)$

$= 4500\times5.02\times10^{-4}(20)$

$= 45.18W$ //.

(9) ⇒ $d = 4cm = 0.04m$

$r = 0.02m$

$T_2 = 25°C$, $T_1 = 80°C$

$\varepsilon = .08$

$h = 10W/m^2K$

$\sigma = 5.67\times10^{-8}\ W/m^2K^4$.

$T_2 = (25+273)K = 298K$

$T_1 = 80+273 = 353K$

$l = 1m$.

$A = 2\pi r l$

$= 2\times\pi\times0.02\times1$

$= \pi/25\ m^2$.

Total heat loss from the unit length of the pipe (H) = ?.

$$\left(\frac{dQ}{dt}\right)_{conv} = hA(T_1 - T_2) \quad \text{——— (1)}$$

$$\left(\frac{dQ}{dt}\right)_{radiation} = \varepsilon \sigma A(T_1^4 - T_2^4) \quad \text{——— (2)}.$$

Total heat transfer $= hA(T_1 - T_2) + \varepsilon\sigma A(T_1^4 - T_2^4)$

$$= 10 \times \frac{\pi}{25}(55) + 0.8 \times 5.67 \times 10^{-8} \times \left(\frac{\pi}{25}\right)\left(7.6 \times 10^{9}\right)$$

$$= 112.67\ W/m$$

(a) For constant pressure work done $(W_{12}) = P(V_2 - V_1)$

(b) For constant temp, $W_{12} = P_1 \cancel{V_2} \ln\left(\frac{V_2}{V_1}\right)$

$$= PV_1 \ln\left(\frac{V_2}{V_1}\right)$$

$$= 800 \times 10^3 \times 0.1 \times \ln\left(\frac{0.5}{0.1}\right)$$

$$= 128755.03 J$$

$$= 128.03 KJ$$

(c) $PV^2 =$ constant:

$P_1 V_1^2 = P_2 V_2^2$ [Both sides ln].

or, $P_2 = \frac{P_1 V_1^2}{V_2^2} = 32000\ Pa$

$= 32 kPa$

$W = \cancel{P_2 V_2 - P_1 V_1}$

$= \cancel{32 \times 0.5}$

$= \cancel{64 KJ}$

$$W = \frac{P_2V_2 - P_1V_1}{n-1}$$

$$= \frac{64 - \cancel{800 \times 0.1}}{\cancel{1}} \qquad \frac{P_2V_2 - P_1V_1}{n-1 \text{ or } (1-n)}$$

$$= 64 - \qquad = \frac{16-80}{\cancel{2-1}\ 1-2}$$

$$= 64\,\text{kJ}$$

(5) ⇒

$\dot{W} = 1\,\text{kW}$

$= 1000\,\text{W}$

$T_1 = 0^\circ C = 273\,\text{K}$

$T_2 = 20^\circ C = 293\,\text{K}$

$dQ_H = \frac{1000\,\text{kJ}}{\text{min}}$

$= \frac{1000 \times 10^3}{60}$

$= 16666.67\,\text{J/sec}$

$$(COP)_{pump} = \frac{1}{1 - \frac{Q_L}{Q_H}} = \frac{\dot{Q}_H}{\dot{W}}$$

$$= \frac{1000\,\text{kJ/min}}{1\,\text{kW}} = 16.67$$

$$(COP)_{rev} = \frac{1}{1 - \frac{T_L}{T_H}} = \frac{1}{1 - \frac{273}{293}} = 14.65$$

$(COP)_{pump} > (COP)_{rev}$

$16.67 > 14.65$ His claim was invalid.

(11) $\Rightarrow$ $T_L = -20^\circ C = 253 K$

$T_H = 22^\circ C = 295 K$

$\dot{Q}_L = 2.5 kW = 2.5 \times 10^3 W$

we know,

$$(COP)_{ref} = \frac{Q_L}{W}$$

$$= \frac{\dot{Q}_L}{\dot{W}}$$

$$= \frac{2.5 \times 10^3}{}$$

$$\text{or, } \frac{1}{\left(\frac{T_H}{T_L} - 1\right)} = \frac{2.5 \times 10^3 W}{\dot{W}}$$

$$\text{or, } \frac{1}{\frac{295}{253} - 1} = \frac{2.5 \times 10^3}{\dot{W}}$$

$$\therefore \dot{W} = \frac{2.5 \times 10^3}{}$$

$$\text{or, } \frac{253}{42} = \frac{2.5 \times 10^3}{\dot{W}}$$

$$\text{or, } \dot{W} = \frac{2.5 \times 10^3 \times 42}{253}$$

$$= 415.01 W$$

$$= 0.415 kW.$$

$m = 5kg$
$A = 30 cm^2$.

$P = \frac{F}{A} = \frac{mg}{A}$

Q no 5 :

$P_{atm} = 100 kPa$
$= 100 \times 10^3 Pa$.

Pressure due to piston $= \frac{F}{A} = \frac{mg}{A} = \frac{5 \times 9.79}{\frac{30}{100 \times 100} m^2}$
$= 16316.66$
$= 16.31 kPa$.

Pressure due to the spring $= \frac{F}{A} = \frac{100N}{\frac{30}{100 \times 100}} = 33.33 kP$

Total pressure inside the cylinder $= 100 + 16.31 + 33.3$
$= 149.64 kPa$

The laser source is so constructed that all the light radiations ~~flours~~ produced by individual atoms. In each are in mutual agreement not only in phase but also in plane of polarization, & direction of emission. Therefore, the laser light is highly monochromatic, coherent & directional and having large intensity.

The word 'LASER' stands for "Light Amplification by stimulated ~~Emmi~~ Emission of Radiation."

Principle of generation of laser light:

(1) Induced absorption:

An atom at ground state (ε_1) can be raised to a higher energy state (ε_2) by imparting it with an external photon of energy ($\varepsilon_2 - \varepsilon_1 = hf$). The incident photon is absorbed by the atom & the atom gets excited to higher energy state. This phenomenon is called induced absorption.

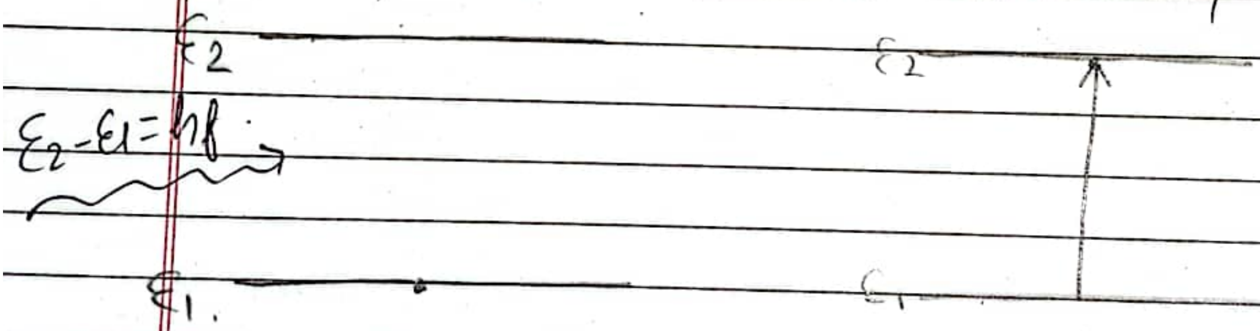


fig: induced absorption

The rate of induced absorption is given by:

$$\boxed{R_{abs} = B_{12} \rho N,}$$

where

B_{12} = Einstein's coefficient for induced absorption.

ρ = energy density of incident radiation.
N_1 = no. of atoms at ground state.

② Spontaneous emission :-

An atom cannot remain for long time in ~~the~~ excited state (E_2). Within the time interval of 10^{-8} sec, the it returns to ground state (E_1) with the emission of photon of energy ($E_2 - E_1 = hf$). This phenomenon is called spontaneous emission.

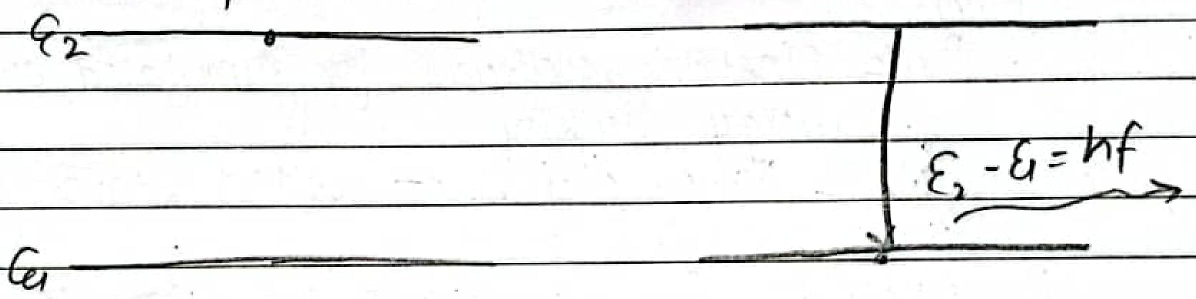


fig: spontaneous emission.

The rate of spontaneous emission is given by:-

$\boxed{R_{sp} = A_{21} N_2}$

where A_{21} = Einstein's coefficient ~~of~~ for spontaneous emission
N_2 = No. of atoms at excited state.

③ Stimulated emission:

An atom at excited state (E_2) can be returned to ground state (E_1) before spontaneous emission by imparting it with an external force of photon of energy ($E_2 - E_1 = hf$). In this case, an extra radiation is produced ~~by~~ along with the incident one. This phenomenon is called stimulated emission.

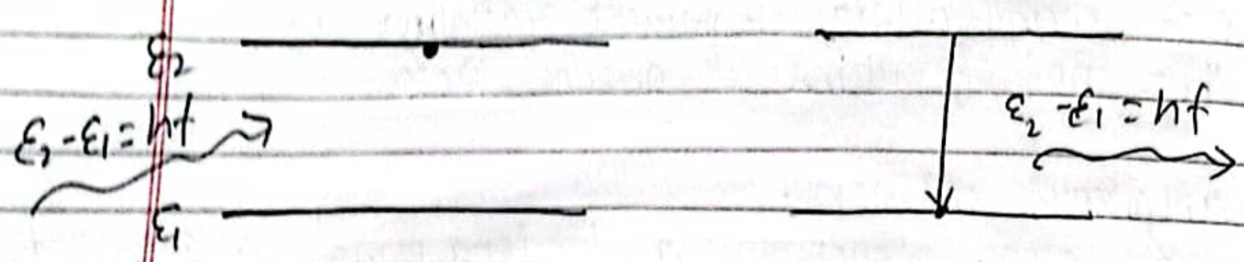


fig: stimulated emission.

The rate of stimulated emission is given by;

$R_{st} = B_{21} \rho N_2$

where,

B_{21} = Einstein's coefficient for stimulated emission

ρ = energy density

N_2 = No. of atoms at excited state.

(4) Population inversion, pumping and metastable states:

For lasing action, stimulated emission is necessary process. To carry out the stimulated emission, large number of atoms should be at excited state. The state of being large number of atoms at excited state than at ground state is known as population inversion.

The process which is applied to achieve population inversion is called pumping. where it is done by light energy, it is called optical pumping, & when it is done by chemical energy, it is called chemical pumping & so on. It is ~~also~~ observed that some higher energy states ~~are~~ of few elements have larger life time of about 10^{-4} sec. Those energy states are called metastable states.

In metastable states, the population inversion & hence, stimulated emission are possible.

Semi- Conductor Laser :-

Semi-conductors lasers are basically PN junction diodes. When a p-type semi-conductor comes into ~~int~~ immediate contact to an n-type semi-conductor, a PN junction forms at the interface.
When they collide, they neutralise each other and emit recombination radiation. For a semi-conductor laser, the energy of the photon emitted as recombination radiation is equal to the band gap of the material. The active medium of the semi-conductor laser is PN-junction. In this laser, mirror is not used as in other resonator or cavity for optical feedback to sustain laser osullation.

He – Ne laser:

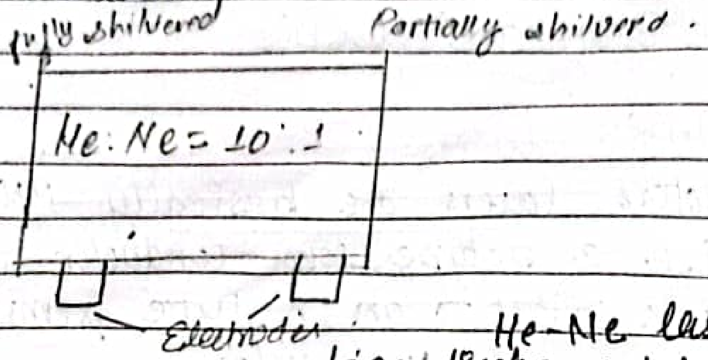


fig: He-Ne laser ~~discharge tube~~:

He-Ne consists of a discharge tube provided with a fully shilvered mirror and a partially shilvered mirror at 2 inch. The mixture of He & Ne gas in the ratio of 10 : 1 is kept inside the tube. When a large potential of about 10kV is applied to electrodes the gas inside a tube gets discharged & the +ve & -ve ions are produced. These ions accelerated towards corresponding electrode with large speed. These ions during their acceleration towards corresponding electrode collide with He atoms & the He-atoms get excited to 2s-energy level. The 2s-energy state is a metastable state for He with energy 20.61 eV. Before spontaneous emission, the He atoms transfer their energy of 20.61 eV along with K.E of 0.05 eV to Ne-atoms. With this energy, the Ne-atoms get excited to 5s-energy level. 5s-energy state is also a metastable state for Ne-atoms in which population inversion can be achieved. The stimulated emission carried out betn the 5s and 3p energy states of Ne atoms produced laser light of wavelength 6328 A°.

After the intensity has reached to an optimum value, a highly monochromatic, coherent, intensed beam of laser light is emerged out by penetrating the partially shilvered mirror.

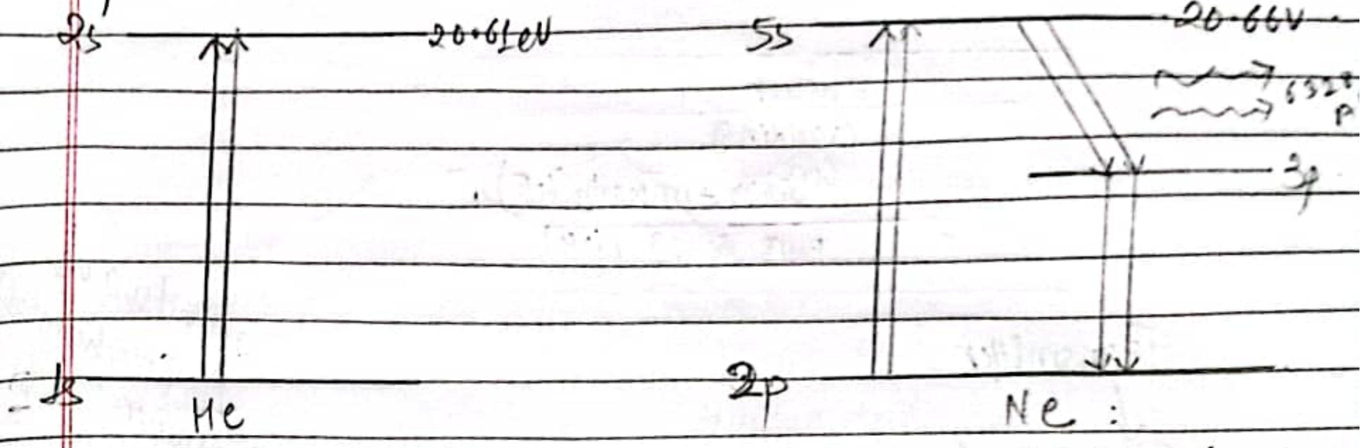


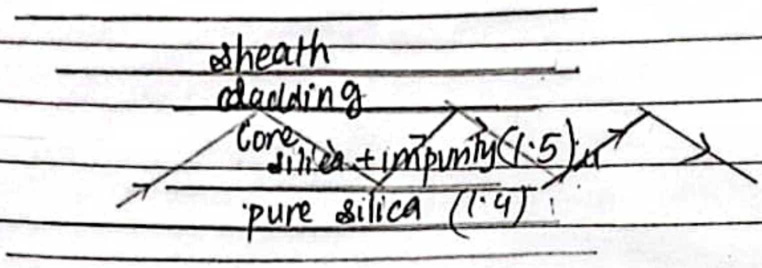
fig: Energy level diagram for He-Ne laser

Uses of laser:
OR
Applications of laser:

- Cutting & drilling of hard materials such as glass, diam
- In scientific research such as Raman spectroscopy, nuclear reactions, astronomy etc.
- to separate isotopes of an element.
- Medical: treatment of cancer, blood less surgery. ~~detached~~ to join detached retina.
- modern electronic devices such as CD-player, laser printe memory cards are based on the principle of laser.
- Used in wars to target missile,
- Holography and fibre optics are the result of laser technology.

Optical fibres:

Construction:



Transmitter

↓

Transducer (mechanical → electrical)

↓

modulation -

(data + light)
(current)

copper cable / Fibre cable

Transmission chanel :

Receiving → (current) (light) → Reject.

Receiving → demodulation → sound accept → Transducer → soundwave (mechanical)

~~Appli~~ Advantages:

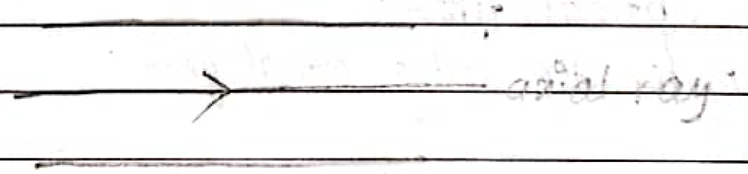
1. The rate of data transmission is very high. i.e. 'c'
2. The band width of light wave is large. So huge amount of data can be transmitted.
3. It is more safer than traditional communication.
4. security purposes.
5. Bad conductor of heat & electricity. so no corrosion & maintained for large time.
6. Light is light in weight.

Types of optical fibre.

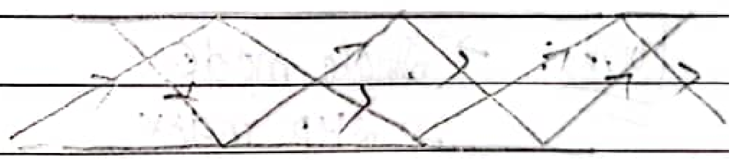
On the basis of modes of propagation, there are 2 types of optical fibre.

1. Single mode optical fibre.

→ Optical fibre having only one mode of propagation is called single mode optical fibre.



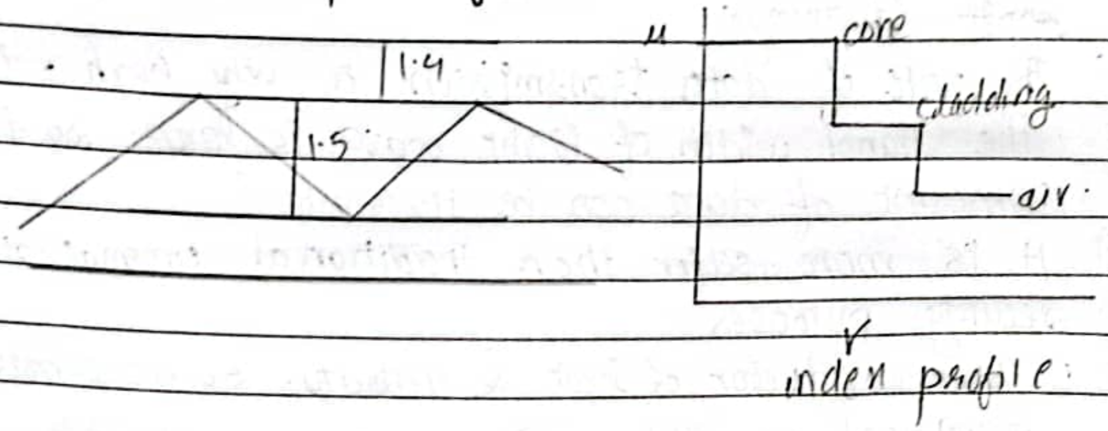
2. Multi mode optical fiber:



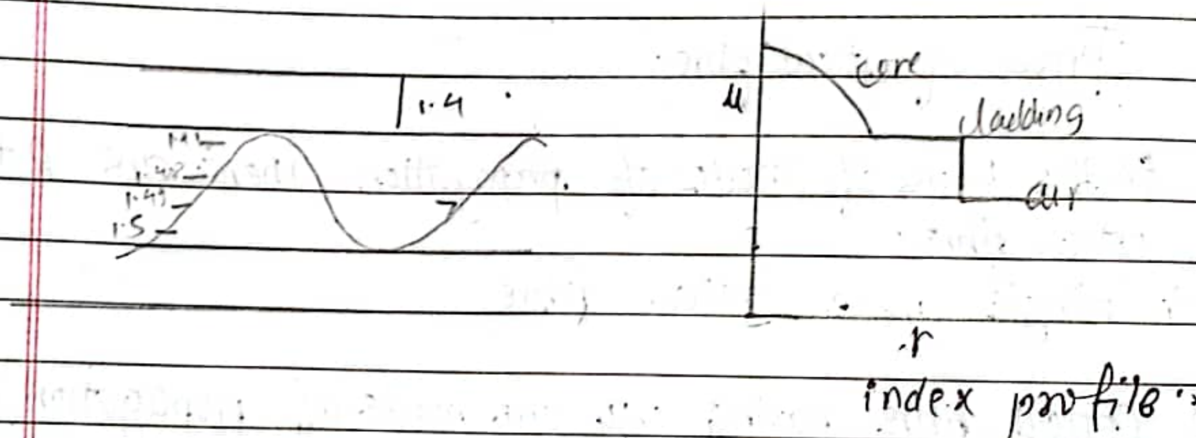
- Optical fibre having ~~th~~ two or more than modes of propagation is called multimode optical fiber.

On the basis of index profile, there are two types of optical fiber:

① Step index optical fiber:



② Graded index optical fibre:



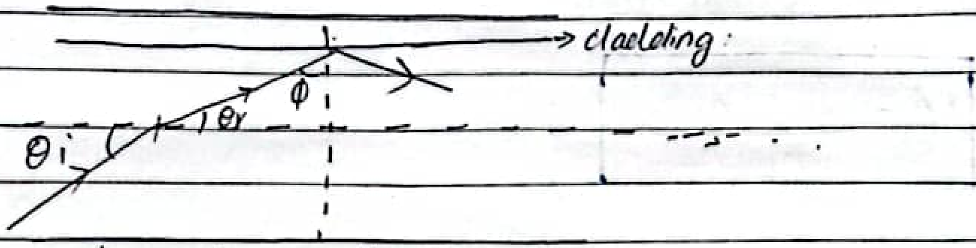
Exam P.O.V: Types of optical fiber:

① Single mode step index optical fibre
② Multi " " " "
③ " " graded " " "

Points: → Single mode
→ step index
→ Figure, → Transmission of scattering.

Principle of propagation of light wave through optical fibre: (Acceptance angle):

$n_0 \rightarrow$ refractive index of air
$n_1 \rightarrow$ refractive index of core
$n_2 \rightarrow$ refractive index of cladding.



$$\theta_r + \phi + 90^\circ = 180^\circ$$

$$\therefore \phi = 90 - \theta_r \ / \ \theta_r = 90 - \phi$$

cos-core

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{n_1}{n_0}$$

$$\sin\theta_i = \frac{n_1}{n_0} \cdot \sin\theta_r$$

or, $$\sin\theta_i = \frac{n_1}{n_0} \sin(90-\phi)$$

$$\boxed{\therefore \sin\theta_i = \frac{n_1}{n_0}\cos\phi}$$

When $\theta_i = \theta_{imax}$, $\phi = \phi_c$

$$\sin\theta_{imax} = \frac{n_1}{n_0}\cos\phi_c \quad \cdots (1)$$

$$\cos\phi_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

or, $$\sin\theta_{imax} = \frac{n_1}{n_0}\sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

$$= \frac{\cancel{n_1}}{n_0}\frac{\sqrt{n_1^2 - n_2^2}}{\cancel{n_1}}$$

core-cladding

when $\theta_i = \theta_{imax}$, then $\phi = \phi_c$

$$\frac{\sin\phi_c}{\sin 90^\circ} = \frac{n_2}{n_1}$$

or, $$\sin\phi_c = \frac{n_2}{n_1}$$

$$\therefore \cos\phi_c = \sqrt{1 - \sin^2\phi_c}$$

$$= \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

Date
Page

or, $\sin\theta_{imax} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$

Acceptance angle;

$$\therefore \boxed{\theta_{imax} = \sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right)}$$

Numerical Aperture (NA) : It is defined as sine of angle of acceptance.

$$\therefore \boxed{NA = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}}$$

optics - ref.

Optical fibre sensors:

→ Thermometer
→ pollution detection.
→ To measure the depth of water level in well, sea etc.

A glass clad fibre is made with core glass of refractive index 1.5 and cladding is dopped to give a fractional index difference of 5×10^{-4} determines (i) cladding index
(ii) critical internal reflection angle
(iii) The external critical acceptance "
(iv) Numerical Aperture.

Solⁿ: ~~Ans~~ = $n_1 = 1.5$

$\Delta = 5\times10^{-4}$

$\Delta = \frac{n_1 - n_2}{n_1}$.

or, $5 \times 10^{-4} = \frac{1.5 - n_2}{1.5}$

or, $7.5 \times 10^{-4} = 1.5 - n_2$

$\therefore n_2 = 1.49$ //.

(ii)

$\sin\phi_c = \frac{n_2}{n_1}$

or, $\phi_c = \sin^{-1}\left(\frac{1.49925}{1.5}\right)$

$= \cancel{87.907^\circ}$ // . 88.18°

(iii) $\theta_{imax} = \sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right)$

$= \sin^{-1}\left(\frac{\sqrt{(1.5)^2 - (1.499)^2}}{n_0}\right) \left[\because n_0 = 1\right]$

$= \sin^{-1}(0.05474)$

$= \cancel{3.139^\circ}$ //. 2.71 //.

(iv) $N \cdot A = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$

$= \cancel{0.0547}$ // 0.0474.

(2)

Soln:

$\Delta = 0.19$

$n_2 = 1.58$

$\Delta = \frac{n_1 - n_2}{n_1}$

or, $0.19 = \frac{n_1 - \cancel{0.19}\ 1.58}{n_1}$

or, $0.19 \times n_1 = n_1 - 1.58$

or, $\cancel{0.19 n_1}\ 1.58 = n_1 - 0.19 n_1$

or, $\frac{1.58}{0.81} = n_1$

$\therefore n_1 = 1.9506$ //

$n_0 = 1.33$

Or, $\theta_{imax} = \sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right)$

$= \sin^{-1}\left(\frac{\sqrt{1.95^2 - 1.58^2}}{1.33}\right)$

$= 59.23^0$ //

(3)

$\Delta = \frac{n_1 - n_2}{n_1}$ —— (1)

$NA = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$ —— (2)

from eqn (1),

$0.0192 = \frac{n_1 - n_2}{n_1}$

Q.21 :

$NA = 0.21.$

or, $n_1 \sqrt{2\Delta} = 0.21.$

$\therefore n_1 = \dfrac{0.21}{\sqrt{2\Delta}}$

$= 1.0716$

Given: $\Delta = 0.0192$

$\dfrac{n_1 - n_2}{n_1} = 0.0192$

or, $n_1 - n_2 = 0.0192$

or, $1.0716 - n_2 = 0.0192$

$\therefore n_2 = 1.052$ //.

Exercise:-

① Write the principle of optical fibre.

F → electric field
$\mathcal{E}$ = emf
ϵ = permittivity

$L \rightarrow \frac{L\,dI}{dt} = V_L$
$C \rightarrow q/C = V_C$
$R \rightarrow IR = V_R$

$\left[\begin{array}{l} q = CV \\ q = C\mathcal{E} \\ q_0 = \cancel{C\mathcal{E}}\ \mathcal{E}C \end{array}\right]$

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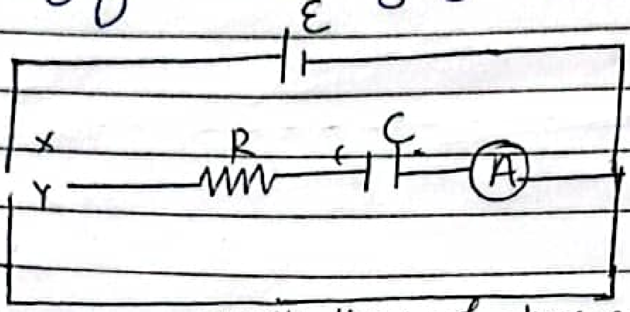
Charging & discharging of capacitor:



circuit diagram of charging & discharging of capacitor.
Fig. 1

Let a capacitor of ~~capict~~ capacitance 'C' is collected with a resistor of resistance R, ammeter (A), a battery of emf '$\mathcal{E}$' and with a two-way key (K) as shown in fig. 1.

When the switch X is on, the charging of capacitor takes place. The positive and negative charges deposited on a plates of capacitor oppose the other flow of current. And, the current goes on diffusing. When the capacitor is fully charged, the current through the circuit finally drops to zero. Applying Kirchhoff's voltage rule on the upper part of circuit of fig. 1.

$$\mathcal{E} = V_R + V_C$$
$$= IR + q/C.$$

or, $\mathcal{E} = \cancel{IR}\ R\frac{dQ}{dt} + q/C$ $\left[\because I = \frac{dQ}{dt} \text{ acc}^n \text{ to def}^n\right]$.

or, $\mathcal{E} - q/C = R\frac{dQ}{dt}$.

or, $\frac{\mathcal{E}C - q}{C} = \frac{RdQ}{dt}$.

or, $\frac{q_0 - q}{C} = \frac{RdQ}{dt}$

$q = VC$
$q_0 = \varepsilon C$

or, $q_0 - q = RC\dfrac{dq}{dt}$ ——— (4).

where $q_0 = \varepsilon C$ is the maximum amount of charge that can be stored in the capacitor.

$$\frac{dq}{q_0 - q} = \frac{1}{RC}dt$$

Integrating;

$$\int_0^q \frac{dq}{q_0 - q} = \int_0^t \frac{1}{RC}dt$$

$$= \int_0^q \frac{-dq}{q_0 - q} = \frac{-1}{RC}\int_0^t dt.$$

or, $-\ln(q_0 - q)\Big|_0^q = \dfrac{t}{RC}.$

or, $\ln(q_0 - q)\Big|_0^q = \dfrac{-t}{RC}.$

or, $\ln(q_0 - q)\Big|_0^q = \dfrac{-t}{RC}$

or, $\ln(q_0 - q) - \ln(q_0) = \dfrac{-t}{RC}$

or, $\ln\left(\dfrac{q_0 - q}{q_0}\right) = \dfrac{-t}{RC}$

Taking anti-log on both sides,

or, $\dfrac{q_0 - q}{q_0} = e^{-t/RC}.$

or, $q_0 - q = q_0 e^{-t/RC}$

or, $q = q_0 - q_0 e^{-t/RC}$

$\therefore q = q_0(1 - e^{-t/RC})$ ......(1) which is the required equation for charging of capacitor //

According to eq^n ①, the variation of charge with time in charging circuit as shown in figure ②.

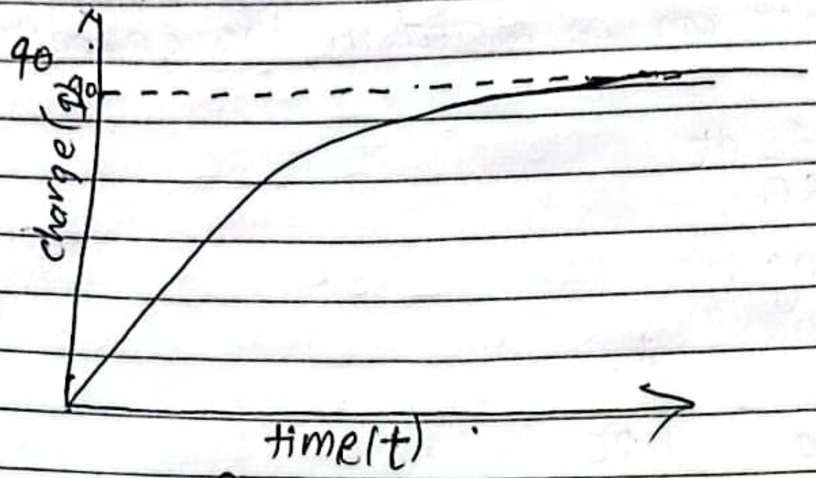


fig. 2 :: Variation of charge with time in charging circuit.

Differentiating eq^n ① with respect to time, we get,

$$\frac{dq}{dt} = \frac{d}{dt}\left(q_0 - q_0 e^{-t/RC}\right)$$

$$I = -q_0 e^{-t/RC}\left(\frac{-1}{RC}\right)$$

$$= \frac{\mathcal{E}C}{RC} e^{-t/RC}$$

$$= \mathcal{E}/R \, e^{-t/RC}$$

or, $I = I_0 e^{-t/RC}$ $\left[\because I_0 = \mathcal{E}/R\right]$

$\therefore$ $I = I_0 e^{-t/RC}$ ..... ② which is the charging eq^n of circuit in terms of charge.

According to eq^n ②, the variation of current with respect to time as shown in fig. 3.

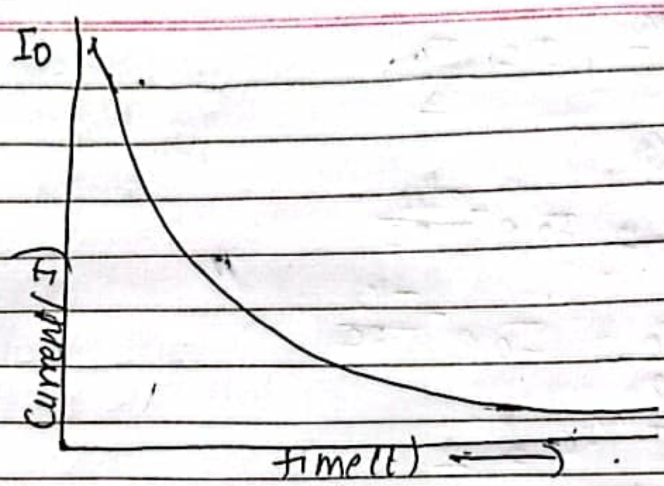


fig. 3.
Variation of current with time 't' in charging circuit.

When $t = RC$

from eq^n (1),

$$q = q_0(1 - e^{-1})$$

$$= q_0\left(1 - \frac{1}{e}\right) = q_0\left(1 - \frac{1}{e}\right)$$

$$\text{or, } q = q_0\left(\frac{e-1}{e}\right) = 0.639q_0$$

$$= 63\% \text{ of } q_0.$$

$$\text{or, } qe = q_0e - q_0$$

$$\text{or, } q_0 = q_0e - qe$$

$$q_0 = e(q_0 - q)$$

$$\therefore e = \frac{q_0}{q_0 - q}$$

Hence, capacitive time constant for the charging circuit is defined as the time required to charge a capacitor ~~of~~ upto 63% of its maximum charge.

The half life time is defined as the time at which the current in the circuit becomes half of the maximum current.

When $t = T^{1/2}$, $I = \frac{I_0}{2}$

From eq^n (2),

$$\frac{I_0}{2} = I_0 e^{-\frac{T^{1/2}}{RC}}$$

$$\text{or, } \frac{I_0}{2} = I_0 e^{-\frac{T^{1/2}}{RC}}$$

$$\text{or, } e^{T^{1/2}/RC} = 2$$

Taking log on both sides,

$$\frac{T^{1/2}}{RC} = \ln(2)$$

$$T^{1/2} = \ln(2) \times RC$$

$$\therefore C = \frac{T^{1/2}}{0.693R} \quad \text{———} \quad (3)$$

The potential drop across the capacitor is given by;

$$V_C = \frac{q}{C}$$

$$\text{or, } V_C = \frac{q_0}{C}\left(1 - e^{-t/RC}\right)$$

$$= \frac{\mathcal{E}C}{C}\left(1 - e^{-t/RC}\right)$$

$$V_C = \mathcal{E}\left(1 - e^{-t/RC}\right) \quad \text{———} \quad (4)$$

Therefore, the variation of V_C with time is similar to those q and t as shown in fig. 2. ~~Again~~,

Again; the potential across the resistor is given by;

$V_R = IR$

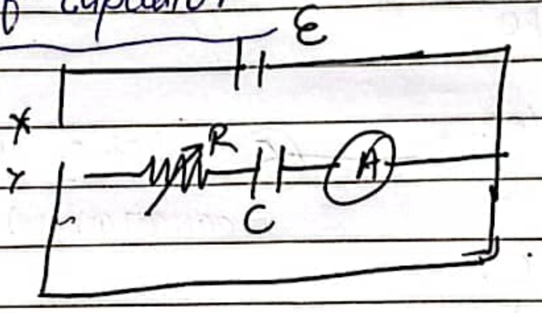
$V_R = R I_0 e^{-t/RC}$

$V_R = \frac{R\varepsilon}{R} e^{-t/RC}$

$\therefore V_R = \varepsilon e^{-t/RC}$ . . . . . . (5)

Therefore, the variation of V_R with time is similar ~~to those~~ as that of I & t as shown in fig: 3.

Discharging of capacitor:



When the switch Y is on, the discharging of capacitor to resistor takes place. Applying Kirchoff's voltage rule on the lower half of the circuit.

$$0 = V_C + V_R \quad [\because \varepsilon = 0]$$

$$\text{or, } V_C = -V_R$$

$$\text{or, } \frac{q}{C} = -IR$$

$$\text{or, } \frac{q}{C} = -\frac{dq}{dt} R$$

$$\text{or, } \frac{dq}{dt} = \frac{RC}{-q}$$

$$\therefore \frac{dq}{q} = -\frac{1}{RC} dt \quad \text{—— (6)}$$

Integrating;

$$\int_{q_0}^{q} \frac{dq}{q} = \frac{-1}{RC}\int_0^t dt$$ (Limit (+) → (-) Discharging)

$$\ln(q)_{q_0}^{q} = \frac{-t}{RC}$$

$$\ln q - \ln q_0 = -t/RC$$

$$\ln\left(\frac{q}{q_0}\right) = -t/RC$$

Taking anti-log on both sides,

$$\frac{q}{q_0} = e^{-t/RC}$$

$$\therefore q = q_0 e^{-t/RC} \quad \text{——— (6)}$$ which is the discharging equation in terms of charge.

when $t = RC$;

$$q = q_0 e^{-1}$$

$$q = \frac{q_0}{e}$$

$$\therefore q_0 = 0.37 q_0$$

$$q = 37\% \text{ of } q_0$$

Hence, capacitive time constant for discharging of circuit is defined as the time required to discharge a capacitor upto 37% of its initial value.

Differentiating eqⁿ (6) with respect to time,

$$\frac{dq}{dt} = q_0 e^{-t/RC}\left(\frac{-1}{RC}\right)$$

$$= -\frac{\varepsilon C}{RC} \cdot e^{-t/RC}$$

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$I = \frac{-\mathcal{E}}{R} e^{-t/RC}$

$\therefore I = -I_0 e^{-t/RC}$ —— (7).

(-ve) charge

(Q) The capacitor of capacitance 'C' through resistor R. Calculate the time at which which the potential across the resistor is equal to potential across capacitor.

$V_C = V_R$

or, $\frac{q}{C} = IR$

or, $\frac{q}{C} = \frac{dq}{dt} R$ $\qquad$ or, $\frac{q_0 e^{-t/RC}}{C} = \frac{dq}{dt} R.$

or, $\cancel{q} \frac{q}{CR} \cdot \frac{dq}{dt} R = \frac{q}{C}$ $\qquad$ or, q_0

or, $\frac{dq}{q} = \frac{dt}{RC}$

or, $\int_0 \frac{dq}{q} = \int \frac{1}{RC} \int dt.$

or, $(\ln q)_0^q = \frac{1}{RC} \int_0^t dt.$

or, $(\ln q)_0^q = \frac{t}{RC}$

or, $\ln q - \ln(0) = \frac{t}{RC}.$

or, $\therefore t = R(\ln(q) - 1$

$V_C = V_R$.

or, $\frac{q}{C} = IR$

or, $\frac{q_0(1-e^{-t/RC})}{C} = R\cdot I_0 e^{-t/RC}$.

or, $q_0 - q_0 e^{-t/RC} = RC\cdot I_0 e^{-t/RC}$.

or, $q_0 = (RCI_0 + q_0) e^{-t/RC}$.

or, $\left(\frac{q_0}{q_0 + RCI_0}\right) = e^{-t/RC}$

$e^{t/RC} = \frac{q_0 + RCI_0}{q_0}$

$e^{t/RC} = 1 + \frac{RCI_0}{q_0}$

or,

or, $\mathcal{E}(1-e^{-t/RC}) = \cancel{R}\cdot\frac{\mathcal{E}}{\cancel{R}} e^{-t/RC}$

or, $\mathcal{E} - \mathcal{E}e^{-t/RC} = \mathcal{E}e^{-t/RC}$

or, $\mathcal{E} = \mathcal{E}e^{t/RC} + \mathcal{E}e^{t/RC}$.

or, $1 = 2e^{t/RC}$

or, $e^{t/RC} = 1/2$

$t/RC = \ln(1/2)$

or, $\frac{t}{\cancel{RC}} = \ln(1/2)\cdot RC$.

or, $t =$

A capacitor of capacitance 'C' is discharged through a resistor of resistance 'R' after How many time constant (RC) is the stored energy becomes one-fourth of its initial value?

Acc^n to qn, $U = U_0/4$.

Sol: $U = \frac{q^2}{2C}$ $\therefore U_0 = 4U$

$= \frac{\left(q_0 e^{-t/RC}\right)^2}{2C}$

$= \cancel{\frac{\varepsilon^2 C^2 e^{-\frac{2t}{RC}}}{2C}}$ $\frac{q_0^2 e^{-2t/RC}}{2C}$ $\left[\because U_0 = \frac{q_0^2}{2C}\right]$

$= U_0 e^{-2t/RC}$

$\cancel{U} = 4\cancel{U} e^{-2t/RC}$

$\therefore e^{2t/RC} = 4$

$\therefore \frac{2t}{RC} = \ln(4)$

$\therefore t = \frac{\ln(4)}{2} \times RC$

$= 0.693RC$.

20. Obtain the charging time constant of a capacitor in a RC circuit such that current through the resistor is decreased by 50% of its peak value in 5 seconds.

Sol^n; $I = \frac{I_0}{2}$ $\therefore I = 50\%$ of I_0.

& $t = 5sec.$

We have,

$$I = I_0 e^{-t/\tau}$$

$$\text{or, } \frac{I}{I_0} = e^{-t/\tau}$$ (First exponentially +ve).

Taking ~~anti~~ log on both sides,

$$\ln\left(\frac{I}{I_0}\right) = \frac{-t}{\tau}$$

$$\text{or, } \ln\left(\frac{1}{2}\right) = \frac{-t}{\tau}$$

$$\text{or, } \ln\left(\frac{1}{2}\right) = \frac{-5}{\tau}$$

$$\therefore \tau = \frac{-5}{\ln\left(\frac{1}{2}\right)} = 7.21 \text{ sec.}$$

~~$\therefore RC =$~~

$$\text{or, } e^{t/RC} = \frac{I_0}{I}.$$

$$\text{or, } \frac{t}{RC} = \ln\left(\frac{I_0}{I}\right)$$

$$\text{or, } RC = \frac{5}{\ln(2)} = 7.21 \text{ sec.}$$

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Dielectric :

A dielectric is an insulating material which does not conduct electricity. Eg. rubber, plastic, wood etc.

dielectric constant (K) :

It is defined as the ratio of capacitance of a capacitor with dielectric to the ~~capacitance~~ capacitance without dielectric. i.e,

$$K = \frac{C}{C_0} \quad \text{i.e,} \quad C = C_0 K \quad \text{——— (1)}.$$

$\therefore \cancel{C_0 = C/K}$

Therefore, when a dielectric of dielectric constant (K) is inserted between the ~~periods~~ plates of a capacitor, its capacitance increases by 'K' times.

i.e. Since, $C = \frac{q}{V}$ & $C_0 = \frac{q}{V_0}$

From eqⁿ (1),

$$C = C_0 K \qquad \frac{q}{V} = \frac{q}{V_0} \cdot K$$

$$\text{or, } V = \frac{V_0}{K} \quad \text{——— (2)}$$

$$\therefore K = \frac{V_0}{V}$$

Again, $\cancel{V = V_0 K}$ $\quad V = V_0/K$

$$\text{or, } E \cdot d = \frac{E_0 d}{K} \quad \text{——— (3)}$$

$$U = \frac{q^2}{2C}$$

$$\text{or, } U = \frac{C^2V^2}{2C}$$

$$= \frac{1}{2} CV^2$$

$$U = \frac{1}{2}(C_0 K)\left(\frac{V_0}{K}\right)^2$$

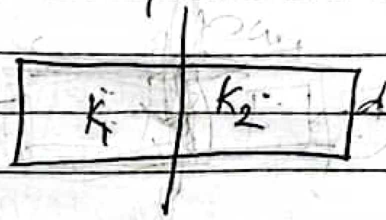
$$U = \frac{1}{K}\left(\frac{1}{2} C_0 V_0^2\right)$$

$$\therefore U = \frac{U_0}{K} \quad \cdots\cdots (4) \; //$$

Numericals:

A parallel plate capacitor is filled with 2-dielectric as shown in figure. Show that the capacitance is given by

$$C = \frac{\epsilon_0 A}{d}\left(\frac{K_1 + K_2}{2}\right)$$



For dielectric k_1,

$$C_1 = \cancel{\frac{\epsilon_0 A}{d}} \quad \frac{\epsilon A/2}{d} = \frac{K_1 \epsilon_0 A/2}{d}$$

$$\Rightarrow C_1 = \frac{K_1 \epsilon_0 A}{2d} \quad \text{——— (1)}$$

For dielectric k_2,

$$C_2 = \frac{K_2 \epsilon_0 A}{2d} \quad \text{——— (11)}$$

Capacitors are in parallel circuit so, equivalent capacitance is given by,

$$C_{eq} = C_1 + C_2$$

$$= \frac{K_1 \epsilon_0 A}{2d} + \frac{K_2 \epsilon_0 A}{2d}$$

$$= \frac{\epsilon_0 A}{2d}(K_1 + K_2)$$

$$\therefore C = \frac{\epsilon_0 A}{d}\left(\frac{K_1 + K_2}{2}\right) \text{ ~~pro~~ showed}$$

N.F.E

dielectric constant & Relative permittivity:
(K) (ϵ_r)

$$k = \frac{C}{C_0}$$

For parallel plate capacitor:

$$C = \frac{\epsilon A}{d} \quad \& \quad C_0 = \frac{\epsilon_0 A}{d}$$

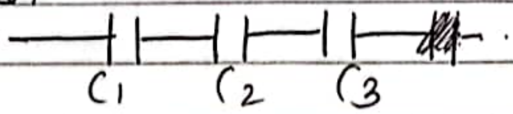
$$k = \frac{\epsilon}{\epsilon_0} = \epsilon_r$$

$$\epsilon = \epsilon_0 \cdot \epsilon_r$$

$$\boxed{\epsilon = k\epsilon_0}$$

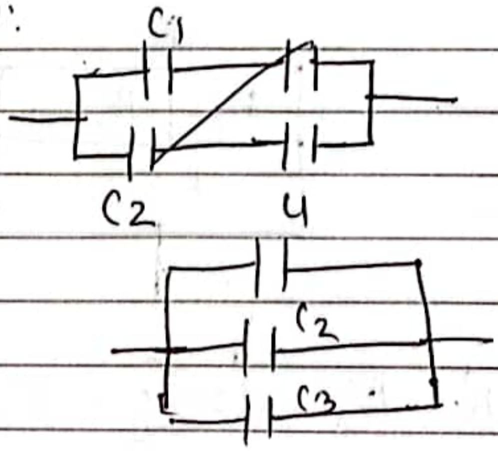
Capacitors in series & parallel circuit:

① Series:



$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

② parallel:



$$\therefore C_p = C_1 + C_2 + C_3 + \dots$$

or, $C_1 = \frac{\epsilon_0 A k_1}{4d}$ — (1)

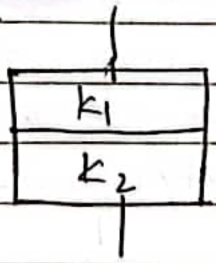
$C_2 = \frac{k_2 \epsilon_0 A}{4d}$ — (2).

In parallel, $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

$\therefore C = \frac{C_1 C_2}{C_1 + C_2}$

$= \frac{\epsilon_0 A}{4d} \frac{(k_1 \cdot k_2)}{(k_1 + k_2)}$

Question:



$C_1 = \frac{K \epsilon_0 A}{d}$

$C_0 = \frac{\epsilon_0 A}{d}$

For k_1, $d' = d/2$

$\therefore C_1 = \frac{k_1 \epsilon_0 A}{d/2}$

$= 2\left(\frac{k_1 \epsilon_0 A}{d}\right).$

For k_2, $d' = d/2$

$C_2 = \frac{k_2 \epsilon_0 A}{d/2}$

$\therefore C_2 = 2\left(\frac{k_2 \epsilon_0 A}{d}\right).$

k_1 & k_2 are in series. So,

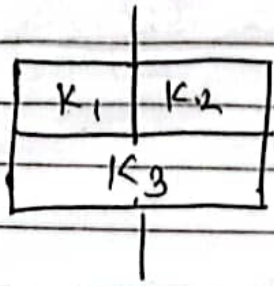
$C = \frac{C_1 C_2}{C_1 + C_2}$

$= \frac{\left(2\frac{k_1 \epsilon_0 A}{d}\right)\left(2\frac{k_2 \epsilon_0 A}{d}\right)}{2\frac{k_1 \epsilon_0 A}{d} + 2\frac{k_2 \epsilon_0 A}{d}}$

$$\Rightarrow \frac{\left(\frac{2\epsilon_0 A}{d}\right)^{\cancel{2}} (K_1 \cdot K_2)}{\cancel{\left(\frac{2\epsilon_0 A}{d}\right)} (K_1 + K_2)}$$

$$= \frac{\frac{2\epsilon_0 A (K_1 \cdot K_2)}{d}}{K_1 + K_2} = \frac{2\epsilon_0 A (K_1 \cdot K_2)}{d(K_1 + K_2)} \quad \text{ans.}$$

Ques. 2.



For K_1 & K_2

$$A' = A/2 \ , \ d' = d/2$$

$$C_1 = \frac{K_1 \epsilon_0 A}{2d/2} \ , \ C_2 = \frac{K_2 \epsilon_0 A}{2d/2}$$

For parallel,

$$C = C_1 + C_2$$

$$= \frac{K_1 \epsilon_0 A}{2d/2} + \frac{K_2 \epsilon_0 A}{2d/2}$$

$$= \frac{\epsilon_0 A (K_1 + K_2)}{2d/2}$$

$$= \frac{\epsilon_0 A (K_1 + K_2)}{d}.$$

For series with K_3.

$$C_3 = \frac{K_3 \epsilon_0 A}{d/2}$$

$$= 2\left(\frac{K_3 \epsilon_0 A}{d}\right)$$

Now, K_3 is in series with (K_1+K_2) ;

$$\cancel{C_3} \quad C = \frac{C_3 \cdot C}{C_3 + C}$$

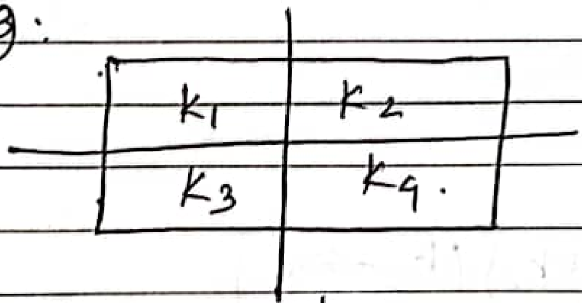
$$= \frac{2\left(\frac{K_3 \epsilon_0 A}{d}\right) \cdot \frac{\epsilon_0 A}{d}(K_1+K_2)}{2\left(\frac{K_3 \epsilon_0 A}{d}\right) + \frac{\epsilon_0 A}{d}(K_1+K_2)}$$

$$= \frac{\left(\frac{\epsilon_0 A}{d}\right)^2 \left[\frac{2K_3}{\cancel{d}}(K_1+K_2)\right]}{\frac{\epsilon_0 A}{\cancel{d}}\left[2K_3 + (K_1+K_2)\right]}$$

$$= \frac{\frac{\epsilon_0 A}{d}\left[2K_3(K_1+K_2)\right]}{2K_3+(K_1+K_2)}$$

$$= \frac{\epsilon_0 A\left[2K_3(K_1+K_2)\right]}{d\left[2K_3+(K_1+K_2)\right]}$$ ans.

Ques. 3:



For K_1 & K_2, $A' = A/2$, $d' = d/2$.

$$C_1 = \frac{K_1 \epsilon_0 A/2}{d/2} = \frac{K_1 \epsilon_0 A}{d}$$

$$C_2 = \frac{K_2 \epsilon_0 A/2}{d/2} = \frac{K_2 \epsilon_0 A}{d}.$$

$$C = C_1 + C_2 = (K_1+K_2)\frac{\epsilon_0 A}{d} \quad //$$

For k_3 and k_4, $A' = A/2$, $d' = d/2$.

$$C_3 = \frac{k_3 \epsilon_0 A}{2d}, \quad C_4 = \frac{k_4 \epsilon_0 A}{2d}$$

$$C' = C_3 + C_4$$
$$= (k_3 + k_4)\frac{\epsilon_0 A}{d}.$$

Now,
C & C' are in ~~parall~~ series,

$$\cancel{C = C}$$

$$\frac{1}{C} = \frac{1}{C} + \frac{1}{C'}$$

$$C = \frac{C \cdot C'}{C + C'}$$

$$\cancel{= \frac{(k_1+k_2)\frac{\epsilon_0 A}{d}}{(k_3+k_4)\frac{\epsilon_0 A}{d}} = \frac{k_1+k_2}{k_3+k_4}.}$$

$$= \frac{(k_1+k_2)\frac{\epsilon_0 A}{d} \cdot (k_3+k_4)\frac{\epsilon_0 A}{d}}{(k_3+k_4)\frac{\epsilon_0 A}{d} + (k_1+k_2)\frac{\epsilon_0 A}{d}}$$

$$\Rightarrow \frac{\left(\frac{\epsilon_0 A}{d}\right)^2 \left[(k_1+k_2)(k_3+k_4)\right]}{\left(\frac{\epsilon_0 A}{\cancel{d}}\right)(k_1+k_2+k_3+k_4)}$$

$$\therefore C = \frac{\frac{\epsilon_0 A}{d}\left[(k_1+k_2)(k_3+k_4)\right]}{(k_1+k_2+k_3+k_4)} \quad \text{ans}$$

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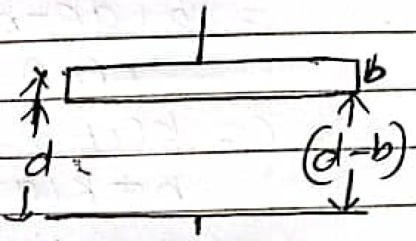
Ques: A dielectric slab of thickness 'b' is inserted betⁿ the plates of a parallel plate capacitor of plate separation 'd' ($b<d$). Show that the capacitance is given by;

$$C = \frac{K\epsilon_0 A}{Kd - b(K-1)}.$$



Solution;

The capacitor are considered to be in series,

One capacitor consists of dielectric 'b' thickness 'd' and Area 'A'; while the other one capacitor consists of thickness 'd-b' and area (A).

$$C_1 = \frac{K\epsilon_0 A}{d} \quad \& \quad C_2 = \frac{\epsilon_0 A}{d-b}$$

Now,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow C = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{\frac{K\epsilon_0 A}{d} \cdot \frac{\epsilon_0 A}{d-b}}{\frac{K\epsilon_0 A}{d} + \frac{\epsilon_0 A}{d-b}} \Rightarrow \frac{(\epsilon_0 A)^2 \left[\frac{K \cdot 1}{d(d-b)}\right]}{\epsilon_0 A \left[\frac{K}{d} + \frac{1}{d-b}\right]}$$

$$\therefore C = \frac{\epsilon_0 A K}{d(d-b)\left(\frac{K}{d} + \frac{1}{d-b}\right)}.$$

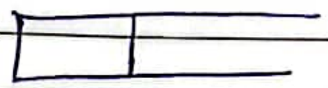
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{b}{k\epsilon_0 A} + \frac{d-b}{\epsilon_0 A}$$

$$= \frac{b + dk - bk}{k\epsilon_0 A}$$

$$\therefore C = \frac{k\epsilon_0 A}{b + k(d-b)}$$ **showed.**

Ques: An air-filled parallel plate capacitor has a capacitance of 10^{-12} C. The separation between the plates is doubled and wax is inserted between them, which increases the capacitance to 2×10^{-2} F. Calculate the dielectric constant of wax.



$C_1 = 10^{-12}$ C.

$d = 2d$

$C_2 = 2\times10^{-2}$ F.

$K = ?$

$\therefore K = \frac{C}{C_0}$.

$C_1 = \frac{\epsilon_0 A}{d}$

$\Rightarrow$

$C_1 = 10^{-12}$ F

$d_1 = d$,

$A_1 = A$

$\epsilon_0 = e$

Now,

$C_2 = 2\times10^{-12}$ F

$d_2 = 2d$

$A_2 = A$

$\epsilon_2 = \epsilon$ (say)

We have,

$$C_2 = \frac{\epsilon_2 A_2}{d}$$

$$= \frac{\epsilon A}{2d} = \frac{k\epsilon_0 A}{2d} \quad \dots (ii).$$

$$\frac{C_2}{C_1} = \frac{4}{2} = K$$

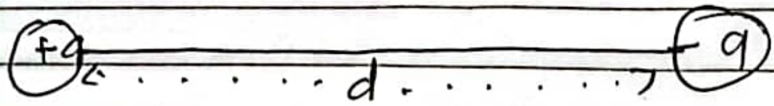
$$\therefore K = \frac{2C_2}{C_1}$$

$$\text{or, } K = \frac{2 \times 2 \times 10^{-12}}{10^{-12}}$$

$$= 4 \text{ ans}$$

Electric dipole:-

The combination of two equal & opposite charge is separated by a finite distance is called electric dipole.



distance between two electric charges is dipole seperation 'd'.

The product of magnitude of one of charges and dipole separation is called electric dipole moment.

i.e.

electric dipole moment (P) = $q \cdot d$.

Electric flux density (D):

It is defined as the surface at density;

i.e. $D = \frac{q}{A}$

$D = \epsilon E$ [$\because \epsilon$ is the permittivity of medium]

Types of dielectric:

- Non-polar dielectric:-

The dielectric in which the +ve and -ve charges have symmetrical charge distribution about their centre is called non-polar dielectric.

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In such cases, a centre of gravity of +ve and -ve charges concide with each other. Hence, the net electric dipole moment is zero.

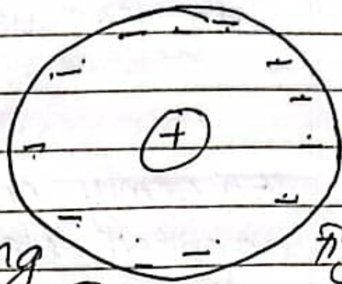


fig ①.

When an external field is applied, the +ve charge centre shifts in the direction of along the direction of applied field. The negative charge centre shift in the direction opposite to that of applied field. There why, creating dipole

In this way, a non-polar dielectric can be polarised. Eg. fig (i).

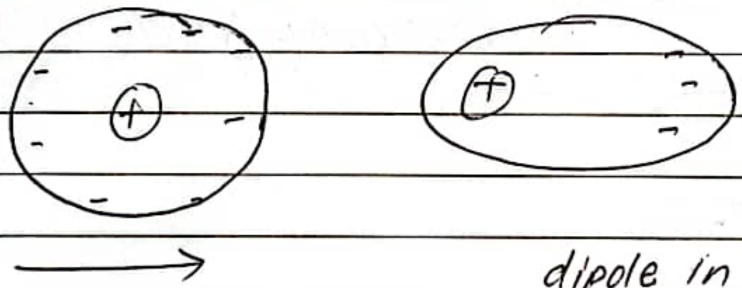


fig. dipole in the absence of electric field

dipole in the presence of electric field.

- Polar dielectric.

The dielectric in which the +ve and -ve charge have ~~assymm~~ asymmetric charge distribution about their centre is called polar dielectric.

In such cases, a centre of gravity of +ve & -ve charges doesn't concide with each other. due to finite separation of betn +ve & -ve charges, such dielectric possess some electric dipole moment. However, in a polar dielectric substance, there are large number of electric dipoles with orientation in different directions.

When an external field is applied, the randomly oriented dipoles tend to align along the applied electric field thereby producing ~~dipole~~

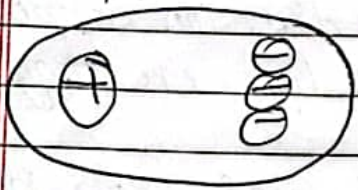


fig: polarmolecule

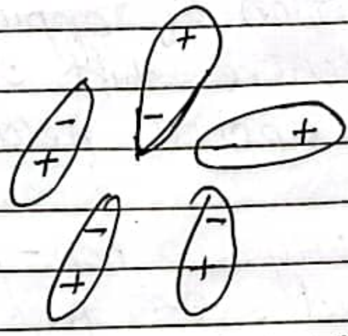


fig. polar dielectric into the absence of field:

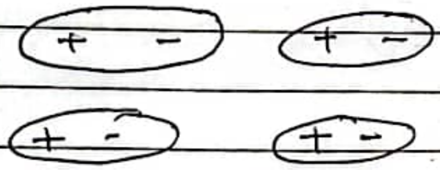
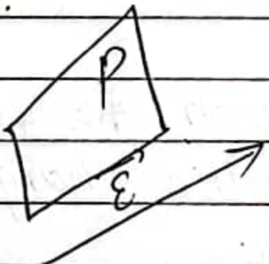
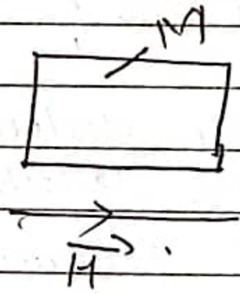


fig: polar dielectric in the presence of electric field.

Polarization:





$D = \epsilon_0 \mathcal{E} + P$.

$D = \epsilon \mathcal{E}$

$B = \mu_0 H + M$

flux density

$B = \mu H$.

$P = \frac{q'}{A}$ —— (1)

P —— z

When a dielectric is introduced between the plates of a capacitor, some charges induced on dielectric. The polarization (P) can be defined as the induced charge within the dielectric per unit Area.

i.e,

$$P = \frac{q'}{A} \quad \text{—— (1)} \quad [\because q' = \text{induced charge}]$$

Therefore, polarization (P) is equivalent to surface polarization charge density.

The polarization of dielectric can also be defined as the dipole moment per unit volume.

$P = \frac{\Sigma p_i}{V}$, p_i is the dipole moment of its molecule.

When dielectric is placed, in an external field, the electric dipoles within it tends to arrange in ~~or~~ such order that dielectric acquires a certain electric moment. This phenomenon is known as polarization.

The polarization is directly proportional to electric field.

$$P \propto E$$

$$\text{or, } P = \alpha E \quad [\because \alpha \text{ is the polarizibility}]$$

proportionality constant

When $E = 1$ unit,

$P = \alpha$

Hence,

~~$P = N\alpha E$ (∵ N is the no. of~~

polarizibility can be defined as the polorization produced by ~~unit~~ field of unit ~~state~~ strength.

$P = N\alpha E$ (where $N = 1, 2, 3, \ldots$)

In free space, the electric flux density is given by,

$D_0 = \epsilon_0 E$ ——— (1) (where, ϵ_0 is the permittivity of free space)

For a dielectric medium, for permittivity ϵ_0, it is given by $D = \epsilon E$ ——— (2)

$P = D - D_0$

$= \epsilon E - \epsilon_0 E$

$= \epsilon_r \cdot \epsilon_0 E - \epsilon_0 E$

The increased in flux density is called polarization. i.e. $P = D - D_0$

$= \epsilon E - \epsilon_0 E$

$= \epsilon_0 \cdot \epsilon_r E - \epsilon_0 E$

$= \epsilon_0 E(\epsilon_r - 1)$

or, $P = \epsilon_0(\epsilon_r - 1)E$

$= \epsilon_0 \chi E$

∴ where χ is susceptibility.

For free space,

$$\epsilon_r = \frac{\epsilon_0}{\epsilon_0} = 1 \quad [\because \epsilon = \epsilon_0]$$

$$\therefore P = 0$$

Dielectrics and Gauss law :-

Consider a parallel plate capacitor of plate area 'A' having charge 'q' on each plate. Let us draw a Gaussian surface which encloses charge 'q' on +ve plate. ~~Let E~~

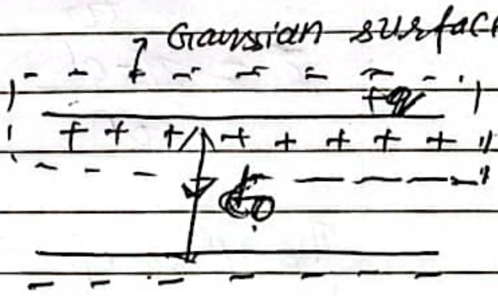


fig. capacitor without dielectric.

Applying Gauss's law,

$$\oint \vec{E_0}\cdot\vec{dA} = \frac{q}{\epsilon_0 \cdot 1} \quad \text{——} (1)$$

($\because E_0$ is the ~~diele~~ without dielectric)

$$E_0 \cdot A = \frac{q}{\epsilon_0}$$

$$E_0 = \frac{q}{A\epsilon_0} \quad ,, \quad \text{——} (2)$$

8.85×10^{-12}

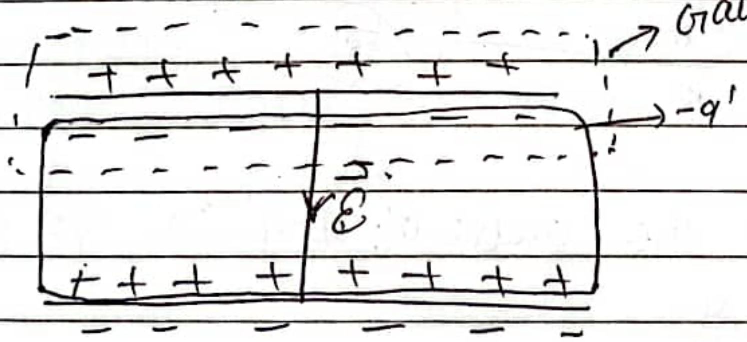


fig: capacitor with dielectric.

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$$\oint \vec{E}\,\vec{dA} = \frac{q-q'}{\epsilon_0}$$

When a dielectric is inserted between the plates of a capacitor, let q' be the amount of charge induced from the surface of dielectric. Taking the same Gaussian surface. Applying Gauss' law,

$$\oint \vec{E}\cdot\vec{dA} = \frac{q-q'}{\epsilon_0} \text{ --- } (3)$$

$$\vec{E}\cdot A = \frac{q-q'}{\epsilon_0}$$

$$\cancel{\vec{E} = \frac{q-q'}{\epsilon_0 A}} \quad \boxed{E = \frac{q-q'}{\epsilon_0 A}} \text{ --- } (4)$$

The effect of dielectric is to reduce electric field by the factor of $1/K$.

i.e,

$$E = \frac{E_0}{K}$$

~~Eq~~ Using equation (2) & (4)

$$\frac{q-q'}{\epsilon_0 A} = \frac{1}{K}\frac{q}{\epsilon_0 A}$$

$$\text{or, } q-q' = \frac{q}{K} \text{ --- } (5)$$

Substituting the value of $q-q'$ in eq^n (3).

$$\oint \vec{E}\,\vec{dA} = \frac{q}{K\epsilon_0}$$

$$\text{or, } K\oint \vec{E}\cdot\vec{dA} = \frac{q}{\epsilon_0} \text{ --- } (6)$$

which is the gauss law in dielectric.
Hence,
Gauss's law of in dielectric can be obtained by introducing dielectric constant 'K' on the left ~~side~~ hand side of a general form.

* To derive $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ $\Big\{ D$ = electric

$$E = \frac{q - q'}{\epsilon_0 A}$$

or, $\epsilon_0 E = \frac{q}{A} - \frac{q'}{A}$

or, $\epsilon_0 E = D - P$

$\therefore D = \epsilon_0 E + P$

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ // desired.

where, $D = \frac{q}{A}$ is the electric flux density also known as displacement vector.

And, $P = \frac{q'}{A}$ is the polarisation vector.

Q) The plates of parallel plate capacitor are 10cm apart and have area = $2m^2$. If the charge on each plate 8.85×10^{-10} C, find the electric field at a point between the plates & the capacitance of a capacitor,

Sol:-

$$E = \frac{V}{d} \quad \text{——— (I)}$$

$$q = VC \quad \text{——— (II)}$$

Here, ~~$q = VC$~~

or,

$$E = \frac{q}{A\epsilon_0}$$

$$= \frac{8.85\times10^{-10}}{2\times8.85\times10^{-12}}$$

$$= \frac{10^{-10+12}}{2} = \frac{10^2}{2} = 50.$$

$E = \frac{V}{d}$ or, $50 = \frac{V}{10\times10^{-2}}$ $\therefore V = 5V.$

$q = VC$

or, $C = \cancel{\frac{V}{q}} = q/V = \frac{8.85\times10^{-10}}{5}$ $= \cancel{1.77}$

$= 1.77\times10^{-10}F$

$= 1.77\times10^{-4}\mu F$

(Q) A parallel plate capacitor has a capacitance of $100\times10^{-12}F$. a plate area of $100cm^2$ mica is used as a dielectric. At 50 volts potential difference, calculate electric field intensity and magnitude of induced charge.

Sol:- $C = 100\times10^{-12}F$

$A = 100cm^2 = 100cm\times1cm$

$= \frac{100}{100}m\times\frac{1}{100}m$

$= \frac{1}{100}m^2$

$V = 50$ Volts. $\qquad E = \frac{q}{K\epsilon_0 A}$. $\qquad k = 5.4$ for mica.

~~V~~ $\qquad q' = q(1 - 1/k)$

$E = \frac{q}{K\epsilon_0 A}$

$= \frac{VC}{k\epsilon_0 A}$

$= \frac{50 \times 100 \times 10^{-12}}{5.4 \times 8.85 \times 10^{-12} \times \frac{1}{100} m^2}$

$= \cancel{1046.24}\ 10462.43$.

$q' = q(1 - 1/k)$. $\qquad q' = q - \frac{q}{K}$

$= 50 \times 100 \times 10^{-12}\left(1 - \frac{1}{5.4}\right)$. $\qquad \left(q - q' = \frac{q}{K}.\right)$.

$= 4.07 \times 10^{-9} C$

(Q) A parallel plate capacitor has a capacitance of $100\mu\mu F$, a plate area of $100 cm^2$ and mica as dielectric. At 50 volts, potential difference, calculate

(a) E in mica.

(b) the free charge on the plates.

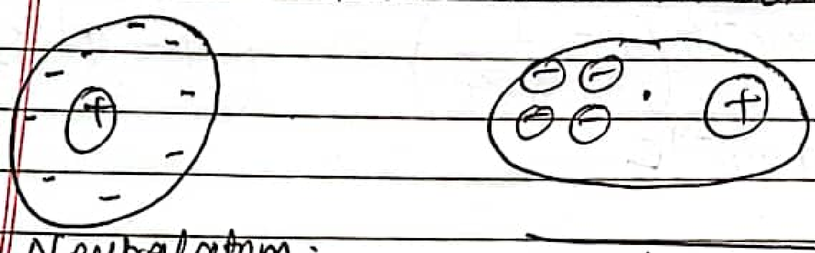
(c) induced charge (Given $k = 5.4$)

Same as above.

Types of polarisation:

(a) Electronic polarization:

In a neutral atom, the centre of gravity of positive charge centre and ~~negative~~ that of negative charge centre coincide with each other. So, there is no net dipole moment. However, when an external field is applied, the positive charge centre shifts in the direction of applied electric field whereas the negative charges shifts in the opposite direction of applied electric field. There why producing polarisation. Since, these type of polarisation ~~in volts~~ involves shifting of electrons ~~show~~ So, it is called electronic polarization.



Neutral atom: electric field.

The electronic polarisibility is given by;

$$\cancel{P = \alpha E} \quad \text{——— } \cancel{(1)}$$

For ~~nth~~ an atom, the electric dipole is given by;

$$p = \alpha E \quad \text{——— (1)}$$

$$\alpha = P/E \quad \text{——— (2)} \qquad [P = q \cdot r = Ze \cdot r]$$

In case of atoms, there exists centripetal force balanced by electrostatic force i.e.,

$$F = \frac{mv^2}{r}$$

or $$qE = \frac{mv^2}{r}$$

or $$~~me~~ eE = \frac{m(\omega r)^2}{r}$$
$$= m\omega^2 r$$

or, $$E = \frac{m\omega^2 r}{e} \quad \text{——— (3)}$$

Now,
$$\alpha = P/E = \frac{zer}{m\omega^2 r} \times e = \frac{ze^2}{m\omega^2}$$

$$\therefore \alpha_e = \frac{ze^2}{m\omega^2} \quad \text{——— (4)}$$

Under electronic polarisation,

Clausius - Mossotti Equation:

The actual field explains by a molecule in a dielectric is called local field. (E_{local}). This is the sum of applied field & field due to polarisation. i.e.

$$E_{local} = E + E_{polarisation}$$
$$= E + \frac{P}{3\epsilon_0} \quad \left(\because E = \frac{P}{3\epsilon_0}\right)$$

Now, the polarisation is given by;
$$P = N\alpha E_{local}.$$
or, $$P = N\alpha\left(E + \frac{P}{3\epsilon_0}\right)$$
$$= N\alpha E + \frac{N\alpha P}{3\epsilon_0}.$$

or, $P\left(1-\frac{N\alpha}{3\epsilon_0}\right) = N\alpha E$

or, $P = \frac{N\alpha E}{1-\frac{N\alpha}{3\epsilon_0}}$ ——— (1).

Also,

$P = \epsilon_0 \chi E$ .... (2)

From eqn (1) & (2), we get:

$\frac{N\alpha E}{1-\frac{N\alpha}{3\epsilon_0}} = \epsilon_0 \chi E.$

or, $\frac{N\alpha E}{E} = \epsilon_0 \chi \left(1 - \frac{N\alpha}{3\epsilon_0}\right)$

or, $N\alpha = \epsilon_0 \chi - \frac{N\alpha \cancel{\epsilon_0} \chi}{3\cancel{\epsilon_0}}$

or, $N\alpha = \epsilon_0 \chi - \frac{N\alpha \chi}{3}$

or, $N\alpha \left(1 + \chi/3\right) = \epsilon_0 \chi$

or, $N\alpha\left(\frac{3+\chi}{3}\right) = \epsilon_0 \chi$

or, $\frac{N\alpha}{3\epsilon_0} = \frac{\chi}{\chi+3}$ ——— (3)

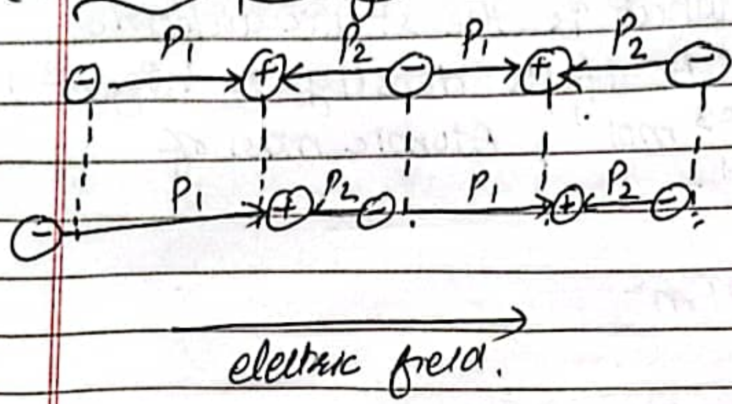
which is the required equation of Clausius - Marotti equation.

Since,

$\frac{N\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$ ..... (4) $(\because \chi = \epsilon_r - 1)$

which is the required equation of Clausius-Mascotti equation in terms of relative permittivity.

(b) Ionic polarization:



$P_{net} = P_1 - P_2 = 0$

$P_{net} = P_1 - P_2 \neq 0.$

$$\boxed{\alpha_i = \frac{ze^2}{m\omega^2}}.$$

The ionic crystals such as NaCl, KCl, LiBr etc have well-defined lattice site in which positive and negative ions are located ~~is~~. Each pairs of +ve and -ve ions, form a dipole. Since, these dipoles are aligned one after another. So, there is no net dipole moment i.e.;

$$P_{net} = P_1 - P_2 = 0$$

When an external field is applied, the +ve ions move in the direction of applied electric field & the -ve ions move opposite to the direction of applied field.

Hence, P_1 increases while P_2 decreases so that ~~there~~ ~~exr~~

$P_{net} = P_1 - P_2 \neq 0$. In this way, an ionic crystal is polarised.

Since these types of ~~ion~~ polarization involves shifting of ions, so it is called ionic polarisation.

$$\therefore \boxed{\frac{N\alpha}{3\epsilon_0} = \frac{\chi}{\chi + 3}}$$ //.

From Clausius Mosotti eq^n:

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The electronic polarizibility of the Ar atom is 1.7×10^{-40} Fm². What is the static dielectric constant of solid Ar if its density is $1.8 g cm^{-3}$? Given $N_A = 6.02 \times 10^{23} mol^{-1}$, Atomic mass of Ar = $39.95 g mol^{-1}$.

Here,

$\alpha_e = 1.7 \times 10^{-40} Fm^2$

$\rho = 1.8 g cm^{-3}$.

$N = \frac{\rho N_A}{M_{At}}$.

$= \frac{6.023 \times 10^{23} \times 1.8}{39.95}$

$= 2.7 \times 10^{22} cm^{-3}$ $\quad 10^{22} cm \times cm \times cm$

$= 2.7 \times 10^{28} m^{-3}$ $\quad = 10^{22} \times 10^2 \times 10^2 \times 10^2$

From Claussius – Massotti equation,

$$\frac{N\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

or, $\cancel{\frac{2.7 \times 10^{28} \times 1.7 \times 10^{-40}}{3 \times}}$

or, $\frac{N\alpha\epsilon_r}{3\epsilon_0} + \frac{2N\alpha}{3\epsilon_0} = \epsilon_r - 1$

or, $\epsilon_r\left(\frac{N\alpha}{3\epsilon_0} - 1\right) = -\left[1 + \frac{2N\alpha}{3\epsilon_0}\right]$

or, $\epsilon_r = \frac{-3\epsilon_0 - 2N\alpha}{N\alpha - 3\epsilon_0}$

$= 1.627$ //.

Ques: Assuming that the polarisibility of an argon atom is equal to $1.43 \times 10^{-40}\ Fm^2$, Calculate the relative dielectric constant at 0°c and 1 atm pressure

Sol^n: $\alpha = 1.43 \times 10^{-40}\ Fm^2$

ϵ_r at $0^\circ C$, 1atm = ?

$$\frac{N\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

or, $6\cancel{.023 \times 10^{23} \times 1.4}$

$$\text{or, } \frac{N\alpha\epsilon_r + 2N\alpha}{3\epsilon_0} = \epsilon_r - 1$$

$$\text{or, } \frac{N\alpha\epsilon_r}{3\epsilon_0} - \epsilon_r = -1 - \frac{2N\alpha}{3\epsilon_0}$$

$$\text{or, } \epsilon_r\left(\frac{N\alpha}{3\epsilon_0} - 1\right) = -1 - \frac{2N\alpha}{3\epsilon_0}$$

$$\text{or, } \epsilon_r = \cancel{-1-2}\ \frac{-3\epsilon_0 - 2N\alpha}{N\alpha - 3\epsilon_0}$$

$$= \frac{(-3 \times 8.85 \times 10^{-12}) - 2(6.023 \times 10^{23} \times 1.43 \times 10^{-40})}{(6.023 \times 10^{23} \times 1.43 \times 10^{-40}) - 3(8.85 \times 10^{-12})}$$

$$=$$

~~No. of atoms~~ per unit volume = $N_A \times \rho$

$$\frac{N\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$\text{or, } \epsilon_r\left(\frac{N\alpha}{3\epsilon_0} - 1\right) = -1 - \frac{2N\alpha}{3\epsilon_0}$$

$$\therefore \epsilon_r = \frac{\cancel{-}(3\epsilon_0 + 2N\alpha)}{\cancel{-}(3\epsilon_0 - N\alpha)}$$

$$= \frac{3\epsilon_0 + 2N\alpha}{3\epsilon_0 - N\alpha}$$

$$\epsilon_r = \frac{2N\alpha + 3\epsilon_0}{3\epsilon_0 - N\alpha e}.$$

$PV = nRT.$

$$o_r,\ P = \frac{\text{No. of atoms per unit volume}}{N_A} \times RT.$$

$$= \frac{n}{V} \times RT. \quad \because n = \frac{\text{Total no. of atoms}}{\text{No. of atoms per mole}} \times RT.$$

$$= \frac{N}{N_A} \times RT$$

$$or,\ P = \frac{N}{N_A} \times RT$$

$$\therefore N = \frac{PN_A}{RT}$$

$$= \frac{1 \times 6.023 \times 10^{23}}{8.31 \times 273}$$

$$= 2.65 \times 10^{20}.$$

$$\therefore \epsilon_r = \frac{(2 \times 2.65 \times 10^{20} \times 1.43 \times 10^{-40}) + 3(8.85 \times 10^{-12})}{3 \times 8.85 \times 10^{-12} - N \times 1.43 \times 10^{-40}}$$

$$= 1.0000000004 //.$$

$PV = nRT$

$$or,\ PV = \frac{\text{Total no. of atoms}}{\text{No. of atoms per mole}} \times RT.$$

$$o_r,\ P = \frac{(N)}{N_A} \times RT$$

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Consider a pure Si crystal that has, $\epsilon_r = 11.9$.
(a) What is the electronic polarisibility due to valence electron per Si atom?.
(b) Suppose that a voltage is applied across Si crystal sample, by how much is the local field greater than applied field? (Given $\epsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}, $N = 5 \times 10^{28} m^{-3}$)

Solution :,

From Clausius Masotti eq^n,

$$\frac{N\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$\cancel{N = \frac{\rho N_A}{M_{at}}}$$

$$=$$

(a) ⇒ $\alpha = \frac{\epsilon_r - 1}{\epsilon_r + 2} \times \frac{3\epsilon_0}{N}$

$$= \frac{11.9 - 1}{11.9 + 2} \times \frac{3 \times 8.85 \times 10^{-12}}{5 \times 10^{28}}$$

$$= 4.16 \times 10^{-40} \, Fm^2$$

(b) ⇒.

$$E_{local} = E + E_p$$
$$= E + \frac{P}{3\epsilon_0}.$$

or $P = \epsilon_0 \times E$.

or, $E_{local} = E + \frac{\epsilon_0 \times E}{3\epsilon_0}$.

$\cancel{\text{or, } E_{local} = \frac{3E + E}{3}}$ or $E_{local} =$

$$E_{local} = E + \frac{P}{3\epsilon_0}$$

$$= E + \frac{\cancel{\epsilon_0} \times E}{3\cancel{\epsilon_0}}$$

$$= E + \frac{(\epsilon_r - 1) E}{3}$$

$$= \frac{3E + (\epsilon_r - 1) E}{3}$$

$$= E \left(\cancel{3} \cdot 1 + \frac{1}{3} (\epsilon_r - 1) \right)$$

$$\therefore \frac{E_{local}}{E} = 1 + \frac{1}{3}(11.9 - 1)$$

$$= 1 + \frac{1}{3}(10.9)$$

$$= 4.63$$

If the refractive index of ~~material~~ medium w.r.t. free space is defined as the ratio of velocity of electromagnetic wave in free space to that in the middle. ~~Show~~ ~~So~~ that the Clausius - Masotti equation becomes

Show

$$\frac{N\alpha}{3\epsilon_0} = \frac{n^2 - 1}{n^2 + 1}$$

~~the~~ Here,

refractive index of medium w.r.t. free space

According to question,

$$n = \frac{c}{v}$$

$\left[\because c\right.$ is speed of light in free space

~~v~~

v is speed of light in medium

The velocity of electromagnetic wave in free space is defined by;

$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad \text{——— (1)}$$

The velocity of electromagnetic wave in medium is given by;

$$V = \frac{1}{\sqrt{\mu \epsilon}} \quad \text{——— (2)}$$

For a non-magnetic medium, $\mu = \mu_0$

$$\therefore V = \frac{1}{\sqrt{\mu_0 \epsilon}} \quad \text{——— (3)}$$

Dividing eq^n (1) by (3),

$$\frac{C}{V} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \times \sqrt{\mu_0 \epsilon}$$

$$= \sqrt{\frac{\cancel{\mu_0} \epsilon}{\cancel{\mu_0} \epsilon_0}}$$

$$= \sqrt{\frac{\epsilon}{\epsilon_0}}$$

or, $$\frac{C}{V} = \sqrt{\frac{\epsilon}{\epsilon_0}}$$

$$\therefore n = \sqrt{\frac{\cancel{\epsilon_0} \cdot \epsilon_r}{\cancel{\epsilon_0}}}$$

or, $n^2 = \epsilon_r$ ——— (4)

So, ~~En~~ In clausius - Mossotti equation //.

$$\frac{N}{N\alpha} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$ [$\because$ substituting the ϵ_r by n^2 from eq^n (4)]

$$\boxed{\frac{N}{N\alpha} = \frac{n^2 - 1}{n^2 + 2}}$$ //. showed //.

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The optical index of a refraction and the dielectric constant per for glass are 1·45 and 6·5 respectively. Calculate the percentage of ionic polarisibility.

Solution: Given,

$n = 1.45$

& $\epsilon_r = 6.5$

Now, $\alpha_i = ?$

$\alpha_e = ?$

We have,

From Clausius – Masotti equation,

$$\frac{N(\alpha_e + \alpha_i)}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \quad \ldots \text{(i)}$$

At optical frequencies, ϵ_r in clausius - massoti equation for electronic polarization is replaced by n^2.

Therefore,

$$\frac{N(\alpha_e \cancel{+\alpha_i})}{3\epsilon_0} = \frac{n^2-1}{n^2+2} \quad \ldots \text{(ii)}$$

Dividing eqn (1) by eqn (2),

$$\frac{\dfrac{N(\alpha_e + \alpha_i)}{3\epsilon_0}}{\dfrac{N\alpha_e}{3\epsilon_0}} = \frac{\dfrac{\epsilon_r - 1}{\epsilon_r + 2}}{\dfrac{n^2-1}{n^2+2}}$$

$$\frac{\alpha_e + \alpha_i}{\alpha_e} = \frac{6.5-1}{6.5+2} \times \frac{1.45^2+2}{1.45^2-1}$$

$$= \frac{253}{51} \quad 2.407$$

or, $$\frac{\alpha_e}{\alpha_e + \alpha_i} = \frac{51}{253} \quad 0.415.$$

or, $253\alpha_e = 51\alpha_e + 51\alpha_i$

or, $202\alpha_e = 51\alpha_i$

or, $\alpha_i = \frac{51}{202}\alpha_e$.

Now,

The percentage of ionic polarisibility.

or, $\frac{\alpha_e}{\alpha_e + \alpha_i} = 0.415$. $= 41.5\%$

or, $\alpha_e = 0.415\alpha_e + 0.415\alpha_i$

or, $0.585\alpha_e = 0.415\alpha_i$

or, $\alpha_i = \frac{0.585\alpha_e}{0.415}$

The percentage of ionic polarisibility;

$$\left(\frac{\alpha_i}{\alpha_i + \alpha_e}\right) \times 100\% = \frac{\frac{0.058\alpha_e}{0.415}}{\frac{0.585\alpha_e}{0.415} + \alpha_e} \times 100\%.$$

$$= 0.058\alpha_e$$

$\Rightarrow$ $100\% - \left(\frac{\alpha_e}{\alpha_e + \alpha_i}\right) \times 100\%$

$$\left(\frac{\alpha_i}{\alpha_i + \alpha_e} \times 100\%\right) = (100 - 41.5)\%$$

$$= 58.5\%.$$

Quantum Mechanics:

De. broglie Electron wave:

The ball of mass 'm' 10gm have velocity 100 cm/s. Calculate the wavelength associated with it. ~~where~~ ~~when~~ Why doesn't this wave nature show up our daily observations.

Given $h = 6.62 \times 10^{-34}$ Js.

Now,

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.62 \times 10^{-34}}{\frac{10}{1000} \times \frac{100}{1000}\ m/s}$$

$$= \frac{6.62 \times 10^{-34}}{\frac{1}{100}\ m/s}$$

$$= 6.62 \times 10^{-34} \times 10^{2}$$

$$= 6.62 \times 10^{-32}\ m.$$

This wavelength is much smaller than the dimensions of the balls therefore in such cases. wave-like properties of matter cannot be observed in our daily life.

Calculate the wavelength associated with an electron subjective to a potential difference of 1.5KV.

Sol^n $\frac{1}{2}mv^2 = eV$ or, ~~$eV = 1.5 \times 10^{-19}$~~

or, $v = \sqrt{\frac{2eV}{m}}$ $e = 1.6 \times 10^{-19} C$
$m = 9.1 \times 10^{-31} kg.$

or, $\lambda = \frac{h}{mv}$

$= \frac{h}{m\sqrt{\frac{2eV}{m}}}$

$= \frac{h}{\sqrt{2eVm}}$

$= \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 10^{-19} \times 1.5 \times 10^{3} V \times 9.1 \times 10^{-31}}}$

$= 3.167 \times 10^{-11} m$

$= 3.167 \times 10^{-10} \times 10^{-1} m$

$= 0.3167 A^0$ //

Find the energy of the neutron in unit of electron volt whose de-Broglie wavelength is $1 A^0$. Given mass of neutron $= 1.674 \times 10^{-27} kg$, $h = 6.62 \times 10^{-34} Js$.

Sol^n $E = hf$

$= \cancel{\frac{hc}{\lambda}}$. $m = 1.674 \times 10^{-27} kg$
$h = 6.62 \times 10^{-34} Js$
$\lambda = 1 A^0 = 1 \times 10^{-10} m$

$\lambda = \frac{h}{mv}$

or, $v = \frac{h}{m\lambda}$.

$\therefore E = \frac{1}{2}mv^2$

$= \frac{1}{2}m\left(\frac{h^2}{m^2\lambda^2}\right)$

$= \frac{1}{2}\frac{h^2}{m\lambda^2}$

$= \frac{1}{2} \times \frac{(6.62\times10^{-34})^2}{1.674\times10^{-24} \times (10^{-10})^2}$

$= \frac{2.677\times10^{-23}}{2}$ J.

$= \cancel{8.}$

$= 1.36\times10^{-20}$ J

$= \frac{1.36\times10^{-20}}{1.6\times10^{-19}}$

$= \cancel{1.9}\ 0.08$ eV //.

(Q) What voltage must applied to an electron microscope to produce electrons of wavelength 0.50A°? Given e = 1.6×10^{-19} C, m = 9.1×10^{-31} kg, h = 6.62×10^{-34} Js.

Sol^n, $eV = \frac{1}{2}mv^2$

$\lambda = \frac{h}{mv}$

or, $v = \frac{h}{m\lambda}$.

or, $V = \frac{\frac{1}{2}mv^2}{e}$

$= \frac{1}{2}\cancel{m}\ \frac{\cancel{m}\,h^2}{m^2\lambda^2 2e}$.

$= \frac{h^2}{2em\lambda^2}$

$= \frac{(6.62\times10^{-34})^2}{2\times1.6\times10^{-19}\times9.1\times10^{-31}\times(0.50\times10^{-10})^2}$

$= 601.92$ V

$= 0.601$ KV //.

$\hbar$ = Reduced Planck's constant

Wave function and its ~~appl~~ significance:

$\psi = Ae^{-i(\omega t - kx)}$

$E = hf = \frac{h}{2\pi} \cdot \frac{h}{h} 2\pi f = \hbar \cdot \omega$

or, $\omega = \frac{E}{\hbar}$

$\lambda = \frac{h}{mv}$ or, $p = \frac{h}{\lambda}$

$= \frac{h}{p}$ $= \frac{h}{2\pi} \cdot \frac{2\pi}{\lambda}$

$= \frac{h}{2\pi} \cdot 2\pi p = \cancel{\hbar}$ $= \hbar \cdot k$

$\therefore k = \frac{p}{\hbar}$

$\psi = Ae^{-i(\frac{E}{\hbar}t - \frac{p}{\hbar}x)}$

$\psi = Ae^{-i/\hbar(Et - px)}$

ψ has no meaning in itself.

* Schrondinger's Wave equation:

(i) ~~Time independent~~ SWE.

$E = K.E + P.E$

$E = \frac{1}{2}mv^2 + V$

or, $E = \frac{1}{2m} \cdot m^2v^2 + V$

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$$\mathcal{E} = \frac{p^2}{2m} + V \quad \text{——— (1)}$$

Multiplying both side by ψ, we get,

$$\mathcal{E}\psi = \frac{p^2\psi}{2m} + V\psi \quad \text{——— (1)}$$

$$\cancel{\psi = Ae^{-\frac{i}{\hbar}(\mathcal{E}t - px)}}$$

$$\cancel{\therefore \frac{d\psi}{dx} = \frac{-i}{\hbar}(-p)\cdot\psi}$$

$$\cancel{\therefore \frac{d\psi}{dx} = \frac{ip\psi}{\hbar}}$$

$$\cancel{\frac{d^2\psi}{dx^2} = \left(\frac{ip}{\hbar}\right)^2 \psi}$$

$$\cancel{= -\frac{1\,p^2}{\hbar^2}\psi}$$

$$\cancel{= \frac{-p^2\psi}{\hbar^2}}$$

$$\cancel{\therefore p^2\psi = -\hbar^2 \frac{d^2\psi}{dx^2}}$$

$$\cancel{\psi = Ae^{-i/\hbar(\mathcal{E}t}}$$

$$\psi = Ae^{-i/\hbar(\mathcal{E}t - px)} \quad \text{——— (2)}$$

Differentiating w.r.t. x, we get,

$$\frac{d\psi}{dx} = \frac{-i}{\hbar}(-p)\cdot\psi$$

$$= \frac{ip\psi}{\hbar}$$

$$\text{or, } \frac{d^2\psi}{dx^2} = \left(\frac{ip}{\hbar}\right)^2 \psi$$

$$= \frac{-p^2\psi}{\hbar^2}$$

$$\therefore p^2\psi = -\hbar^2 \frac{d^2\psi}{dx^2} \quad \text{——— (3)}$$

Again,

differentiating eq^n ① w.r.t t, we get

$$\frac{d\psi}{dt} = \left(\frac{-i}{\hbar}\right) \cdot E \cdot \psi.$$

$$= \frac{-iE}{\hbar}\psi$$

or, $E\psi = \frac{-\hbar}{i}\left(\frac{d\psi}{dt}\right)$

$$E\psi = +\frac{i^2 \hbar}{i}\left(\frac{d\psi}{dt}\right)$$

or, $E\psi = i\hbar\frac{d\psi}{dt}$ ——— ④.

(i) Time independent of S.W.E.

Substituting the value of $p^2\psi$ from eq^n ③ in eq^n ①,

$$E\psi = \frac{-\hbar^2 \cdot \frac{d^2\psi}{dx^2}}{2m} + V\psi$$

$$= \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi$$

or, $E\psi = \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi$

or, $\frac{d^2\psi}{dx^2} + \frac{2m(E-V)}{\hbar^2}\psi = 0$. ——— ⑤

(ii) Time dependent of S.W.E.

Substituting the value of $p^2\psi$ and $E\psi$ from eq^n ③ & ④ in eq^n ①.

$$i\hbar \frac{d\psi}{dt} = \frac{-\hbar^2}{2m} \cdot \frac{d^2\psi}{dx^2} + V\psi \quad \text{—— (6)}$$

or, $$\boxed{E\psi = \hat{H}\psi} \quad \text{—— (7)}$$

where,

$$\hat{H} = \frac{-\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V$$

This is Hamiltonian

where,

$$E\psi = \hat{H}\psi$$

eigen function

operator

eigen value.

Hence,

Eq^n (7) is eigen value function.

(Q) Derive the time independent Schrondinger's wave equation starting with classical ~~mechanica~~; ~~y + wave~~ equation $y = \cancel{A\sin(2\pi}$ $A\sin 2\pi\left(ft - \frac{x}{\lambda}\right)$ where symbols has its usual meaning.

Energy and momentum operator:

$$\psi = Ae^{-i/\hbar(Et-px)} \quad \text{—— (1)}$$

$$\frac{d\psi}{dt} = \frac{-i}{\hbar} \cdot E\psi$$

$$\text{or, } E\psi = \frac{-\hbar}{i}\frac{d\psi}{dt}$$

$$\text{or, } E\psi = \frac{i^2\hbar}{i}\frac{d\psi}{dt}$$

$$\text{or, } E\psi = i\hbar\frac{d\psi}{dt}$$

$$\boxed{\hat{E} = i\hbar\frac{d}{dt}}$$

on $\cancel{\frac{d\psi}{dx}}$

on ~~dw~~ differentiating (1) w.r.t. x, we get,

$$\frac{d\psi}{dx} = \frac{-i}{\hbar}(-p)\psi$$

$$= \frac{ip\psi}{\hbar}$$

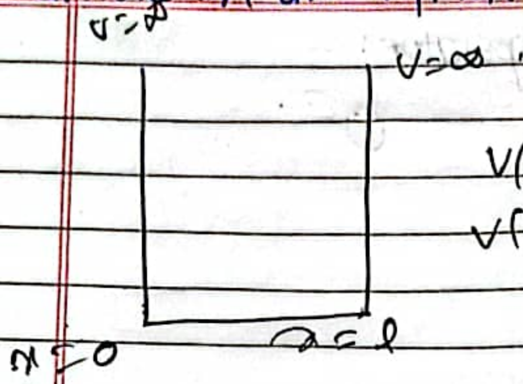
$$\text{or, } p\psi = \frac{\hbar}{i}\frac{d\psi}{dx}$$

$$p\psi = -\frac{i^2\hbar\, d\psi}{dx \cdot i}$$

$$= \cancel{-\hbar} \cdot -i\hbar\frac{d\psi}{dx}$$

$$\Rightarrow \therefore \boxed{\hat{p} = -i\hbar\frac{d}{dx}}$$

Particle in an Infinitely Deep Potential Well



$V(x)=0$ for $0<x<l$
$V(x)=\infty$ for $x<0,\ x>l$.

$$\frac{d^2\psi}{dx^2}+\frac{2m(E-V)}{\hbar^2}\psi=0$$

or, $$\frac{d^2\psi}{dx^2}+\frac{2mE}{\hbar^2}\psi=0$$

or, $$\frac{d^2\psi}{dx^2}+k^2\psi=0 \quad \text{--- (1)}.$$

where $k^2=\frac{2mE}{\hbar^2}$ --- (2),

$$\psi=A\sin kx+B\cos kx \quad \text{--- (3)}$$

Using boundary condition,

At $x=0$, $\psi=0$

$0=A\sin k\times 0+B\cos k\times 0$

$=0+B$

$\therefore B=0$

Now, From eq^n (3),

$\psi=A\sin kx$.

Again,

$\psi=0$ at $x=l$.

$\therefore 0=A\sin kl+0$

or, $\sin kl=0$

or, $\sin kl=\sin n\pi \ \{n=0,1,2,3\cdots\}$.

$kl=n\pi$ or, $k=\frac{n\pi}{l}$ --- (4)

Comparing eq^n (2) and (4), we get,

$$K^2 = \frac{2mE}{\hbar^2}$$

or, $$\frac{(n\pi)^2}{(l)^2} = \frac{2mE}{\hbar^2}$$

or, $$\frac{n^2\pi^2}{l^2} = \frac{2mE}{\hbar^2}$$

or, $$E = \frac{n^2\pi^2\hbar^2}{2ml^2}$$

Hence, energy of the particle in an infinitely deep potential is quantised.

which is the equation for energy eigen value for a particle in an infinitely deep potential well.

Now,

The wave function is given by,

$$\psi = A\sin kx$$

$$= A\sin \frac{n\pi}{l}x \quad [\because \text{from eq}^n\ 4]$$

Using normalising condition,

$$\int_0^l |\psi|^2 \cdot dx = 1.$$

$$\int_0^l A^2 \sin^2 \frac{n\pi}{l}x \cdot dx = 1$$

or, $$A^2 \int_0^l \sin^2 \frac{n\pi}{l}x \, dx = 1.$$

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$$\left[\begin{array}{l} \cos 2x = 1 - 2\sin^2 x \\ \sin^2 x = \dfrac{1-\cos 2x}{2} \end{array}\right]$$

or, $\dfrac{A^2}{2}\int_0^l \left(1 - \cos\dfrac{2n\pi x}{l}\right) dx = 1.$

or, $\dfrac{A^2}{2}\int_0^l dx - \dfrac{A^2}{2}\int_0^l \cancel{\cos\dfrac{2n\pi x}{l}\, dx}^{\,0} = 1$

or, $\dfrac{A^2}{2}\cdot l = 1$

or, $A = \sqrt{\dfrac{2}{l}}$

$\boxed{\psi = \sqrt{\dfrac{2}{l}} \cdot \sin\dfrac{n\pi x}{l}}$ and

Note*

$2^2 = 2\times 2^* = 2\times 2 = 4$

$(a+ib)^2 = (a+ib)(a+ib)^*$

$= (a+ib)(a-ib)$

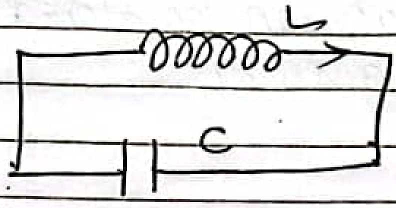
So, $\psi^2 = \psi \cdot \psi^*$

Chapter-6

Electromagnetism

LC-oscillation :

Consider a circuit of inductor with inductance 'L' and capacitor 'C' as shown in fig.(i).



$$L\frac{dI}{dt} + \frac{Q}{C} = 0$$

or, $$L\frac{d}{dt}\left(\frac{dQ}{dt}\right) + \frac{Q}{C} = 0$$

or, $$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0$$

or, $$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0 \quad \text{———} \quad (i)$$

Comparing with $\frac{d^2y}{dx^2} + \omega^2 x = 0$, of SHM, we get,

$Q = Q_0 \sin(\omega t + \phi)$ $\qquad \omega = \frac{1}{\sqrt{LC}}$

or, $2\pi f = \frac{1}{\sqrt{LC}}$

$\therefore f = \frac{1}{2\pi\sqrt{LC}}$

which is the ~~natural~~ frequency of LC-oscillation

Oscillation of electric and magnetic energy in LC circuit.

$$U_E = \frac{q^2}{2C}$$

The electric energy that can be stored in a capacitor at any time 't' is given by,

$$U_E = \frac{q^2}{2C}$$

$$= \frac{(q_0 \sin \omega t)^2}{2C} = \frac{\{q_0 \sin(\omega t + \phi)\}^2}{2C}$$

$$= \frac{q_0^2 \sin^2(\omega t + \phi)}{2C} \quad \text{——— (1)}$$

$$U_B = \frac{1}{2} L I^2$$

$$= \frac{1}{2} L \left(\frac{dq}{dt}\right)^2$$

$$= \frac{1}{2} L \left[\frac{d}{dt}\{q_0 \sin(\omega t + \phi)\}\right]^2$$

$$= \frac{1}{2} L \{q_0 \, \omega \cos(\omega t + \phi)\}^2$$

$$= \frac{1}{2} L q_0^2 \omega^2 \cos^2(\omega t + \phi) \qquad \left(\because \omega = \frac{1}{\sqrt{LC}}\right)$$

$$= \frac{1}{2} L \left(\frac{1}{LC}\right) q_0^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2C} q_0^2 \cos^2(\omega t + \phi)$$

$$= \frac{q_0^2 \cos^2(\omega t + \phi)}{2C} \quad \text{——— (2)}$$

at $\phi = 0$, $t = n \cdot \frac{T}{2}$, $n = 0, 1, 2, 3 \ldots$

$U_E = 0$ — min

$U_B = \frac{q_0^2}{2C}$ — max

Again, at $\phi = 0$, $t = \frac{(2n+1)}{2} \cdot T/_2$ where $n = 0, 1, 2, \ldots$

$U_E = \frac{q_0^2}{2C}$ — max

$U_B = 0$ — min

$U_{Total} = U_E + U_B$

$= \frac{q_0^2}{2C} \left(\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right)$

$= \frac{q_0^2}{2C}$ //

(Q) A radio tuner has a frequency range from 500kHz to 5MHz. If its LC-circuit has an effective inductance of 400 μH. What is the range of variable capacitor?

$f = \frac{1}{2\pi\sqrt{LC}}$

soln $f_1 = \frac{1}{2\pi\sqrt{L_1 C_1}}$ — (i)

$f_2 = \frac{1}{2\pi\sqrt{LC_2}}$ — (ii).

or, $500 \times 10^3 = \frac{1}{2\pi\sqrt{400 \times C_1}}$

or, $\sqrt{400 \times C_1} = \frac{1}{500\times10^3 \times 2\pi}$

or, $C_1 = \left(\frac{1}{500\times10^3\times2\pi}\right)^2 \cancel{\times 400\times10^{-6}}$

$= \cancel{6.6} \; \cancel{6.33\times10}$

$= \left(\frac{1}{500\times10^3\times2\pi}\right)^2 \times \frac{1}{400\times10^{-6}}$

$= 2.533\times10^{-10}\,F.$

for $f_2 = 5MHz$

$= 5\times10^6\,Hz.$

$C_2 = \left(\frac{1}{5\times10^6\times2\pi}\right)^2 \times \frac{1}{400\times10^{-6}}$

$= 2.535\times10^{-12}\,F$

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(2) You are given an inductor of 1mH. If you are asked to make it oscillate with a frequency of 1MHz, How can you make such an oscillatory device?

Sol^n $L = 1mH = 1 \times 10^{-3} H$

$f = 1MHz = 10^6 Hz$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

or, $\sqrt{LC} = \frac{1}{2\pi f}$

or, $LC = \left(\frac{1}{2\pi f}\right)^2$

or, $C = \left(\frac{1}{2\pi f}\right)^2 \times \frac{1}{L}$

$= \left(\frac{1}{2\pi \times 10^6}\right)^2 \times \frac{1}{10^{-3}}$

$= 2.53 \times 10^{-11}$ F $_{//}$

(3) What inductance must be connected to 17pF capacitor in an oscillator capable of ~~500n~~ 550nm electromagnetic wave?

Sol^n ~~$L = 17p$~~ $\lambda = 550nm$

$= 550 \times 10^{-9} m$.

$C = 17pF = 17 \times 10^{-12} F$

We have,

$c = \lambda = \frac{c}{v}$ $\therefore v = 5.45 \times 10^{14}$

or, $c = f\lambda$ ~~$f - v = f\lambda$~~

$\therefore f = 1.009 \times 10^{-21}$ Hz or, $f = \frac{c}{\lambda} = \frac{3\times10^{8}}{550\times10^{-9}} = 5.45\times10^{14}\ Hz.$

$\cancel{\omega}\ f = \frac{1}{2\pi\sqrt{LC}}$

or, $LC = \left(\frac{1}{2\pi f}\right)^2$

or, $L = \left(\frac{1}{2\pi f}\right)^2 \times \frac{1}{C}$

$= \left(\frac{1}{2\pi\times5.45\times10^{14}}\right)^2 \times \frac{1}{17\times10^{-12}}$

$= 5.016\times10^{-21}\ H$ //.

(4) If 10mH inductor and two capacitors of 5uF and 2uF are given. find the two resonant frequencies that can be ~~observed~~ obtained by connecting these elements in different ways.

$L = 10mH = 10\times10^{-3}H.$
$C_1 = 5uF = 5\times10^{-6}F$
$C_2 = 2uF = 2\times10^{-6}F.$

Sol^n $f = \frac{1}{2\pi\sqrt{LC}}$ —— ①

$f_1 = \frac{1}{2\pi\sqrt{LC_1}}$

$= \frac{1}{2\pi\sqrt{10\times10^{-3}\times5\times10^{-6}}}$

$=$

At series,

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

or, $C = \frac{C_1C_2}{C_1+C_2}$

$= \frac{5\times2}{5+2} = \frac{10}{7}\ uF.$

$$f_1 = \frac{1}{2\pi\sqrt{10\times10^{-3}\times\frac{10}{7}\times10^{-6}}}$$

$$= 1331.58\,Hz.$$

At parallel,

$$C_e = C_1 + C_2$$
$$= 5+2 = 7\,\mu F.$$

$$f_2 = \frac{1}{2\pi\sqrt{LC_2}}$$
$$= \frac{1}{2\pi\sqrt{10\times10^{-3}\times 7\times10^{-6}}}$$
$$= 601.54\,Hz.$$

(5) . In an oscillating LC circuit, what value of charge expressed in terms of maximum charge is present on the capacitor when the energy is shared equally between the electric and magnetic fields? At what time will this condition occur, assuming the capacitor to be fully charged initially? Assume that $L = 10\,mH$ and $C = 1.0\,\mu F$

$$U = \frac{q^2}{2C} \quad \text{——} \quad (1)$$

$$U_E = U_B$$

or, $$\frac{q_0^2 \sin(\omega t+\phi)}{2C} = \frac{q_0^2 \cos^2(\omega t+\phi)}{2C}$$

According to question,

$$U_E = U_B$$

$U_E = U_B$.

or, $U_{max} = U_E + U_B$

$= \frac{q_0^2}{2C}$ $2U_E$.

$U_E = \frac{1}{2}\frac{q_0^2}{2C}$ $\frac{U_{max}}{2}$

$= \frac{q_0^2}{4C}$ $= \frac{q_0^2}{4C}$ or, $\frac{q^2}{2C} = \frac{q_0^2}{2 \cdot 4C}$

$L = 10mH = 10 \times 10^{-3} H$ or, $q = \sqrt{\frac{q_0^2}{2}}$

$C = 1.0 \mu F = 1.0 \times 10^{-6} F$ $= \frac{q_0}{\sqrt{2}}$

$q = \frac{q_0}{\sqrt{2}}$

Initially $\phi = 0$

$q = q_0 \sin(\omega t + \phi)$

$= q_0 \sin \omega t$

or, $\frac{q_0}{\sqrt{2}} = q_0 \sin \omega t$

or, $\sin \omega t = \frac{1}{\sqrt{2}}$ $\Longrightarrow$ (x).

or, $\sin \omega t = \sin\left(\frac{\pi}{4}\right)$

or, $\omega t = \pi/4$.

or, $t = \frac{\pi}{4\omega}$

$= \frac{\pi}{4 \times \frac{1}{\sqrt{LC}}} = \frac{\pi}{4 \times \frac{1}{\sqrt{10^{-3} \times 10 \times 10^{-6}}}}$

$= 7.85 \times 10^{-5}$ sec

$$\omega t = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

or, ~~$\omega t = \pi$~~

or, $\frac{1}{\sqrt{LC}} \times t = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

The electrical - mechanical analogy:-

	Block-spring system	LC osullatory
Energy	$P.E = \frac{1}{2}Kx^2$ (spring)	$U_C = \frac{Q^2}{2C}$
	$K.E = \frac{1}{2}mv^2$ (block)	$U_B = \frac{1}{2}LI^2$
parameters	K	$1/C$
	x	Q
	$v = dx/dt$	$I = dQ/dt$.
	m	L
diffⁿ equation	$\frac{d^2x}{dt^2} + \frac{K}{m}x = 0$	$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0.$
Solution :	$x = A\sin(\omega t + \phi)$	$Q = Q_0\sin(\omega t + \phi)$
Angular frequency	$\omega = \sqrt{\frac{K}{m}}$	$\omega = \frac{1}{\sqrt{LC}}$

LC osullation with Resistance (Damped oscillation in LCR circuit);

$$L\frac{dI}{dt} + IR + \cancel{VC}\ \frac{q}{c} = 0$$

or, $$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{c} = 0 \quad \text{--- (1)}$$

$$m\frac{d^2x}{dt^2} + b\cdot\frac{dx}{dt} + kx = 0 \quad \text{--- (2)}$$

Also, By comparing,

$$m = L,\ b = R,\ k = \frac{1}{C}$$

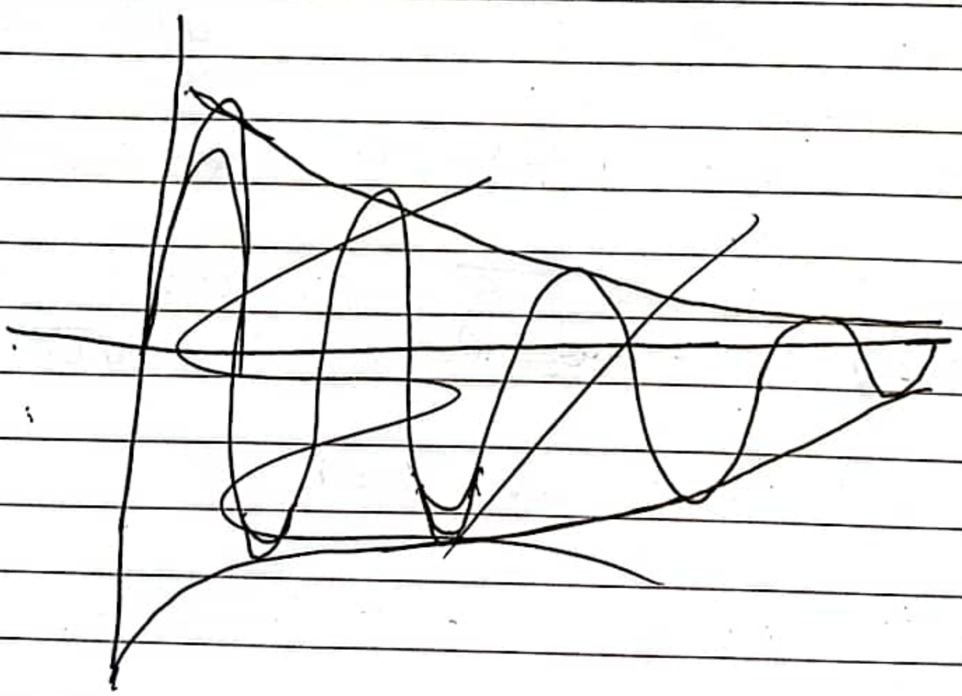
$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

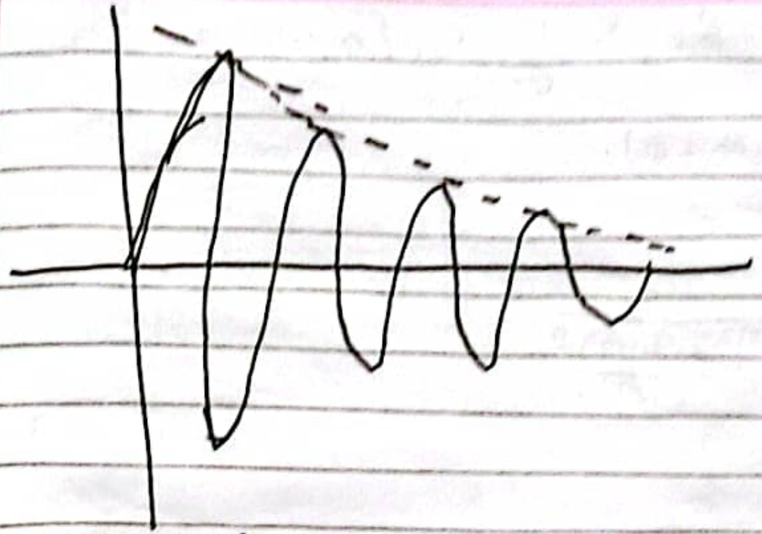
or, $$\omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$x = x_0 e^{-bt/2m} \sin(\omega t + \phi)$$

$$\cancel{Q = Q_0 e^{-\frac{bt}{2}}}$$

$$Q = Q_0 e^{-\frac{Rt}{2L}} \sin(\omega t + \phi)$$



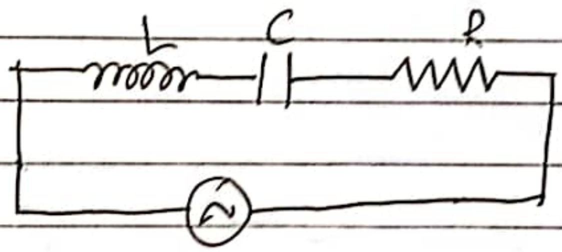


$\frac{1}{LC} > \left(\frac{R}{2L}\right)^2 \Rightarrow$ oscillatory. $(\omega > 0)$

$\frac{1}{LC} = \left(\frac{R}{2L}\right)^2 =$ critically damped. $(\omega = 0)$

$\frac{1}{LC} < \left(\frac{R}{2L}\right)^2 \Rightarrow$ non-oscillatory. $(\omega < 0)$.

Forced LCR oscillation [Forced electromagnetic (em) oscillation].



figure(1).

$$L\frac{dI}{dt} + IR + \frac{Q}{C} = E_0 \sin\omega t$$

or, $$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E_0\sin\omega t \quad \text{—— (1)}$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\sin\omega t \quad \text{—— (2)}$$

$m = L, \quad b = R, \quad k = \frac{1}{C}, \quad \varepsilon_0 = F_0.$

$\rightarrow q = q_0 \sin(\omega t + \phi)$

$$q_0 = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega_d^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$

$$Q_0 = \frac{\varepsilon_0/L}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{R\omega}{L}\right)^2}}$$

$Q = Q_0 \sin(\omega t + \phi)$

$$I = \frac{dQ}{dt}$$

$$= \frac{d}{dt}\left(Q_0 \sin(\omega t + \phi)\right)$$

$$= Q_0 \omega \cos(\omega t + \phi)$$

$\therefore I = I_0 \cos(\omega t + \phi).$

where, $I_0 = Q_0 \omega$

$$I_0 = \frac{\varepsilon_0 \frac{\omega}{L}}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{R\omega}{L}\right)^2}}$$

$$\therefore I_0 = \frac{\varepsilon_0 \omega / L}{\frac{\omega}{L}\sqrt{\frac{L^2}{\omega^2}(\omega_0^2 - \omega^2)^2 + R^2}}$$

$$= \frac{\varepsilon_0}{\sqrt{\left(\frac{L}{\omega}\cdot\omega_0^2 - \frac{L}{\omega}\cdot\omega^2\right)^2 + R^2}}$$

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$$= \frac{\mathcal{E}_0}{\sqrt{\left(\frac{L\omega_0^2}{\omega} - L\omega\right)^2 + R^2}}$$

$$= \frac{\mathcal{E}_0}{\sqrt{\left(\frac{L}{\omega}\cdot\frac{1}{LC} - L\omega\right)^2 + R^2}}$$

$$= \frac{\mathcal{E}_0}{\sqrt{\left(\frac{1}{\omega C} - L\omega\right)^2 + R^2}} = \frac{\mathcal{E}_0}{\sqrt{(X_C - X_L)^2 + R^2}}$$

$$\left[\because X_C = \frac{1}{\omega C},\quad X_L = L\omega\right]$$

$$I_0 = \frac{\mathcal{E}_0}{\sqrt{(X_L - X_C)^2 + R^2}} = \frac{\mathcal{E}_0}{Z}$$

where, $Z = \sqrt{(X_C - X_L)^2 + R^2}$ which is the impedance · is the total resistance of the circuit.

Resonance, $X_C = X_L$

or, $\frac{1}{\omega C} = L\omega$

or, $\frac{1}{LC} = \omega^2$

$\therefore \omega = \frac{1}{\sqrt{LC}}$

or, $f = \frac{1}{2\pi\sqrt{LC}}$ is called resonance frequency.

Electro-magnetic Wave

Maxwell's Equation:

① Gauss Law in Electrostatics:

$$\phi_E = \frac{q}{\epsilon_0}$$

$$\oint \vec{E}\cdot\vec{dA} = \frac{q}{\epsilon_0}$$

Faraday's law of em induction

$$\varepsilon = -\frac{d\phi_B}{dt}$$

$$\oint \vec{E}\cdot\vec{dl} = \frac{-d\phi_B}{dt}$$

There are four Maxwell equations:

① Gauss law in electrostatics

It states that, "Total electric flux through a closed surface is equal to $\frac{1}{\epsilon_0}$ times the electric charge enclosed by that surface. i.e,

$$\phi_E = \frac{q}{\epsilon_0}$$

$$\text{or, } \oint \vec{E}\cdot\vec{dA} = \frac{q}{\epsilon_0}$$

It confirms the existence of single charge (electric monopole).

② Gauss law in magnetism

It states that, "Total magnetic flux through a closed surface is equal to 0". i.e;

$$\phi_B = 0$$

$$\text{Or } \oint \vec{B}\cdot\vec{dA} = 0$$

[It confirms the non-existence of magnetic monopole]

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(3) Faraday's law of electromagnetic induction:

It states that, " Induced emf in a circuit is equal to the negative of a time rate change in magnetic flux.

$$\varepsilon = -\frac{d\phi}{dt}.$$

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt}.$$

It tells, change in magnetic flux produces electric field.

(4) Ampere's - Maxwell Law:

This is the modification of Ampere's law by Maxwell. According to this law, there are at least two ways of setting of magnetic field

(a) by means of steady current

i.e. According to Ampere's law, $\oint \vec{B} \cdot \vec{dl} = \mu_0 I$

(b) By means of change in electric flux.

i.e. According to Maxwell law of induction

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \epsilon_0 \frac{d}{dt}(\phi_E)$$

Therefore, $\oint \vec{B} \cdot \vec{dl} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$

$$= \mu_0 \left(I + \epsilon_0 \frac{d\phi_E}{dt} \right)$$

$$= \mu_0 (I + I_d)$$

where $I_d = \epsilon_0 \frac{d\phi_E}{dt}$

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Displacement Current:

Although no charge actually moves across the gap between the plates of a capacitor, the displacement current can be considered as fictitious (imaginary) current which is equivalent to change in electric flux ~~due to the~~ during the charging & discharging of capacitor.

Conversion of Maxwell Equations into Differential form:

① $\oint \vec{E}\,\vec{dA} = \frac{q}{\epsilon_0}$

From Gauss divergence theorem,

$\oint \vec{E}\cdot\vec{dA} = \int \nabla E \cdot dv$

Now,

$q = \int \rho\, dv$

$\oint \nabla E \cdot dv =$

Note*

$i = 0, 1, 0, 0$

$j = 0, 1, 0$

$k = 0, 0, 1$

$\left(\nabla = \frac{i\partial}{\partial x}, \frac{j\partial}{\partial y}, \frac{k\partial}{\partial z}\right)$

$\nabla^2 = \nabla\cdot\nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$\rho = \frac{q}{V}$ $\therefore q = \rho V$

①.

Gauss law in electrostatic is given by;

$$\oint \vec{E}\cdot\vec{dA} = \frac{q}{\epsilon_0} \quad \text{———} ①$$

From Gauss divergence theorem,

$$\oint \vec{E}\cdot\vec{dA} = \int \nabla E \cdot dv$$

We know,

$$q = \int \rho\, dv$$

$\int dA$ - 6

Substituting above values in eq^n ①.

$$\int \nabla \cdot E \cdot dV = \frac{1}{\epsilon_0} \int \rho \, dV$$

$$\Rightarrow \int \nabla \cdot E \cdot dV = \int \frac{\rho}{\epsilon_0} dV$$

$$\Rightarrow \boxed{\nabla \cdot E = \frac{\rho}{\epsilon_0}} \text{ ——— (ii)}$$

② Gauss law in magnetism is given by;

$$\oint \vec{B} \cdot \vec{dA} = 0 \text{ ——— ①}$$

From Gauss divergence method,

$$\oint \vec{B} \cdot \vec{dA} = \int \nabla \cdot B \, dV$$

Therefore,

$$\int \nabla \cdot \vec{B} \, dV = 0$$

$$\therefore \boxed{\nabla \cdot \vec{B} = 0}$$ //.

③ From Faraday's law of electromagnetic induction.

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt} \text{ ——— ①}$$

From Stoke's theorem, $\oint \vec{E} \cdot \vec{dl} = \int \nabla \times \vec{E} \cdot dA$

$$\frac{d\phi_B}{dt} = \frac{d}{dt} \int \vec{B} \cdot \vec{dA}$$

$$= \int \frac{dB}{dt} \vec{dA}.$$

Reverse.

Substituting,

$$\int \nabla\times\vec{E}\, \vec{dA} = -\int \frac{dB}{dt}\, \vec{dA}$$

Therefore,

$$\boxed{\nabla\times E = -\frac{dB}{dt}}$$

$$\nabla E = \frac{-dB}{dt}$$

$$\int \nabla \vec{E}\cdot\vec{dA} = -\int \frac{dB}{dt}\, \vec{dA}$$

$$= \int \nabla\vec{E}\cdot\vec{dA} = \int \frac{-dB}{dt}\, \vec{dA}$$

$$\frac{d\Phi_B}{dt} = \frac{dB}{dt}\oint \vec{E}\cdot\vec{dl} = \int \nabla\vec{E}\cdot\vec{dA}$$

(B) Ampere's Maxwell's law is given by:

$$\oint \vec{B}\cdot\vec{dl} = \mu_0\epsilon_0 \frac{d\Phi_E}{dt} \quad \text{——— (1)}$$

$$\oint \vec{B}\cdot\vec{dl} = \mu_0(I + I_d), \quad I_d = \epsilon_0 \left(\frac{d\Phi_E}{dt}\right)$$

From Stoke's theorem,

$$\oint \vec{B}\cdot\vec{dl} = \int \nabla\times\vec{B}\, \vec{dA}$$

$$\oint \vec{B}\cdot\vec{dl} = \int \nabla\times\vec{B}\cdot\vec{dA}$$

$$\left[J = \frac{I}{A}\right.$$

$$I = J\cdot A$$

Since, $I = \int J\, dA$.

& ~~where~~, $\boxed{I_d = \epsilon_0 \frac{d\Phi_E}{dt}} = \epsilon_0 \frac{d}{dt}\int \vec{E}\cdot\vec{dA}$

Therefore,

$$\int \nabla\times\vec{B}\cdot\vec{dA} = \mu_0\left(\int J\,dA + \epsilon_0 \frac{d}{dt}\int \vec{E}\cdot\vec{dA}\right)$$

$$= \mu_0\left(\int J\,dA + \epsilon_0 \int \frac{dE}{dt}\, \vec{dA}\right)$$

$$= \mu_0\int J\,dA + \mu_0\epsilon_0 \int \frac{dE}{dt}\cdot\vec{dA}$$

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$$\int \mu_0 J\, dA + \mu_0 \int \epsilon_0 \frac{dE}{dt} \cdot d\vec{A}$$

$$= \int \mu_0 \left(J + \epsilon_0 \frac{dE}{dt}\right) dA$$

$$\nabla \times \vec{B} = \mu_0 \left(J + \epsilon_0 \frac{dE}{dt}\right) //$$

Wave Equations in free space:-

In free space, the charge density and current density are zero, i.e. $\rho = 0$, $J = 0$. Therefore, maxwell equations can be written as;

$$\nabla \cdot \vec{E} = 0 \quad \text{---} (1)$$

$$\nabla \cdot \vec{B} = 0 \quad \text{---} (2)$$

$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt} \quad \text{---} (3)$$

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{dE}{dt} \quad \text{---} (4)$$

Note:

$\nabla \cdot \vec{K} =$ divergence

$\nabla \times \vec{K} =$ curl

$\vec{A} \times \vec{B} \times \vec{C} = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

From eqn (3), we get,

$$\nabla \times \nabla \times \vec{E} = -\frac{d}{dt}(\nabla \times \vec{B}) \quad [\because \nabla \cdot \vec{E} = 0]$$

or, $$\nabla(\nabla \cdot \vec{E}) - (\nabla \cdot \nabla)\vec{E} = -\frac{d}{dt}\left(\mu_0 \epsilon_0 \frac{dE}{dt}\right)$$

or, $$-\nabla^2 E = -\mu_0 \epsilon_0 \frac{d^2 E}{dt^2}$$

or, $$\nabla^2 E = \mu_0 \epsilon_0 \frac{d^2 E}{dt^2} \quad \text{---} (5)$$

Taking the ~~curl~~ curl of eq^n 3 , in terms of magnetic field.

or, $\nabla^2 B = \mu_0 \epsilon_0 \frac{d^2 B}{dt^2}$ —— 6

or, $E = E_0 \sin(\omega t - kx)$
$B = B_0 \sin(\omega t - kx)$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{d^2 y}{dt^2}$$

$\frac{1}{v^2} = \mu_0 \epsilon_0$.

$v^2 = \frac{1}{\mu_0 \epsilon_0}$

$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$\mu_0 = 4\pi \times 10^{-7}$

$\epsilon_0 = 8.85 \times 10^{-12}$

$v = 3 \times 10^8 \, m/s$.

(*)

eq^n 5 & eq^n 6 are the wave equation in free space.

The solution of eq^n 5 & 6 can be written as;

Comparing eq^n 5 & 6 with general wave equation.

$E = E_0 \sin(\omega t - kx)$

$B = B_0 \sin(\omega t - kx)$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{d^2 y}{dt^2} \; //.$$

(*) → which is the same as velocity of light in free space.

This shows that light is an electromagnetic wave.

Wave Equations in Non-conducting (dielectric) medium:

Consider a dielectric medium with permeability μ, permittivity ϵ, the charge density and current density are zero. ∴ the

The Maxwell equations can be written as,

$$\nabla \cdot \vec{E} = 0 \quad \text{—— (1)}$$

$$\nabla \cdot \vec{B} = 0 \quad \text{—— (2)}$$

$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt} \quad \text{—— (3)}$$

$$\nabla \times \vec{B} = \mu\epsilon \frac{dE}{dt} \quad \text{—— (4)}$$

Taking curl of equation (3), we have,

$$\nabla \times \nabla \times \vec{E} = \nabla \times -\frac{d\vec{B}}{dt}$$

$$= -\frac{d}{dt}(\nabla \times \vec{B})$$

or, $\nabla(\nabla \cdot \vec{E}) - (\nabla \cdot \nabla)\vec{E} = -\frac{d}{dt}\left(\mu\epsilon \frac{d\vec{E}}{dt}\right)$ [∵ using eqⁿ (4)]

or, $-\nabla^2\vec{E} = -\mu\epsilon \frac{d^2\vec{E}}{dt^2}$ [∵ $\nabla \cdot \vec{E} = 0$]

or, $\nabla^2\vec{E} = \mu\epsilon \frac{d^2\vec{E}}{dt^2}$ —— (5)

Similarly, taking the curl of equation (4) and proceeding in the same way as above we get,

$$\nabla^2\vec{B} = \mu\epsilon \frac{d^2\vec{B}}{dt^2} \quad \text{—— (6)}$$

Eqⁿ (5) and (6) are the equations of electromagnetic wave motion in non-conducting medium.

Comparing the above equations with general wave equation,

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}, \text{ we get,}$$

$$v = \frac{1}{\sqrt{\mu \epsilon}}$$

<u>Ques</u> Using Maxwell equation, prove that, $c = \frac{E_m}{B_m}$ where symbols carry their usual meaning.

Solution: From third Maxwell Equation, we have, In one-dimensional form, we can write as,

$$\nabla \times E = -\frac{dB}{dt}$$

$$\frac{dE}{dx} = -\frac{dB}{dt} \quad \text{---} \quad (1)$$

We know,

$$E = E_m \sin(\omega t - kx)$$

$$\frac{dE}{dx} = E_m \cos(\omega t - kx)(-k) \quad \text{---} \quad (2)$$

Again,

$$B = B_m \sin(\omega t - kx)$$

$$\frac{dB}{dt} = B_m \cos(\omega t - kx)(\omega) \quad \text{---} \quad (3)$$

Using eq^n (2) & (3) in eq^n (1), we get,

$$-k E_m \cos(\omega t - kx) = -B_m \cos(\omega t - kx)\, \omega$$

$$\text{or, } E_m k = \omega \cdot B_m.$$

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or, $\frac{E_m}{B_m} = \frac{\omega}{k}$

or, $\frac{E_m}{B_m} = \frac{2\pi f}{\frac{2\pi}{\lambda}}$

$= f \times \lambda$

$= c$

$\therefore$ $c = \frac{E_m}{B_m}$

Again,

$\frac{E}{B} = \frac{E_m}{B_m}$

$\therefore$ $\boxed{c = \frac{E}{B}}$.

① Prove charge conservation theorem,

② derive continuity equation

③ show that $\nabla \cdot J + \frac{d\rho}{dt} = 0$, where symbols have their usual meanings.

From 4th maxwell equation,

$\nabla \times B = \mu_0 \left(J + \epsilon_0 \frac{dE}{dt}\right)$

Taking divergence on both sides;

$\nabla \cdot (\nabla \times B) = \nabla \left(\mu_0 J + \mu_0 \epsilon_0 \frac{dE}{dt}\right)$

Since, divergence of curl of any vector is zero;

$0 = \nabla \mu_0 J + \mu_0 \epsilon_0 \frac{d(\nabla \cdot E)}{dt}$

$\Rightarrow \nabla J + \epsilon_0 \frac{d}{dt}\left(\frac{\rho}{\epsilon_0}\right) = 0 \quad \left[\because \nabla \cdot E = \frac{\rho}{\epsilon_0}\right]$

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$$\boxed{\nabla \cdot J + \frac{d\rho}{dt} = 0}$$ //.

(Q) What is the displacement current for a capacitor having radius 5cm with variable electric field 8.9×10^{12} volts /m·s.?

Sol^n $r = 5cm = 5 \times 10^{-2} m = 0.05m$

$\frac{dE}{dt} = 8.9 \times 10^{12}$ volts /m·s.

$$I_d = \epsilon_0 \frac{d\phi_E}{dt}$$

$$= \epsilon_0 \frac{d}{dt}(E \cdot A)$$

$$= \frac{d\, \epsilon_0 A\, dE}{dt}$$

$$= 8.85 \times 10^{-12} \times \frac{\pi \times 0.1^2}{4} \times 8.9 \times 10^{12}$$

$$= 0.61 A_{//}$$

(Q2) A parallel plate capacitor has capacitance 20μF. At what rate, the potential difference between the plates must be changed to produce displacement current of 1.5A?

$C = 20\mu F$

$\frac{dV}{dt} = ?$

$I_d = 1.5A$

We have,

$$I_d = \epsilon_0 \frac{d\phi_E}{dt}$$

$$= \epsilon_0 A \frac{dE}{dt}$$

$$= \epsilon_0 A \frac{d}{dt}\left(\frac{V}{d}\right)$$

$$= \frac{\epsilon_0 A}{d} \frac{dV}{dt}.$$

$$= C \frac{dV}{dt}$$

or, $1.5 = 20 \times 10^{-6} \times \frac{dV}{dt}$.

$\therefore \frac{dV}{dt} = 75{,}000$

$= 7.5 \times 10^4$ volts/sec.

(Q3) A sinusoidal voltage is applied across $8\mu F$ capacitor. The frequency of the voltage is 3 kHz and the voltage amplitude is 30V. Find the maximum value of displacement current.

Soln: According to question, $V = V_0 \sin\omega t$

$C = 8\mu F$.

$f = 3kHz$

$V_0 = 30V$

$I_d(max) = ?$