3D Viewing Pipeline

At the first step, a scene is constructed by transforming object descriptions from **modeling coordinates** to **world coordinates**.

Next, a view mapping convert: the world descriptions to **viewing coordinates**.

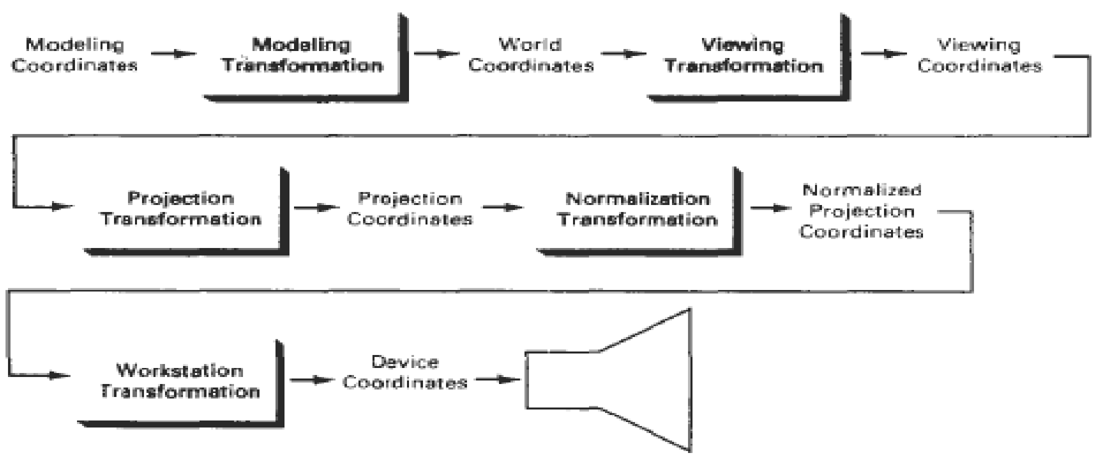
At the projection stage, the viewing coordinates are transformed to **projection coordinates**, which effectively converts the view volume into a rectangular parallelepiped.

Then, the parallelepiped is mapped into the unit cube, a normalized view volume called the **normalized projection coordinate system**.

The mapping to normalized projection coordinates is accomplished by transforming points within the rectangular parallelepiped into a position within a specified three-dimensional viewport, which occupies part or all of the unit **cube.**

Finally, at the workstation stage, normalized projection coordinates are converted to **device coordinates** for display. The normalized view volume is a region defined by the planes

$$x = 0, \quad x = 1, \quad y = 0, \quad y = 1, \quad z = 0, \quad z = 1$$



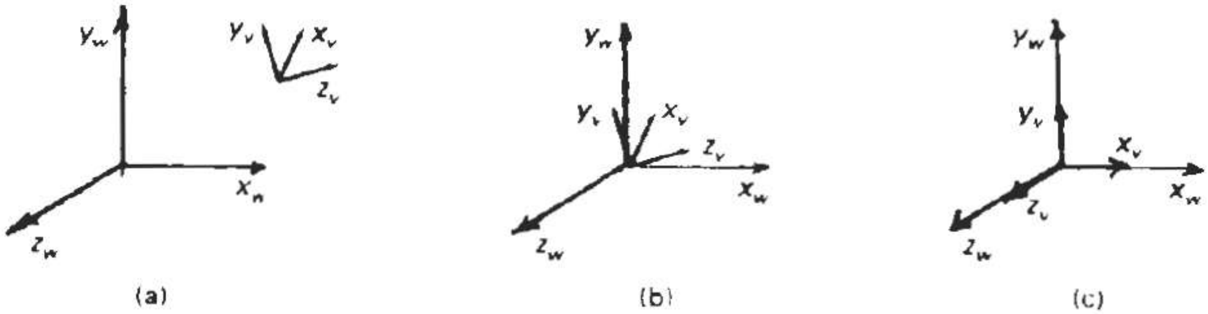
4.4 Three Dimensional Viewing Transformation

Before object descriptions can be projected to the view plane, they must be transferred to viewing coordinates.
Conversion of object descriptions from world to viewing coordinates is equivalent to a transformation that superimposes the viewing reference frame onto the world frame using the basic geometric translate - rotate operations.
This transformation sequence is
1 . **Translate** the view reference point to the origin of the world-coordinate system.
2. Apply **rotations** to align the x_v, y_v, and z_v axes with the world x_w, y_w, and z_w axes, respectively.
If the view reference point is specified at world position (x_0,y_0,z_0), this point is translated to the world origin with the matrix transformation

$$T = \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The rotation sequence can requires three coordinate-axis rotations, depending on the direction of N.
If N is not aligned with any world-coordinate axis, superimpose the viewing and world systems with the transformation sequence $\mathbf{R_x} \cdot \mathbf{R_y} \cdot \mathbf{R_z}$
First rotate around world x_w axis to bring z_v into the $x_w z_w$ plane.

Then, rotate around world $\mathbf{y_w}$ axis to align the z_w and z_v axes.
Finally rotate about $\mathbf{z_w}$ axis to align $\mathbf{y_w}$ and y_v axes.



The composite transformation matrix is then applied to world-coordinate descriptions to transfer them to viewing coordinates.

Alternative Approach
Calculate unit u v n vectors and form the composite rotation matrix directly
Given vectors N and V, these unit vectors are calculated

$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = (n_1, n_2, n_3)$$

$$\mathbf{u} = \frac{\mathbf{V} \times \mathbf{N}}{|\mathbf{V} \times \mathbf{N}|} = (u_1, u_2, u_3)$$

$$\mathbf{v} = \mathbf{n} \times \mathbf{u} = (v_1, v_2, v_3)$$

This method automatically adjusts the direction for V so that v is perpendicular to n.
The composite rotation matrix for the viewing transformation is then

$$\mathbf{R} = \begin{bmatrix} u_1 & u_2 & u_3 & 0 \\ v_1 & v_2 & v_3 & 0 \\ n_1 & n_2 & n_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

It transforms u onto world x_w axis, v onto y_w axis, and n onto z_v axis.
This matrix automatically performs the reflection necessary to transform a left-handed viewing system onto right-handed world system.
The complete world-to-viewing coordinate transformation matrix is obtained as the matrix product

$M_{MW.VC} = R.T$

This transformation is then applied to coordinate descriptions of objects in the scene to transfer them to the viewing reference frame.