

## Chapter-8

### Fundamentals of thermodynamics and heat transfer

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#### 8.3. Heat transfer:

Heat is a common form of energy which is continuously being transferred from one body to another. In earlier time, during the development of concept of heat and temperature, people had only the concept of inequality for flow of heat. According to this concept, heat always flows from a body of higher temp. to a body of lower temp. Actually, heat always flows until there is difference in temp.

There are three different modes of transfer of heat. Transfer of heat due to actual motion of molecules in fluid (liquid and gas) is called convection, e.g. boiling of water. Transfer of heat due to molecular vibrations in a solid is called conduction, e.g. heating of metal rod. Transfer of heat without support of any material medium is called radiation, e.g. receiving heat from sun.

These modes of heat transfer are described as following:

##### ① Conduction:

Conduction is the process of transfer of heat from one point to another point of a body carried out by means of collisions between rapidly vibrating atoms at hotter region and slowly vibrating atoms at colder region. There is no actual transfer of particles during conduction.



When one end of a solid is heated, atoms at hotter end vibrate with greater amplitude and have more K.E. than neighbouring atoms at colder part. The atoms at hotter region collide with and give up some energy to the neighbouring atoms in colder regions. Similar process continues between each set of neighbouring atom upto next end. As a result transfer of heat takes place and previously cold end also gets heated. This method of transfer of heat is called conduction.

### ② Convection:

Convection is the process of heat transfer of heat in fluids by means of actual motion of heated particles from higher temperature region to lower temp. region.

Heated particles carry heat and move from hot region to cold region but cold particles move in opposite direction. The current set up in the process is called convection current. This method is not possible in solid and vacuum. Heating of water, land breezes, sea breezes, wind etc. are some examples of convection.

~~Explain~~

### ③ Radiation:

Radiation is the process of transmission of heat from one point to another without need of any material medium. Radiation does not heat the medium through which heat energy passes.

In radiation, heat is transformed in the form of e.m. radiation (wave) and travel with speed of  $3 \times 10^8$  m/sec. in vacuum. Heat energy coming to the earth from the sun, and transmission of heat around fire are the examples of radiation. Heat energy radiated by an object is called radiant energy or thermal energy.

#### ② Statement and assumption of Fourier law of thermal conductivity:

The ability of anybody to conduct heat is measured in term of thermal conductivity. Therefore, Ability which measures the thermal conduction of material is called thermal conductivity. Thermal conductivity generally occurs in solids.

#### Statement of Fourier law of Thermal Conductivity:

The Fourier law states that the rate of heat flow in solids is directly proportional to the cross-section area perpendicular to the flow axis and negative of temp. gradient over the length of conductor.

According to Fourier's law, the rate of heat flow through a homogeneous solid is directly proportion -al to the area A of the section at the right angles to the direction of heat flow, and to the temp. difference  $\Delta T$  along the path of heat flow.

$$\text{i.e. } Q = -kA \frac{dT}{dx} \quad \text{---(1)}$$

where

$\dot{Q}$  = rate of heat transfer (watt)

$K$  = Thermal Conductivity ( $\text{W/m}\cdot\text{K}$ )

$A$  = Area of cross-section ( $\text{m}^2$ )

$dT$  = change in temp. along the dirn. of heat flow

$dx$  = thickness of the object.

In the fourier heat conduction eq<sup>n</sup> the -ve sign implies that heat is flowing from higher temp. to lower temp. therefore it is provided to compensate for the negative nature of the temp. gradient.

Assumption in fourier law of heat conduction:

Following are the assumptions of fourier law of heat conduction:

- (1) The thermal conductivity of the material is constant throughout the material.
- (2) There is no internal heat generation that occurs in the body.
- (3) The temp. gradient is considered as constant.
- (4) The heat flow is unidirectional and takes place under Steady-state conditions.
- (5) The surfaces are Isothermal.

(6) Temperature gradient

The rate of fall in temp. w.r.t. distance along the dirn. of heat flow is called the temp. gradient.

It is expressed as  $-\frac{dT}{dx}$  and its unit is Kelvin per meter (K/m). Here  $dT$  is the small change in temp. over a small distance  $dx$  and -ve sign indicates that temp. falls with distance.

One dim! steady state heat conduction through plane wall

Consider parallel sided plane wall of

- (b) thickness  $L$  and uniform cross-section area  $A$ , as shown in figure.

Let  $K$  be the thermal conductivity of wall material through which heat is freezing only in  $x$ -direction.

Let  $T_1$  and  $T_2$  are the temp. of

higher temp. face-1 and lower temp. [Fig: plane wall]

face-2 The small change of temp.  $dT$  when the particles conduct ~~far~~ very small distance.

According to Fourier law, the rate of heat transfer is given by

$$\frac{d\phi}{dt} = -kA \frac{dT}{dx}$$

$$\gamma \frac{d\phi}{dt} dx = -KAdT \quad \text{--- (1)}$$

To obtain the rate of flow of heat through this whole wall is obtained by integrating eqn(1), we get

$$\int_L \frac{dQ}{dt} dx = -kA \Delta T$$

$$\text{or } \frac{d\theta}{dt} = -kA \int_{T_1}^{T_2} dx$$

$$g \frac{dg}{dT} [L_0] = -KA[T_2 - T_1]$$

$$\text{or } \frac{d\phi}{dt} = \frac{\kappa A (T_1 - T_2)}{L}$$

$$\text{Ansatz: } \frac{d\varrho}{dt} = \frac{(T_1 - T_2)}{L/kA} \quad \text{---(2)}$$

- Where  $\frac{L}{KA}$  = Thermal resistance  $R_{th}$

$$\therefore \frac{d\phi}{dt} = \frac{T_1 - T_2}{R_{th}} \quad \text{---(3)}$$

This eq<sup>n</sup> ③ gives the rate of flow of heat through the wall. It increases,  $\frac{dQ}{dt}$  decreases and vice-versa.

## Black body and black body radiation

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A perfect black body is one which absorbs heat radiation of all wavelengths falling on it and emits radiations of all possible wavelengths when heated.

For a perfect black body absorptance ( $\alpha$ ) = 1 and emissivity ( $\epsilon$ ) = 1. When a body absorbs all radiant energy, it neither reflects nor it transmits. As a result body appears black and known as black body. Sun emits radiations of all wavelengths. So it can be considered as a black body. A good absorber is a good emitter. Therefore, a black body is a good emitter and can emit the radiation of all possible wavelengths.

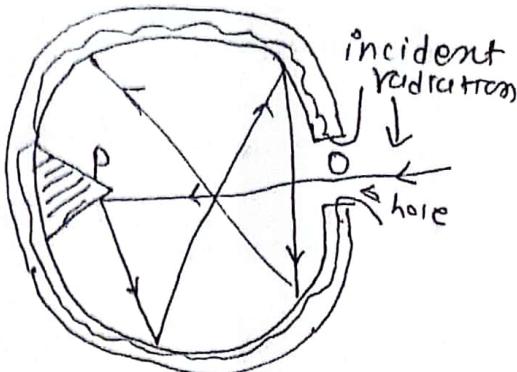
A perfect black body cannot be realized in actual practice. The nearest approach to a perfectly black body is a surface coated with lamp black or platinum black. Such a surface absorbs 96% to 98% of the incident radiation. Ferry's body is an example of perfectly black body realized in practice.

### Ferry's black body:

Ferry's black body is a close approximation of perfectly black body which can be realized in practice. Ferry's body consists of a double walled hollow copper sphere with a small fine hole O and a conical projection P opposite to O as shown in figure.

A heating filament is kept in between two walls of the sphere and inner surface of sphere is coated with lamp black or platinum black.

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[Fy: Ferry's black body]

When heat radiation falls on the conical projection P through hole O, projection P prevents the direct reflection of incident radiation rather it causes multiple reflections of radiations inside the sphere. Most of the incident radiation is absorbed by lamp black or platinum black and remaining part is also lost by multiple reflection. When the sphere is heated by passing electric current to the heating filament, radiations of all possible wavelengths come out from the hole O. Hence, the hole O behaves as a black body.

### Emissive power

It is the amount of heat radiations radiated by a unit area of a black body per unit time at a particular temp.

It is denoted by E. Mathematically,

$$\text{Emissive power } E = \frac{\text{Energy radiated } (\Omega)}{\text{Area } (A) \times \text{time } (t)} = \frac{\Omega}{A \times t}$$

$$\therefore E = \frac{\Omega/t}{A} = \frac{P}{A}$$

$$\text{unit of } E = \frac{\text{J/see}}{\text{m}^2} = \text{W/m}^2.$$

According to Stefan's-Boltzmann law,

$$E = \sigma T^4 \quad \text{where } \sigma = \text{Stefan-Boltzmann constant} = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4.$$

### Emissivity ( $\epsilon$ ):

Emissivity of a body is defined as the ratio of emissive power of a given body to the emissive power of a perfectly black body of same size at the same temp. It is denoted by  $\epsilon$ .

$$\therefore e = \frac{\text{emissive power of body}}{\text{emissive power of perfect black body.}} = \frac{E}{E_b} \quad \text{of}$$

Emissivity has no unit and dim? Value of  $e$  depends upon the nature of the surface of the emitting body. Its value ranges from 0 to 1.

For a perfectly black body,  $e = 1$

" a black body  $e < 1$

For a perfectly reflecting surface  $e = 0$ .

### Stefan's-Boltzmann's law of black body radiation:

Stefan's declared experimentally that energy radiated by a body depends only on its temp. and he put forward a law known as Stefan's law.

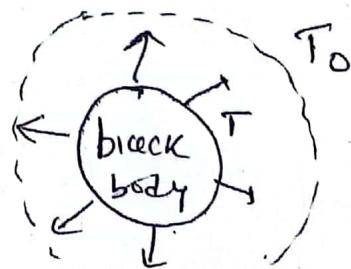
Stefan's-Boltzmann law states that " emissive power of a black body is directly proportional to the fourth power of absolute temp. of surface of the body!"

Let  $E$  be the emissive power of a black body at absolute temp.  $T$ . Then, according to Stefan's-Boltzmann law  $E \propto T^4$

or  $E = \sigma T^4 \rightarrow ①$  where  $\sigma$  is proportionality constant is called Stefan's Const. The value of  $\sigma$  in S.I. system is  $5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^4$ .

If a body is not perfectly black, and has its emissivity  $e$ . Then,

$$E = e \sigma T^4 \rightarrow ②$$



Fy: Stefan's law of 7

$$\frac{Q}{A+t} = \epsilon \sigma T^4 \text{ or } \frac{Q/t}{A} = \epsilon \sigma T^4 \Rightarrow \frac{\text{power}}{A} = \epsilon \sigma T^4$$

$$\therefore P = \epsilon A \sigma T^4. \quad (3)$$

The relations given above are valid only if the temp. of the surrounding is zero kelvin. But, If there is a surrounding at temp.  $T_0$  (more than 0K) then, surrounding also emits heat radiation, which is absorbed by black body. Hence, relation is modified by Boltzmann for the net emissive power as

$$E' = \epsilon(T^4 - T_0^4) \quad (4) \text{ This is called Stefan's-Boltzmann law.}$$

If a body is not perfectly black body, the net emissive power  $E' = \epsilon\epsilon(T^4 - T_0^4)$

$$\frac{Q'}{A+t} = \epsilon\epsilon(T^4 - T_0^4)$$

$$\text{or } \frac{Q'/t}{A} = \epsilon\epsilon(T^4 - T_0^4)$$

$$\therefore P' = \epsilon A \epsilon (T^4 - T_0^4) \quad (5)$$

This gives power of radiated energy of a black body.

### Some Numerical problems

- ① An insulating material having a thermal conductive of  $0.08 \text{ W/m-K}$  is used to limit the heat transfer of  $80 \text{ W/m}^2$  for a temp. of  $15^\circ\text{C}$  across the opposite faces. Find the thickness of the material.

$$\text{Given: } k = 0.08 \text{ W/m-K}, \frac{Q}{A} = 80 \text{ W/m}^2, \Delta T = 15^\circ\text{C} = 15\text{K}$$

$$\text{dx} = ? \Rightarrow \frac{Q}{A} = \frac{KA\Delta T}{dx} \Rightarrow dx = \frac{Q}{KA\Delta T} \Rightarrow dx = \underline{\underline{0.15\text{m}}}$$

- (2) A brick wall 12 cm thick and  $5\text{m}^2$  surface area exposed to  $50^\circ\text{C}$  acetone face and  $20^\circ\text{C}$  to another face. If the thermal conductance of the material is  $1.5 \text{W/m}\cdot\text{K}$ . Find the heat transfer rate.

$$\text{Soln: } dx = 12\text{cm}, A = 5\text{m}^2, dT = 50^\circ\text{C} - 20^\circ\text{C} = 30^\circ\text{C} = 30\text{K.}$$

$$k = 1.5 \text{W/m}\cdot\text{K}, \frac{dQ}{dt} = ? \quad \frac{dQ}{dt} = KA \frac{dT}{dx} = 1875 \text{watt.}$$

- (3) Find the rate of heat lost from a brick wall ( $k = 0.7 \text{W/m}\cdot\text{K}$ ) of length 5m, height 4m and 0.25 m thick. The temp. of the inner surface  $50^\circ\text{C}$  and that of outer surface is  $30^\circ\text{C}$ . Also, calculate the distance from the inner surface at which the temp. is  $40^\circ\text{C}$

$$\text{Soln: } k \text{ of brick wall} = 0.7 \text{W/m}\cdot\text{K.}$$

$$\text{Area } A = 5\text{m} \times 4\text{m} = 20\text{m}^2,$$

$$dx = 0.25\text{m}, dT = 50^\circ\text{C} - 30^\circ\text{C} = 20^\circ\text{C} = 20\text{K}$$

$$\frac{dQ}{dt} = KA \frac{dT}{dx} \boxed{\approx 1120 \text{watt}}$$

$$\text{Again, } dx' = ? \text{ where } \frac{dQ}{dt} = \frac{kA(T_1 - T_2)}{dx'} \Rightarrow 1120 = \frac{0.7 \times 20(50 - 40)}{dx'} \Rightarrow 0.11\text{m}$$

- (4) A 1.2m long tube with outer diameter of 4cm outside temp of  $120^\circ\text{C}$  is exposed to an ambient air at  $20^\circ\text{C}$ . If the heat transfer coefficient between the tube surface and the air is  $20 \text{W/m}^2\cdot\text{K}$ . Find the rate of heat transfer from the tube to the air.

$$\text{Soln: } dx = 1.2\text{m}$$

$$dT = 120^\circ\text{C} - 20^\circ\text{C} = 100^\circ\text{C} = 100\text{K}, \text{Heat transfer rate } h = \frac{k}{dx} = \frac{20 \text{W/m}^2\cdot\text{K}}{0.12\text{m}} = 2000 \text{W/m}^2\cdot\text{K}.$$

$$\frac{dQ}{dt} = \frac{k}{dx} dT \cdot A = 301.593 \text{watt}$$

- (5) A furnace inside temp of  $2250\text{K}$  has a glass circular viewing of 6cm diameter. If the transmissivity of glass is  $0.08$  makes calculations for the heat loss from the glass window due to radiation.

$$\text{Soln: } A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (6 \times 10^{-2})^2 \Rightarrow E = \epsilon T^4 \Rightarrow \frac{dQ}{dt} = \epsilon T^4 e \cdot A = 328.5 \text{watt.}$$

## Chapter-8

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### 8.1 Fundamentals of thermodynamics

#### Introduction:

When two stones strike together spark may be produced. When two dry woods are rubbed together fire may be when we rub our hands each other, we feel warm. In both cases heat is produced in the expense of mechanical work-energy. Also, when certain mass of ice falls from a height, it may melt. This is due to conversion of gravitational potential energy into heat energy. We have other examples such as in cars, bikes, airplanes etc. fuel is burnt to produce heat which causes the expansion of gases inside the cylinder and hence mechanical work is observed. In all cases, we mentioned the principle of conservation of energy is not violated. Transformation of heat into mechanical work and vice-versa has provided us the ~~found~~ foundation for the construction of heat engines (like bike, scooter, car etc) and refrigerators. Thermodynamics is the branch of physics which deals with the interrelationship between heat and mechanical work. Now a days thermodynamics has been more applicable in other fields like electrical, chemical and magnetic fields.

#### Note:

Thermodynamics  $\Rightarrow$  Thermo+dynamics = Heat + Study of motion.  
It is the branch of physics which deals with the transformation of heat energy into mechanical work and vice-versa. Thermodynamics deals macroscopically i.e. it deals with the bulk system but not with the individual molecule.

#### Thermodynamic System (Types of thermodynamic system)

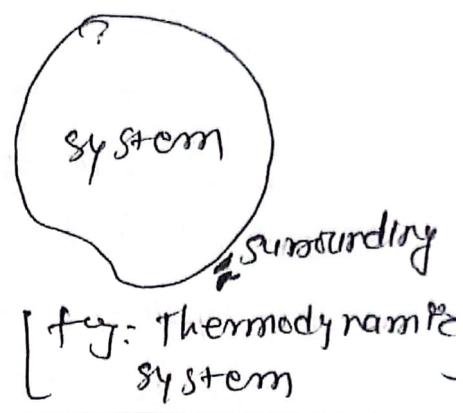
An assembly of large number of particles bounded by a closed surface (boundary surface) having certain value of

macroscopic parameters (pressure, volume and Temp) is called thermodynamic system. Anything outside the boundary surface of system is called its surrounding. Thermodynamic system interacts with its surrounding either by exchanging energy and matter in different ways or by performing work.

On the basis of exchange of energy

(heat) and matter, systems are classified into three types. They are:

- ① Open system: Thermodynamic system which can exchange both matter and energy with its surrounding is known as open system. e.g. gas contained in an open cylinder.
- ② Closed system: Thermodynamic system which can exchange only energy but not matter with its surroundings is called closed system. e.g. gas contained in a perfectly closed and conducting cylinder fitted with piston.
- ③ Isolated system: Thermodynamic system which can exchange neither energy nor matter with its surrounding is called isolated system. e.g. gas contained in a perfectly insulated cylinder fitted with piston.



### Thermodynamic Variables and properties

An equilibrium state of a thermodynamic system is specified by the value of its parameters like pressure ( $p$ ), Volume ( $V$ ), and Temp. ( $T$ ) etc. These parameters are called Thermodynamic Variables.

On the other hand, thermodynamic property is any characteristic of thermodynamic variable and other additional terms like mass, internal energy, total energy, enthalpy, entropy, viscosity, thermal conductivity etc.

Thermodynamic properties are generally classified into two groups. They are:

(1) Intensive properties: The properties which are independent of the mass of the system ~~are~~ called intensive properties. e.g. Temp., pressure, density etc. are examples.

(2) Extensive properties: The properties which depend on the size or extent of the system are called extensive properties. e.g. Total mass, total volume and total momentum.

### (#) Thermodynamic equilibrium:

Thermodynamic deals with equilibrium states. Equilibrium means a state of balance. An equ<sup>m</sup> state, there is no existed unbalanced potentials within a system. A system will be in a state of thermal equ<sup>m</sup>, if the condition for the following three types of equ<sup>m</sup> are satisfied.

(1) Mechanical equ<sup>m</sup>: A system is in mechanical equ<sup>m</sup> if there is no change in pressure at any point of the system with time.

(2) Thermal equ<sup>m</sup>: A system is said to be in thermal equ<sup>m</sup> if its all parts of it exhibit the same value of temp.

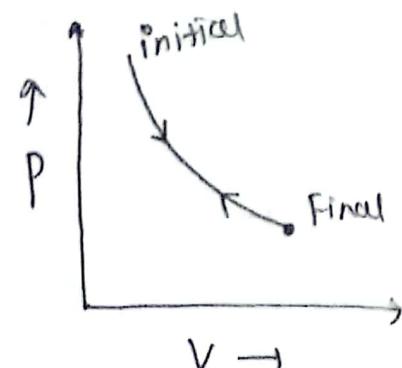
(3) Chemical equ<sup>m</sup>: A system is in chemical equ<sup>m</sup> if its chemical composition does not change with time, that is no chemical reaction occur.

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## Thermodynamic process (Reversible and Irreversible process)

### (a) Reversible process:

If the effects produced by a process on the system as well as on the surrounding can be completely restored to their initial states and no changes are left in any of the system taking part in the process or in the surrounding then the process is said to be a



[Fig: Reversible process]

reversible process. Any process, taking place slowly and controlled manner. All the mechanical process taking place under the action of conservative force are reversible process. In reversible process, a system can retrace in the opposite dir<sup>n</sup> so that it passes through exactly the same states in the respect as in the direct process.

### Conditions for the reversible process

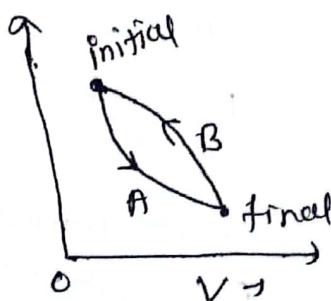
- ① The process should be infinitely slow
- ② There should be no loss of energy during the process.
- ③ System and Surrounding should be in thermodynamic eqm during the process.
- ④ No dissipative force of friction, viscosity, electrical resistance etc should be presented.

### (b) Irreversible process

The process which produce a permanent change in the thermodynamic state of the system and cannot be retraced

in the opposite order are called

Irreversible process. Almost all most real processes are in the nature of irreversible process.



[Fig: Irreversible process]

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E.g. rusting of iron, decay of matter, heat produced by friction etc. are irreversible process.

8.2

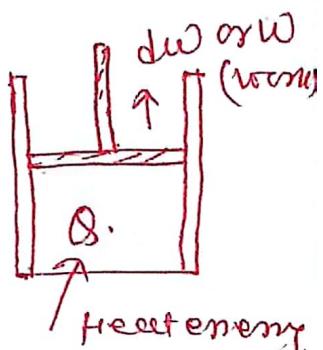
## Laws of thermodynamics

First law of thermodynamics is based on the principle of conservation. It states that "energy can neither be created nor can be destroyed but can change one form to another form". This law can be expressed in closed and open systems.

For a closed system 1st law of thermodynamics can be explained based on the energy conservation. In open system, it can be explained based on the mass conservation and energy conservation. They are described as following:

### First law of thermodynamics for Closed system (For control mass):

In the closed system, energy can be transferred between system and surrounding but not matter.



Consider a closed system, amount of heat energy  $Q$  is given as an input and external work  $(W)$  as an output as shown in figure.

Using the law of conservation of energy.

Energy entering the system - Energy leaving the system = Change in total energy of the system.

$$\text{i.e. } E_{\text{in}} - E_{\text{out}} = \Delta E$$

$$Q - W = \Delta E \quad \text{--- (1)}$$

The total energy of the system (i.e. change of energy + the sum of K.E., P.E. and internal energy.)

$$\therefore \Delta E = \Delta K.E + \Delta P.E + \Delta U \quad \text{where } \Delta U = \text{Change in internal energy of the system.} \quad \text{--- (2)}$$

In the case of closed system, change in  $h_f$  and  $P_E$  are negligible as compared to change in internal energy. Therefore, from eq<sup>n</sup> (2), we get

$$\Delta E = \Delta U - (3)$$

using eq<sup>n</sup> (3) in eq<sup>n</sup> (1) we get

$$Q - w = \Delta U$$

$$\text{or } dQ - dw = dU \quad \left\{ \begin{array}{l} \\ \end{array} \right.$$

$$\text{or } dQ = dU + dw \quad \left\} - (4) \right.$$

This is an eq<sup>n</sup> of 1st law of thermodynamics.

Case-1: If the closed system undergoes in cyclic process; The conservation of energy eq<sup>n</sup> can be written as,

$$\oint(dQ - dw) = \oint dU$$

For cyclic process  $\oint dU = 0$

$$\therefore \oint(dQ - dw) = 0$$

$$\text{or } \oint dQ = \oint dw$$

Therefore, net amount of applied heat energy ( $Q$ ) is equal to net amount of external work done ( $w$ ).

Case-2: For any thermodynamical process from one to another ~~systems~~ of a closed systems.

$$Q_{12} = w_{12} = U_2 - U_1$$

$$Q_{12} = w_{12} + U_2 - U_1.$$

## Application of 1st law of thermodynamics of a Closed system

Some applications of 1st law of thermodynamics are described as following:

### ① Constant Volume (Isochoric) process:

It is a thermodynamic process in which vol<sup>m</sup> of gas remains constant as the gas is kept in a non-expanding chamber.

Consider the working substance (gas) is contained in a rigid vessel in a constant volume process, making the system's boundaries immovable and preventing any work from being done on or by the system. The rigid vessel as shown in figure.

When the heat energy is added in the system, there is no change of vol<sup>m</sup>. So,  $dV = V_2 - V_1 = 0$

Total work done  $dw = pdV = 0$

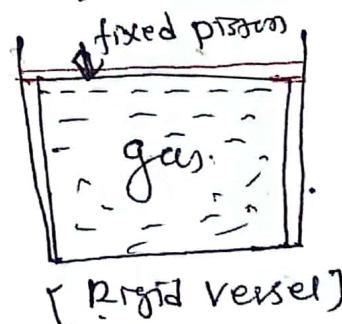
We know that, from 1st law of thermodynamics,

$$\Delta U_{12} = Q_{12} + U_2 - U_1$$

$$Q_{12} = U_2 - U_1 \text{ Since } dw = w_{12} = 0$$

$$\text{Thus, } \Delta U_{12} = (ΔU)_{12}$$

Hence, heat transferred to control mass during a constant volume process is equal to the increase in internal energy of the system.



[ Rigid vessel ]

## ② Isobaric process:

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It's a thermodynamics process in which the working substance is taken in an expanding chamber and heat is supplied to it so that volume and temp. of system change without any change in pressure.

In the Isobaric process, work done

$$\{ dw = w_{12} = \int_{v_1}^{v_2} p dv = p(v_2 - v_1) \text{ where } p_1 = p_2 = p \\ \therefore w_{12} = p v_2 - p v_1 \quad \text{---(1)}$$

From 1st law of thermodynamics,

$$Q_{12} = w_{12} + u_2 - u_1$$

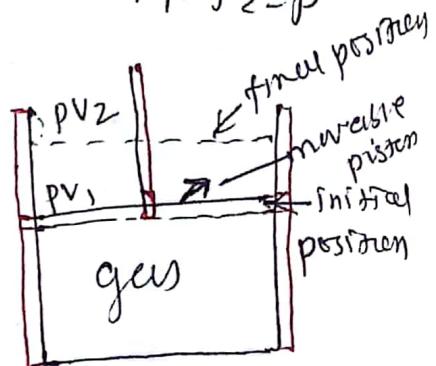
$$\text{or } Q_{12} = p v_2 - p v_1 + u_2 - u_1$$

$$\text{or } Q_{12} = B(u_2 + p v_2) - (u_1 + p v_1)$$

$$\text{or } Q_{12} = H_2 - H_1 \quad \text{where } H_1 = u_1 + p v_1 \text{ and } H_2 = u_2 + p v_2$$

Since  $H = U + pV$  is called enthalpy is equal to the sum of internal energy and product of pressure and volume of the gas.

From eqn (1), we conclude that heat transferred to a system during a const. pressure process is equal to the increase in the enthalpy of the system.



### 33 Isothermal process

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It is an isothermal process in which temp of a system remains constant throughout the process. There is free exchange of heat between system and surrounding. So it's a closed system.

Consider an ideal gas is

Confined in the conducting cylinder as shown in figure. Let  $p$ ,  $v$  and  $T$  are pressure, volume and Temp of a gas. In isothermal process temp remains constant. Therefore.

Eqn of 1st law of thermodynamics

$$dQ = dU + dW$$

$$\text{or } Q_{12} = W_{12} + U_2 - U_1 \quad \dots \textcircled{1}$$

We know that internal energy is a function of temp.

i.e.  $U_2 - U_1 = dU = C_V dT = 0$  since at const temp  $T_2 - T_1 = dT = 0$

$\therefore$  Eqn  $\textcircled{1}$  will be

$$Q_{12} = W_{12} + 0 \Rightarrow Q_{12} = W_{12} \quad \dots \textcircled{2}$$

To find the work done during an isothermal process,

We consider an ideal gas filled in the conducting cylinder fitted with piston.

The amount of work done during isothermal process

$$\text{or } dW = pdv \Rightarrow W = \int_{V_1}^{V_2} pdv$$

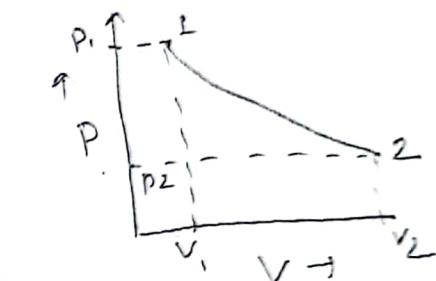
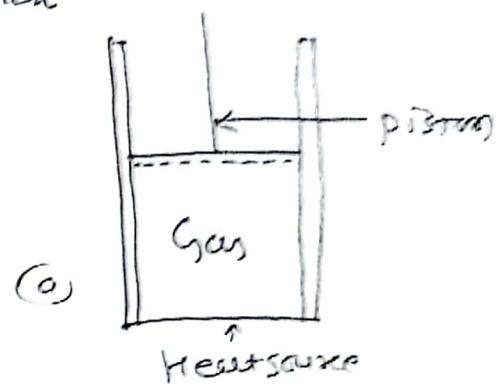
$$\text{or } W = \int_{V_1}^{V_2} pdv \quad \dots \textcircled{3} \text{ same in the process of work}$$

Now, volm. change  $V_1$  to  $V_2$  by taking const Temp.

We know that  $pv = nRT = \text{const.} (C)$

$$\therefore p = \frac{C}{V}$$

$$\text{Then Eqn } \textcircled{3} \text{ will be. } W = \int_{V_1}^{V_2} C \frac{dv}{V} \Rightarrow W = C \left[ \ln \frac{V_2}{V_1} \right] = W = C \ln \left( \frac{V_2}{V_1} \right) \quad \text{L4}$$



we know that for isothermal process

$$P_1 V_1 = P_2 V_2 = \text{const} (C.) \quad \text{Thus, } \frac{P_1}{P_2} = \frac{V_2}{V_1} \quad \text{or}$$

$$\therefore \delta_{12} = w_{12} = \cancel{\text{or}} \quad P_1 V_1 \ln \left[ \frac{V_2}{V_1} \right]$$

or

$$\delta_{12} = w_{12} = P_2 V_2 \ln \left( \frac{V_2}{V_1} \right)$$

Hence heat transferred to a control mass for an ideal gas during a constant temp. process is equal to work done by the sys.

### (a) Adiabatic (polytropic) process

It is an thermodynamic process in which pressure, volume and temp. of the system change without exchange of heat between system and surrounding. Therefore, from law of thermodynamics

$$dQ = dU + dw \quad \text{or} \quad \delta_{12} = w_{12} + u_2 - u_1 \quad \text{--- (1)}$$

~~$\Rightarrow du = dw$  where  $dQ = 0$~~ . Suppose an n moles of an ideal gas taken in a insulated cylinder as shown in figure

~~Also, we noted that~~

$$\delta_{12} = w_{12} + u_2 - u_1$$

$\Rightarrow$

$$\therefore w_{12} = u_2 - u_1$$

The cylinder has perfectly insulated walls and fitted with frictionless, massless con-conducting piston. In this case, there will be no exchange of heat in between

System and surrounding. Suppose gas expand

at  $(P_1, V_1, T_1)$  to  $(P_2, V_2, T_2)$ . If  $P$  is the pressure of the gas at an instant of time then the work done for small change in vol.  $dv$  is given by  $dw = pdv$ . The total work done at

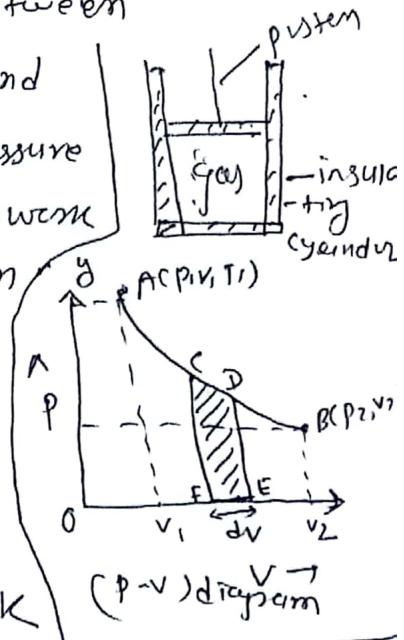
during whole process is

$$w = \int_{V_1}^{V_2} P dv \quad \text{--- (2)}$$

For an adiabatic process  $PV^\gamma = \text{const.} = K$

$$\therefore P = \frac{K}{V^\gamma} \quad \text{--- (3)}$$

using eqn (3) in eqn (2) we get



(17-11)

$$\textcircled{3} \quad w = \int_{V_1}^{V_2} \frac{K dV}{V^\gamma}$$

$$\text{or } w = K \int_{V_1}^{V_2} V^{-\gamma} dV$$

$$\text{or } w = K \left[ \frac{V^{-\gamma+1}}{-\gamma+1} \right]_{V_1}^{V_2}$$

$$\text{or } w = \frac{K}{1-\gamma} \left[ V_2^{1-\gamma} - V_1^{1-\gamma} \right]$$

$$\text{or } w = \frac{1}{(1-\gamma)} (K V_2^{1-\gamma} - K V_1^{1-\gamma}) \quad \text{--- (4)}$$

We hence treat for adiabatic process

$$P_1 V_1^\gamma = P_2 V_2^\gamma = K \text{ so eqn (4) will be}$$

$$w = \frac{1}{(1-\gamma)} \left[ P_2 V_2^{1-\gamma} - P_1 V_1^{1-\gamma} \right]$$

$$\text{or } w = \frac{1}{(1-\gamma)} (P_2 V_2 - P_1 V_1) \quad \text{--- (5)}$$

→ For ideal gas law  $P_1 V_1 = n R T_1$ ,  
 $P_2 V_2 = n R T_2$

So, eqn (5) will be

$$w = \frac{1}{(1-\gamma)} (n R T_2 - n R T_1)$$

$$\therefore w = \frac{n R}{(1-\gamma)} (T_2 - T_1) \quad \text{--- (6)}$$

where  $n$  = mole of gas.

Using eqn (6) in eqn (1) we get

$$\delta_{12} = \frac{n R}{(1-\gamma)} (T_2 - T_1) + n c_v (T_2 - T_1)$$

where  $dU = n c_v (T_2 - T_1)$  or  $n c_v dT$ .  
 or and  $C_v$  = molar sp. heat capacity  
 of a gas at constant volume.

$$\therefore \delta_{12} = n \left[ \frac{R}{1-\gamma} + C_v \right] T_2 - T_1$$

$$\text{or } \delta_{12} = n C_n (T_2 - T_1) \text{ where } C_n = \left( \frac{R}{1-\gamma} + C_v \right) \text{ is called}$$

polytropic specific heat capacity of mole of gas.

### (5) Specific heat capacity?

⇒ Specific heat of a gas is defined in terms of standard mass system (kg), it is called principial specific heat capacity (C).

⇒ If specific heat of gas defined in terms of molar mass,  
 is called molar specific heat capacity (C)

specific heat capacity of a gas is explained by two ways

i.e. taking Volume constant ( $C_V$ ) and pressure constant ( $C_P$ )

They are described as following:

### (1) SP. heat capacity at constant Volume:

(a) principial SP. heat capacity at gas at constant volume ( $C_V$ ):  
 It is defined as the quantity of heat required to raise the  
 temp. of 1 kg of a gas through 1 Kelvin temp. at const. volume  
 In SI unit  $C_V \text{ is } \text{J kg}^{-1} \text{ K}^{-1}$

### (b) molar specific heat capacity at constant volume ( $C_V$ ):

$C_V$  is defined as the amount of heat required to raise the temp. of one mole of gas through one kelvin temp. at const. volume. Its SI unit is  $J\text{mol}^{-1}\text{K}^{-1}$ . M be the molar mass of gas in kg. Then

$$C_V = mc_V.$$

specific heat of an ideal gas at constant volume can be defined in term of change in internal energy. So, internal energy of an ideal gas can be determined if its temp. and specific volume are given.

$$\therefore U = U(T, V)$$

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

For constant volume,  $dV = 0$

$$\therefore dU = \left(\frac{\partial U}{\partial T}\right)_V dT$$

$$\text{or } dU = C_V dT \text{ where } C_V = \left(\frac{\partial U}{\partial T}\right)_V.$$

For n mole of gas,  $dU = n C_V dT$

in m mole of gas  $dU = m C_V dT$

∴ Specific heat at constant volume is defined as a change in specific internal energy per degree change in temp.

### (d) Specific heat Capacity at constant pressures

#### (a) principal specific heat capacity at constant pressure ( $C_P$ ):

$C_P$  is defined as the quantity of heat required to raise the temp. of 1 kg of gas through 1 K at constant pressure. In SI unit,  $C_P$  has  $J\text{kg}^{-1}\text{K}^{-1}$ .

#### (b) molar specific heat capacity at constant pressure ( $C_p$ ):

$C_p$  is defined as the amount of heat required to raise the temp. of 1 mole of gas through 1 K at constant pressure. If its units is  $J\text{mol}^{-1}\text{K}^{-1}$ . If m be the molar mass of a gas in kg. Then  $C_p = m C_P$ .

The sp heat capacity of an ideal gas at constant pressure can be defined in terms of change in enthalpy. So we have, specific enthalpy

$$H = H(T, P)$$

Change in sp. enthalpy  $\tau_p$  given as

$$dH = \left(\frac{\partial H}{\partial T}\right)_P dT + \left(\frac{\partial H}{\partial P}\right)_T dP$$

For constant pressure  $dP = 0$

$$\therefore dH = \left(\frac{\partial H}{\partial T}\right)_P dT$$

$\Rightarrow dH = C_p dT$  where  $C_p = \left(\frac{\partial H}{\partial T}\right)_P$  is called sp. heat at constant pressure and  $\tau_p$  defined as the change in sp. heat enthalpy per degree change in temp.

H.W Show that  $C_p > C_v$  [form + 2 mark]

② Derive the relation  $[C_p - C_v = R]$  [form + 2 mark]

### Second law of thermodynamics

### Limitations of first law of thermodynamics

First law of thermodynamics is an important tool for understanding physical processes. Also no any process can violate it but has four limitations. They are given as following:

- ① When two bodies at different temp. are brought in thermal contact, heat flows always from hot body to cold body but first law of thermodynamics is silent to the question why heat doesn't flow from cold body to hot body. Therefore, first law of thermodynamics does not indicate the direction of heat flow.
- ② Heat can not be converted into useful work form a

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(1)

Single heat reservoir like sea. But why this is impossible? First law of thermodynamics has no answer to this question. That is first law of thermodynamics does not provide the condition under which heat can be converted into work.

③ When a bullet strikes a target, K.E. of bullet converts into heat energy but target cannot provides energy to the bullet for fly back losing its heat why? First law of thermodynamics has no answer that whether the mechanical work and heat energy are interconvertible or not.

④ In a heat engine, allowed by first law of thermodynamics, no 100% heat energy converts into mechanical work, why? First law of thermodynamics has no answer to this. That is first law of thermodynamics does not indicate that up to what extent, heat energy can be converted into mechanical work.

### Second law of thermodynamics

Second law of thermodynamics deals with the nature or quality of heat energy and defines the direction in which the system can proceed. Hence, the main feature of 2nd law of thermodynamics is that it defines the directions of the processes.

2nd law of thermodynamics defines the process's direction in terms of a system property known as entropy. Thus, entropy is the randomness or disorder of a thermodynamics system. It is an extensive property of a system that is used to measure the amount of energy that is unavailable for work.

impossible

∴ que it is denoted by  $S$ . The equation of change of entropy is

$$\Delta S = \frac{Q}{T} \quad \text{--- (1)}$$

where,  $Q$  = amount of heat energy transfers during a process

$T$  = absolute temp at which the process takes place

We know that, Entropy change is equal to the sum of entropy change of system and surroundings. Therefore, Total Change of entropy

$$(\Delta S)_T = (\Delta S)_{\text{system}} + (\Delta S)_{\text{surrounding}} \quad \text{--- (2)}$$

If the system loses an amount of heat  $Q$  at a temp.  $T_1$  which is received by surroundings at temp.  $T_2$ .

So,  $(\Delta S)_{\text{total}}$  can be calculated

$$(\Delta S)_{\text{system}} = -\frac{Q}{T_1} \quad \text{and} \quad (\Delta S)_{\text{surrounding}} = +\frac{Q}{T_2}$$

$$\therefore (\Delta S)_{\text{total}} = -\frac{Q}{T_1} + \frac{Q}{T_2} \quad \text{--- (3)}$$

If  $(\Delta S)_{\text{total}} = +ve$ , the process is spontaneous

"  $(\Delta S)_{\text{total}} = -ve$ , " " " non-spontaneous

"  $(\Delta S)_{\text{total}} = 0$ , the process is at equilibrium.

## II) Statements of 2<sup>nd</sup> law of thermodynamics

### ① Kelvin Statement (About heat engine):

" It is impossible to get continuous work from a body by cooling it to a temp. lower than that of its surroundings."

This means that in heat engine, working substance absorbs heat from source and performs mechanical work then rejects certain amount of heat to the sink. But, no any heat engine converts all heat extracted from the source into work without rejecting to the sink. Entire

In the other words, presence of colder body is required for ~~otherwise~~ continuous work. This means single heat reservoir cannot transfer heat into mechanical work continuously.

## (2) Clausius Statement (about refrigerators)

"It is impossible to make fire of heat from colder body to hotter body without doing external work on the working substance".

This means, in a refrigerator, working substance absorbs heat from the sink, certain external work is added on it so that finally large amount of heat is rejected to the source. But, no one refrigerator transfer heat from colder body to hotter body without adding external work on it.

## (3) Heat engine:

Heat engine is a device which converts heat into mechanical work continuously. For example: the gasoline engines in automobile, airplane, steam turbine or steam engine are heat engines. Heat engine consists of following components.

I) Source: It is a body of infinite heat capacity maintained at heat temp. Source has greater temp. ( $T, K$ ) than that of sink. Working substance can absorb any amount of heat from source keeping source temp. constant.

II) Working Substance: The material body of heat engine which burn to produce heat so that it performs mechanical work.

~~Qd Xes~~ Ideal gas in Carnot's engine, 2% petrol vapour and 98% air in petrol engine are examples of working substance.

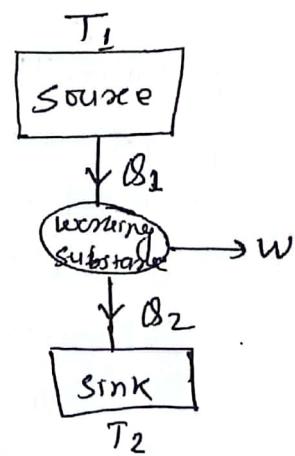
(IV) Sink: It is a body of infinite heat capacity maintained at fuel temp. Sink has lower temp. ( $T_2$  K) than that of source. Any amount of heat can be rejected to sink keeping sink temp. constant.

### Thermal efficiency of heat engine.

Heat engine operates on a cyclic process and receive heat from a high temp. reservoir (source) convert part of this heat to work and then reject the remaining part of heat to a lower temp. reservoir (sink) during the cycle.

Suppose, working substance

extract  $\dot{Q}_1$  amount of heat from the source at temp.  $T_1$  K, perform certain mechanical work  $w$  and rejects remaining amount of heat  $\dot{Q}_2$  to the sink at lower temp.  $T_2$  K as shown in figure.



Therefore,  $(\dot{Q}_1 - \dot{Q}_2)$  is the amount [Fig: Block diagram of] of heat converted into mechanical work. That is work done

$$w = \dot{Q}_1 - \dot{Q}_2 \quad \text{--- (1)}$$

Now, efficiency of heat engine is defined as the ratio of the external work obtained to the heat energy absorbed by the working substance from the source. It is denoted by  $\eta$ . So,

$$\eta = \frac{\text{External work done (w)}}{\text{Heat energy absorbed from the source}} \quad \text{(Q. 1)}$$

$$\therefore \eta = \frac{\dot{Q}_1 - \dot{Q}_2}{\dot{Q}_1} \Rightarrow \eta = 1 - \frac{\dot{Q}_2}{\dot{Q}_1} \text{ since } \dot{Q}_1 - \dot{Q}_2 = W.$$

In general efficiency is expressed in %. So,

$\eta = \left(1 - \frac{\dot{Q}_2}{\dot{Q}_1}\right) \times 100\%$ . Since  $\dot{Q}_1 > \dot{Q}_2$ , efficiency is always less than 100%. For cent percentage efficiency is possible only when  $\dot{Q}_2 = 0$  but  $\dot{Q} = 0$  is impossible because to drive engine, heat must be lost to the sink compulsorily.

In term of temp. efficiency of heat engine can be expressed as

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$$

### ④ Refrigerator:

Refrigerator operates on a cyclic process which takes heat from a low-temp. reservoir at temp.  $T_2$  and delivers it into the high-temp. reservoir at temp.  $T_1$  with the help of external work. Refrigerator maintains the temp. of desired space lower than that of surroundings.

A refrigerator is essentially a heat engine working in the reverse direction. That means in refrigerator, working substance takes heat from cold bay (interior of refrigerator), compressor supplies the necessary mechanical work input and sum of two energies is rejected to the hot reservoir (surrounding air). The schematic operation of a refrigerator is shown in figure below.

Opposite working substance absorbs  $Q_2$  amount of heat from sink at temp  $T_2$  on adding external work  $w$  in unit by motor.

Finally, large amount of heat  $Q_1$  is rejected to the source at higher temp.

$$T_2 \text{ Now, } Q_1 = Q_2 + w. \quad \textcircled{1}$$

Here, Input work (or energy)  $w = Q_1 - Q_2$

Output work (or energy) =  $Q_2$

The efficiency of different refrigerators [Fig: Block diagram of refrigerator] can be compared by their coefficient

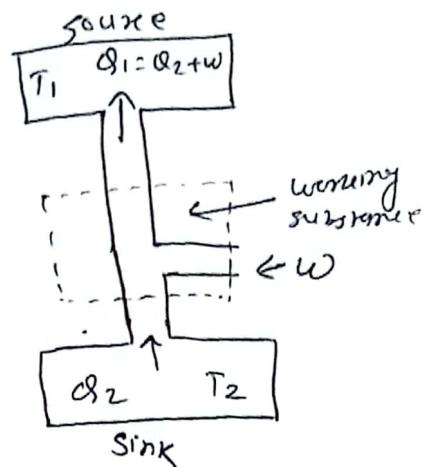
of performance (COP). Coefficient

of performance ( $\beta$ ) is defined as the ratio of heat extracted from the cold body (refrigerant) to work done by the motor.

$$\therefore \beta = \frac{\text{Output}}{\text{Input}} = \frac{Q_2}{Q_1 - Q_2} \Rightarrow \frac{1}{\frac{Q_1 - Q_2}{Q_2}} \Rightarrow \frac{1}{\left(\frac{Q_1}{Q_2} - 1\right)}$$

$$\therefore \beta = \frac{1}{\left(\frac{T_1}{T_2} - 1\right)} \Rightarrow \beta = \frac{T_2}{T_1 - T_2}$$

This shows that all reversible refrigerating machines working between the same two temp. have the same coefficient of performance. It is clear from expression, that if  $T_1 - T_2$  is small,  $\beta$  will be higher. The best refrigerator removes maximum heat  $Q_2$  with least possible work input by motor. If no work were needed to operate a refrigerator, the coefficient of performance  $\beta$  would become infinite which is rather impossible. For practical refrigerator,  $\beta$  is always less than  $\frac{T_2}{T_1 - T_2}$ .



## (II) Heat pump

(M-2)

Heat pump operates on a cyclic process which takes heat from a low temp. reservoir (surroundings) at  $T_2$  and delivers into the high temp. reservoir (desired space) at  $T_L$  with the help of external work done. Heat pump maintains the temp. of desired space higher than that of surroundings.

Consider a heat pump takes  $\dot{Q}_2$  amount of heat from sink at temp.  $T_2$  and supply  $\dot{Q}_1$  to desired space at  $T_L$  with the help of external work done (W). performance of heat pump  $\beta_p$  measured in terms of coefficient of performance (COP)  $\beta$ , which is defined as the ratio of desired effect and work supplied. i.e.

$$\beta = \frac{\text{Desire effect}}{\text{Work input}}$$

where Desire effect = amount of heat supplied to the desired space ( $\dot{Q}_1$ ).

$$\therefore (\beta)_{\text{heat pump}} = \frac{\dot{Q}_1}{W} = \frac{\dot{Q}_1}{\dot{Q}_1 - \dot{Q}_2} \Rightarrow \frac{1}{1 - \frac{\dot{Q}_2}{\dot{Q}_1}}$$

In term of Temp,

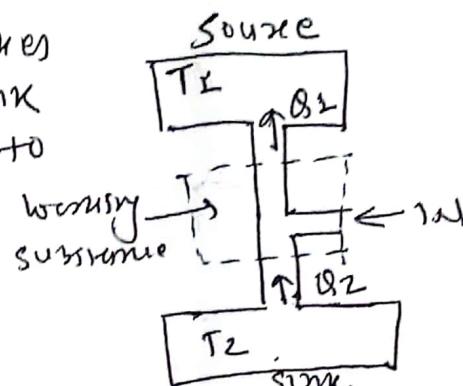
$$(\beta)_{\text{heat pump}} = \frac{T_L}{T_L - T_2} \Rightarrow \frac{1}{1 - \frac{T_2}{T_L}}$$

## (II) Components of refrigerator and heat pump:

OR.

Construction and Working of Refrigerator.

$\Rightarrow$  A refrigerator and heat pump operates on a cyclic process called refrigeration cycle. The refrigeration cycle is the



[Block diagram of heat pump]

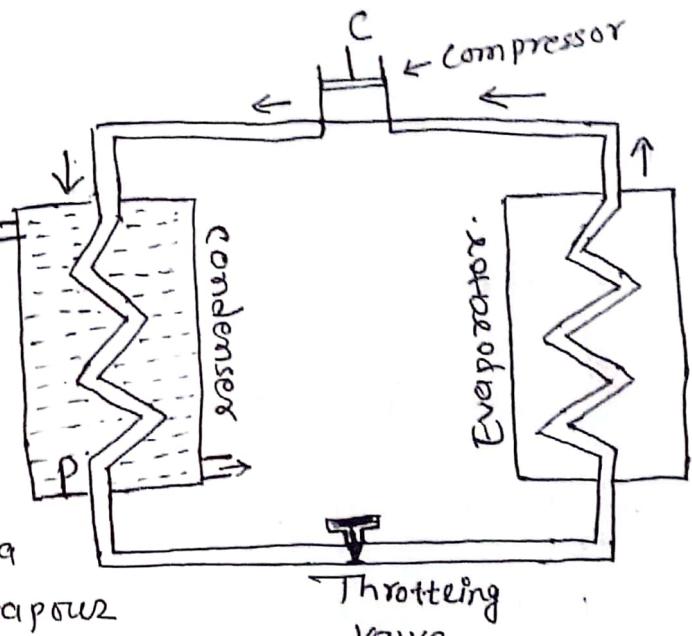
widely used for heating and cooling of desired space.

The working principle of common refrigerator has been explained with the support of schematic diagram as shown in figure below. Four components of refrigerator are

- (i) Compressor
- ii) Condenser.
- iii) Throttling valve, and
- iv) Evaporator.

The working procedure of refrigerator along with its components are described as following.

The working substance in a refrigerator is usually the vapour of some low boiling point liquid, such as chloro-fluoro carbon ( $\text{CCl}_2\text{F}_2$ ) or even ammonia,



[Fig: Schematic diagram of refrigerator.]

The compressor (c) compresses the working substance adiabatically to a high pressure with the support of motor (external work is done here) so that gas becomes quite hot. This hot and high pressurized gas is passed through condenser. Condenser is a spiral pipe (P) to which cold water is circulated. Now gas gets liquefied. This liquid is passed through the throttling valve (V) and it expands suddenly by the process of called throttling process. This process causes the decrease in the temp. of gas. Then the gas is passed through the evaporator (spiral tube  $\rightarrow$  covering department containing objects to be cooled like food items) where

liquid evaporators and gets cooled by taking heat from food items. Now, working substance leaves the cooling unit and then goes to the compressor (C) for fresh cycle.

### Note:

Compressor  $\rightarrow$  high pressure - Temp.  $\rightarrow$  p, T and H increases but entropy (S) remains constant.

Condenser  $\rightarrow$  high pressure - low temp.  $\rightarrow$  p = const, H and S decreases.

Throttling valve  $\rightarrow$  high press - low temp  $\rightarrow$  H = const,  $p \propto T$  decreases and S = increases.

Evaporator  $\rightarrow$  low pressure - low temp  $\rightarrow$  p = const,

H & S = increases.

### # Numerical problems

① A gas is contained in a piston cylinder device initially at a pressure of 150 kPa and a volume of 0.04 m<sup>3</sup>. Calculate the work done by the gas when it undergoes the following process to a final volume of 0.1 m<sup>3</sup>.

(a) Constant pressure. (b) Constant temp. (c)  $PV^{1.35} = \text{constant}$ .

Soln: we have given,

$$\text{initial pressure } (P_1) = 150 \text{ kPa}$$

$$\text{Volume } (V_1) = 0.04 \text{ m}^3$$

$$\text{final } \text{Volume } (V_2) = 0.1 \text{ m}^3$$

work done by the gas = ?

(a) At const. pressure, work done  $W = PdV = P_1(V_2 - V_1)$

$$\therefore W = 150 \times 10^3 (0.1 - 0.04) \Rightarrow W = 9000 \text{ J}$$

(b) At constant temp, work done  $W = P_1 V_1 \ln \left( \frac{V_2}{V_1} \right)$

$$= 150 \times 10^3 \times 0.04 \ln \left( \frac{0.1}{0.04} \right) = 5497 \text{ J}$$

(c) For adiabatic process,  $PV^\gamma = \text{const.}$  since  $\gamma = 1.35$ .

$$P_1 V_1^\gamma = P_2 V_2^\gamma \Rightarrow P_2 = P_1 \left( \frac{V_2}{V_1} \right)^\gamma = 1.35$$

$$P_2 = 150 \times 10^3 \left( \frac{0.04}{0.1} \right)^{1.35} \Rightarrow 43538.38 \text{ Pa.}$$

$\frac{1}{\gamma-1}$  (Used the value at ambient condition)  
find ...

- Q ② An ideal engine has an efficiency of 25%. If the source temp. is increased by  $200^\circ\text{C}$ , its efficiency gets doubled. Determine its source and sink temp.

Soln. Let Temp. of source =  $T_1$ ,

$$\therefore \text{Sink} = T_2$$

$$\text{efficiency } \eta = 1 - \frac{T_2}{T_1} \Rightarrow 0.25 = 1 - \frac{T_2}{T_1} \Rightarrow T_2 = 0.75 T_1. \quad (1)$$

$$\eta \text{ is double, i.e. } \eta_2 = 2\eta = 2 \times 0.25 \Rightarrow \eta_2 = 0.50.$$

$$\therefore \eta_2 = 1 - \frac{T_2}{T_1}$$

$$0.50 = 1 - \frac{\cancel{T_2}}{T_1 + 200}$$

$$\frac{T_2}{T_1 + 200} = 0.5 \quad (2)$$

Using eqn(1) in eqn(2), we get

$$\frac{0.75 T_1}{T_1 + 200} = 0.5$$

$$\therefore T_1 = 2100 \text{ K.}$$

$$\text{and } T_2 = 0.75 \times 2100 = 3150 \text{ K.}$$

- Q ③ The source reservoir of a heat engine is at temp. of  $4000^\circ\text{C}$  and takes  $400\text{J}$  of heat and rejects  $20\text{J}$  of heat to the sink reservoir in each cycle. What are the efficiency of engine and temp. of sink?

Soln Hint:  $T_1 = 4000 \text{ K.}$ ,  $T_2 = ?$ ,  $Q_1 = 400\text{J}$ ,  $Q_2 = 20\text{J}$ ,  $\eta = ?$

$$\eta \% = \frac{Q_1 - Q_2}{Q_1} \times 100\% = \frac{400 - 20}{400} \times 100\% = 95\%.$$

$$\text{Again, } \frac{Q_1}{Q_2} = \frac{T_1}{T_2} \Rightarrow \frac{400}{20} = \frac{400}{T_2} \Rightarrow T_2 = 20 \text{ K.}$$

- Q ④ An ideal heat engine operates between two reservoirs at two temp in order to achieve  $30\%$  efficiency when the temp. of the sink is  $50^\circ\text{C}$ . What should be the temp. of the source?

Soln. Hint  $\eta = 30\%$ ,  $T_2 = 50^\circ\text{C} = (50 + 273) \text{ K} = 323 \text{ K}$

$$T_1 = ? \text{ we have } \frac{T_1 - T_2}{T_1} \times 100\% = \eta \% \Rightarrow T_1 = 461.43 \text{ K.}$$

## Electromagnetic wave

Pj-9

### (F) Vector algebra:

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

① Dot product:  $\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$

$$= A_x B_x + A_y B_y + A_z B_z$$

② Vector product:  $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = i(A_y B_z - B_y A_z) + j(A_z B_x - B_z A_x) + k(A_x B_y - B_x A_y)$

### (G) Vector operator:

$$\nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$$

This quantity gives the variation of the various quantities quantitatively along different directions.

### (H) New terms:

- ① Gradient of a scalar
- ② Divergence of a vector
- ③ Curl of a vector.

#### (i) Gradient of a scalar:

Let  $\phi$  be a scalar function, then gradient of  $\phi$  is given by

$$\nabla \phi \text{ and it is given as } \nabla \phi = \left( \hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z} \right) \phi$$

$$\nabla \phi = \hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z}$$

Hence, Gradient of a scalar is a vector quantity.

#### Physical significance of gradient:

Physical significance of gradient is maximum slope at point. The change gradient of a scalar is maximum slope at point is known as gradient of vector with respect to vector position is known as gradient of physical quantity. Gradient gives the magnitude and direction of maximum change to the quantity along the particular direction.

#### (ii) Divergence of a vector:

Let  $\vec{A}$  be a vector quantity. its divergence is denoted by  $\nabla \cdot \vec{A}$  and it is given by  $\nabla \cdot \vec{A} = (\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}) \cdot (\hat{i} A_x + \hat{j} A_y + \hat{k} A_z)$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Hence, divergence of a vector quantity is a scalar quantity.

## Physical significance of Divergence:

Divergence of any vector quantity gives scalar. There are three cases. They are

(i) case-I: Divergence of vector is +ve

i.e.  $\nabla \cdot \vec{A} > 0$  it means, it is a source, i.e. flow of flux will be increased.

(ii) case-II: Divergence of vector is -ve,

i.e.  $\nabla \cdot \vec{A} < 0$ , it means it is a sink, i.e. flow of flux will be decreased.

(iii) case-III: Divergence of vector is zero

i.e.  $\nabla \cdot \vec{A} = 0$ , it means, there is neither flow of flux neither increase nor decrease, and going similar out of given input.

## Curl of a vector:

Let  $\vec{A}$  be a vector quantity, then curl of  $\vec{A}$  is denoted by

$$\nabla \times \vec{A} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{pmatrix} = \hat{i} \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{j} \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{k} \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

## Physical significance of curl of a vector:

Curl of any vector gives the rotation of the quantity.

## Gauss divergence theorem:

It states that the volume integral of divergence of a vector  $\vec{A}$  is equal to the surface integral of  $\vec{A}$  taken over the surface which encloses the volume.

$$\therefore \oint_V (\nabla \cdot \vec{A}) dv = \oint_S \vec{A} \cdot \hat{n} ds$$

## Stoke's Theorem:

It states that the surface integral of a curl of a vector  $\vec{A}$  taken over the surface is equal to the line integral of vector  $\vec{A}$  taken over periphery of  $\vec{A}$ .

$$\text{i.e. } \oint_S (\nabla \times \vec{A}) \cdot d\vec{s} = \oint_L \vec{A} \cdot d\vec{l}$$

## (1) Continuity equation [imp]

Current is given by  $I = -\frac{dq}{dt}$  —① where -ve sign shows that charge decreases with time. Since  $\oint \vec{J} \cdot d\vec{s}$

where,  $\vec{J}$  is the electric current density.

$$\text{since } \vec{J} = \frac{\vec{I}}{A} \Rightarrow I = \vec{J} \cdot \vec{A} = \oint \vec{J} \cdot d\vec{s}$$

$$\text{Also, } q = \int_S s dv \text{ since } s = \frac{q}{V} \Rightarrow q = s \cdot V = \oint_S s dv$$

Equation ① becomes

$$I = \oint_S \vec{J} \cdot d\vec{s} = -\frac{d}{dt} \oint_S s dv —②$$

Using gauss divergence theorem.

$$\oint_S \vec{J} \cdot d\vec{s} = \int_V (\nabla \cdot \vec{J}) dv$$

So, eq<sup>n</sup> ② will be

$$\int_V (\nabla \cdot \vec{J}) dv = -\frac{d}{dt} \int_V s dv$$

$$\text{or } \int_V (\nabla \cdot \vec{J}) dv = -\int_V \left( \frac{ds}{dt} \right) dv$$

$$\text{or } \int_V (\nabla \cdot \vec{J}) dv = \int_V \left( -\frac{\partial s}{\partial t} \right) dv$$

$$\text{or } \nabla \cdot \vec{J} = -\frac{\partial s}{\partial t}$$

$$\boxed{\therefore \nabla \cdot \vec{J} + \frac{\partial s}{\partial t} = 0} —③ \text{ This is the equation of continuity which explains the law of conservation of charge.}$$

For the steady state condition, i.e., for static charge,

$$\frac{\partial s}{\partial t} = 0 \text{ so, eq<sup>n</sup> ③ will be } \nabla \cdot \vec{J} = 0, \text{ thus divergence}$$

of current will be zero means, there is no source and sink.