Chapter - 5

Capacitor and Dielectric

1. Introduction:

A specially designed device to store charge and electric energy in a small space is called capacitor. Capacitor consists of two conducting surfaces (in the form of foil, thin film, electrolyte etc) which are called plates. Plates are separated by insulating medium (air, plastic films, mica, glass, ceramics, papers, etc) called dielectrics. The plates connected to a battery is called collector plate and that earthed is called condenser plate. Effect of capacitor (capacity to hold charges and energy) is called capacitance. Non-conducting medium (dielectric) are used to increase capacitance. There are various types of capacitors. The symbol of capacitor is —||—

2. Capacitance of capacitor:

Capacitance of a capacitor is an ability to store (hold) electric charge. Series of experiments show that charge given to a capacitor is directly proportional to electric potential developed across its plates. That is

$$q \propto V$$

or $q = CV$ where C = proportionality const. is known as capacitance of capacitor.

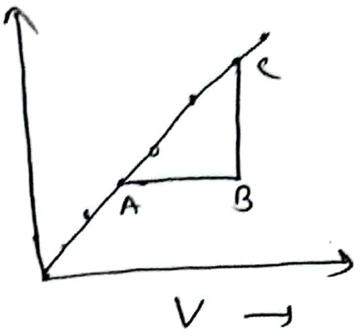
$$\therefore C = \frac{q}{V} \quad \text{--- (1)}$$

Thus, capacitance of capacitor can be defined as the ratio of amount of charge given to a capacitor to potential difference developed across its plates.

Capacitance of a capacitor is also determined by the plot of charge (q) vs potential difference (V) as shown in figure.

In figure, slope $= \frac{BC}{AB} = \frac{\Delta q}{\Delta V}$

$\therefore$ Slope $= \frac{q}{V} = C$ (capacitance).



Unit of C:

$C = \frac{q}{V} = \frac{\text{Coulomb}}{\text{Volts}} =$ Farad (F) [Graph. both q vs V]

$\therefore$ Capacitance can be measure in Faraday (F) unit.

~~1F~~ = C can also be measure in mF, μF, pF, nF etc.

~~1F =~~ $1mF = 10^{-3}F$, $1\mu F = 10^{-6}F$, $1pF = 10^{-12}F$, $1nF = 10^{-9}F$

Defination of 1F:

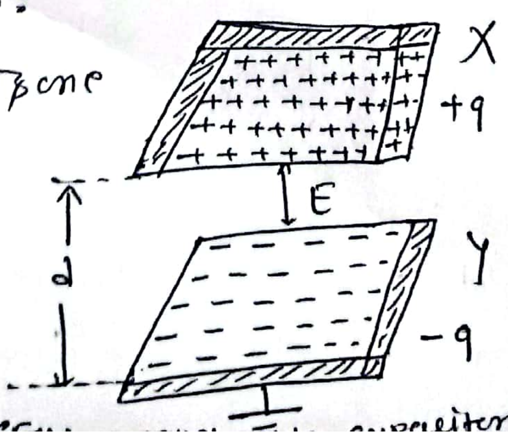
The capacitance of a capacitor is said to be 1F if the application of 1C charge causes the rise in electric potential between its plates through 1V.

3) Types of capacitors:

There are various types of capacitors. Among them here, we are discussed three types of capacitor and then calculated their capacitance respectively.

a) parallel plate capacitor:

parallel plate capacitor is one of the simplest forms of capacitor and consists of two (collector plate and condenser plate) parallel



metal plates separated by dielectric medium. Suppose two parallel metal plates X and Y each of area A are separated by distance (d) as shown in figure.

Assume that plate X is charged positively with charge +q so that plate Y gets equal number of induced negative charge (−q) on its inner surface. The outer surface of Y is earthed. Let distance 'd' is very small as compared to area (A) so that electric field (E) setup by charge +q and −q is uniform between plates. If σ be the surface charge density and ϵ is permittivity of medium between plates, then, from Gauss's law

$$E = \frac{\sigma}{\epsilon} \quad \text{—(1)} \quad (\text{since } E \cdot A = q/\epsilon)$$

The potential difference between two plates is given by $V = Ed$ —(2) Using eqn (1) in eqn (2) we get,

$$\text{or } V = \frac{\sigma}{\epsilon} d$$

$$\text{or } V = \frac{q}{A\epsilon} d \quad \text{where } \sigma = \frac{q}{A} = \text{surface charge density}$$

—(3)

we know that capacitance of a capacitor $C = \frac{q}{V}$

So, eqn (3) becomes

$$C = \frac{q}{V} \Rightarrow C = \frac{\cancel{q}}{\frac{\cancel{q}d}{A\epsilon}} \Rightarrow C = \frac{A\epsilon}{d} \quad \text{—(4)}$$

we know that $K = \frac{\epsilon}{\epsilon_0} \Rightarrow \epsilon = K\epsilon_0$ where K = dielectric const.

So, eqn (4) will be,

$$C = \frac{AK\epsilon_0}{d} \quad \text{—(5)}$$

This eqn (5) gives the capacitance of a capacitor where dielectric medium is placed in both two plates.

If the space betn the plates is air or vacuum, then eqn (5) will be

$$C_0 = \frac{A\epsilon_0}{d} \quad \text{where } \epsilon_0 = \text{permittivity of vacuum.} \quad (6)$$

These eqn. (5) and (6) gives the capacitance of parallel plate capacitor.

(II) Factor affecting the capacitance of capacitor:

Capacitance of a capacitor is independent of nature of material of plate, amount of charges given to the plate and potential across the plates but depends on the following factors.

(i) The distance between the plates d (i.e. $C \propto \frac{1}{d}$); inversely proportional relation.

(ii) Area of each plate (A) (i.e $C \propto A$); directly proportional relation

(iii) Nature of dielectric (medium separating the plates) ϵ (i.e. $C \propto \epsilon$ or $C \propto \epsilon_0 k$); directly proportional relation.

(III) Energy stored in a charged capacitor:

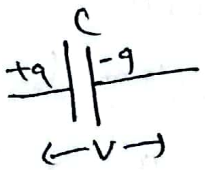


Fig: energy stored in a capacitor

When a capacitor is connected to a battery, charging goes to some extent and then further charging is opposed by electric field setup between plates of the capacitor. To charge the capacitor furthermore, the battery has to do work against developed electric field.

This work done is stored as an electric potential energy and is obtained at the cost of chemical energy stored in the battery.

Suppose a capacitor of capacitance C has charge q and potential difference V at any time of its charging as shown in figure. If dw is small work done to store dq amount of charge to the capacitor then,

$$dw = (V+dV)\,dq$$

or $dw = V\,dq$ (since by storing small charge dq, $dV \simeq 0$ and $V+dV \simeq V$)

or $dw = \frac{q}{C}\,dq$ since $C = \frac{q}{V}$

Now, total work done require to store q amount of charge to the capacitor,

$$W = \int dw = \int_0^q \frac{q}{C}\,dq$$

$$\text{or } W = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^q$$

$$\text{or } W = \frac{1}{2}\frac{q^2}{C}$$

$$\therefore W = \frac{q^2}{2C} \quad \text{—(1)}$$

This work done is stored in the capacitor as its electric potential energy (U) i.e.

$$U = \frac{1}{2}\frac{q^2}{C} \Rightarrow U = \frac{1}{2}\frac{V^2C^2}{C} \Rightarrow U = \frac{1}{2}CV^2 \quad \text{—(2)}$$

This gives the work done is stored in capacitor as its electric potential U.

Energy density:

Energy stored per unit volume of the space between the plates of a capacitor is called energy density. ~~If~~ A be the area of each plate of parallel plate capacitor with dielectric of permitivity ϵ and d be the distance between plates. Then, energy density of capacitor is given by $U_d = \frac{\text{energy stored}}{\text{volume}}$

$$U_d = \frac{\frac{1}{2}CV^2}{A \cdot d}$$ (Since Vol^m (V) = Area×distance)

or $$U_d = \frac{\frac{1}{2}\left(\frac{A\epsilon}{d}\right)\cdot V^2}{A \cdot d}$$ Since $C = \frac{A\epsilon}{d}$

or $$U_d = \frac{1}{2}\epsilon\left(\frac{V}{d}\right)^2$$

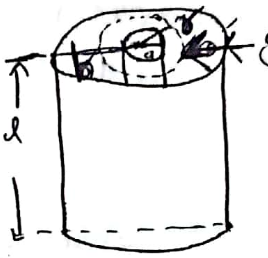
or $$U_d = \frac{1}{2}\epsilon E^2$$ where $E = V/d$

$$\therefore U_d = \frac{1}{2}\epsilon_0 K E^2 \quad \text{--- (3)}$$

Hence, eq^n (3) is an expression of energy density.

(B) Cylindrical Capacitor:

Consider two cylinder of radii a and b are placed co-axially each of length l separated by small distance (b−a) form a cylindrical capacitor. As a Gaussian surface we chose a cylinder of length l and radius r that enclosed just the charge q on the positive inner cylinder.



From Gauss's law, $EA = \frac{q}{\epsilon_0} \Rightarrow E = \frac{q}{A\epsilon_0}$

or $$E = \frac{q}{2\pi l r\epsilon_0}$$ where $A = 2\pi r l$.

Now $$V = \int_a^b E \cdot dr$$

or $$V = \int_a^b \frac{q}{2\pi l r\epsilon_0}\cdot dr \Rightarrow V = \int_a^b \left(\frac{q}{2\pi\epsilon_0 l}\right)\frac{1}{r}dr$$

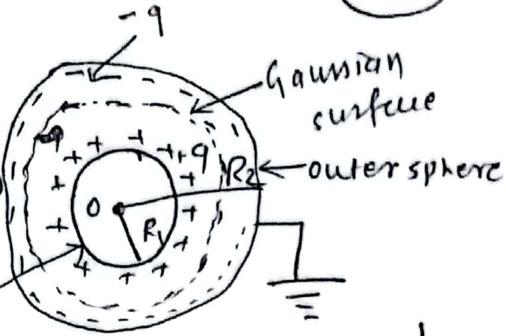
or $$V = \frac{q}{2\pi\epsilon_0 l}[\ln b - \ln a] \Rightarrow V = \frac{q}{2\pi\epsilon_0 l}\ln(b/a)$$

$$C = \frac{q}{V} \Rightarrow C = \frac{q}{\frac{q}{2\pi\epsilon_0 l}\ln(b/a)} \Rightarrow C = \frac{2\pi\epsilon_0 l}{\ln(b/a)}$$

This gives the capacitance of a cylindrical capacitor.

(c) Spherical Capacitor:

A spherical capacitor consists of two concentric conducting spheres where outer one is earthed as is shown in figure.



(Spherical capacitor.)

Let R_1 and R_2 be the radii of inner and outer sphere respectively. when +q charge is given to inner sphere then -q charge is induced on inner surface of ~~outer~~ outer sphere. The outer surface is earthed so it does not store charge and electric potential due to it ($V_0 = 0$). If there is vacuum or air between two spheres, then
The potential at inner sphere (V_i) = potential due to (+q) charge + potential due to (-q) charge.

$$V_i = \frac{1}{4\pi\epsilon_0}\frac{q}{R_1} + \frac{1}{4\pi\epsilon_0}\frac{(-q)}{R_2}$$

$$\text{or } V_i = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\text{or } V_i = \frac{q}{4\pi\epsilon_0}\left(\frac{R_2 - R_1}{R_1 \cdot R_2}\right)$$

Now potential difference between inner and outer sphere is $V = V_i - V_0$

$$\text{or } V = \frac{q}{4\pi\epsilon_0}\left(\frac{R_2 - R_1}{R_1 R_2}\right) - 0$$

$$\therefore V = \frac{q}{4\pi\epsilon_0}\left(\frac{R_2 - R_1}{R_1 \cdot R_2}\right) \quad \text{—(1)}$$

Thus, capacitance of spherical capacitor is

$$C = \frac{q}{V} \Rightarrow \frac{q}{\frac{q}{4\pi\epsilon_0}\left(\frac{R_2 - R_1}{R_1 \cdot R_2}\right)} \Rightarrow C = 4\pi\epsilon_0\left(\frac{R_1 R_2}{R_2 - R_1}\right) \quad \text{—(2)}$$

This eqn (2) gives the capacitance of spherical capacitor.

(H) Applications of capacitors.

Some applications of capacitors are given as following:

1. Capacitors are used to store the energy (charge)
2. Capacitors are used in high frequency microwave system.
3. Capacitors are used in electrical measuring equipment such as in sensors.
4. Capacitors are used in electronics and telecommunications equipment.
5. Capacitors are required in both residential and commercial appliances for example batteries, cameras, electronic chargers, LED lights, Audio equipment etc.
6. Capacitors are used in power system to control voltage and enhance the quality of the power supply.

imp (4) Charging and discharging of capacitor

Let us consider charging of a capacitor of capacitance C through a resistor R in series as shown in figure (a). Suppose, E ~~for an instant, q~~ be the emf of the source having negligible internal resistance. Let a time $t=0$, charge in a capacitor $q=0$.

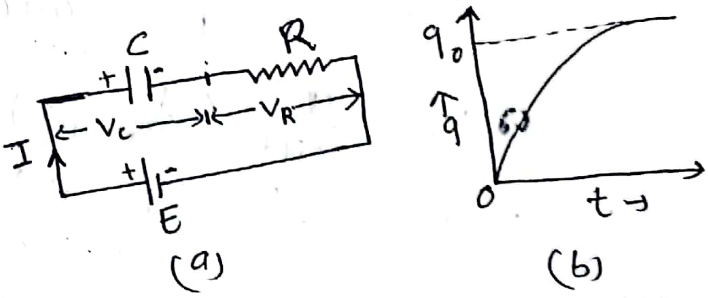


Fig: (a) charging of a capacitor, (b) graph showing the charging nature of capacitor

Suppose for an instant, q be the charge on capacitor and I be the current through R. Let, V_c and V_R are the potential difference across the capacitor C and resistor R respectively.

Therefore, $E = V_c + V_R$

$$\text{or } E = \frac{q}{C} + IR$$

$$\text{or } E = \frac{q}{C} + \frac{dq}{dt} \cdot R \qquad \text{Since } I = \frac{dq}{dt}.$$

or $\frac{dq}{dt}R = E - \frac{q}{C}$

or $\frac{dq}{dt} = \frac{EC-q}{RC}$

or $\frac{dq}{EC-q} = \frac{dt}{RC}$

or $\frac{dq}{q_0-q} = \frac{dt}{RC}$ — (1) ~~where~~

where $q_0 = EC$ is maximum Charge that can be stored in the capacitor. Here integrating eqn (1) we get

$$\int_0^q \frac{dq}{q_0-q} = \int_0^t \frac{dt}{RC}$$

or $-\log(q_0-q)\Big|_0^q = \frac{1}{RC}[t]_0^t$

or $-\log(q_0-q) + \log q_0 = \frac{t}{RC}$

or $\log\left(\frac{q_0}{q_0-q}\right) = \frac{t}{RC}$

or $\log\left(\frac{q_0-q}{q_0}\right) = -\frac{t}{RC}$

or $\frac{q_0-q}{q_0} = e^{-t/RC}$

or $q_0 - q = q_0 e^{-t/RC}$

or $q = q_0(1-e^{-t/RC})$

or $q = q_0(1-e^{-t/T})$ — (2)

where $T = RC$ is called R-C time constant. or charging time constant. of the capacitor

This eqn (2) is called an eqn. of Charging of a capacitor.

→ Charging time constant (relaxation time of circuit):

when $t = T$ then

$q = q_0(1-e^{-T/T})$

or $q = q_0(1-e^{-1})$

or $q = q_0(1-\frac{1}{e})$

or $q = q_0(1-0.37)$

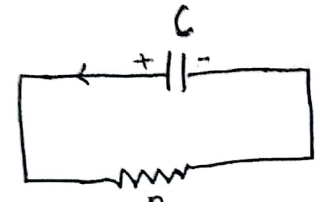
or $q = 0.63q_0$

Therefore, $q = 0.63q_0$ when $t = RC = T$

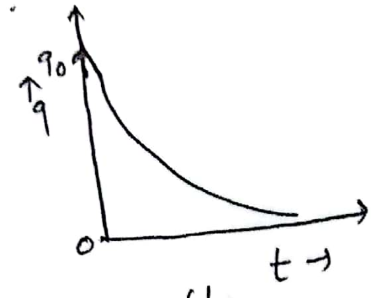
Hence, charging time constant (R-C time constant) of a capacitor is defined as the interval at which capacitor gets charge equal to 63% of maximum charge that it can ~~store~~. Store.

If we plot a graph in between charge q and time t as shown in figure (b). Initially at $t = 0$ sec $q = 0$, and at $t = T$, $q = q_0$ is maximum charge stored in a capacitor.

Discharging of a capacitor:



(a)



(b)

[Fig: (a) Discharging of a capacitor (b) graph showing the discharging nature of a capacitor]

Consider a capacitor of capacitance C is initially charged up to potential V_0 and charge q_0. Then, $q_0 = CV_0$

Let us connect the two plates of the capacitor through a resistor R as shown in figure (a). So that capacitor starts discharging. Suppose after time 't' potential across plates becomes V and remaining charge becomes q. If I be the current through resistor R, then we have

$$V_C = V_R$$

$$\frac{q}{C} = IR$$

or $$\frac{q}{C} = -\frac{dq}{dt}R$$

where - sign indicates that q decreases with increase in time.

or, $$\frac{dq}{q} = -\frac{dt}{RC} \quad \text{--- (1)}$$

Integrating eqn (1) we get

$$\int_{q_0}^{q}\frac{dq}{q} = \int_{0}^{t}\frac{-dt}{RC}$$

or $$[\log q]_{q_0}^{q} = -\frac{1}{RC}[t]_0^t$$

or $$\log\left(\frac{q}{q_0}\right) = -\frac{t}{RC}$$

or $$q = q_0 e^{-t/RC}$$

or $$q = q_0 e^{-t/T} \quad \text{--- (2)}$$

This eqn (2) gives the eqn of discharging of capacitor.

Discharging time constant:

The term $RC = T$ is called discharging time constant. If RC is high, it takes longer time for discharging. If RC is small, the capacitor takes shorter time for discharging.

If $t = T = RC$, then eqn (2) will be

$$q = q_0 e^{-t/t}.$$

or $$q = q_0 e^{-1}$$

or $$q = 0.37\, q_0 \quad \text{--- (3)}$$

Thus, the discharging time constant of a capacitor is defined as the time interval at which the charge on the capacitor is equal to 37% of the initial charge on the capacitor. If we plot a graph in between charge q and time t as shown in figure (b). Where discharging takes place with time.

Dr. Hari Neupane

5.3 Dielectric

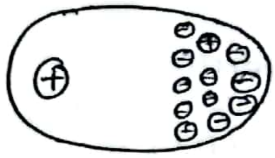
Introduction

A dielectric is an insulator in which all the electrons are tightly bounded to the specific atoms or molecules so that there are no free electrons to carry current on the dielectric. A substance whose basic electric property is the ability to be polarized and in which an electrostatic field can exist. A dielectrical material is used to prevent the leakage of electric charge in electrical engineering devices. There are two types of dielectric substances (molecules). They are given as following;

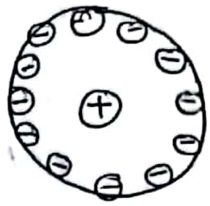
① polar dielectric substance (molecule):
The molecule in which two charge centers (positive and negative) are displaced from each other and has permanent electric dipole moment is called polar dielectric substance or molecules and this process is called polarization. In polar dielectric substance, there is permanent displacement even in the absence of external field. The dipole will tend to align along the direction of applied field. In this case, centre of gravity (C.G) of positive and negative ions separated by a certain distance.
e.g. H_2O, NaCl, NO_2 etc.

(2) Non-polar dielectric substance (molecule) (Q-12)

The molecule in which two charge centers (positive and negative) are coincided in molecules and has no resultant electric dipole moment is called non-polar molecules or substances. In the non-polar dielectrics, no permanent dipole moment exist, however the dipole moment will be produced only after the application of external electric field by induction. In this case, the centre of gravity (C.G) of positive and negative charges are coincided in non-polar dielectrics. eg. H_2, O_2, N_2 etc.



[polar molecule]



(non-polar molecule)

Note:
The product of distance between positive and negative charge centres of a molecules and magnitude of either charge is called dipole moment. ($\mu = q \times d$)

(III) Dielectric constant (K)

The dielectric constant (K) of a plastic or dielectric or insulating material can be defined as the ratio of the charge stored in an insulating material placed between two metallic plates to the charge that can be stored when the insulating material is replaced by vacuum or air. It is also called relative permitivity i.e. $\epsilon_r = K = \frac{\epsilon}{\epsilon_0}$.

Dielectric Constant characterizes the ability of insulators to store electrical energy.

In the other words, dielectric constant can also be defined as the ratio of the capacitance induced by two metallic plates with an insulator between them, to the capacitance of the same plates with air or a vacuum between them; i.e.

$$K = \frac{C}{C_0} \quad -(1)$$

where C = capacitance of capacitor when dielectric is placed in both plates
C_0 = capacitance of capacitor when air or vacuum is placed in both plates.

~~where~~ $C = q/V$ and $C_0 = q/V_0$.

~~or $K = \frac{q/V}{q/V_0}$~~

Consider a capacitor with magnitude of charge q on each plate and potential difference V_0. when an uncharged medium is inserted between the plates of electric potential V. Therefore,

$$K = \frac{q/V}{q/V_0} \quad \Rightarrow K = \frac{V_0}{V} \Rightarrow V = \frac{1}{K} \times V_0 \quad -(2)$$

$$\text{or } K = \frac{E_0}{E} = \frac{\text{electric field before dielectric is introduced}}{\text{electric field after dielectric is introduced}}$$

since $E = V/d$.

$$\text{or } E = \frac{1}{K} \times E_0 \quad -(3)$$

From above eqn. (1), (2) and (3), we can conclude the following points:

(i) Capacitance of capacitor increases by K times.

(ii) Potential difference between the plates of capacitor decreases by a factor $\frac{1}{K}$ times.

(iii) Electric field across the plates of capacitors decreases by a factor $\frac{1}{K}$ times.

Again we know that energy stored in a capacitor

$U = \frac{1}{2} CV^2$ for dielectric is placed in both plates

$U_0 = \frac{1}{2} C_0 V_0^2$ for air or vacuum " " " "

or $U = \frac{1}{2} C_0 K \left(\frac{V_0}{K}\right)^2$

or $U = \frac{1}{2} \frac{C_0 V_0^2}{K}$

or $U = \frac{U_0}{K}$ —

Therefore. (4), energy stored in dielectric is decreases by a factor $\frac{1}{K}$ times.

Note: $K = \frac{C}{C_0} = \frac{\left(\frac{\epsilon A}{d}\right)}{\left(\frac{\epsilon_0 A}{d}\right)} = \frac{\epsilon A}{d} \times \frac{d}{\epsilon_0 A} = \frac{\epsilon}{\epsilon_0} \Rightarrow K = \frac{\epsilon}{\epsilon_0} = \epsilon_r.$

where parallel plate capacitor $C = \frac{\epsilon A}{d}$ and $C_0 = \frac{\epsilon_0 A}{d}$

(II) Polarization and polarizability:

The state of dielectric acted by electric field can be described by two quantities. They are density of electric field E and polarization vector p.

Let us consider the parallel plates of area A having surface charge density σ. One plate being +ve and other being -ve charge. If there is vacuum between the two plates, the electric field will exist in between these plates. Therefore, +q and -q are polarization charges separated by small distance d, so polarization

$p = \frac{q}{A} = \sigma$. Therefore, polarization vector is equivalent to surface charge density.

When dielectric is placed in an external electric field, the electrically charged particles (atoms, ions, molecules) get ~~arranged~~ arranged in such order that dielectric acquires a certain electric moment, such phenomenon is called polarization. Therefore, polarization is directly proportional to applied electric field,

$P \propto E$

$\boxed{P = \alpha E}$ — (1) where, α is called polarizibility.

If $E = 1$ unit,

$P = \alpha$ means, α is dipole moment produced by field of unit strength.

If there is N dipoles,

$\boxed{P = N\alpha E}$ — (2)

Let D_0 and D are electric flux densities when air and insulatory dielectric medium are placed in between the parallel plates capacitor respectively. Therefore,

$$\left.\boxed{\begin{array}{l} D_0 = \epsilon_0 E \\ D = \epsilon E \end{array}}\ \text{and}\right\} \quad (3)$$

The increase (change) in flux density

$P = D - D_0 \Rightarrow P = \epsilon E - \epsilon_0 E$ since $\epsilon_r = \epsilon/\epsilon_0$.

or $P = \epsilon_r \epsilon_0 E - \epsilon_0 E$

or $P = \epsilon_0(\epsilon_r - 1)E$.

or $\boxed{P = \epsilon_0 \chi E}$ — (4) where $\chi = (\epsilon_r - 1)$ = called dielectric ~~[illegible]~~ susceptibility.

Case-1: In vacuum, $\chi = 0$, $P = 0$ (since $\epsilon_r \cdot \epsilon_0 = \epsilon$. In vacuum $\epsilon = \epsilon_0$ $\therefore$ $\epsilon_r = 1$ $\chi = (1-1) = 0$)

Therefore, there is no polarization in free space.

(II) Gauss's law in Dielectric [imp]

Consider a parallel plate capacitor of area A having charge q on each plate as shown in fig. a. where air is used as a dielectric in both two parallel plate. let E_0 be the of the field and ϵ_0 be the perminivity of air between two plates. Then according to gauss's law

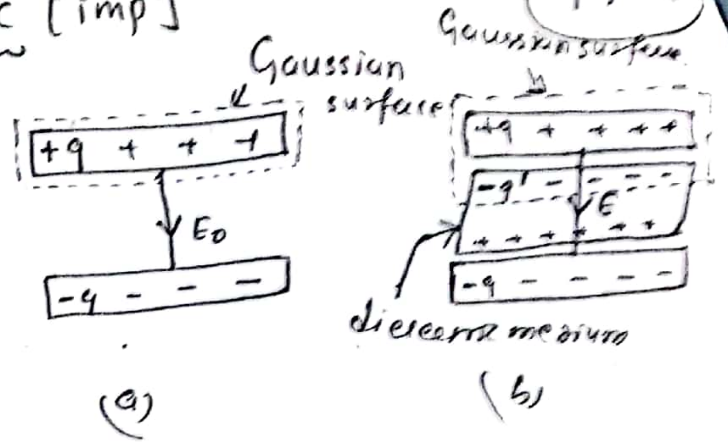


Fig: (a) parallel plate capacitor with air as a dielectric (b) parallel plate capacitor with insulating dielectric medium

$$E_0 \cdot A = \frac{q}{\epsilon_0} \Rightarrow \boxed{E_0 = \frac{q}{A\epsilon_0}} \quad \text{--- (1)}$$

when a dielectric medium is placed in between two parallel plate capacity as shown in figure (b). -q' charge is induced due to +q charge on dielectric medium. Take a Gaussian surface which enclosed net charge $q + -q' \Rightarrow q - q'$. shown in fig (b).

Using Gauss's law.

$$E \cdot A = \frac{q-q'}{\epsilon_0}$$ where E be the electric field ~~generated in both plate~~ across capacitor with dielectric.

$$\text{or } \boxed{E = \frac{q-q'}{A\epsilon_0}} \quad \text{--- (2)}$$

we know that, from the defination of dielectric,

$$K = \frac{E_0}{E} \Rightarrow \boxed{E = \frac{E_0}{K}} \quad \text{--- (3)}$$

Using eqn (3) in eqn (2) we get

$$\boxed{\frac{q-q'}{A\epsilon_0} = \frac{E_0}{K}} \quad \text{--- (4)}$$

Using eqn (1) in eqn (4) we get

$$\frac{q-q'}{\cancel{A\epsilon_0}} = \frac{q}{\cancel{A\epsilon_0}K} \Rightarrow \boxed{q - q' = \frac{q}{K}} \quad \text{--- (5)}$$

Using eqn (5) in eqn (2) we get

$$E = \frac{q}{A\epsilon_0 K}$$

$$\text{or } \boxed{KE \cdot A = \frac{q}{\epsilon_0}} \quad \text{--- (6)}$$

This eqn (6) is an ~~eqn of~~ expression of Gauss's law in dielectric.

~~Therefore,~~

(II) Derive the relation between electric displacement vector D, polarization P and electric field E:

From eqn (2) $\vec{E}\epsilon_0 = \frac{q}{A} - \frac{q'}{A}$

or $\vec{E}\epsilon_0 = \vec{D} - \vec{P}$ $\Rightarrow \boxed{\vec{D} = \vec{P} + \epsilon_0\vec{E}}$

(#) Different types of polarizability (polarization)

There are three types of polarizability. They are;

(i) Electronic polarizability (polarization)

(ii) Ionic polarizability (polarization)

(iii) Orientational (dipolar) polarizability (polarization)

Among these three types of polarizability, here we described only electronic and ionic polarizability.

(i) Electronic polarizability:

The electronic polarizability arises due to the displacement of electron cloud of an atom relative to its nucleus in the presence of an applied electric field as shown in figure.

let us consider an electron is acted ~~by~~ upon by an electric field E_{loc}, then force equation is given by

$$F = \frac{mv^2}{r} = \frac{m(r\omega_0)^2}{r} = m\omega_0^2 r \approx m\omega_0^2 x \quad \text{since } r = x$$

$$F = m\omega_0^2 x \quad \text{---(1)}$$

we know that $F = -eE$ ---(2) where $-e$ = electron charge

Equating (1) and (2), we get

$$m\omega_0^2 x = -eE_{loc}$$

$$\text{or } x = \frac{-eE_{loc}}{m\omega_0^2} \quad \text{---(3)}$$

Here, 'x' is displacement of electron due to the applied electric field. Now, dipole moment p is given by

$p = -e \cdot x$

or $p = \frac{e^2 E_{loc}}{m\omega_0^2}$ — (4)

We know that $p = \alpha E_{loc}$

$\therefore \alpha = \frac{p}{E_{loc}}$ is called polarizability. — (5)

Using eqn (4) in eqn (5) we get

$$\boxed{\alpha = \frac{e^2}{m\omega_0^2} \quad \text{— (6)}}$$ ← [In syllabus upto this]

To obtain the frequency dependence of electronic polarizability, we treat the system as a simple harmonic oscillator. If ω is the frequency of the local field, the field at any time 't' is given by $E_{loc} \sin\omega t$. The equation of motion is given by

$$m\frac{d^2x}{dt^2} + m\omega_0^2 x = -e E_{loc} \sin\omega t \quad \text{— (7)}$$

where ω_0 = resonant frequency.

put $x = x_0 \sin\omega t$, we obtain eqn (7)

$$m(-\omega^2 + \omega_0^2) x_0 = -e E_{loc}$$

$$\text{or } x_0 = \frac{-e E_{loc}}{m(\omega_0^2 - \omega^2)} \quad \text{— (8)}$$

The amplitude of dipole moment is given by

$$p_0 = -e x_0 = \frac{e^2 E_{loc}}{m(\omega_0^2 - \omega^2)} \quad \text{— (9)}$$

We know that

$$\alpha_{electronic} = \frac{p_0}{E_{loc}} = \frac{e^2}{m(\omega_0^2 - \omega^2)} \quad \text{— (10)}$$

This expression gives the frequency dependence of electronic polarizability.

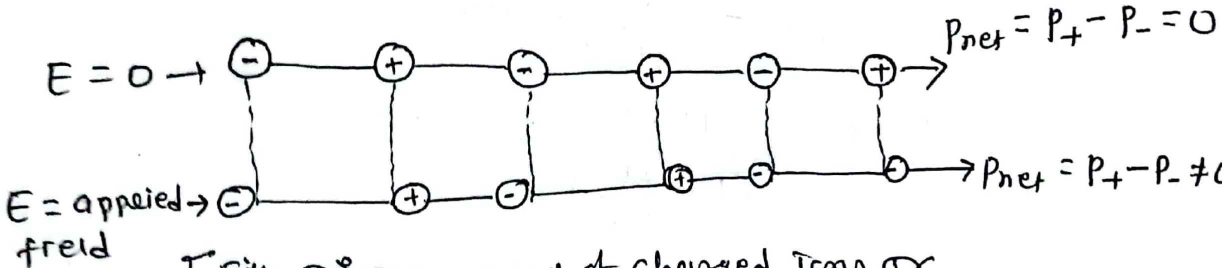
(ii) Ionic polarizability (polarization):

Ionic crystal (crystal having ionic bonding) exhibits ionic polarization (polarizability). The ionic polarizability arises due to the displacement of charged ion relative to other ions in solid. Examples of ionic crystal are NaCl, KCl, LiBr etc. The ionic crystal have well defined lattice site in which positive and negative ions are located. Each pairs of positive and negative ion form a dipole. Since, these dipoles are lined one after another so there is no net dipole moment,

Thus $P_+ - P_- = 0$ — (1)

When an external field is applied, the negative ions move in the direction opposite to applied field and positive ions move in the direction of applied field. Hence, P_+ increases and P_- decreases. So that $P_+ - P_- \neq 0$. — (2)

~~The chain~~ In this way, the ionic crystal is polarized. The displacement of charged ions are shown in figure.



[Fig. Displacement of charged ions of Na^+Cl^- chain in the presence and absence of electric field]

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Clausius-Mossotti relation (For non-polar molecules)

The relation between the microscopic parameters like atomic polarizability (α), molecular polarizability (ν) to the macroscopic parameters like dielectric constant (K) and susceptibility (χ) is called Clausius-Mossotti relation.

The actual field experienced by a molecule in dielectric is called local field (E_{local}). This is the sum of applied field and field of polarization. So,

$$E_{local} = E + \frac{P}{3\epsilon_0} \quad \text{—(1)}$$

The induced polarization is now given by

$$P_{in} = \alpha E_{local}$$

polarization due to N molecules

$$P = N p_{in} = N\alpha E_{local} = N\alpha \left(E + \frac{P}{3\epsilon_0}\right) \text{—(8)}$$

$$\text{or } P = N\alpha E + \frac{N\alpha P}{3\epsilon_0}$$

$$\text{or } P\left(1 - \frac{N\alpha}{3\epsilon_0}\right) = N\alpha E$$

$$\therefore P = \frac{N\alpha E}{\left(1 - \frac{N\alpha}{3\epsilon_0}\right)} \quad \text{--(2)}$$

we know that, $P = \epsilon_0 \chi E$ —(3)

Comparing eqn (2) and (3), we get

$$\epsilon_0 \chi E = \frac{N\alpha E}{\left(1 - \frac{N\alpha}{3\epsilon_0}\right)}$$

or $\epsilon_0 \chi \left(1 - \frac{N\alpha}{3\epsilon_0}\right) = N\alpha$

or $\epsilon_0 \chi - \frac{N\alpha\chi}{3} = N\alpha$

or $\epsilon_0 \chi = N\alpha + \frac{N\alpha\chi}{3}$

or $\epsilon_0 \chi = N\alpha \left(1 + \frac{\chi}{3}\right)$

or $\epsilon_0 \chi = N\alpha \left(\frac{3+\chi}{3}\right)$

or $\frac{N\alpha}{3\epsilon_0} = \frac{\chi}{3+\chi}$

or $\frac{N\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r - 1 + 3}$ since $\chi = \epsilon_r - 1$

$\therefore \frac{N\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$

or $\alpha = \frac{3\epsilon_0(\epsilon_r - 1)}{N(\epsilon_r + 2)} \Rightarrow \alpha = \frac{3\epsilon_0(k-1)}{N(k+2)}$ — (4)

This eqn (4) is called Clausius-Mossotti equation. This α is called atomic polarizability. But for molecular polarizability, $\nu = \frac{\alpha}{\epsilon_0}$

$\therefore$ eqn (4) becomes.

$$\nu = \frac{3(k-1)}{N(k+2)} \quad \text{— (5)}$$

Case-I: For electronic polarizability, Clausius-Mossotti relation, $\alpha_e = \frac{3\epsilon_0(k-1)}{N(k+2)}$ or $\frac{3\epsilon_0(\epsilon_r - 1)}{N(\epsilon_r + 2)}$

and Ionic polarizability: $\alpha_i = \frac{3\epsilon_0(k-1)}{N(k+2)}$ or $\frac{3\epsilon_0(\epsilon_r - 1)}{N(\epsilon_r + 2)}$

Case-II: For electronic polarizability, Clausius-Mossotti equation can be written in term of optical frequency

$\alpha_e = \frac{3\epsilon_0(\eta^2 - 1)}{N(\eta^2 + 2)}$ where $\eta^2 = \epsilon_r = k$, η = optical freq.

(II) Limitations of Clausius- Mossotti relation

Following are the limitations of Clausius-mossotti relation,

(i) The relation holds best for the dilute substances like gas.

(ii) It holds approximately valid for the solids and liquids of their dielectric constant is very high.

(iii) It holds for monoatomic non-polar molecules.

(iv) Not holds for liquid of strong solution.

Worked out examples

(1) In dielectric prove that $K = (1 + \chi_e)$.

Soln: we know that $\vec{D} = \vec{P} + \epsilon_0 \vec{E}$

$$\vec{D} = \epsilon_0 \vec{E} + \epsilon_0 \chi_e \vec{E} \quad \text{since } \vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$\text{or } \vec{D} = \epsilon_0 \vec{E} (1 + \chi_e).$$

$$\text{or } \epsilon \vec{E} = \epsilon_0 \vec{E} (1 + \chi_e) \quad \text{since } \vec{D} = \epsilon \vec{E}$$

$$\text{or } \frac{\epsilon}{\epsilon_0} = (1 + \chi_e)$$

$$\therefore K = (1 + \chi_e) \quad \text{since } K = \epsilon/\epsilon_0.$$ proved,

(2).