ASSIGNMENT

CALCULUS I

CHAPTER 1

LIMIT, CONTINUITY AND DERIVATIVE

Continuity

1. Define removable discontinuity. Show that the function

$$f(x) = \begin{cases} 3x^2 & \text{for } x < 1 \\ 2 & \text{for } x = 1 \\ x^2 + 1 & \text{for } x > 1 \end{cases}$$ is discontinuous at $x = 1$. What type of discontinuous is it and how it can be redefined to make it continuous?

2. Define continuity of a function. Test the continuity of the function

   (i) $$f(x) = \begin{cases} (1 + 3x)^{1/x} & \text{for } x \neq 0 \\ e^3 & \text{for } x = 0 \end{cases}$$ at $x = 0$.

   (ii) $$f(x) = \begin{cases} x \sin\frac{1}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}.$$

3. A function is defined as

$$f(x) = \begin{cases} 2 - x^2 & \text{for } x < 2 \\ 3 & \text{for } x = 2 \\ x - 4 & \text{for } x > 2 \end{cases}.$$

Is the function continuous at $x = 2$? If not why? Restate to make it a continuous function.

4. Find the value of k for which the function

$$f(x) = \begin{cases} \dfrac{\sin 2x}{5x} & \text{for } x \neq 0 \\ k & \text{for } x = 0 \end{cases}$$ is continuous at $x = 0$.

Derivatives

1. Define derivative of a function at a point. Examine the continuity and differentiability of the function

$$f(x) = \begin{cases} 1 & \text{when } x \in (-\infty, 0) \\ 1 + \sin x & \text{when } x \in \left[0, \frac{\pi}{2}\right) \\ 2 + \left(x - \frac{\pi}{2}\right)^2 & \text{when } x \in \left[\frac{\pi}{2}, \infty\right) \end{cases}$$

at $x = 0$ and $x = \frac{\pi}{2}$.

2. Let a function be defined as $$f(x) = \begin{cases} 5x - 4 & \text{for } 0 < x \leq 1 \\ 4x^2 - 3x & \text{for } 1 < x < 2 \\ 3x + 4 & \text{for } x \geq 2 \end{cases}$$

Show that $f(x)$ is continuous as $x = 1$ and 2. Also prove that $f'(x)$ exists for $x = 1$ but not at $x = 2$.

3. If $$f(x) = \begin{cases} 3 + 2x & \text{for } -\frac{3}{2} < x \leq 0 \\ 3 - 2x & \text{for } 0 < x < \frac{3}{2} \end{cases}$$

Show that $f(x)$ is continuous at $x = 0$ but is not differentiable at that point.

4. Show that if a function is differentiable at a point it is necessarily continuous at that point but converse may not be true.

5. Show that the function $f(x) = |x| + |x - 1|$ is continuous at $x = 1$ but is not differentiable at that point.

Higher Order Derivatives

1. State Leibniz theorem. If $y = a \cos(\log x) + b \sin(\log x)$. Then prove that

(i) $x^2 y_2 + x y_1 = 0$

(ii) $x^2 y_{n+2} + (2n+1) x y_{n+1} + (n^2+1) y_n = 0$.

2. If $y^{1/m} + y^{-1/m} = 2x$, show that

$(x^2 - 1) y_{n+2} + (2n+1) x y_{n+1} + (n^2 - m^2) y_n = 0$.

3. If $y = \sin(m \sin^{-1} x)$, prove that :

(i) $(1 - x^2) y_2 - x y_1 + m^2 y = 0$

(ii) $(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} + (m^2 - n^2) y_n = 0$.

4. If $y = \sin^{-1} x$, show that

(i) $(1 - x^2) y_2 - x y_1 = 0$

(ii) $(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} - n^2 y_n = 0$

(iii) $(y_{n+2})_0 = n^2 (y_n)_0$. Find also the value of $(y_n)_0$.

5. If $y = \log(x + \sqrt{a^2 + x^2})$, show that

(i) $(a^2 + x^2) y_2 + x y_1 = 0$

(ii) $(a^2 + x^2) y_{n+2} + (2n+1) x y_{n+1} + x^2 y_n = 0$

6. If $y = (x^2 - 1)^n$, show that

(i) $(x^2 - 1) y_1 + 2(n - 1) x y_1 - 2ny = 0$

(ii) $(x^2 - 1) y_{n+1} + 2x y_{n+1} - n(n+1) y_n = 0$

7. If $y = e^{a \tan^{-1} x}$, prove that

(i) $(1 + x^2) y_2 + (2x - a) y_1 = 0$

(ii) $(1 + x^2) y_{n+2} + (2nx + 2x - a) y_{n+1} + n(n+1) y_n = 0$

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CHAPTER 2

Application Of Derivative

Mean Value theorem

1. State and prove Rolle’s theorem. Write the geometrical interpretation of Rolle’s theorem.
2. Verify Rolle’s theorem for
   (i) $f(x) = \dfrac{\cos x}{e^x}$, $x \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$
   (ii) $f(x) = e^x(\sin x - \cos x)$, $x \in \left[\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right]$
   (iii) $f(x) = \log\left(\dfrac{x^2 + ab}{(a + b)x}\right)$, $x \in [a, b]$, $a > 0$.
3. If $f(x) = (x - a)^m(x - b)^n$; m and n are positive integers, show that c in Rolle’s theorem divides the segment $a \le x \le b$ in the ratio $m : n$.
4. If $f(x) = \tan x$, $x \in [0, \pi]$. Is Rolle’s theorem applicable to the function $f(x)$ in $[0, \pi]$?
5. State and prove Lagrange’s mean value theorem. Write the geometrical interpretation of Lagrange’s mean value theorem.

6. Verify Lagrange's Mean Value Theorem for
   (i) $f(x) = x^2 - 2$ in [1, 4]
   (ii) $f(x) = e^x$ in [0, 1]
   (iii) $f(x) = \log x$ in $[1, e]$
7. For what value of a, m, b does the function
$$f(x) = \begin{cases} 3 & \text{for } x = 0 \\ -x^2 + 3x + a & \text{for } 0 < x < 1 \\ mx + b & \text{for } 1 \le x \le 2 \end{cases}$$
   satisfy the hypothesis of the mean value theorem in the interval [0, 2].
8. Show that: $\dfrac{b-a}{b} < \log\left(\dfrac{b}{a}\right) < \dfrac{b-a}{a}$ by using Lagrange's mean value theorem.

Higher Order mean value theorem

1. Assuming the validity of expansion expand $f(x) = e^x$ in ascending powers of x.
2. Assuming the validity of expansion expand $f(x) = e^{\sin x}$ in ascending powers of x.
3. Assuming the validity of expansion expand $f(x) = e^x$ in ascending powers of $x - 1$.
4. Assuming the validity of expansion, find Maclaurin’s series expansion of f(x) = log(sec x). Find the expansion of tanx.

Asymptotes

1. Define asymptote to the curve.
2. Obtain the horizontal and vertical asymptotes, if any, of the following curves:
   (i) $x^2y^2 - 4(x - y)^2 + 2y - 3 = 0$
   (ii) $y = \dfrac{2x - 3}{x^2 - 3x + 2}$
   (iii) $x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$
3. Find the asymptotes, if any, of the following curves.
   (i) $xy^2 - x^2y = a^2(x + y) + b^2$
   (ii) $(x^2 - y^2)y - 4y^2 + 5x - 7 = 0$
   (iii) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

(iv) $x^3 + x^2y - xy^2 - y^3 + x^2 - y^2 - 2 = 0$

(v) $x^2 (x - y)^2 - a^2 (x^2 + y^2) = 0.$

Tracing of curves

Trace the following curves

(i) $y^2 (a - x) = x^2(a + x)$

(ii) $x^2 (x^2 + y^2) = a^2 (x^2 - y^2)$

(iii) $a^2b^2 + x^2y^2 = a^2x^2$

(iv) $x^2 (a^2 + y^2) = y^2(a^2 - y^2)$

Curvature

1. Define curvature and radius of curvature of a curve at a point. Find the radius of curvature at any point (x, y) of the cartesian curves.

   (i) $y^2 = 4ax$ (ii) $xy = c^2$

   (iii) $y = a\cosh\left(\frac{x}{a}\right)$

2. Find the radius of the curvature for the parametric curves at the points indicated.

   (i) $x = at^2, y = 2a$ at (x, y)

   (ii) $x = a\cos^3\theta, y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$

3. Find the radius of curvature for the polar curves at the points indicated:

   (i) $r = ae^{\theta\cot\alpha}$ at (r, θ)

   (ii) $r^2 = a^2 \cos 2\theta$ at $\theta = 0$

4. Find the radius of curvature of the following curves at the origin

   (i) $x^2 - 3xy + y^2 - 3y = 0$

   (ii) $3x^2 + 4y^2 = 2x$

   (iii) $x^4 + y^2 = 6a (x + y)$

5. Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point where it cuts the line $y = x$.

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CHAPTER 3

Integral Calculus

1. Evaluate the following integrals

(i) $\int \frac{dx}{4\sin x - 5\cos x}$ (ii) $\int \sqrt{4x^2 - 4x + 5}\, dx$

(iii) $\int x^5 e^x\, dx$ (iv) $\int \frac{x}{(x-3)(x+1)}\, dx$

2. Prove the following

(i) $\int_0^{\pi/2} \log\sin x\, dx = \frac{\pi}{2}\log\frac{1}{2}$ (ii) $\int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}} = \frac{\pi}{4}$

(iii) $\int_0^1 \frac{\log x}{\sqrt{1-x^2}}\, dx = \frac{\pi}{2}\log\left(\frac{1}{2}\right)$ (iv) $\int_0^{\pi/2} \log(1+\tan\theta)\, d\theta = \frac{\pi}{8}\log 2$

3. Evaluate the following improper integrals

(i) $\int_2^{\infty} \frac{x\,dx}{x^2-1}$ (ii) $\int_0^{\infty} e^{-ax}\sin bx\, dx$ (iii) $\int_1^{\infty} \frac{dx}{(1+x)^{3/2}}$

(iv) $\int_0^{\infty} xe^{-x^2}\, dx$ (v) $\int_0^{\infty} \frac{x\,dx}{(1+x^2)^2}$

4. Determine whether the following integrals are convergent or divergent

(i) $\int_a^{\infty} \frac{\cos^2 x}{x^2}\, dx\ (a > 0)$ (ii) $\int_3^{\infty} \frac{dx}{x - e^{-x}}$ (iii) $\int_2^{\infty} \frac{x^3\,dx}{\sqrt{x^7 - 1}}$

Reduction formula

1. Obtain the reduction formula for $\int x^n e^{-ax} dx$, $(n \neq -1)$ and hence find $\int x^3 e^{-ax} dx$.

2. Obtain the reduction formula for $\int \cot^n x\, dx$. and hence find the value $\int \cot^5 x\, dx$.

3. Find the reduction formula for $\int \cos^n x\, dx$ and hence evaluate

(i) $\int \cos^7 x\, dx$ (ii) $\int_0^{\pi/2} \cos^7 x$

4. Find the reduction formula for $\int \sec^n x\, dx$ and hence evaluate $\int \sec^7 x\, dx$.

Beta function and Gamma function

1. Define Beta function and Gamma function.
2. By using Gamma function show that

(i) $\int_0^{\pi/2} \sin^3 x \cos^5 x dx = \frac{1}{24}$ (ii) $\int_0^{\pi/6} \cos^4 3\theta \sin^2 6\theta \, d\theta = \frac{5\pi}{192}$

(iii) $\int_0^1 x^6 \sqrt{1-x^2}\, dx = \frac{5\pi}{256}$ (iv) $\int_0^{2a} x^{9/2} \sqrt{2a-x}\, dx = \frac{63}{8}\pi a^5$

3. Prove that

(i) $$\int_0^{\pi/2} \sin^m x \cos^n x \, dx = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\,\Gamma\left(\frac{m+n+2}{2}\right)}$$

(ii) $$\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right), \quad p, q > -1$$

4. Prove that $\int_0^{\infty} \sqrt{x}\, e^{-x^2} dx \times \int_0^{\infty} \frac{e^{-x^2}}{\sqrt{x}} dx = \frac{\pi}{2\sqrt{2}}$

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CHAPTER 4

Application Of Integral

Arc Length

1. Find the arc length of the following curves
   (i) $y = x^2; \quad -1 \le x \le 2.$
   (ii) $x^2 + y^2 = a^2$
   (iii) $y^2 = 4ax$ to the points cut off by line $3y = 8x$
   (iv) from vertex of the parabola $y^2 = 4x$ to one end of the extremity of the latus rectum.

Area

1. Find the area bounded by the given curves.
   (i) the area bounded by $y = x^2$ between $x = 1$ and $x=3$
   (ii) the circle $x^2 + y^2 = a^2$
   (iii) the curve $y^2 = 4x$ and line $y = x$
   (iv) the area bounded by x-axis and $y = 2x - x^2$
   (v) the curve $y = 2 - x^2$ and $y = -x$
   (vi) the curve $x = y^2$ and $x = -2y^3 + 3$

Area of surface of revolution

1. Find the area of the surface obtained by rotating the curve $y = x^3$ between $0 \le x \le 2$ about x-axis.
2. Find the area of the surface obtained by rotating the curve $y = x^{1/3}$ between $1 \le y \le 2$ about y-axis.
3. Find the area of the surface obtained by rotating the curve $y = \frac{x^2}{2} + \frac{1}{2}$ between $0 \le x \le 1$ about y-axis.
4. Find the area of the surface obtained by rotating the curve $y = \frac{1}{3}(x^2 + 2)^{3/2}$ between $0 \le x \le 3$ about y-axis.
5. The loop of the curve $9x^2 = y(3 - y)^2$ is rotaed about x-axis. Find the surface area generated.

Volume of solid of revolution

1. (i) Find the volume of the solid in the region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ revolved about x-axis.
   (ii) Find the volume of the solid generated by revolving the region bounded by the parabola $y = x^2$ and the line $y = 1$ about the line $y = -1$.
   (iii) Find the volume of the solid generated by revolving the region bounded between $x^2 = 4y$ and $y = |x|$ about $y = -2$.

2. (i) Find the volume of the solid generated by revolving the region enclosed by triangle with vertices (1,0), (2,1) and (1,1) about y-axis.
   (ii) The region in the first quadrant bounded by the parabola $y = x^2$ and the line $y = x$ is revolved about y-axis. Find the volume of the solid thus generated.
   (iii) Find the volume of the solid in the region in the first quadrant bounded by the curve $y = x^2$ below the x-axis and on the right by the line $x = 1$ about the line $x = -1$.

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CHAPTER 5

PARTIAL DERIVATIVE

1. Prove that

(i) If $u = e^{x/y} + e^{y/z} + e^{z/x}$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

(ii) If $u = \sqrt{x^2 + y^2 + z^2}$ show that $u_{xx} + u_{yy} + u_{zz} = \frac{2}{u}$.

(iii) If $u = log(x^2 + xy + y^2)$ show that $xu_x + yu_y = 2$.

2. If $u = log(x^3 + y^3 + z^3 - 3xyz)$, show that

(i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x + y + z}$

(ii) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x + y + z)^2}$

3. (i) If $u = tan^{-1}\left(\frac{y}{x}\right)$, Show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

(ii) If $u = log\sqrt{(x^2 + y^2 + z^2)}$, show that

$(x^2 + y^2 + z^2)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 1$

(iii) If $v = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$, show that $v_{xx} + v_{yy} + v_{zz} = 0$.

4. Define homogeneous function. State and prove Euler's theorem for homogeneous function of two independent variables.

5. Verify the Euler's theorem for the following functions.

(i) $u = ax^2 + 2hxy + by^2$ (ii) $x^3 + y^3 + 3x^2y + y^3 + 3xy^2$

6. (i) If $u = \sin^{-1}\frac{x^3 + y^3}{x + y}$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

(ii) If $u = \sin^{-1}\left[\frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}\right]$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

(iii) If $u = \log\left(\frac{x^5 + y^5 + z^5}{x^2 + y^2 + z^2}\right)$ prove $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 3$

(iv) If $u = \cos\left[\frac{xy + yz + zx}{x^2 + y^2 + z^2}\right]$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$

7. Find $\frac{dy}{dx}$ by using partial differentiation

(i) $x^y = y^x$ (ii) $x^{2/3} + y^{2/3} = a^{2/3}$

8. Find $\frac{dz}{dt}$ if $z = x^2 + y^2$, $x = at^2$, $y = 2at$.

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CHAPTER 6

APPLICATION OF PARTIAL DERIVATIVE

1. Find the stationary points and the extreme values of the following functions $f(x\ ,y)$

   (i) $6x^2 - 2x^3 + 3y^2 + 6xy$ (ii) $xy + \frac{a^3}{x} +$

   (iii) $x^2y^2 - 5x^2 - 8xy - 5y^2$ (iv) $x^2 + y^2 + (ax + by + c)^2$

2. Find the extreme values of the functions

   (i) $f(x, y, z) = y^2 + 2z^2 - 5x^4 + 4x^5$

   (ii) $f(x, y, z) = -3x^2 + 6xz + 4y - 2y^2 - 6z^2$

3. (i) Show that the function $(x + y)^4 + (x - 3)^6$ has a minimum at (3, –3).

   (ii) Show that the function $f(x\ y) = 4x^2y - y^2 - 8x^4$ is maximum at (0, 0).

4. (i) Find the extreme values of $f = 48 - (x - 5)^2 - 3(y - 4)^2$ subject to the constraint $x + 3y = 9$.

   (ii) If $u = x^2 + y^2$ subject to $x + y = 1$, find the point where u is minimum and also minimum value of u.

   (iii) Use Lagrange's multiplier to find the maxima and minimum values of the function $f(x, y) = ax + by$ subject to the constraint $x^2 + y^2 = 1$ where a and b positive constants.

5. (i) Find the extreme values of the function $f(x\ y\ z) = x^2 + y^2 + z^2$ under the constraint $x + y + z = 3a$.

   (ii) Find the minimum value of $x^2 + y^2 + z^2$ when $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.

   (iii) Find the extreme value of $x^2 + y^2 + z^2$ when $xyz = a^3$.

7. (i) Find the extreme point of the function $v = xyz$ under $2x + 3y + z = 10$. Find the maximum value of v.

   (ii) Find the extreme value of $u = xyz$ under $x + y + z = 6$.

9. (i) Find the dimensions of the rectangular box open at the top of maximum capacity whose surface is 432 sq. cm.

   (ii) Prove that the cube has the least surface among all the parallelepiped of equal volumes.

   (iii) If the sum of the dimensions of a rectangular swimming pool is given. Prove that the amount of water in the pool is maximum when it is a cube.

   (iv) A rectangular box open at the top is to have a volume 32 cc. Find the dimension of the box requiring least material for the construction.

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CHAPTER 7

First Order Ordinary Differential Equation

1. Define the following terms
   - (i) Ordinary differential equations
   - (ii) Order and degree of differential equations
   - (iii) General and particular solution of differential equations
   - (iv) Initial value problem
   - (v) Exact differential equation
   - (vi) Homogeneous differential equation
   - (vii) Linear differential equation
   - (viii) Bernoulli's differential equation
   - (ix) Riccati's differential equation

2. Solve the following differential equations

   (i) $\sqrt{1-x^2}\, dy + \sqrt{1-y^2}\, dx = 0$

   (ii) $ydx = (e^x + 1)\, dy$

   (iv) $\dfrac{dy}{dx} + \dfrac{1+\cos 2y}{1-\cos 2x} = 0$

3. Solve the following initial value problems

   (i) $xy'\, y = 0,\ y(1) = 1$

   (ii) $y'\cos^2 x - \sin^2 y = 0,\ \ y(0) = \dfrac{\pi}{2}$

3. Solve the following differential equations

   (i) $(x + y + 1)\dfrac{dy}{dx} = 1$ (ii) $\sin^{-1}\left(\dfrac{dy}{dx}\right) = x + y$

4. Solve the following differential equations

   (i) $\dfrac{dy}{dx} = \dfrac{x^2 y}{x^3 + y^3}$ (ii) $\dfrac{dy}{dx} + \dfrac{x-2y}{2x-y} = 0$

5. Solve the following differential equations

   (i) $(3x - 2y + 1)\, dx + (3y - 2x - 1)\, dy = 0$ (ii) $\dfrac{dy}{dx} = \dfrac{2x-y+1}{x+2y-3}$

6. Solve the following differential equations

   (i) $\cos x \dfrac{dy}{dx} + y \sin x = \sec^2 x$ (ii) $x \log x \dfrac{dy}{dx} + y = 2 \log x$

   (iii) $(1 + x^2)\dfrac{dy}{dx} + y = e^{\tan^{-1} x}$

7. Solve the following initial value problems

   (i) $y' + 2y = 4x,\ y(0) = -1$ (ii) $x^2 y' + 2xy - x + 1 = 0,\ \ y(1) = 0$

8. Solve the following differential equations

   (i) $\dfrac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ (ii) $\dfrac{dy}{dx} + \dfrac{y}{x}\log y = \dfrac{y}{x^2}(\log y)^2$

9. (i) Show that the substitution $y = y_1 + u$ where y_1 is the solution of Riccati equation, reduces the Riccati equation to a Bernoulli's equation.

   (ii) Show that the substitution $y = y_1 + \dfrac{1}{u}$ where y_1 is the solution of Riccati equation, reduces the Riccati equation to a linear differential equation.

   (iii) Solve the Riccati equation $2\cos x \dfrac{dy}{dx} = 2\cos^2 x - \sin^2 x + y^2$.

10. (i) United Nations Organization (UNO) report the world population in 1998 was approximately 5.9 billion and growing rate of about 1.33% per year. Formulate a differential equation for world population and estimate the world population at the end of the year 2025. When will the population be double of 1998 ?

(ii) A tank initially contains 4kg of salt dissolved into 100 liter of water. Suppose that the brine (salt solution) 2kg os salt per liter is allowed to enter the tank at a rate of 5 liter/min and the uniform solution is drained from the tank at the same rate. Find the amount of salt in the tank after 10 minutes.

(iii) The growth rate of a culture of bacteria is proportional to the number of bacteria present. After one day it is 1.5 times of original number. Find after how many days it will be (a) double (b) triple.

(iv) Suppose that you turn off the heater in your home at night 2 hours before yoy go to bed. If temperature of thr room is 66˚F when you turn off the heater. The temperature of the room fall to 63˚F at the time you go to bed. What will the temperature of the room in the morning after 8 hours you go to bed ? Assume that the outside atmospheric temperature is 32˚F.

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CHAPTER 8

Second Order Ordinary Differential Equation

1. Solve the following differential equations:
   - (i) $y'' + 3y' + 2y = 0$
   - (ii) $y'' + 2y' + 5 = 0$
   - (iii) $y'' + 2y' + y = 0$

2. Solve the following initial value problems:
   - (i) $y'' - 3y' + 2y = 0$, $y(0) = 1$, $y'(0) = 1$
   - (ii) $y'' - 4y' + 4 = 0$, $y(0) = 3$, $y'(0) = 1$
   - (iii) $y'' - 4y' + 5y = 0$, $y(0) = 1$, $y'(0) = 2$

2. Find the general solution of the following differential equations
   - (i) $x^2y'' - 4xy' + 6y = 0$
   - (ii) $x^2y'' - xy' + 2y = 0$
   - (iii) $x^2y'' + 7xy' + 9y = 0$

3. Find the general solution of the following differential equations by method of undetermined coefficients
   - (i) $y'' - 5y' + 6y = 3e^{-x}$
   - (ii) $y'' - 5y' + 6y = 3e^{2x}$
   - (iii) $y'' - 4y' + 4y = 3e^{2x}$
   - (iv) $y'' - y' - 2y = 10\cos x$
   - (v) $y'' - y' - 2y = 10x^2$
   - (vi) $y'' - y' - 2y = e^x - 7\cos 2x$
   - (vii) $y'' + 4y = 7\cos 2x$
   - (viii) $y'' + 4y = 7\cosh x$
   - (ix) $y'' - 2y' = e^x \sin x$
   - (x) $y'' + 3y' = 4x^2$

5. Find the general solution of the following differential equations by method of variation of parameter.
   - (i) $y'' - 4y' + 4y = 3e^{2x}$
   - (ii) $y'' - y' - 2y = 10\cos x$
   - (iii) $y'' - y' - 2y = 10x^2$
   - (iv) $x^2y'' - 4xy' + 6y = 3x^{-2}$
   - (v) $x^2y'' - xy' + 2y = 2x^5$

6. If a series circuit has capacitor of 1.6×10^{-6} F and the inductor of L = 0.4H, then find the resistance R so that the circuit is critically damped.

7. If a parallel circuit has capacitor of 1.6×10^{-6} F and the inductor of L = 0.4H, then find the resistance R so that the circuit is critically damped.

8. Solve the second order differential equation of the parallel LCR Circuit $\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{1}{LC}V = 0$, $V(0) = 6$, $V'(0) = -12$, $R = 20\Omega$, $L = 50H$, $C = 6 \times 10^{-1}$ F.

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