CHAPTER - 1
Introduction

Role of Electricity in Modern Society

Progress of human ~~ci~~ civilization has historically been in proportion to the ability of humans to use energy. The operation of our technology society depends upon the production and use of large amount of energy. The use of electrical energy has reached such a scale that it is unimaginable to live without it. Per capita consumption of energy has become an indicator of the development of a country. The per capita consumption of energy of countries like USA, Canada, UK and Russia are in the range of 4000 - 6000 kWh per annum whereas the consumption in developing countries are merely in the range of 30-60 kWh per annum and least developed countries get hardly 10 kWh per annum per capita.

The problem is not ~~a~~ that of having insufficient energy, it is incredibly abundant, rather infinite. What is needed is exploring these available forms of energy into beneficial applications and use it for development. As mentioned earlier, energy plays a vital role in the development process. We need energy for any activities right from lighting or cooking food to any high technological processes, Thus tapping off ~~of~~ energy available in the nature and challenging it into benificial applications or requirements to the maximum extent is the prime need for any society or country to be developed.

Need of Energy:-

1. Lighting.
2. Cooling, heating and ventilation.
3. Transportation system.
4. Communication system.
5. Industrial and manufacturing processes
6. Construction applications
7. Agriculture productions.

Advantages of Electrical Energy

1. It can be controlled efficiently.
2. It can be transmitted at the speed of light.
3. It is inherently pollution free.
4. Conversion to other forms is directly and easy.
5. It can be stored efficiently.
6. It can be converted to other forms at typically high efficiencies.

It is because of these advantages that electrical energy has become the most common forms of energy and its generation, transmission, distribution has become of fundamental importance to a modern society and to any developing country.

Electrical energy Resources and production

There are some basic energy sources that can be used to produce bulk electrical energy. The only practical device for generation of electrical energy in large scale is called "generator" which essentially converts mechanical energy into electrical energy.

A brief discussion of some sources are as follows:-

1. Thermal:-

a) Coal:-

Coal has been the major energy source for power generation in the countries where its diposits are substaintial. Because of low cost and good calorific value, it is used in most of thermal power stations of the world. However, in Nepal, we donot have any diposits of this source and therefore there is no power station using coal as the source.

b) Oil and Natural gas:-

These sources, espicially liqurd fuels like petroleum and diesel oil are used in internal combustion engines. Because of the cost and scarcity and its major use in automobiles, these fuels are not used for bulk electrical energy generation.

iii) Nuclear fission:-

Nuclear fission of atomic materials such as 'Uranium' produces heat which is utilized to produce steam to run steam turbines. Such a power station is called a nuclear power station. The speciality of this source of energy is the quantity of fuel required. About 3000 metric tons of coal produces the same amount of heat as 1 kg of nuclear fuel.

When an atom of U-235 ($^{235}_{92}U$) is bombarded with slow moving neutrons, it breaks up into two smaller atoms and three fast neutrons releasing heat. This process is called nuclear fission.

Nuclear power station:-

In nuclear reactor, the chain reaction is controlled by movable control rods made of boron. These absorb neutrons to slow down or stop the chain reaction. The heat produced is used to make steam to drive generators in a power station.

Solar:-

It is possible to collect solar energy directly and concentrate it on boilers for steam production. The major problem is its diffuse nature requiring large amount of land for collectors and the unreliability caused by atmospheric and weather conditions.

Geothermal:-

Heat from the earth's interior and sub-surface water is combined to produce natural steam, which can be used to run a turbine and thereby produce electrical energy. This source is also rare and not of much importance as far as commercial power generation is concerned. One such installation is in California operating at a capacity of about 400 MW, which is the biggest of its kind.

2) Non Thermal:-

Hydropower:-

It has historically been an economical and pollution free source of energy. Especially in the countries like Nepal where there are number of high current flowing rivers, streams and water falls, This source of energy is extremely important. Almost 95% of the electrical energy generation in Nepal is by hydropower. The initial capital investment in dams, transmission and generation is quite high but recurring cost afterward is very low, so that the overall system will be very economical in the long run.

Tidal:

A dam with large gates is made across the mouth of the bay in the ocean and low head water turbines are used for generation of electric power. At the time of high tide, the gates are opened and after the water level reaches its maximum extent, the gates are closed and water is trapped is allowed to pass through turbine generating electricity.

Wind:-

The wind can be used to drive turbines (air turb..) that, in turn, drive generators to produce electricity. Bec.. of its inherent intermittent characteristic, the use of this source of energy is also rare. In certain places where the strong winds are a common feature, this source of energy may be very attractive. China's wind power generation is developing very fast.

Direct solar Conversion:-

Semiconductors exposed to solar radiation produce electricity through the so-called photovoltaic mechanism. Because of the high cost of solar cells and low efficiences of the conversion, this source for the commericial production of electrical energy has not been feasible.

Generation, Distribution and Consumption of electrical energy.

Generating stations, transmission lines and distribution systems are the main components of an electrical power system. Generating stations and distribution system are connected through transmission lines, which also connect one power system to another. A distribution system connects all the loads in a particular area to the transmission lines. Electric power is generated at a voltage of 11 to 25KV which then is stepped up to the transmission levels in the range of 66 to 132KV. As the transmission capability of a line is proportional to the square of its voltage, research is continously being carried out to raise the transmission voltage. Some of the countries are already employing 750KV for transmisson The voltages are expected to raise to 1000KV in near future in advanced countries. In Nepal, 132KV is the highest voltage used for transmission of electricity.

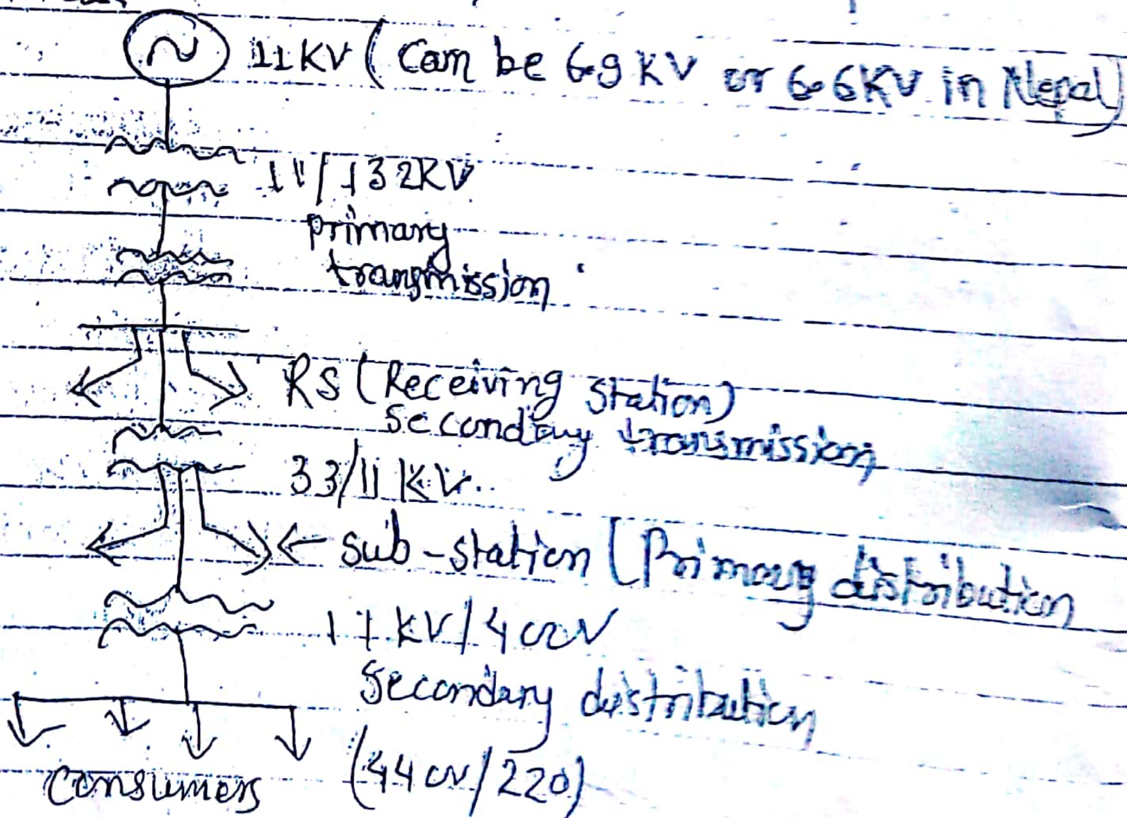
For very long distances (over 400km) it is economical to transmit bulk power by dc transmission at 400KV and above. The line is connected to the AC systems at the two ends through a transformer and converting and inverting equipment (silicon controlled rectifiers are employed for this purpose). Several dc transmission lines have been constructed in Europe, the USA and in Asia.

The first step-down of voltage from the transmission level is to a range of 33 to 66 KV depending upon the transmission line voltage. Some industries may require power at these levels. This step down is from the transmission and grid level to sub transmission level.

The next step down in voltage is at the distribution sub stations. Normally, two distribution voltage levels are employed:

- The primary or feeder voltage (11KV)
- The secondary or consumer voltage (440V) Three phase / 230V single phase.

The distribution system, fed from the distribution transformer stations, supplies power to domestic or industrial and commercial consumers. Thus, a power system operates at various voltage levels seperated by transformers.



INTRODUCTION.

CHAPTER-2

DC CIRCUIT ANALYSIS.

The flow of electrons through a conductor constitutes an electron current; and the path of electric current is known as electric circuit, which always is a closed path. If the flow of electric current, polarity is unidirectional, then the path is called a d.C. circuit. In this, polarity does not change with time.

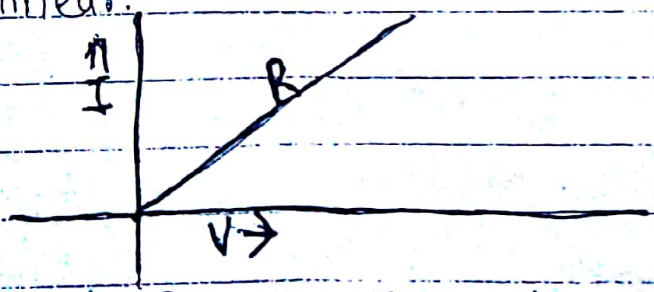
Linear and non-linear parameter

The passive element which resist the flow of electric current or controls the flow of current is known as resistor.

Resistors are of two types:-

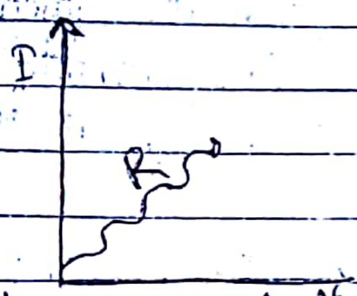
(i) Linear resistor:-

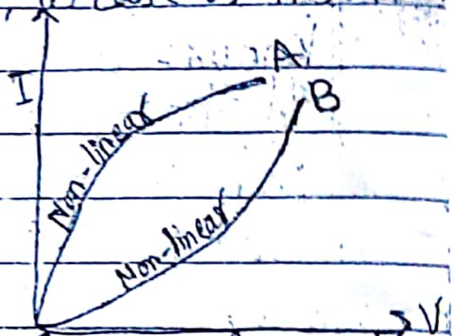
A resistor (or resistance) in which current produced is directly proportional to the applied voltage, is called linear resistor. In other words, the resistance of a linear resistor remains constant and a graph of current versus voltage is linear.



In other words, It follows ohm's law.

(ii) Non-linear resistor:-
A resistor, whose current does not change linearly with changes in applied voltage, is called non linear resistor.
The reason for such a behaviour is that flow of current always results in the production of some amount of heat, which either:
(i) increases resistance of the resistor, eg. in case of most me
(ii) decreases resistance of the resistor eg. in case of insulator





Thus, the resistor which does not follow ohm's law is known as non-linear resistor.

Types of non-linear resistors:-

Non-linear resistors are of two types:
(1) Those through which current increases more than proportionality with applied voltage. In other words, resistance of such a non-linear resistance decreases with rise in temperature. Eg. thyristor.

(2). Those through which current increases less than proportionality with applied voltage.
Resistance of such a resistor increases with rise of temperature.
Eg. semi-conductor

Active and passive circuits

Those elements which can take energy, consume power but cannot deliver or energige the other elements are called passive elements.

Eg. resistor, capacitor, inductors etc.

Those elements which can take energy, consume power and also can deliver or energige the other elements are called active elements.

Eg. voltage source and current sources.

Active elements:-

All sources are divided into two parts:-

(a) Ideal
(b) Practical.

Ideal voltage source.

An ideal voltage source is that voltage source which gives a fixed or constant load voltage despite infinite variation in load.

An ideal voltage source possesses ~~zero~~ minimum internal resistance (neglegible).

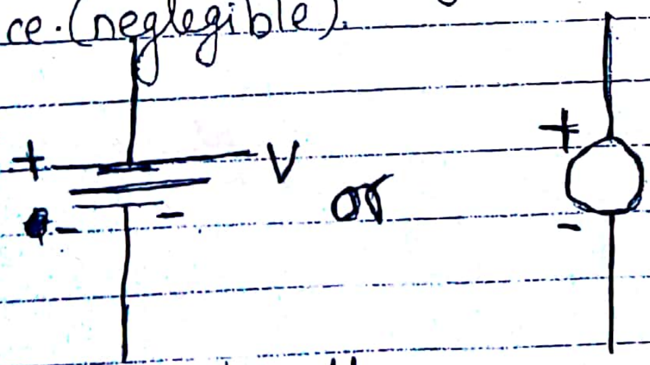


Fig (a) Ideal voltage source.

In fact, an ideal voltage source is not in use and not possible.

Practical voltage source

The voltage sources which has got some internal resistance, ~~which~~ that makes the load voltage vary with load or load current due to drop in internal resistance of the source is known as practical voltage source.

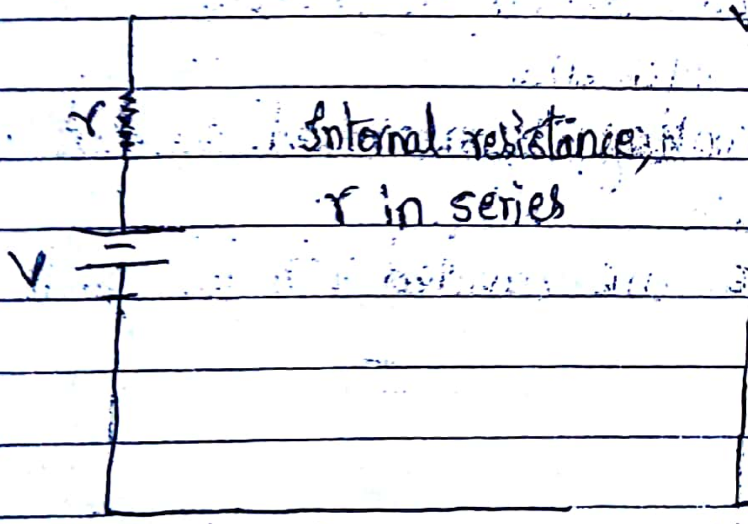
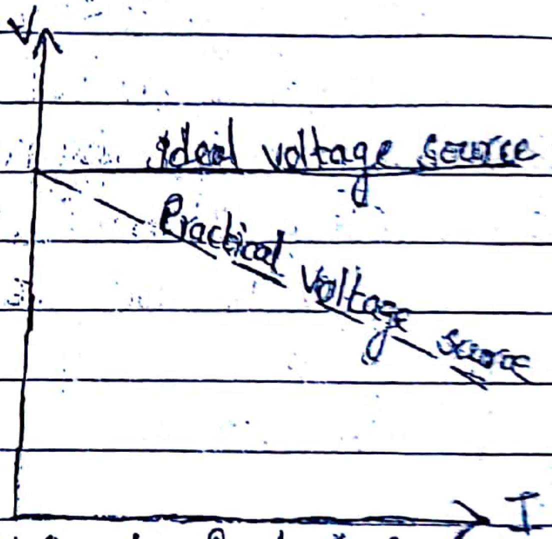


fig (i): Practical voltage source



(ii) Graph of ideal & practical source

Ideal current source:-

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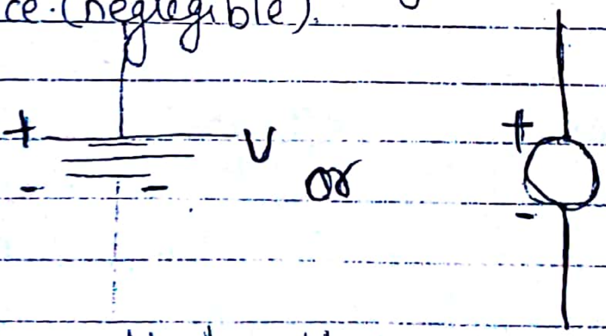

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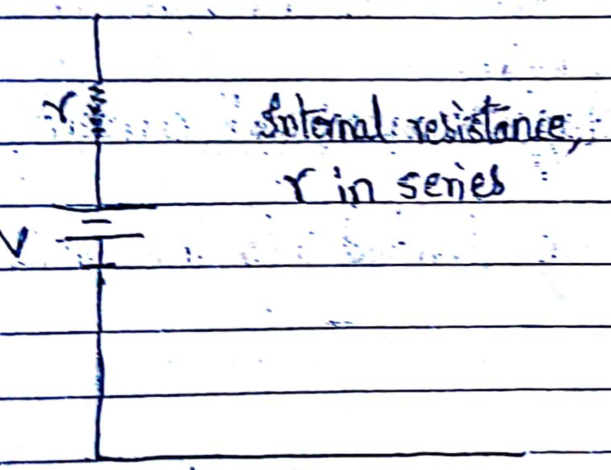

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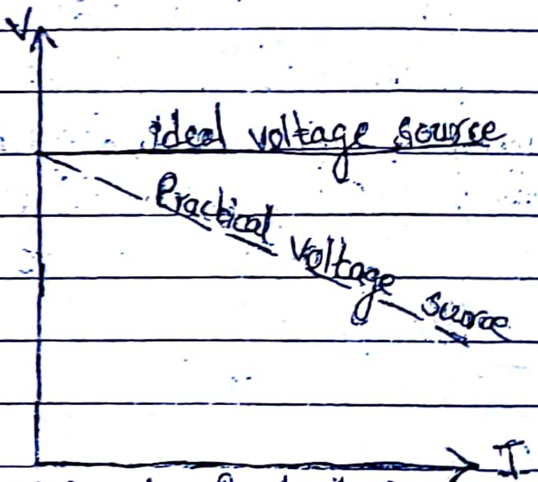


(ii) Graph of ideal & practical source

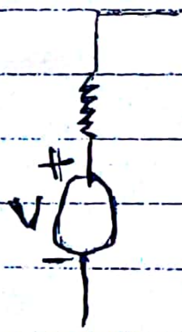
Ideal current source:-

Types of voltage and current source:-

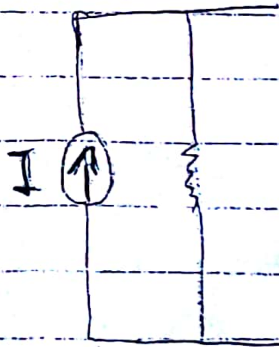
1. Independent type.
2. Dependent type.

1. Independent type:-

The sources which does not depends upon the quantities like voltage and current found anywhere in the circuit are known as independent type.



Independent Volt.



Independent current source.

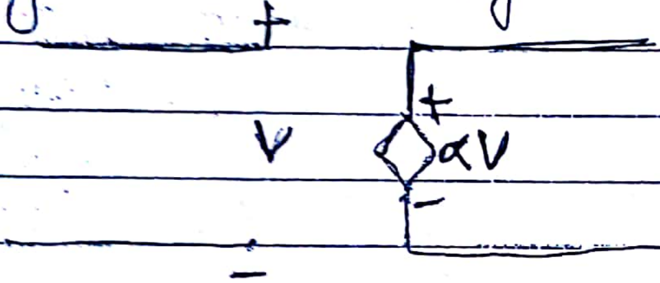
(2) Dependent type:-

The sources which are dependent upon the quantities like voltage and current found anywhere in the circuit are known as dependent sources.

Dependent sources are of four types:-

(a) Voltage Controlled Voltage Source (VCVS):-

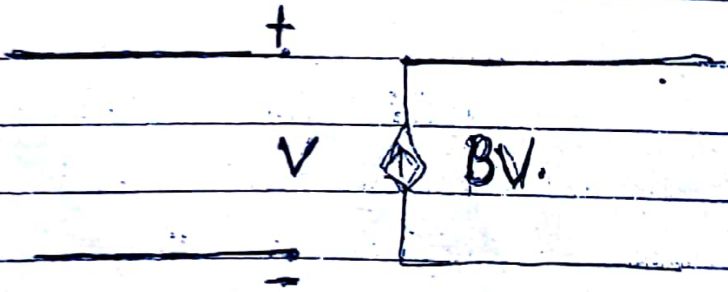
The voltage source which depends on the voltage found anywhere in the circuit is called voltage controlled voltage source.



α = constant

(b) Voltage Controlled Current Source (VCCS):-

The current source which depends on the voltage found anywhere in the circuit is known as voltage controlled current source.



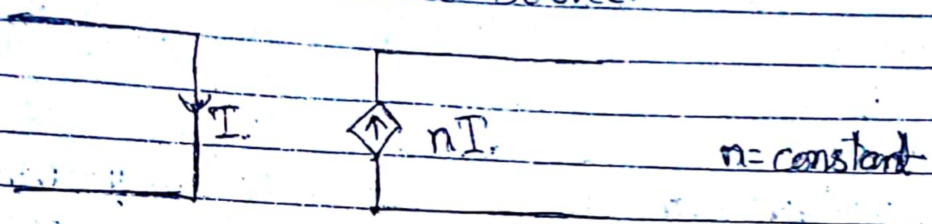
β = constant

(c) Current controlled voltage source (CCVS):-

It is that type of voltage source which depends upon current found anywhere in the circuit.

m = constant

(d) Current controlled current source (CCCS):-
The current source which depends upon the current found anywhere in the circuit is known as current controlled current source.



Open and short circuit

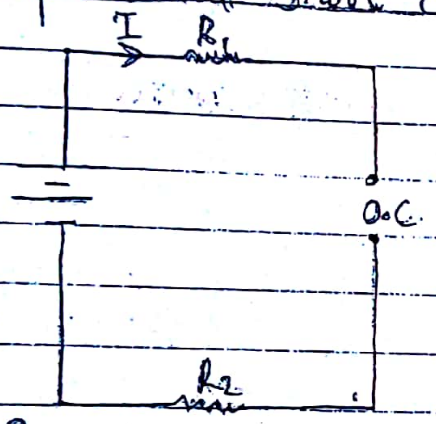
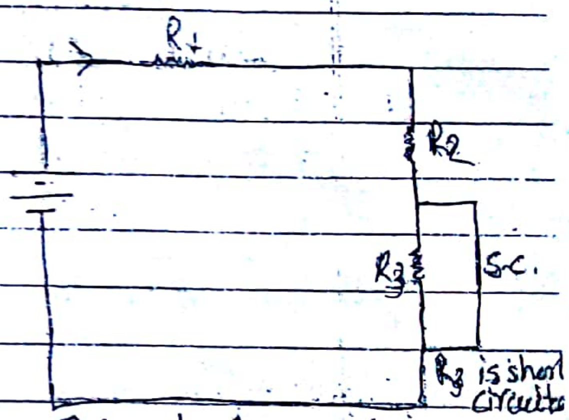


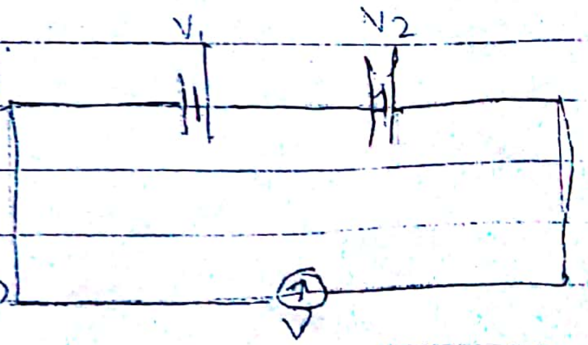
Fig.(1) Open circuit
Current, I = 0
Resistance, R = ∞



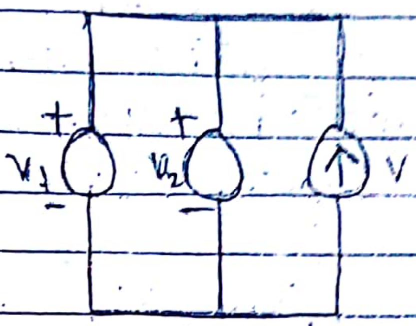
Fig(2) Short circuit
If R_3, I = maximum &
R = minimum.

Voltage Source in Series:

- If opposite terminals are connected then $V = V_1 + V_2$
- If same terminals are connected then
  $V = V_2 - V_1$ or $V_1 - V_2$



Voltage source in parallel



$V_1 = V_2 = V$.

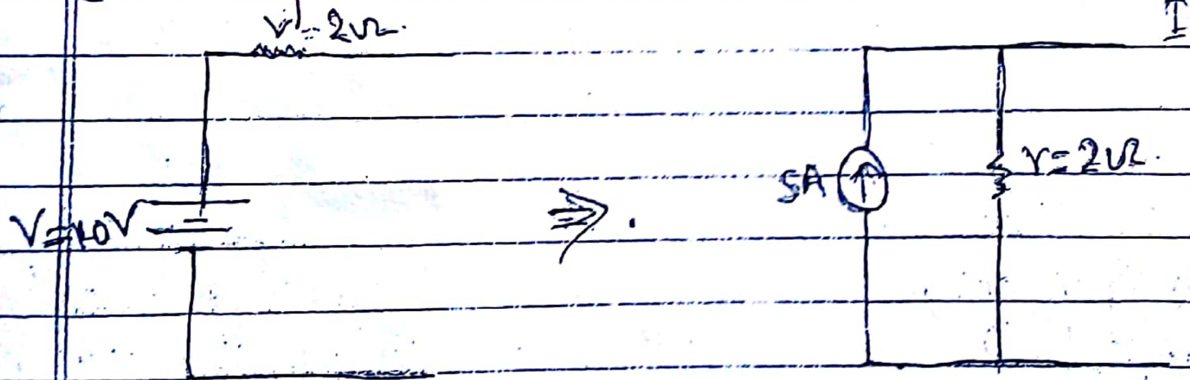
voltages are said to be connected in parallel if all of them are connected across two common point.

Source conversion:

(1) From voltage source to current source.

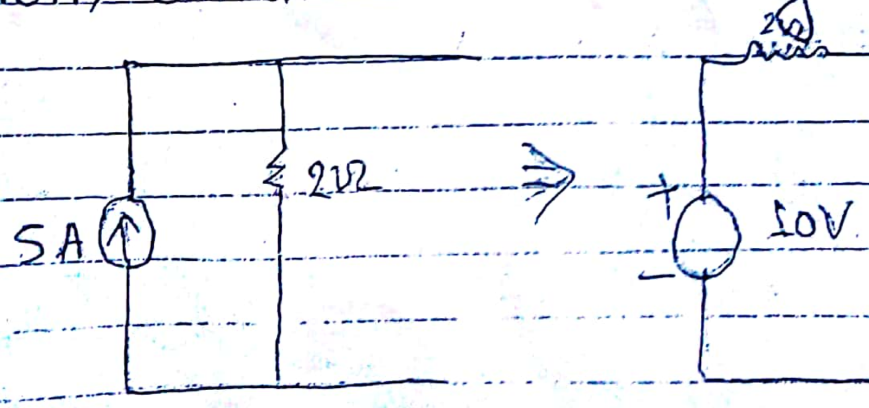
(a) Independent.

$I = \frac{V}{R} = \frac{10}{2} = 5A$



Independent.

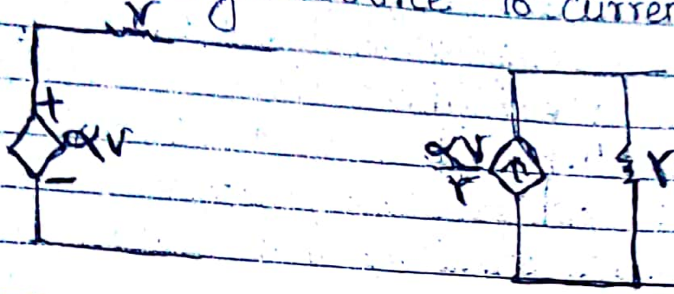
(2) From current source to voltage source.



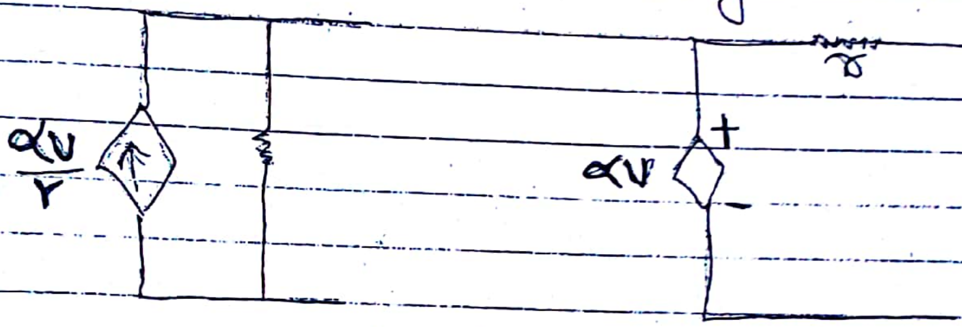
$V = IR$
$= 5 \times 2 = 10V$

(b) Dependent source:-

(i) From voltage source to current source:



(ii) From current source to voltage source.

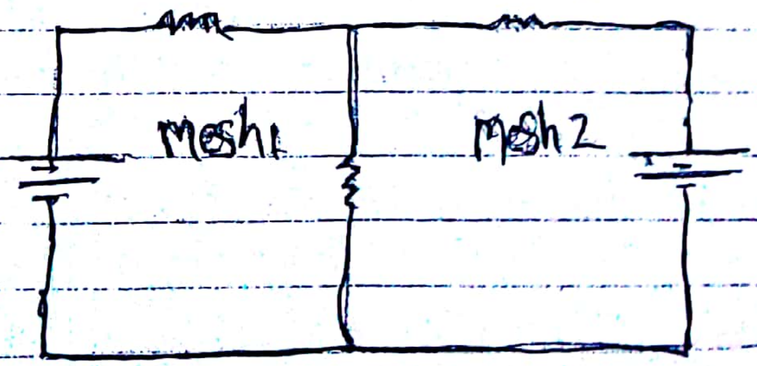


Mesh and loop

Mesh:- A closed circuit in which another mesh cannot be adjusted.

Loop:- A closed circuit in which another mesh can be adjusted.

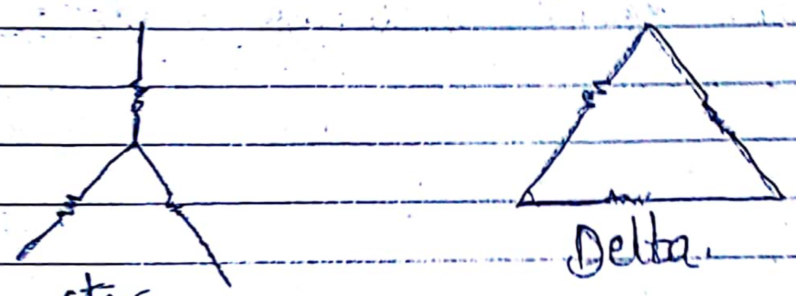
Both mesh and loop are closed circuit.



Mesh = 3
loop = 1.

Star and Delta Representation and Conversion

In circuit analysis, the topology of some networks is so complicated that it is very difficult to reduce into an equivalent structure of series parallel combinations. By means of delta-star transformation, most of such networks can be converted into an equivalent structure that will allow series parallel reduction.



(1) Delta-star conversion:-

Delta-star conversion is a method of obtaining equivalent star of a delta.

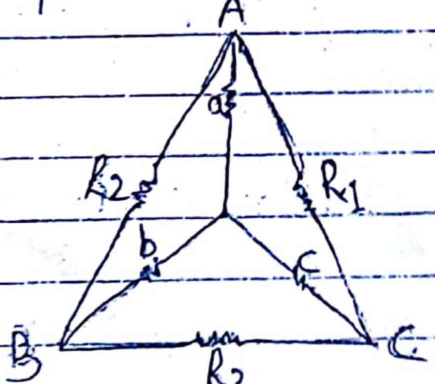


fig (1) Delta to star conversion

Let us consider a delta circuit ABC made up of three resistors R_1, R_2 and R_3 as shown in fig (1). Suppose its equivalent star circuit be represented by a, b and c.

The resistance between terminals A and B in delta connection is

$$= \frac{R_1 \times (R_2 + R_3)}{R_1 + (R_2 + R_3)}$$

and the resistance between the same terminal A and B in the star connection is

$$= a + b.$$

Since the terminals are same, so:

$$a + b = \frac{R_1 \times (R_2 + R_3)}{R_1 + (R_2 + R_3)} \quad \ldots (1)$$

Similarly, by solving for terminals B and C, and C and A, we get,

$$b + c = \frac{R_2 \times (R_3 + R_1)}{R_2 + (R_3 + R_1)} \quad \ldots (2)$$

and,

$$c + a = \frac{R_3 \times (R_1 + R_2)}{R_3 + (R_1 + R_2)} \quad \ldots (3)$$

On solving eqn.(1), (2) and (3), we get,

$$a = \frac{R_1 \times R_2}{R_1 + R_2 + R_3}, \quad b = \frac{R_2 \times R_3}{R_1 + R_2 + R_3} \quad \& \quad c = \frac{R_1 \times R_3}{R_1 + R_2 + R_3}.$$

Thus, in delta-star conversion, "the resistance of any arm of the star is equal to the product of the resistances of the two delta sides meeting it divided by the sum of the three delta resistances."

2) Star to delta conversion:-

Star to delta conversion is a method of obtaining equivalent delta of a star.

The total resistance ~~must be same~~ between

B and C must be same for both star and delta connections. So:

$$b+c = R_3 // (R_2+R_1)$$

$$= \frac{R_3 \cdot R_2 + R_3 \cdot R_1}{R_3 + R_2 + R_1} \quad \dots (1)$$

Similarly, we will get,

$$c+a = R_1 // (R_3+R_2)$$

$$= \frac{R_3 \cdot R_1 + R_1 \cdot R_2}{R_1+R_2+R_3} \quad \dots (2)$$

and, $a+b = R_2 // (R_1+R_3)$

$$= \frac{R_2 \times R_1 + R_2 \times R_3}{R_1+R_2+R_3} \quad \dots (3)$$

Adding eqn (1), (2) & (3), we get,

$$2(a+b+c) = \frac{2(R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1)}{R_1+R_2+R_3}$$

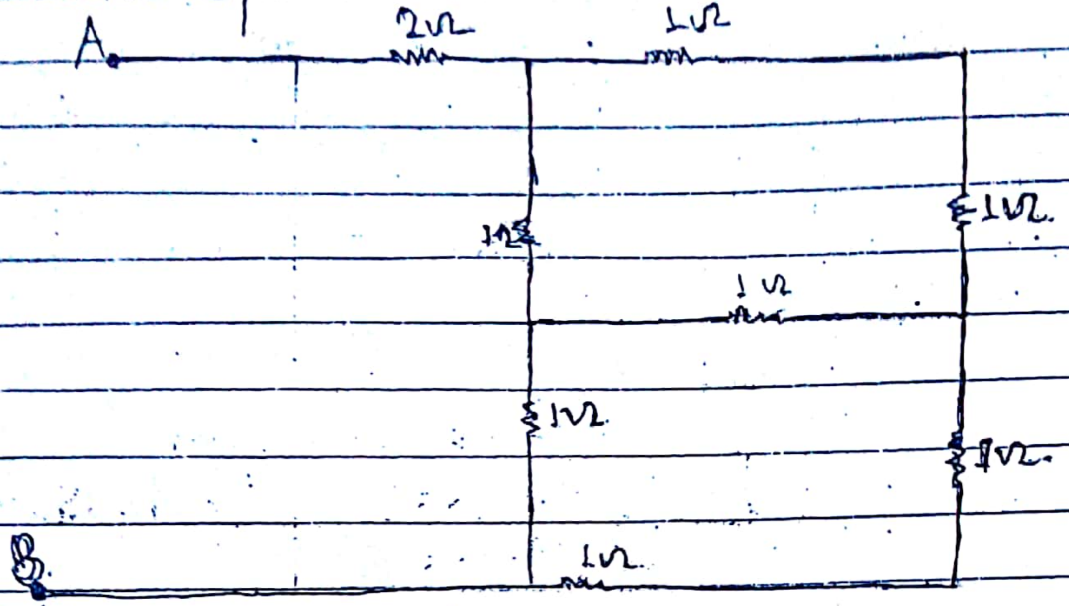
or, $$a+b+c = \frac{R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1}{R_1+R_2+R_3} \quad \dots (4)$$

$\therefore$ $R_1 = \frac{a\times b + b\times c + c\times a}{b}$ $\qquad$ $R_2 = \frac{a\times b + b\times c + c\times a}{c}$

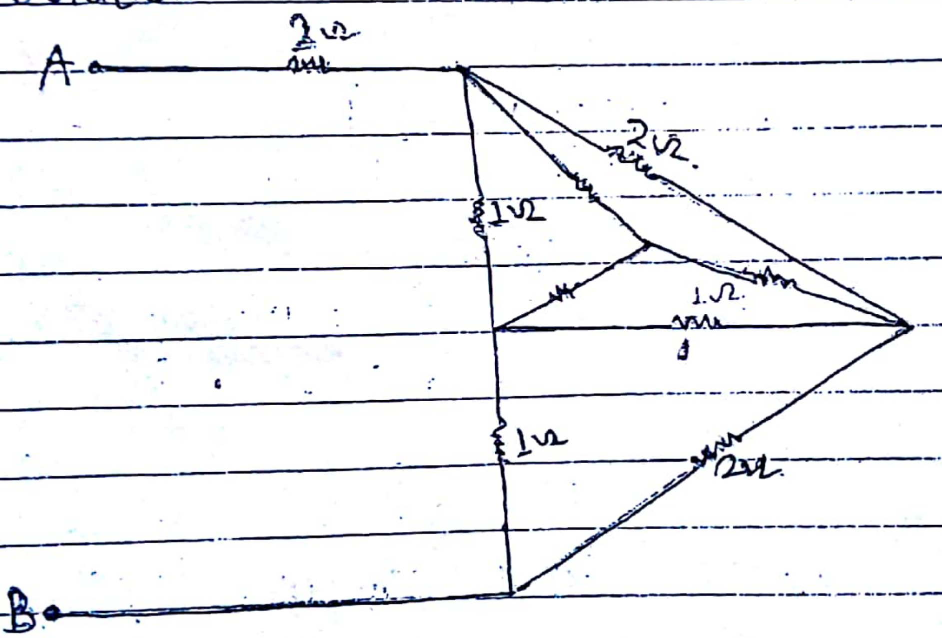
and $R_3 = \frac{a\times b + b\times c + c\times a}{a}$.

Thus, in star to delta transformation, the equivalent delta resistance between any two terminals is the sum of star resistances between the involved resistance plus the ratio of the product of these two star resistances and the remaining star resistances.

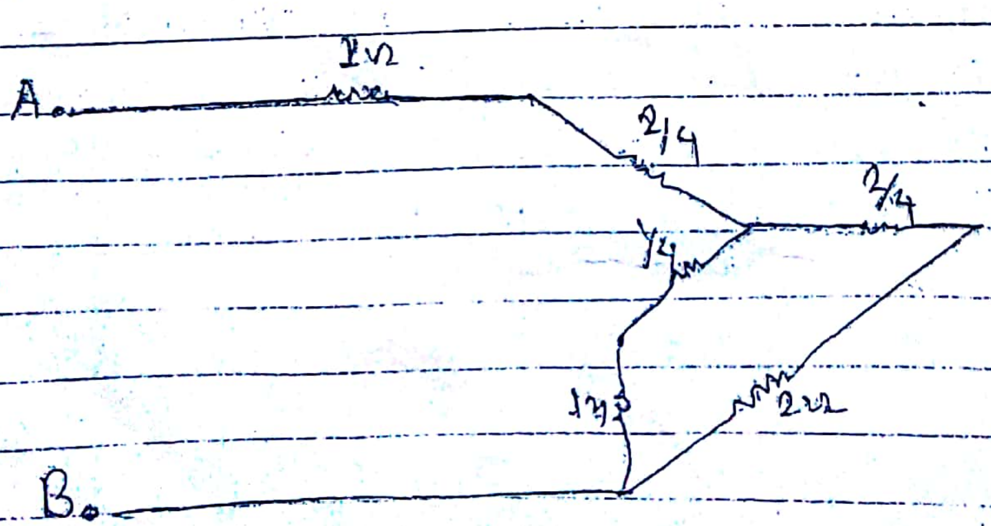
2. Find the equivalent resistance across AB terminals.

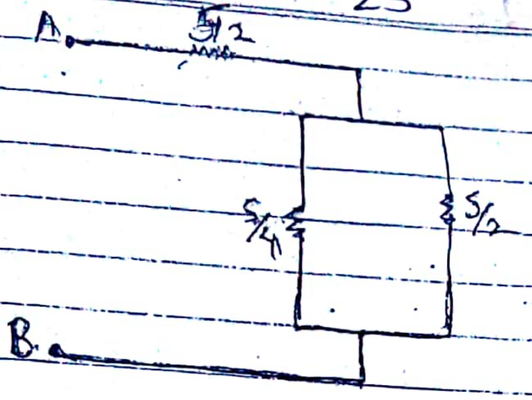


→ Solution:-



$1+1 = 2\Omega$

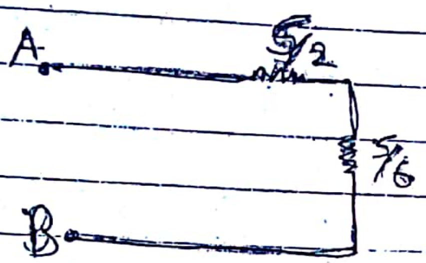




$2 + \frac{2}{4} = \frac{10}{4} = \frac{5}{2}$

$\frac{1}{4} + 1 = \frac{5}{4}$

$\frac{2}{4} + 2 = \frac{10}{4} = \frac{5}{2}$

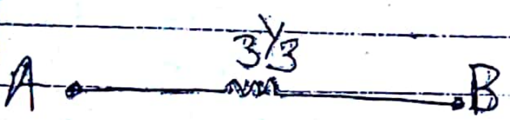


$\frac{5}{4} \parallel \frac{5}{2}$

$= \dfrac{\frac{5}{4} \times \frac{5}{2}}{\frac{5}{4} + \frac{5}{2}} = \dfrac{\frac{25}{8}}{\frac{15}{4}} = \frac{25}{8} \times \frac{4}{15} = \frac{5}{6}$

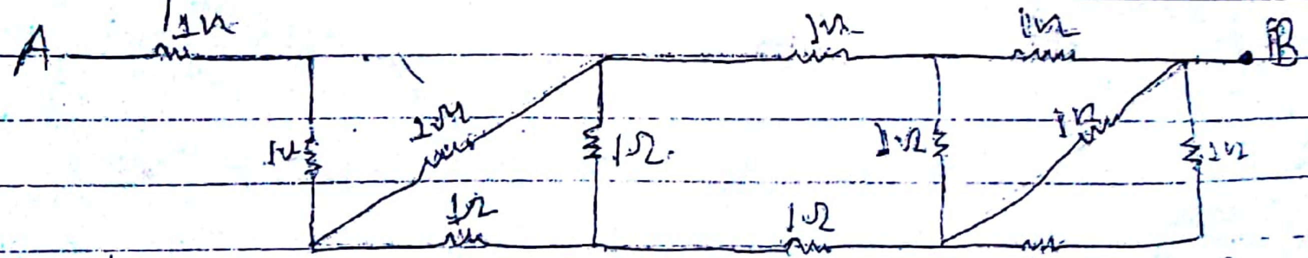
$\frac{5}{2} + \frac{5}{6} = \frac{15+5}{6} = \frac{20}{6}$

$= \frac{10}{3} = 3\frac{1}{3}$

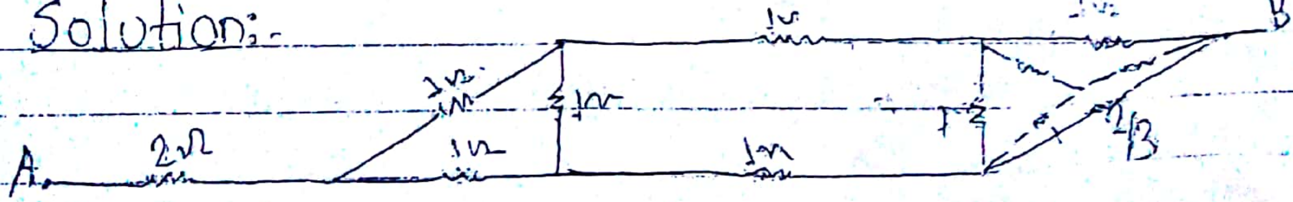


∴ The equivalent resistance is $3\frac{1}{3}\ \Omega$.

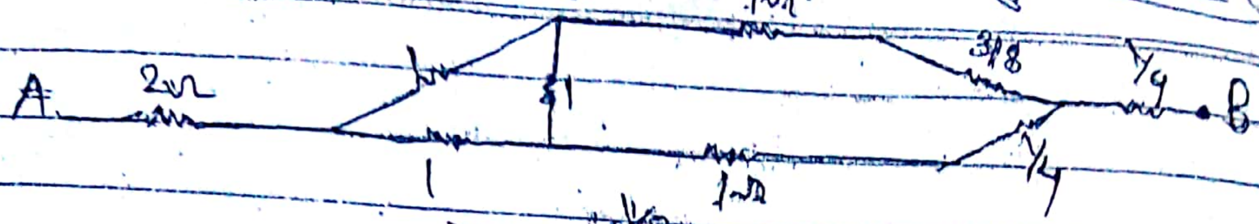
2. Find equivalent resistance.

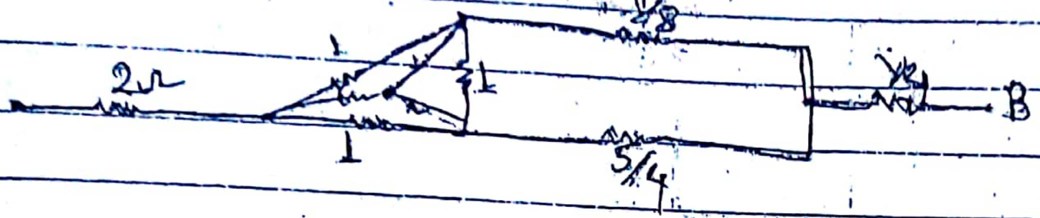


→ Solution:-



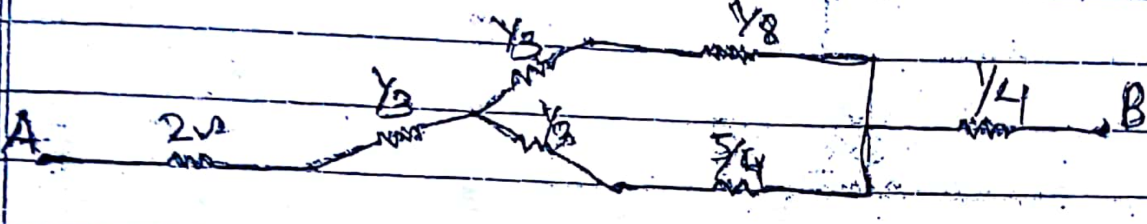
$2 \parallel 1 = \frac{2}{3}$





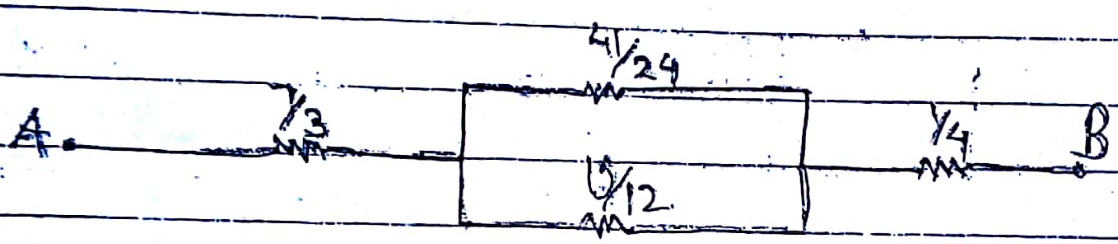
$\frac{2}{3}+2=\frac{8}{3}$

$\frac{\frac{2}{3}}{\frac{8}{3}}=\frac{1}{4}$



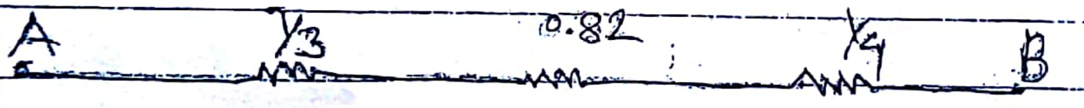
$1+\frac{3}{8}=\frac{11}{8}$

$1+\frac{1}{4}=\frac{5}{4}$

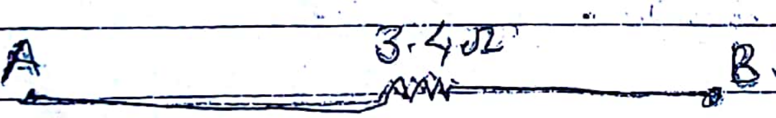


$\frac{1}{3}+\frac{11}{8}=\frac{8+33}{24}=\frac{41}{24}$

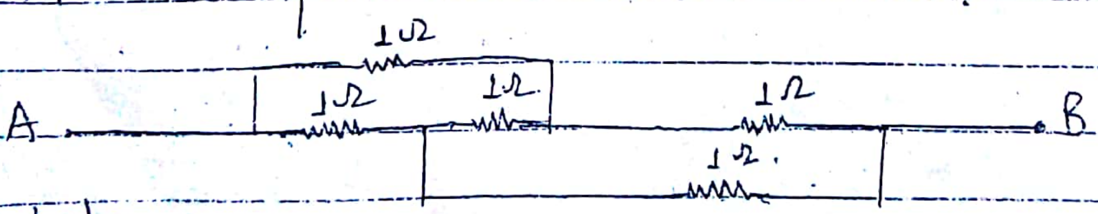
$\frac{1}{3}+\frac{5}{4}=\frac{4+15}{12}=\frac{19}{12}$



$\frac{\frac{41}{24}\times\frac{19}{12}}{\frac{41}{24}+\frac{19}{12}}=\frac{2.7}{3.29}$

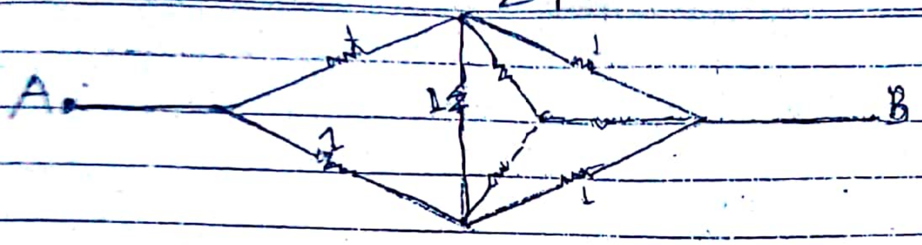


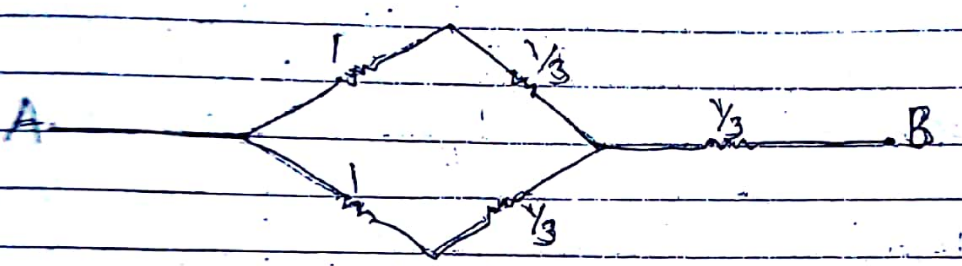
2. Find the equivalent resistance across AB terminal.



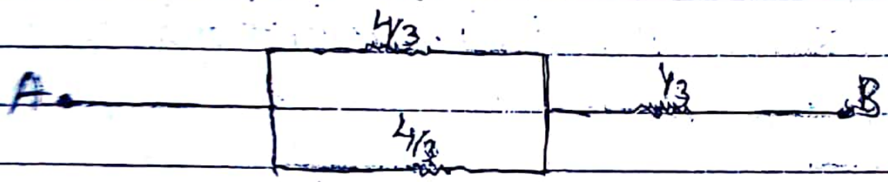
→ Solution:-

The above figure can be rearranged into following:

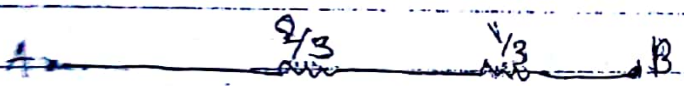


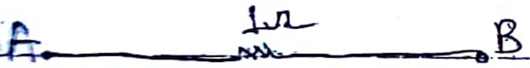


$\frac{1}{3}+1=\frac{4}{3}$

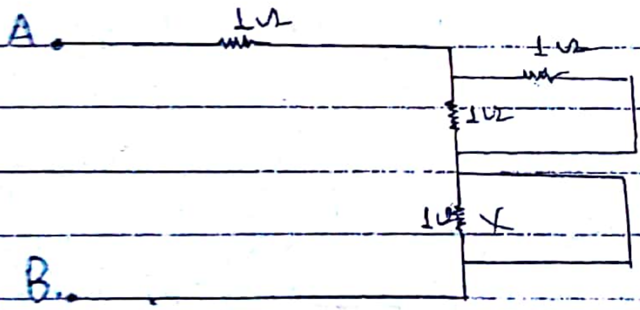


$$\frac{\frac{4}{3}\times\frac{4}{3}}{\frac{4}{3}+\frac{4}{3}} = \frac{16}{9}\times\frac{3}{8} = \frac{2}{3}$$





②



→ Solution:-

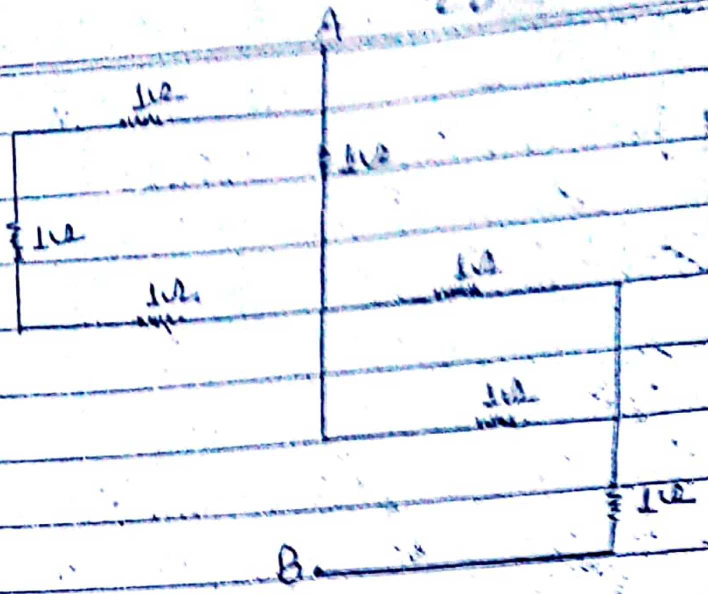
Due to barrier, 1Ω is short circuit.

So, equivalent resistance is

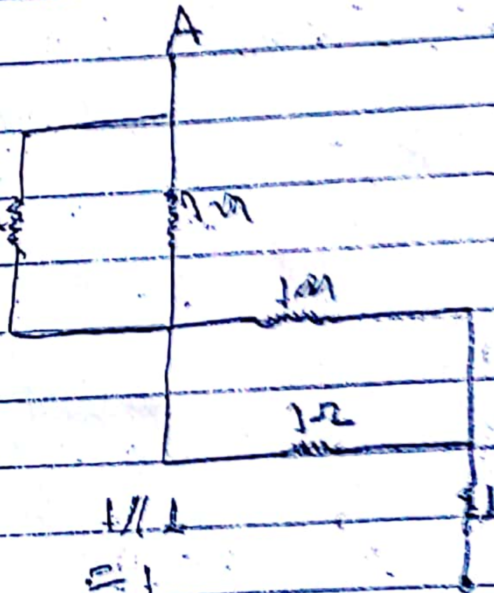
$$R_{eq.} = 1+(1 \parallel 1)$$

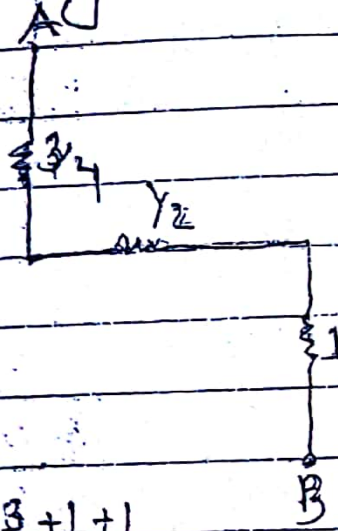
$$= 1+\frac{1}{2} = \frac{3}{2}\,\Omega.$$

3.



→ Solution:-
The above circuit can be rearranged as.

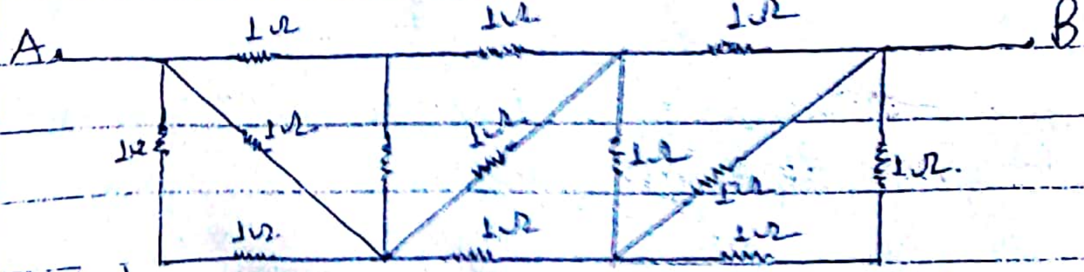




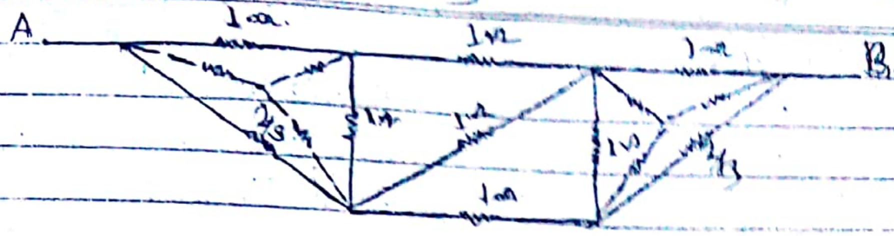
$3 \parallel 1 = \frac{3}{4}$ $\quad 1 \parallel 1 = \frac{1}{2}$

$\frac{3}{4} + \frac{1}{2} + 1 = \frac{3+2+4}{4} = \frac{9}{4}$

4.



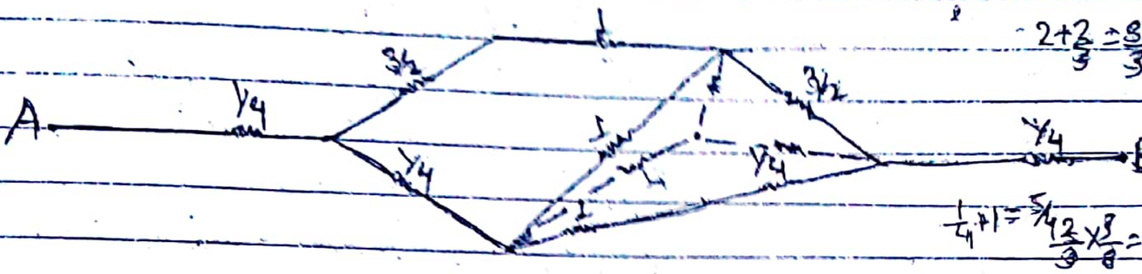
→ Solution:-
The above figure can be rearranged as:



$(1+1)\parallel 1$

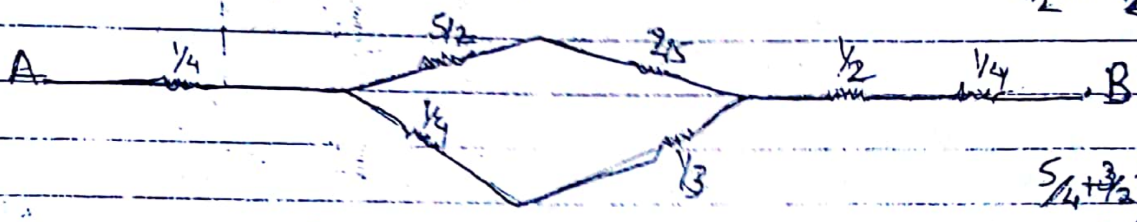
$= \frac{2\times1}{2+1} = \frac{2}{3}$

$2+\frac{2}{3}=\frac{8}{3}$



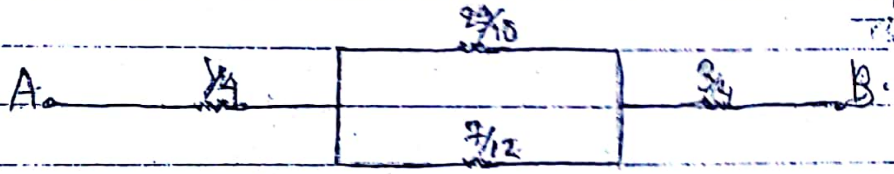
$\frac{1}{4}+1=\frac{5}{4}$ $\quad \frac{2}{3}\times\frac{3}{8}=\frac{1}{4}$

$\frac{3}{2}+1=\frac{5}{2}$



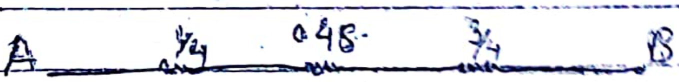
$\frac{5}{4}+\frac{3}{2}+1$

$\frac{1\times5}{4}\times\frac{4}{15}=$ $\quad = \frac{5+6+4}{4}=\frac{15}{4}$



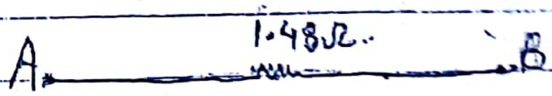
$\frac{\frac{3}{2}\times\frac{5}{2}}{\frac{15}{4}} = \frac{15\times4}{8\times15}$

$\frac{1}{4}+\frac{1}{2}=\frac{3}{4}$ $\quad \frac{5}{2}+\frac{2}{5}=\frac{29}{10}$

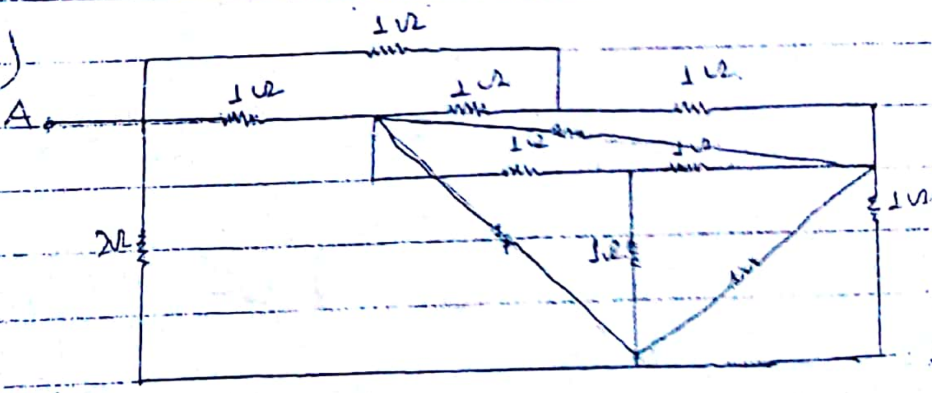


$\frac{\frac{29}{10}\times\frac{7}{12}}{\frac{29}{10}+\frac{7}{12}}$ $\quad \frac{1}{4}+\frac{1}{3}=\frac{7}{12}$

$=\frac{203}{418}=0.48$

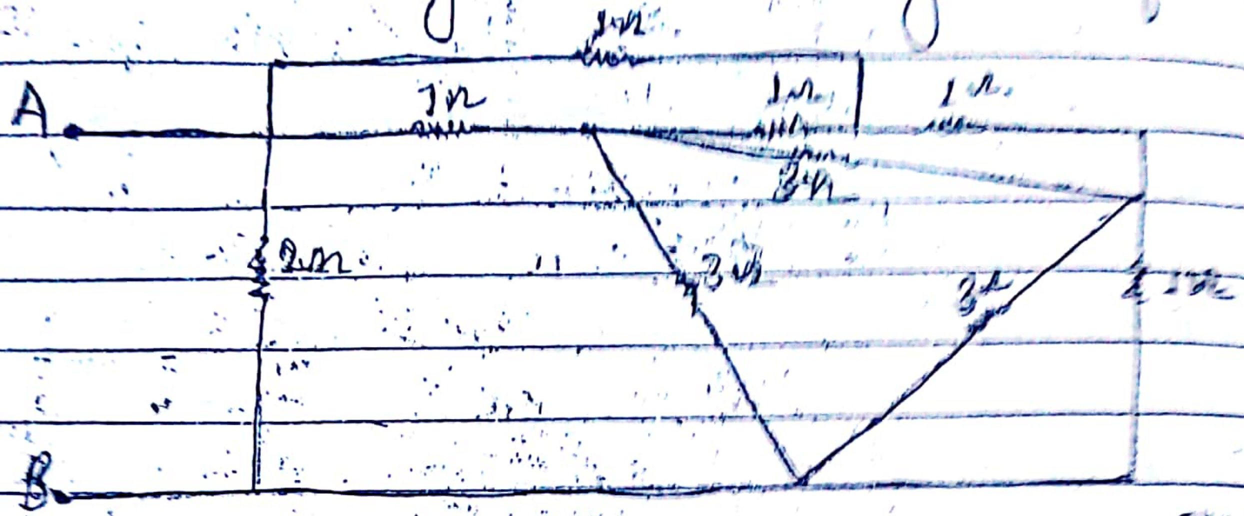


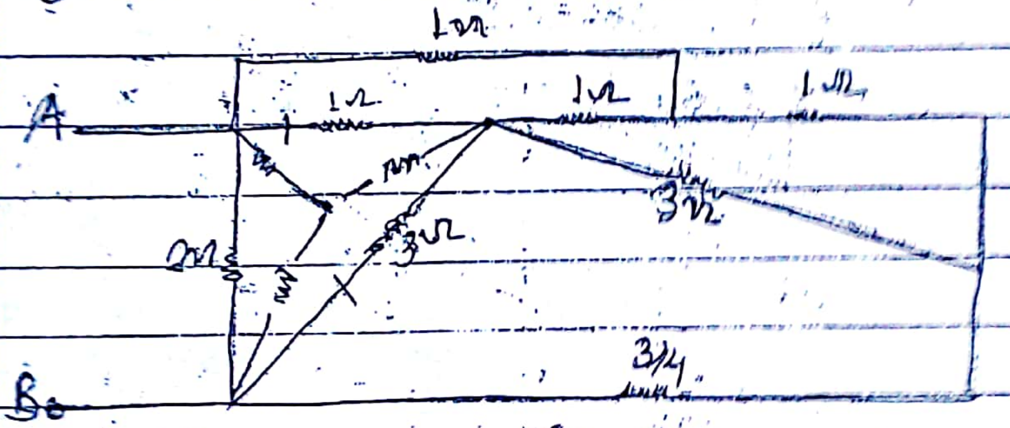
Q 2013 (spring) 7 (marks)



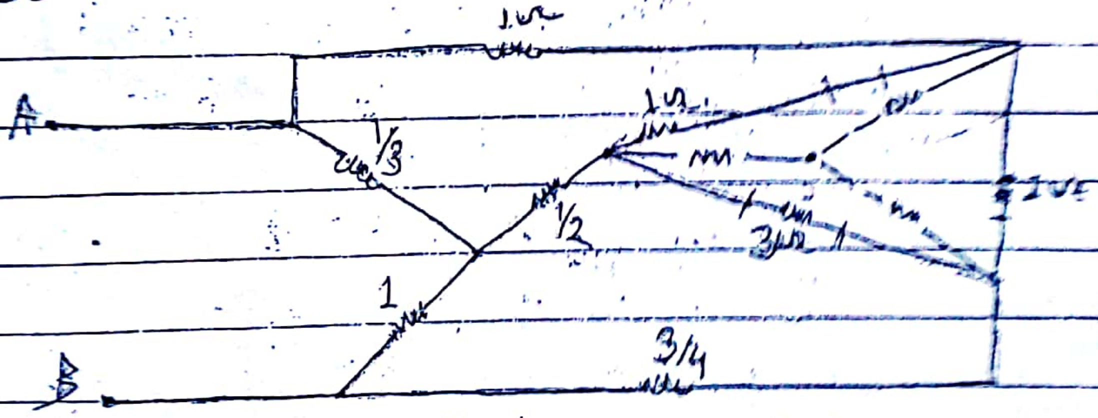
→ Solution:-

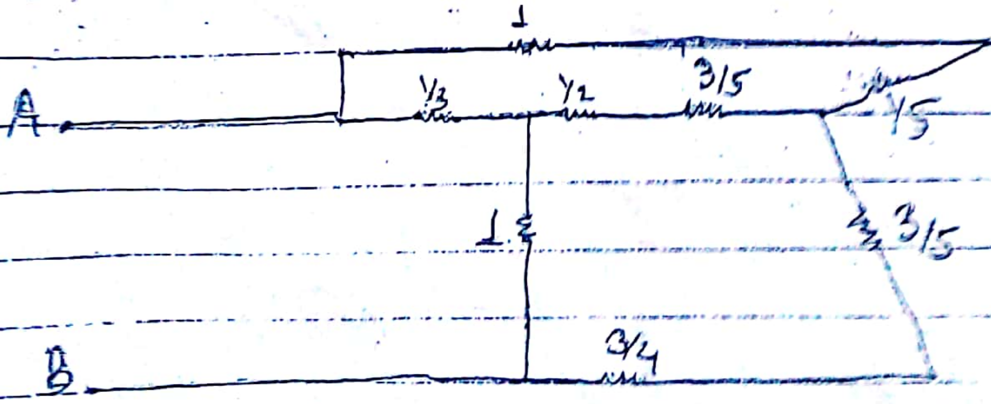
The above figure can be rearranged into following





$3/11 = 3/4$



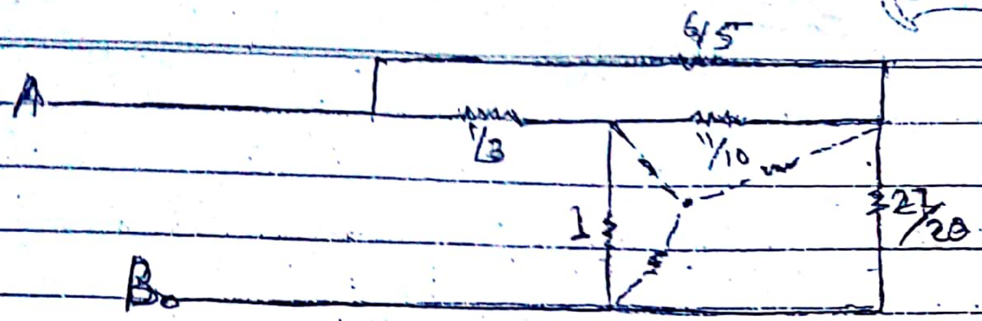


$\frac{1}{2}+\frac{3}{5}$

$1+\frac{1}{5}=\frac{6}{5}$

$\frac{3}{5}+\frac{3}{4}$

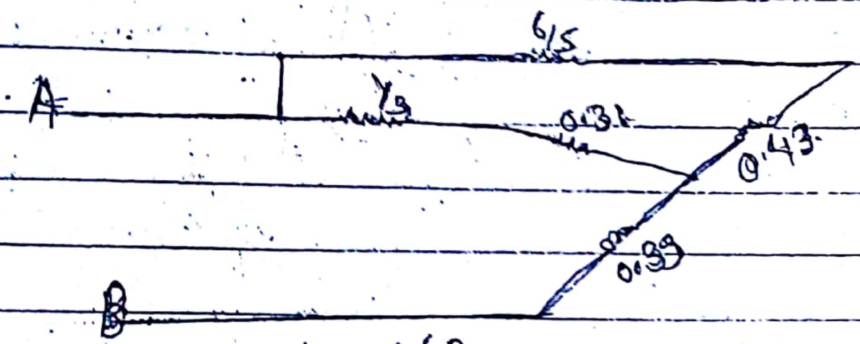
$=\frac{12+15}{20}$



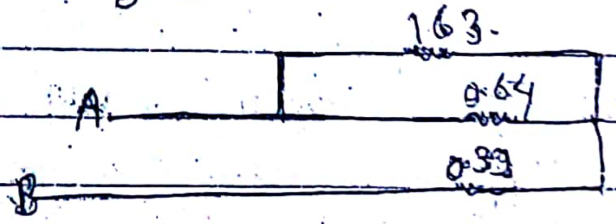
3.45

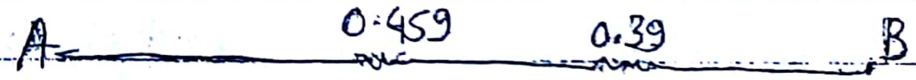
$\frac{11}{10}$

$\frac{\;}{3.45}$



2.27





Kirchhoff's law

Kirchhoff's voltage Law (Mesh analysis)

Statement:-

"The algebric sum of voltages within a closed circuit is always equal to zero."

In other words, the voltage rise and voltage drops in a closed circuit is always equal.

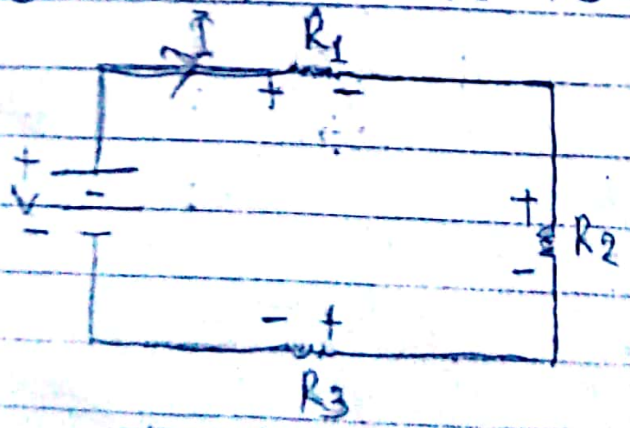
Sign rule for KVL

Give positive sign to all rise in voltage and negative sign to voltage drop. Thus, if we move from negative (-ve) terminal of a battery to positive (+ve) terminal, a positive sign should be given, since there is a rise in voltage. On the other hand, if we go from positive (+ve) terminal to negative (-ve) terminal, a negative sign should be given, since there is a drop in voltage.

In short,

rise { current from − to +ve = +

fall { current from + ve −ve = −



Let resistors R_1, R_2 and R_3 are connected in series with a voltage source V as shown in figure above.

Using Kirchoff's voltage law,

$$+V - IR_1 - IR_2 - IR_3 = 0$$

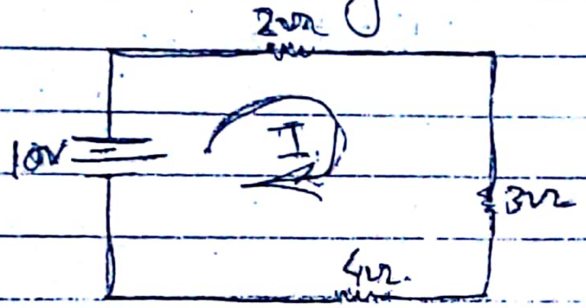
or, $V = IR_1 + IR_2 + IR_3$

or, $\boxed{V = V_1 + V_2 + V_3}$

where $V_1 = IR_1$, $V_2 = IR_2$ & $V_3 = IR_3$.

One mesh example:-

Find current through 4Ω using mesh analysis.



→ Solution:-

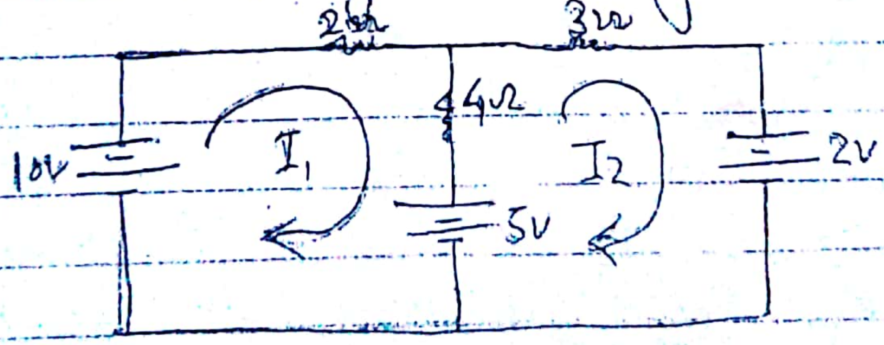
Using KVL in mesh, we get

$$-2I - 3I - 4I + 10 = 0$$

or, $I = 10/9\ A$.

$\therefore$ Current through $4\Omega = 10/9\ A$.

Q. Find current through 4Ω using mesh analysis.



→ Solution:-

Using KVL in mesh I, we get

$$-2I_1 - 4(I_1 - I_2) - 5 + 10 = 0$$

or, $-6I_1 + 4I_2 + 5 = 0$ ....(1)

Similarly in mesh 2, we get,

$$-3I_2 - 2 + 5 - 4(I_2 - I_1) = 0$$

or, $-7I_2 + 4I_1 + 3 = 0$ ....(2)

Solving eqn (1) and (2), we get,

$$\left.\begin{aligned} -6I_1 + 4I_2 + 5 &= 0 \\ 4I_1 - 7I_2 + 3 &= 0 \end{aligned}\right\} \begin{aligned} &\times 4 \\ &\times 6 \end{aligned}$$

$$-24I_1 + 16I_2 + 20 + 24I_1 - 42I_2 + 18 = 0$$

or, $-26I_2 = -38$

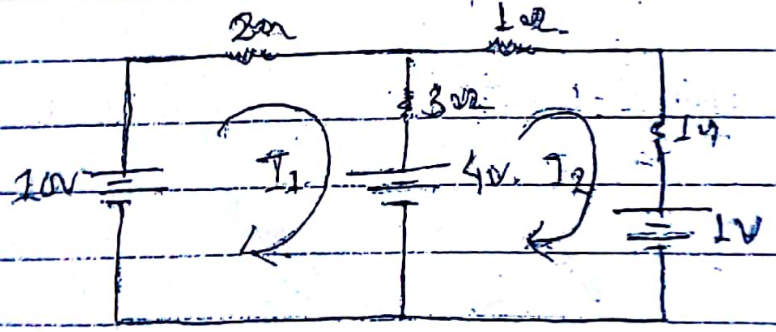
or, $I_2 = \frac{38}{26} = \frac{19}{13}$.

& $\cancel{I_1 =}$ $-7 \times \frac{19}{13} + 4I_1 + 3 = 0$

or, $I_1 = \frac{94}{13}$

$\therefore$ Current through $4\Omega = 4\left(\frac{94}{13} - \frac{19}{13}\right) = \frac{75}{13}A = \cancel{\frac{300}{13}A}$

Q. Find the current in each resistance using mesh analysis.



→ Solution:-

Using KVL in 1st mesh,

$$-2I_1 - 3(I_1 - I_2) - 4 + 10 = 0$$

or, $-5I_1 + 3I_2 + 6 = 0$ ....(1)

Similarly, in mesh II, we get,

$$-3(I_2 - I_1) - I_2 - I_2 - 1 + 4 = 0$$

or, $-5I_2 + 3I_1 + 3 = 0$ ..... (2)

Solving eqⁿ. (1) and (2), we get,

$$\left.\begin{aligned} -5I_1 + 3I_2 + 6 &= 0 \\ 3I_1 - 5I_2 + 3 &= 0 \end{aligned}\right\} \begin{aligned} &\times 3 \\ &\times 5 \end{aligned}$$

$-15I_1 + 9I_2 + 18 + 15I_1 - 25I_2 + 15 = 0$

or, $I_2 = \frac{33}{16} = 2.06A.$

& $-5I_1 + 3\times\frac{33}{16} + 6 = 0$

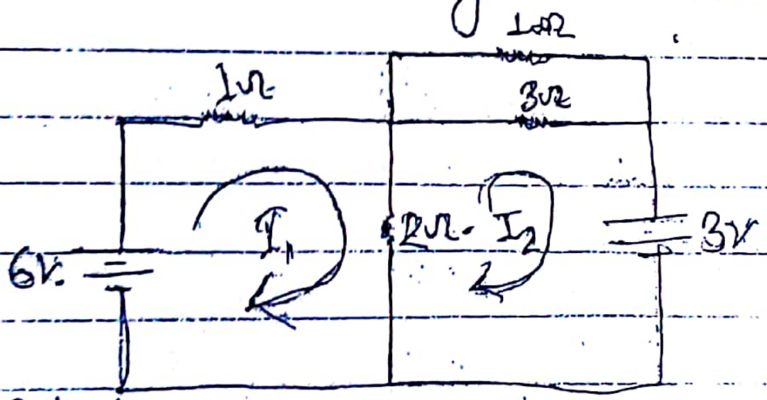
$I_1 = 2.43.$

$\therefore$ Current through $1\Omega = I_2 = 2.06\,A$

Current through $2\Omega = I_1 = 2.43\,A$ ~~×2 = 4.87A~~

Current through $3\Omega = \cancel{3}(I_1 - I_2) = \cancel{3}(2.43 - 2.06) = \cancel{1.11}$ 0.37.

Q. Find current through 2Ω resistance using mesh analysis.



→ Solution:-

Since 3//1, so $R_{eq} = \frac{3}{4}$.

Using mesh analysis in mesh 1, we have

$-I_1 - 2(I_1 - I_2) + 6 = 0$

or, $-3I_1 + 2I_2 + 6 = 0$ .... (1)

Similarly, in mesh 2, $-\frac{3}{4}(I_2) - 3 - 2(I_2 - I_1) = 0$

or, $-\frac{11}{4}I_2 + 2I_1 - 3 = 0$ .... (2)

Solving eqn. (1) and (2), we get

$2I_2 - 3I_1 + 6 = 0$ } × 2
$-\frac{11}{4}I_2 + 2I_1 - 3 = 0$ } × 3

$4I_2 - 6I_1 + 12 - \frac{33}{4}I_2 + 6I_1 - 9 = 0$

or, $\frac{16-33}{4} I_2 = -3$

or, $I_2 = \frac{12}{17} = 0.7\,A$

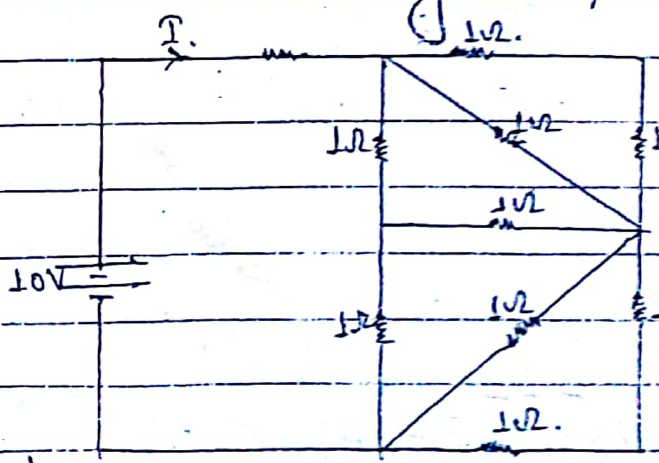
and $\frac{2 \times 12}{17} - 3I_1 + 6 = 0$

or, $\frac{24+102}{17} = 3I_1$

or, $I_1 = \cancel{2.47A}$ 1.52A

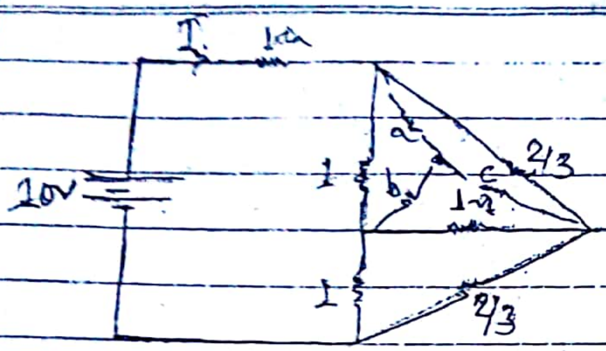
∴ Current through 1Ω = (1.52 − 0.7) = ~~1.62A~~ 0.81A

Q. Find current I using star/delta concept.



→ Solution:-
The above figure can be rearranged as below:

$R_{eq} = 2 \parallel 1 = \frac{2}{3}$

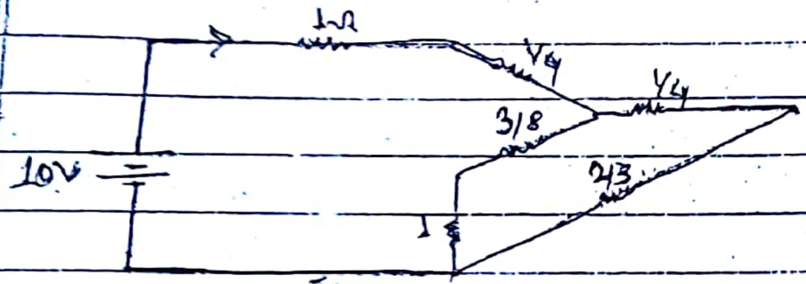


$\frac{2}{3} + 2 = \frac{8}{3}$

$a = \frac{2}{3} \times \frac{3}{8} = \frac{1}{4}$

$b = \frac{1}{8/3} = 3/8$

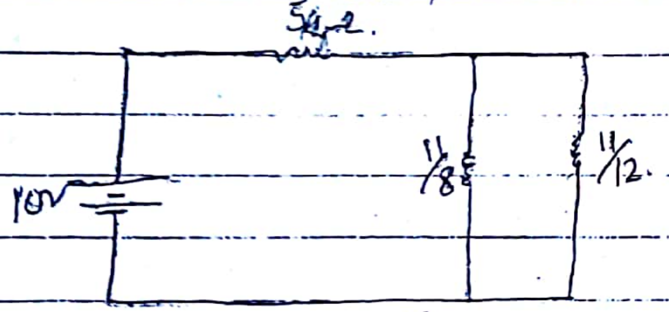
$c = 2/3 \times 3/8 = 1/4$.



$1 + \frac{1}{4} = 5/4$

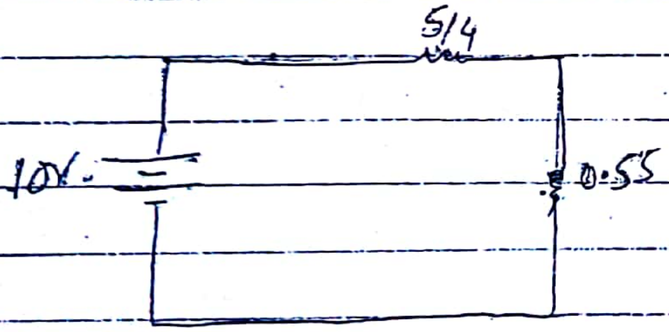
$3/8 + 1 = \frac{11}{8}$

$\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$



$$\frac{\frac{11}{12} \times \frac{11}{8}}{\frac{11}{12} + \frac{11}{8}} = \frac{\frac{121}{96}}{\frac{88+132}{96}} = 0.55$$

1.80

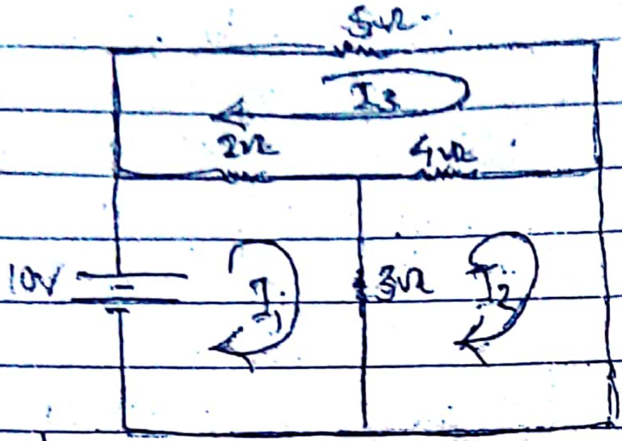


$\therefore I = \frac{V}{R_{eq}} = \frac{10}{1.80} = 5.55A.$

$\therefore$ The required current is 5.55A.

Scanned by CamScanner

Q. Find current in each resistance using mesh analysis.



→ Solution:-

Using mesh analysis in mesh 1, we get,

$$-2(I_1 - I_3) - 3(I_1 - I_2) + 10 = 0$$

or, $-2I_1 + 2I_3 - 3I_1 + 3I_2 + 10 = 0$

or, $-5I_1 + 3I_2 + 2I_3 + 10 = 0$ .....(1)

Similarly,

Using mesh analysis in mesh 2, we get,

$$-4(I_2 - I_3) - 3(I_2 - I_1) = 0$$

or, $-4I_2 + 4I_3 - 3I_2 + 3I_1 = 0$

or, $\cancel{-7I_2}$ $3I_1 - 7I_2 + 4I_3 = 0$ ...(2)

Using mesh analysis in mesh 3, we get,

$$-5I_3 - 4(I_3 - I_2) - 2(I_3 - I_1) = 0$$

or, $-5I_3 - 4I_3 + 4I_2 - 2I_3 + 2I_1 = 0$

or, $2I_1 + 4I_2 - 11I_3 = 0$ .....(3)

Solving eqn. (1), (2) & (3) by matrix method.

$$I_1 = \frac{\begin{vmatrix} -10 & 3 & 2 \\ 0 & -7 & 4 \\ 0 & 4 & -11 \end{vmatrix}}{\begin{vmatrix} -5 & 3 & 2 \\ 3 & -7 & 4 \\ 2 & 4 & -11 \end{vmatrix}} = \frac{-10\begin{vmatrix} -7 & 4 \\ 4 & -11 \end{vmatrix} - 0}{-130} = 4.69A$$

$$I_2 = \frac{\begin{vmatrix} -5 & -10 & 2 \\ 3 & 0 & 4 \\ 2 & 0 & -11 \end{vmatrix}}{-130} = \frac{390}{130} = 3A.$$

$$I_3 = \frac{\begin{vmatrix} -5 & 3 & -10 \\ 3 & -7 & 0 \\ 2 & 4 & 0 \end{vmatrix}}{-130} = \frac{-5\begin{vmatrix} 7 & 0 \\ 4 & 0 \end{vmatrix} - 3\begin{vmatrix} 3 & 0 \\ 2 & 0 \end{vmatrix} - 10\begin{vmatrix} 3 & -7 \\ 2 & 4 \end{vmatrix}}{-130}$$

$$= \frac{-10(12+14)}{-130} = \frac{-260}{-130} = 2A.$$

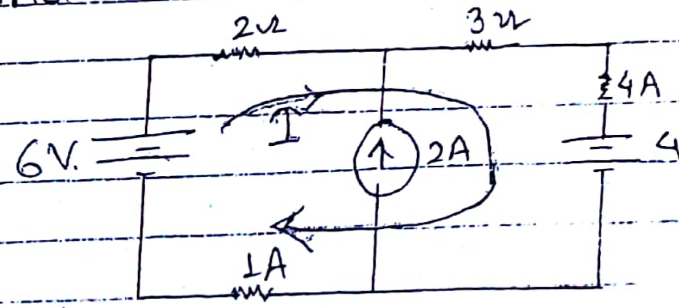
∴ Current through $2\Omega = (I_1 - I_3) = 2.69A.$

Current through $3\Omega = (I_1 - I_2) = 1.69$

Current through $4\Omega = (I_2 - I_3) = 1A$

Current through $5\Omega = I_3 = 2A.$

Q. Find current in each resistance using mesh analysis.



→ Solution:

Concept:-

Supermesh:

When there is current source between meshes, both meshes combine to form a single mesh.

$-2I - 3(I+2) - 4(I+2) - 4 - I + 6 = 0$

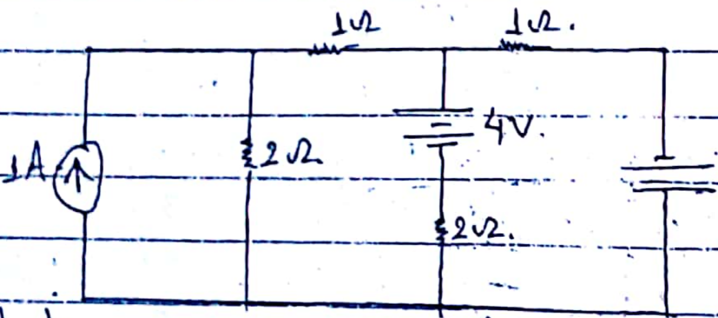
or, $-2I - 3I - 6 - 4I - 8 - I + 2 = 0$

or, $-10I - 12 = 0$

or, $I = -1.2$.

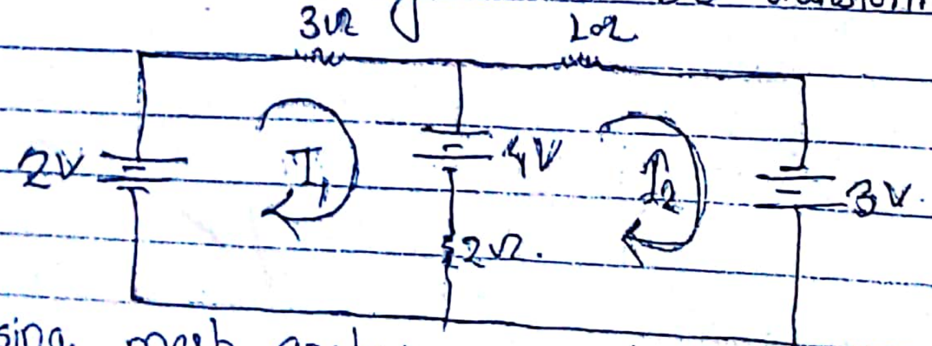
∴ Current through $1\Omega = I = -1.2A$.
Current through $2\Omega = I = -1.2A$
Current through $3\Omega = (I+2) = 0.8A$
Current through $4\Omega = (I+2) = 0.8A$
~~Cur~~

Q. Find current in each resistor using mesh analysis



→ Solution:-

The above diagram can be transformed into:-



Using mesh analysis in mesh 1, we get,

$-3I_1 - 4 - 2(I_1 - I_2) + 2 = 0$

or, $-5I_1 + 2I_2 - 2 = 0$ ...(1)

Similarly,

$-I_2 + 3 - 2(I_2 - I_1) + 4 = 0$

or, $-3I_2 + 2I_1 + 7 = 0$ .....(2)

On solving eq^n (1) and (2), we get,

$$2I_1 - 3I_2 + 7 = 0 \quad \times 2$$
$$-5I_1 + 2I_2 - 2 = 0 \quad \times 3$$

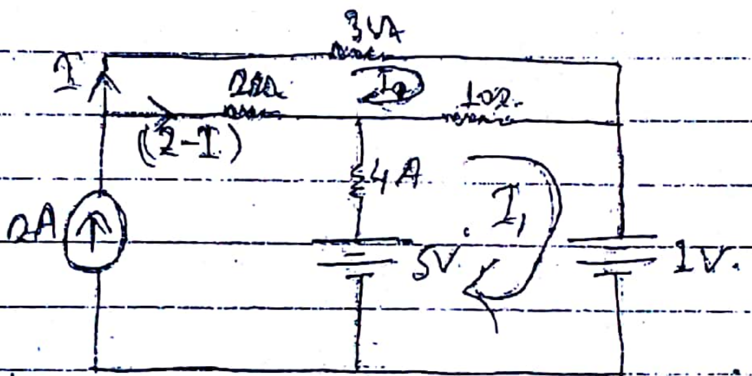
$$4I_1 - 6I_2 + 14 - 15I_1 + 6I_2 - 6 = 0$$

or, $I_1 = \frac{8}{11}$ A.

$$-\frac{2\times 8}{11} - 3I_2 + 7 = 0$$

or, $\frac{-16+77}{11} = 3I_2 \Rightarrow I_2 = 1.84$ A.

Q. Find current in each resistance using mesh analysis.



→ Solution:-

Using mesh analysis in mesh 1,

$$-1(I_1 - I) - \cancel{1} + 5 - 4(I_1 - 2) = 0$$

or, $-I_1 + I + 4 - 4I_1 + 8 = 0$

or, $-5I_1 + I + 12 = 0 \quad \ldots\ldots (1)$

Similarly,

$$-3I - 1(I - I_1) - 2(-(2 - I)) = 0$$

or, $-3I - I + I_1 - 2(I - 2) = 0$

or, $-3I - I + I_1 - 2I + 4 = 0$

or, $-6I + I_1 + 4 = 0 \quad \ldots\ldots (2)$

$$-5I_1 + I + 12 = 0 \quad \} \times 1$$
$$I_1 - 6I + 4 = 0 \quad \} \times 5$$

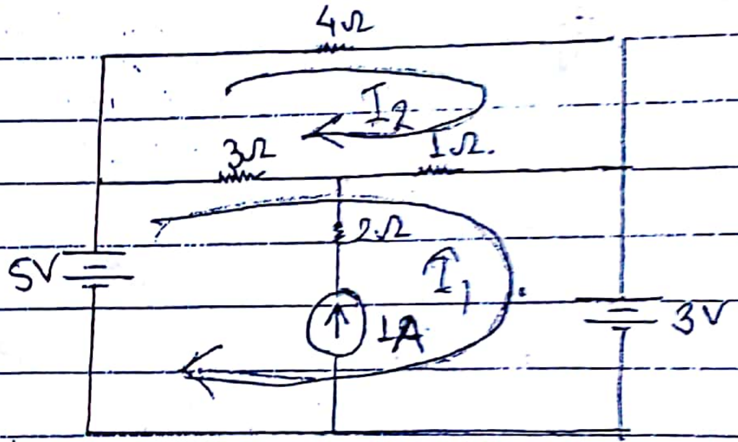
$$-5I_1 + I + 12 + 5I_1 - 30I + 20 = 0$$
$$-29I = -32$$
$$\therefore I = \frac{32}{29} = 1.1.$$

& $$I_1 - 6 \times \frac{32}{29} + 4 = 0$$

or, $I_1 = 2.62$

$\therefore$ Current through $2\Omega = 2 - I = 2 - 1.1 = 0.9$ A.

Q. Find currents.



→ Solution:-

Using supermesh in mesh 1, we get,

$$-3(I_1 - I_2) - (I_1 + 1 - I_2) - 3 + 5 = 0$$

or, $-3I_1 + 3I_2 - I_1 - 1 + I_2 + 2 = 0$

or, $-4I_1 + 4I_2 + 1 = 0$ . . . . (1)

Similarly, in mesh 2, we get,

$$-4I_2 - 1(I_2 - (I_1 + 1)) - 3(I_2 - I_1) = 0$$

or, $-4I_2 - (I_2 - I_1 - 1) - 3(I_2 - I_1) = 0$

or, $-4I_2 - I_2 + I_1 + 1 - 3I_2 + 3I_1 = 0$

or, $-8I_2 + 4I_1 + 1 = 0$ . . . . (2)

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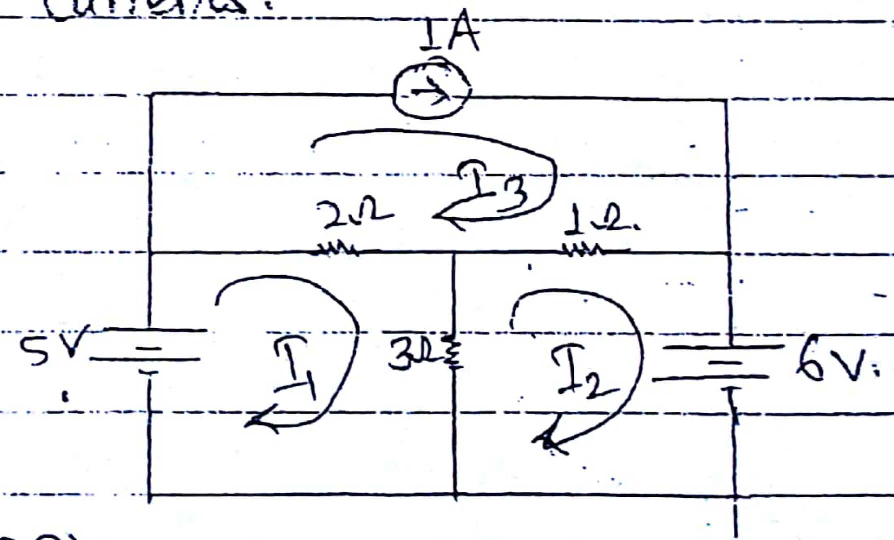
solving eq^n (1) & (2), we get,

$$-4I_1 + 4I_2 + 1 = 0$$
$$4I_1 - 8I_2 + 1 = 0$$
$$-4I_2 = -2$$
or, $I_2 = \frac{1}{2}$

$$-4I_1 + 4\times\frac{1}{2} + 1 = 0$$
or, $-4I_1 + 3 = 0$

or, $I_1 = \frac{3}{4}$

Q. Find Currents.



→ Solution:-

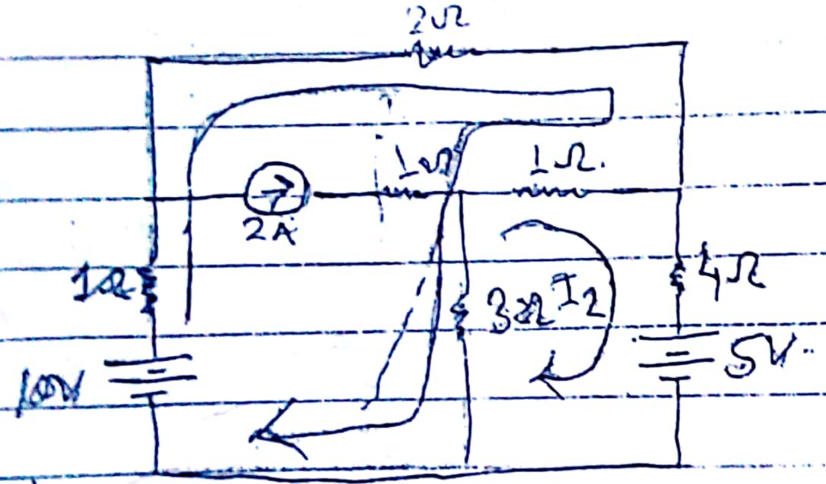
$I_3 = 1A$.

In mesh I,

$$-2(I_1 - 1) - 3(I_1 - I_2) + 5 = 0$$

or,

Q. Find current in each resistance using mesh analysis



→ Solution:-

Using supermesh, we get,

$-I_1 - 2(I_1-2) - 1(I_1-2-I_2) - 3(I_1-I_2) \cancel{=0} + 10 = 0$

or, $-I_1 - 2I_1 + 2 - I_1 + 2 + I_2 - 3I_1 + 3I_2 + 10 = 0$

or, $-7I_1 + 4I_2 + 16 = 0$ ..... (1)

Similarly,

$-(I_2 - I_1 + 2) - 4I_2 - 5 - 3(I_2 - I_1) = 0$

or, $-I_2 + I_1 - 2 - 4I_2 - 5 - 3I_2 + 3I_1 = 0$

or, $-8I_2 + 4I_1 - 7 = 0$ .... (2)

Solving eq^n (1) and (2), we get,

$$\left.\begin{aligned} -7I_1+4I_2+16=0 \\ 4I_1-8I_2-7=0 \end{aligned}\right\} \times 2$$

$$-14I_1+8I_2+32+4I_1-8I_2-7=0$$

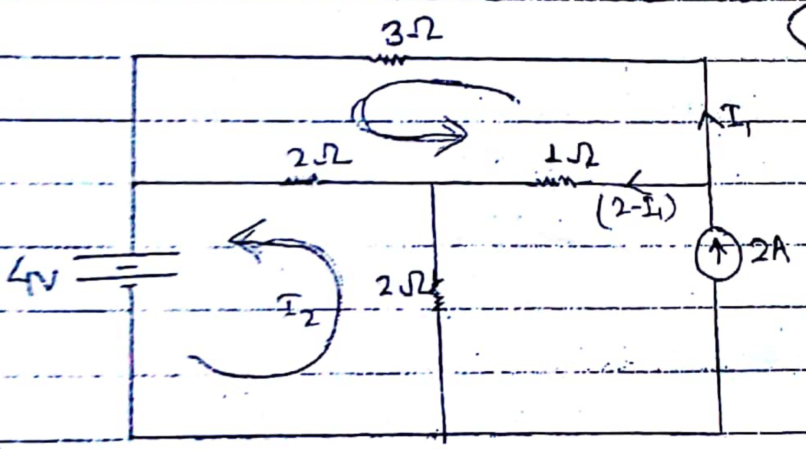
or, $-10I_1=-25$

$\therefore I_1=2.5A.$

& $4\times 2.5-8I_2-7=0$

or, $I_2=\frac{3}{8}=0.375A$

Q. Find current in each resistances using mesh analysis.



In mesh I, we get,

$-3I_1-2(I_1-I_2)-1(-(2-I_1))=0$

or, $-3I_1-2I_1+2I_2-(-2+I_1)=0$

or, $-5I_1+2I_2+2-I_1=0$

or, $-6I_1+2I_2+2=0$ ... (1)

Similarly, in mesh 2,

$-2(I_2-2)-2(I_2-I_1)-4=0$

or, $-2I_2+4-2I_2+2I_1-4=0$

or, $-4I_2+2I_1=0$

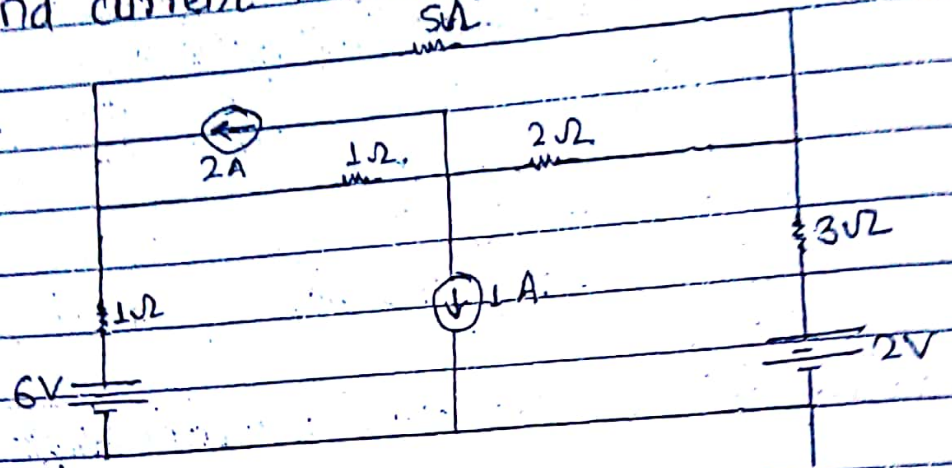
or, $I_1=2I_2$ ... (2)

Solving eq^n (1) & (2), we get,

$-6(2I_2)+2I_2+2=0$

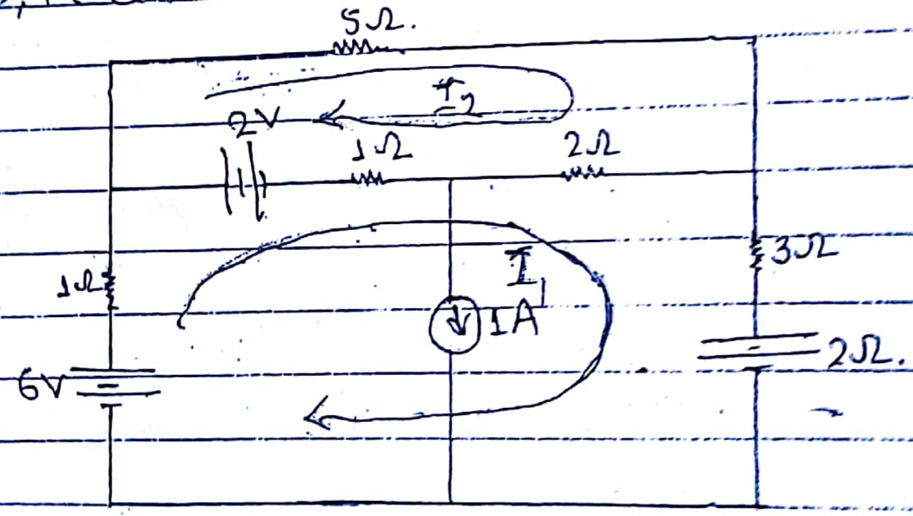
$\therefore I_2 = \frac{1}{5} A.$

Q. Find current in each resistance using KVL.



→ Solution:-

The current 2A is in parallel with 1Ω resistance. so, it can be converted as:



Using mesh in supermesh, we get,

$$-I_1 - 2 - (I_1 - I_2) - 2(I_1 - 1 - I_2) - 3(I_1 - 1) - 2 + 6 = 0$$

or, $-I_1 - 2 - I_1 + I_2 - 2I_1 + 2 + 2I_2 - 3I_1 + 3 + 4 = 0$

or, $-7I_1 + 3I_2 + 7 = 0$ .... (1)

Similarly in mesh 2, we get,

$$-5I_2 - 2(I_2 - (I_1 - 1)) - 1(I_2 - I_1) + 2 = 0$$

or, $-5I_2 - 2I_2 + 2I_1 - 2 - I_2 + I_1 + 2 = 0$

or, $-8I_2 + 3I_1 = 0$

or, $I_2 = \frac{3}{8} I_1 \quad \ldots (2)$

Substituting value of I_2 in eqn.(1), we get,

$-7I_1 + 3 \times \frac{3}{8} I_1 + 7 = 0$

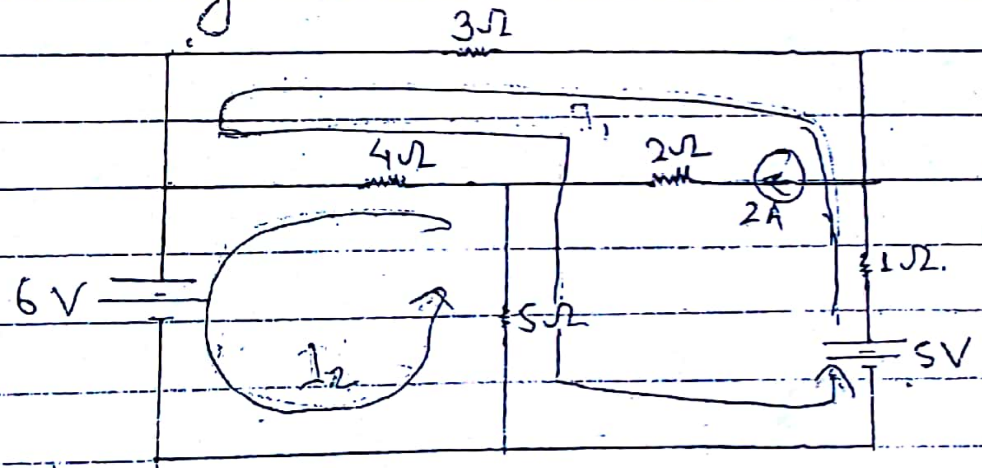
or, $-7I_1 + \frac{9}{8} I_1 + 7 = 0$

or, $(-56+9) I_1 = -56$

or, $I_1 = \frac{56}{47}$ A.

& $I_2 = \frac{3}{8} \times \frac{56}{47} = \frac{21}{47}$ A

Q. Find currents and voltages in each resistance using mesh analysis.



→ Solution:-

In supermesh, we get,

$-I_1 - 3(I_1-2) - 4(I_1-2-I_2) - 5(I_1-I_2) \cancel{-6} + 5 = 0$

or, $-I_1 - 3I_1 + 6 - 4I_1 + 8 + 4I_2 - 5I_1 + 5I_2 + 5 = 0$

or, $-13I_1 + 9I_2 + 19 = 0 \quad \ldots (1)$

Similarly, in mesh 2, we get,

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$$-4(I_2-(I_1-2))-6-5(I_2-I_1)=0$$

or, $-4I_2+4I_1-8-6-5I_2+5I_1=0$

or, $-9I_2+9I_1-14=0$ ....(2)

Solving eq^n (1) & (2), we get,

$$-13I_1+9I_2+19=0$$
$$9I_1-9I_2-14=0$$
$$-4I_1 \quad +5=0$$

or, $I_1=\frac{5}{4}=1.25A.$

and, $9\times1.25-9I_2-14=0$

or, $I_2=0.3A$

∴ Current through $1\Omega = I_1 = 1.25A$

Current through $2\Omega = 2A$

Current through $3\Omega = I_1-2 = 1.25-2 = -0.75A$

Current through $4\Omega = 0.45A$

Current through $5\Omega = 1.25-0.3 = 0.95A.$

And,

Voltage across $1\Omega = 1.25\times1 = 1.25V.$

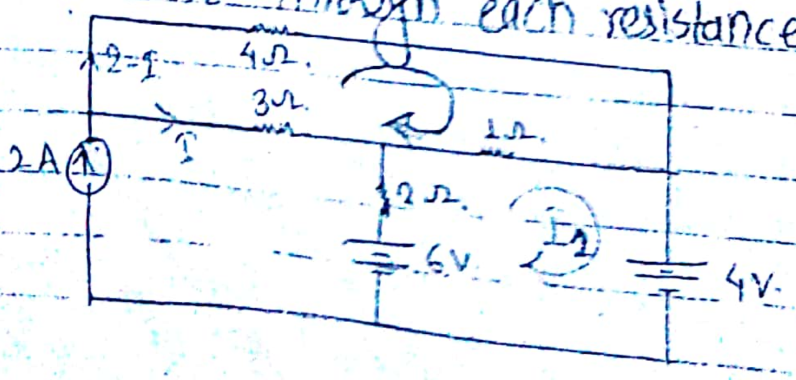
Voltage across $2\Omega = 2\times2 = 4V$

voltage across $3\Omega = 0.75\times3 = 2.25V.$

voltage across $4\Omega = 0.45\times4 = 1.8V$

voltage across $5\Omega = 0.95\times5 = 4.75V.$

Q. Find current through each resistance using mesh analysis



→ Solution:-

In mesh 1, we get,

$$-4(2-I) - 1(2-I-I_1) + 3I = 0$$

or, $-8+4I-2+I+I_1+3I = 0$

or, $8I + I_1 - 10 = 0$

or, $I_1 = 10 - 8I \quad \ldots(1)$

Similarly,

$$-1(I_2-(2-I)) - 4 + 6 - 2(I_2-2) = 0$$

or, $-(I_2-2+I) + 2 - 2I_2 + 4 = 0$

or, $-I_2 + 2 - I + 2 - 2I_2 + 4 = 0$

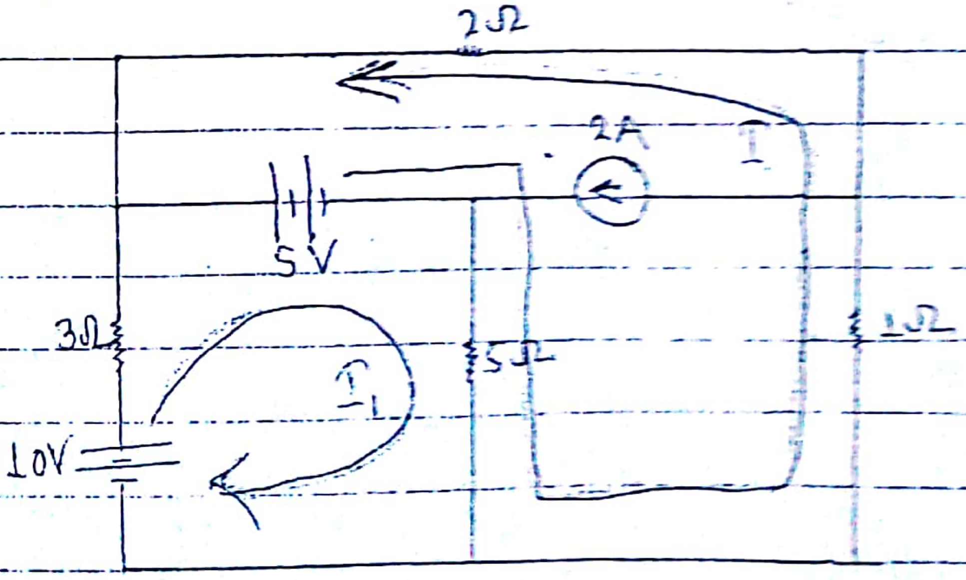
or, $-3I_2 - I + 8 = 0 \quad \ldots(2)$

or, $-3(10-8I) - I + 8 = 0$

or, $-30 + 24I - I + 8 = 0$

or,

Q. Find currents.



Date
Page

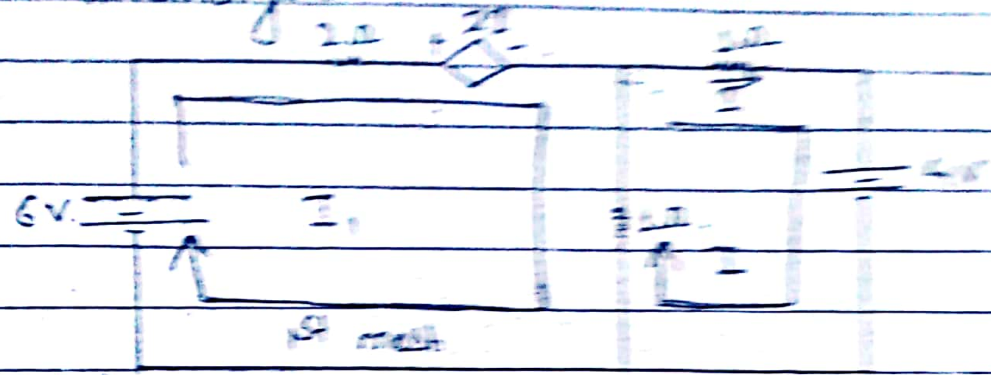
$-5-5(I+2+I_1) -1(I+2) - 2I = 0$

Or, $-5-5I-10-5I_1 - I - 2 - 2I = 0$

Or,

Use of dependent Source

Q. Find current and voltage in each resistance using mesh analysis



> Solution:-

Using KVL in 1st mesh, we get

$-2I_1 - 2I - 1(I_1 - I) + 6 = 0$

or, $-2I_1 - 2I - I_1 + I + 6 = 0$

or, $-3I_1 - I + 6 = 0$ ....(1)

Similarly, in 2nd mesh, we get

$-I - 4 - (I - I_1) = 0$

or, $-2I + I_1 - 4 = 0$ ... (2)

From eqn (1) and (2), we get

$$\left.\begin{array}{l} -3I_1 - I + 6 = 0 \\ I_1 - 2I - 4 = 0 \end{array}\right\} \times 3$$

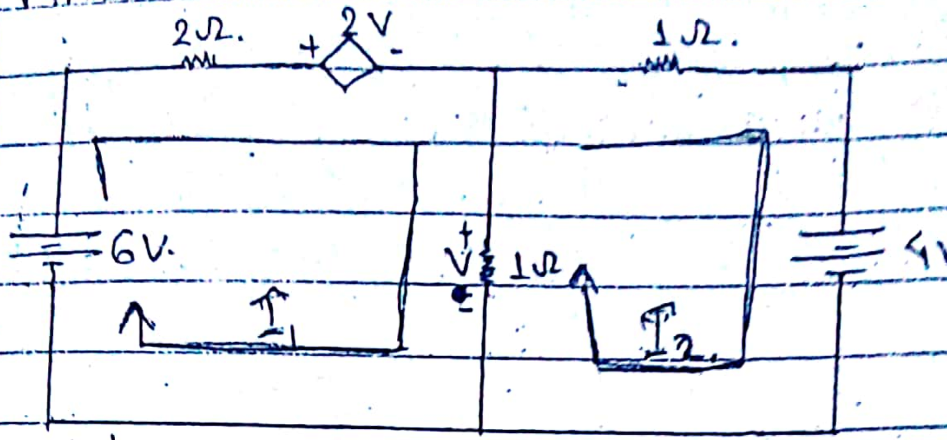
$-3I_1 - I + 6 + 3I_1 - 6I - 12 = 0$

or, $I = -\frac{6}{7}$ A

& $-3I_1 + \frac{6}{7} + 6 = 0$

or, $I_1 = \frac{16}{7}$ A

Q. Find currents.



→ Solution:-

Using KVL in 1st mesh, we get,

$$-2I_1 - 2V - 1(I_1 - I_2) + 6 = 0$$

or, $-2I_1 - 2V - I_1 + I_2 + 6 = 0$

or, $-3I_1 + I_2 - 2V + 6 = 0$ ..... (1)

Using KVL in 2nd mesh, we get,

$$-I_2 - 4 - (I_2 - I_1) = 0$$

or, $-2I_2 + I_1 - 4 = 0$ ..... (2)

Again,

$V = 1(I_2 - I_1)$ ..... (3)

putting V in eqn (1), we get,

$$-3I_1 + I_2 - 2(I_2 - I_1) + 6 = 0$$

~~or,~~ $\cancel{I_1 + 2I_2 - 4 = 0}$ .... (4)

~~From eqn (2) & (4), we get,~~

$\cancel{-2I_2 + I_1 - 4 = 0}$ } ×5

$\cancel{3I_2 - 5I_1 + 6 = 0}$

$\cancel{-10I_2 + 5I_1 - 20 + 3I_2 - 5I_1 + 6} = 0$

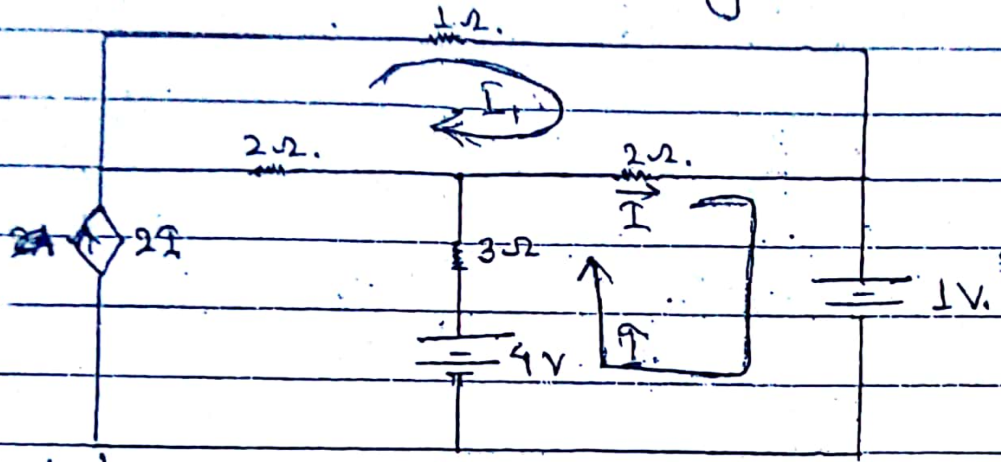
~~or,~~ $\cancel{I_2 = -2}$

or, $-I_1 - I_2 + 6 = 0$. — (4)

Solving eqn (2) & (4), we get,

$I_2 = \frac{2}{3}$ & $I_1 = \frac{16}{3}$ A.

Q. Find current and voltage in each resistance.



→ Solution:-

Using KVL in 1st mesh, we get,

$$-I_1 - 2(I_1 - I) - 2(-(2I - I_1)) = 0$$

$$-2(I - I_1) - 1 + 4 - 3(I - 2I) = 0$$

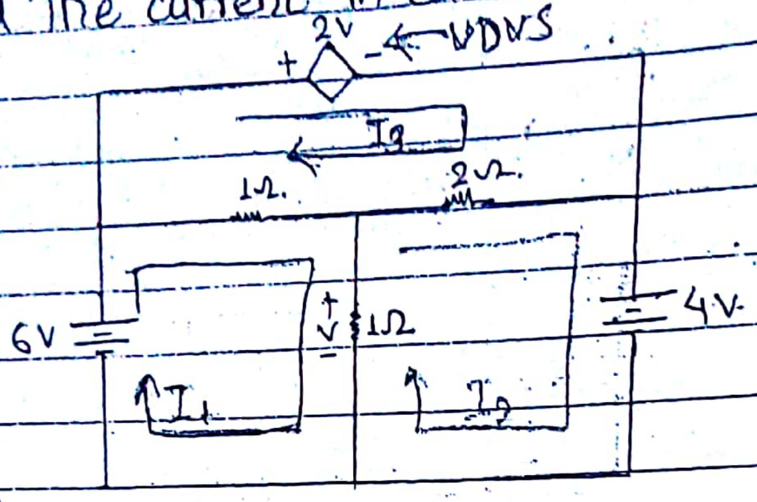
or, $-2I + 2I_1 + 3 + 3I = 0$

or, $2I_1 + I + 3 = 0$

or,

54.

Q. Find the current in each resistance using mesh analysis.



→ Solution:-

Applying KVL in 1st mesh, we get,

$-(I_1-I_3)-(I_1-I_2)+6=0$

or, $-2I_1+I_2+I_3+6=0$ ....(1)

Similarly in mesh 2, we get,

$-2(I_2-I_3)-4-(I_2-I_1)=0$

or, $-2I_2+2I_3-4-I_2+I_1=0$

or, $-3I_2+2I_3+I_1-4=0$ .... (2)

Similarly, in mesh 3, we get,

$-2V-2(I_3-I_2)-(I_3-I_1)=0$

or, $-2V-2I_3+2I_2-I_3+I_1=0$

or, $+I_1-2(I_1-I_2)-3I_3+2I_2=0$

or, $I_1-2I_1+2I_2-3I_3+2I_2=0$

or, $-I_1+4I_2-3I_3=0$ .... (3)

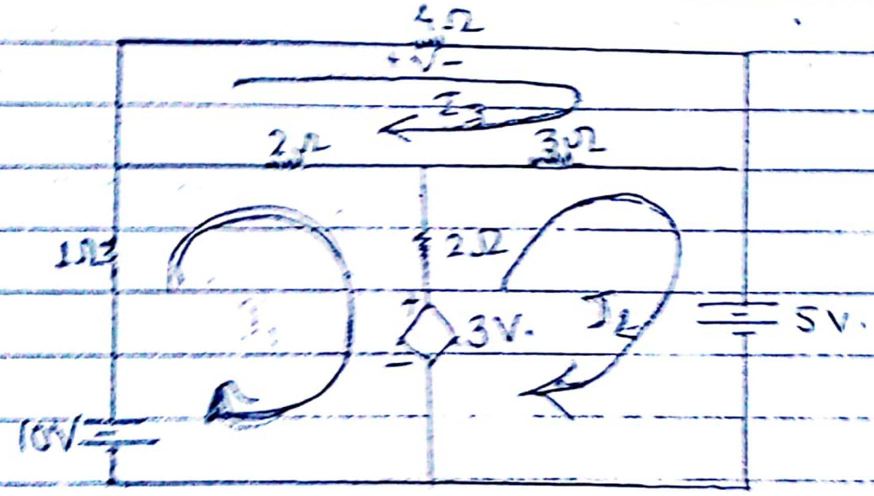
Solving by matrix method, we get,

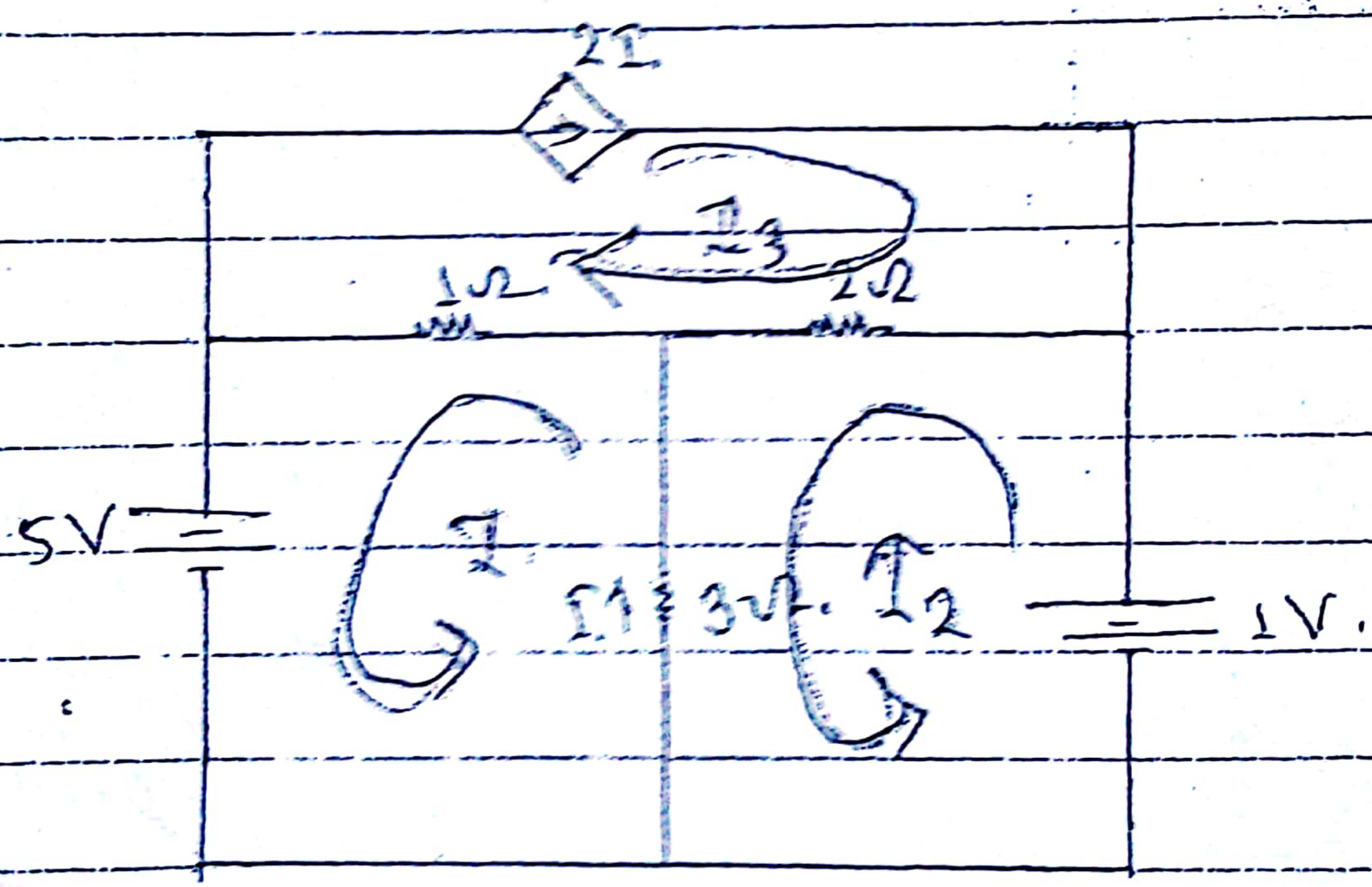
$$I_1=\frac{\begin{vmatrix}-6 & 1 & 1\\ 4 & -3 & 2\\ 0 & 4 & -3\end{vmatrix}}{\begin{vmatrix}-2 & 1 & 1\\ 1 & -3 & 2\\ -1 & 4 & 3\end{vmatrix}}=\frac{-6\begin{vmatrix}-3 & 2\\ 4 & -3\end{vmatrix}-4\begin{vmatrix}1 & 1\\ 4 & -3\end{vmatrix}+0}{-2\begin{vmatrix}-3 & 2\\ 4 & -3\end{vmatrix}-1\begin{vmatrix}1 & 1\\ 4 & -3\end{vmatrix}-1\begin{vmatrix}1 & 1\\ -3 & 2\end{vmatrix}}$$

$$= \frac{-6(3-8) - 4(-3-4)}{-2(3-8) - 1(-3-4) - 1(2+3)} = \frac{-6+28}{-2+7-5} = \frac{22}{0} = \infty.$$

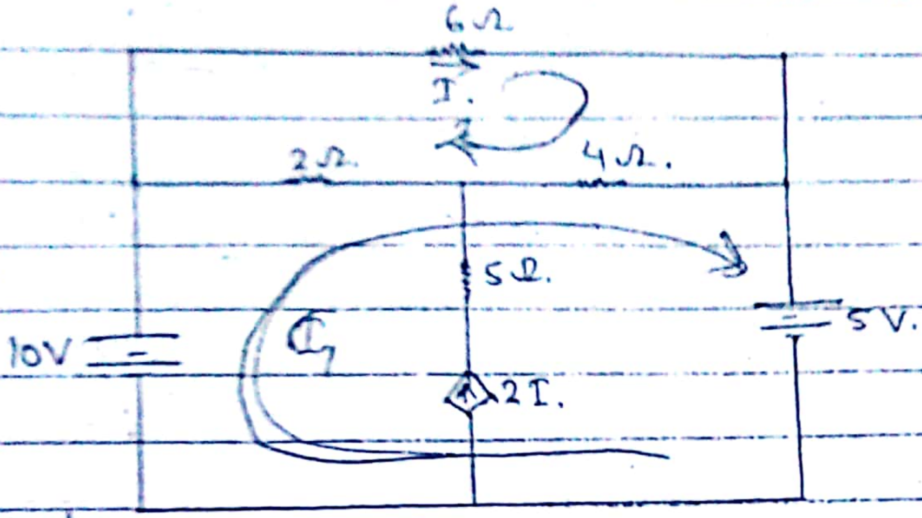
$\therefore I_1 = I_3 = I_2 = \infty.$

Q Find currents in each resistance.




2I
I_3
1Ω
2Ω
5V
I
I1
3Ω
I_2
1V.

Q. Find the currents.



→ Solution:-

Using KVL in supermesh, we get,

$10 - 2(I_1 - I) - 4(I_1 + 2I - I) - 5 = 0$

or, $5 - 2I_1 + 2I - 4I_1 - 4I = 0$

or, $-6I_1 - 2I + 5 = 0$ ... (1)

Using KVL in mesh(1), we get,

$-6I - 4\{I - (2I + I_1)\} - 2(I - I_1) = 0$

or, $-6I - 4(-I - I_1) - 2I + 2I_1 = 0$

or, $-4I + 6I_1 = 0$

or, $I = \frac{6}{4} I_1$ ....(2)

or, $5 - 6I_1 - 2I_1 = 0$

~~or,~~ $\cancel{5 - 6I_1 - 2\times\frac{6}{4} I = 0}$ $\cancel{5 - 6\times\frac{6}{4}}$

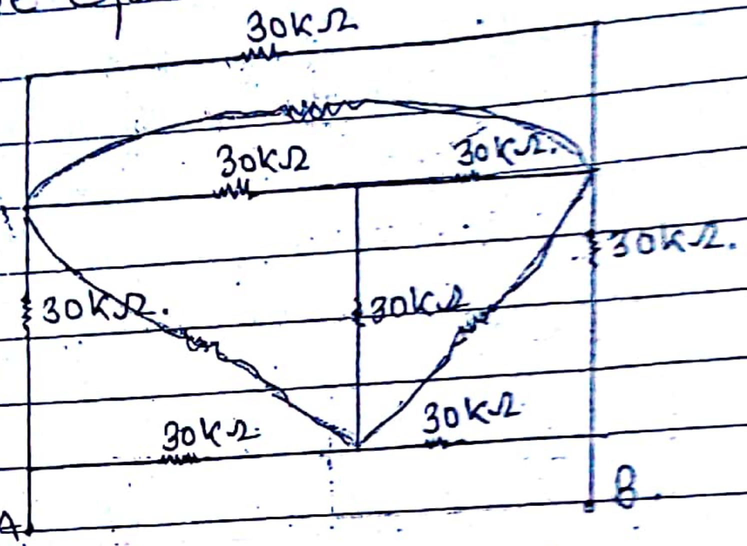
~~or,~~ $\cancel{5 = 9I}$. $5 - 6I_1 - 2\times\frac{6}{4} I_1 =$

∴ $\cancel{I = \frac{5}{9} A}$ or, $I_1 = \frac{5}{9}$ A.

and

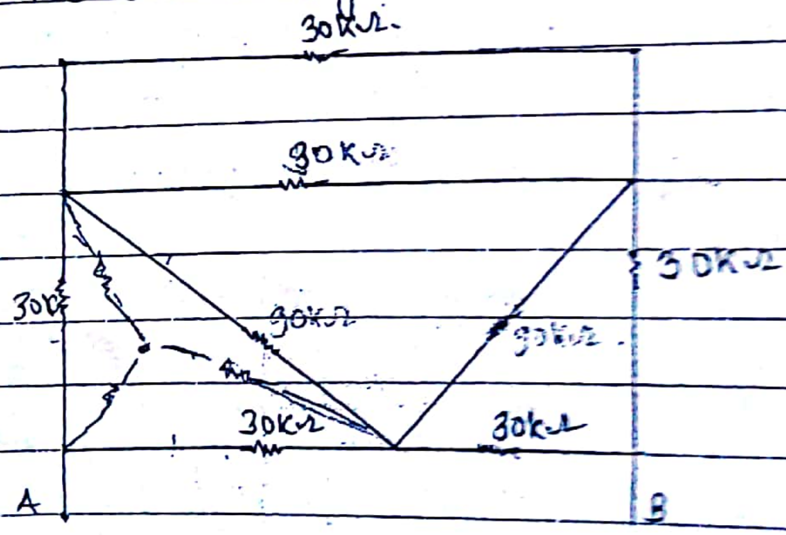
$I = \frac{6}{4} \times \frac{5}{9} = \frac{5}{6}$ A.

Find the equivalent resistance across AB terminal



Solution:-

The above figure can be rearranged as:



$$\frac{30\times30+30\times30+30\times30}{30}$$

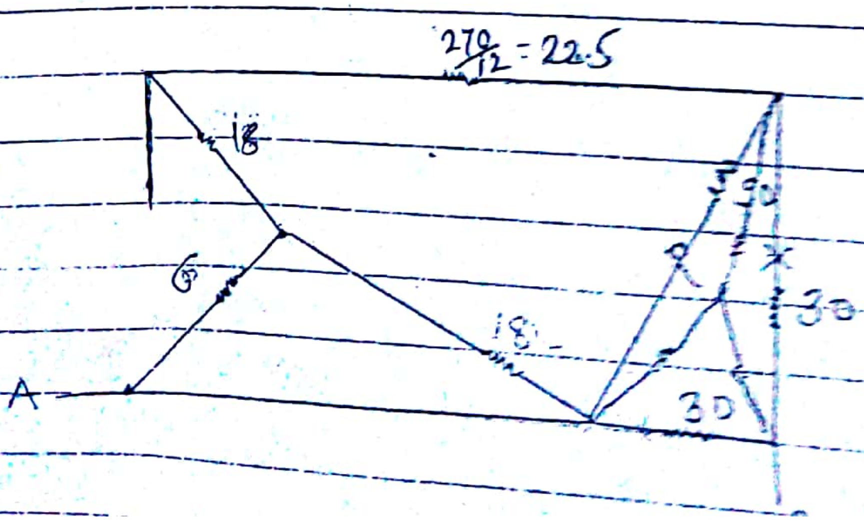
$$= \frac{2700}{30} = 90$$

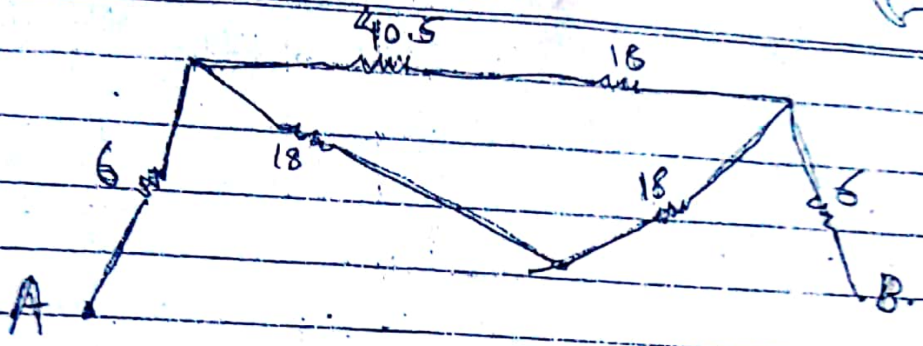
$90 // 30$

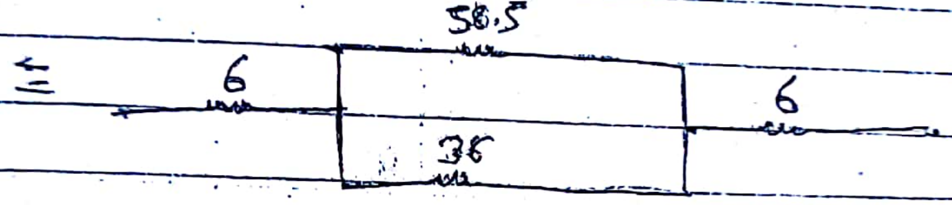
$$= \frac{90\times30}{90+30} = \frac{270}{12}$$

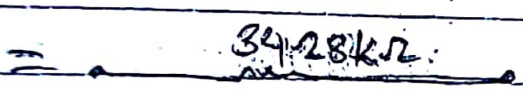
$$\frac{30\times90}{150} = \frac{2700}{150}$$

$$\frac{900}{150}$$





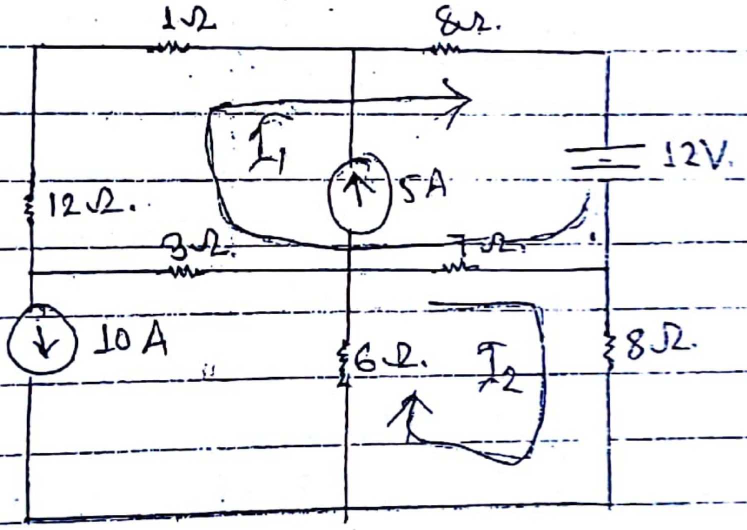




$\frac{58.5 \times 36}{58.5+36} = \frac{2106}{94.5} =$

$= 22.28 + 12 = 34.28$

Q. Find the currents.



→ Solution:

In supermesh, we get,

$-1(I_1) - 8(I_1+5) - 12 - 7(I_1+5-I_2) - 3(I_1+10) - 12I_1 = 0$

or, $-I_1 - 8I_1 - 40 - 12 - 7I_1 - 35 + 7I_2 - 3I_1 - 30 - 12I_1 = 0$

or, $-31I_1 + 7I_2 - 125 = 0$ —— (1)

In 2nd mesh, we get,

$-7\{I_2 - (I_1+5)\} - 8I_2 - 6(I_2+10) = 0$

or, $-7(I_2 - I_1 - 5) - 8I_2 - 6I_2 - 60 = 0$

or, $-7I_2 + 7I_1 + 35 - 8I_2 - 6I_2 - 60 = 0$

or, $-21I_2 + 7I_1 - 25 = 0$ ......(2)

Solving eqn. (1) & (2), we get,

$$\left.\begin{array}{l} -31I_1 + 7I_2 - 125 = 0 \\ 7I_1 - 21I_2 - 25 = 0 \end{array}\right\} \times 3$$

$$-93I_1 + 21I_2 - 375 + 7I_1 - 21I_2 - 25 = 0$$

or, $-86I_1 = 400$

or, $I_1 = 4.65\,A$.

and,

$$7 \times 4.65 - 21I_2 - 25 = 0$$

or, $I_2 = 0.36\,A$.

Kirchhoff's current law (Nodal Analysis)

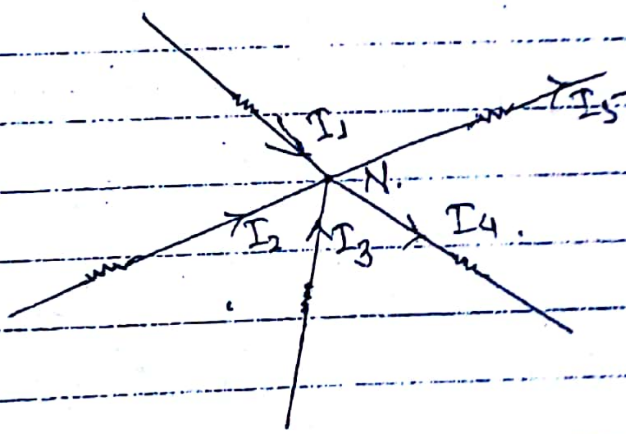
The algebric sum of the total currents at a junction or node is always equal to zero.

In other words, the incoming currents to the node is equal to the outgoing currents from the node.

Sign convection:-

All the incoming currents may be taken as positive; while all the outgoing currents taken as -ve.

Explanation:-

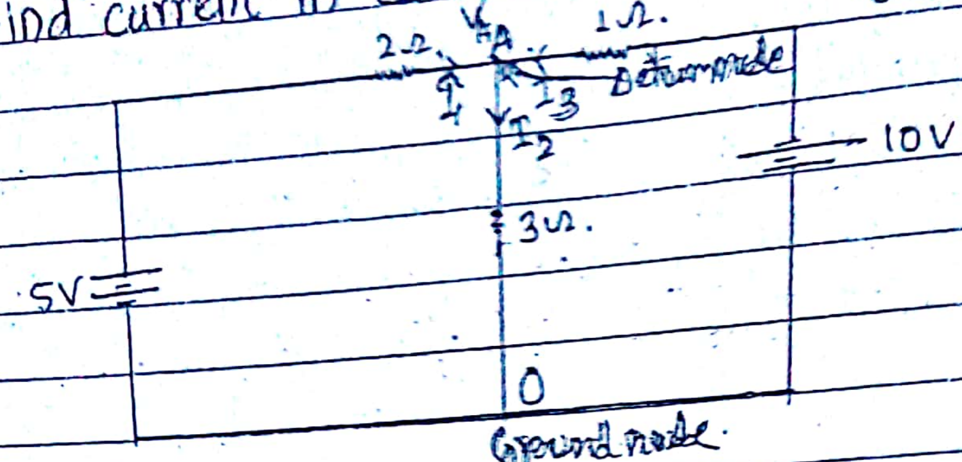


Let us consider the incoming currents be I_1, I_2 and I_3. Also, I_4 and I_5 be the outgoing currents. Then, at the node, according to Kirchhoff's current law, we can write,

$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

or, $I_1 + I_2 + I_3 = I_4 + I_5$.

Q. Find current in each resistance using Nodal Analysis



Working procedure:-
1. Identify the number of nodes.
2. Give name to the nodes.
3. Give the nodal potential.
4. Assume the currents in the branches.
5. Write the KCL equation.

Solution:-

At node A, using KCL,

Suppose incoming currents be +ve & outgoing currents be -ve.

$$I_1 - I_2 + I_3 = 0 \quad \ldots (1)$$

$$I_1 = \frac{5 - V_A}{2}, \quad I_2 = \frac{V_A - 0}{3} \quad \& \ I_3 = \frac{-10 - V_A}{1}$$

Putting I_1, I_2 and I_3 in eqn (1), we get

$$\frac{5 - V_A}{2} - \frac{V_A}{3} + \frac{-10 - V_A}{1} = 0$$

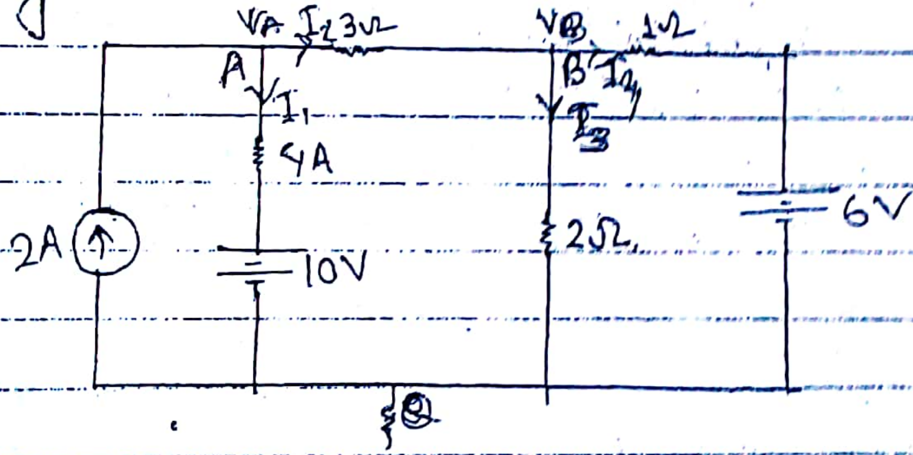
or, $\frac{15 - 3V_A - 2V_A - 60 - 6V_A}{6} = 0$ or, $V_A = -\frac{45}{11}$

$$\therefore I_1 = \frac{5 + \frac{45}{11}}{2} = \frac{55 + 45}{11} \times \frac{1}{2} = \frac{50}{11} = 4.54\,A.$$

$$I_2 = -\frac{45}{11} \times \frac{1}{3} = \frac{-15}{11}\,A.$$

$$\& \; I_3 = \frac{-10 + 45}{11} = \frac{-110 + 45}{11} = \frac{-65}{11}\,A.$$

Q. Find currents in each resistances using nodal analysis.



→ Solution:-

At node A, using KCL,

$$2 - I_2 - I_1 = 0 \quad \ldots\ldots (1)$$

$$I_1 = \frac{V_A - 10}{4} \quad \& \quad I_2 = \frac{V_A - V_B}{3}$$

Putting I_1 & I_2 in eq^n.(1), we get

$$2 - \frac{V_A - 10}{4} - \frac{V_A - V_B}{3} = 0$$

or, $$\frac{24 - 3V_A + 30 - 4V_A + 4V_B}{12} = 0$$

or, $-7V_A + 4V_B = -54$ ....(1)

At node B, using KCL,

$$I_2 - I_3 - I_4 = 0$$

$$I_2 = \frac{V_A - V_B}{3} \quad I_3 = \frac{V_B}{2} \quad \& \quad I_4 = \frac{V_B - 6}{1}$$

$$\therefore \frac{V_A - V_B}{3} - \frac{V_B}{2} - \frac{V_B - 6}{1} = 0$$

or, $\frac{2V_A - 2V_B - 3V_B - 6V_B + 36}{6} = 0$

or, $2V_A - 11V_B + 36 = 0$ ....(2)

On solving eqn.(1) and (2), we get,

$$\left.\begin{aligned} -7V_A + 4V_B + 54 = 0 \\ 2V_A - 11V_B + 36 = 0 \end{aligned}\right\} \begin{aligned} \times 2 \\ \times 7 \end{aligned}$$

$$-14V_A + 8V_B + 108 + 14V_A - 77V_B + 252 = 0$$

or, $V_B = \frac{360}{69} = 5.21V$

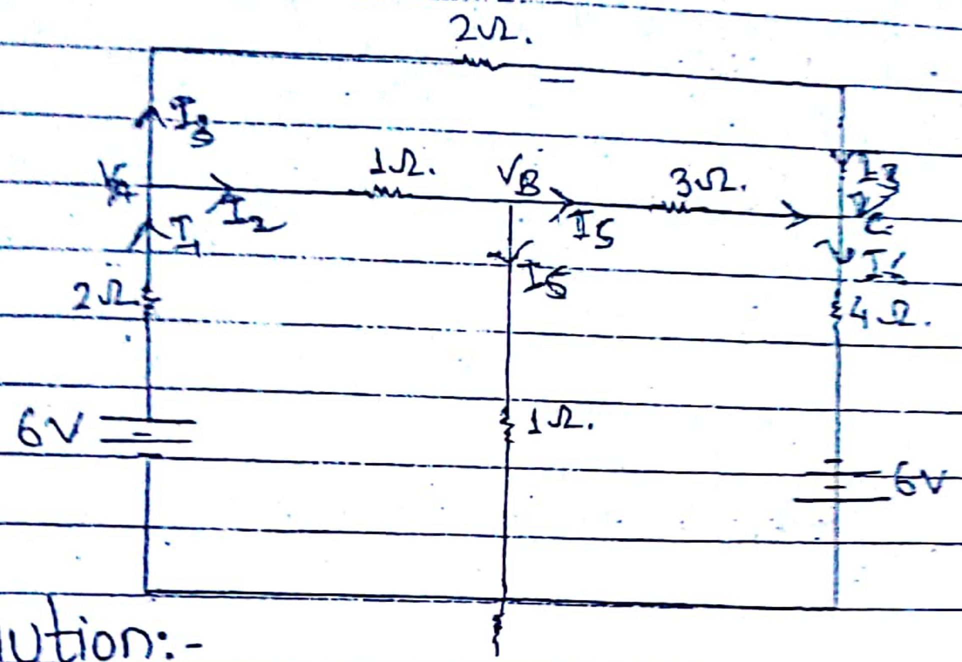
and $2V_A - 11 \times \frac{360}{69} + 36 = 0$

or, $V_A = 10.69V$

$$\therefore I_1 = \frac{V_A - 10}{4} = 0.1725A$$

$I_2 = \frac{V_A - V_B}{3} = 1.82A$ & $I_3 = \frac{V_B}{2} = \frac{5.21}{2} = 2.6A$

Find the currents.



Solution:-

At node A, using KCL,

$$I_1 - I_2 - I_3 = 0 \quad \ldots (1)$$

$$I_1 = \frac{6-V_A}{2} \qquad I_2 = V_A - V_B \quad \& \quad I_3 = \frac{V_A - V_C}{2}$$

$$\frac{6-V_A}{2} - V_A + V_B - \frac{V_A - V_C}{2} = 0$$

or, $6 - V_A - 2V_A + 2V_B - V_A + V_C = 0$

or, $6 - 4V_A + 2V_B + V_C = 0 \quad \ldots (2)$

Again,

At node B, using KCL,

$$I_2 - I_5 - I_4 = 0 \quad \ldots (3)$$

$$I_2 = V_A - V_B \qquad I_4 = V_B \qquad I_5 = \frac{V_B - V_C}{3}$$

or, $V_A - V_B - V_B - \dfrac{V_B - V_C}{3} = 0$

or, $3V_A - 7V_B + V_C = 0 \quad \ldots (4)$

And, at node C, we get,

$$I_5 + I_3 - I_6 = 0 \quad \ldots (5)$$

$$I_5 = \frac{V_B - V_C}{3} \qquad I_3 = \frac{V_A - V_C}{2} \quad \& \quad I_6 = \frac{V_C + 6}{4}$$

$$\frac{V_B - V_C}{3} + \frac{V_A - V_C}{2} - \frac{V_C + 6}{4} = 0$$

or, $$\frac{4V_B - 4V_C + 6V_A - 6V_C - 3V_C - 18}{12} = 0$$

or, $$4V_B - 13V_C + 6V_A - 18 = 0 \quad \ldots (6)$$

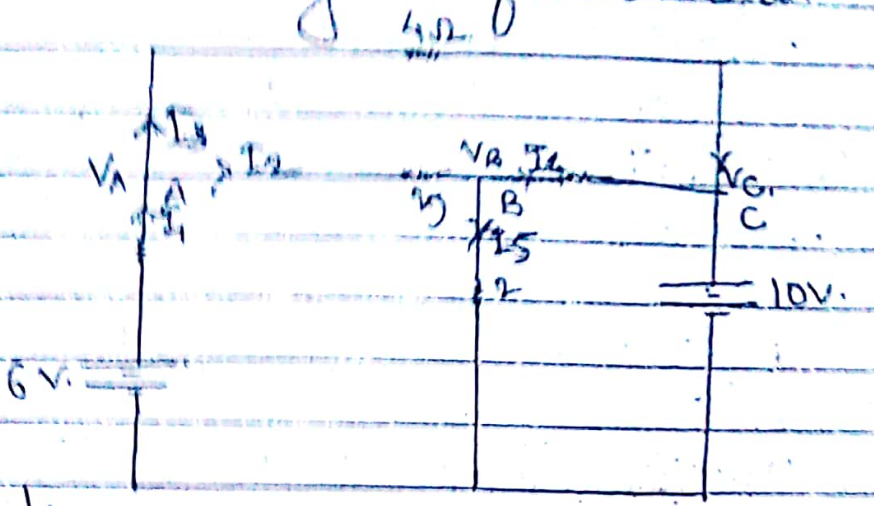
Solving eqn (2), (4) & (6) by matrix method, we get,

$$V_A = \frac{\begin{vmatrix} -6 & 2 & 1 \\ 0 & -7 & 1 \\ 18 & 4 & -13 \end{vmatrix}}{\begin{vmatrix} -4 & 2 & 1 \\ 3 & -7 & 1 \\ 6 & 4 & -13 \end{vmatrix}} = \frac{-6(87) + 18(2+7)}{-4(87) - 3(-26-4) + 6(2+7)} = \frac{-522 + 162}{-348 + 90 + 54} = 1.76$$

$$V_B = \frac{\begin{vmatrix} -4 & -6 & 1 \\ 3 & 0 & 1 \\ 6 & 18 & -13 \end{vmatrix}}{-204} = \frac{-4(-18) - 3(78-18) + 6(-6)}{-204} = 0.70$$

$$\& \quad V_C = \frac{\begin{vmatrix} -4 & 2 & -6 \\ 3 & -7 & 0 \\ 6 & 4 & 18 \end{vmatrix}}{-204} = 0.35$$

Q. Use nodal analysis to find currents.



$V_C = 10V$

→ Solution:-

Here, $V_C = 10V$.

At node A, using KCL,

$$I_1 - I_2 - I_3 = 0 \quad \dots (1)$$

$$I_1 = \frac{6 - V_A}{2}, \quad I_2 = \frac{V_A - V_B}{3} \quad \& \quad I_3 = \frac{V_A - 10}{4}$$

$$\therefore \frac{6 - V_A}{2} - \frac{V_A - V_B}{3} - \frac{V_A - 10}{4} = 0$$

$$\frac{36 - 6V_A - 4V_A + 4V_B - 3V_A + 30}{12} = 0$$

or, $66 - 13V_A + 4V_B = 0 \quad \dots\dots (2)$

Similarly,

At node B, using KCL, we get,

$$I_2 - I_5 - I_4 = 0 \quad \dots (3)$$

$$I_2 = \frac{V_A - V_B}{3} \quad I_4 = V_B - 10 \quad \& \quad I_5 = \frac{V_B}{2}$$

or, $\frac{V_A - V_B}{3} - V_B + 10 - \frac{V_B}{2} = 0$

or, $2V_A - 11V_B + 60 = 0 \quad \dots (4)$

Solving eqn. (1) and (2), we get,

$$\left.\begin{aligned} -13V_A + 4V_B + 66 &= 0 \\ 2V_A - 11V_B + 60 &= 0 \end{aligned}\right\} \begin{aligned} &\times 11 \\ &\times 4 \end{aligned}$$

$$-143V_A + 44V_B + 726 + 8V_A - 44V_B + 240 = 0$$

or, $-135V_A = -966$

$\therefore V_A = 7.15\,V$

& $2\times 7.15 - 11V_B + 60 = 0$

or, $V_B = 6.75\,V.$

$\therefore I_1 = \frac{6-V_A}{2} = \frac{6-7.15}{2} = \frac{-1.15}{2} = -0.575\,A.$

$I_2 = \frac{V_A - V_B}{3} = 0.133\,A$

$I_3 = \frac{V_A - 10}{4} = -0.7125\,A.$

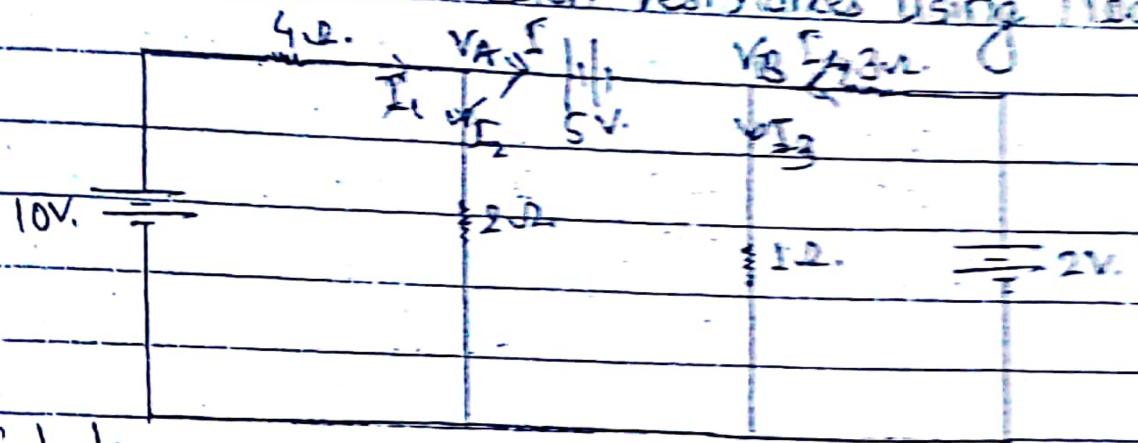
$I_4 = V_B - 10 = 6.75 - 10 = -3.25\,A.$

& $I_5 = \frac{V_B}{2} = \frac{6.75}{2} = 3.375\,A.$

Supernode Concept:-

When there is voltage source between two nodes we apply supernode concept. This means to combine both nodes together.

Find currents in each resistance using Nodal Analysis



Solution:-

At node A, using nodal analysis or KCL,

$$I_1 - I_2 - I = 0 \quad \ldots (1)$$

$$I = I_1 - I_2 \quad \ldots (1)$$

$$I_1 = \frac{10 - V_A}{4} \qquad I_2 = \frac{V_A - 0}{2}$$

$$I = \frac{10 - V_A}{4} - \frac{V_A}{2} = \frac{3V_A - 10}{4} \quad \ldots (2)$$

Similarly at node B, we get,

$$I + I_4 - I_3 = 0$$

$$I = I_3 - I_4 \quad \ldots (3)$$

$$I_3 = V_B \quad \& \quad I_4 = \frac{2 - V_B}{3}$$

$$\therefore I = V_B - \frac{2 - V_B}{3}$$

$$\therefore I = \frac{4V_B - 2}{3} \quad \ldots (4)$$

From eq^n (3) & (4), we get,

$$-9V_A - 16V_B + 38 = 0 \quad \ldots (5)$$

From fig;

$$V_A - V_B = 5 \quad \ldots (6)$$

or, $V_A = 5 + V_B$.

or, $-9(5+V_B) - 16V_B + 38 = 0$

or, $-9V_B - 45 - 16V_B + 38 = 0$

or, $V_B = \frac{-7}{25}$.

$\therefore V_A = 5 - \frac{7}{25} = \frac{125-7}{25} = \frac{118}{25}$

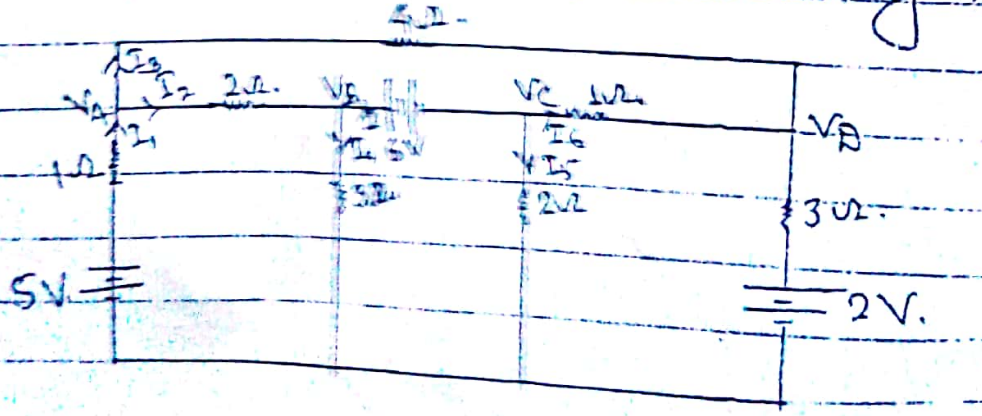
$\therefore I_1 = \frac{10-V_A}{4} = \frac{10-\frac{118}{25}}{4} = \frac{250-118}{25} \times \frac{1}{4} = 1.32A.$

$I_2 = \frac{V_A}{2} = \frac{118}{25\times 2} = \frac{59}{25}$ A

$I_3 = V_B = -7/25$ A

& $I_4 = \frac{2-V_B}{3} = \frac{2+7/25}{3} = \frac{19}{25}$ A

Q. Find current in each resistance using nodal analysis.



Solution:-

At node A, using KCL,

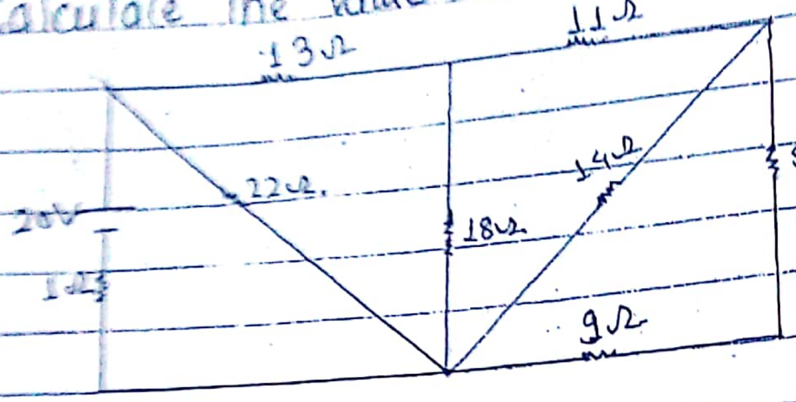
$$I_1 - I_2 - I_3 = 0$$

or, $I_1 = \frac{5 - V_A}{1}$ $\quad I_2 = \frac{V_A - V_B}{2}$ & $I_3 = \frac{V_A - V_D}{4}$

or, $5 - V_A - \frac{V_A - V_B}{2} - \frac{V_A - V_D}{4} = 0$

Solution of unit Test - 1.

Calculate the value of current from 20V source.



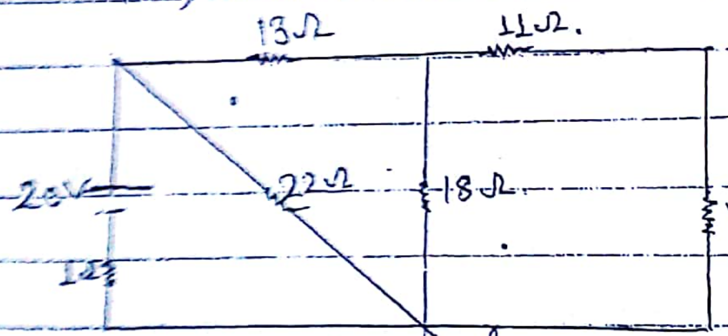
Solution:-

In the given figure above,
since 5Ω and 9Ω are in series,
so Requi. = 5+9 = 14Ω,

Again, 14Ω and 14Ω are in parallel, so,

$$Req. = \frac{14 \times 14}{14+14} = 7\Omega.$$

And, the circuit becomes,



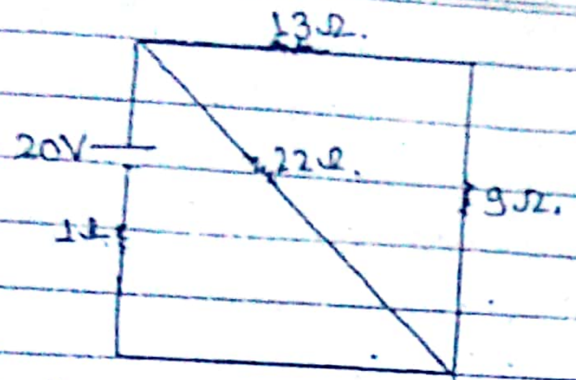
Again, in above figure,
Resistances 11Ω and 7Ω are in series,
so, Req. = 11+7 = 18Ω.

And,

18Ω and 18Ω are in parallel,

$$so, Req. = \frac{18 \times 18}{18+18} = 9\Omega.$$

Now, the circuit becomes,



Again, since 13Ω and 9Ω are in series

So, $R_{eq} = 13+9 = 22\Omega$.

Again, since $22\Omega \parallel 22\Omega$.

So, $R_{eq} = \frac{22\times22}{22+22} = 11\Omega$

And, since 11Ω is in series with 1Ω,

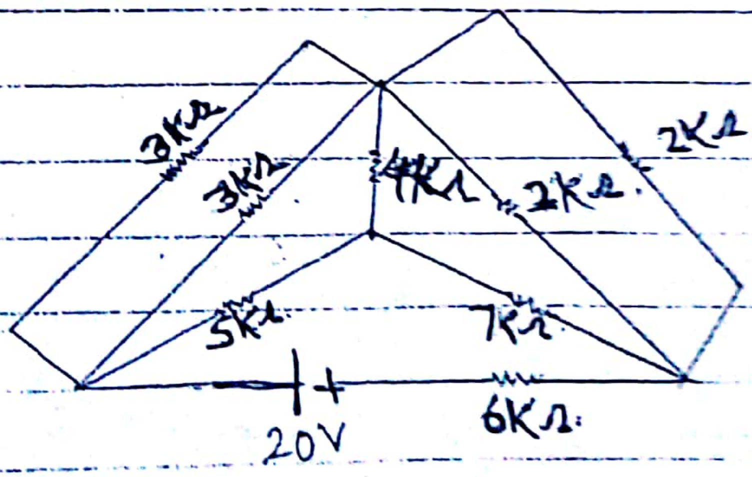
So, the equivalent Resistance is $11+1 = 12\Omega$.

Now,

current (I) from 20V is given by.

$I = \frac{V}{R_{eq}} = \frac{20}{12} = 1.67A$.

Q.2.(a) Find current from 20V source.



⇒ Solution:-

In above figure,

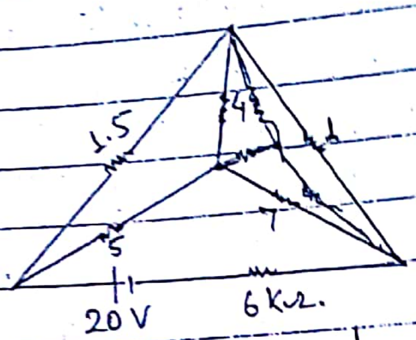
Since 3KΩ & 3KΩ are in parallel,

So, $R_{eq} = \frac{3 \times 3}{3+3} = 1.5\,k\Omega$.

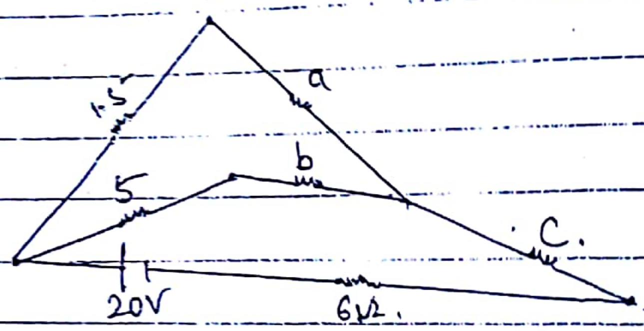
Also, $2k\Omega // 2k\Omega$,

So, $R_{eq} = \frac{2 \times 2}{2+2} = 1\,k\Omega$.

Now, the circuit becomes,



Now, converting delta to star, we get,



$a = \frac{4 \times 1}{4+1+7} = \frac{4}{12}\,k\Omega. = 0.33\,k\Omega.$

$b = \frac{7 \times 4}{12} = \frac{28}{12}\,k\Omega. = 2.33\,k\Omega.$

$c = \frac{1 \times 7}{12} = \frac{7}{12}\,k\Omega. = 0.58\,k\Omega$

In above circuit, since $1.5k\Omega$ and a are in series

$R_{eq_1} = 1.5 + 0.33 = 1.83\,k\Omega.$

Also, $5k\Omega$ & b are in series.

So, $R_{eq_2} = 5 + 2.33 = 7.33\,k\Omega$

Again,
Since $Req_1 \parallel Req_2$, so,

$$\therefore Req_3 = \frac{1.83 \times 7.33}{1.83 + 7.33} = 1.46k\Omega.$$

Again,
Since Req_3 are in series with c and $6k\Omega$,
So, net equivalent resistance becomes,

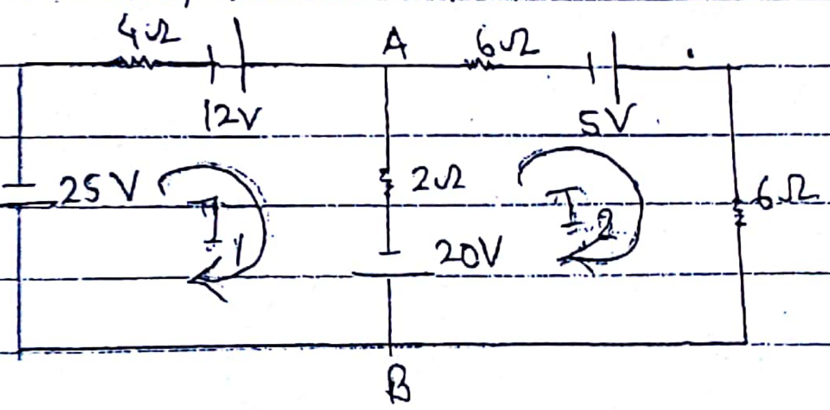
$$R_{net} = 1.46 + 0.58 + 6$$
$$= 8.04k\Omega.$$

Now, the total current is given by

$$I = \frac{V}{R_{net}} = \frac{20}{8.04 \times 10^3} = 2.48 \times 10^{-3} A.$$

Hence, current from 20V is 2.48×10^{-3} A.

Using KVL, find current in each resistor.



Solution:-
Using mesh analysis in mesh 1, we get,

$$-4I_1 + 12 - 2(I_1 - I_2) + 20 - 25 = 0$$

or, $7 - 6I_1 + 2I_2 = 0$ ....(1)

Similarly, in mesh 2, we get,

$$-6I_2 + 5 - 6I_2 - 20 - 2(I_2 - I_1) = 0$$

or, $-14I_2 + 2I_1 - 15 = 0$ ....(2)

$-4I_2 - 3(I_2 - I_1) - 1(I_3 - I_1) + 5 = 0$

or, $-4I_2 - 3I_2 + 3I_1 - I_3 + I_1 + 5 = 0$

or, $-7I_2 + 4I_1 - I_3 + 5 = 0$

or, $-7I_2 + 4I_1 - 2 - I_2 + 5 = 0$

or, $-8I_2 + 4I_1 + 3 = 0$ ....(2)

And,

$-3(I_1 - I_2) + 10 - 1(I_1 - I_3) = 0$

or, $-3I_1 + 3I_2 + 10 - I_1 + I_3 = 0$

or, $-3I_1 + 3I_2 + 10 - I_1 + 2 + I_2 = 0$

or, $-4I_1 + 4I_2 + 12 = 0$ .... (3)

On solving eq^n (2) & (3), we get,

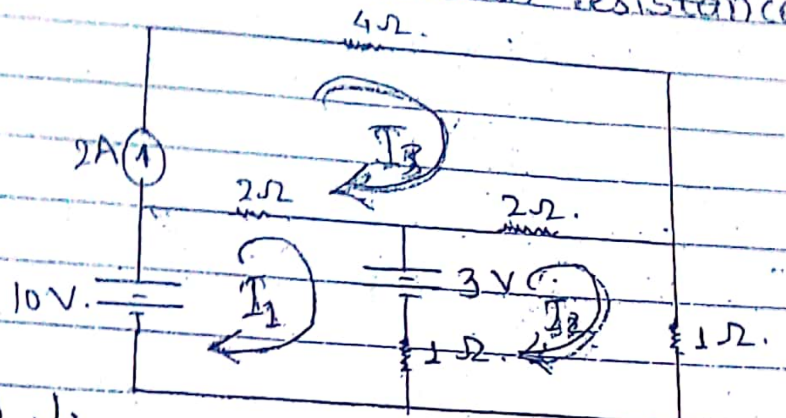
$$-8I_2 + 4I_1 + 3 = 0$$
$$4I_2 - 4I_1 + 12 = 0$$
$$-4I_2 \quad + 15 = 0$$

$\therefore I_2 = 3.75A$

and $I_1 = 6.75A$

$I_3 = 2 + I_2 = 2 + 3.75 = 5.75A$

Find currents in each resistance.



Solution:-

In above mesh, $I_3 = 2A$.

In mesh 1, we have,

$$-2(I_1-2)-3-1(I_1-I_2)+10=0$$

or, $-2I_1+4-3-I_1+I_2+10=0$

or, $-3I_1+I_2+11=0 \quad \ldots(1)$

And,

In mesh (2) we get,

$$-2(I_2-2)-1(I_2)-1(I_2-I_1)+3=0$$

or, $-2I_2+4-I_2-I_2+I_1+3=0$

or, $-4I_2+I_1+7=0 \quad \ldots(2)$

Solving eqn. (1) & (2), we get,

$$\left.\begin{array}{r} -3I_1+I_2+11=0 \\ I_1-4I_2+7=0 \end{array}\right\} \begin{array}{l} \\ \times 3 \end{array}$$

$$-3I_1+I_2+11+3I_1-12I_2+21=0$$

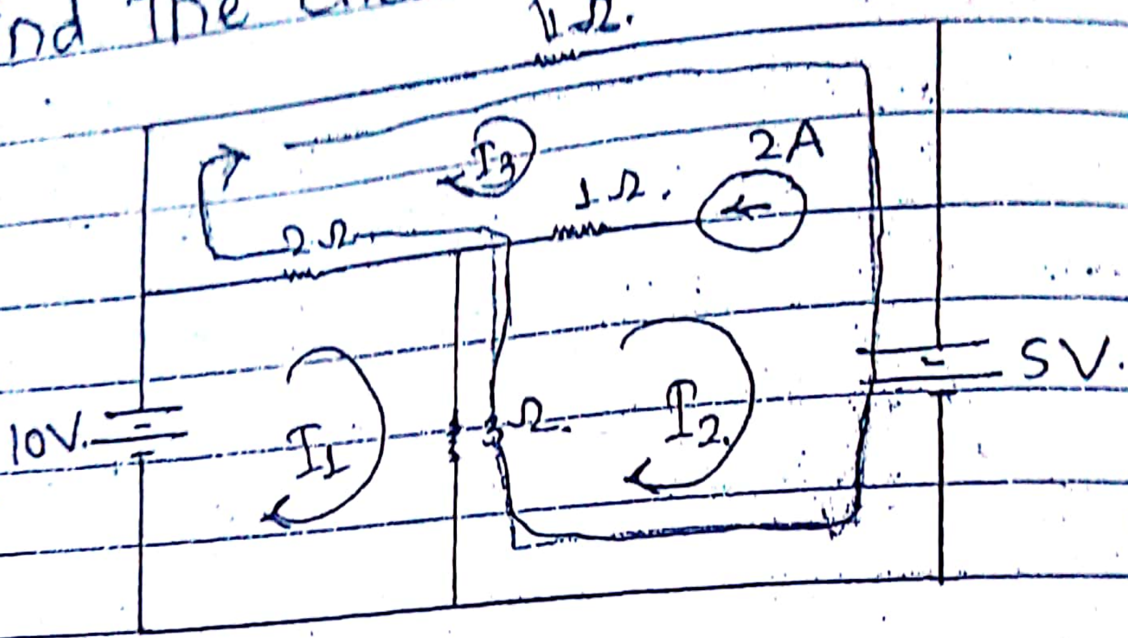
or, $-11I_2=-32$

$\therefore I_2 = 32/11 \text{ A}$.

& $I_1 - 4\times\frac{32}{11}+7=0$

or, $I_1 = \frac{77-128}{11} = -\frac{51}{11} \text{ A}$.

Find the circuit currents.



Solution:-

Here, $I_3 - I_2 = 2A$ ....(1)

or, $I_3 = 2 + I_2$.

In supermesh,

$-I_3 - 5 - 3(I_2 - I_1) - 2(I_3 - I_1) = 0$

or, $-I_3 - 5 - 3I_2 + 3I_1 - 2I_3 + 2I_1 = 0$

or, $-3I_3 - 3I_2 + 5I_1 - 5 = 0$

or, $-3(2 + I_2) - 3I_2 + 5I_1 - 5 = 0$

or, $-6 - 6I_2 + 5I_1 - 5 = 0$

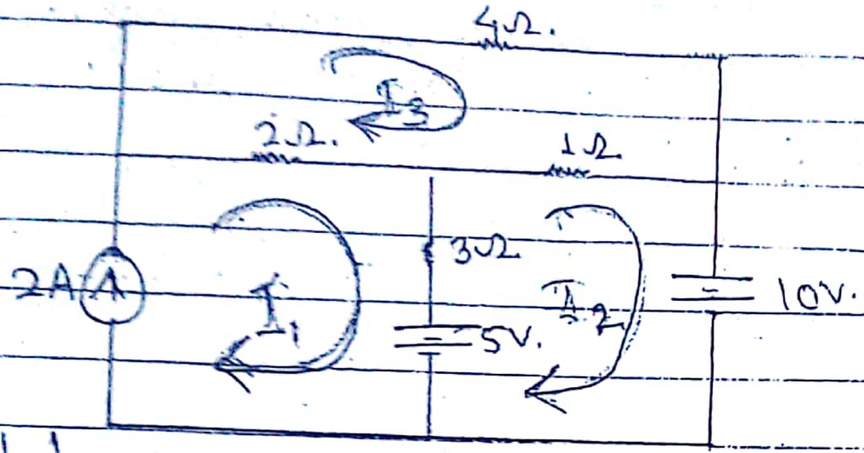
or, $-6I_2 + 5I_1 - 11 = 0$ ——— (2)

Also,

$-2(I_1 - I_3) - 3(I_1 - I_2 + 2) + 10 = 0$

or,

Find currents.



Solution:-

Here, $I_1 = 2A$.

In 2nd mesh,

$-1(I_2 - I_3) - 10 + 5 - 3(I_2 - 2) = 0$

or, $-I_2 + I_3 - 5 - 3I_2 + 6 = 0$

or, $-4I_2 + I_3 + 1 = 0$ .... (1)

In 3rd mesh,

$-4I_3 - 1(I_3 - I_2) - 2(I_3 - 2) = 0$

or, $-4I_3 - I_3 + I_2 - 2I_3 + 4 = 0$

or, $-7I_3 + I_2 + 4 = 0$ ... (2)

On solving eqn. (1) and (2), we get,

$I_3 - 4I_2 + 1 = 0$

$-7I_3 + I_2 + 4 = 0 \} \times 4$

$I_3 - 4I_2 + 1 - 28I_3 + 4I_2 + 16 = 0$

or, $-27I_3 = -17$.

$\therefore I_3 = 17/27$ A. 0.629

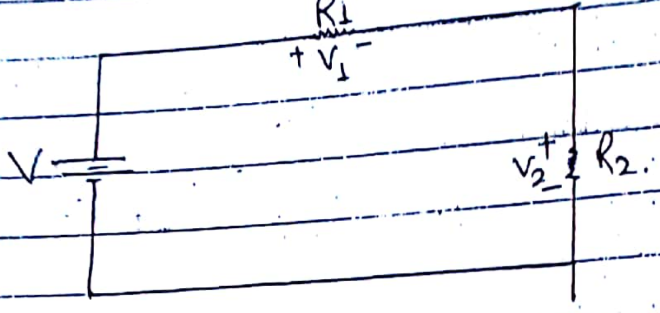
And,

$\frac{17}{27} + 1 = 4I_2$ or, $I_2 = 0.4A$

Voltage Divider Rule.

Voltage divides in series circuit. We can find the voltage across an element directly using voltage divider rule.

The rule explains:-



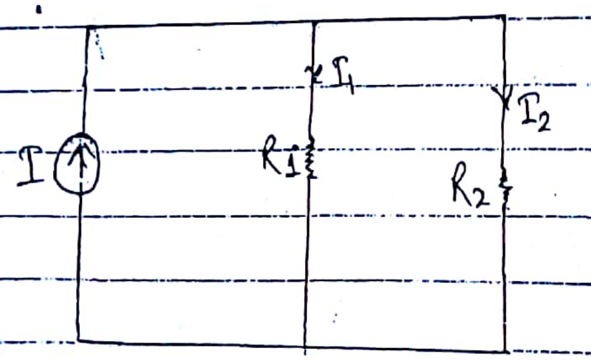
$$V_2 = V \times \frac{R_2}{R_1 + R_2}$$

$$V_1 = V \times \frac{R_1}{R_1 + R_2}$$

Current Divider Rule:-

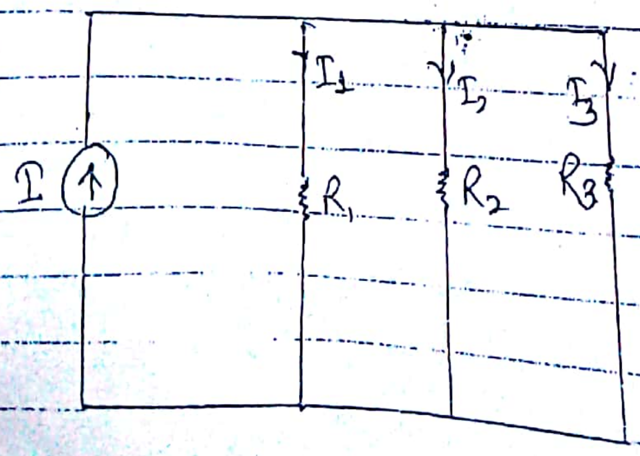
Current divides in parallel circuit. The current of an element can be found out by using current divider rule.

The rule explains as:-



$$I_2 = I \times \frac{R_1}{R_1 + R_2}$$

$$I_1 = I \times \frac{R_2}{R_1 + R_2}$$



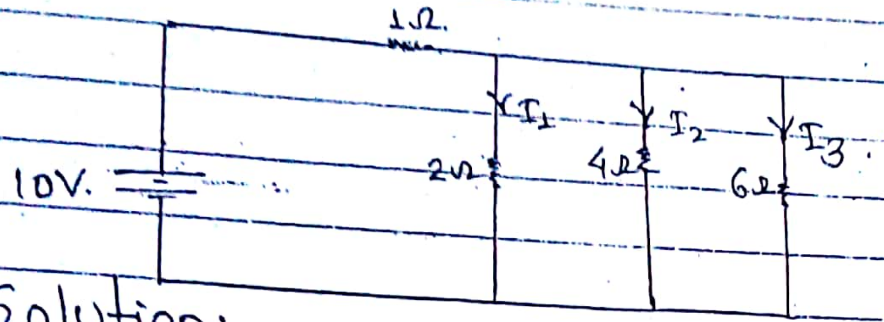
$$I_1 = I \times \frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

$$I_2 = I \times \frac{R_1 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

$$I_3 = I \times \frac{R_2 R_1}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

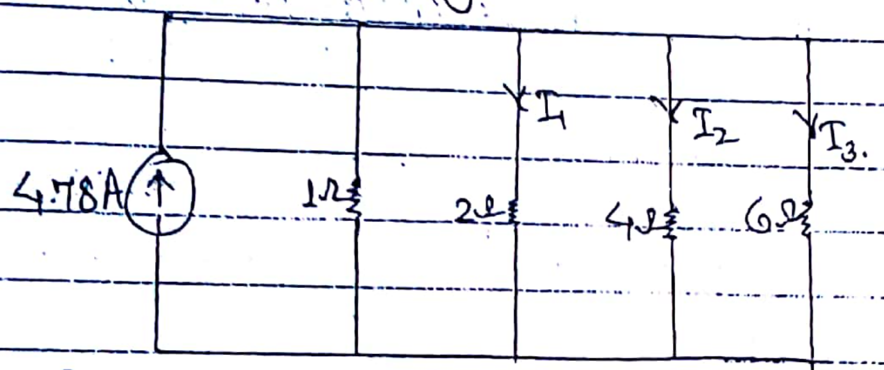
83

Find current through 2Ω, 4Ω and 6Ω using current divider rule.



Solution:-

The above figure can be transformed into:-



$$R_{eq} = \frac{2\times4\times6}{2\times4+2\times6+4\times6} = \frac{48}{44} = 1.09+1 = 2.09\,\Omega.$$

$$\therefore\ I = \frac{V}{R_{eq}} = \frac{10}{2.09} = 4.78\ A.$$

Now,

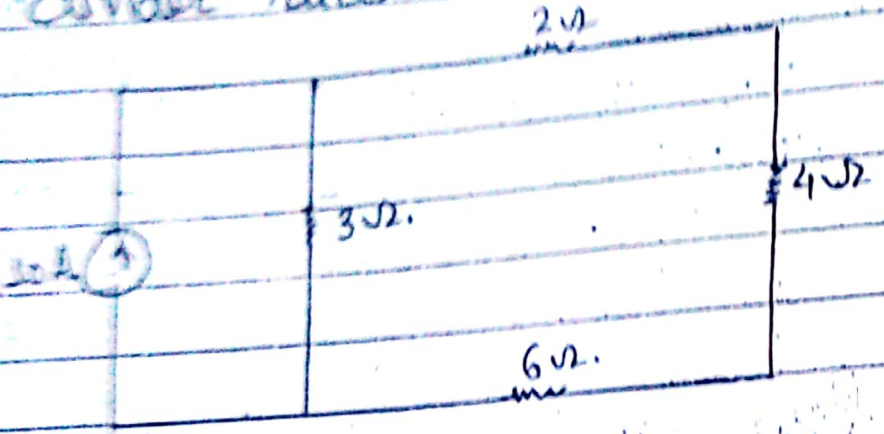
Current through $2\Omega = I_1 = \dfrac{4.78\times4\times6}{2\times4+4\times6+6\times2} = 2.6\ A.$

Current through $4\Omega = I_2 = \dfrac{4.78\times 2\times6}{44} = 1.3\ A$

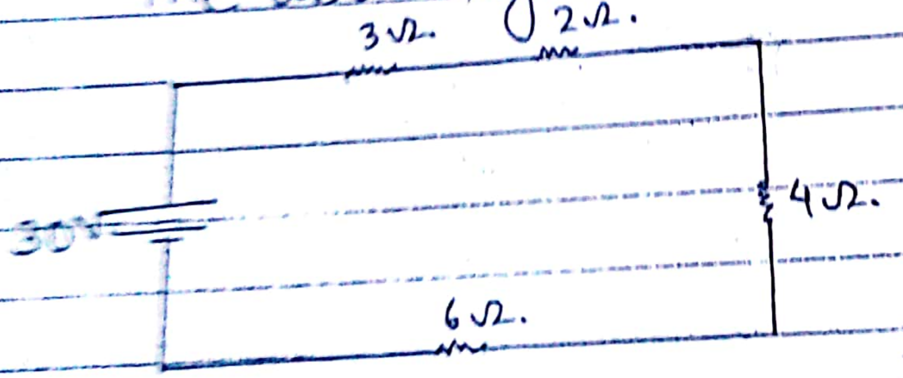
& Current through $6\Omega = I_3 = 4.78\times\dfrac{2\times4}{44} = 0.86\ A.$

84

1. Find voltage across 2Ω, 4Ω and 6Ω using voltage divider rule.



→ Solution:-
The above figure can be arranged as:



$V = IR$
$= 10 \times 3$
$= 30V$

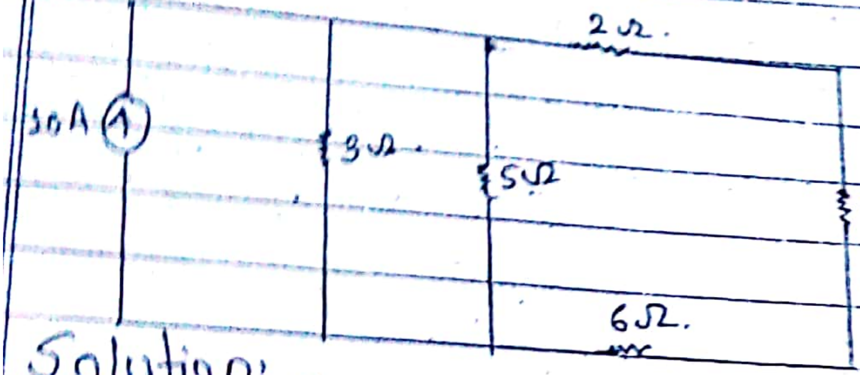
$\therefore$ Voltage across $2\Omega = 30 \times \frac{2}{2+3+4+6} = \frac{60}{15} = 4V$

Voltage across $4\Omega = 30 \times \frac{4}{15} = 8V$

And, voltage across $6\Omega = 30 \times \frac{6}{15} = 12V$

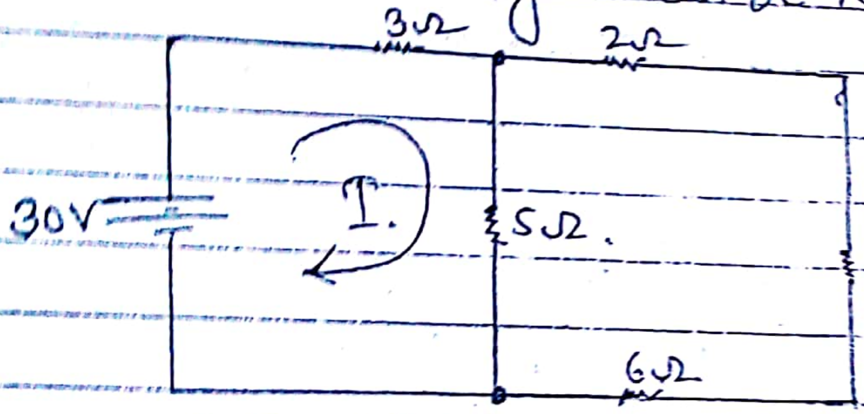
85

Find the voltage across 2Ω, 4Ω and 6Ω using voltage divider rule.



Solution: -

The above figure can be rearranged as:-



$-3I - 5I + 30 = 0$

or, $I = \frac{30}{8}$ A. $\therefore V_5 = \frac{30}{8} \times 5 = \frac{150}{8}$ A.

$\therefore$ V across 2Ω $= \frac{2}{12} \times \frac{150}{8} = 3.125$ V

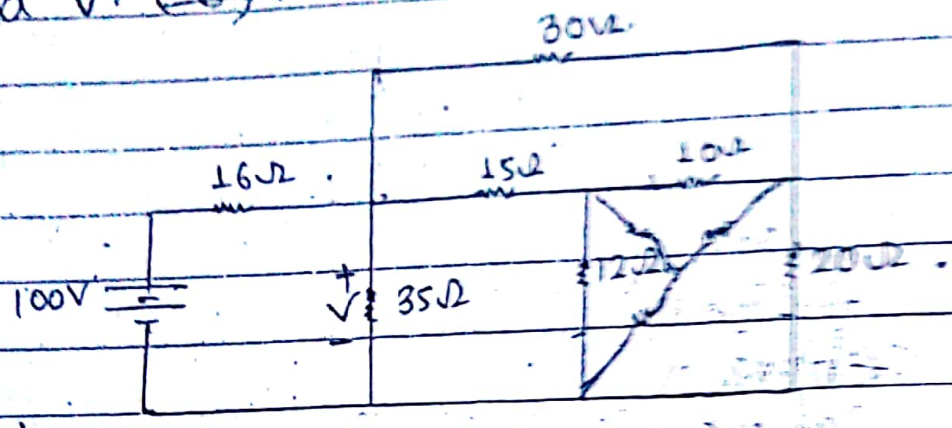
[Because Voltage across 5Ω flows in all resistance]

V across 4Ω $= \frac{150}{8} \times \frac{4}{12} = 6.25$ V.

And,

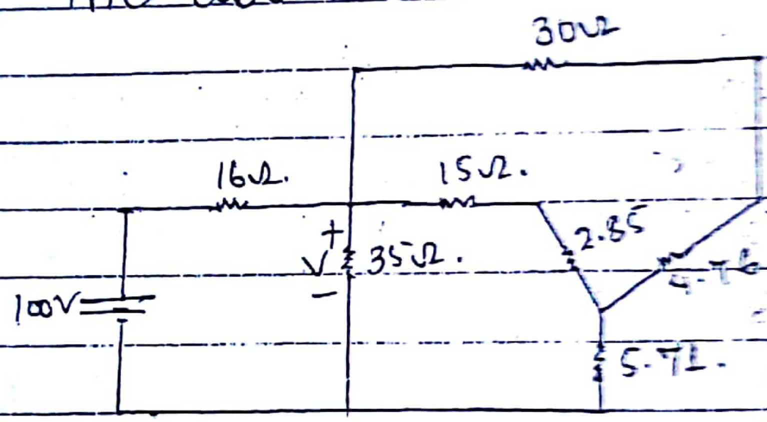
Voltage across 6Ω $= \frac{150}{8} \times \frac{6}{12} = 9.375$ V.

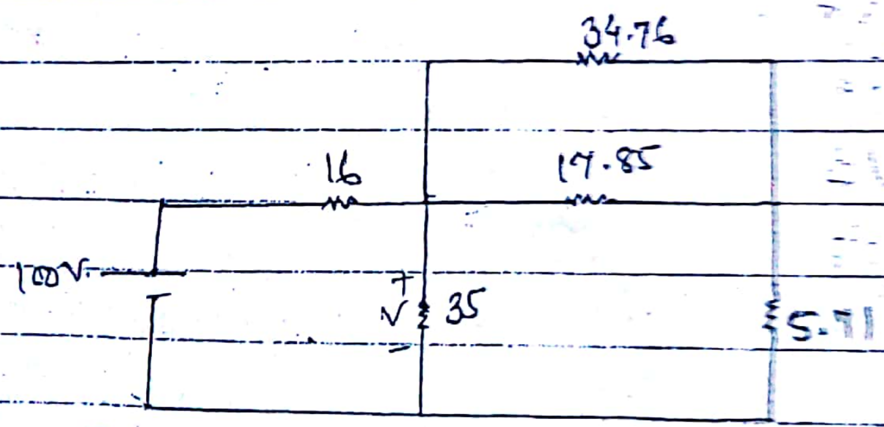
Obtain the equivalent resistance and use it to find source current for the circuit shown below. Also find V. ~~(8)~~. (8)



Solution:-

The above circuit becomes.





Since 34.76 // 17.85, so

$$R_{eq} = \frac{34.76 \times 17.85}{34.76 + 17.85} = \frac{620.466}{52.61} = 11.79$$

and, $11.79 + 5.71$ (series combination)

$= 17.5\,\Omega$.

Again $17.5\,\Omega$ and $35\,\Omega$ are in parallel, so

$$R_{eq} = \frac{17.5 \times 35}{17.5 + 35} = \frac{612.5}{52.5} = 11.66\,\Omega.$$

And, $11.66\,\Omega$ is in series with $16\,\Omega$.

So,

$$R_{eq} = 11.66 + 16 = 27.67\,\Omega.$$

Now,

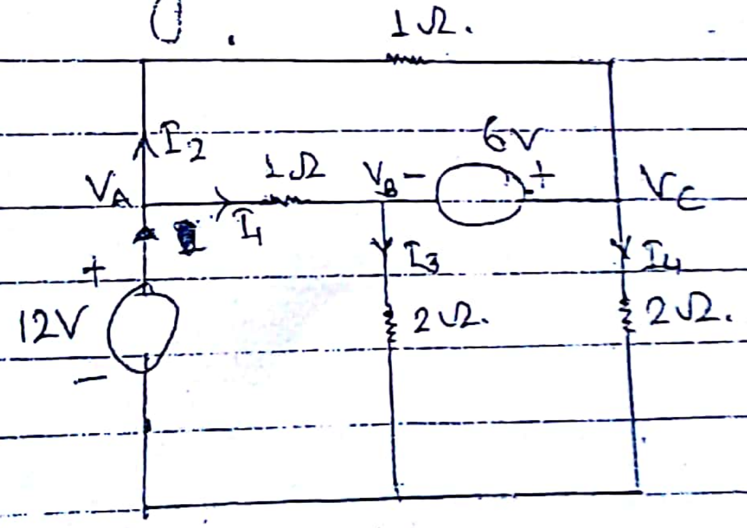
$$I = \frac{V}{R_{eq}} = \frac{100}{27.67} = 3.61\,A.$$

And, ~~$V = I R_{eq} = 37\ 3.61 \times 27.67$~~

$$I_1 = I \times \frac{17.50}{35 + 17.5} = \frac{63.175}{52.5} = 1.2\,A.$$

$$\therefore V = IR = 1.2 \times 35 = 42\,V.$$

Calculate the power absorbed or delivered by 6 source for the network shown in figure using nodal analysis.



Solution:-

In above figure,

$$V_A = 12\,V.$$

$$V_C - V_B = 6\,V$$

At node V_A,

$$-I_1 - I_2 = 0 \quad \ldots (1)$$

90

$$-10I_1 + 10 \times \frac{7}{2} - 10 = 0$$

or, $I_1 = 2.5A$.

And, $I_2 = 2 + 2.5A = 4.5A$.

And,

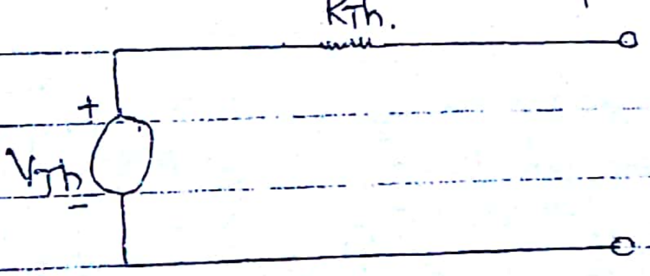
Thevenin's Theorem:-

Statement:-

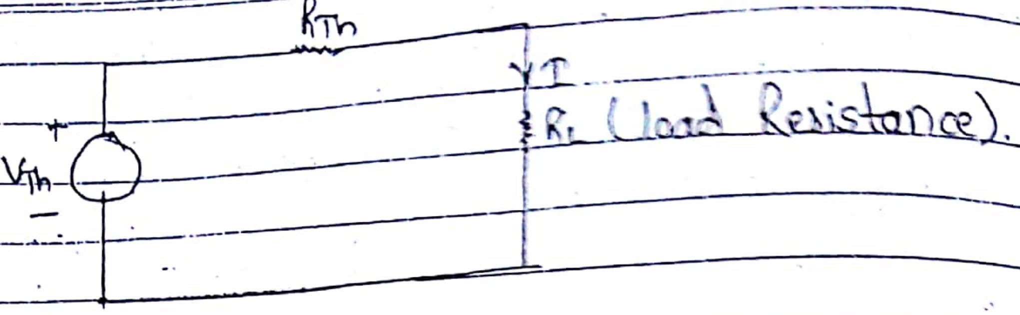
Any linear complex circuit can be converted into a simple circuit which consists of a voltage source (V_{Th} or V_{oc}) and a resistance (R_{Th}) in series with that voltage source. The theorem can be applied to electric circuits to find the current, voltage and power in a particular elements by converting the circuit into the simple voltage source and resistance circuit which is known as Thevenin's equivalent circuit. The circuit can be converted to Thevenin's equivalent circuit by applying following steps:

or, The steps to Thevenize a circuit are:-

1. Remove temporarily the resistance through ~~wich~~ which current, voltage and power is required.
2. Find the voltage V_{Th} or V_{oc} (Thevenin's voltage or open circuit) voltage across the terminals from where the element was removed.
3. Find the equivalent resistance R_{Th} across the same terminal by short circuiting the voltage source and opening the current source.
4. Draw the Thevenin's equivalent circuit as given below.



5) Connect the removed element at the terminals and find current, voltage and power as required.

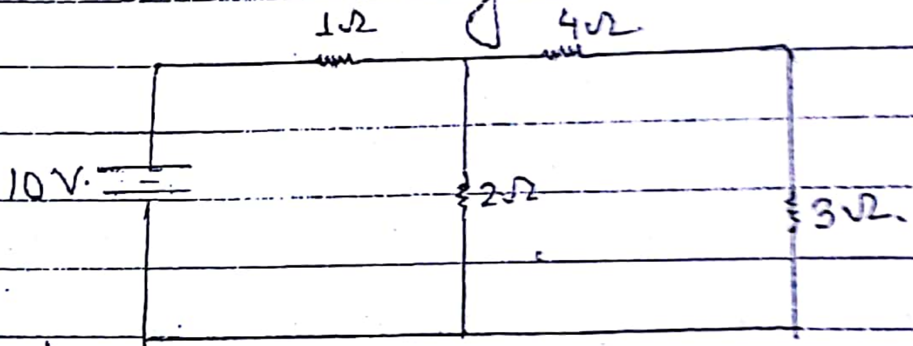


$I = \dfrac{V_{Th}}{R_{Th}+R_L}$

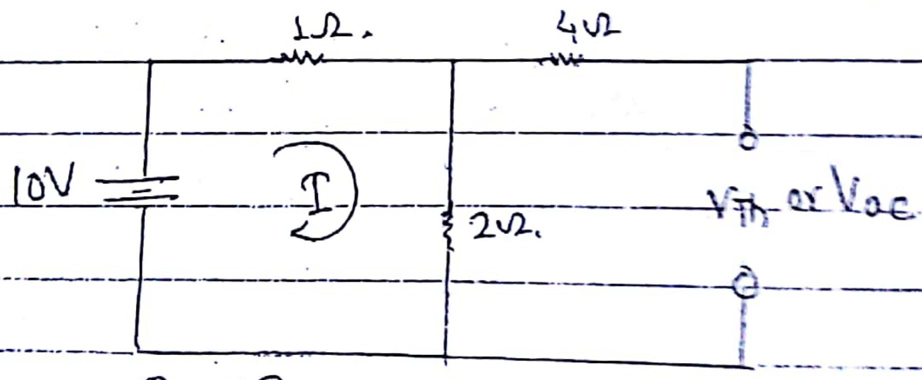
$V_L = IR_L$

$P_L = I^2R_L$

Find the current, voltage and power across the 3Ω resistance using Thevenin's theorem.



Solution:-



$-I - 2I + 10 = 0$

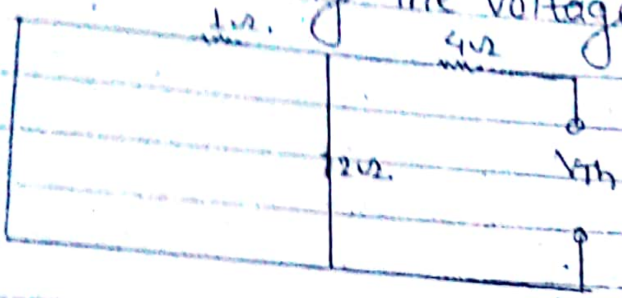
or, $I = 10/3 = 3.33A$.

∴ Voltage across 2Ω = IR = 3.33×2 = 6.66V.

∵ V_{Th} is the voltage across 2Ω,

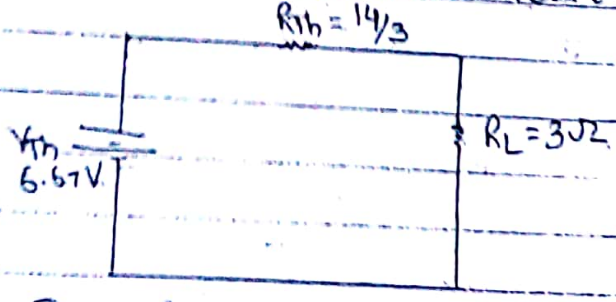
So, $V_{Th} = 6.66V$.

Now, short circuiting the voltage source, we get,



$$R_{Th} = 1//2 + 4 = \frac{14}{3}\,\Omega.$$

And, The thevenin's circuit becomes,



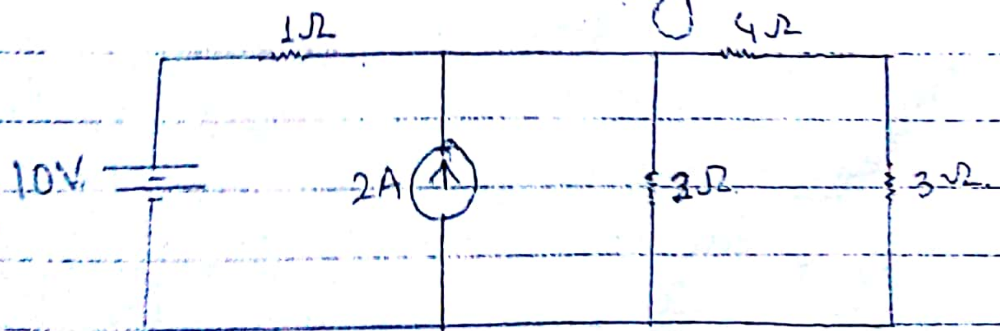
$$\therefore I = \frac{6.66}{4.66+3} = 0.86\,A$$

Voltage across $3\Omega = IR = 0.86 \times 3 = 2.58\,V$

And,

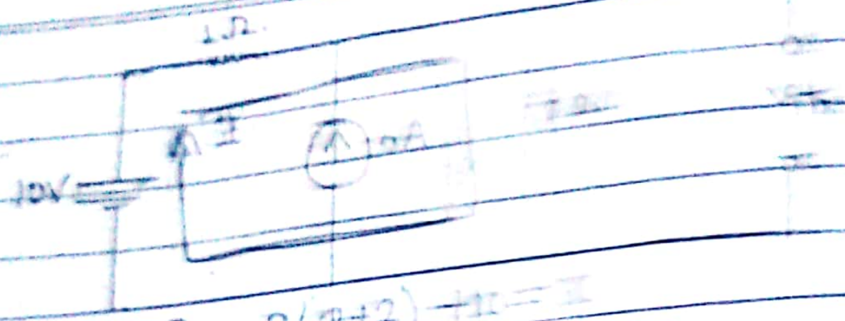
$$P = I^2R = (0.86)^2 \times 3 = 2.21\,W.$$

Q.2. Find I, V and P using Thevenin's theorem in 3Ω.



→ Solution:-

Removing temporaling the 3Ω resistance, we get,



$-I - 2(I+2) + 10 = I$

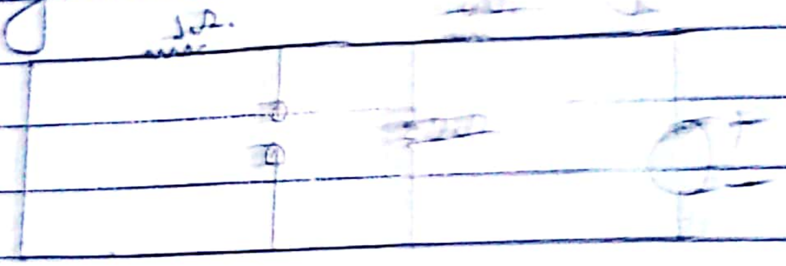
or, $-3I + 6 = I$

or, $I = 2A$

∴ Voltage across $2\Omega = 2(I+2) = 2\times4 = 8V$.

∴ $V_{Th} = 8V$

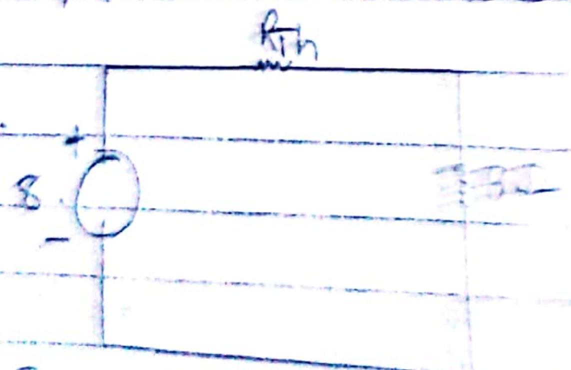
Now, short circuiting voltage source and opening current source we get,



And,

$R_{Th} = 1//2 + 4 = \frac{14}{3} =$

And, the Thevenin's circuit developed into

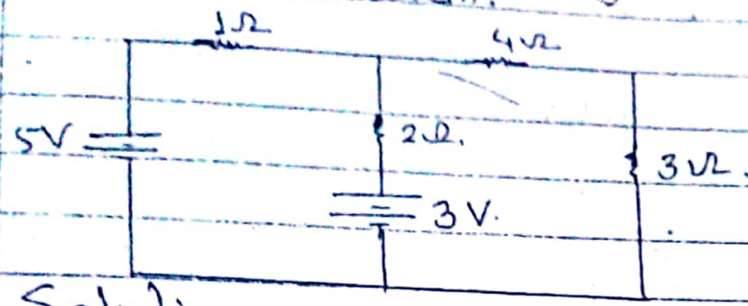


∴ $I = \frac{8}{4.66+3} = 1.04A$

$V = IR = 1.04\times3 =$

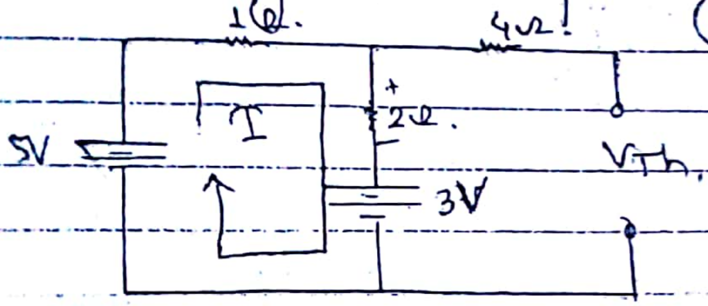
& $P = I^2R = (1.04)^2\times3 =$

Q. Find the current, voltage and power across 3Ω using Thevenin's theorem.



→ Solution:-

Removing 3Ω temporarily, we get,



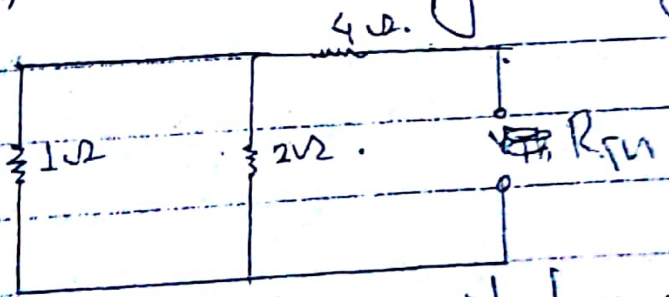
$$5 - I - 2I - 3 = 0$$

or, $I = 2/3\ A.$

and, Voltage across 2Ω = $2 \times \frac{2}{3} = \frac{4}{3}$ V.

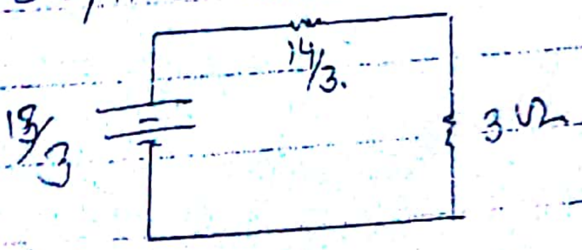
and $V_{Th} = \frac{4}{3} + 3 = \frac{13}{3}$

Now, short circuiting voltage source,



$R_{Th} = 1 \| 2 + 4 = \frac{14}{3}\ \Omega.$

So, the thevenin's circuit becomes.

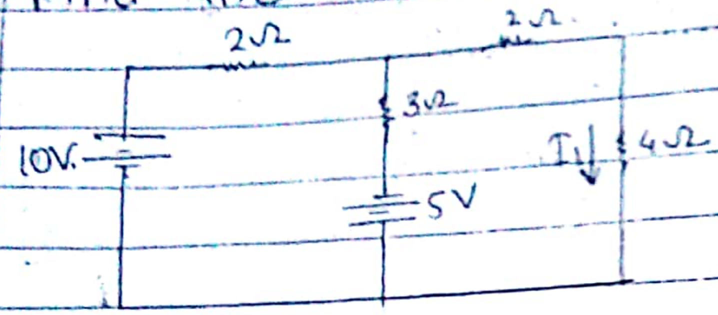


I =

V =

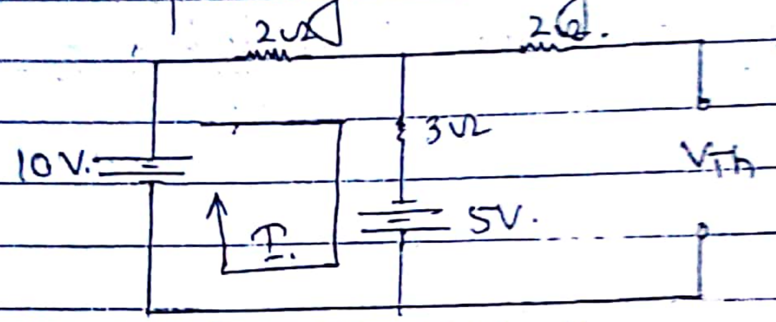
P =

Q. Find the load current I_L using Thevenin's theorem



→ Solution:-

Temporaliry removing the resistance 4Ω,



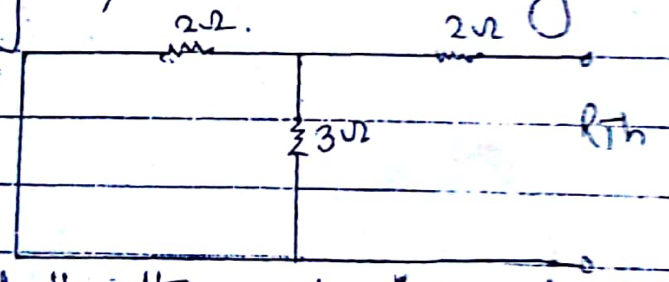
$$-2I - 3I + 5 + 10 = 0$$

or, $I = 3A$.

$\therefore V = IR = 3\times3 = 9V.$

$V_{Th} = 9 - 5 = 4V.$

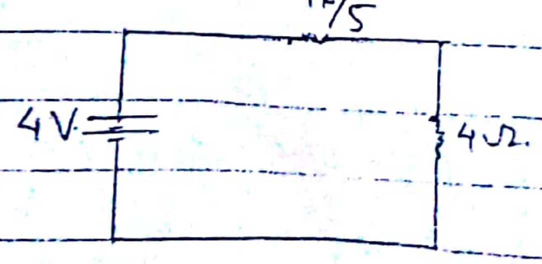
Again, short circuiting voltages sources, we get,



$$\therefore R_{eq} = 2 // 3 + 2$$

$$= \frac{2\times3}{2+3} + 2 = \frac{16}{5}$$

And, the thevenin's circuit becomes

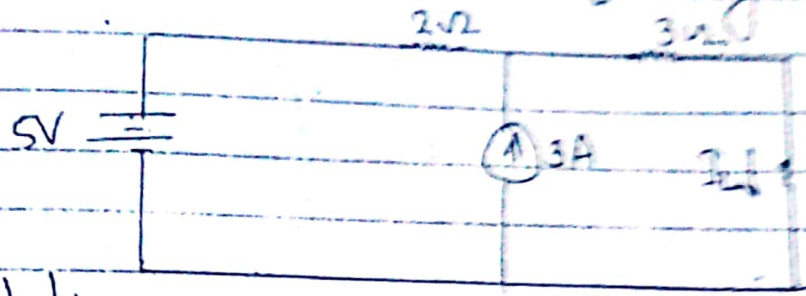


$$I_L = \frac{V_{Th}}{R_{Th}+R_L} = \frac{4}{3.2+4} = 0.55A$$

$$V_L = I R_L = 0.55\times4 = 2.22V$$

& $P = I^2R = 1.21W.$

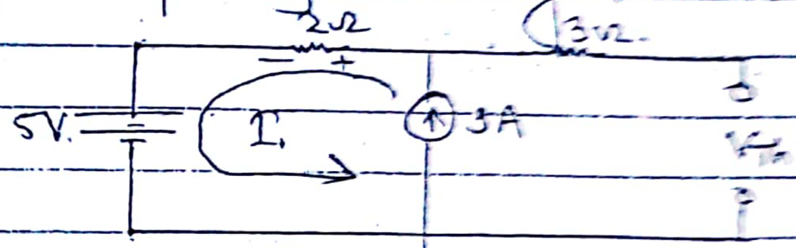
Q. Find the load current I_L using Thevenin's Theorem



Vol. enter = high potential
Vol. exit = low pot. (-ve)

> Solution:-

Temporaliry removing load resistance, we get.

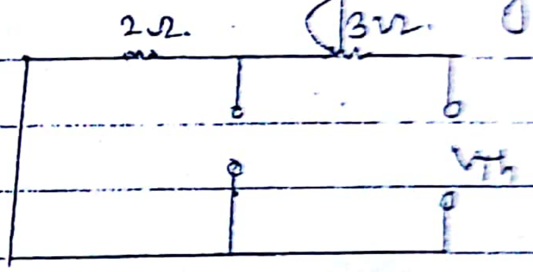


$I = 3A.$

$\therefore V = IR = 3 \times 2 = 6V.$

$\therefore V_{Th} = 6 + 5 = 11V$

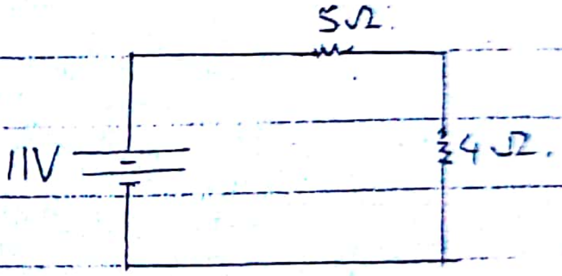
Short circuiting voltage source and opening current,



$R_{Th} = 2 + 3$
$= 5\Omega.$

And,

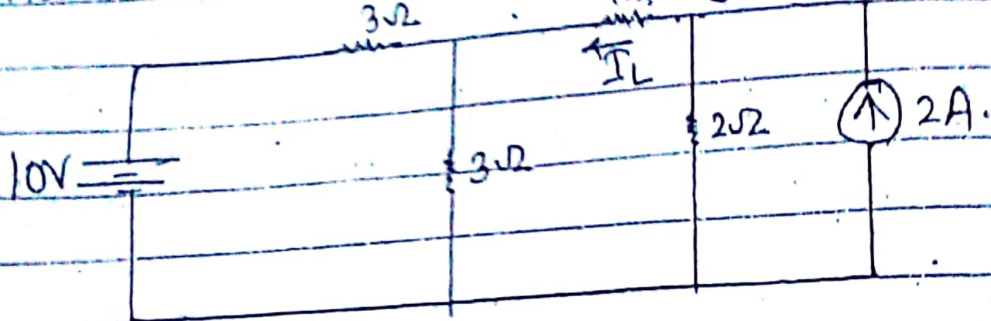
The Thevenin's circuit becomes



$\therefore I = \frac{11}{5+4} = \frac{11}{9} = 1.22A$

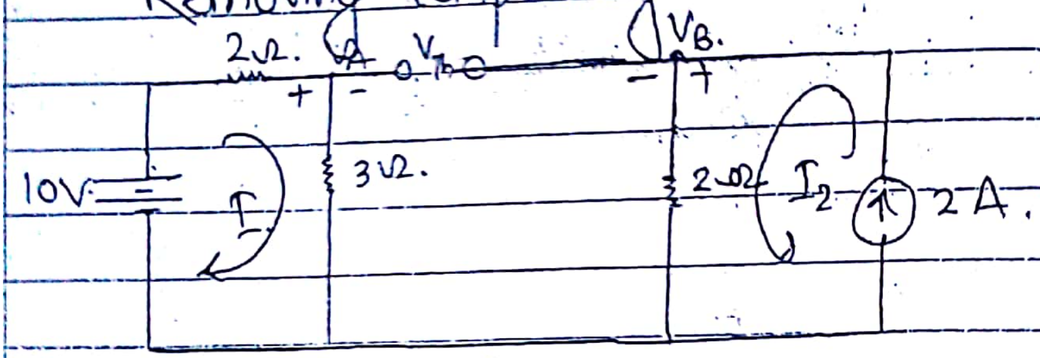
$V = IR_L = 1.22 \times 4 = 4.88V$

Find the load current I_L using Thevenin's theorem.



Solution:-

Removing temporaliry the load resistance, we get,



$V_{Th} = V_A - V_B$.

$-2I - 3I + 10 = 0$

or, $I = 2A$

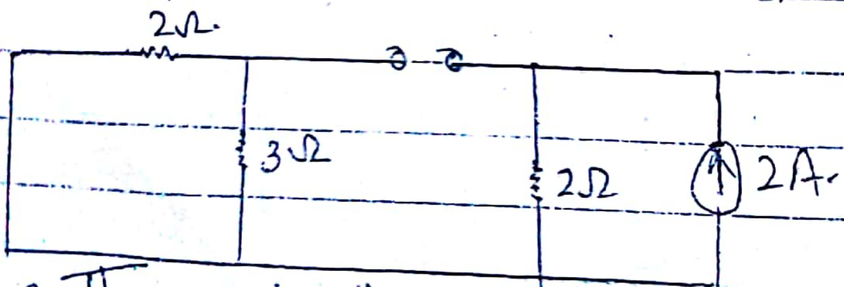
$I_2 = 2A$.

$\therefore V_A = IR = 3 \times 2 = 6V$.

$V_B = 2 \times 2 = 4V$.

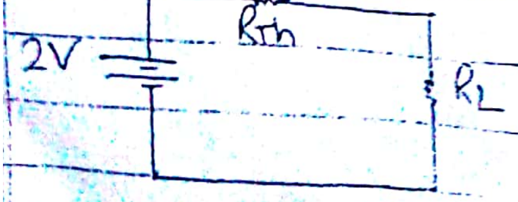
$\therefore V_A - V_B = 2V = V_{Th}$.

Now, the circuit becomes,



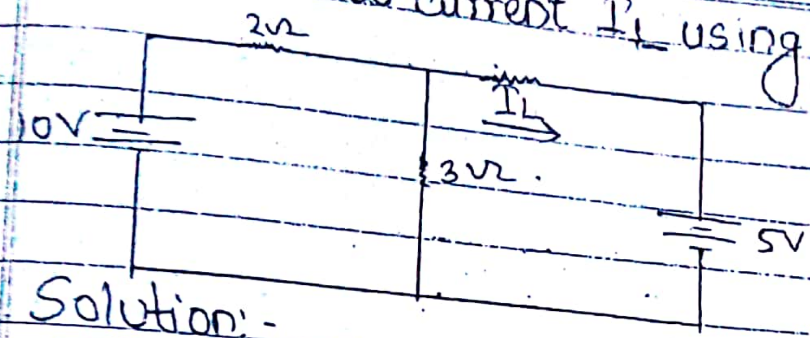
$R_{Th} = \frac{3 \times 2}{3+2} + 2 = \frac{16}{5}$

The Thevenin's theorem becomes



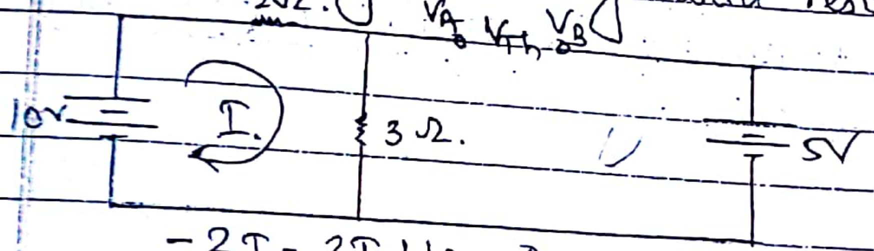
$\therefore I_L = \frac{2}{3.2+4} = 0.27A$.

Q. Find the load current I_L using Thevenin's theorem.



→ Solution:-

Temporarily removing load resistance, we get,



$$-2I - 3I + 10 = 0$$

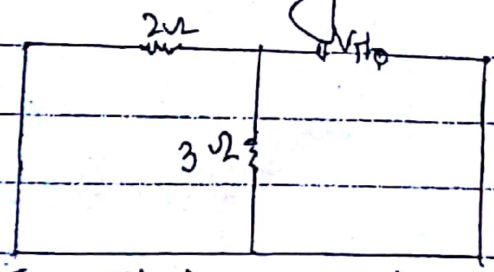
or, $I = 2A$.

$\therefore V = IR = 2 \times 3 = 6V$.

$\therefore V_{Th} = V_A - V_B = 6 - 5 = 1V$

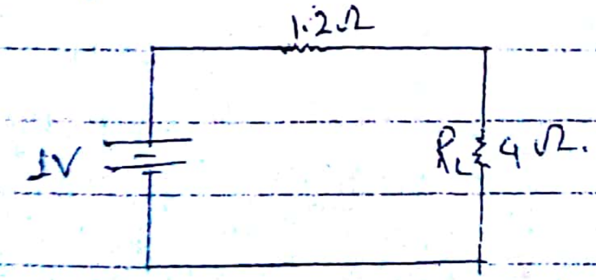
And, ~~Removin~~

short circuiting voltage source, we get,



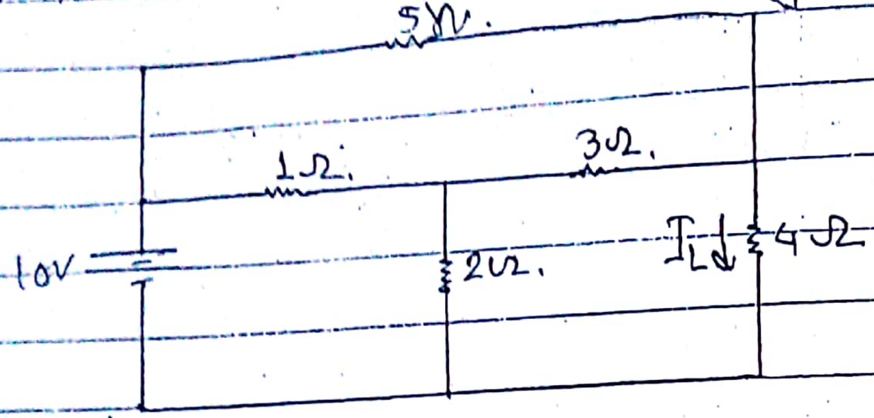
$\therefore R_{eq} = 2 // 3 = \frac{6}{5}$

And, The Thevenin's circuit becomes,



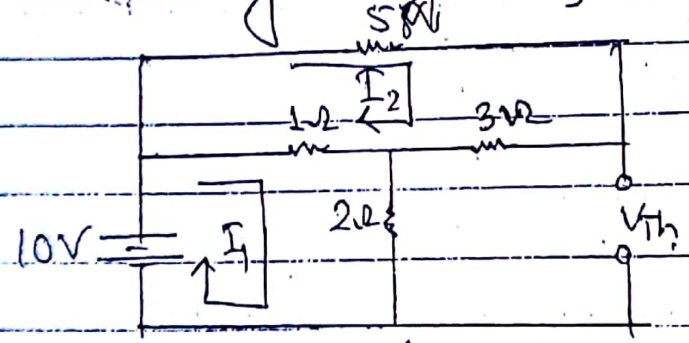
$$\therefore I_L = \frac{1}{1.2+4} = \frac{1}{5.2} = 0.19A.$$

Find the load current I_L using Thevenin's theorem.



Solution:-

Removing Temporalily the current resistance, we get,



$-5I_2 - 3I_2 - 1(I_2 - I_1) = 0$

or, $-9I_2 + I_1 = 0$

or, $I_1 = 9I_2$ .... (1)

And,

$-1(I_1 - I_2) - 2I_1 + 10 = 0$

or, $-I_1 + I_2 - 2I_1 + 10 = 0$

or, $-3I_1 + I_2 + 10 = 0$

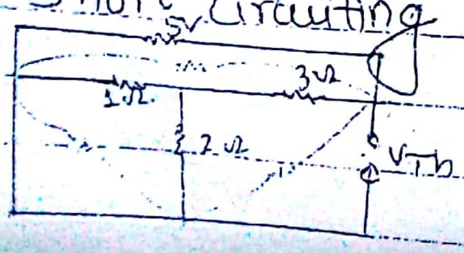
or, $-3 \times 9I_2 + I_2 + 10 = 0.$

$\therefore$ $I_2 = 10/26 = 0.38$ A.

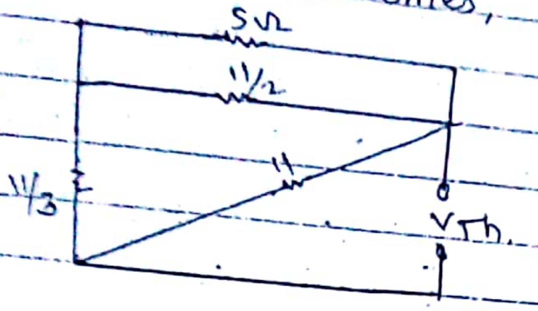
& $I_1 = 9 \times 0.38 = 3.46$ A.

$\therefore$ $V_{Th} = I_1 R = 2 \times 3.46 = 6.92$ V.

~~Remo~~ Short circuiting voltage source, we get,



The circuit becomes,



$3 \times 2 + 2 \times 1 + 1 \times 3 = 6+2+3 = 11$

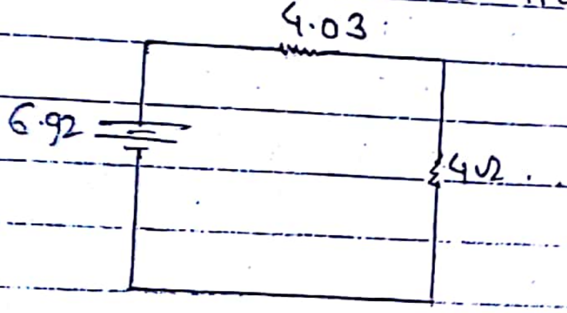
$5.2 // 5 = 2.69\,\Omega$

& $2.69 + 11/3 = 6.36\,\Omega$

& $6.36 // 11$

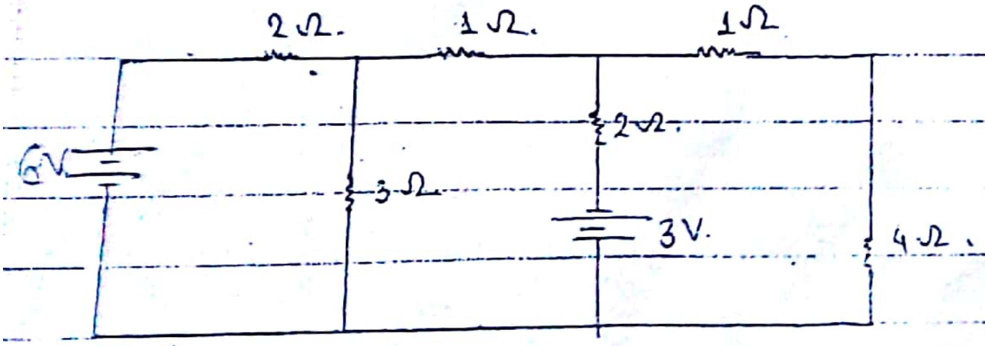
$\therefore R_{Th} = 4.03\,\Omega$.

And, the Thevenin's theorem becomes,



$\therefore I_L = \dfrac{6.92}{4.03+4} = 0.86\,A$.

Find current, voltage and power through 4Ω using Thevenin's theorem.



Solution:-

Temporaling removing the 4Ω resistance, we get

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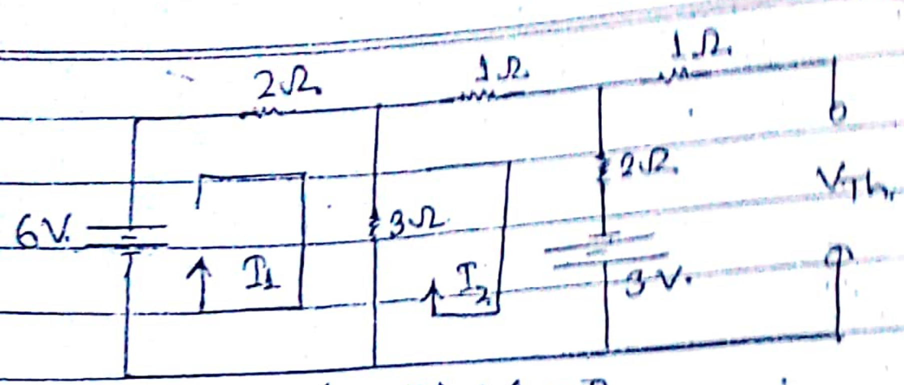


$-2I_1 - 3(I_1 - I_2) + 6 = 0$

or, $-2I_1 - 3I_1 + 3I_2 + 6 = 0$

or, $-5I_1 + 3I_2 + 6 = 0$ ...(1)

$-I_2 - 2I_2 + 3 - 3(I_2 - I_1) = 0$

or, $-3I_2 + 3 - 3I_2 + 3I_1 = 0$

or $-6I_2 + 3I_1 + 3 = 0$ ...(1)

On solving eq^n(1) & (2), we get,

$3I_1 - 6I_2 + 3 = 0$

$-5I_1 + 3I_2 + 6 = 0 \} \times 2$

$3I_1 - 6I_2 + 3 - 10I_1 + 6I_2 + 12 = 0$

or, $-7I_1 = -15$

or, $I_1 = 15/7 = 2.14A.$

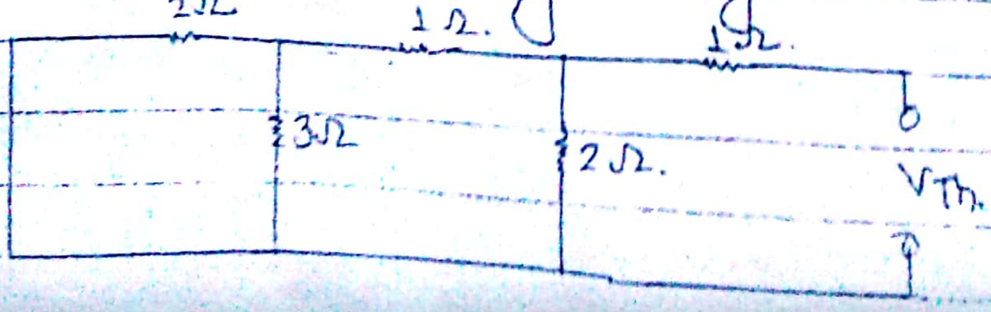
& $-5 \times \frac{15}{7} + 3I_2 + 6 = 0$

or, $I_2 = 1.57A.$

$\therefore$ V across $2\Omega = 2 \times 1.57 = 3.14V.$

$\therefore V_{Th} = 3.14 - 3 = 0.14 V.$

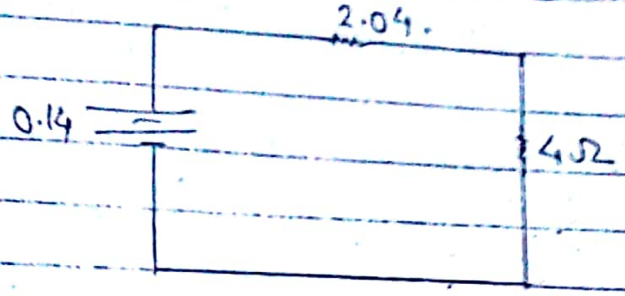
And, short circuiting voltage source, we get,



$\therefore R_{Th} = \frac{43}{21}$

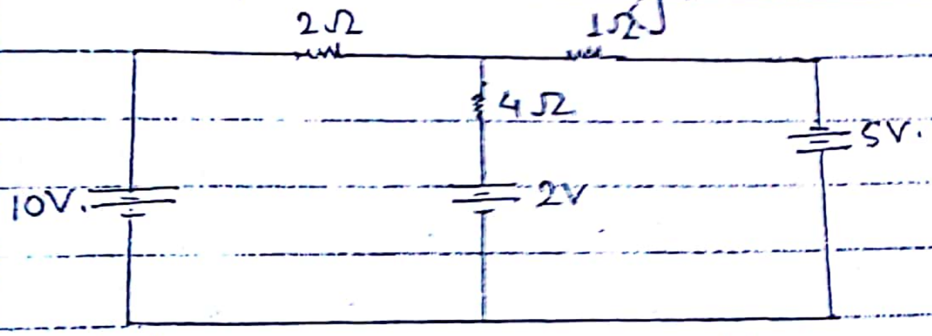
$= 2.04\Omega$

And The Thevenin's circuit becomes.



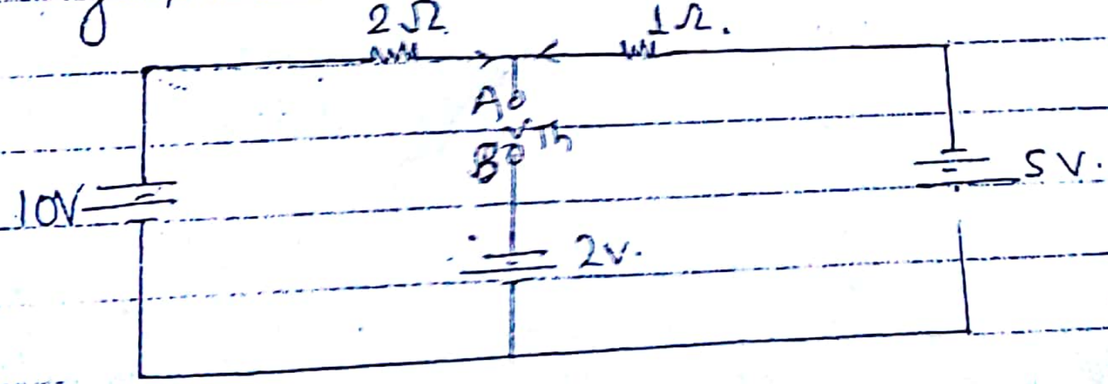
∴ $I_L = \frac{0.14}{2.04+4} = 0.023A.$

Find I, V and P through 4Ω using Thevenin's theor



Solution:-

Temporarily removing the 4Ω resistance and findin voltage:



Here, $V_B = 2V.$

$I_1 + I_2 = 0$ ....(1)

or, $I_1 = \frac{10-V_A}{2}$ $\quad I_2 = \frac{-5-V_A}{1}$

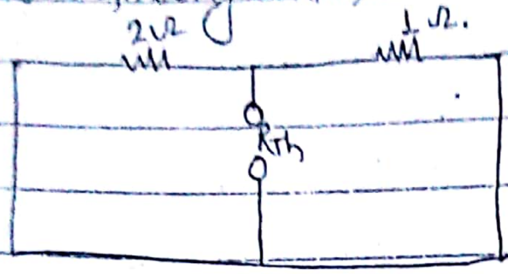
∴ $\frac{10-V_A}{2} - 5 - V_A = 0$

or, $10 - V_A - 10 - 2V_A = 0$

or, $V_A = 0$.

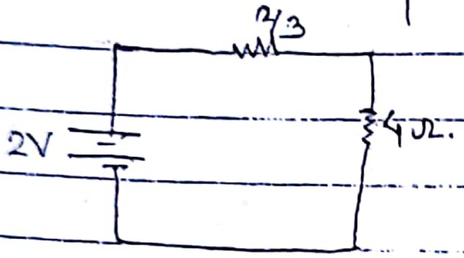
$\therefore V_{Th} = 2 - 0 = 2\Omega$.

And for finding R_{Th},



$R_{Th} = 1 // 2 = \frac{2}{3}\Omega$.

$\therefore$ The Thevenin's equivalent circuit becomes

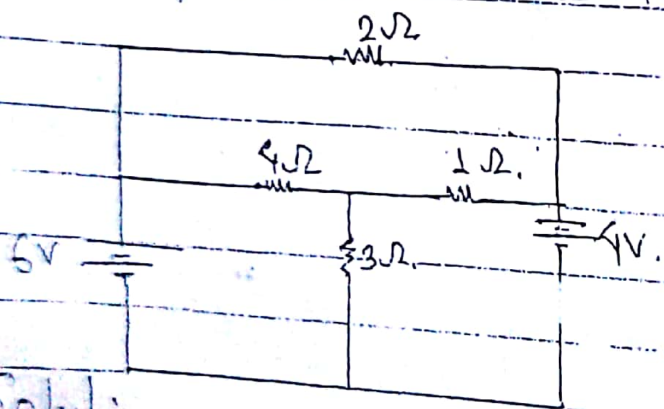


$\therefore I_L = \frac{2}{0.66+4} = 0.42A$.

$V = IR_L = 0.42 \times 4 = 1.71V$

& $P = I^2R_L = 0.70W$.

Find I, V and P across 4Ω using Thevenin's Theo



Solution:-

Since we have to find across 4Ω, so temporarily removing 4Ω and calculating Thevenin's equivalent voltage

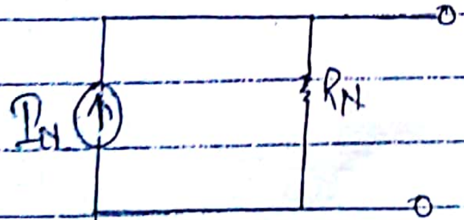
Norton's Theorem

Statement

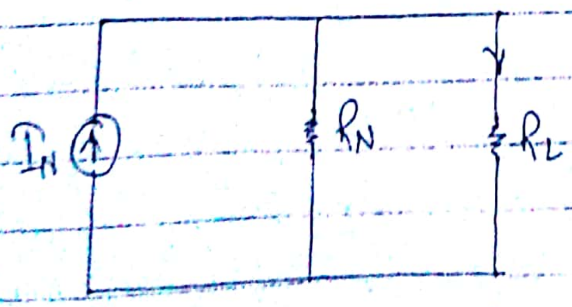
A linear complex circuit consisting of several elements and sources can be converted into a simple circuit which consists of a current source (I_N or I_{sc}) and a resistance (R_N) in parallel with that current source.

The steps to nortorize a given circuit are as follows :-

- Short circuit the elements through which current, voltage or power is to be found out.
- Find the current I_N or I_{sc} (Norton's current or short circuit current) through the shorted branch.
- Find the equivalent resistance R_N across the terminals by short circuiting all voltage ~~sh~~ source and opening all current sources.
- Draw the Norton's equivalent circuit as



Connect the load resistance R_L in parallel with R_N and find current,

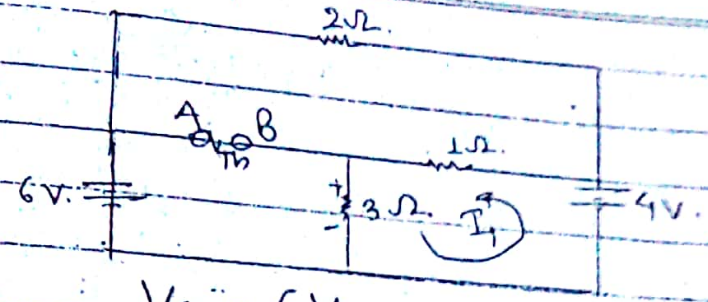


By current dividing rule,

$$I = I_N \times \frac{R_N}{R_N + R_L}$$

$$V \text{ across } R_L = I R_L$$

$$P \text{ across } R_L = I^2 R_L$$



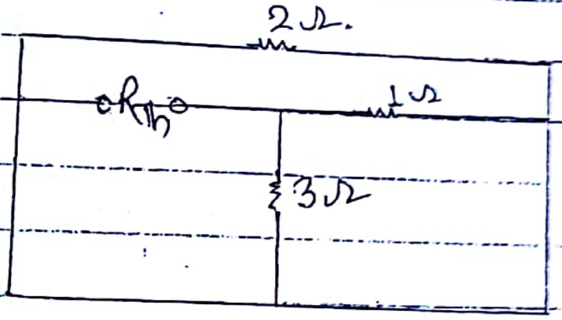
$V_A = 6V.$

$-I_1 - 3I_1 + 4 = 0$

or, $I_1 = 1.$

and, $V = IR = 3 \times 1 = 3V.$

$\therefore V_{Th} = 6 - 3 = 3V.$

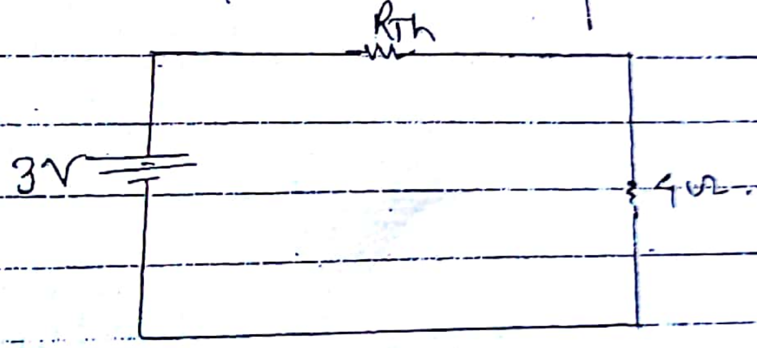


$\therefore R_{Th} = 3 // 1 + 2$

$= \frac{3}{4} + 2 = \frac{11}{4} \Omega$

And,

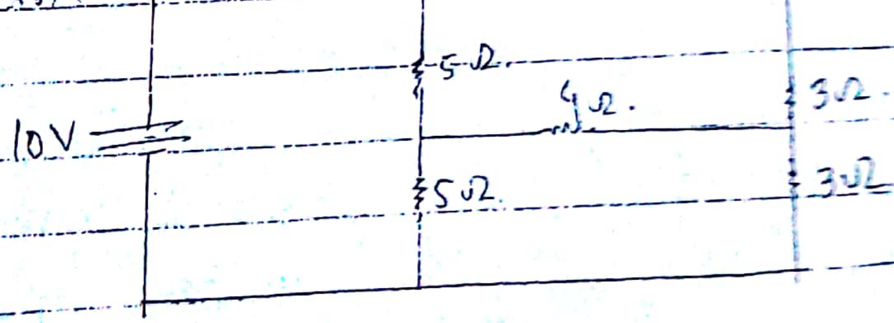
~~Ea~~ Thevenin's equivalent circuit becomes



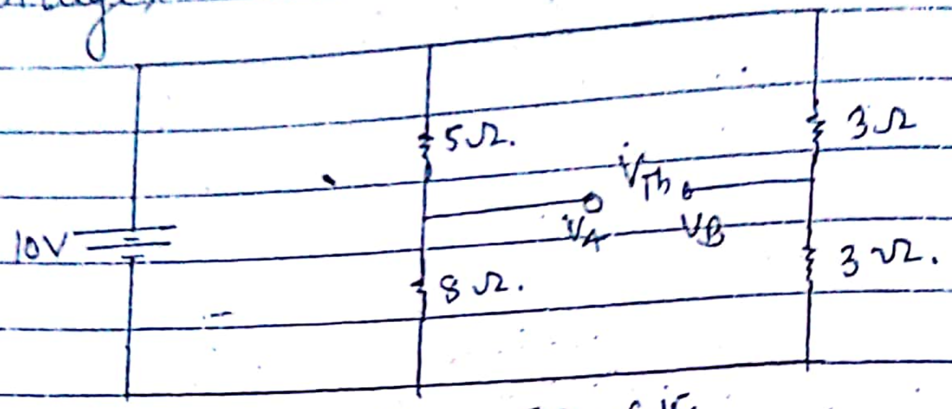
$\therefore I = \frac{3}{2.75 + 4} = 0.44 A.$

$V = IR_L = 0.44 \times 4 = 1.77 V$

Find I, V and power across 4Ω using Thevenin's theorem.



→ Solution:-
Temporality removing 4Ω and finding thevenin's voltage.

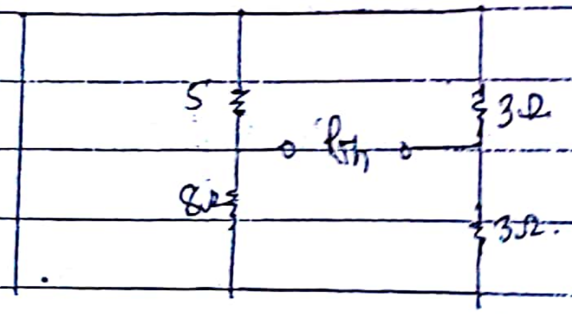


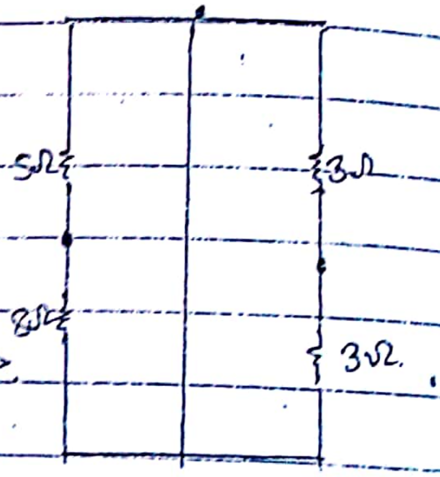
$V_A = 10 \times \frac{8}{8+5} = \frac{80}{13} = 6.15$

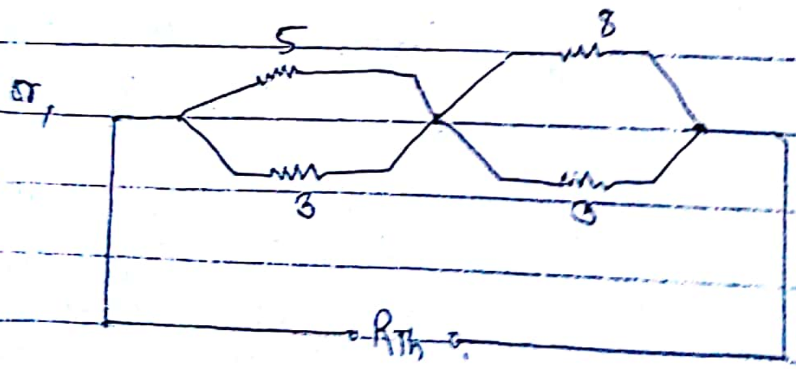
$\therefore V_{Th} = 6.15 - 5 = 1.15V$

$V_B = 10 \times \frac{3}{6} = 5V$

Now, to find R_{Th},

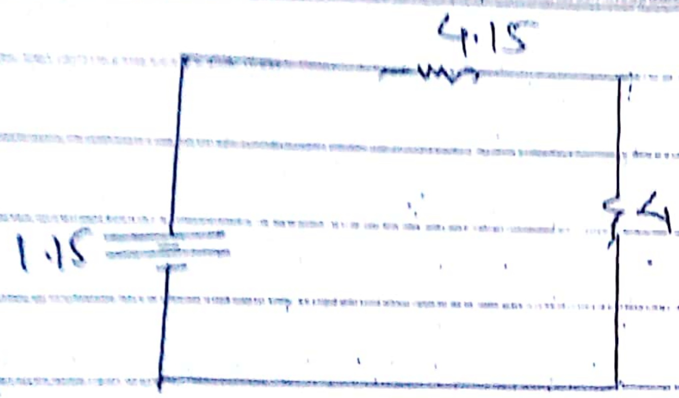






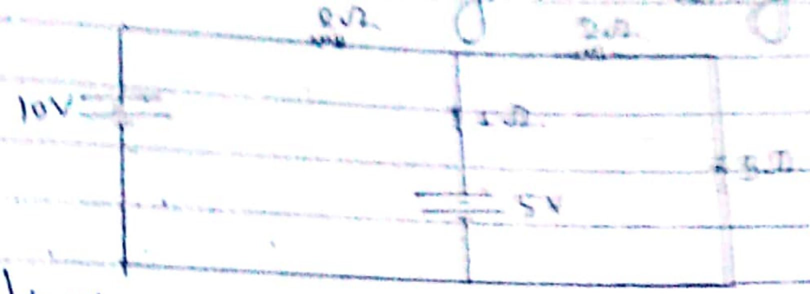
$\therefore R_{Th} = \frac{5 \times 3}{5+3} + \frac{8 \times 3}{8+3} = \frac{15}{8} + \frac{24}{11} = 1.875 + 2.18 = 4.15\Omega$

And, the thevenin's equivalent circuit becomes,



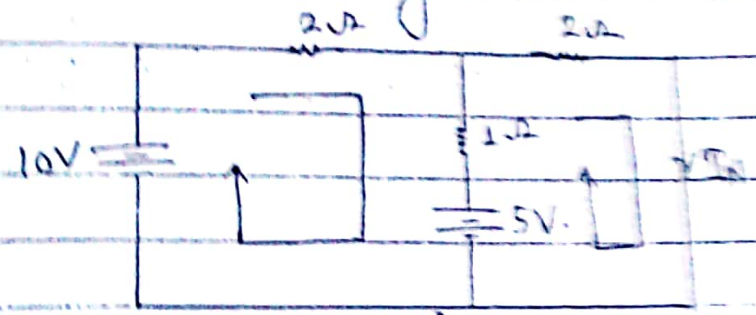
$$I = \frac{1.15}{8.15} = 0.14\,A.$$

Find the current through 3Ω using Norton's Theorem.



Solution:-

Sort shorting 3Ω resistance we get,



$-2I - (I - I_N) - 5 + 10 = 0$

or, $-2I - I + I_N + 5 = 0$

or, $-3I + I_N + 5 = 0$ ... (1)

Also,

$-2 \times I_N + 5 - 1(I_N - I) = 0$

or, $-2I_N + 5 - I_N + I = 0$

or, $-3I_N + I + 5 = 0$ ... (2).

Solving eq^n (1) & (2), we get,

$-3I + I_N + 5 = 0$

$I - 3I_N + 5 = 0 \quad \} \times 3$

$I_N - 9I_N + 5 + 15 = 0$

or, $-8I_N = -20$

$\therefore I_N = 5/2\ A = 2.5\ A$

And,

To find resistance we have,

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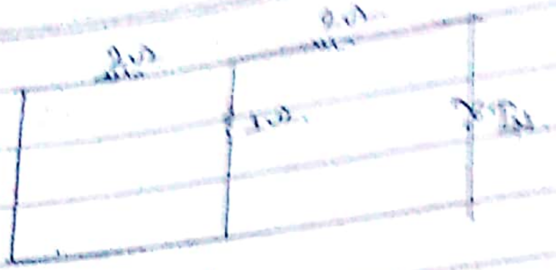
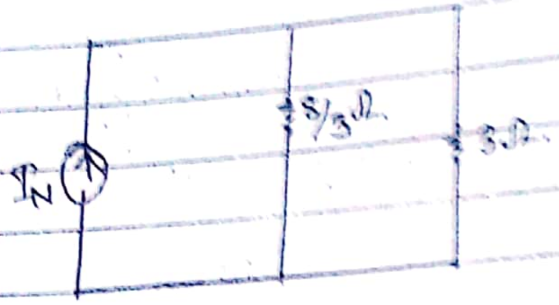


$\therefore R_N = (2 \| 1) + 2$
$= 8/3\ \Omega$

And, the final circuit becomes

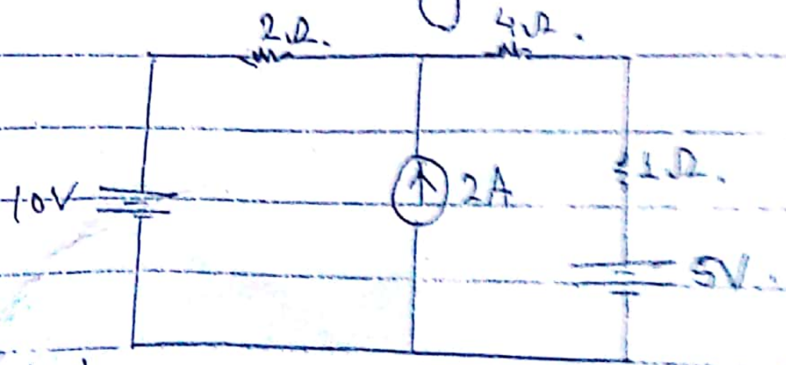


$\therefore I = \dfrac{5/2 \times 8/3}{8/3 + 3} = \dfrac{20}{3} \times \dfrac{3}{17} = \dfrac{20}{17}\ A$

$V = IR_L = \dfrac{20}{17} \times 3 = \dfrac{60}{17}\ V.$

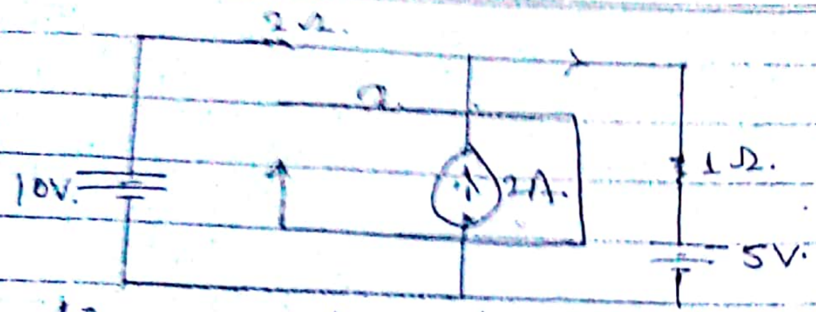
and $P = I^2 R_L = \left(\dfrac{20}{17}\right)^2 \times 3 = 4.15W.$

Find the value of current, voltage and power across 4Ω using Norton's theorem.



Solution:-

Sorting 4Ω to find current, we get,



Using mesh analysis,

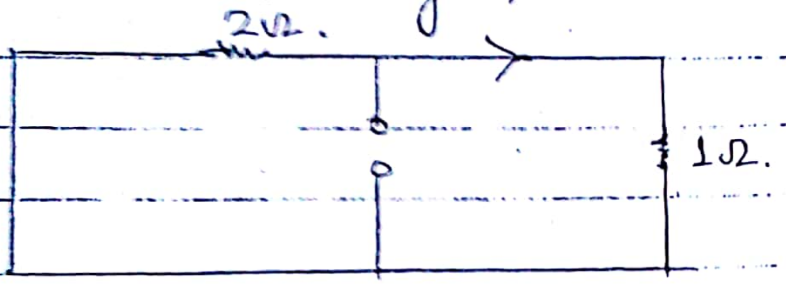
$-2I - (I+2) - 5 + 10 = 0$

or, $-3I = -3$

or, $I = 1$

$\therefore I_N = 2 + I = 3A$

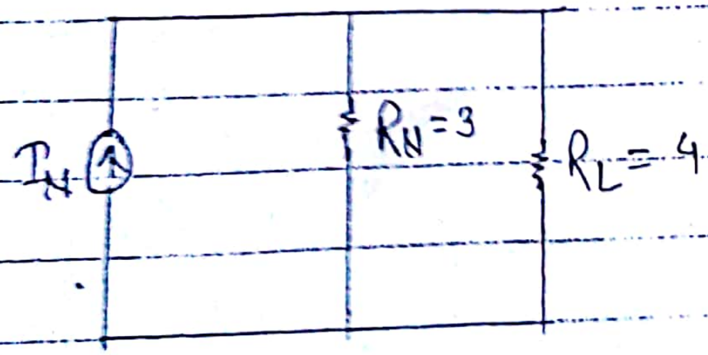
Now, short circuiting voltage source and opening current source, we get,



$R_N = 3\Omega$

And,

Norton's equivalent circuit becomes,

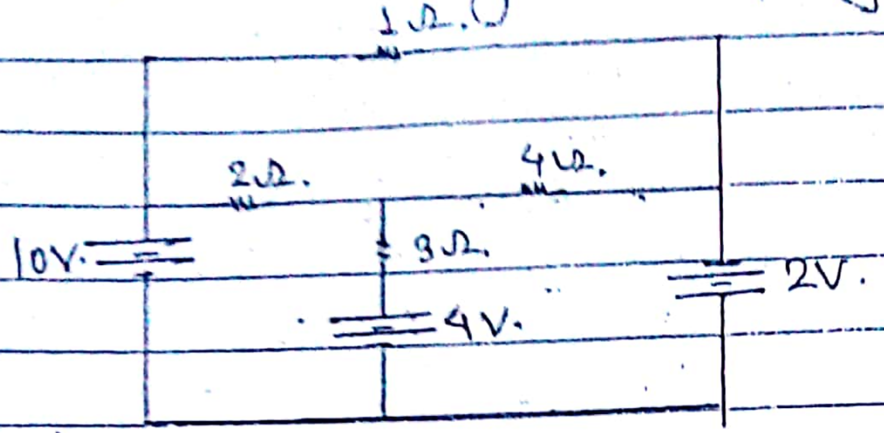


$$\therefore I = 3 \times \frac{3}{3+4} = \frac{9}{7} A$$

$$V = IR_L = \frac{9}{7} \times 4 = \frac{36}{7} V$$

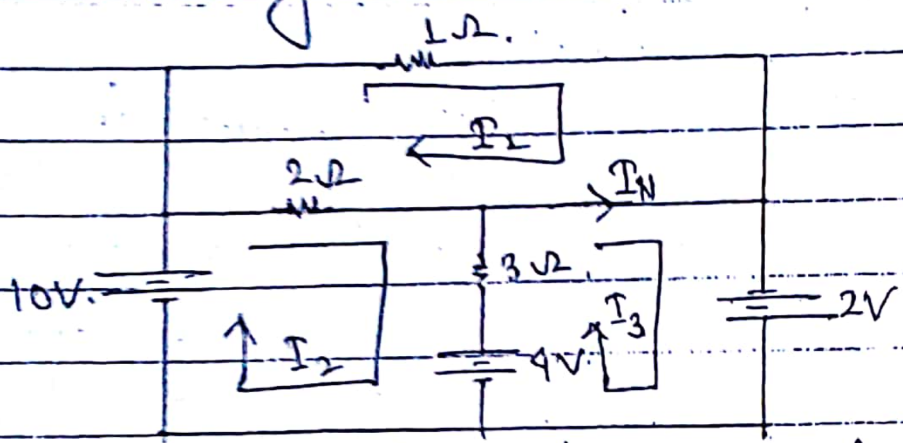
$$\& \ P = I^2 R_L = \frac{81}{49} \times 4 = 6.61 W$$

Find current through 4Ω using Norton's theorem



Solution:-

Shorting 4Ω resistance, we get,



Using mesh analysis, we get,

$-I_1 - 2(I_1 - I_2) = 0$

or, $-3I_1 = -2I_2$

or, $I_1 = \frac{2}{3} I_2$ ... (1)

In 2nd mesh, we get,

$-2(I_2 - I_1) - 3(I_2 - I_3) - 4 + 10 = 0$

or, $-2I_2 + 2I_1 - 3I_2 + 3I_3 + 6 = 0$

or, $-5I_2 + 2I_1 + 3I_3 + 6 = 0$

or, $-5I_2 + 2 \times \frac{2}{3} I_2 + 3I_3 + 6 = 0$

or, $-11I_2 + 9I_3 + 18 = 0$ ... (2)

And,

$-3(I_3 - I_2) + 2 + 4 = 0$

or, $-3I_3 + 3I_2 + 6 = 0 \quad \ldots (3)$

On solving eqⁿ (2) + (3), we get,

$$-3I_3 + 3I_2 + 6 = 0 \} \times 3$$
$$9I_3 - 11I_2 + 18 = 0$$

$$9I_2 + 18 - 11I_2 + 18 = 0$$
or, $-2I_2 = -36$

or, $I_2 = 18\,A$.

and, $I_1 = \frac{2}{3} I_2 = \frac{2}{3} \times 18 = 12\,A$.

and, $-3I_3 + 3\times18 + 6 = 0$

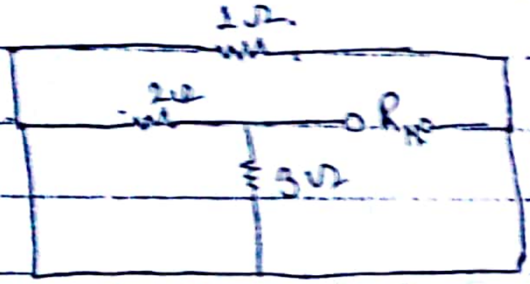
or, $-3I_3 = -60$

or, $I_3 = 20\,A$.

$\frac{6}{5} \times \frac{5}{11}$

$\therefore I_N = I_3 - I_1 = 20 - 12 = 8\,A$.

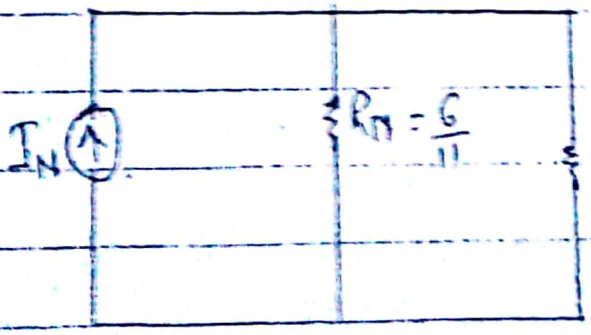
And short circuiting all voltage sources, we get,



$(3 // 2) // 1$

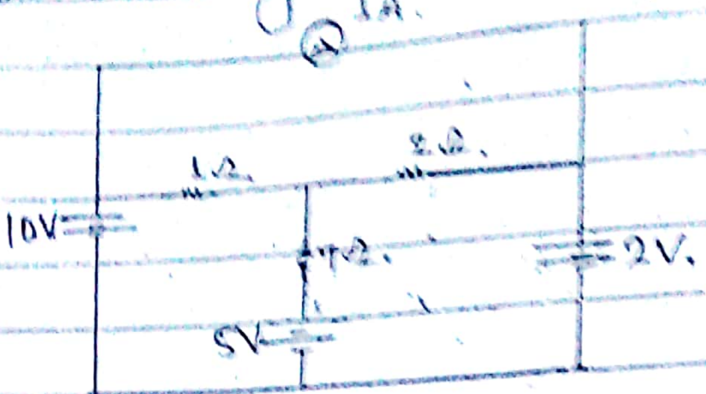
$= \frac{6}{5} // 1 \Rightarrow \frac{6}{5} \times \frac{5}{11} = \frac{6}{11}\,\Omega$.

And, the Norton's equivalent circuit become



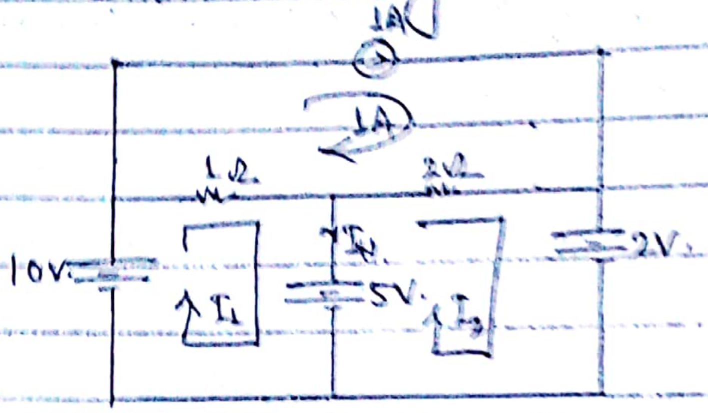
$$\therefore I = \frac{8 \times \frac{6}{11}}{\frac{6}{11} + 4} = \frac{48}{11} \times \frac{11}{50} = \frac{48}{50}\,A.$$

Find I through 4Ω using Norton's theorem.



Solution:-

Short circuiting 4Ω we get,



Here, $-(I_1-1)-5+10=0$

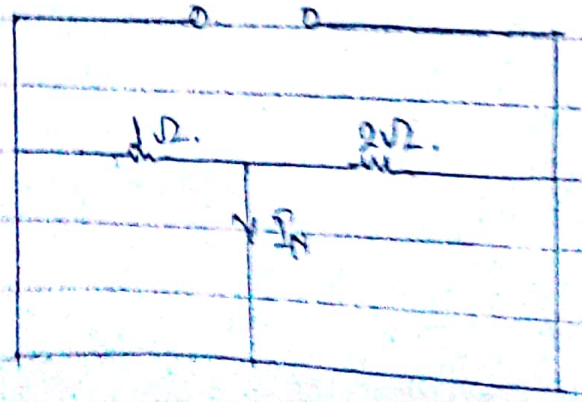
or, $I_1=6$

& $-2(I_2-1)-2+5=0$

or, $I_2=5/2$

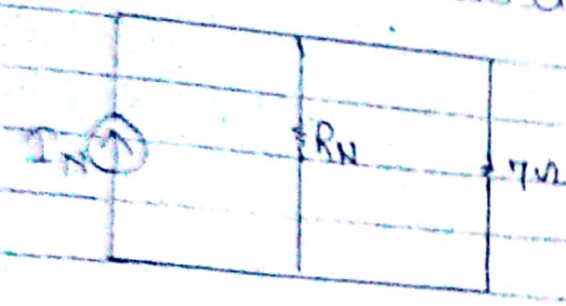
$\therefore I_N = 6-\frac{5}{2} = \frac{12-5}{2} = \frac{7}{2}$ A

To find equivalent resistance, we get,



$\therefore R_N = 1||2 = \frac{2}{3}$

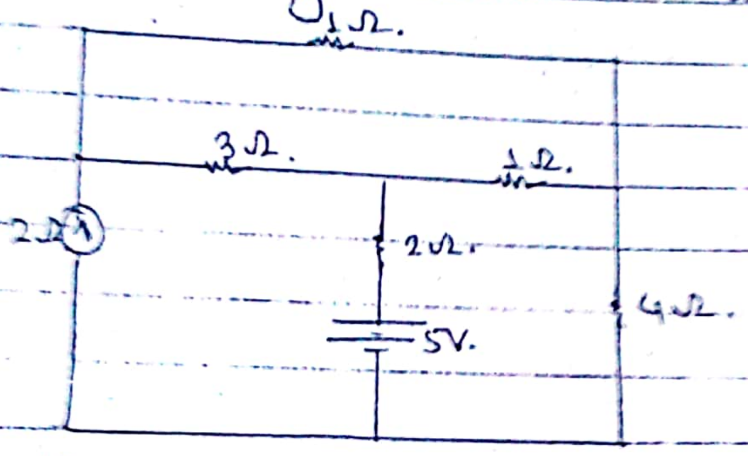
And, the Norton's th circuit becomes,



$$\therefore I = \frac{\frac{7}{2} \times \frac{2}{3}}{\frac{2}{3} + 7} = \frac{\frac{7}{3}}{\frac{23}{3}} = \frac{7}{23} \text{ A.}$$

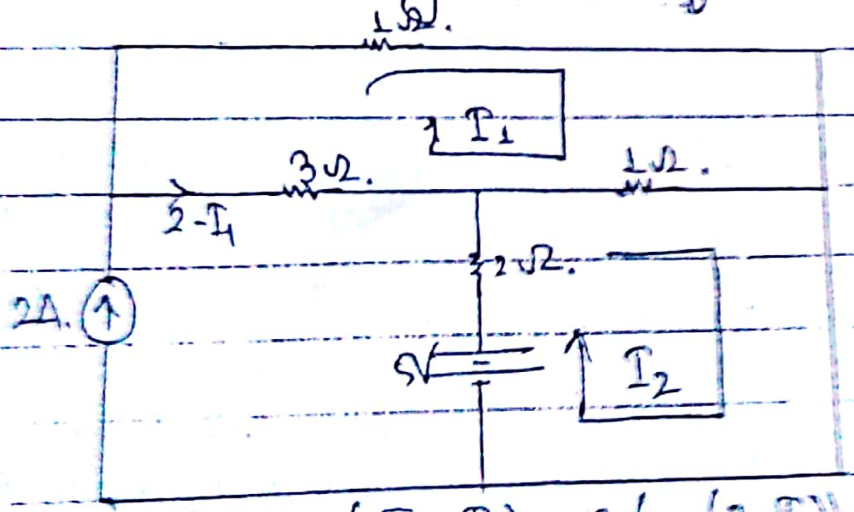
$$V = IR_L = \frac{7}{23} \times 7 = \frac{49}{23} \text{ V.}$$

Find the voltage across 4Ω using Norton's theorem.



Solution:-

Short circuiting 4Ω to find norton's current,



$$-I_1 - 1(I_1 - I_2) - 3(-(2 - I_1)) = 0$$

or, $-I_1 - I_1 + I_2 + 6 - 3I_1 = 0$

or, $-5I_1 + I_2 + 6 = 0$ ... (1)

From 2nd mesh, we get,

$$-1(I_2 - I_1) + 5 - 2(I_2 - 2) = 0$$

$$or,\ -I_2 + I_1 + 5 - 2I_2 + 4 = 0$$

$$or,\ -3I_2 + I_1 + 9 = 0 \quad \ldots (2)$$

$\therefore$ On solving eq^n (1) & (2), we get,

$$-3I_2 + I_1 + 9 = 0$$

$$\underline{I_2 - 5I_1 + 6 = 0\ \} \times 3}$$

$$I_1 + 9 - 15I_1 + 18 = 0$$

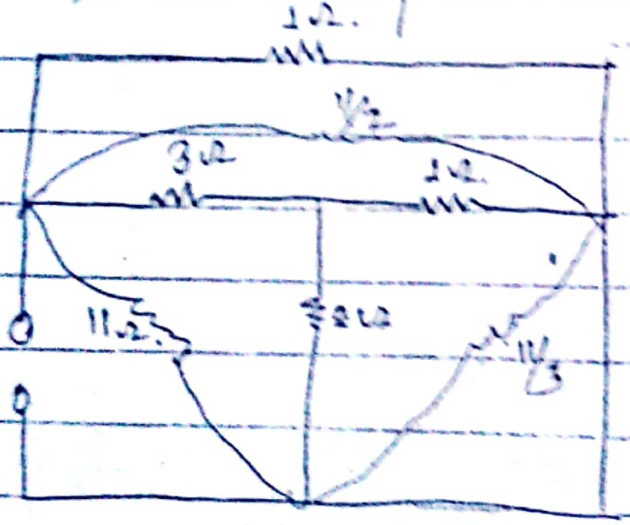
$$or,\ -14I_1 = -27$$

$$or,\ I_1 = \frac{27}{14} \qquad \& \ -5\times\frac{27}{14} + I_2 + 6 = 0$$

$$or,\ I_2 = 3.64$$

$$\therefore I_N = 3.64A.$$

And, to find equivalent resistance,

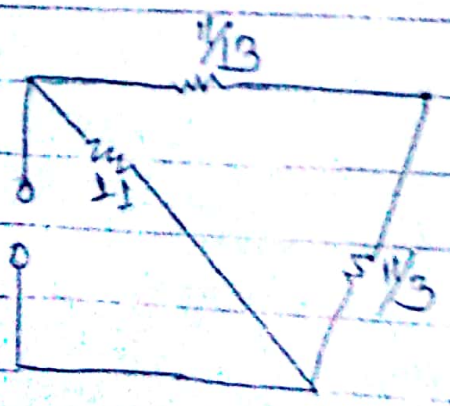


$$3\times2 + 2\times1 + 1\times3 = 11$$

$$\frac{11}{2} // 1$$

$$= \frac{\frac{11}{2}}{\frac{11}{2}+1} = \frac{\frac{11}{2}}{\frac{13}{2}} = \frac{11}{13}$$

$$\frac{11}{13} + \frac{11}{3} = \frac{33+143}{39} = 4.51$$

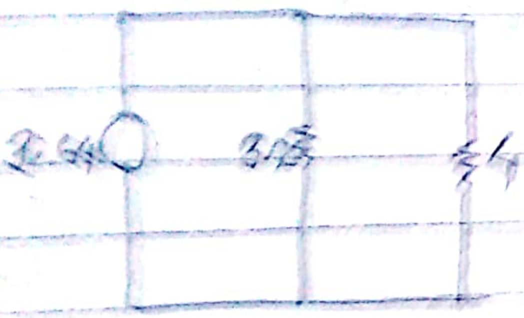


$$4.51 // 11.$$

$$\frac{4.51\times11}{4.51+11} = \frac{49.64}{15.51}$$

$$\therefore R_N = 3.2\Omega.$$

And, the Norton's equivalent circuit becomes,



$$\therefore I = \frac{3.64 \times 3.2}{3.2+4} = 1.61A.$$

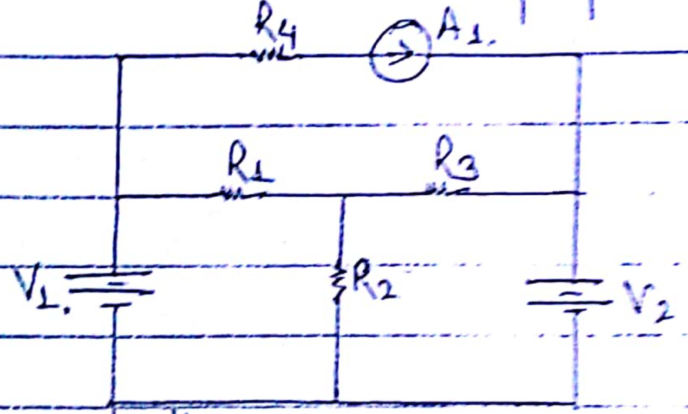
$$V = IR_L = 1.61 \times 4 = 6.47\Omega.$$

Superposition Theorem.

Statement

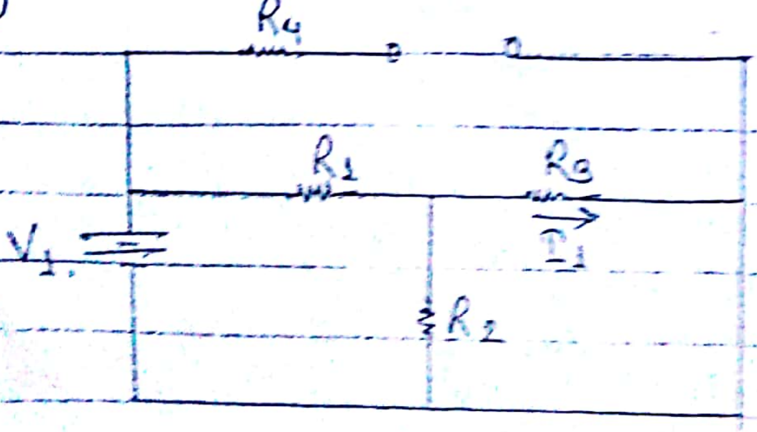
The current or voltage in an element of a linear circuit can be found by adding the currents or voltages found by considering the source individually by short circuiting other voltage sources and opening other current sources when consider a single source.

Example:- To illustrate superposition theorem,



Suppose three sources are connected as shown in figure.

Consider V_1 source. to find suppose current through R_3.



Similarly,

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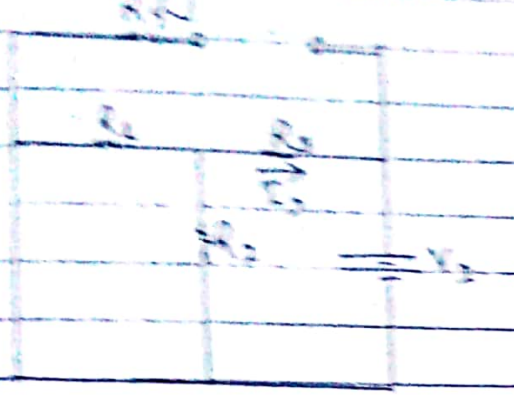
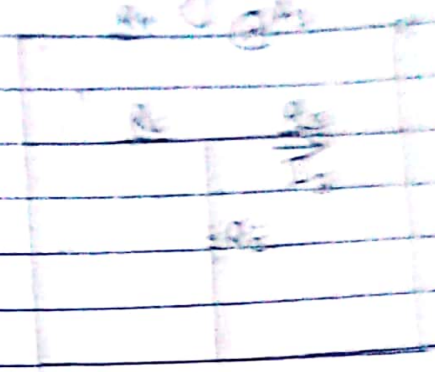
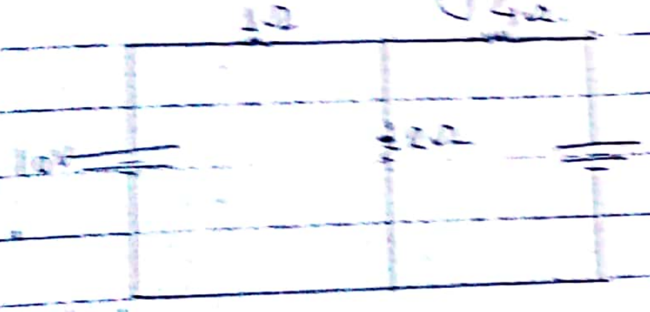
Considering V_2 source   Considering [illegible] source





According to superposition theorem,
current through $R_3 = I = I_1 + I_2$

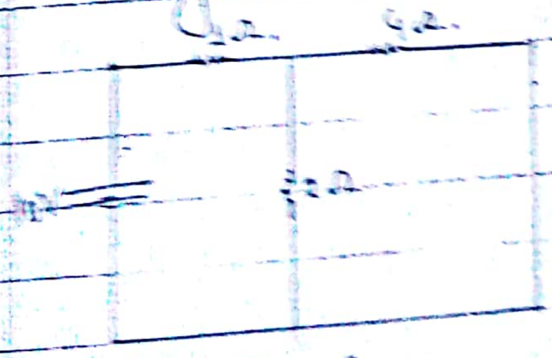
Q. Find current through 4Ω using superposition theorem

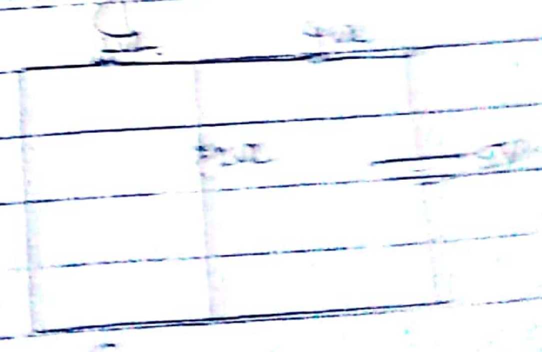


$R_{eq} = (4 \| 2) + 1 = \frac{[illegible]}{[illegible]} = [illegible]$

$\therefore I = \frac{V}{R} = \frac{10}{[illegible]} = [illegible]$

→ Solution:-
through 10V,   through 5V source.





$I_1 = I \times \frac{2}{4+2}$

$= 4.29 \times \frac{2}{6}$

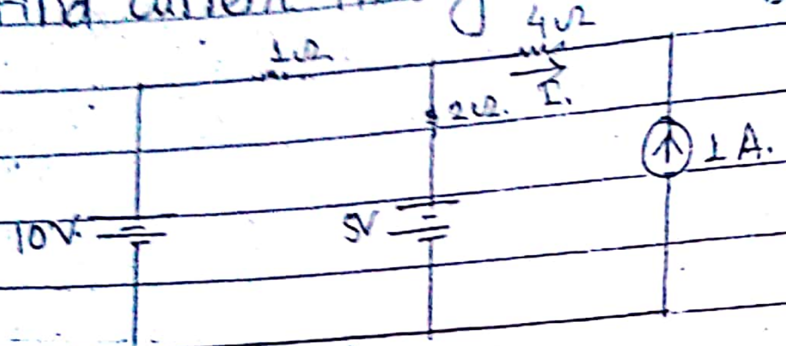
$= 1.43A$

$R = [illegible]$

$I_2 = \frac{5}{[illegible]}$

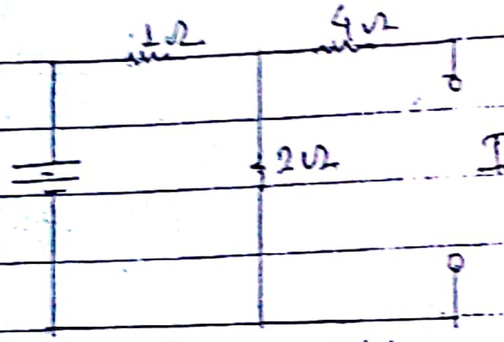
$\therefore$ Current through $4\Omega = I_1 - I_2 = \frac{10}{7} - \frac{15}{14} = \frac{5}{14}$ A.

Find current through 4Ω using superposition theorem



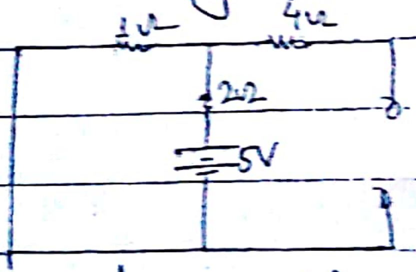
Solution:-

To find current through 4Ω, considering 10 V,



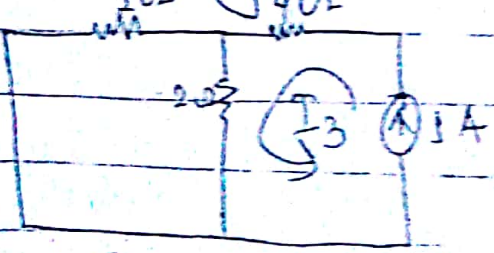
$I_1 = 0.$

And, considering 5V,



$I_2 = 0$

And, considering 1A,

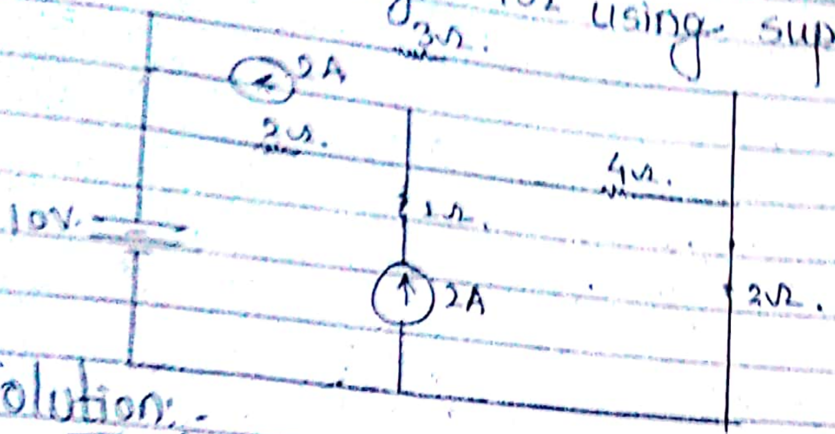


$I_3 = 1A$

$\therefore$ from Superposition theorem, we have,

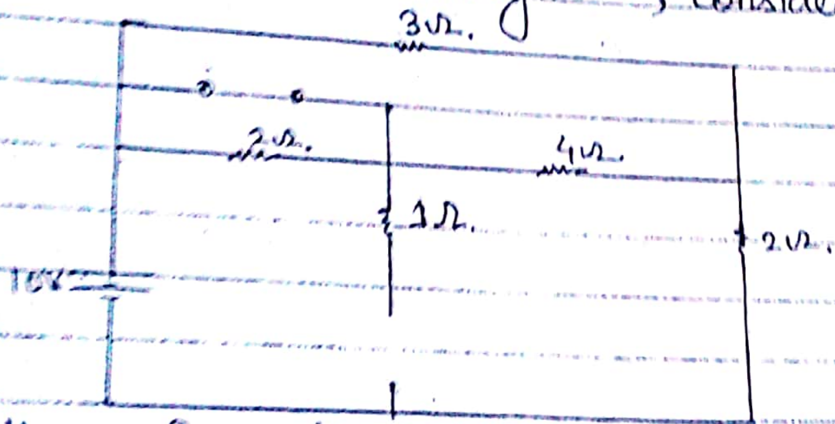
Current through $4\Omega = I_1 + I_2 + I_3 = 1A.$

Q. Find current through 4Ω using superposition theorem.



> Solution:

To find current through 4Ω, considering 10 V,

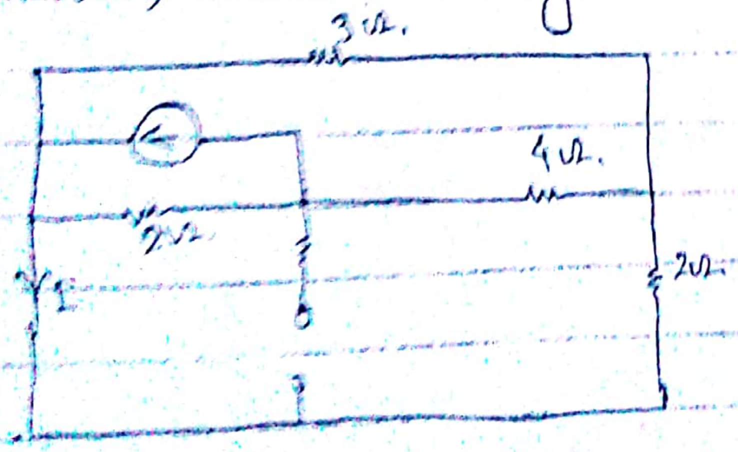


Here, $R_{eq} = (3 // 6) + 2 = 4\Omega$

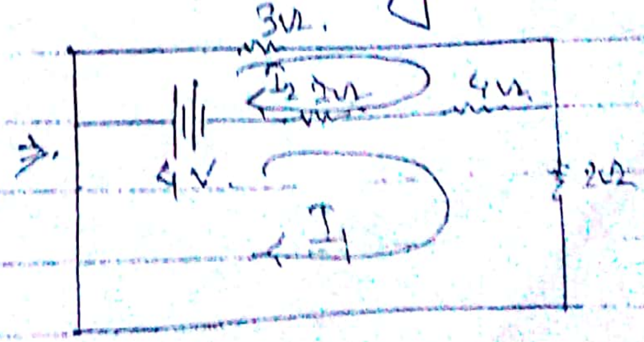
and, $I = \frac{V}{R} = \frac{10}{4} = \frac{5}{2}$ A.

$\therefore I_1 = I \times \frac{3}{6+3} = \frac{5}{2} \times \frac{1}{3} = \frac{5}{6}$ A.

Also, current through 2A,



After converting current source,



$\therefore -3I_2 - 4(I_2 - I_1) - 2(I_2 - I_1) + 4 = 0$

or, $-9I_2 + 6I_1 + 4 = 0$ ——(1)

$-2(I_1 - I_2) - 4(I_1 - I_2) - 2I_1 - 4 = 0$

or, $-8I_1 + 6I_2 - 4 = 0$ ——(2)

On solving eq^n (1) & (2), we get,

$$\left.\begin{aligned} -9I_2 + 6I_1 + 4 &= 0 \\ 6I_2 - 8I_1 - 4 &= 0 \end{aligned}\right\} \begin{aligned} &\times 6 \\ &\times 9 \end{aligned}$$

$-\cancel{54I_2} + 36I_1 + 24 + \cancel{54I_2} - 72I_1 - 36 = 0$

or, $-36I_1 - 12 = 0$

or, $I_1 = \frac{12}{36} = \frac{-1}{3} = 0.33$ A.

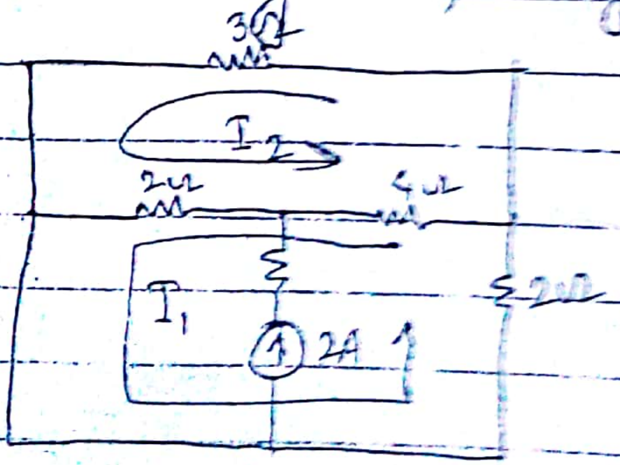
$\cancel{I_2}$ $6 \times \frac{-1}{3} - 9I_2 + 4 = 0$

or, $-2 - 9I_2 + 4 = 0$

or, $I_2 = \frac{2}{9}$

$\therefore$ I through $4\Omega = 0.33 - 0.22 = 0.11$ A.

~~Ctr~~ Considering 2A, we get,



$-4(I_1 - I_2) - 2(I_1 + 2 - I_2) - 2I_1 = 0$

or, $-8I_1 + 6I_2 - 4 = 0$ —(1)

Also,

$-2(I_2 - (I_1 + 2)) - 4(I_2 - I_1) = 0$

or, $-2(I_2 - I_1 - 2) - 4I_2 + 4I_1 = 0$

or, $-6I_2 + 6I_1 + 4 = 0$

On solving eq^n (1) & (2), we get,

$$-8I_1 + 6I_2 - 4 = 0$$
$$6I_1 - 6I_2 + 4 = 0$$
$$\overline{-2I_1 = 0}$$
$$\therefore I_1 = 0$$

& $6I_2 - 4 = 0$

or, $I_2 = \frac{4}{6} = \frac{2}{3}$ A.

$\therefore$ Current through $4\Omega = \frac{2}{3}$.

From superposition theorem,

$$I = I_1 + I_2 + I_3$$
$$= \frac{5}{6} + \frac{2}{9} + \frac{2}{3} =$$

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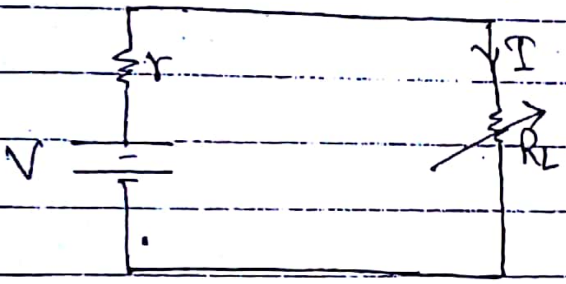
$\delta = \frac{1}{2\sigma L}$

Maximum Power Theorem.

Statement

The maximum power transfer theorem states that the source supplies maximum external power to a load resistance (R_L) when the value of the load resistance equals the value of internal resistance of the source. There is no maximum efficiency of power but there is maximum of power draws by the load resistance.

The circuit ressembles to Thevenin's equivalent circuit.



$$I = \frac{V}{R_L + r}$$

Proof:-

Maximum power, $P_{max} = I^2 R_L$.

For maximum,

$$\frac{d}{dR_L}(P_{max}) = 0$$

$$\text{or, } \frac{d(I^2 R_L)}{dR_L} = 0$$

$$\text{or, } \frac{d\left[\frac{V^2}{(R_L+r)^2} R_L\right]}{dR_L} = 0$$

$$\text{or, } V^2 \left[\frac{d\left(\frac{R_L}{(R_L+r)^2}\right)}{dR_L} \right] = 0$$

$$\text{or, } \left[\frac{(R_L+r)^2 \frac{dR_L}{dR_L} - R_L \cdot \frac{d(R_L+r)^2}{d(R_L+r)} \times \frac{d(R_L+r)}{dR_L}}{\{(R_L+r)^2\}^2} \right] = 0$$

$$\text{or, } \frac{R_L^2 + 2R_L r + r^2 - R_L \times 2(R_L+r) \times 1}{(R_L+r)^4} = 0$$

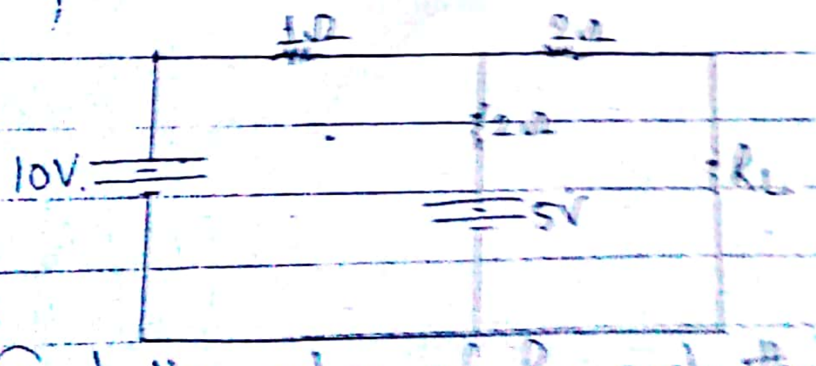
$$\text{or, } R_L^2 + 2R_L r + r^2 - 2R_L^2 - 2R_L r = 0$$

$$\text{or, } -R_L^2 + r^2 = 0$$

$$\text{or, } \boxed{R_L = r}$$

Thus, to draw maximum power, this condition should be satisfied.

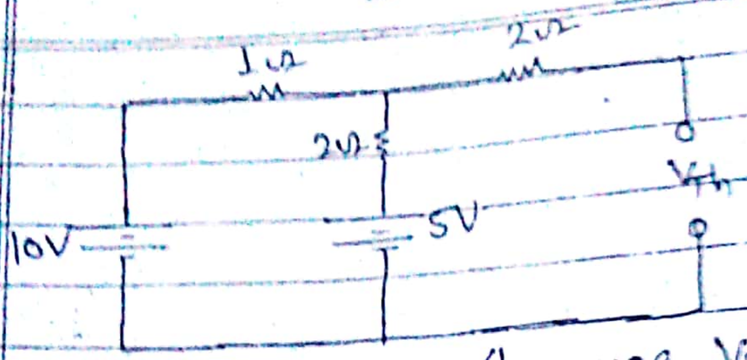
Example:-



Find the value of R_L such that it absorbs maximum power. Also find maximum power.

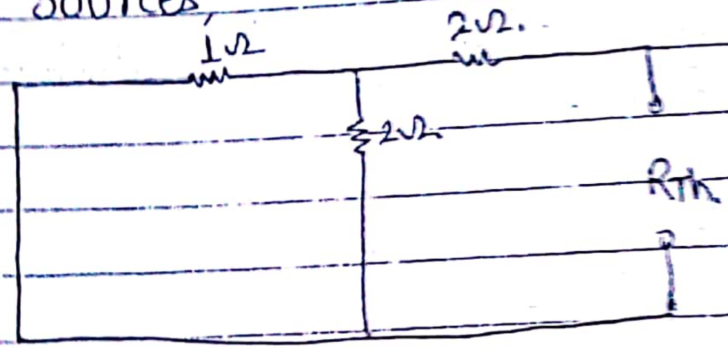
Solution:-

Finding voltage through R_L, we have



$V_{Th} = 5V$ (because voltage through 2Ω equals to V_{Th})

And, to find ~~res.~~ equivalent resistance, short circuiting voltage sources,



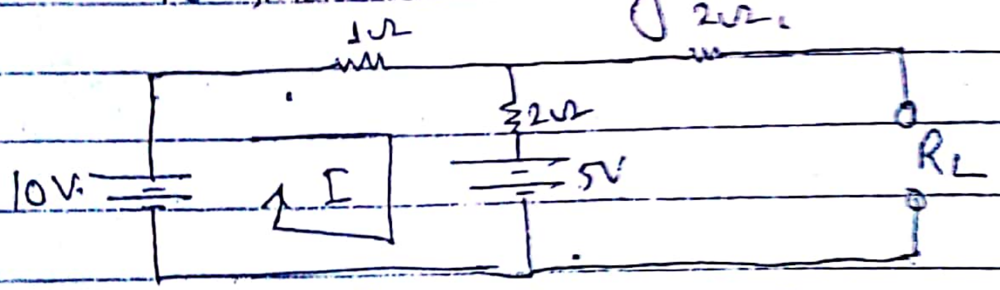
$R_{Th} = (1 \| 2) + 2$

$= \frac{8}{3}\ \Omega$

$= R_L.$

And,

To find total voltage in whole circuit.



$-1 \times I - 2 \times I - 5 + 10 = 0$

or, $I = 5/3.$

and, $V_2 = IR = \frac{5}{3} \times 2 = \frac{10}{3} = 3.33V.$

$\therefore$ Total voltage $V = 3.33 + 5 = 8.33V.$

Since, to achieve maximum power, internal resistance = load resistance.

or, $\frac{8}{3}\ \Omega = R_L.$

And,

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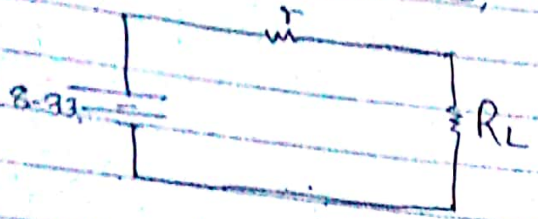
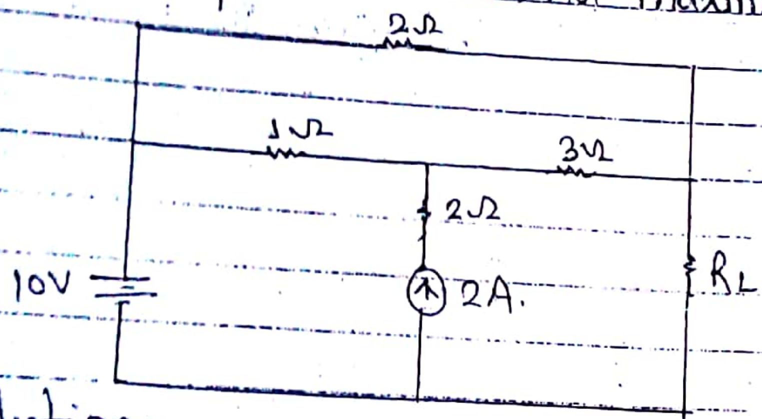
The circuit becomes,



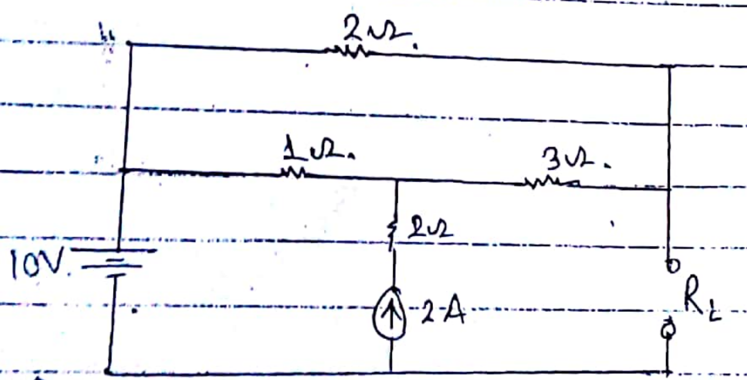
$$\therefore I = \frac{8.33}{\frac{8}{3}+\frac{8}{3}} = 1.56\,A.$$

$$P = I^2 R_L = 6.52\,W.$$

Find the value of R_L such that it consume maximum power and find maximum power.

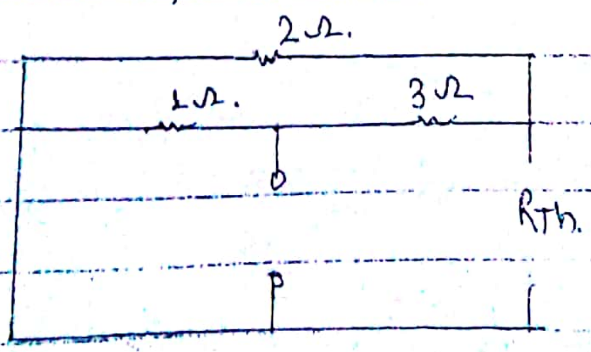


Solution:-



[since 2A flows in circuit, so I=2A]

To find R_{Th}, we have,

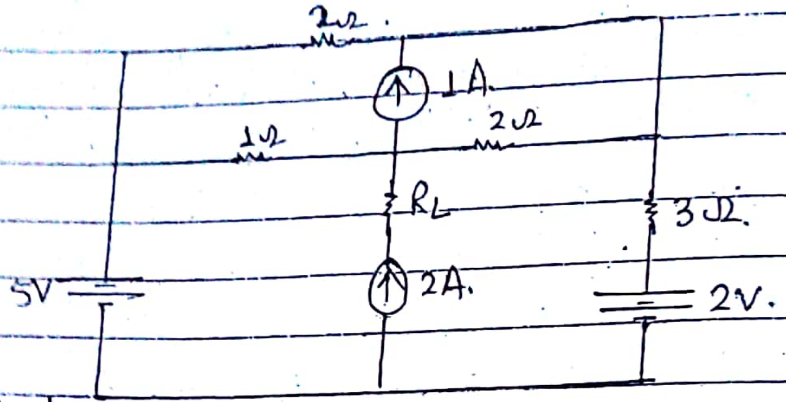


$$R_{Th} = (1+3) \parallel 2.$$

$$= \frac{4\times 2}{4+2} = \frac{4}{3}\,\Omega = R_L.$$

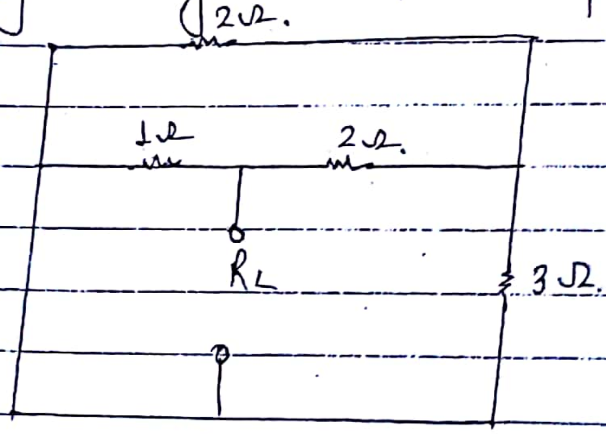
$$P = I^2 R_L = 2^2 \times \frac{4}{3} = \frac{16}{3}\,W.$$

Find the value of R_L and thus find maximum power.

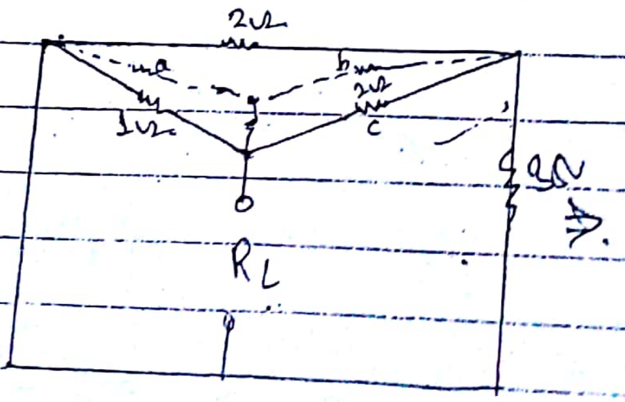


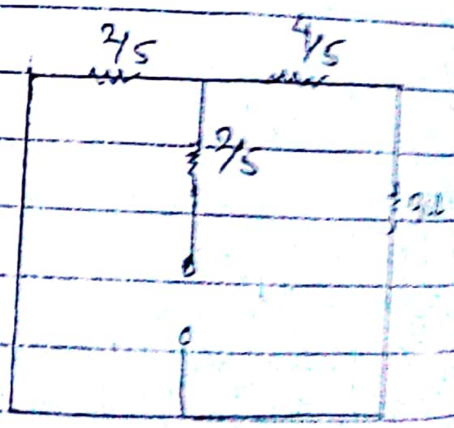
Solution:-

~~Opening R_L,~~ we have to calculate R_{Th} by short circuiting voltage source and opening current source,



or,





$$\therefore R_{Th} = \left[\left(\tfrac{4}{5}+3\right) \parallel \tfrac{2}{5}\right] + \tfrac{2}{5}$$

$$= \left(\frac{\frac{19}{5} \times \frac{2}{5}}{\frac{19}{5} + \frac{2}{5}}\right) + \frac{2}{5} = \frac{38}{25} \times \frac{5}{21} + \frac{2}{5} = \frac{38}{105} + \frac{2}{5}$$

$$= \frac{38+42}{105} = \frac{80}{105} = 0.76\Omega$$

And, current $(I) = 2A$.

$\therefore R_L = 0.76\,\Omega$

and $P_{max} = I^2 R_L = 2^2 \times 0.76 = 3.04\,W$.

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Alternating Current:

Such current in which the direction of current reverses with respect to time or the polarity changes over time is known as alternating current:

Representation

How is alternating current generated?

Ans:- Alternating current is generated in following ways:

a) By rotating electrical coil in a magnetic field.

b) By rotating magnet in the electrical coil.

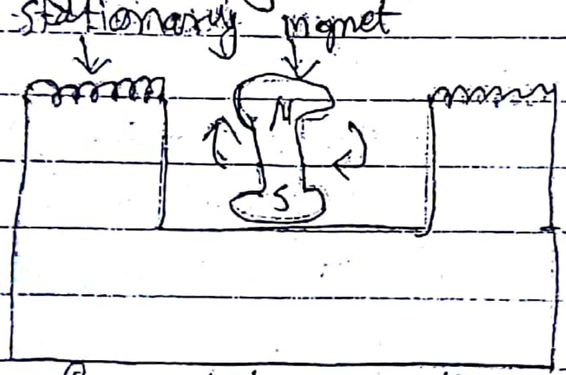


Fig 1 (No current)

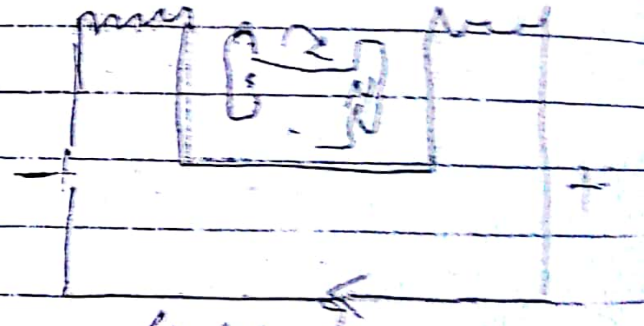


Fig (2)

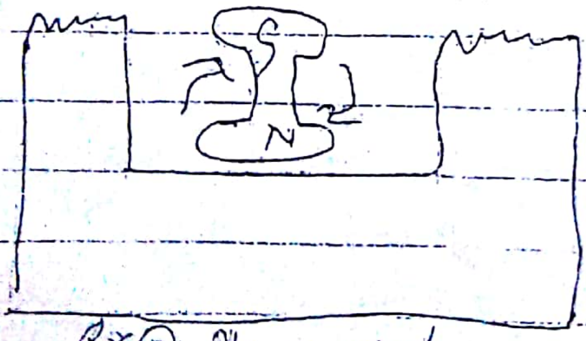


Fig (3) No current.

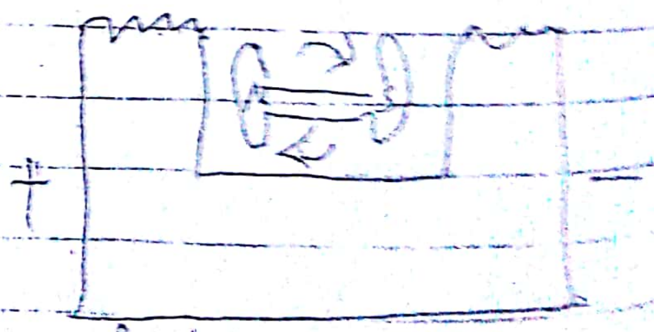


Fig (4)

Differences between AC and DC

Alternating Current	Direct Current
1. Both voltage and current reverses periodically.	1. Both voltage and current remains constant
2. Low cost of production.	2. Higher cost of production.
3. An A.C. generator is almost free from dissipation of energy.	3. An D.C generator undergoes dissipation of energy
4. Cost of transmitting A.C. power can be reduced by using step-up transformers.	4. No such provision can be made
5. A.C. voltages can be lowered or raised as desired by using transformers.	5. No such provision can be made.
6. A.C. circuit current can be decreased by using choke or capacitor without any appreciable power loss.	6. For decreasing D.C circuit current, a resistance has to be used, whose power dissipation factor (= I^2R) is larger
7. A.C. can be converted into D.C. by using a device called converter (rectification circuit)	7. D.C can be converted into A.C by using [illegible] (inverter).
8. A.C. ~~cat~~ cannot be used	8. D.C can be used directly in

directly for electroplating, electrotyping, etc.	carrying such operations
9. A.C motors and other appliances are more robust, and durable.	9. DC motors and appliances are less durable.
10. A.C. attracts a person, so faulty insulation of A.C. are more dangerous, since maximum of A.C is $\sqrt{2}$ times its specified value.	10. D.C. gives a repelling shock to a person, so faulty insulation of D.C are less dangerous.

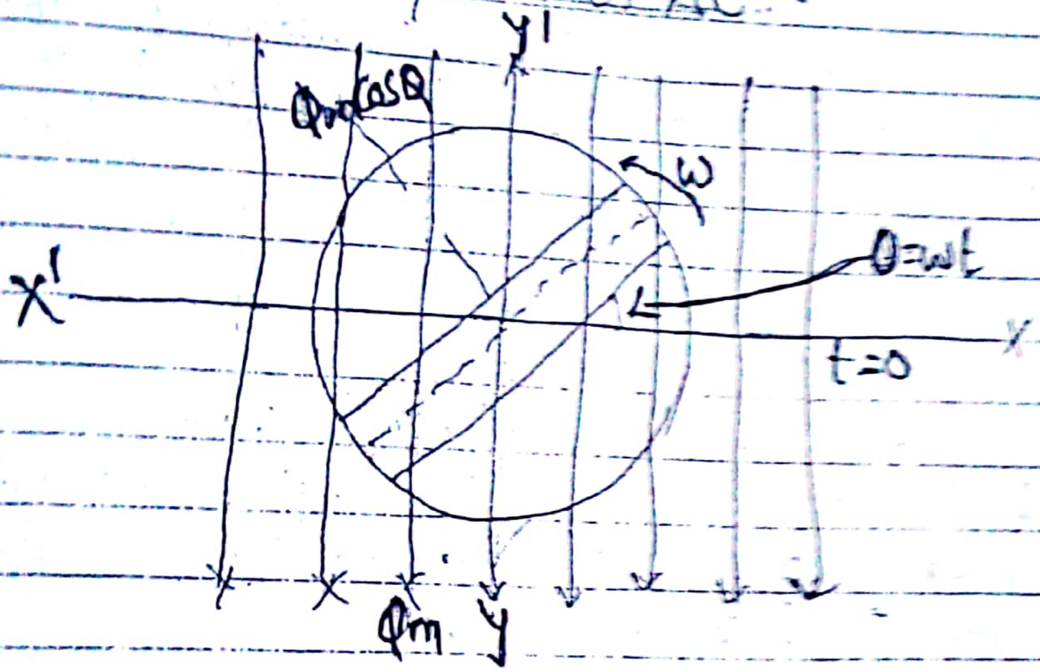
Advantages of A.C. over D.C.

A.C. can be generated at higher voltage.
By the use of transformer, we can raise, and lower the alternating voltage easily, and efficiently.
Cost of generating A.C. is less.
A.C. can be sent to a great distance economically. High voltage A.C. transmission is economical, through the use of transformers.
A.C. induction motors are cheap; and are used for general purpose.
A.C. motors and appliances are most robust and durable.
The maintainance cost of A.C. equipments is less.
A.C. can be converted into D.C. by using a device, called convertor (rectification circuit).

Disadvantages:

1. A.C. cannot be used directly for electroplating, electrotyping etc.
2. A.C attracts a person, so faulty insulation of A.C is more dangerous!

Mathematical equation of AC:-



Let us consider a rectangular coil having turn N and rotating with angular velocity ω rad/sec in a uniform magnetic field. Let, time measured from X-axis and maximum flux ϕ_m is linked with the X-axis. In time t, the coil rotates through an angle $\theta = \omega t$. In the deflected position, the component of flux which is perpendicular to the plane of the coil is $\phi = \phi_m \cos\omega t$. Hence, the maximum flux linked is $N\phi = N\phi_m \cos\omega t$

According to Faraday's law of electromagnetic induction, emf induced in the coil is given by

$$e = -\frac{d}{dt}(N\phi)$$

$$= -N\frac{d\phi}{dt}$$

$$= -N\frac{d}{dt}(\phi_m \cos\omega t)$$

or, $e = \omega N \phi_m \sin\omega t$ ......(1)

This is maximum if $\sin\omega t = 1$,

i.e. $\omega t = \theta = 90°$,

$\therefore E_m = N\phi_m \omega$

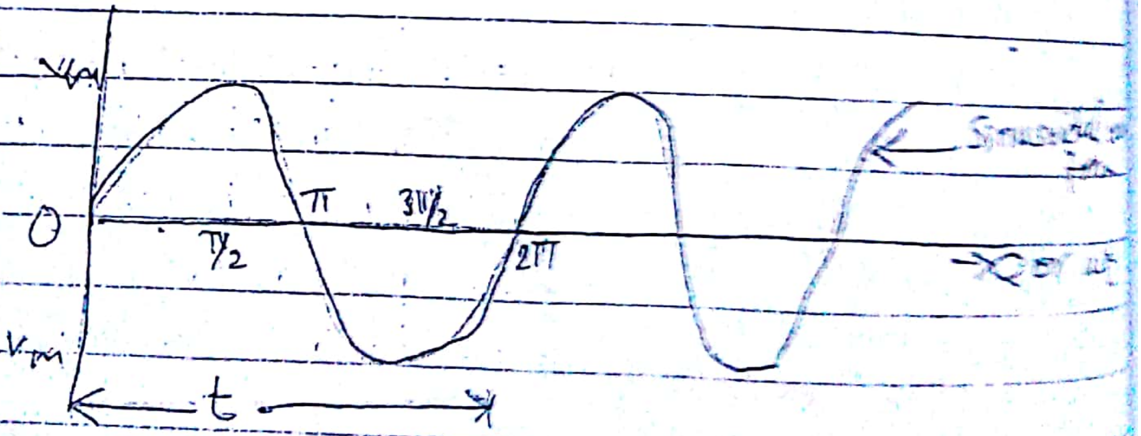
putting in eqn (1), we get,

$$e = E_m \sin\omega t$$

$$= E_m \sin\theta$$

This is in form of voltage, i.e. $e = E_m \sin\theta = E_m \sin\omega t$

Similarly, in current, $i = I_m \sin\theta = I_m \sin\omega t$



$V = V_m \sin\omega t$ or $I = I_m \sin\omega t$

where, V or I = Instantaneous voltage or current value voltage or current at any particular time.

V_m or I_m :- maximum value or peak value.
It is the maximum value of the waveform at any instant of time.

~~ω~~ = Angular velocity(ω) :- The velocity by which the waveform spins.

$$\boxed{\omega = 2\pi f}$$

frequency(f) :-
It is the number of cycles completed in one second. The unit of frequency is Hertz (Hz) or cycles/sec.

Time period :-
The total time taken by waveform to complete one cycle.

$$\boxed{T = \frac{1}{f}}$$

Phase :-
Phase of an alternating quantity is the function of the time period or cycle that has elapsed since it last passed from the chosen zero position on origin. The phase at time t from chosen origin is given by t/T, where T is the time period of alternating quantity.

Phase angle(ϕ) :- The position of the waveform with respect to time and use in comparision with other waveforms

Note:-

- The waveform which leads, achieve the maximum and zero value first. It is represented by +ve.
- The waveform which lags, achieve the maximum and zero value later on. It is represented by -ve.

eg.

$I = 6\sin(314t + 30^\circ)$

Find instantaneous value at 5 sec

Find frequency, maximum value.

Solution:-

$I_m = 6$

$\omega = 314$

or, $2\pi f = 314$

$\therefore f = \frac{314}{2\pi}$ Hz.

Also, $\phi = 30^\circ$

and instantaneous value,

$I = 6(\sin(314 \times 5 + 30^\circ)$

Find the resultant current of a wire if two current I_1 and I_2 flow in that wire

$I_1 = 5\sin\omega t$

$I_2 = 10\sin(\omega t + 30^\circ)$

Solution:-

$I = I_1 + I_2$

$= 5\sin\omega t + 10\sin(\omega t + 30^\circ)$

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$= 5\sin\omega t + 10\{\sin\omega t \cos 30^\circ + \cos\omega t \sin 30^\circ\}$

$= 5\sin\omega t + 5\sqrt{3}\sin\omega t + 5\cos\omega t$

$= 5\sin\omega t + 5\times1.732\sin\omega t + 5\cos\omega t$

$= 13.66\sin\omega t + 5\cos\omega t$

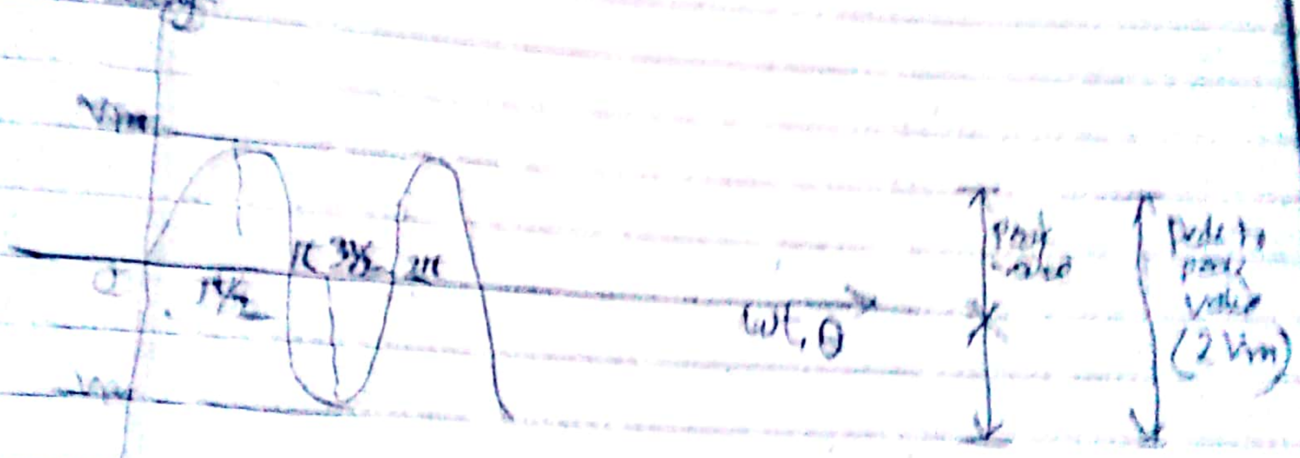
$= 14.54\left\{\dfrac{13.66}{14.54}\sin\omega t + \dfrac{5}{14.54}\cos\omega t\right\}$ $\quad \left[\sqrt{13.66^2 + 5^2} = 14.54\right]$

$= 14.54(\cos\alpha \sin\omega t + \sin\alpha \cos\omega t)$

$= 14.54\sin(\alpha + \omega t)$

$= 14.54\sin(30 + \omega t)$

3. Find Vavg:



→ Solution -

$$V_{avg} = \frac{\int_0^T V\,d\theta}{T} = \frac{\int_0^{2\pi} V_m \sin\theta\, d\theta}{2\pi}$$

$$= \frac{V_m}{2\pi}\left[\cos\theta\right]_0^{2\pi}$$

$$= \frac{V_m}{2\pi} \times 0 = 0$$

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Measurement of AC quantity

Average value:

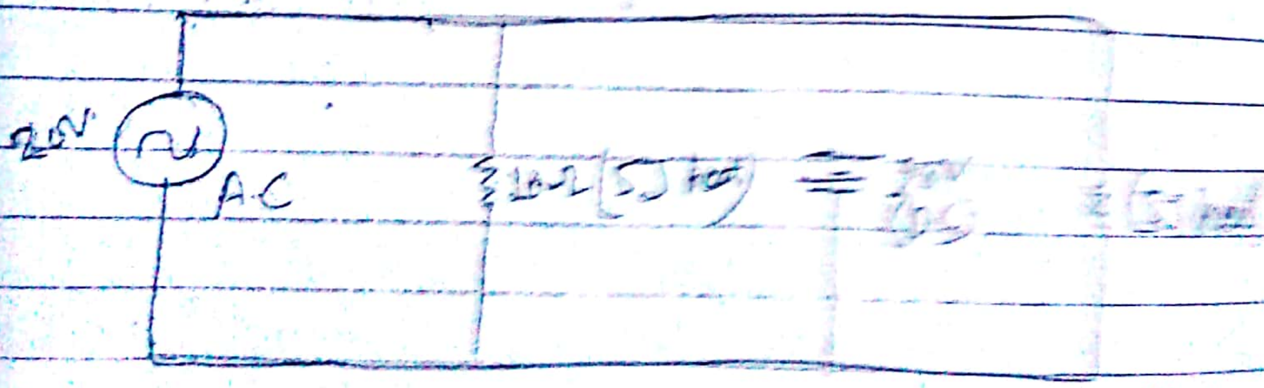
The average value of an AC is the measure of the supply of charge by a steady source (DC) to a conductor as equivalent to the supply by the AC for the same time.

i.e. V_{avg} or $I_{avg} = \frac{1}{T_2-T_1}\int_{T_1}^{T_2} V(t)$ or $I(t)\,dt$

RMS value or equivalent value or effective value

The root mean square value or effective value of the A.C. is the measurement of the production of heat by a steady source (DC) on a conductor as equivalent to the production by an AC ~~during~~ during the same time as DC applies.

i.e RMS value = $\sqrt{\frac{1}{T}\int_0^T I^2\,dt}$



Form factor:-

It is the ratio of rms value to the average value of an alternating quantity

i.e. form factor $(K_f) = \frac{\text{RMS value}}{\text{Average value}}$

For sinusoidal wave, form factor

$$K_f = \frac{0.707\,I_m}{0.637\,I_m} = 1.11.$$

Importance of form factor :-

1. A knowledge of form factor enables us to evaluate the RMS value from the average value and vice-versa.
2. The value of form factor for less peaky wave is less than 1.11 and approaches to 1.0; while for more peaky wave, it is more than 1.11. Thus, for a square wave, the value of form factor is 1.0; while for a triangular wave, it is 1.15.

Crest or Peak or Amplitude factor (K_a) :-

It is the ratio of the peak (or maximum) value to the rms value of an alternating quantity. Thus, for a sine wave,

$$K_a = \frac{I_m}{I_{rms}} = \frac{I_m}{0.707 I_m} = 1.414.$$

Importance :-

- for measuring iron losses, since they depend upon the peak value of flux, and
- in dielectric insulation testing since dielectric stresses during insulation is proportional to the peak value of applied voltage.

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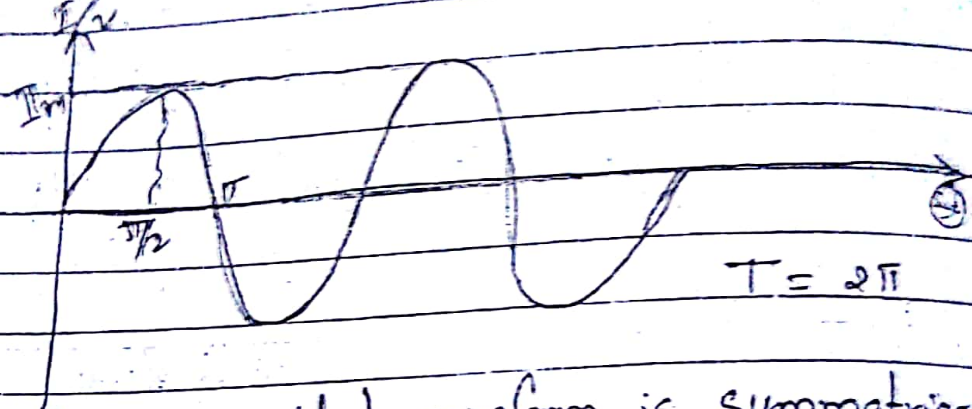
Find the average value, effective value(rms), form factor and crest factor of sinusoidal alternating current.

Solution:-



Since the sinusoidal waveform is symmetrical, its average value comes to be zero. So, we have to take only half cycle for average value.

$$\therefore I_{avg} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi} \qquad [-\cos 180 - \cos 0] = (-1 - 1)$$

$$= \frac{2I_m}{\pi} = 0.637 I_m \qquad \frac{2I_m}{\pi}$$

$$\text{RMS value} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I^2 \, d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(I_m \sin\theta)^2 \, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I_m^2 \sin^2\theta \, d\theta} = \sqrt{\frac{I_m^2}{2\pi}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}}$$

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$= \frac{I_m}{\sqrt{2}} = \frac{1}{\sqrt{2}} I_m = 0.707 I_m$

Form factor $= \frac{\text{rms}}{\text{Vavg value}} = \frac{I_m}{\sqrt{2}} \times \frac{\pi}{2 I_m} = \frac{\pi}{2\sqrt{2}} = 1.11.$

Crest factor $= \frac{\text{Peak value}}{\text{rms value}} = \frac{I_m}{I_m/\sqrt{2}} = 1.414.$

Q. Find the average value, effective value, form factor and peak factor of the waveform. $V = 20\sin\omega t$

→ Solution:-

We have,

$$V = 20\sin\omega t \quad \ldots\ldots (1)$$

Comparing with $V = V_m \sin \omega t$, we get,

$$V_m = 20$$
$$T = 2\pi$$

$$\therefore V_{avg} = \frac{1}{T}\int_0^T v\,dT$$

$$= \frac{1}{\pi}\int_0^{\pi} 20\sin\omega t = \frac{20}{\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$= -\frac{20}{\pi} \times 2 = \frac{40}{\pi}$$

Also,

$$\text{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2\, d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} 400\sin^2\omega t}$$

$$= \sqrt{\frac{400}{2\pi}\left[\frac{1-\cos 2\pi}{2}\right]_0^{2\pi}}$$

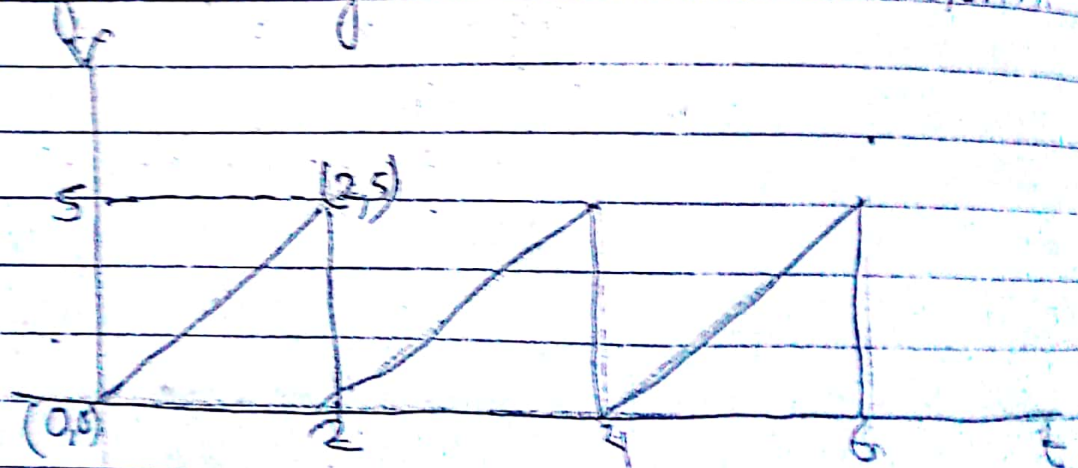
$$= \sqrt{\frac{400}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{400}{4\pi} \times 2\pi} = \sqrt{200} = 10\sqrt{2}.$$

$$\text{Form factor} = \frac{\text{rms}}{\text{avg}} = \frac{10\sqrt{2}}{40/\pi} = \frac{\sqrt{2}\pi}{4} = \frac{\pi}{2\sqrt{2}} = 1.1$$

$$\text{crest factor} = \frac{\text{Peak value}}{\text{rms value}} = \frac{20}{10\sqrt{2}} = \sqrt{2} = 1.414.$$

Q. Find the average value, rms value, form factor & peak factor from given saw-tooth waveform.



> Solution:-

Given that, $T = 2\text{sec}$.

$$\therefore y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\text{or, } V - 0 = \frac{5-0}{2-0}(t-0)$$

or, $v = \frac{5}{2}t$

$$V_{avg} = \frac{\int_0^T v\,dt}{T} = \frac{\int_0^2 \frac{5}{2}t\,dt}{2} = \frac{5}{4}\left[\frac{t^2}{2}\right]_0^2$$

$$= \frac{5}{4} \times \frac{4}{2} = \frac{5}{2}$$

Also,

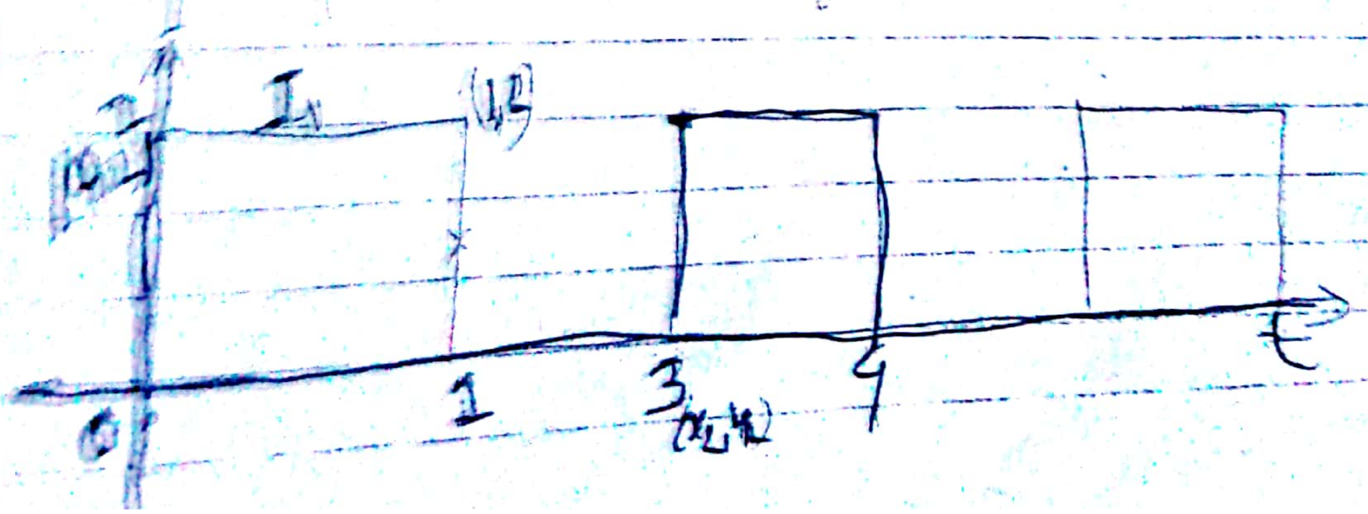
$$V_{RMS} = \sqrt{\frac{1}{2}\int_0^2 \frac{25}{4}t^2\,dt} = \sqrt{\frac{1}{2} \times \frac{25}{4}\left[\frac{t^3}{3}\right]_0^2}$$

$$= \sqrt{\frac{25}{8} \times \frac{8}{3}} = \frac{5}{\sqrt{3}}$$

Form factor $= \frac{rms}{avg} = \frac{5}{\sqrt{3}} \times \frac{2}{5} = \frac{2}{\sqrt{3}}$

Crest factor $= \frac{\text{Peak value}}{rms} = \frac{5}{5} \times \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$.

Find the average value, rms(effective value), form factor and peak(crest factor) of the waveform



Solution:-

Time period (T) = 3

Eq^n of waveform is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\text{or, } I_1 - 2 = \frac{2-2}{1-0}(t-0)$$

or, $I_1 = 2$ [= always maximum value for horizontal line]

Also, $$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\text{or, } I_2 - 0 = \frac{0-0}{1-0}(t-0)$$

$$I_2 = 0$$

$$\therefore I_{avg} = \frac{\int_0^T I\,dt}{T} = \frac{1}{3}\int_0^3 I\,dt$$

$$= \frac{1}{3}\left[\int_0^1 I_1\,dt + \int_1^3 I_2\,dt\right]$$

$$= \frac{1}{3}\left[2t\right]_0^1 + 0$$

$$= \frac{2}{3}\left[t\right]_0^1 = \frac{2}{3}$$

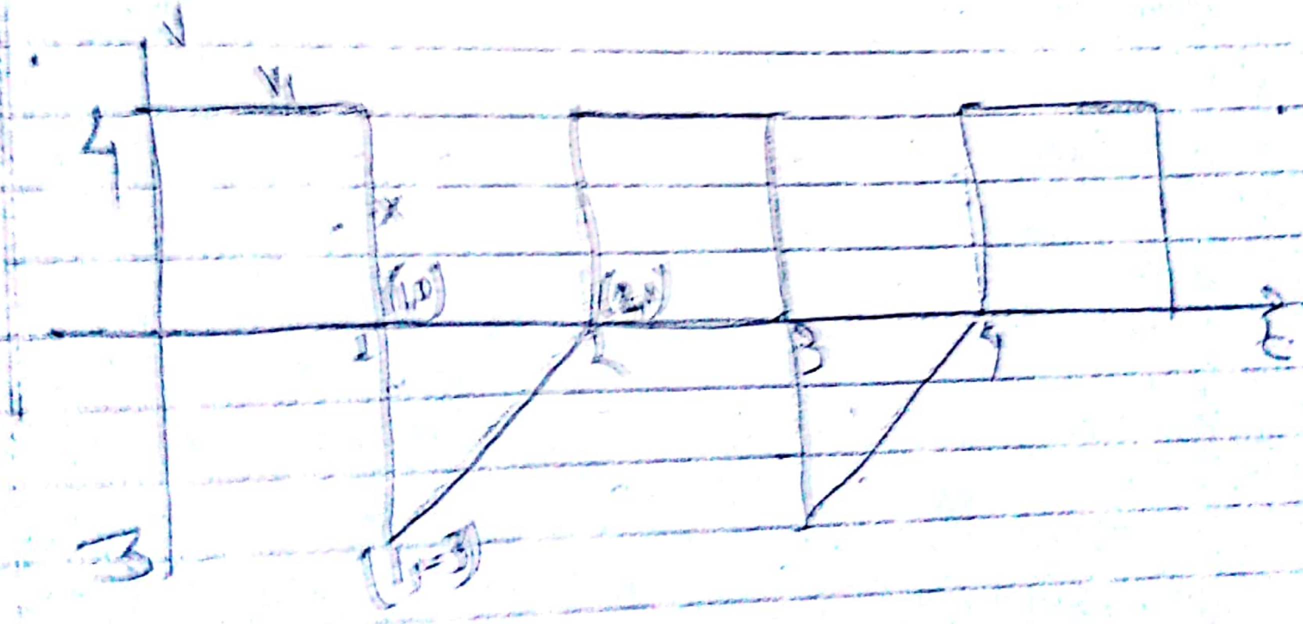
Also, rms value $= \sqrt{\frac{1}{T}\int_0^T I^2\,dt}$

$$= \sqrt{\frac{1}{3}\int_0^1 I_1^2\,dt + \int_1^3 I_2^2\,dt}$$

$$= \sqrt{\frac{1}{3}\left[4t\right]_0^1} = \frac{2}{\sqrt{3}}$$

Form factor $= \frac{\text{rms}}{\text{avg}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3}.$

Peak factor $= \frac{\text{Peak value}}{\text{rms value}} = \frac{2}{2/\sqrt{3}} = \sqrt{3}.$



→ Solution:

$T = 2$

$v_1 = 4 \quad 0 \le t \le 1$

Also, $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Also, rms value $= \sqrt{\frac{1}{T}\int_0^T v^2 dt}$

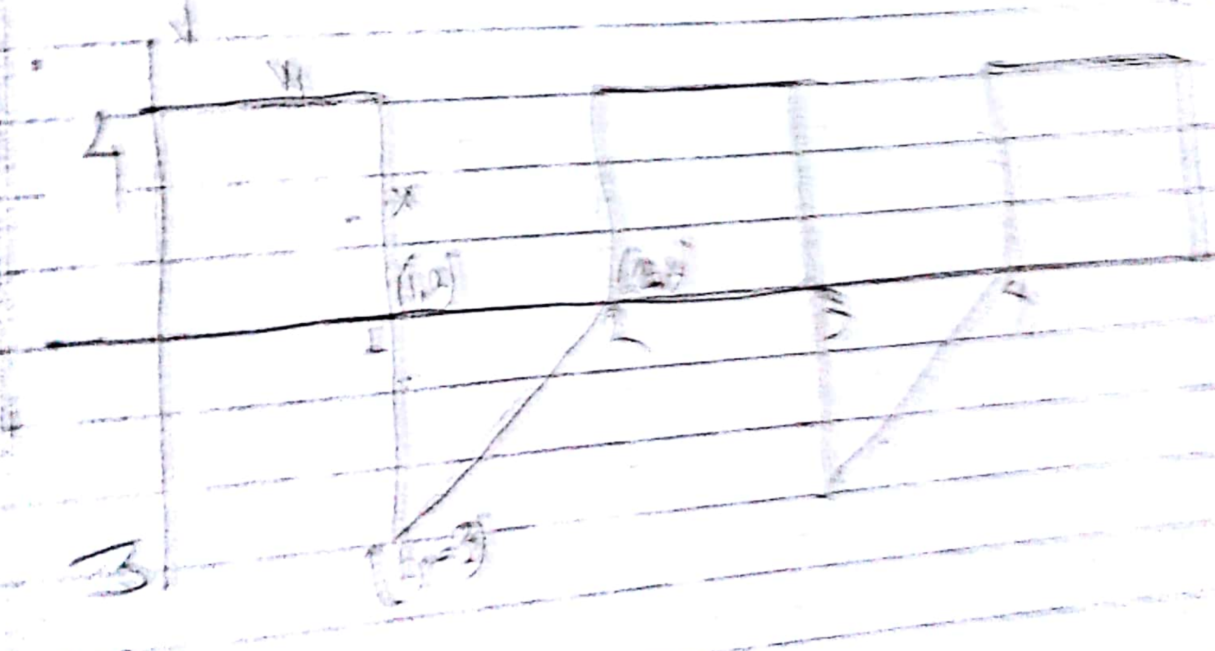
$= \sqrt{\frac{1}{3}\left[\int_0^1 v^2 dt + \int_1^3 v_2^2 dt\right]}$

$= \sqrt{\frac{1}{3}\left[4t\right]_0^1} = \frac{2}{\sqrt{3}}$

Form factor $= \frac{\text{rms}}{\text{avg}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3}$

Peak factor $= \frac{\text{Peak value}}{\text{rms value}} = \frac{2}{2/\sqrt{3}} = \sqrt{3}$

Q.



→ Solution:

$T = 2$

$v_1 = 4 \quad 0 \le t \le 1$

Also,

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

or, $v_2 + 3 = \frac{0+3}{2-1}(t-1)$

or, $v_2 + 3 = 3t - 3$

or, $v_2 = 3t - 6$ $(1 \le t \le 2.$

Now,

$$V_{avg} = \frac{\int_0^T v\,dt}{T} = \frac{1}{2}\left[\int_0^1 4\,dt + \int_1^2 (3t-6)\,dt\right]$$

$$= \frac{1}{2}\left\{[4t]_0^1 + \left[\frac{3t^2}{2} - 6t\right]_1^2\right\}$$

$$= \frac{1}{2}\left\{4 + 6 - 12 - \frac{3}{2} + 6\right\}$$

$$= \frac{1}{2}\left\{4 - \frac{3}{2}\right\} = \frac{1}{2} \times \frac{5}{2} = \frac{5}{4}$$

Also,

$$Rms = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$$

$$= \sqrt{\frac{1}{2}\int_0^1 4^2\,dt + \int_1^2 (9t^2 - 36t + 36)\,dt}$$

$$= \sqrt{\frac{1}{2}\left(16 + \left[\frac{9t^3}{3} - \frac{36t^2}{2} + 36t\right]_1^2\right.}$$

$$= \sqrt{\frac{1}{2}(16 + 24 - 72 + 72 - 3 + 18 - 36)} = \sqrt{\frac{19}{2}}$$

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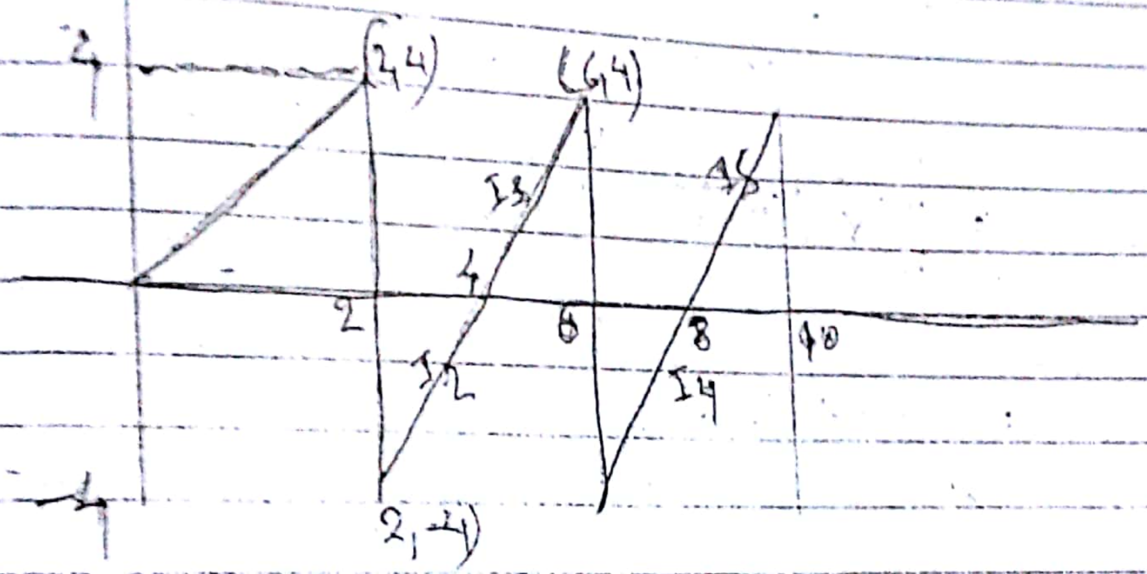
And, form factor $= \frac{rms}{avg} = \sqrt{\frac{19}{2}} \times \frac{4}{5} = 2.46$

crest factor $= \frac{4}{3.08} = 1.29$.



Solution:-

$T = 4$ sec.

The equation of the waveform is,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\text{or, } I_1 - 0 = \frac{4-0}{2-0}(t-0)$$

$$I_1 = 2t \quad \{0 \le t \le 2\}$$

$$I_2 + 4 = \frac{4+4}{6-2}(t-0)$$

$$\text{or, } I_2 + 4 = 2t - 4$$

$$\text{or, } I_2 = 2t - 8 \quad (2 \le t \le 4)$$

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Now,

$$V_{avg} = \frac{1}{4}\left[\int_0^2 (2t)\,dt + \int_2^4 (2t-8)\,dt\right]$$

$$= \frac{1}{4}\left[4 + \left[\frac{2t^2}{2} - 8t\right]_2^4\right]$$

$$= \frac{1}{4}\left[4+16-32-4+16\right]$$

$$= \frac{1}{4}[0] = 0.$$

Since, it is symmetrical, so,

$$V_{avg} = 2\times\frac{1}{4}\int_0^2 2t\,dt$$

$$= \frac{1}{2}\left[t^2\right]_0^2 = \frac{1}{4}\times 4 = 2.$$

$$rms = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}.$$

$$= \sqrt{\frac{1}{4}\int_0^2 4t^2\,dt + \int_2^4 (4t^2-32t+64)\,dt}$$

$$= \sqrt{\frac{1}{4}\left[4\times\frac{t^3}{3}\right]_0^2 + \left[\frac{4t^3}{3} - \frac{32t^2}{2} + 64t\right]_2^4}$$

$$= \sqrt{\frac{1}{4}\left(\frac{32}{3} + \left(\frac{256}{3} - 256 + 256 - \frac{32}{3} + 64 - 128\right)\right)}$$

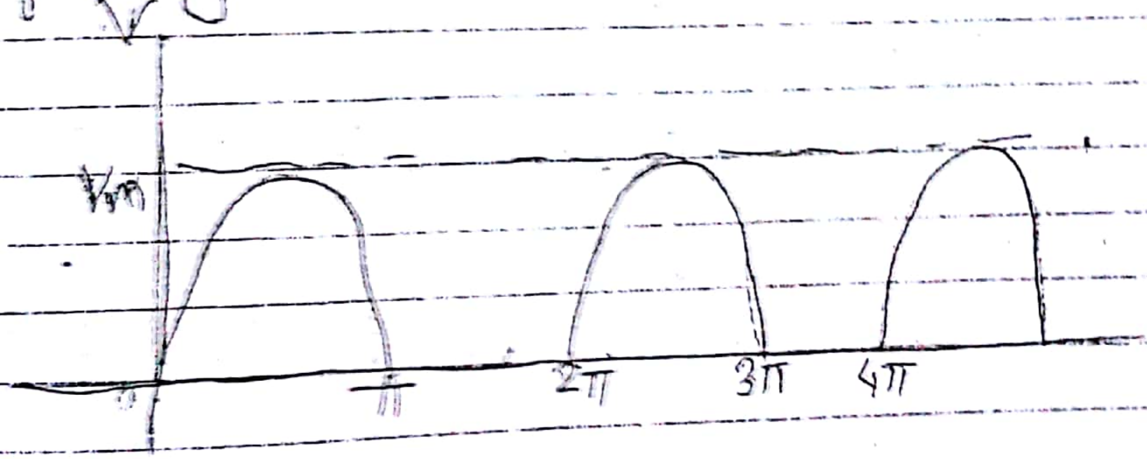
$$= \sqrt{\frac{1}{4}\left(\frac{32}{3} + \frac{256}{3} - \frac{32}{3} - 64\right)}$$

$$= \sqrt{\frac{1}{4}\left(\frac{256-192}{3}\right)} = \sqrt{\frac{1}{4} \times \frac{64}{3}} = \frac{4}{\sqrt{3}}$$

$$\text{Form factor} = \frac{\text{rms}}{\text{avg}} = \frac{4}{\sqrt{3}} \times \frac{1}{2} = 1.15.$$

$$\text{Crest factor} = \frac{\text{Peak value}}{\text{rms}} = \frac{4}{2.30} = 1.73.$$

2. Find V_{avg}, rms, form factor and crest factor for following half-wave rectified sinusoidal waveform.



→ Solution:-

$T = 2\pi.$

$$V_{avg} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin\theta \, d\theta + \int_{\pi}^{2\pi} 0 \, d\theta.$$

$$= -\frac{1}{2\pi}\left[V_m \cos\theta\right]_0^{\pi} = \frac{-1}{2\pi} V_m[-1-1]$$

$$= \frac{V_m}{\pi} = 0.318 V_m$$

$$\text{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\theta \, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \left[\frac{1-\cos 2\theta}{2}\right] d\theta}$$

$$= V_m \sqrt{\frac{1}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}}$$

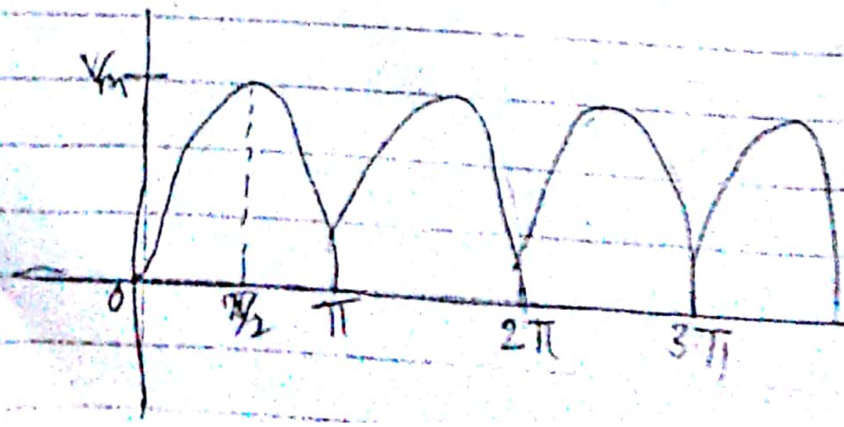
$$= V_m \sqrt{\frac{1}{4\pi}(2\pi - 0)}$$

$$= \frac{V_m}{2}$$

Form factor $= \frac{\text{rms}}{\text{avg}} = \frac{V_m}{2} \times \frac{\pi}{V_m} = 1.57$

Crest factor $= \frac{\text{Peak value}}{\text{rms}} = \frac{V_m}{V_m/2} = 2$

Full wave rectified sinusoidal waveform



Solution:-

$T = \pi$

$\therefore V_{avg} = \frac{1}{\pi} \int_0^{\pi} V_m \sin\theta \, d\theta.$

$= -\frac{1}{\pi} \left[V_m \cos\theta \right]_0^{\pi}$

$= -\frac{V_m}{\pi} (\cos\pi - \cos 0)$

$= \frac{2V_m}{\pi}.$

$rms = \sqrt{\frac{1}{\pi} \int_0^{\pi} V_m^2 \sin^2\theta \, d\theta.}$

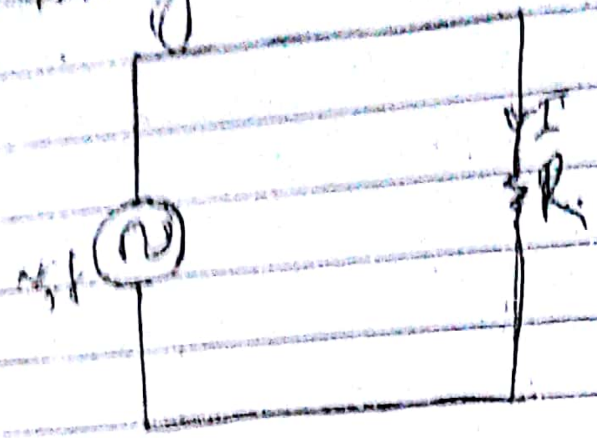
$= V_m \sqrt{\frac{1}{\pi} \int_0^{\pi} \frac{1 - \cos 2\theta}{2} \, d\theta}$

$= V_m \sqrt{\frac{1}{2\pi} \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\pi}}$

$= V_m \sqrt{\frac{1}{2\pi} (\pi +}$

$= \frac{V_m}{\sqrt{2}} = 0.707 V_m$

AC through Resistance



Suppose an ac supply of voltage V and frequency f connected in series to a resistance R.

The supply voltage in $V = V_m \sin\omega t$ .... (1)

According to Ohm's law,

$$V = IR$$

$$\text{or, } I = \frac{V}{R} = \frac{V_m \sin\omega t}{R}$$

$$\text{or, } I = \frac{V_m}{R} \sin\omega t \quad \ldots$$

I comes to be maximum when $\sin\omega t = 1$.

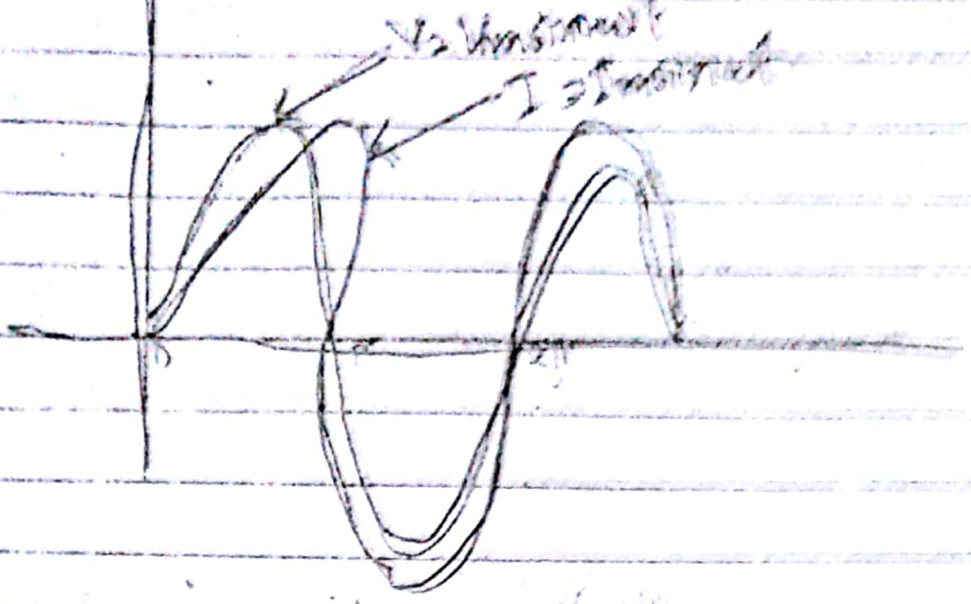
$$\therefore I_m = \frac{V_m}{R}$$

Now,

$$I = I_m \sin\omega t \quad \ldots\ldots (2)$$

Comparing eq^n (1) and (2), we get voltage and current (I) are in phase.

i.e.



In case of resistance, maximum current is independent of frequency.

Vector representation:-

phasor diagram:

Power = voltage x current

$= V_m \sin\omega t \times I_m \sin\omega t$

$= V_m I_m \left(\frac{1 - \cos 2\omega t}{2} \right)$

$= \frac{V_m I_m}{2} - \frac{V_m I_m \cos 2\omega t}{2}$

Power consists of two terms,

$\frac{V_m I_m}{2}$ and $\frac{V_m I_m \cos 2\omega t}{2}$

In complete cycle, $\frac{V_m I_m}{2} \cos 2\omega t$ becomes zero.

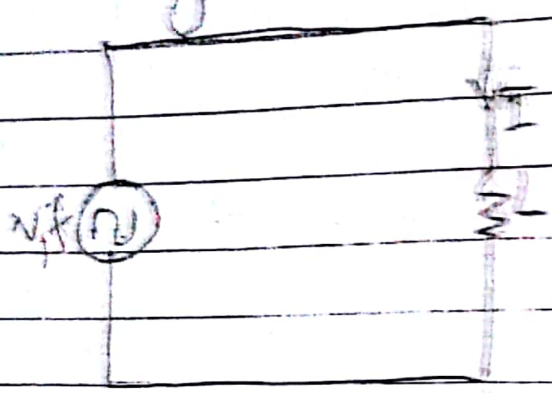
$\therefore$ Power (P) = $\frac{V_m I_m}{2}$

$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$

$= V_{rms} \times I_{rms}$

or, $P = VI$

AC Through inductor :-



Suppose an ac supply of voltage V and frequency f is connected in series to a inductance L.

∴ The voltage across inductor is

$$V \propto \frac{dI}{dt}$$

$$\text{or, } V = L\frac{dI}{dt}$$

where L is inductance in Henry (H)

or, $L\,dI = V\,dt$

or, $dI = \frac{V}{L}\,dt$

We know, $V = V_m \sin\omega t$

or, $dI = \frac{V_m \sin\omega t}{L}\,dt$

or, $dI = \frac{V_m}{L} \sin\omega t \, dt$.

Integration on both sides, we get,

$$\int dI = \frac{V_m}{L} \int \sin\omega t \, dt$$

or, $I = \frac{V_m}{\omega L} (-\cos\omega t)$

$$= \frac{V_m}{X_L} (-\cos\omega t)$$

where $X_L = \omega L$ (inductive reactance)
[unit = Ω (ohm)]

or, $V = V_m \sin\omega t$ ......(1)

Also,
$$I = \frac{V_m}{X_L} \left[\sin\left(\omega t - \pi/2\right) \right]$$

I gets maximum value when $\sin\left(\omega t - \frac{\pi}{2}\right) = 1$.

$$\therefore I_m = \frac{V_m}{X_L}$$

Now, $I = I_m \sin(\omega t - \pi/2)$ .... (2)

Thus, comparing eqn(1) and (2), we get,

$\boxed{\text{Current lags voltage by } \pi/2}$

or, V leads I with $\pi/2$

Phasor diagram.

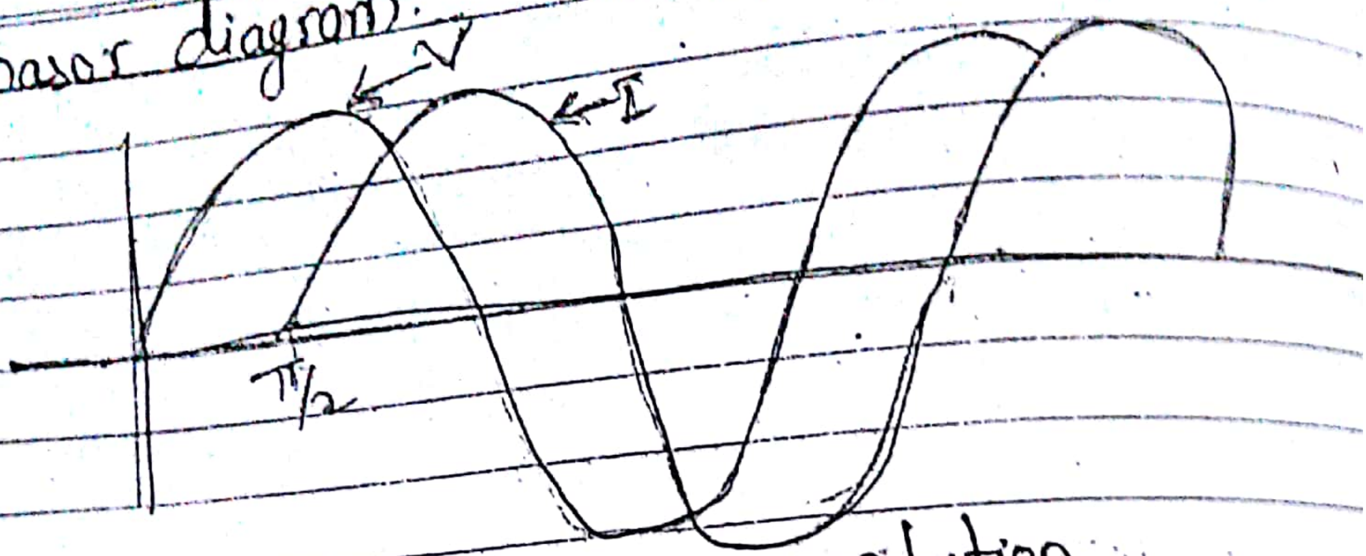
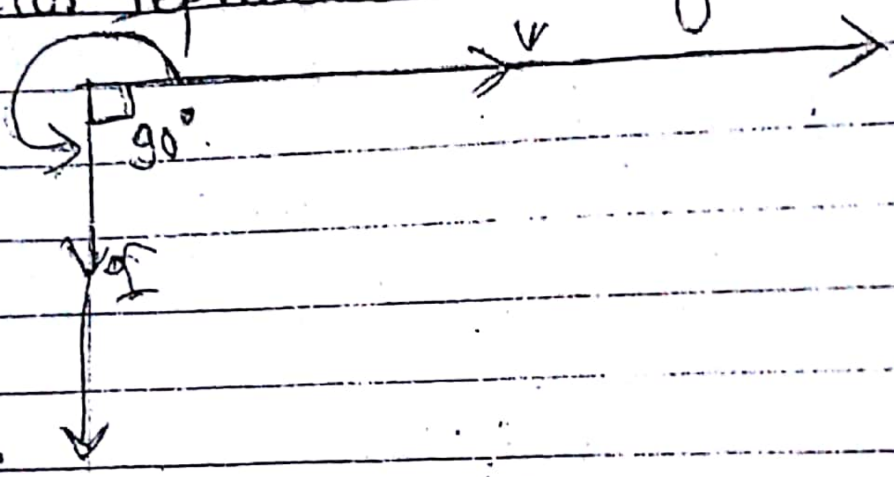


fig. 1. waveform representation.

Vector representation diagram.



Power = Voltage × Current

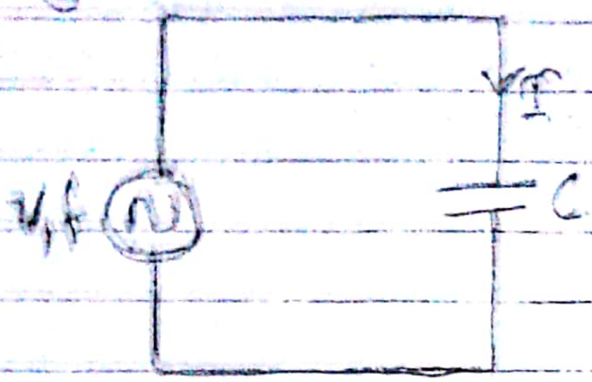
$= V_m \sin\omega t \times I_m \sin(\omega t - \pi/2)$

$= V_m I_m \cdot - \sin\omega t \cdot \cos\omega t$.

$= -V_m \cdot I_m \times \dfrac{2\sin\omega t \cdot \cos\omega t}{2}$

$= -\dfrac{V_m I_m}{2} \cdot \sin 2\omega t$.

AC through capacitor



Suppose an ac supply of voltage V and frequency f is connected in series to a capacitor C.

Then, we have,

$$Q = CV$$

$$\text{Current, } I = \frac{dQ}{dt}$$

$$\text{or, } I = \frac{d(CV)}{dt}$$

$$= \frac{C.d(V)}{dt}$$

$$= \frac{C\, d(V_m \sin\omega t)}{dt}$$

$$= CV_m \cos\omega t . \omega$$

$$= \frac{V_m}{1/\omega C} . \cos\omega t .$$

$$\text{or, } I = \frac{V_m}{X_C} \cos\omega t .$$

where $X_C = \frac{1}{\omega C}$ = capacitive reactance.

$$\therefore I = \frac{V_m}{X_c} \sin(\omega t + \pi/2)$$

I gets maximum value when $\sin(\omega t + \pi/2) = 1$.

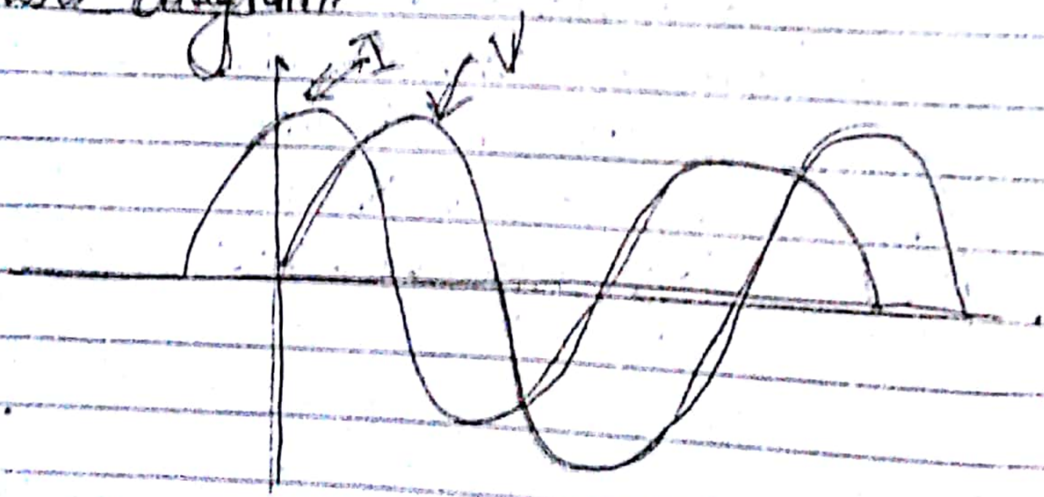
$$\text{or, } I_m = \frac{V_m}{X_c}$$

$$\text{or, } I = I_m \sin(\omega t + \pi/2)$$

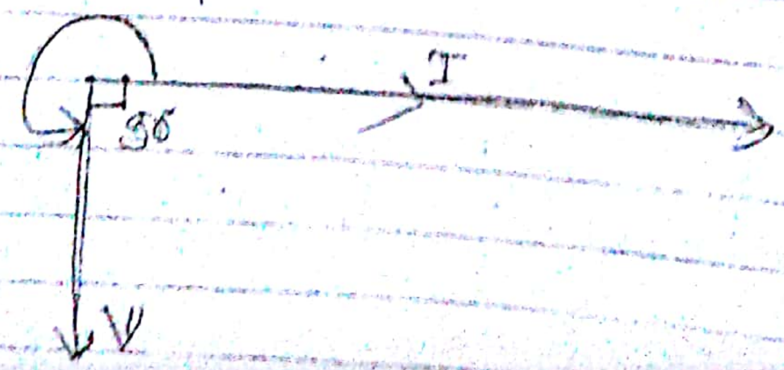
Comparing with $V = V_m \sin \omega t$, we conclude that

$\boxed{\text{current leads voltage with } \pi/2.}$

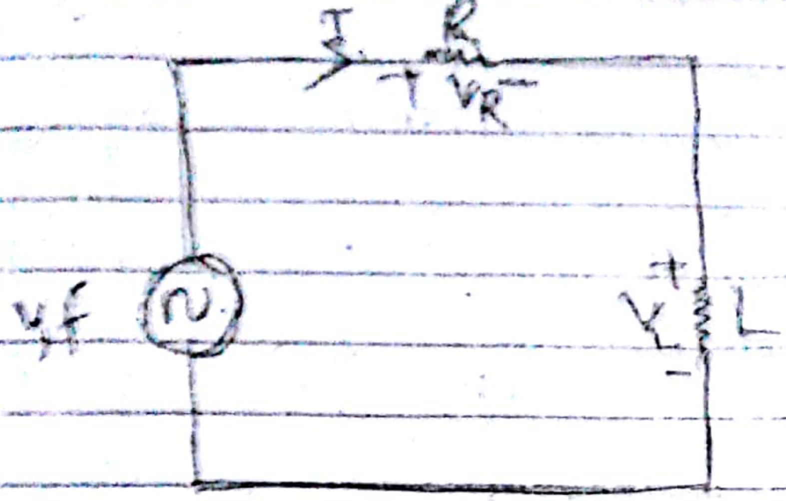
Phaser diagram.



Vector representation:-



Series RL circuit in AC :-



Suppose a resistance R and an inductance L are connected in series with ac supply of voltage V and frequency f.

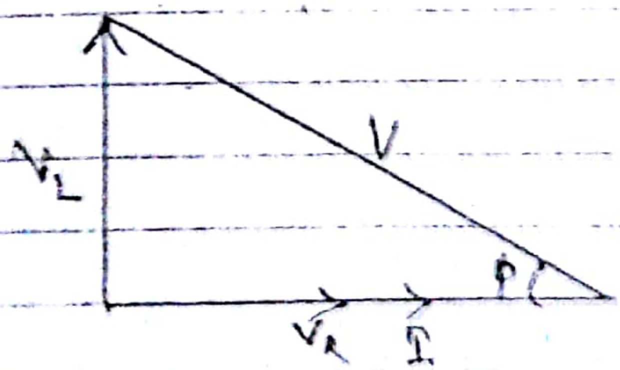
The supply voltage in $V = V_m \sin\omega t$.

Due to flow of current I, there will be voltage drop across resistance and inductor.

$\therefore$ The voltage across $R(V_R) = IR$.

The voltage across $L(V_L) = IX_L$.

Phaser diagram.



$$\text{Resultant}(V) = \sqrt{V_L^2 + V_R^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I\sqrt{R^2 + X_L^2}$$

$$= IX_R Z.$$

where Z is known as impedence and is measured in ohm (Ω).

Phase angle(ϕ):

$$\phi = \tan^{-1}\left(\frac{V_L}{V_R}\right)$$

$$= \tan^{-1}\left(\frac{IX_L}{IR}\right) = \tan^{-1}\left(\frac{X_L}{R}\right)$$

The angle ϕ is known as power factor if it comes with cosine angle.

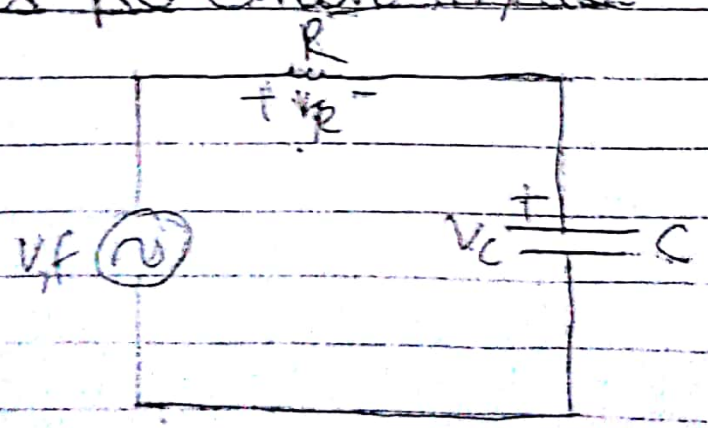
i.e. $\cos\phi$ = power factor.

But in circuit, $\cos\phi = \frac{R}{Z}$

$$Z = \sqrt{R^2 + X_L^2}$$

$|Z| = R + jX_L$ (j = complex operation)

Series RC circuit in AC



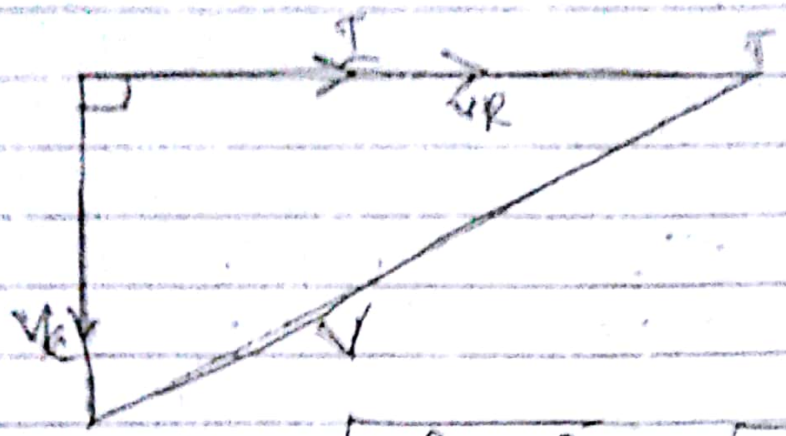
$\boxed{\frac{R}{Z}}$

Suppose a resistance R and a capacitor C are connected in series with an ac mains of supply.

voltage V and frequency f.

V across resistance (V_R) = IR

V across capacitor (V_C) = IX_C.



$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{R^2 + (-X_C)^2}$$

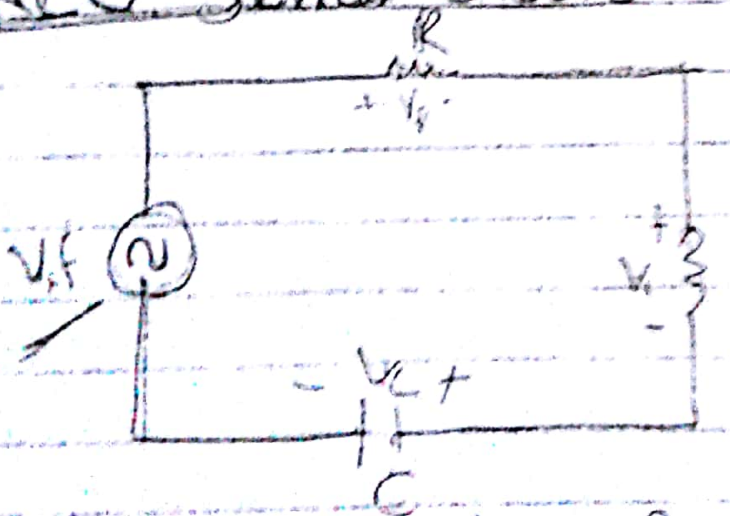
change is taken as discharging, so, $X_C = (-ve)$.

where, $Z = \sqrt{R^2 + X_C^2}$ = Impedence.

$$|Z| = R - jX_C.$$

$$\text{phase angle } (\phi) = \frac{V_C}{V_R} = \tan^{-1}\left(\frac{-IX_C}{IR}\right) = \tan^{-1}\left(-\frac{X_C}{R}\right)$$

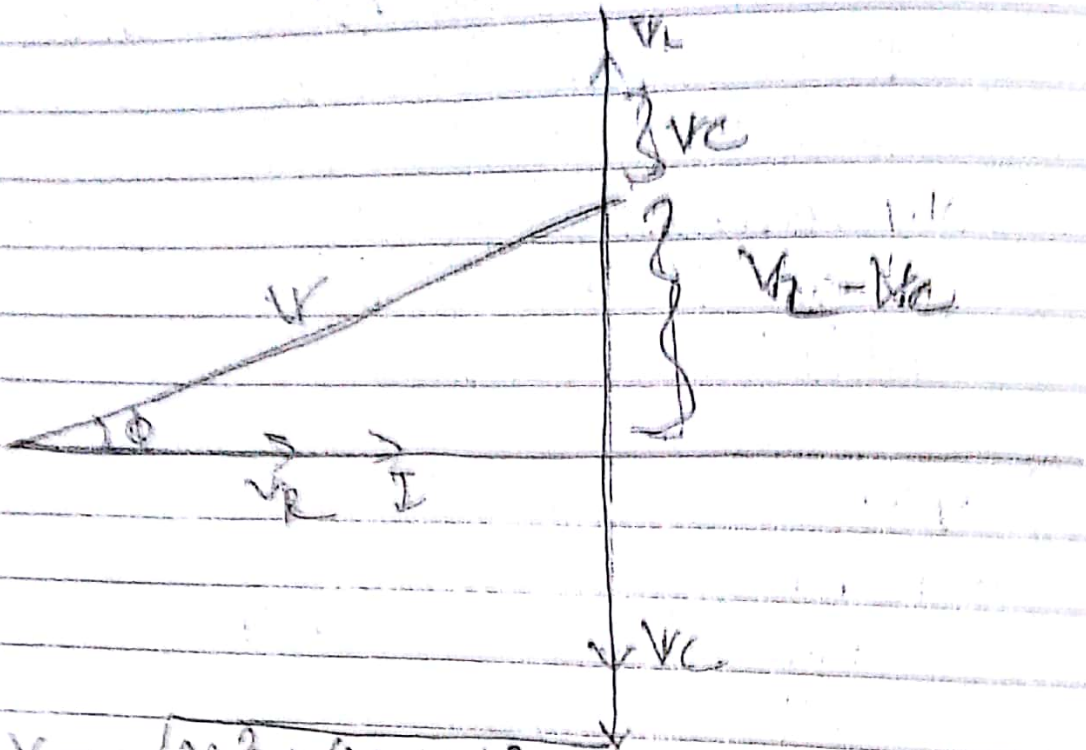
RLC series circuit.



$V = V_m \sin\omega t$

Suppose a resistance R, inductance L and a capacitor C are connected in series with an ac means of supply voltage V and frequency f.

∴ V across resistance (V_R) = IR.
V across inductance (V_L) = IX_L
V across capacitor (V_C) = IX_C.



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2 - 2IX_L \cdot IX_C + (IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$= IZ$, where Z is impedence.

$$\therefore |Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

or, $Z = R + j(X_L - X_C)$

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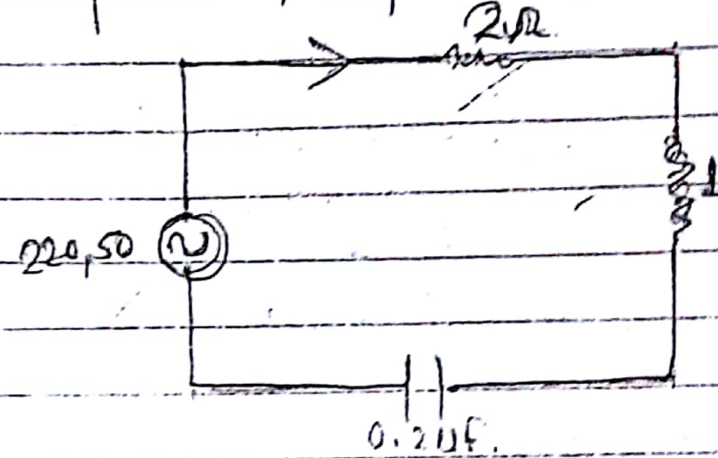
phase angle (ϕ) = $\tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

Suppose a resistance of 2Ω, inductor 1H and capacitance of $0.2\mu F$ are connected in series with 220V, 50Hz ac supply. Find:

a) Voltage across three phases, Impedence
b) Reactance
c) Supply current
d) Phase angle
e) Power factor



→ Solution:-

Given that,

$R = 2\Omega$ $\quad C = 0.2\times10^{-6}F$

$L = 1H$ $\quad V = 220V$ $\quad f = 50Hz.$

we have,

$X_L = \omega L = 2\pi \times 50 \times 1 = \cancel{3.14.11}\ 314.15.$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 0.2\times10^{-6}} = 1.5\times10^{4}$

$\therefore Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{2^2 + (314.15 - 15915.49)^2} = \sqrt{4 + 243401809}$

$Z = 15601.34$

$\therefore$ ~~V = IZ~~

Now,

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a) Voltage across resistance, $V_R = IR = 0.014 \times 2 = 0.028V$
voltage across resistance, $V_L = IR_L = 0.014 \times 1 = 0.014V$
f
Voltage across resistance, $V_C = IR_C = 0.014 \times 0.2 \times 10^{-6}$
$= 2.8 \times 10^{-9}$ V.

b). We have,

$V = IZ$

or, $I = \frac{V}{Z} = \frac{220}{15601.34} = 0.014$

phase angle $(\phi) = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

$= \tan^{-1}(\cancel{89}9 - 7342.925)$

$= \cancel{89.99}\ -89.99°$

and,

power factor $= \cos\phi = \cos(-89.99°) = 0.00017$

Q. An ac of 220v, 50Hz supplied to a series combination of two impedances $(2+j1)\Omega$ and $(1-0.5j)\Omega$. Find,

(i) Current.
(ii) Total impedance
(iii) Phase angle.
(iv) Power factor.

→ Solution:-

$Z_1 = \sqrt{2^2 + 1^2} = \sqrt{5}$

$Z_2 = \sqrt{1^2 + (0.5)^2} = \frac{\sqrt{5}}{2}$

Reactance$(R) = 3$

as the power is known as reactive power. This is the losses in the circuit. It is represented by Q and is given by

$$Q = VI \sin\theta$$

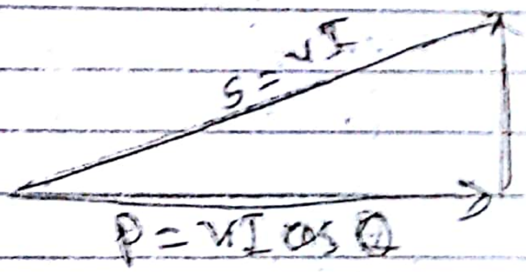
unit: Volt Ampere Reactive (VAR)

3) <u>Apparent power:-</u>

It is the product of voltage rms value and current rms value represented by S.

$$S = VI$$

unit: (VA) volt-Ampere).



$$S = \sqrt{P^2 + Q^2}$$

$$= \sqrt{(VI)^2 \cos^2\theta + (VI)^2 \sin^2\theta}$$

$$\therefore S = VI.$$

Q. What are the significance of power factor?

1. The current needed to obtain a given power is very high. This in turn results of the resistive losses, thereby decreases efficiency.
2. It causes a poor voltage regulation.
3. It limits the output of generators and transformer.
4. It causes fall in terminal voltage.
5. It causes more power loss in the line.
6. It reduces kwh, since the true power is less.
7. It reduces the torque of the consumer motor.

$\therefore Z = \sqrt{5} + \frac{\sqrt{5}}{2} = \frac{3\sqrt{5}}{2}$

$I = \frac{V}{Z} = \frac{220}{3\sqrt{5}} \times 2 = \frac{88\sqrt{5}}{3}$

phase angle $(\phi) = \tan^{-1}\frac{1 - 0.5}{2} = 14.0362°$

power factor $= \cos\phi = \frac{R}{Z} = \frac{2}{\sqrt{5 + 5/2}} = \frac{2\sqrt{30}}{15}$

Power :-

1. Active power/True power/real power/wattful power.
2. Reactive power
3. Apparent power.

1). Active power :-

In one complete cycle, there is stored energy in the circuit. If the energy flows in constant direction to the load as power, this is known as the active or true power. It is measured in watt. It is given by,

$$P = VI\cos\theta \ (\text{unit: watt})$$

where, $\cos\theta$ = power factor.

2). Reactive power :- (wattless power)

The energy stored in any half cycle of voltage and current ~~to~~ flows towards the source

Some Complex operation.

Cartesian form = $z = a+ib = \sqrt{a^2+b^2}$

Polar form = $r\angle\theta$

where r = magnitude and

θ = phase angle.

Conversion of cartesian form to polar form

$$r = \sqrt{a^2+b^2} \quad \& \quad \theta = \tan^{-1}(b/a)$$

Conversion of polar form to cartesian form.

$$r\angle\theta = r(\cos\theta + j\sin\theta)$$
$$= r\cos\theta + jr\sin\theta.$$

For cartesian form:-

1. Multiplication:-

$$z_1 \cdot z_2 = (2+j3)\cdot(1+j2)$$
$$= 2+4j+3j-6$$
$$= -4+7j.$$

2. Division:-

$$\frac{z_1}{z_2} = \frac{2+3j}{1+2j} = \frac{2+3j}{1+2j} \times \frac{1-2j}{1-2j}$$

For polar form:-

$$z_1 = 2\angle 30^\circ$$
$$z_2 = 3\angle 60^\circ$$

1) Addition (not possible) : $z_1 + z_2 \neq$

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4) Multiplication:-

$$Z_1 \cdot Z_2 = 2\angle 30^\circ \times 3\angle 60^\circ$$
$$= 6\angle 30+60^\circ$$
$$= 6\angle 90^\circ$$

5) Division:-

$$\frac{Z_1}{Z_2} = \frac{2\angle 30}{3\angle 60} = \frac{2}{3}\angle 30^\circ - 60^\circ$$

Q. An ac supply of 220V, 50Hz in supply to a series combination of 5Ω resistance, 1H inductor and 0.5F capacitor. Find the following.

1. Impedance
2. Current
3. Power factor
4. Active power
5. Reactive power
6. Apparent power.
7. Nature of circuit.

→ Solution:-

Given that,

voltage (V) = 220V

f = 50 Hz

Resistance (R) = 5Ω

Inductor (L) = 1H

Capacitor (C) = 0.5 F

We have,

$$X_L = \omega L = 2\pi f L = 100\pi.$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = 6.36\times 10^{-3} = 0.0063\Omega.$$

Now,

(1) Impedance $(Z) = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{5^2 + (314.1 - 0.0063)^2}$

$$= 314.13\,\Omega$$

ii) $I = \frac{V}{Z} = \frac{220}{314.1} = 0.70\,A.$

iii) Power factor $(\cos\theta) = \frac{R}{Z} = \frac{5}{314.1} = 0.015.$

iv). Active power (P_{active}) $= VI\cos\theta$

$= 220 \times \frac{220}{314.1} \times \frac{5}{314.1} = 2.31\,W.$

v) Reactive power $= VI\sin\theta = 220 \times 0.70 \times \sqrt{1-(0.015)^2}$

$= 154.07\,VAR.$

vi) Apparent power $= VI = 220 \times 0.7 = 154.09\,VA.$

vii) ~~Curit~~ Circuit is resistive in nature.

Losses in transformer

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Hysteresis losses:
It is due to reversal of magnetization in the core. It depends upon volume & grade of iron, frequency of reversal & flux density.

Eddy current loss:
In primary magnet

Note:- $x + j1 \rightarrow$ Inductive in nature

$x - j1 \rightarrow$ capacitive

Q. An ac supply of $220\angle 30°$, 50Hz in supply to a series combination of two impedance $(3+2j)\Omega$ and $(4-1j)\Omega$. Find:

i) Impedance
ii) Current.
iii) Power factor
iv) Active power
v) Reactive power
vi) Apparent power
vii) Nature of the circuit.

> Solution:-

Given that,

$V = 220\angle 30°$ $\qquad f = 50\,Hz.$

i) Impedance (Z) $= (3+2j) + (4-1j) = (7+j)\,\Omega.$

ii) Current (I) $= \frac{V_{rms}}{Z} = \frac{220\angle 30°}{7+j} = \frac{220\angle 30}{\sqrt{50}\,(\tan^{-1}(1/7))} = 31.11\angle$

iii) Power factor $(\cos\theta) = \frac{R}{Z} = \frac{7}{|7+j|} = \frac{7}{\sqrt{50}} = 0.99$

iv) Active power $= VI\cos\theta$

$= 220 \times 31.11 \times 0.99$

$= 6775.75\,W.$

v). Reactive power $= VI\sin\theta = 220 \times 31.11 \times \sqrt{1-(0.99)^2}$

$= 68.44\,VAR.$

vi) ~~Active~~ Apperaut power $= VI = 220 \times 31.11 = 6844.2\,VA$

vii) Nature = inductive

Q. An ac supply of $V = 220 \sin(\omega t + 60^\circ)$, 50Hz is supplied to a parallel combination of two impedances $(3+2j)\Omega$ and $(4-j)\Omega$. Find the following.

→ Solution:-

(a)

$V = 220 \sin(\omega t + 60^\circ)$

$f = 50Hz.$

$Z_1 = (3+2j)\Omega$

$Z_2 = (4-1j)\Omega.$

(i) $\text{Impedance}(Z) = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} = \frac{(3+2j)(4-j1)}{(7+j)}$

$= \frac{12-3j+8j+2}{7+j}$

$= \frac{14+5j}{7+j} = \frac{\sqrt{14^2+5^2}\ \angle \tan^{-1}(5/14)}{\sqrt{50}\ \angle \tan^{-1}(1/7)}$

$= 2.106 \angle 11.56.$

ii) $\text{Current}(I) = \frac{V_{rms}}{Z} = \frac{V}{\sqrt{2}} \times \frac{1}{Z} = \frac{220 \angle 60}{\sqrt{2} \times 2.106 \angle 11.56}$

$= 73.86 \angle 48.44$

iii) $Z = 2.106 \angle 11.56$

$= 2.106 (\cos 11.56 + j \sin 11.56)$

$= \underbrace{2.106 \times \cos 11.56}_{R} + j \cdot 2.106 \sin 11.56$

$= 2.063.$

$\therefore \cos\theta = \frac{R}{Z} = \frac{2.063}{2.106} = 0.979$

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iv) Active power $= VI\cos\theta = \frac{220}{\sqrt{2}} \times 73.86 \times 0.979$

$= 11248.63\,W.$

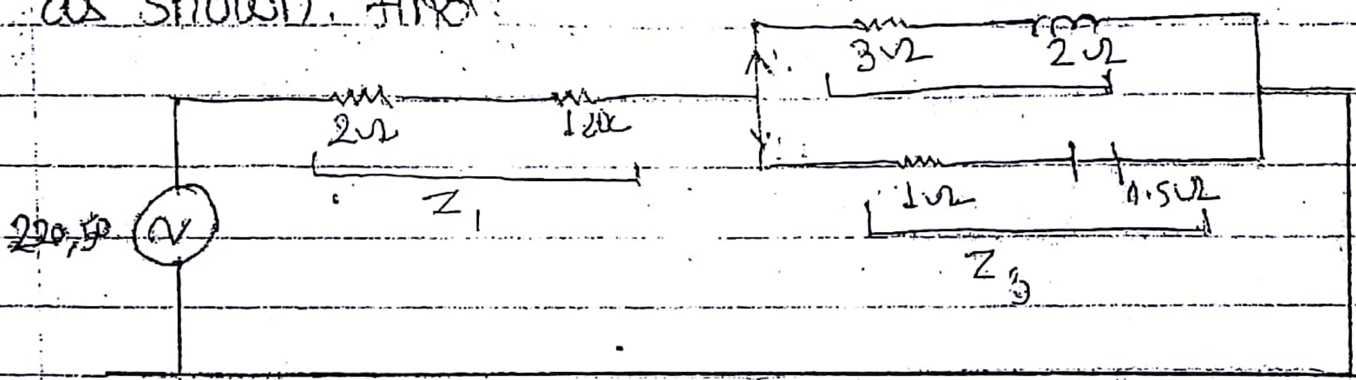
v) Reactive power $= VI\sin\theta = \frac{220}{\sqrt{2}} \times 73.86 \times \sqrt{1-(0.979)^2}$

$= 2342.33\,VAR$

vi) Apparent power $= VI = \frac{220}{\sqrt{2}} \times 73.86 = 11489.91\,VA$

vii) Nature = inductive.

Q. An ac supply of 220V, 50Hz is supplied to the c as shown. find:



a) Total impedence
ⓑ Current (I)
ⓒ Currents (I_1 and I_2)
ⓓ Power factor
ⓔ Active power
ⓕ Reactive power
ⓖ Apparent power.

→ Solution:-

Given, That,

$Z_1 = (2+j1)\,\Omega$

$Z_2 = (3+j2)\,\Omega$

$Z_3 = (1-0.5j)\,\Omega$

$\therefore$ a) Impedance $(Z) = Z_1 + \left(\frac{Z_2 \times Z_3}{Z_1 + Z_3}\right)$

$$= 2+J1 + \frac{(3+J2)(1-0.5J)}{(3+J2)+(1-0.5J)}$$

$$= 2+J1 + \frac{3-1.5J+J2+1}{4+1.5J}$$

$$= 2+J1 + \frac{4+0.5J}{4+1.5J}$$

$$= \frac{10.5+7.5J}{4+1.5J}$$

$$= \frac{\sqrt{(10.5)^2+(7.5)^2}\ \angle \tan^{-1}(7.5/10.5)}{\sqrt{(4)^2+1.5^2}\ \angle \tan^{-1}(1.5/4)}$$

$$= \frac{12.9\ \angle 35.53}{4.27\ \angle 20.55}$$

$$= 3.02\ \angle 14.98 \simeq 3.02\ \angle 15^\circ$$

b) $I = \frac{V}{Z} = \frac{220\angle 0^\circ}{3.02\angle 15^\circ} = 72.84\angle -15^\circ$

c) $I_1 = I \times \frac{Z_2}{Z_2+Z_3} = 72.84\angle -15 \times \frac{\cancel{1-0.5J}\ 1-0.5J}{(3+J2)+(1-0.5J)}$

$=$

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$$I_2 = I \times \frac{Z_3}{Z_2+Z_3} =$$

iv) $\cos\theta = \frac{R}{Z} = \frac{2.91}{3.02} = 0.96$

$Z = 3.02\angle 15^\circ$
$= 3.02(\cos 15^\circ + j\sin 15^\circ)$
$= 3.02 \times \cos 15^\circ + 3.02 \times \sin 15^\circ j$
$= 2.91 + 0.78j$

v) Active power $= VI\cos\theta = 220 \times 72.84 \times 0.96$
$= 15383.8W.$

vi) Reactive power $= VI\sin\theta = 220 \times 72.84 \times \sqrt{1-(0.96)^2}$
$= 4486.944 VAR.$

vii) Apparant power $= VI = 220 \times 72.84$
$= 16024.8VA.$

Old Q. A coil of inductance 0.08 H and neglegible resistance is connected in series with a 15Ω non-inductive resistance. The combined circuit is energised from a 220V, 50Hz supply. Calculate:

(a) Reactance of coil.
(b) Current in the circuit
(c) Voltage across circuit
(d) Power absorbed by ckt.
(d) Power factor.
(e) Impedance of circuit.
(f) Voltage across resistance.

→ Solution:-

inductance $(L) = 0.08H$

Resistance $(R) = 15\Omega$

voltage $(V) = 220V.$

(i) $X_L = \omega L = 2\pi f L = 8\pi = 25.13\Omega.$

(ii) $I = \frac{V}{Z} = \frac{240}{\sqrt{R^2+X_L^2}} = \frac{240}{\sqrt{15^2+(25.13)^2}} = 8.2A.$

(iii) $V_{X_L} = IX_L = 8.2 \times 25.13 = 206.07V$

(iv). Power factor $(\cos\theta) = \frac{R}{Z} = \frac{15}{\sqrt{15^2+(25.13)^2}} = 0.51.$

(v). $Z = \sqrt{(15)^2+(25.13)^2} = 29.26$

(vi). $V_R = IR = 8.2 \times 15 = 123V$

(vii) Absorbing power = active power $= VI\cos\theta$

$= 240 \times 8.2 \times 0.51$

$= 1003.68$ W.

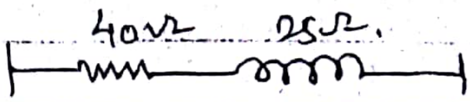
Q. ~~VIII~~ A coil of resistance 40Ω and inductive reactance of 25Ω is connected in series to 220V, 50Hz supply. Calculate:-

(a). Active and reactive component.

b). Total power of the circuit.

> Solution:-

Given that,



Resistance (R) = 40Ω

Reactance (L) = 25Ω.

Voltage (V) = 220 V.

We have,

$$I = \frac{V}{Z} = \frac{220}{\sqrt{40^2+25^2}} = 4.66 A.$$

and,

$$\cos\theta = \frac{R}{Z} = \frac{40}{47.16} = 0.84.$$

∴ i) Active component = $VI\cos\theta$

$= 220 \times 4.66 \times 0.84$

$= 861.16$ W

Reactive component = $VI\sin\theta$ $= 1025.2 \times 4.66 \times \sqrt{1-(0.84)^2}$

$= 556.25$ VAR

ii) Total power of the circuit = VI

$= 220 \times 4.66$

$= 1025.2$ VA.

Q. In an RLC series circuit, current supplied by the single phase ac source is $15\angle{-38^\circ}$. Determine the value of all three kind of power if $R = 100\Omega$, $X_L = 35\Omega$ and $X_c = 25\Omega$.

> Solution:-

Given that, $I = 15\angle{-38^\circ}$

$R = 100\Omega$ $X_L = 35\Omega$ $X_c = 25\Omega$

$\therefore Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{100^2 + 10^2} = 100.49$

Also,

$V = IZ$

$= 15\angle{-38} \times 100.49$

$= 1507.35\angle{-38^\circ}$

And,

$\cos\theta = \frac{R}{Z} = \frac{100}{100.49} = 0.99$

Now,

Active power $= VI\cos\theta = 1507.35 \times 15 \times 0.99$

$= 22500\ W.$

Reactive power $= VI\sin\theta = 3189.56\ VAR.$

and,

Apparent power $= VI = 22610.25\ VA$

Q. An ac supply of 220V, 50Hz is supplied to a resistance of 5Ω and an inductance of 0.02H in series. Find the following:

(a) Impedence
(b) Current
(c) Power factor
(d) Voltage across resistor
(e) Voltage across inductor
(f) Active power or real power
(g) Apparent power
(h) Reactive power
(i) Quality factor (Q)

→ Solution:-

Given that,

$V = 220V$ $R = 5\Omega$

$f = 50\ Hz$ $L = 0.02H$

a) $Z = \sqrt{R^2 + X_L^2} = \sqrt{5^2 + (\omega L)^2} = \sqrt{25 + (2\pi f L)^2}$

$= \sqrt{25 \times (2\pi \times 50 \times 0.02)^2} = 8.027\,\Omega$

b) Current $(I) = \frac{V}{Z} = \frac{220}{8.027} = 27.40\,A.$

(c) power factor $(\cos\theta) = \frac{R}{Z} = \frac{5}{8.027} = 0.62$

(d) $V_R = IR = 27.40 \times 5 = 137\,V.$

(e) $V_L = IL = 27.40 \times 2\pi \times 50 \times 0.02 = 172.15\,V.$

(f) Active power $= VI\cos\theta$

$= 220 \times 27.40 \times 0.62$

$= 3737.36\,W$

(g). Apparent power $= VI = 220 \times 27.40 = 6028\,VA$

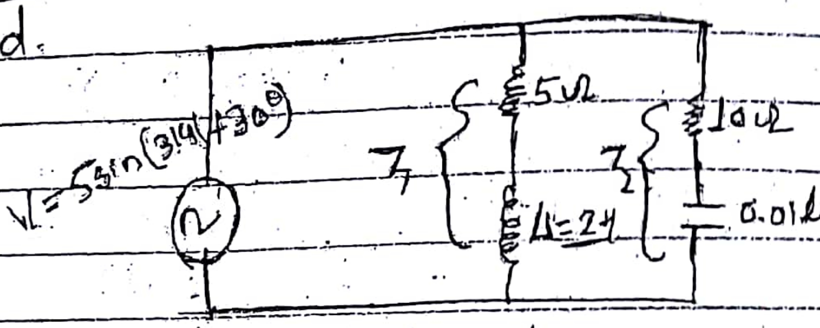
h) Reactive power $= VI\sin\theta$

$= 220 \times 27.40 \times \sqrt{1 - (0.62)^2}$

$= 4729.57\,VAR.$

i) Quality factor $= \frac{Z}{R} = \frac{8.027}{5} = 1.6054.$

Q. A voltage of $\boxed{V = 5\sin(314t + 30°)}$ is supplied in the given circuit. Find
1) Impedence
11) Current
3) powers, power factor
4) Draw phaser diagram.



inductor = +
capacitor = -

> Solution:-
We have,

$$V = 5\sin(314t + 30°)$$

$$Z_1 = 5 + jX_L$$

$$Z_1 = 5 + j\omega L$$
$$= 5 + j \cdot 2\pi f L$$
$$= 5 + j\,314 \times 2$$
$$Z_1 = 5 + j\,628\,\Omega.$$

$$Z_2 = 10 - jX_C$$
$$= 10 - j\frac{1}{\omega_L}$$
$$= 10 - j\frac{1}{314 \times 0.01 \times 10^{-6}}$$
$$= 10 - j\,318.47)\,\Omega$$

$\therefore$ Total impedence $(Z) = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}$ (in parallel)

=

$5V$

$V_{rms} = \frac{5}{\sqrt{2}}$

$V_{rms} = 220 V$

Current (I) = $\frac{V_{rms}}{Z} = \frac{V_m / \sqrt{2}}{Z} = \frac{5/\sqrt{2} \angle 30^\circ}{Z} =$

Powers

Active power =

If ~~R~~ = $2\,\Omega$

$Z_1 = 5 + j2$

If $L = 2H$

$x_L = 2\pi f L$

$=$

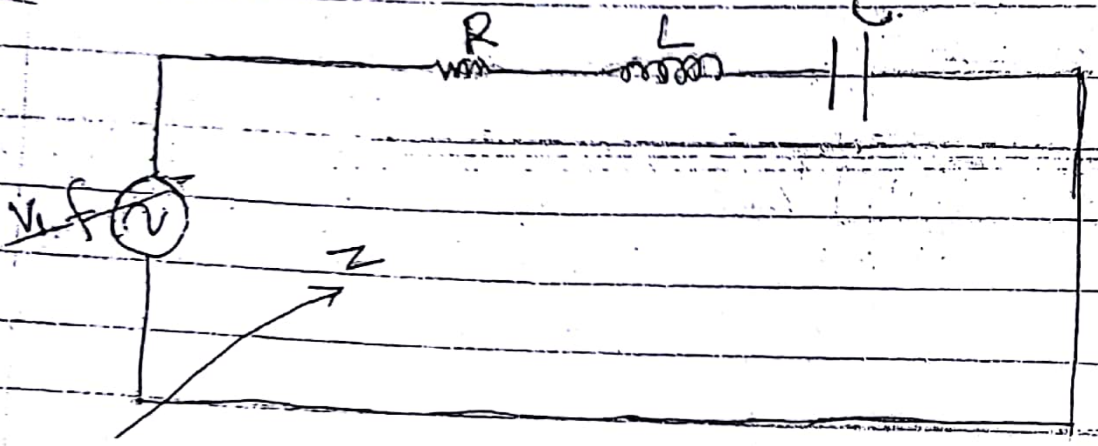
$Z_1 = 5 + j2(\pi f L)$

Series Resonance

2010 f:

Resonance:-

A state when the electrical energy and magnetic energy transferred from one form to another is known as resonance.



When a constant voltage of variable frequency is applied to a series combination of LRC, a state comes when the net reactance cancel each other. The state is known as resonance. The frequency at which such happen is known as resonance frequency.

In series RLC,

impedence $Z = \sqrt{R^2 + (X_L - X_C)^2}$

or, $R + j(X_L - X_C)$

i.e. at resonance,

$$X_L - X_C = 0$$

or, $X_L = X_C$.

or, $\omega_L = \frac{1}{\omega C}$

$$\text{or, } \omega^2 = \frac{1}{LC}$$

$$\text{or, } (2\pi f_0)^2 = \frac{1}{LC}$$

$$\text{or, } 4\pi^2 f_0^2 = \frac{1}{LC}$$

$$\text{or, } f_0^2 = \frac{1}{4\pi^2 LC}$$

$$\text{Resonance } (f_0) = \frac{1}{2\pi\sqrt{LC}}$$

At resonance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

but $X_L - X_C = 0$

$$\text{or, } Z_0 = \sqrt{R^2} = R \text{ (minimum)}$$

i.e. the circuit behaves as purely resistive.

And,

current at resonance, $I_0 = \frac{V}{Z}$.

At resonance, $Z_0 = R$

$$\text{or, } I_0 = \frac{V}{R} \text{ (maximum)}$$

Power factor,

$$\cos\theta = \frac{R}{Z}$$

At resonance, $Z_0 = R$

$\therefore$ Power factor $\cos\phi = \frac{R}{R} = 1$ (max)

Series Resonance (Band width):

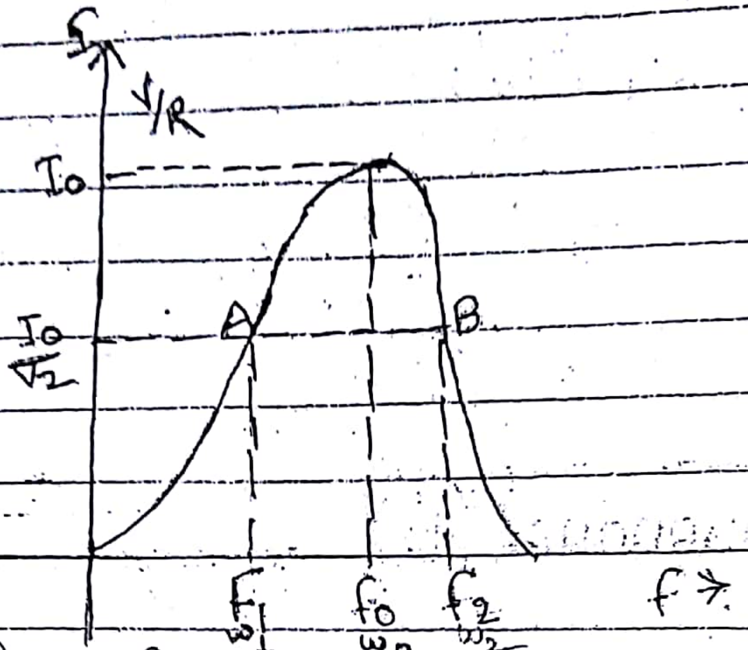


Fig. 1. Resonance curve.

A and B points are known as half power point where the resonant circuit falls to $\frac{I_0}{\sqrt{2}}$

$\therefore$ Bandwidth, $\Delta F = f_2 - f_1$.

where f_1 and f_2 are known as corner frequencies.

We know,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$= \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad \ldots\ldots (1)$$

Also, we know that,

$$I = \frac{I_0}{\sqrt{2}} = \frac{V}{R\sqrt{2}} \quad \ldots\ldots (2)$$

Now, equating ① and ② we get,

$$\frac{V}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}} = \frac{V}{R\sqrt{2}}$$

$$or, \; R^2+\left(\omega L-\frac{1}{\omega C}\right)^2 = 2R^2$$

$$or, \; \left(\omega L-\frac{1}{\omega C}\right)^2 = R^2$$

$$or, \; \left(\omega L-\frac{1}{\omega C}\right)^2 = R^2$$

$$or, \; \omega L-\frac{1}{\omega C} = \pm R$$

For ω_1 and ω_2 we get,

$$\omega_2 L-\frac{1}{\omega_2 C} = R \quad —(3)$$

$$\omega_1 L-\frac{1}{\omega_1 C} = -R \quad —(4)$$

Now adding eqn. (3) and (4), we get,

$$(\omega_1+\omega_2)L-\frac{1}{C}\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right) = 0$$

$$or, \; (\omega_1+\omega_2)-\frac{1}{LC}\left(\frac{\omega_1+\omega_2}{\omega_1\omega_2}\right) = 0$$

common

$$or, \; (\omega_1+\omega_2)\left(1-\frac{1}{LC}\frac{1}{\omega_1\omega_2}\right) = 0$$

$$\text{or, } 1 - \frac{\omega_0^2}{\omega_1 \omega_2} = 0 \qquad \left(\because \frac{1}{LC} = \omega_0^2\right)$$

$$\text{or, } \omega_0^2 = \omega_1 . \omega_2$$

$$\text{or, } \omega_0 = \sqrt{\omega_1 . \omega_2} \qquad \ldots (5)$$

Subtracting eqn. (4) and eqn. (3), we get.

$$(\omega_2 - \omega_1)L + \frac{1}{C}\left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right) = 2R.$$

$$\text{or, } (\omega_2 - \omega_1) + \frac{1}{LC}\left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right) = \frac{2R}{L}$$

$\because \Delta\omega = \omega_2 - \omega_1$ in radian form (Bandwith).

$$\text{or, } \Delta\omega + \omega_0^2 \frac{\Delta\omega}{\omega_0^2} = \frac{2R}{L}$$

$$\text{or, } 2\Delta\omega = \frac{2R}{L}$$

$$\text{or, } \boxed{\Delta\omega = \frac{R}{L}}$$

$$\text{or, } \Delta 2\pi f = \frac{R}{L}$$

$$* \quad \therefore \boxed{\Delta f = \frac{R}{2\pi L}}$$

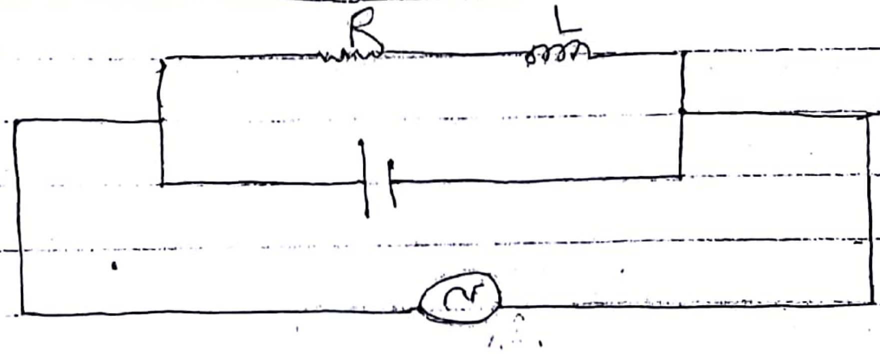
* Corner frequency $f_1 = f_0 - \frac{R}{4\pi L}$

$$f_2 = f_0 + \frac{R}{4\pi L}$$

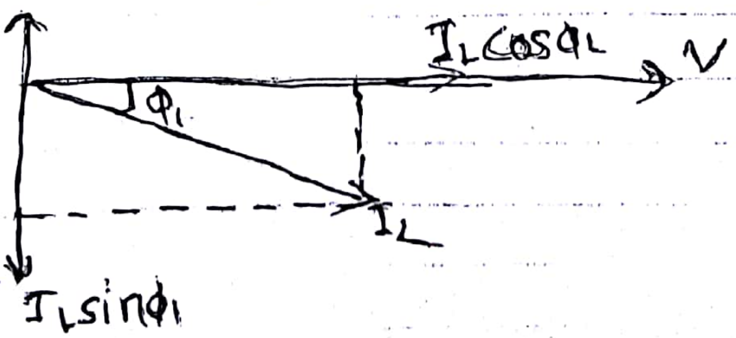
Quality factor (Q):-

This represents the magnification factor and is given by $Q = \frac{1}{\omega_0 CR} = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$.

Parallel resonance (RLC) circuit:- (Imp)



such a circuit is said to be in electrical resonance when the reactive (or wattless) component of line current becomes zero. The frequencies at which this happens is known as resonant frequency.



net reactive or wattless component is $I_C - I_L \sin\phi_L$.
As at resonance value is zero, hence

$$I_C - I_L \sin\phi_L = 0$$

$$\text{or, } I_L \sin\phi_L = I_C \quad \ldots (1)$$

Now, $I_L = \frac{V}{Z}$, $\sin\phi_L = \frac{X_L}{Z}$ and $I_C = \frac{V}{X_C}$

so, from eqn. (i)

$$\frac{V}{Z} \times \frac{X_L}{Z} = \frac{V}{X_C}$$

$$\text{or, } X_L \times C = Z^2$$

$$\text{or, } \frac{\omega L}{\omega C} = Z^2$$

$$\text{or, } \frac{L}{C} = R^2 + X_L^2 = R^2 + (2\pi f L)^2$$

$$\text{or, } (2\pi f_0 L)^2 = \frac{L}{C} - R^2$$

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

This is resonance frequency and if R is negligible we get,

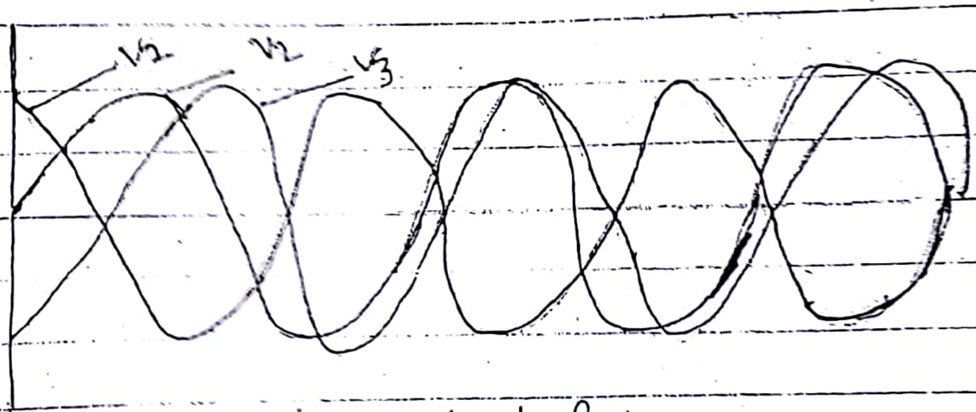
$$\boxed{f_0 = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}}$$

Polyphase AC.

3- Phase AC (3ϕ)

Advantages of 3 phase ac over single phase.

1. 3 phase ac is more efficient than single phase a.c.
2. 3-ϕ ac consumes less materials (eg. copper or aluminium) than the single phase.
3. 3ϕ a.c. equipments are self-starter.



3ϕ voltages V_1, V_2 & V_3.

4. The waveform in 3ϕ never (longertime) falls to the zero value.
5. Polyphase motors are more compact, whereas single-phase motors requires more space for conveying the same power.
6. Polyphase generators work in parallel without any difficulty.

Polyphase	Single phase.
1. Power in it never falls to zero.	1. Power in it may fall to zero.
2. It requires less copper for transmission of same power.	2. It requires more copper for transmitting the same power.
3. Polyphase motors have uniform torque.	3. It has fluctuating torque which is serious disadvantages in large machin
4. Its motors are more compact.	4. Its motor requires more space for conveying the same power.
5. Its motors can be made self-starting.	5. Single phase motor posses no starting torque, so they must be provided with additional starting device.
6. Its generators can be worked in parallel, without any difficulty.	6. Its generators cannot be operated in parallel.
7. Transmission and distribution of polyphase power is cheap, and more efficient.	7. Transmission and distribution of single phase power is costly and less efficient.
8. Its efficiency is high.	8. Its efficiency is low.
9. Its more economical in use.	9. Its more costly in use.
10. For a given size of frame, its output is high.	10. For a given size of frame, its output is low.

Star (Y) connection	Delta (Δ) connection.
1. $V_L = \sqrt{3} V_{ph}$ & $I_L = I_{ph}$	1) $V_L = V_{ph}$ and $I_L = \sqrt{3} I_{ph}$,
2. Line voltages are 30° ahead of the respective phase voltages.	2. Line currents are 30° behind the respective phase currents.
3. For a balanced system, the resultant potential at the neutral point is zero.	3. In the balanced system, the resultant in the closed circuit is zero.
4. System can be arranged to suit both lighting and power circuits simultaneously.	4. System can be arranged only to suit either lighting or power.
5. Star connection transformer work less satisfactory.	5. Its connection transformers work more satisfactory.
6. This type of connections are not suitable for rotary converters.	6. It is only connection suitable for rotary converters.
7. Used mostly for high voltage 3-ϕ generators	7. Used mostly for small & low voltage 3-ϕ motors.

Similarities :-

1. Line voltages are 120° apart.
2. Apparent power $= \sqrt{3} V_L I_L$.
3. $P_{total} = \sqrt{3} V_L I_L \cos\phi$.

4. Angle between line currents and corresponding line voltage is $(30^\circ + \phi)$.

Power drawn in a.c. systems

Types of power	Single-phase	Three-phase.
1. Active power or P (in W)	$VI\cos\phi$	$\sqrt{3}V_L I_L \cos\phi$
2. Volt ampere (in VA)	VI	$\sqrt{3}V_L I_L$
3. Reactive volt ampere (VAr)	$VI\sin\phi$	$\sqrt{3}V_L I_L \sin\phi$.

Note:-

1. In 3-ϕ star connection,

$$I_L = I_{ph} \text{ and } V_L = \sqrt{3}V_{ph}.$$

2. In 3ϕ delta connection,

$$V_L = V_{ph} \text{ and } I_L = \sqrt{3}I_{ph}.$$

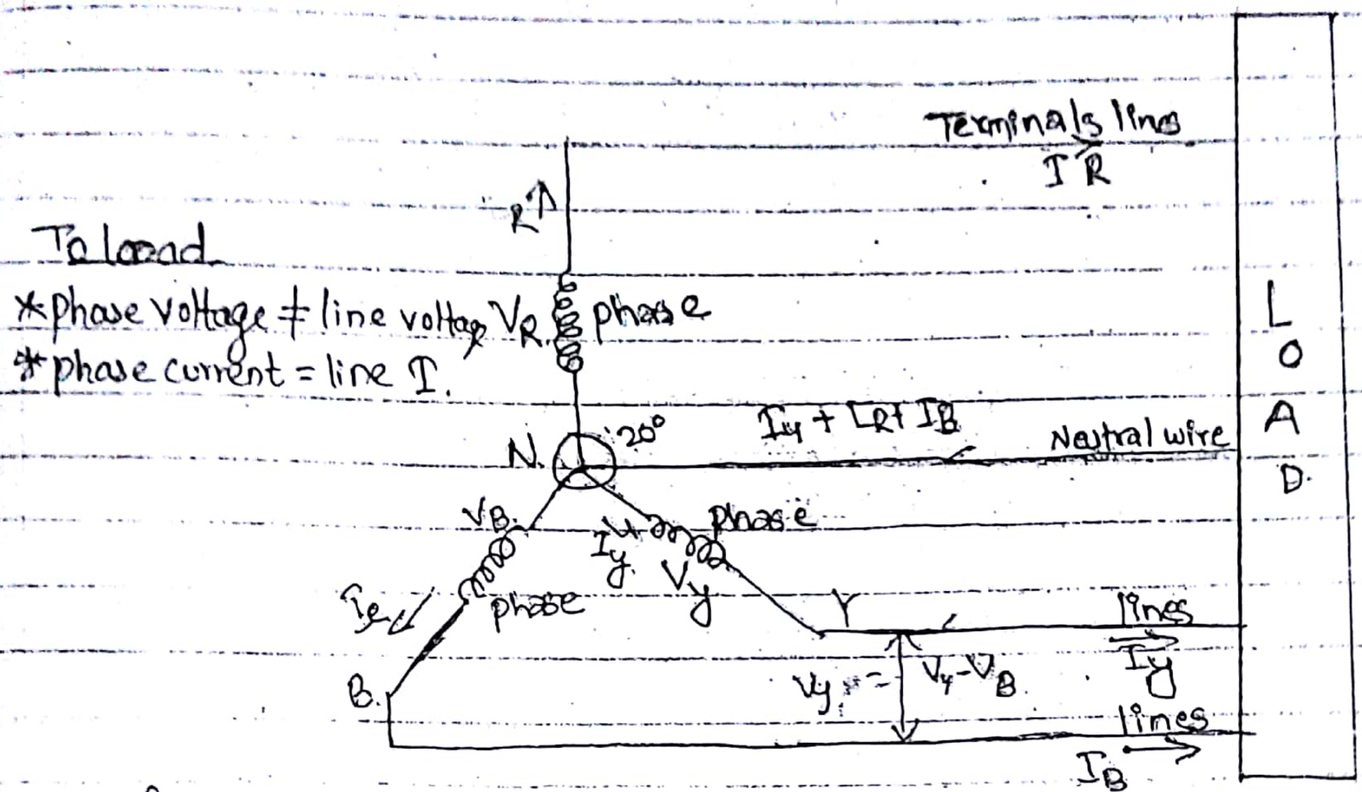
Connections of 3-ϕ a.c.

It can be connected by two methods:

1) wye (Y) or star connection.

2. Delta (Δ) connection.

1. Star connection:-



If an additional neutral wire drawn from the neutral point (N) the system is known as 3 Φ, 4 wire system other wire 3 Φ, 3 wire system.

In star connection,

phase current = line current

or, $I_{ph} = I_L$

We have,

$$V_R = V\sin(\omega t)$$

$$V_Y = V\sin(\omega t - 120)$$

$$V_B = V\sin(\omega t - 240^\circ)$$

From Kirchoff's law, we have,

$$V_R + V_Y + V_B = 0$$

or, $V\sin(\omega t) + V\sin(\omega t - 120^\circ) + V\sin(\omega t - 240^\circ) = 0$

or, $V\sin\omega t + V(\sin\omega t.\cos 120° - \cos\omega t.\sin 120°) +$
$V(\sin\omega t.\cos 240° - \cos\omega t.\sin 240°)$

or, $V\sin\omega t + V\left(\sin\omega t \times -\frac{1}{2} - \cos\omega t \times \frac{\sqrt{3}}{2}\right) +$
$V\left(\sin\omega t \times -\frac{1}{2} - \cos\omega t \times -\frac{\sqrt{3}}{2}\right)$

or, $V\sin\omega t - \frac{V}{2}\sin\omega t - \cancel{\frac{\sqrt{3}}{2}V\cos\omega t} - \frac{V}{2}\sin\omega t + \cancel{\frac{\sqrt{3}}{2}V\cos\omega t}$

or, $V\sin\omega t - V\sin\omega t$
$= 0.$

Phase voltage and current:-

V_R	I_R
V_Y	I_Y
V_B	I_B.

Line current & voltage

I_R	$V_{RY} = V_R - V_Y$
I_Y	$V_{RB} = V_R - V_B$
I_B	$V_{YB} = V_Y - V_B$

Phase voltage Vs line voltage

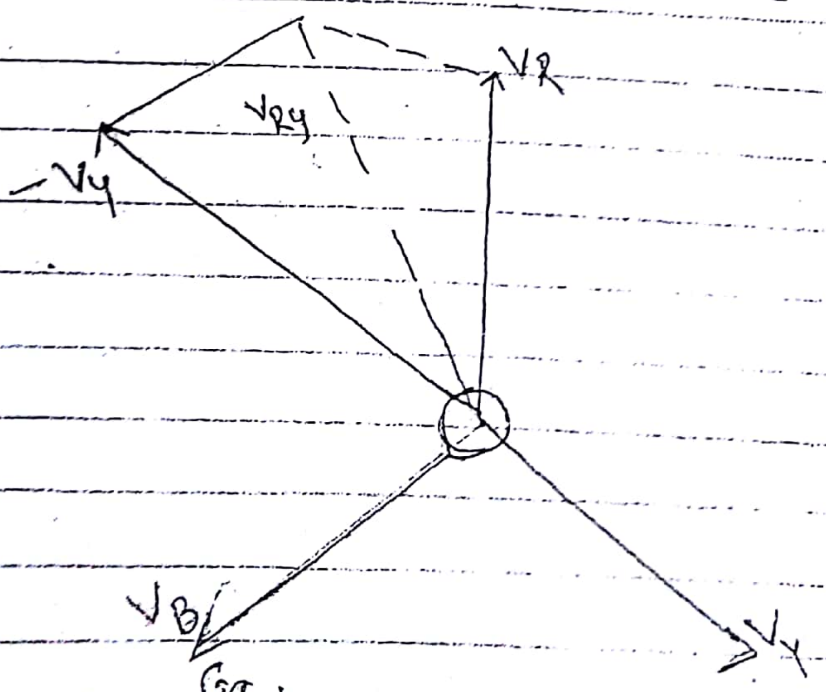


fig 1

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Let us consider the balanced condition.

i.e. $V_R = V_Y = V_B = V_{ph}$.

From fig. applying law of parallelogram,

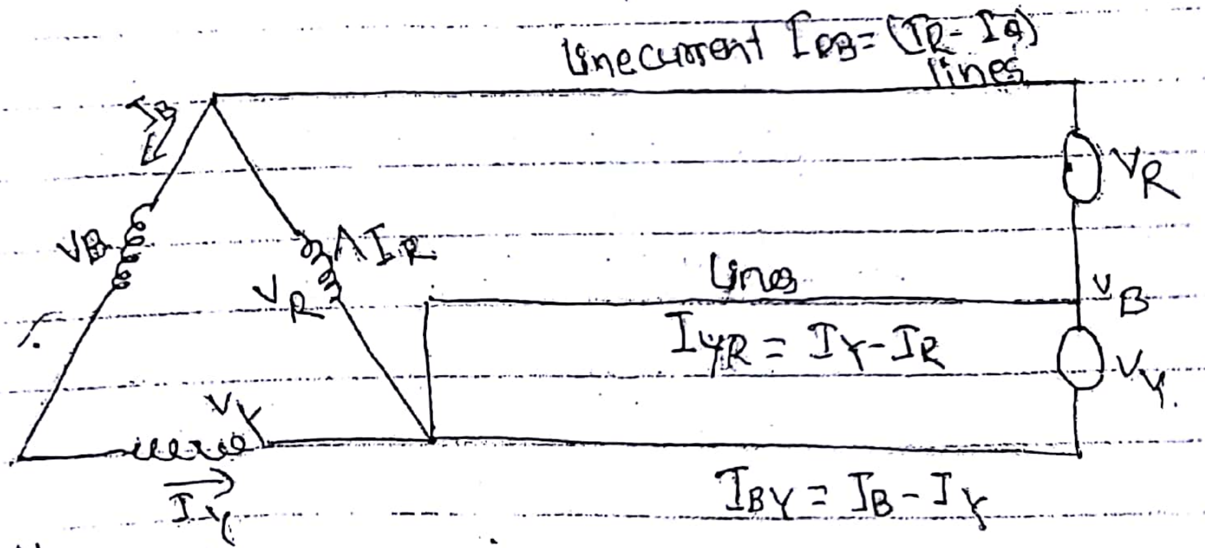
$$V_{RY} = \sqrt{(V_R)^2 + (-V_Y)^2 - 2V_R(-V_Y)\cos 60^\circ}$$

$$= \sqrt{V_R^2 + V_Y^2 + 2V_RV_Y\cos 60^\circ}$$

$$= \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}^2 \times \frac{1}{2}}$$

$$\text{or, } V_L = \sqrt{3V_{ph}^2} = \boxed{\sqrt{3}V_{ph}}$$

For delta connection:-

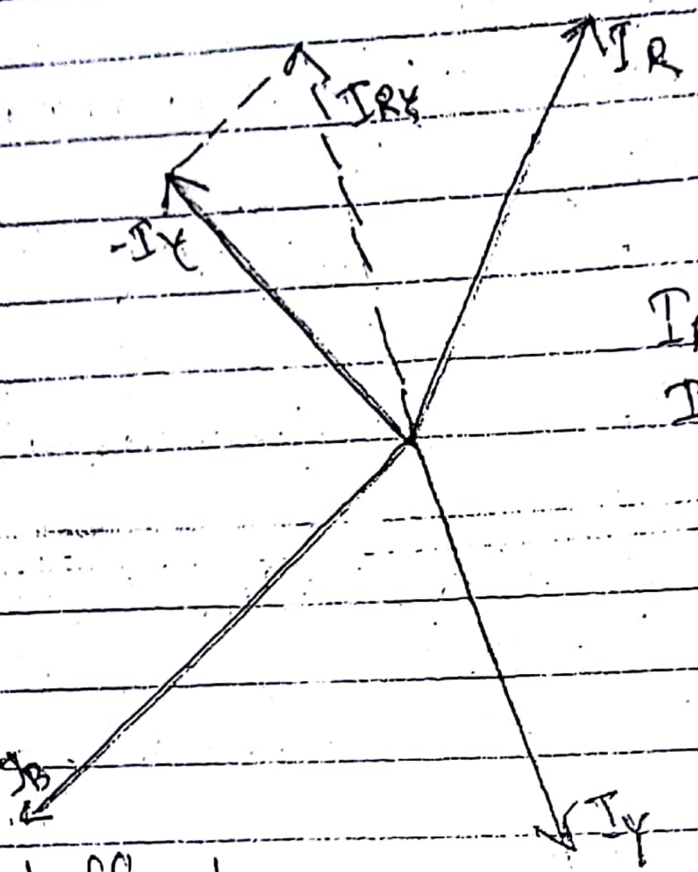


In delta:-

phase voltage = line voltage

$V_{ph} = V_L$.

Line currents Vs phase current.



$I_{RY} = \sqrt{3} I_{ph}$

$I_L = \sqrt{3} I_{ph}$

From Kirchoff's law;

$\therefore I_R = I_Y = I_B = I_{ph}$

Now,

from fig, applying law of parallelogram,

$$I_{RY} = \sqrt{(I_R)^2 + (-I_Y)^2 - 2 I_R(-I_Y)\cos 60^\circ}$$

$$\Rightarrow \sqrt{I_R^2 + I_Y^2 + 2 I_R I_Y \cos 60^\circ}$$

$$= \sqrt{I_{ph}^2 + I_{ph}^2 + 2 I_{ph}^2 \times \frac{1}{2}}$$

$$= \sqrt{3 I_{ph}^2}$$

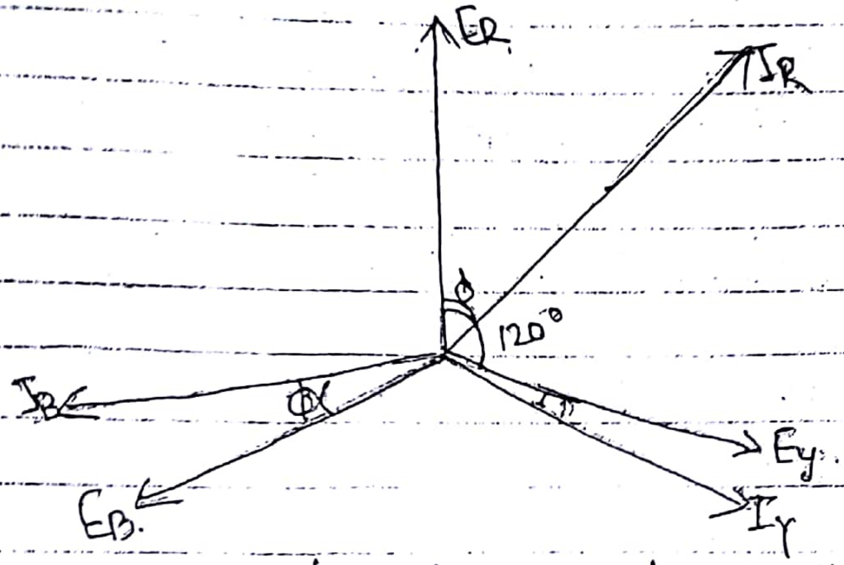
$$\therefore I_L = \sqrt{3}\, I_{ph}.$$

Imp

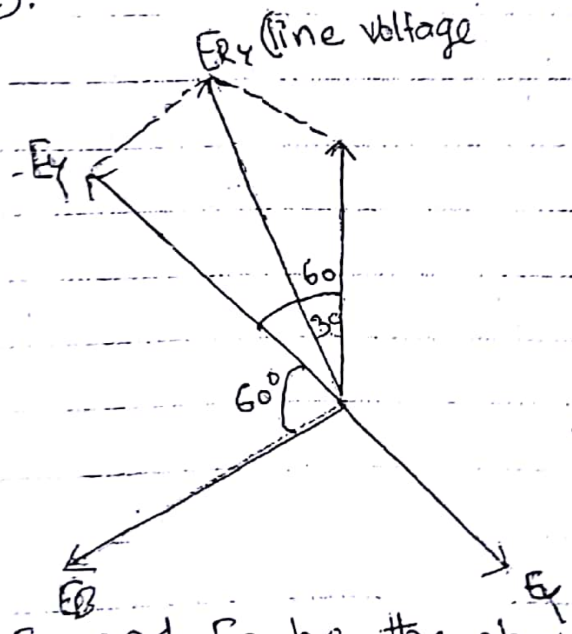
#1. Prove that power consumed in 3ϕ is $\sqrt{3} V_L I_L \cos\phi$.

→ Solution:-

We know that in 3ϕ system either in star or delta, the phasor diagram is given by.



Now by star (we have to do in voltage) (in delta by current).



Let E_R, E_Y and E_B be the phase voltages.

The line voltage E_{RY}, E_{RB} and E_{YB} are the vector differences of the phase voltage as,

$$E_{RY} = E_R - E_Y$$

$E_{RB} = E_R - E_B.$

$E_{YB} = E_Y - E_B.$

Let load be balanced in which $E_R = E_Y = E_B = E_{ph}$ (say phase voltage).

Now, using law of parallelogram, we get,

$$E_{RY} = 2 E_{ph} \cos 30^\circ$$

$$= 2 E_{ph} \frac{\sqrt{3}}{2}$$

or, $E_{RY} = \sqrt{3} E_{ph}$

~~Also~~,

$\therefore$ line voltage $(E_L) = \sqrt{3} E_{ph}$ (phase voltage)

Similarly, we get

$$E_{RB} = \sqrt{3} E_{ph}.$$

$$E_{YB} = \sqrt{3} E_{ph}.$$

Now, the power (active) is the power given by all three phases.

$P = 3 \times E_{ph} \cdot I_{ph} \cos\phi$ $(V = E)$ $E_L = \sqrt{3} E_{ph}$

$P = 3 \times \frac{E_L}{\sqrt{3}} \times I_L \cos\phi$ $E_{ph} = \frac{E_L}{\sqrt{3}}$

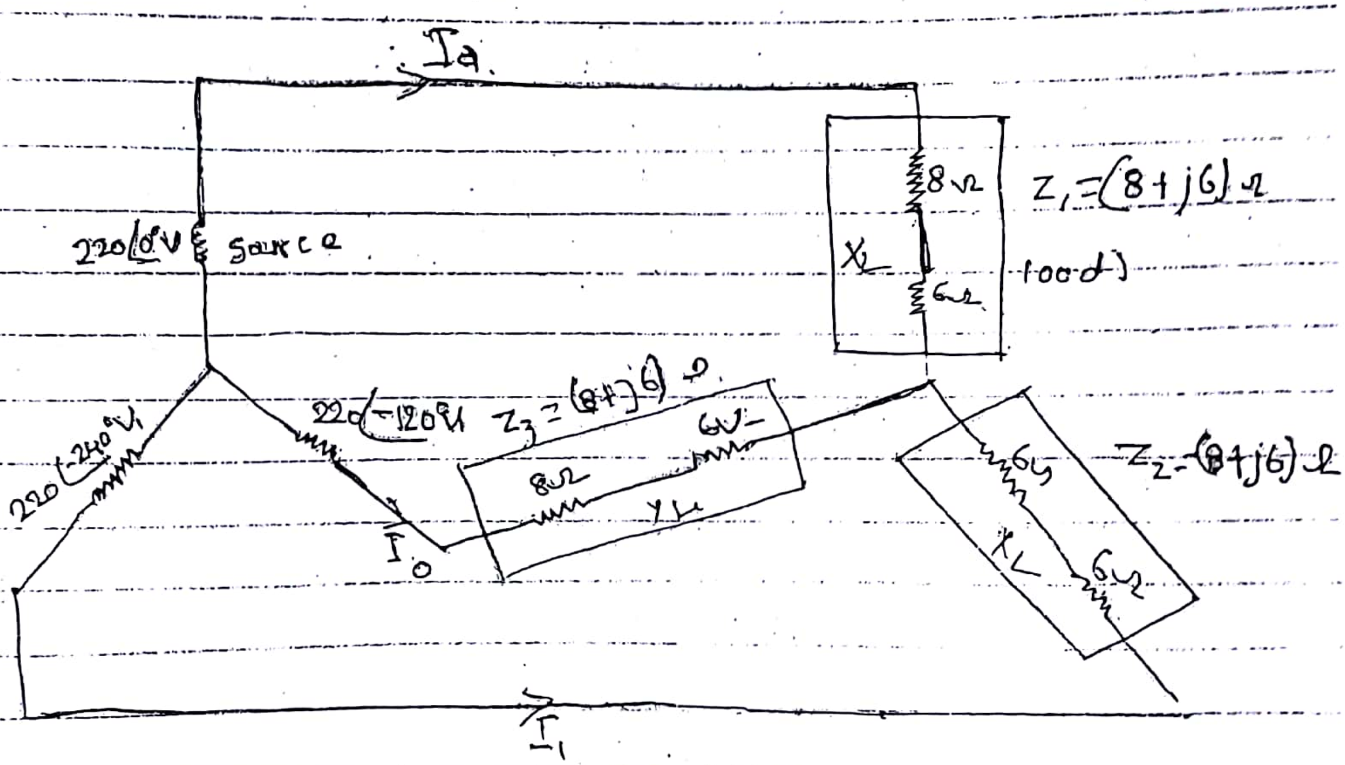
or, $P = \sqrt{3} E_L I_L \cos\phi$ (in star connection, $I_{ph} = I_L$)

or, $\boxed{P = \sqrt{3} V_L I_L \cos\phi.}$

proved.

Numerical for 8 marks

Q. Find I_a, I_b & I_c.



Here,

$$I_a = \frac{V}{Z_1} = \frac{220\angle 0^\circ}{\sqrt{8^2+6^2}\ \tan^{-1}\left(\frac{6}{8}\right)} =$$

$$I_b = \frac{220\angle -120^\circ}{\sqrt{8^2+6^2}\ \tan^{-1}(6/8)} =$$

$$I_c = \frac{220\angle -240^\circ}{\sqrt{8^2+6^2}\ \tan^{-1}(6/8)} =$$

If voltage name is not given then this is always line voltage.

Old. Q. 2010 (E). A 400V, balanced Y-connected supply is connected to three equal impedance $(80+60j)\Omega$ in a Y-formation. Calculate: phase current, line current, power factor and total power.

→ Soln. -

In star connection.

$V_L = 400V$

$Z = (80+60j)\Omega$.

$I_{ph} = ?$

$I_L = ?$

$P.F. (\cos\phi) = ?$

Total power$(P) = ?$

i) Phase current $I_{ph} = \frac{V_{ph}}{Z_{ph}}$

In star connection,

$V_L = \sqrt{3}\, V_{ph}$.

$V_{ph} = \frac{V_L}{\sqrt{3}}$

Now, $I_{ph} = \frac{V_L}{\sqrt{3} Z_{ph}} = \frac{400}{\sqrt{3} \times \sqrt{80^2+60^2}} = \frac{400}{173.205}$

$= 2.309A$.

ii) $I_L = I_{ph} = 2.309A$.

iii) $P.F. (\cos\phi) = \frac{R}{Z} = \frac{80}{\sqrt{80^2+60^2}} = \frac{80}{100} = 0.8$

m) Total power = $\sqrt{3} I_L V_L \cos\phi$

$= \sqrt{3} \times 2.309 \times 400 \times 0.8$

$= 1280 W.$

Three similar coils, each having a resistance of 20Ω and 2012F an inductance of 0.05 H are connected in star 3-ϕ, 50 Hz supply with 400V between lines. Calculate the total power absorbed and the line current in each case (Delta also). Find the magnitude of current flowing in the neutral wire.

→ Solution:-

$R = 20\Omega$

$L = 0.05 H$

$X_L = \omega L = 2\pi f L = (2 \times 3.14 \times 50 \times 0.05) \doteq 15.7\Omega.$

$Z_{ph} = (R + jL) = (20 + j15.7)\Omega$

$V_L = 400V.$

Total power (P) = $\sqrt{3} V_L I_L \cos\phi$

In star connection,

$I_{ph} = \frac{V_{ph}}{Z_{ph}}$

$V_L = \sqrt{3} V_{ph}$

$\therefore V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{V_L}{\sqrt{3} Z_{ph}} = \frac{400}{\sqrt{3}\sqrt{20^2 + (15.7)^2}} = 9.08 A.$

In star connection;

$I_L = I_p = 9.08A.$

$$\cos\theta = \frac{R}{Z} = \frac{20}{9.08} = 0.78.$$

$$\text{Power}(P) = \sqrt{3} \times V_L \times I_L \cos\theta \ (W)$$

$$= \sqrt{3} \times 400 \times 9.08 \times 0.78 = 9908.99 W.$$

For Delta,

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{\sqrt{20^2+(15.7)^2}} = 15.73.$$

& $I_L = \sqrt{3} I_{ph}.$

$$\therefore I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 15.73.$$

$$\cos\theta = \frac{R}{Z} = 0.78.$$

And, $\text{Power}(P) = \sqrt{3} V_L I_L \cos\theta$

$$= \sqrt{3} \times 400 \times 27.25 \times 0.78.$$

$$= 14725.89 W.$$

In neutral wire,

$$I_L = I_R + I_Y + I_B$$

$$= 3I_L$$

$$= 3 \times 27.66$$

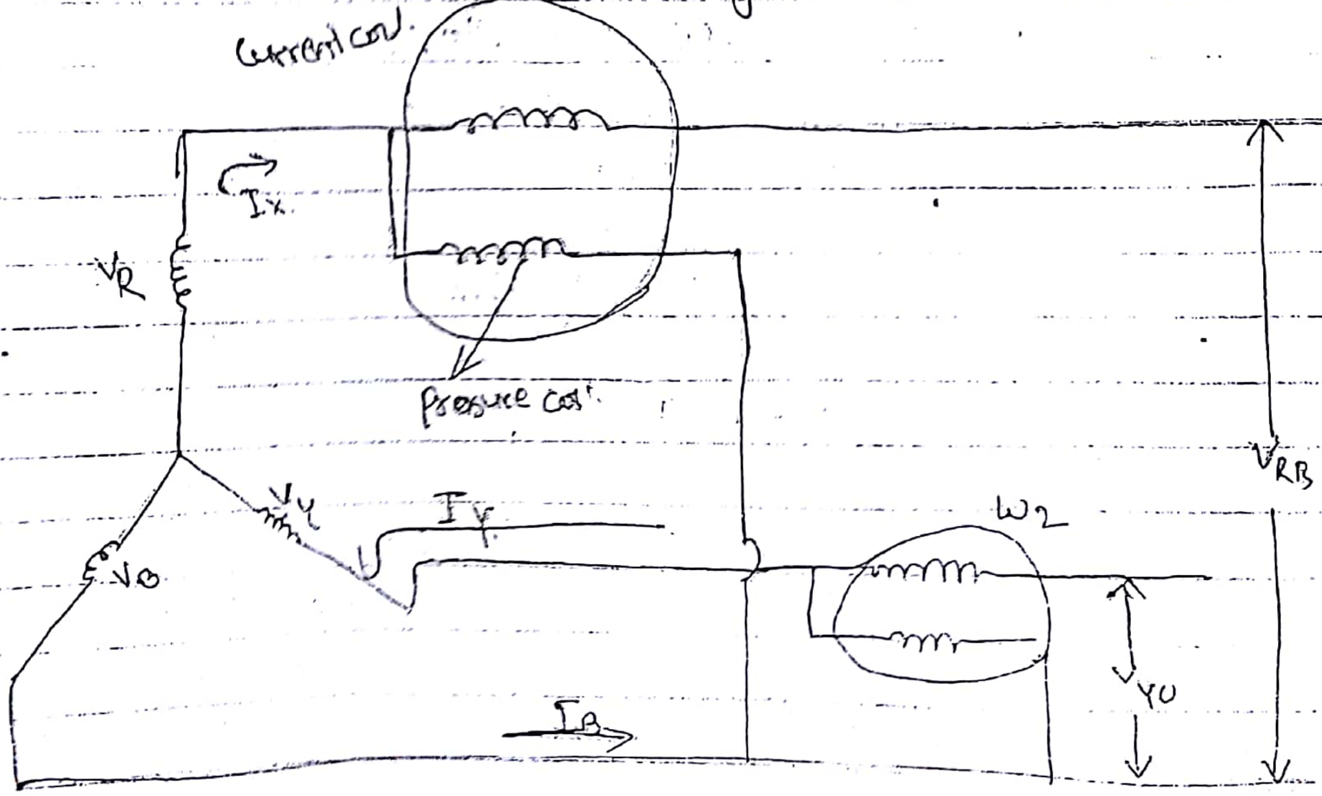
$$= 82.98 A.$$

Imp. Power measurement in 3-ϕ a.c.

1. One wattmeter method
2. Two wattmeter method
3. Three wattmeter method..

2) Two wattmeter method:-

In this process, two wattmeters are connected on any two terminals of the star or Delta connection. The current coil of the wattmeter is placed in series with the phase circuit and the pressure or potential coil of both wattmeter is commonly connected to the third line. The current coil measures the phase current and pressure coil measures the line voltage between terminals. The



product of phase current and line voltages gives the power. The total power consumed in the circuit in the addition of the

readings of the two wattmeters.
i.e. the total power is the sum of the powers of each phase.

Reading provide by $W_1 = I_R V_{RB}$
$= I_R(V_R - V_B)$

Reading provided by $W_2 = I_Y V_{YB}$.
$= I_Y(V_Y - V_B)$

$\therefore$ Total power $(W_1 + W_2) = I_R(V_R - V_B) + I_Y(V_Y - V_B)$
$= I_R V_R - I_R V_B + I_Y V_Y - I_Y V_B$

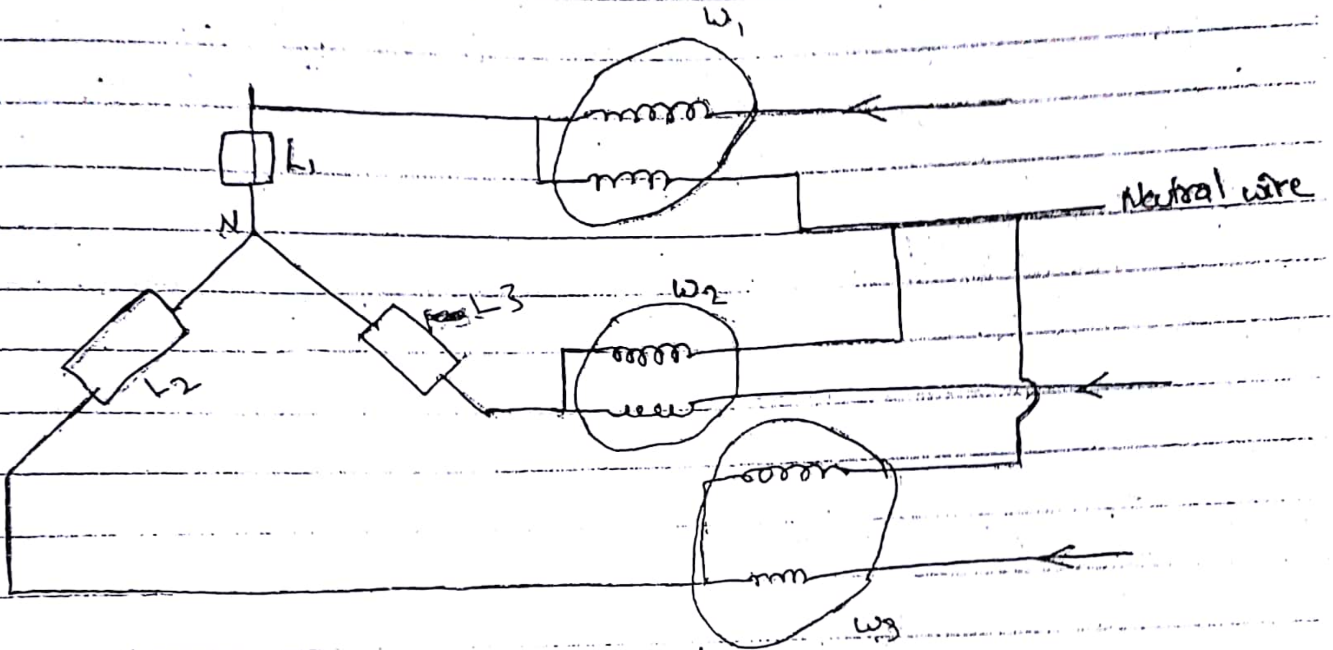
$= I_R V_R + I_Y V_Y - V_B(I_R + I_Y)$

But in balanced condition,
$I_R + I_Y + I_B = 0$
putting this value in above equation, we get -

$P = I_R V_R + I_Y V_Y + I_B V_B$

$\boxed{\text{i.e. } P = P_1 + P_2 + P_3}$

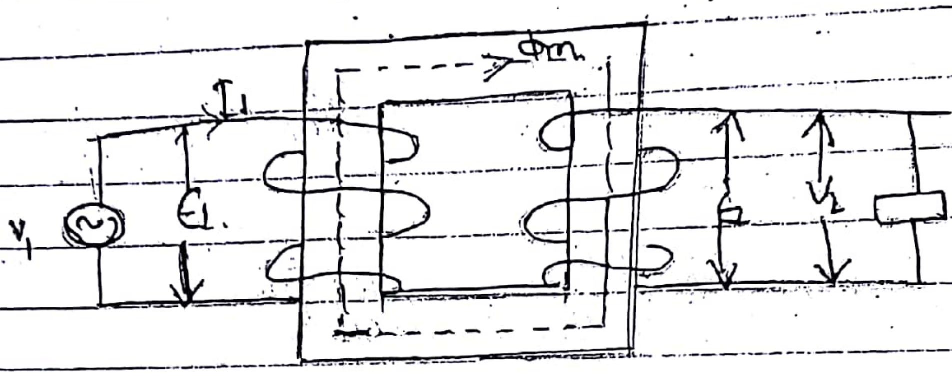
Three wattmeter method:-
In this method three wattmeter are inserted in each of the 3 phase of load whether delta connected or Y. The current coil of each wattmeter carries current of one phase only and pressure coil measures the phase voltage of this phase. Hence, each wattmeter measures the power in a single phase. The algebric sum of the reading of 3 wattmeter



gives the total power in the load.

Transformer

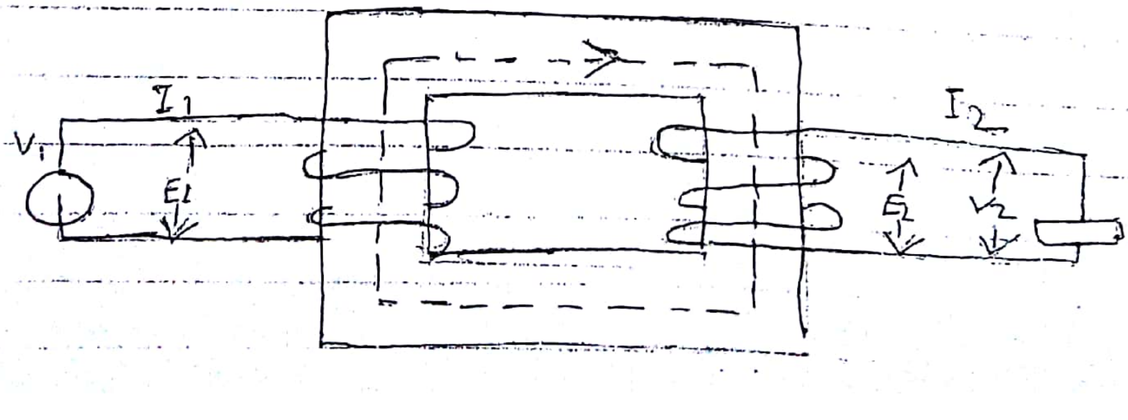
A static device that transfers the electrical energy from one circuit to another circuit without changing the frequency.



Properties of ideal transformer:-

1. winding should be purely inductive i.e. neglegible resistance and zero resistance.
2. winding permeability (μ) should be infinite.
3. No losses (core or iron loss and copper loss, hysteresis eddy current).
4. 100% efficiency.

Working principle of single phase transformer.



When a primary is connected to a sinusoidal voltage V_1, a primary current I_1 flows through the coil which is in charging with time. This charge of current produces a magnetic flux ϕ_m which in then produces an emf in the primary coil by self induction. This flux again produces the other emf E_2 in secondary by mutual induction. This emf supplies secondary current I_2 to the load.

Emf Equation of Transformer

Let, the flux generated in the coil of transformer be $\phi = \phi_m \sin\omega t$

Now, the total flux generated in the coil of N turns $= N\phi$

Hence, the emf generated is given by, (Faraday's law)

$$\varepsilon = \frac{-d(N\phi)}{dt}$$

$$= \frac{-d(N\phi_m \sin\omega t)}{dt}$$

$$= -N\phi_m\omega\cos\omega t$$

$$= N\phi_m\omega\sin(\omega t - \pi/2)$$

or, $\varepsilon = E_m \sin(\omega t - \pi/2)$

Where, $E_m = N\phi_m\omega$ ↳ Maximum

The rms value of emf. $E_{rms} = \frac{E_m}{\sqrt{2}} = \frac{N\phi_m\omega}{\sqrt{2}} = \frac{N\phi_m 2\pi f}{\sqrt{2}}$

$= 4.44\, N\phi_m f$

For primary: $E_1 = 4.44 N_1 \phi_m f$

For secondary: $E_2 = 4.44 N_2 \phi_m f$

Flux density $B_m = \frac{\phi_m}{A}$

$\phi_m = AB_m$ (A cross sectional area of coil)

Emf/turn (E_1/N_1) $= 4.44\,\phi_m f$

Emf/turn (E_2/N_2) $= 4.44\,\phi_m f$

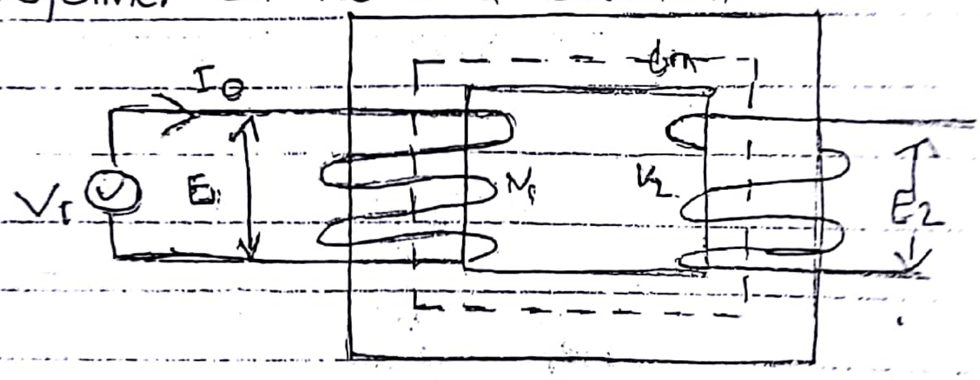
$$\frac{E_1}{N_1} = \frac{E_2}{N_2}$$

or, $\frac{N_2}{N_1} = \frac{E_2}{E_1} = K$ (transformation ratio)

if $N_2 > N_1 \longrightarrow$ step up

$N_2 < N_1 \longrightarrow$ step down

Transformer on no load condition



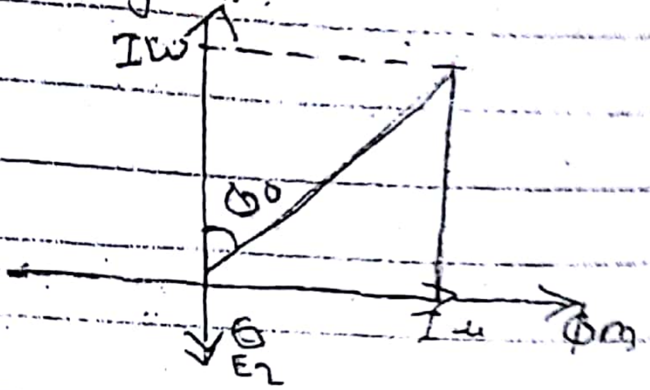
no load current I_0 has two ~~compounds~~ components.

$I_\mu \longrightarrow$ Reactive components / magnetising component or wattless component

Which magnetizes the core by producing magnetic flux ϕ_m and is in phase with ϕ_m,

$I_w \longrightarrow$ active component / wattful component and produces the hystesis and eddy current loss.

phasor diagram :-



$I_w = I_0 \cos\phi_0$

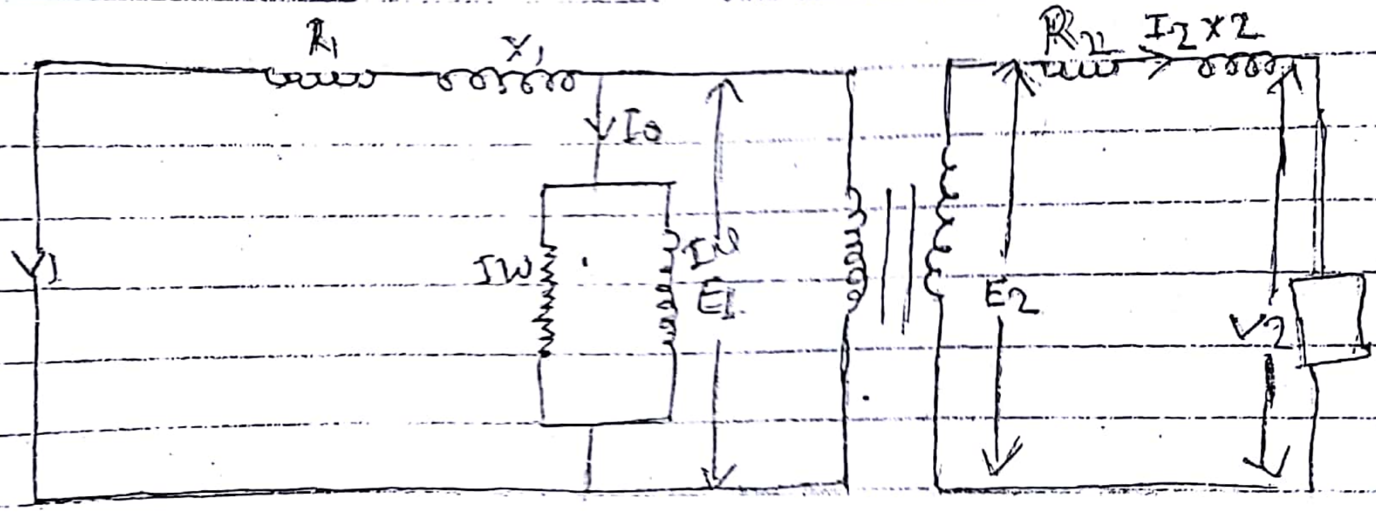
$I_\mu = I_0 \sin\phi_0$

$I_0 = \sqrt{I_w^2 + I_\mu^2}$

$I_w = I_0 \cos\phi_0$

$I_\mu = I_0 \sin\phi_0$.

Equivalent elect of Transformer : Ideal transformer



Secondary → Primary (referred to primary)

Primary → secondary (referred to secondary)

To transformers from secondary to primary

(referred to primary)

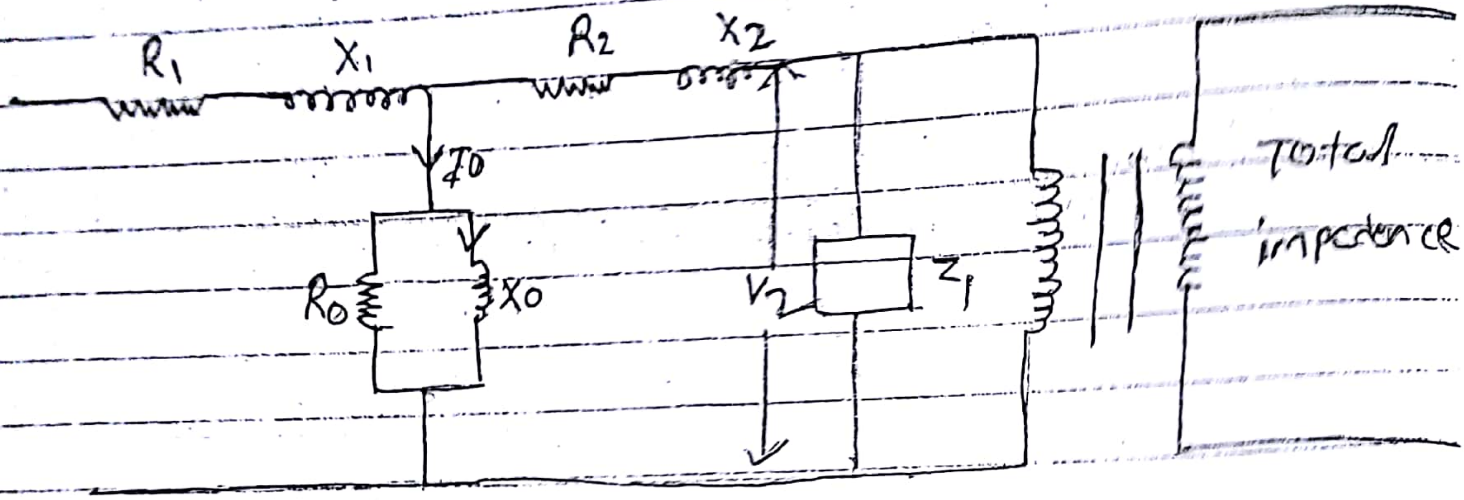
① Resistance , rectance , loads and impedences divides by k^2.

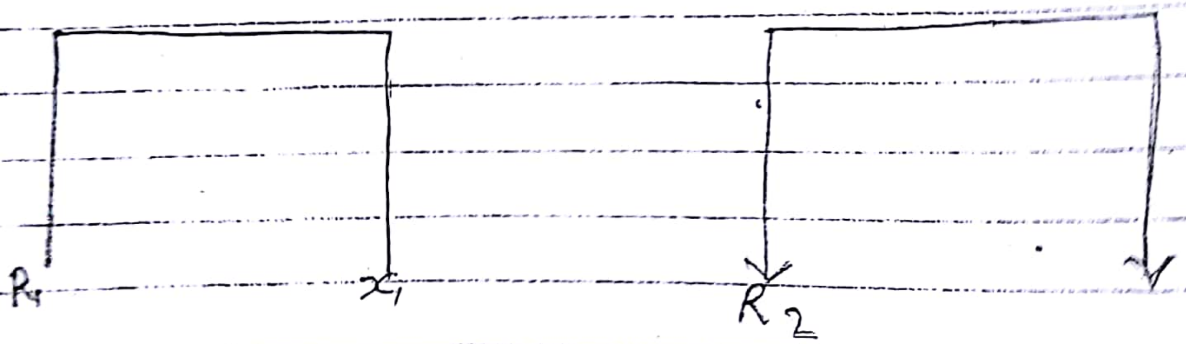
② Voltage (E_2 & V_2): divide by k

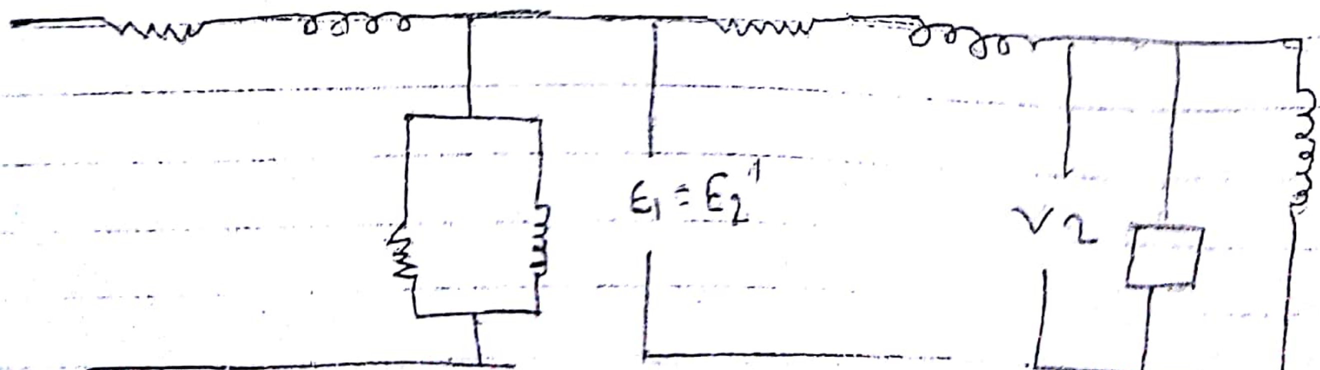
③ For current multiply by k

$I_2' = kI_2$

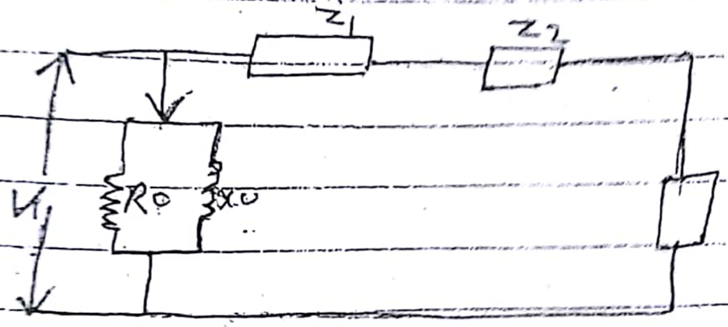
Equivalent ckt. of the Referred to primary







The exact Equivalent ckt is



Secondary Equivalent emf refer to primary $E_2' = E_2/K$

Secondary Equivalent voltage refer to Primary $V_2' = V_2/K$

Secondary Equivalent Current refer to Primary $I_2' = kI_2$

Secondary Equivalent resistance refer to primary $R_2' = R_2/K^2$

Secondary Equivalent reactance refer to Primary $X_2' = X_2/K^2$

Secondary Equivalent load refer to primary $Z_L' = \frac{Z_L}{K^2}$

Transformer Tests :

To find voltage regulation, efficiency and working of transformer. All these knows after knowing the constaints of transformer i.e., R_0, X_0, R, X and impedence.

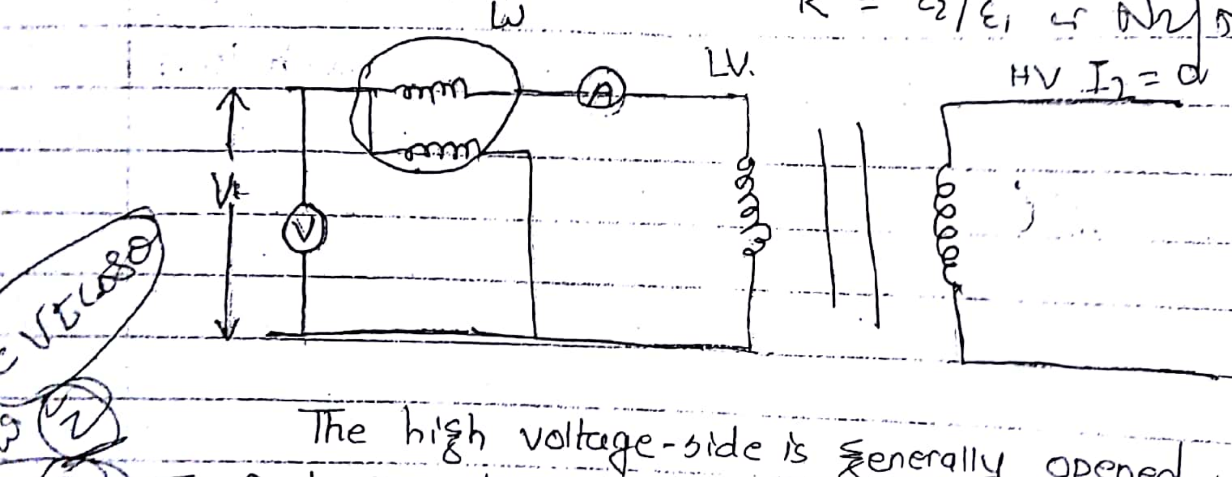
(i) Open circuit test or no load test
(ii) Short circuit test or Impedence test

Test Equipments
(i) voltmeter
(ii) Ammeter
(iii) wattmeter.

(i) Open Circuit Test

Generally, this test is having on low voltage side of the transformers. Eg- 10KVA, 250/500V, 50 HZ

$K = E_2/E_1$ or N_2/N_1



The high voltage-side is generally opened. Objective:-

(2) To find R_0 and X_0 → loss component / Reactance component
→ Magnetizing component

Ammeter gives no load current $= I_0$
Voltmeter gives the primary voltage $= V_1$
Wattmeter gives the reading of core or iron loss

$$P = V_1 \times I_0 \cos\phi$$

$$\therefore \cos\phi = \frac{P}{V_1 I_0}$$

Differences between Induction motor & transformer.

Induction motor.	Transformer.
1. Lower efficiency, due to higher losses.	1. Higher efficiency, due to lower losses.
2. Uses rotating winding.	2. Uses stationary ~~winding~~ coils.
3. Ratio of stator to rotor currents is not equal to turns ration (N_2/N_1)	3. $\frac{I_1}{I_2} = \frac{N_2}{N_1}$
4. Magnetic leakage.	4. leakage resistance of rotor are higher.
5. Per unit value of magnetising current is much higher.	5. Per unit value of magnetising current is much lower.

Differences between 3 ϕ synchronous & Induction motor

3 ϕ Synchronous motor	3 ϕ Induction motor.
1. It is not self starting.	1. It is self-starting and car starts up at rest.
2. Starting device is required.	2. starting device is not required
3. It runs only at synchronous speed.	3. It runs depending on the frequency of A.C. used.
4. Its speed is constant.	4. Its speed decreases with load
5. It has to be synchronized.	5. It has not to be synchronized
6. It more costly and complicated.	6. It is very simple, reliable and low in cost.
7. The change in applied voltage do not cause much effects on its torque.	7. The change in applied voltage causes much effect on its torque.
8. The breakdown torque is approximately proportional to applied voltage.	8. The breakdown torque depends on the square of the applied voltage.

$R_0 = V_1/I_w$

$I_w = I_0 \cos\phi$

$X_0 = V_1/I_\mu = \frac{V_1}{I_0 \sin\phi}$

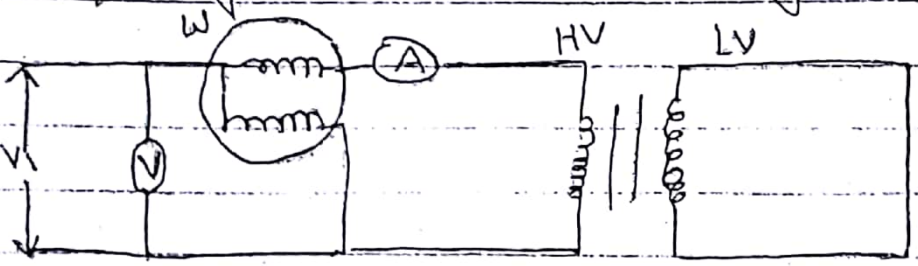
Now, we get, $R_0 = \frac{\text{Voltage}}{\text{Current}} = \frac{V_1}{I_w} = \frac{V_1}{I_0 \cos\phi}$

$$X_0 = \frac{V_1}{I_\omega} = \frac{V_1}{I_0 \sin\phi}$$

(ii) Short Ckt Test

Objective: To find resistance, reactance impedance and copper loss (ohmic loss): -

Generally, This test is on the high voltage side.



SC
$V_2 = 0$.

i.e., low voltage side is short ckted.

Ammeter gives short ckt current
$= I_{sh}$

Voltmeter gives short ckt voltage $= V_{sh}$

Wattmeter reading gives $P = I_{sh}^2 R_{sh}$

$$R_{sh} = \frac{P}{I_{sh}^2}$$

$$Z_{sh} = \frac{V_{sh}}{I_{sh}}$$

Reactance $X_{sh} = \sqrt{Z_{sh}^2 - R_{sh}^2}$

$Z^2 = R^2 + X^2$

$X = \sqrt{Z^2 - R^2}$

Numerical

- A 220/440 V, 50 Hz transformer give the following data on test obtain the parameter of the equivalent circuit and draw the equivalent circuit

O.C test	220V,	0.6 A,	66 W	measured L.V side.
refer to S.C test.	17V,	10 A,	80 W	measured H.V side,

Primary.

Soln

From O.C $\Rightarrow R_0$ and X_0

$V_1 = 220\,V$

$I_0 = 0.6$ (no load current)

$P = 66\,W$ (core loss)

$P = V_1 I_0 \cos\phi$

$$\cos\phi = \frac{66}{220 \times 0.6}$$

$\cos\phi = 0.5$

$$R_0 = \frac{V_1}{I_W} = \frac{220}{I_0 \cos\phi} \qquad (I_W = I_0 \cos\phi)$$

$$= \frac{220}{I_0 \cos\phi} = \frac{220}{0.6 \times 0.5}$$

$= 733.33\,\Omega$

$$X_0 = \frac{V_1}{I_\mu} = \frac{V_1}{I_0 \sin\phi} \qquad (I_\mu = I_0 \sin\phi)$$

$$= \frac{V_1}{I_0(\sqrt{1-\cos^2\phi})}$$

$$= \frac{220}{0.6\sqrt{1-0.5^2}}$$

$= 423.390\,\Omega$

from oc test.

$$V_{sh2} = 17V$$

$$I_{sh2} = 10A$$

$$P = 80W$$

$$P = I^2_{sh2} R_{sh}$$

$$R_{sh} = \frac{P}{I^2_{sh}} = \frac{80}{100} = 0.8\,\Omega$$

$$Z_{sh} = \frac{V_{sh}}{I_{sh}} = \frac{17}{10} = 1.7\,\Omega$$

$$X_{sh} = \sqrt{Z^2 - R_{sh}^2}$$

$$= \sqrt{1.7^2 - 0.8^2}$$

$$= 1.5\,\Omega$$

To draw equivalent circuit converting should be done.

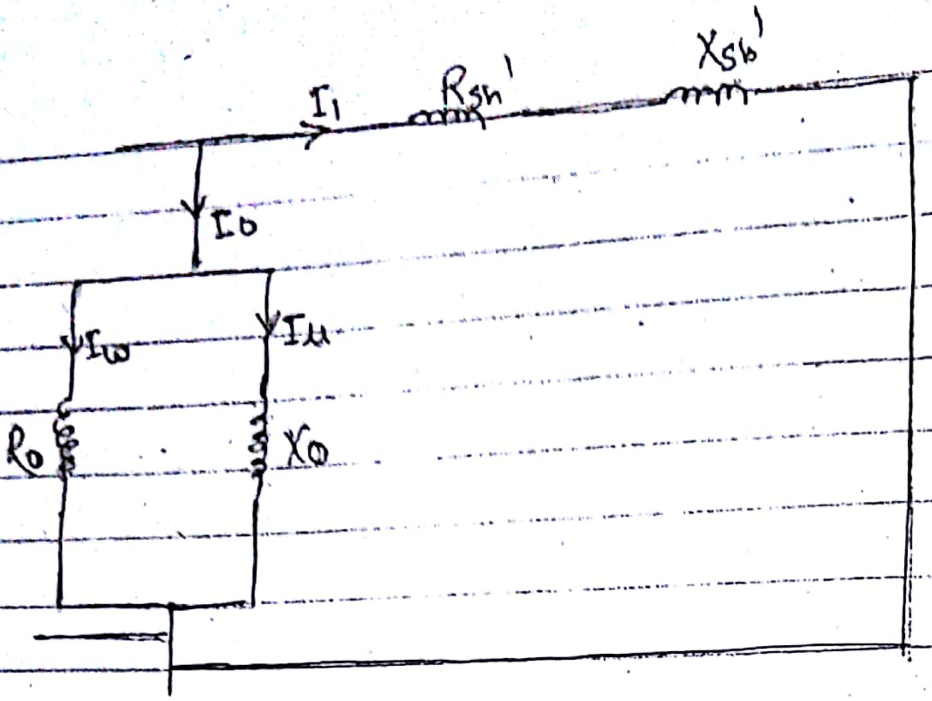
$$K = \frac{E_2}{E_1}$$

$$= \frac{440}{220} = 2$$

$$R_{sh}' = \frac{R_{sh}}{K^2} = \frac{0.8}{4} = 0.2$$

$$Z_{sh}' = \frac{Z_{sh}}{k^2} = \frac{1.7}{4} = 0.425$$

$$X_{sh}' = \frac{1.5}{K^2} = 0.375$$



Primary full load current,

$$I_1 = \frac{P}{E_1} \quad \left(\frac{\cancel{V}I}{\cancel{V}} = I \right)$$

(11) A 30 KVA, 240/120, 50 Hz transformer has high voltage winding resistance of 0.1 Ω and leakage resistance of 0.22 Ω. The ~~two~~ low voltage winding resistance of 0.35 Ω and leakage reactance is 0.12 Ω. Find the Equivalent winding resistance, reactance and impedence refer to high voltage & low voltage.

Soln,

$P = 30$ KVA

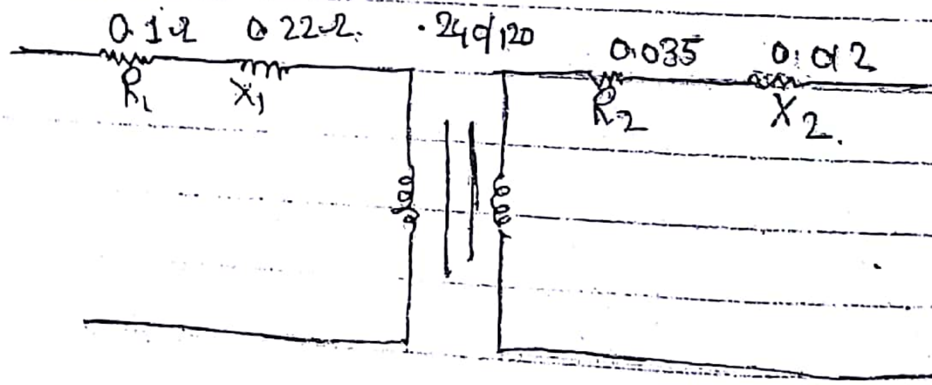
$R = ?$

$x = ?$

$Z = ?$

$H.V = ?$

$L.V = ?$



For H.V,

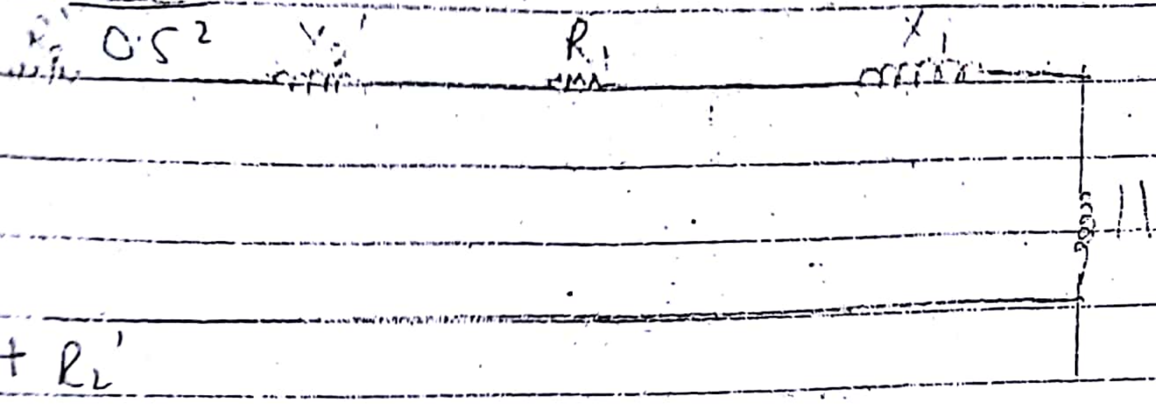
$$R_2' = \frac{R_2}{K^2} = \frac{0.035}{0.25} = 0.14\,\Omega$$

$K = E_2/E_1$

$= 120/240$

$= 0.5$

$$X_2' = \frac{X_2}{K^2} = \frac{0.012}{0.5^2} = 0.048\,\Omega$$



$$R_{01} = R_1 + R_2'$$
$$= 0.1 + 0.14 = 0.24\,\Omega$$

$$X_{01} = X_1 + X_2'$$
$$= 0.22 + 0.048 = 0.268\,\Omega$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$
$$= \sqrt{0.24^2 + 0.268^2}$$
$$= \sqrt{0.129424}$$
$$= 3.59\times10^{-1}$$

$$R_1' = K^2 R_1$$
$$X_1' = K^2 X_1$$
$$R_{02} = R_2 + R_1'$$
$$X_{02} = X_2 + X_1'$$

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$$

Q. The following data refer to a 50KVA, 220/110V, 50 Hz transformer O.C test with H.V open : let 350W, 9.5A, 110V SC test with L.V Shrted : 820W, 22.7A and 100V determine the equivalent circuit parameter refer to 2V side. Also find the primary current and secondary current.

Soln.

$$\text{Power } (P) = 50KVA$$

$$E_1 = 220$$

$$E_2 = 110$$

From O.C test:-

$$P = 380W$$

$$I_o = 9.5A$$

$$V = 110V$$

$$P = VI_o \cos\phi$$

$$\cos\phi = \frac{P}{VI_o}$$

$$= \frac{380}{110 \times 9.5}$$

$$= 3.6 \times 10^{-1}$$

$$R_o = \frac{V_1}{I_w} = \frac{V_1}{I_o \cos\phi} = \frac{110}{9.5 \times 3.6 \times 10^{-1}} \doteq 3.22 \times 10 = 32.2\,\Omega$$

$$X_o = \frac{V_1}{I_\mu} = \frac{110}{I_o \sin\phi} = \frac{110}{9.5\sqrt{1-0.36^2}}$$

At H.V

$$R_{sh} = \frac{P}{I_{sh}^2} = \frac{826}{22.7^2} = 1.59\,\Omega$$

$$Z_{sh} = \frac{V_{sh}}{I_{sh}} = \frac{100}{22.7} = 4.41\,\Omega$$

$$X_{sh} = 4.69\,\Omega$$

$$\therefore\ K = \frac{E_2}{E_1}$$

$$= \frac{110}{220}$$

$$= 0.5$$

Now converting,

$$R_{sh}' = 1.59 \times 0.5^2 =$$

$$Z_{sh}' = 4.41 \times 0.5^2 =$$

$$X_{sh}' = 4.69 \times 0.5^2 =$$

Q. A 12KVA transformer having primary voltage of 2000 V at 50 Hz has $N_1 = 180$, $N_2 = 30$ find (i) the full load primary and secondary current (ii) the no load secondary induced emf (iii) the maximum flux in core

Primary $I_1 = P/V_1$

$$\frac{N_2}{N_1} = \frac{E_2}{E_1}$$

$$E_1 = 4.44\,\phi_m N_1 f$$

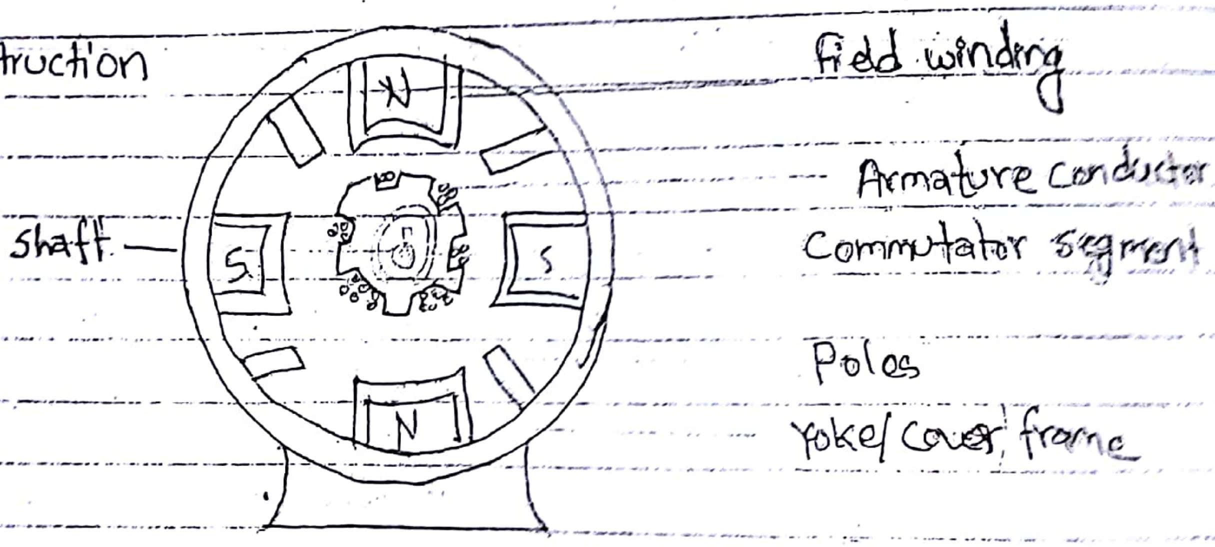
$$\phi_m = \frac{E_1}{4.44 N_1 f}$$

DC MACHINES

Generators and Motors } 15 marks (1 theory, 1 numerical)

Generators
-> Mechanical to electrical energy converter.
*· -> Principle:
When a conductor is rotated in field, a voltage will be generated in the corner.

-> Construction

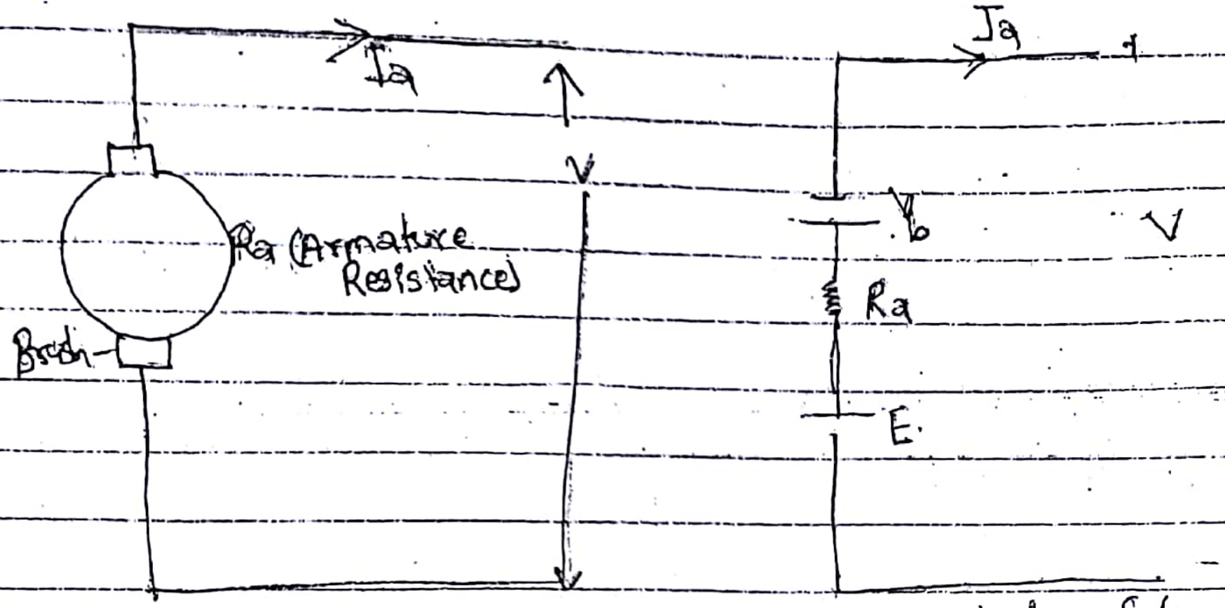


It has mainly four parts:

(i) Magnetic ckt system
(ii) Armature -> rotating part
(iii) Commutators -> Collect the DC and rotate unidirectionally, AC voltage and connect to d.c voltage.
(iv) Brusher

(Armature reaction): effect two thing.
(i) Distorts / weakens
(ii) Cross magnetization

Equivalent circuit of armature:



E = generated emf / voltage
Ra = Armature Resistance
Vb = brush drop voltage

$\boxed{E = V + I_a R_a + R_b}$ ← for Generator.

$V = E - I_a R_a - R_b.$

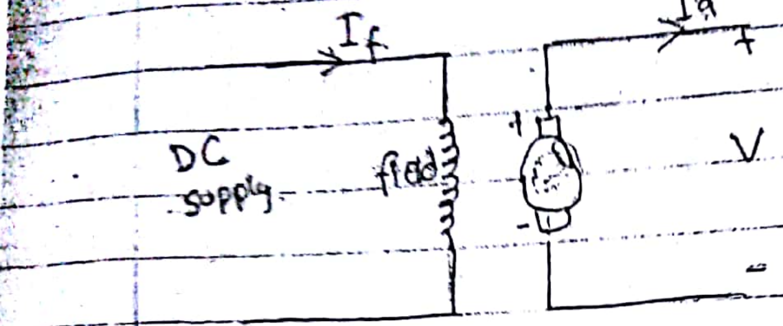
Types of Dc machines

Actually current through the field windind produce the flux. This process is known as excitation and based on this excitation, dc machine are of two types:-

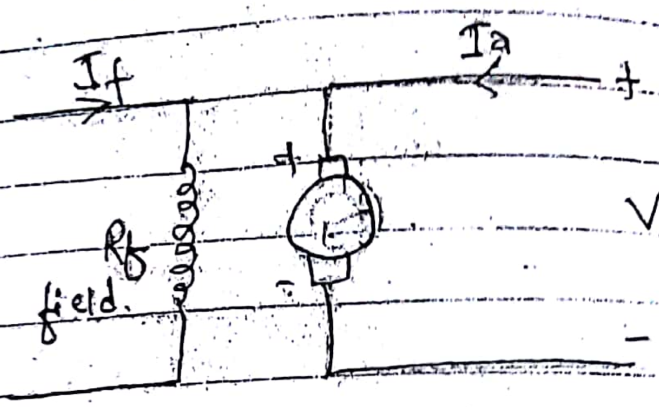
Dc machine (Generator and Motors)

- separately excited machine
- self excited machine
  - shunt wound machine
  - series wound machine
  - compound wound machine
    - Short wound compound
    - long wound compound

1. Separately excited machine:-

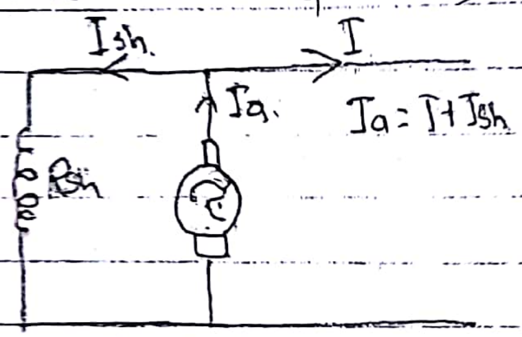


Generator

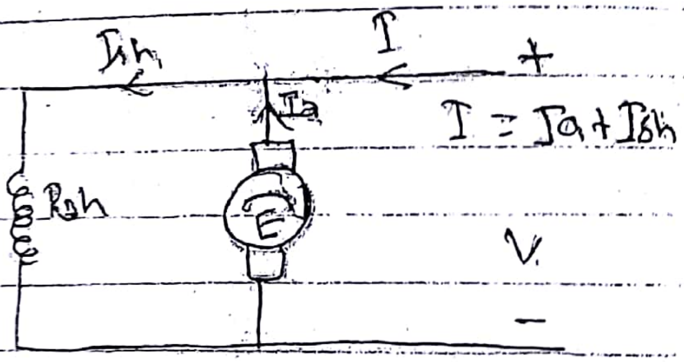


motor

2. Shunt wound (parallel).

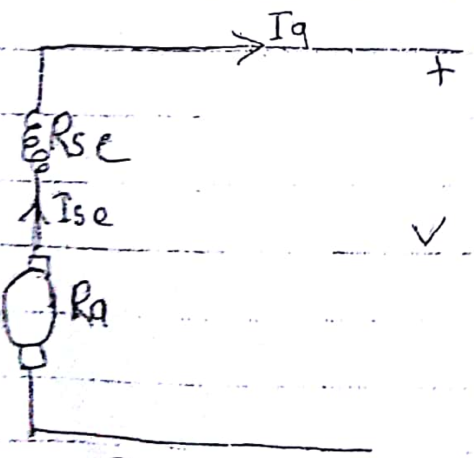


Generator

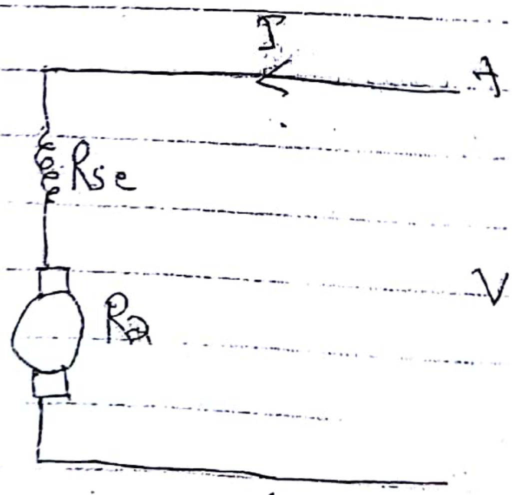


Motor

3. Series wound:-

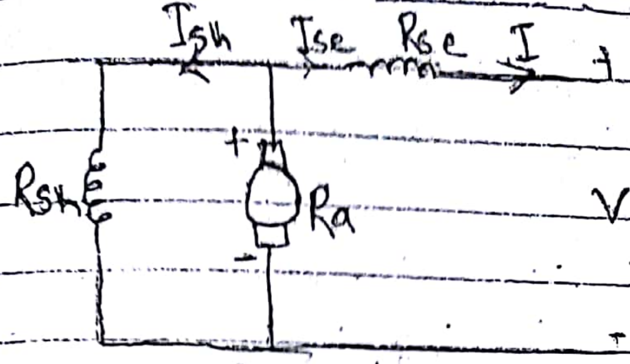


Generator

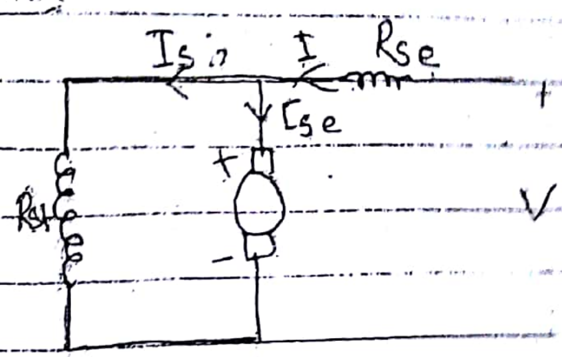


motor

4. Short shunt wound compound:-

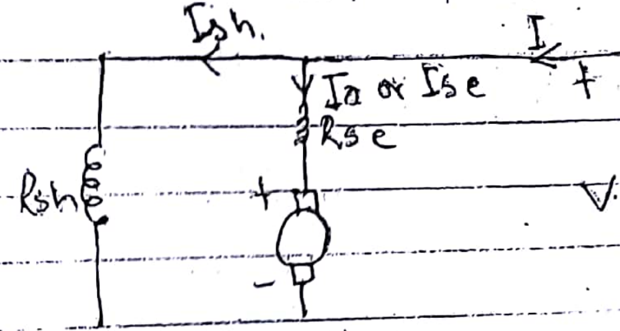


Generator



motor

4. Long shunt wound compound:-



motor.

Emf Equation of a DC machine

let,

ϕ = useful flux pole (webes)

P = Total no. of poles

Z = Total no. of conductors in armature

n = speed of rotation in sps

A = no. of Paralled paths

Z/A = no. of armature conductor In series per Each parallel path,

Now, the emf is given by,

$$E = \frac{Pn\phi Z}{A}$$

For lap winding:
$A = P$

For wave winding:-
$A = 2$

Numericals:

(I) A four pole, wave wound armature have 720 conductors and is rotated at 1000 rpm. If the flux is 20 mwb. Calculate the Generated voltage.

$P = 4$

$A = 2$

$Z = 720$

$\phi = 20 \times 10^{-3}$ wb

$n = 1000$ rpm

$= \frac{1000}{60}$ rps $= 16.67$ rps

Generated voltage (emf) $E = \frac{Pn\phi Z}{A}$

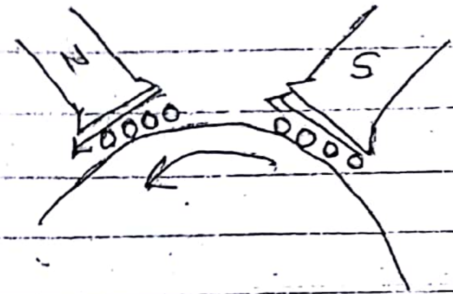
$= \frac{4 \times 16.67 \times 20 \times 10^{-3} \times 720}{2}$

$= 480096 \times 10^{-3}$

$= 480.096$ V.

(II) A shunt machine delivers 450 A at 230 V and the resistance of shunt field and armature are 50 Ω 0.03 Ω respectively. Calculate generated emf when acts as Generator and motor

Motor working Principle

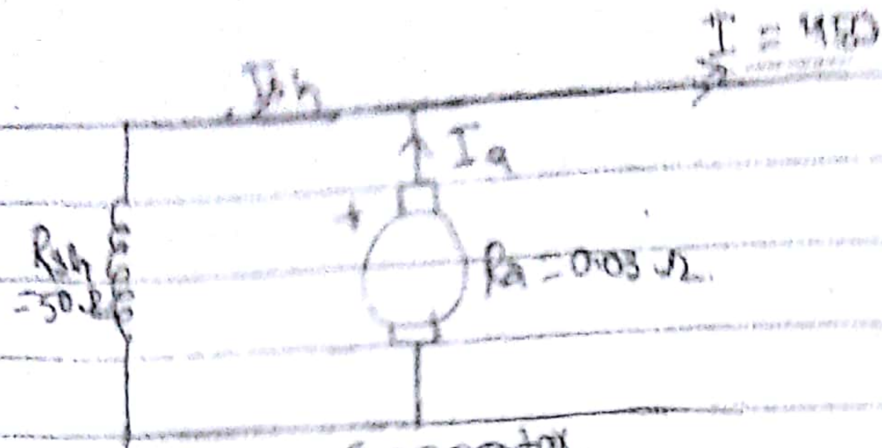


It's action is based on the principle that when a current carrying conductor is placed in a magnetic field it experiences a mechanical force whose direction is given by Flemings left hand rule and whose magnitude is given by.

$$F = BIlN$$

Constructionally there is no basic difference betn a dc generators and dc motors.

Where it's field magnet are excited and it's armature conductors are supplied with current from the supply, they experience a force tending to rotate the armature. Armature conductor under N pole are assumed to carry current downwards and those under S pole to carry current upwards. Applying flemming's left hand rule the direction of rotation can be found. Each conductors experiences a force which tends to rotate the armature in an anticlock wise direction.



Generator.

as Generator,

$$I_{sh} = \frac{V}{R_{sh}}$$

$$= \frac{230}{50}$$

$$= 4.6\ A$$

$$I_a = I_{sh} + I$$

$$= 4.6 + 450$$

$$= 454.6\ A$$

$$E = V + I_a R_a$$

$$= 230 + 454.6 \times 0.03$$

$$= 243.64$$

as motor, $I = I_a + I_{sh}$

$$I_a = I - I_{sh}$$

$$= 450 - 4.6$$

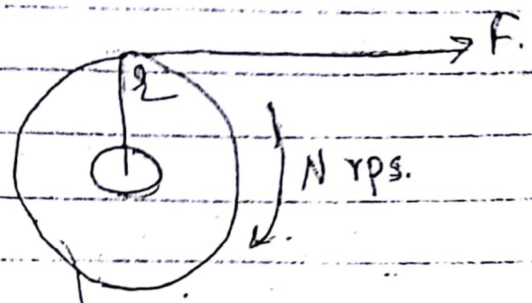
$$= 445.4\ A$$

$$E = V - I_a R_a$$

$$= 230 - 445.4 \times 0.03$$

$$= 216.64\ V$$

Torque.



Torque $T = (F \times R)$ (Nm)

now,

workdone by this force in one rev = F × distance

$= F \times 2\pi r$

now,

the power developed $= F \times 2\pi r \times N$

$= (F \times r) \times 2\pi N$ $(\omega = 2\pi N)$

$= T \times \omega$ where, ω is the angular velocity. — (I)

Now,

The power generate in armature $= E_b \times I_a$ — (II)

Equating Eqn (I) & (II)

$T \times 2\pi N = E_b I_a$

$T \times 2\pi \cancel{N} = \frac{P \cancel{N} \phi Z I_a}{A}$

Now, armature torque,

$$\boxed{T_a = \frac{PZ \phi I_a}{2\pi A}}$$ Torque equation.

$= K \phi I_a$ where, $K = \left(\frac{PZ}{2\pi A}\right)$

$\boxed{T_a \propto \phi I_a}$

Case (i) for series motor / generator $\phi \propto (I_a)$ before saturation

$$T_a \propto I_a^2$$

Case (ii) for shunt (ϕ is practically constant)

$$T_a \propto I_a$$

$$E = \frac{P\phi N Z}{A}$$

$$N \propto E/\phi$$

$$N = \frac{EA}{P\phi Z}$$

$$N_1 \propto \frac{E_1}{\phi_1}$$

$$= \frac{A}{PZ}\frac{E}{\phi}$$

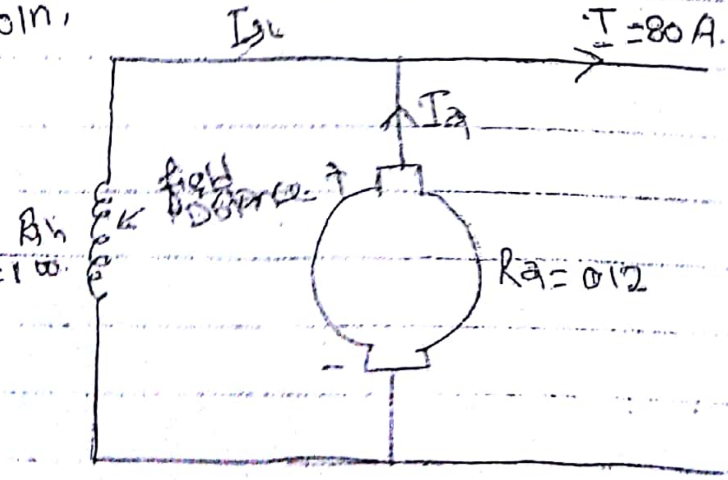
$$N_2 \propto \frac{E_2}{\phi_2} \qquad \frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\phi_1}{\phi_2}$$

$$N = KE$$

Numerical

(1) A DC shunt machine connected to 250 volt supply has an armature resistance of 0.12 Ω and the H.C resistance of the field circuit 100 Ω. Find the ratio of the speed as a generator to the speed to a motor. The line current in each case is 80 A.

Soln,



$V = 250$ as Generator

Now,

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{100} = 2.5$$

$$I_a = I + I_{sh}$$
$$= 80 + 2.5$$
$$= 82.5\ A$$

Now,

Generated voltage in generator

$$E_g = V + I_a R_a$$
$$= 250 + 82.5 \times 0.12$$
$$= 259.9\ V$$

As motor,

$$I_{sh} = \frac{V}{R_{sh}} = 2.5$$

$$I = I_a + I_{sh}$$

$$I_a = I - I_{sh} = \underline{(80 - 2.5)}\ A$$
$$77.5$$

$$E_m = \underline{V - I_a R_a}$$
$$= 250 - 77.5 \times 0.12$$
$$= 240.7$$

We have,

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\phi_2}{\phi_1} \rightarrow \text{Practical constant.}$$

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} = \frac{259.9}{240.7} = 1.08$$

⑥ A armature resistance of a 200 V shunt motor is 0·4 Ω and no load current is 2A. When loaded and taking an armature current of 50 A, the speed is 1200 is rpm. Find appropriately no load speed.

$I_{a_1} = 2A$

$I_{a_2} = 50A$

$R_a = 0.4A$

$V = 200V$

$N_2 = 1200\ rpm$

$E_2 = V - I_{a_2}R_a$

$= 200 - (50 \times 0.4)$

$= 180$

$E_1 = V - I_{a_1}R_a$

$= 200 - (2 \times 0.4)$

$= 199.2$

$\frac{N_1}{N_2} = \frac{E_1}{E_2}$

$N_1 = \frac{199.2 \times 1200}{180}$

$= 1328\ rpm$

#/ A 220V dc series motor is running at a speed of 800rmp and draws 100A. Calculate at what speed the motor will run when developing half the torque, total resistance of field and armature is 0.1Ω

→ Solution:-

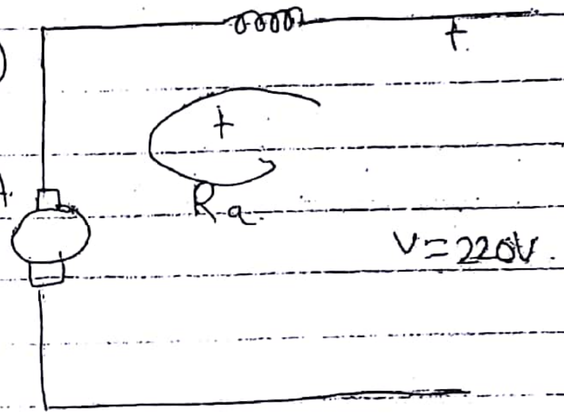
Given that,

motor voltage = 220V (series)

speed (N_1) = 800rpm

Armature current $(I_a)_1 = 100A$

$N_2 = ?$



Let first torque = T_1

second torque $T_2 = \frac{1}{2} T_1$.

$$or, \frac{T_2}{T_1} = \frac{1}{2}$$

Total resistance $(R_a + R_{se}) = 0.1\Omega$.

we have,

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\phi_1}{\phi_2}$$

For series motor,

Torque, $t \propto I_a^2$

and $\phi_1 \propto I_{a_1}$ & $\phi_2 \propto I_{a_2}$.

$$\therefore \frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{I_{a_1}}{I_{a_2}}$$

and $T_1 = I_{a_1}^2$

$T_2 = I_{a_2}^2$.

$$Also, \frac{T_2}{T_1} = \frac{I_{a_2}^2}{I_{a_1}^2}$$

$$or, \frac{1}{2} = \frac{I_{a_2}^2}{10000} \Rightarrow I_{a_2} = \frac{100}{\sqrt{2}}.$$

$$\frac{N_2}{N_1} = \frac{V - I_{a_2}(R_a + R_{se})}{V - I_{a_1}(R_a + R_{se})} \times \frac{100}{100} \times V_2.$$

$$\text{or, } N_2 = \frac{220 - \frac{100}{\sqrt{2}}(0.1) \times \sqrt{2} \times N_1}{200 - 100(0.1)}$$

$$= \quad \times 800 \text{ rpm.}$$

\# A 230V shunt motor has an armature resistance of 0.1Ω and shunt field resistance of 275Ω. It runs at speed of 1000rpm when drawing armature current of 75 A. Calculate the addition resistance to be inserted in the field circuit to raise motor speed to 1200rpm at armature current of 125 A. 2011F

→ Solution:-

Given That,

$V = 230$ V

$R_a = 0.1\,\Omega$

$R_{sh} = 275\,\Omega$

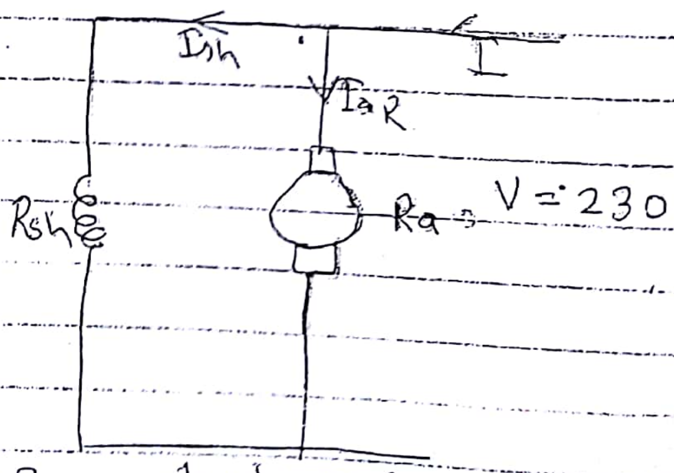
$N_1 = 1000$ rpm

$I_{a_1} = 75$ A

$N_2 = 1200$ rpm

additional resistance (R) = ?

$I_{a_2} = 125$ A.



shunt motor.

For this we have,

$I = I_a + I_{sh}$

or, $I_a = I - I_{sh}$.

we know,

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\phi_1}{\phi_2}$$

or, $$\frac{N_2}{N_1} = \frac{V - I_{a_2}(R_a + R)}{V - I_{a_1}(R_a)}$$

$$\frac{1200}{1000} = \frac{230 - 125(0.1 + R)}{230 - 75 \times 0.1}$$

or, $R =$

Q. Write

1) 3ϕ induction motor (construction and explanation)
2) synchronous motor (construction & explanation)

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VIP

E_1 E_2

Q. Obtain equivalent current of a 200/400 V, 50 Hz single phase transformer from the following test data as referred to primary and secondary.

O.C. test: 220V, 0.7 A, 70W on LV side.

S.C. test: 15V, 10 A, 85W on HV side.

Note:- low voltage side (LV side = Primary side).
high voltage side (HV side = Secondary side).

→ Solution:-

Transformation ratio $(K) = \frac{E_2}{E_1} = \frac{400}{200} = 2$ Ans.

From O.C. Test,

wattmeter reading $= V_1 I_0 \cos\phi_0$

or, $70 = 200 \times 0.7 \times \cos\phi_0$

or, $\cos\phi_0 = 0.5$.

working current, $I_w = I_0 \cos\phi_0 = 0.7 \times 0.5$
$= 0.35 A$

Magnetic current $I_\mu = I_0 \sin\phi = 0.7\sqrt{1-\cos^2\phi}$
$= 0.7\sqrt{1-(0.5)^2} = 0.6 A$.

$R_0 = \frac{V_1}{I_w} = \frac{200}{0.35} = 571.4 \Omega$.

$X_0 = \frac{V_1}{I_\mu} = \frac{200}{0.6} = 333.33 \Omega$.

For S.C. test,

$V_{sc} = 15 V$

$I_{sh} = 10 A$

$W = 85 W$

from wattmeter reading, $W = I_{sh}^2 \times R_{02}$

or, $R_{02} = \frac{W}{I_{sh}^2} = \frac{85}{100} = 0.85 \Omega$

$$Z_{o_2} = \frac{V_{sc}}{I_{sh}} = \frac{15}{10} = 1.5\,\Omega.$$

$$X_{o_2} = \sqrt{Z_{o_2}^2 - R_{o_2}^2} = \sqrt{0.85^2 - 1.5^2} = 1.23\,\Omega.$$

Now,

equivalent resistance as reffered to primary,

$$R_{o_2}' = \frac{R_{o_2}}{K^2} = \frac{0.85}{4} = 0.2125\,\Omega.$$

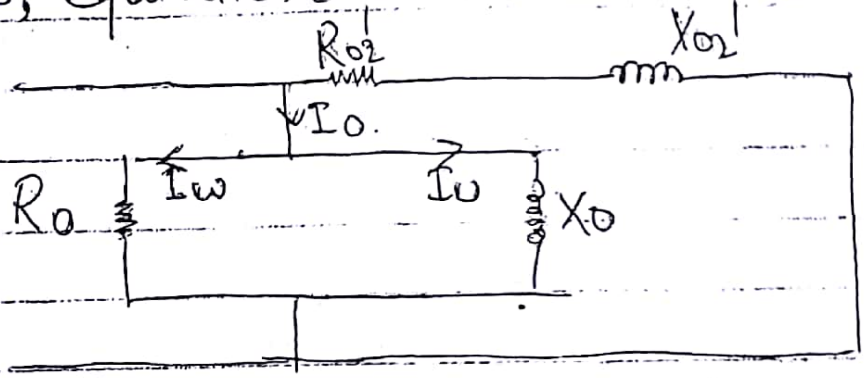
Equivalent impedence as reffered to primary,

$$Z_{o_2}' = \frac{Z_{o_2}}{K^2} = \frac{1.5}{4} = 0.375\,\Omega$$

Equivalent reactance as reffered to primary,

$$X_{o_2}' = \frac{X_{o_2}}{K^2} = \frac{1.23}{4} = 0.3075\,\Omega$$

Now, equivalent circuit as reffered to primary.



Magnetising reactance as reffered to secondary

$$X_o' = K^2 X_o$$

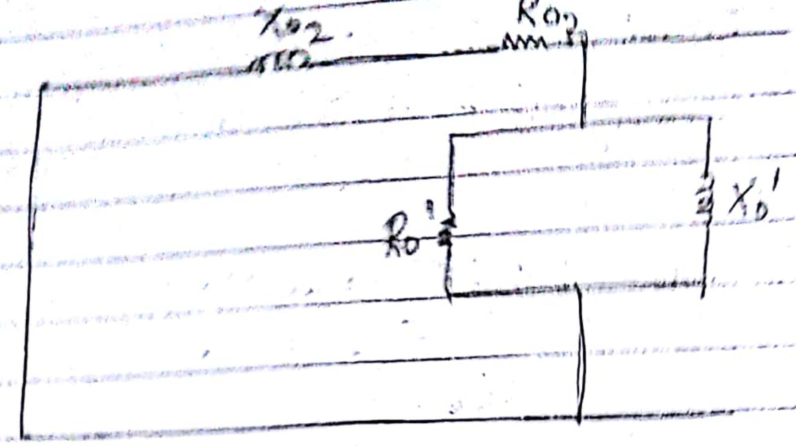
$$=$$

Working resistance as reffered to secondary,

$$R_o' = K^2 R_o$$

$$=$$

And equivalent circuit as reffered to secondary is,



Also, find secondary voltage when delevering 5KW at 0.8 p.f. lagging, the primary voltage being 200V.

(secondary voltage) $V_2 = I_2 (Ro_2 \cos\phi_0 + Xo_2 \sin\phi_0)$

Now,

$I_2 = \frac{5 \times 10^3}{400} = 12.5$ $\left[\begin{array}{l} P = E_2 I_2 \\ I_2 = \frac{P}{E_2} \end{array} \right]$

η of transformer $= \frac{O/P \text{ power}}{o/p + \text{total losses}} \times 100\%$

old in: ~~cold~~

\#. A 10 kVA, 200/1000V, 50Hz single phase transformer gave the following test result: 2014F

OC test (LV side) : 200V, 2.4A 100 W

SC test (HV side) : 50V 10A 150W

a) Calculate the parameter of the equivalent circuit reffered to LV side.

b) Calculate the efficiency for ½ rated 0.8 p.f lagging and load current for which it gives maximum efficiency.

→ Solution:-

$\frac{E_2}{E_1} = K = \frac{1000}{200} = 5$ [∵ High voltage is secondary side because $V_H = 1000$]

From O.C. test,

wattmeter reading $= V_1 I_0 \cos\phi_0$

or, $100 = 200 \times 2.4 \cos\phi_0$

$\cos\phi_0 = 0.20.$

$I_w = I_0 \cos\phi_0 = 2.4 \times 0.20 = 0.48.$

$I_\mu = I_0 \sin\phi_0 = 2.4\sqrt{1-(0.20)^2} = 2.35.$

so, $R_0 = \frac{V_1}{I_w} = \frac{200}{0.48} = 416.66\,\Omega.$

$X_0 = \frac{V_1}{I_\mu} = \frac{200}{2.35} = 85.10\,\Omega.$

For SC. test,

wattmeter $= I_{sh}^2 R_{O_2}$

$\therefore R_{O_2} = \frac{150}{10^2} = 1.5.$

Impedence, $Z_{O_2} = \frac{V_{sc}}{I_{sh}} = \frac{50}{10} = 5\,\Omega.$

Reactance, $X_{O_2} = \sqrt{Z_{O_2}^2 - R_{O_2}^2} = \sqrt{5^2 - 1.5^2} = 4.76\,\Omega.$

a) Now,

resistance reffered to primary, $R_{O_2}' = \frac{R_{O_2}}{K^2} = \frac{1.5}{25} = 0.06.$

impedence reffered to primary, $Z_{O_2}' = \frac{Z_{O_2}}{K^2} = \frac{5}{25} = 0.2$

Reactance reffered to primary $X_{O_2}' = \frac{X_{O_2}}{K^2} = \frac{4.76}{25} = 0.19.$

And,

b). $\eta = \frac{o/p \text{ power}}{o/p \text{ power} + \text{losses}} \times 100$

Copper loss $= I^2R = 100\,W$

Iron loss $= 150\,W$

$\therefore$ Total loss $= 100 + 150 = 250\,W$,

Rated output power $= 10 \times 10^3 \times 0.8 \times \frac{1}{2} = 4000$

$\therefore \eta = \frac{4000}{4000+250} \times 100 = 94.11$

Rated output $=$ pf $\times$ watt

$I_2 = \frac{\text{Rated power}}{\text{secondary voltage}} = \frac{10 \times 10^3}{1000} = 10\,A.$

Q. A 250/500 V, 50 Hz transformer gave following results. 5KVA, 2012 F

S.C. test = 20V, 12 A, 100W with LV short circuited.
O.C test = 250V, 1 A; 80W with HV side opened.
Determine circuit constaints. Calculate the efficiency when output is 10 A at 500V & 0.8 p.f. lagging.

> Solution:-

$\frac{E_2}{E_1} = \frac{500}{250} = 2 = K.$

Now,

for s.c. test,

$R_{O2} = 100/12^2 = 0.6944\,\Omega$

$Z_{O2} = \frac{V}{I_{sh}} = 20/12 = 1.666\,\Omega$

Reactance $X_{O2} = \sqrt{(0.6944)^2 + (1.666)^2} = 1.5143\,\Omega$

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$XO_2 = 1.514\,\Omega$

For O.C. test,

wattmeter reading $= V I_0 \cos\phi$.

or, $80 = 250 \times 1 \cos\phi$

$\therefore \cos\phi = \frac{80}{250} = 0.32$.

And, $I_w = I_0 \cos\phi = 0.32$.

$I_\mu = I_0 \sin\phi = 1\sqrt{1-(0.32)^2} = 0.0974$

Now,

$R_0 = \frac{V}{I_w} = \frac{250}{0.32} = 781.25$.

$X_0 = \frac{V}{I_\mu} = \frac{250}{0.974} = 256.67$.

output power $= 500 \times 10 = 5000$ VA.

$\therefore \eta = \frac{500 \times 10 \times 0.8}{500 \times 10 \times 0.8 + (100+80)} = \frac{4000}{580} \times 100$

$= 95.69$.

And,

$I_2 =$

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Q. A 220V dc shunt motor with armature resistance 0.2Ω runs with 500rpm by taking 50A current. Calculate the additional resistance that should be placed in series with armature to reduce speed to 250 rpm.

→ Solution:-

Given that,

$R_a = 0.2\,\Omega$ $V = 220\,V$ $N_1 = 500\,rpm$

$I = 50\,A$ $N_2 = 250\,rpm$

Let R be an additional resistance in series with armature.

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\phi_1}{\phi_2}$$

But motor is shunt; $\therefore \phi =$ constant

$$\therefore \frac{N_2}{N_1} = \frac{E_2}{E_1} \Rightarrow \frac{N_2}{N_1} = \frac{V - I_a(R_a + R)}{V - I_a R_a}$$

$$\text{or, } \frac{250}{500} = \frac{220 - 50(0.2 + R)}{220 - 50 \times 0.2}$$

$$\text{or, } 220 - 50(0.2 + R) = 105$$

$$\text{or, } R = 2.3 - 0.2$$

$$\therefore R = 2.1\,\Omega$$

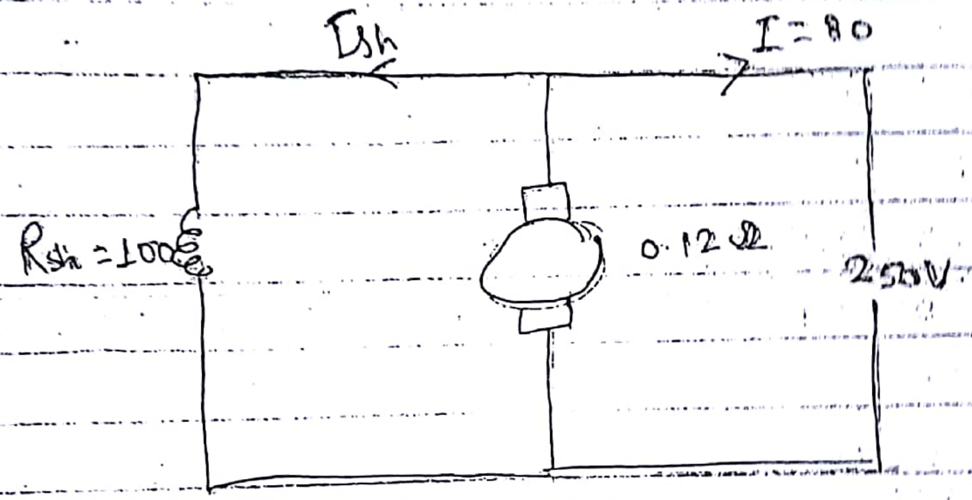
Hence, the additional resistance is 2.1 Ω.

Q. A DC shunt machine connected to 250V supply has an armature resistance of 0.12 Ω and the resistance of the field current is 100 Ω. Find the ratio of the speed as a generator to the speed of a motor. The line current is 80 A.

Solution:-

Given that,

$V = 250\,V$ $R = 100\,\Omega$ $r = 0.12\,\Omega$ $I = 80\,A$.



$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{100} = 2.5$

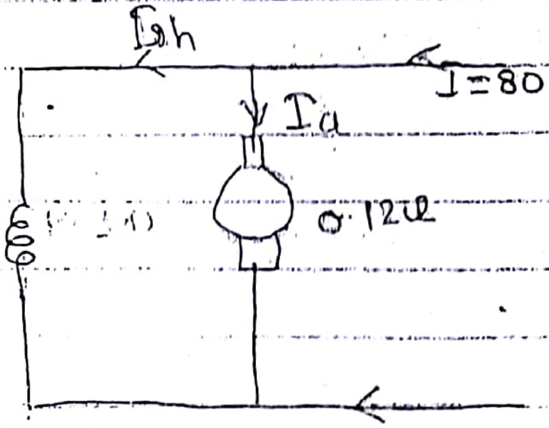
$I_A = I + I_{sh} = 80 + 2.5 = 82.5$.

$E_g = V + I_a R_a = 250 + 82.5 \times 0.12 = 259.9$.

For motor:-

$I_{sh} = \frac{250}{100} = 2.5$

$I_a = I - I_{sh}$
$= 80 - 2.5 = 77.5$.



& $E_m = V - I_a R_a$
$= 250 - 77.5 \times 0.12 = 240.7$.

∴ $\frac{N_m}{N_G} = \frac{E_m}{E_G} = \frac{240.7}{259.9} = 0.92$.

Q. A 460V series motor runs at 500rpm taking a current of 40A. Calculate the speed and % change in torque if the load is reduced so that motor is taking 30 A. Total resistance of armature and field current is 0.8Ω.

→ Solution:-

$N_1 = 500$ rpm.

$I_{a_1} = 40A$

$N_2 = ?$

% change in torque $(I_{a_2} = 30A) = ?$

$R_a + R_{se} = 0.8\Omega$.

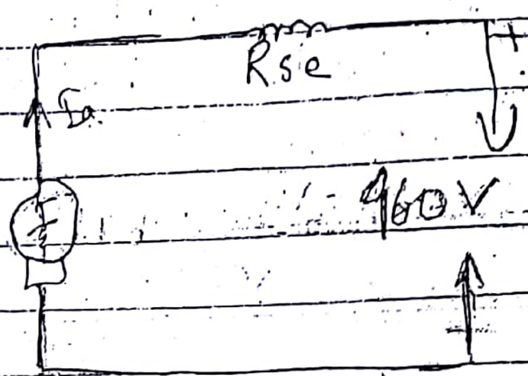


Fig. Series motor

We have,

~~$N_2 = 460 - 30(0.8$~~

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\phi_1}{\phi_2} = \frac{V - I_{a_2}(R_a + R_{se})}{V - I_{a_1}(R_a + R_{se})} \times \frac{I_{a_1}}{I_{a_2}}$$

$$\therefore N_2 = \frac{460 - 30(0.8)}{460 - 30(0.8)} \times \frac{40}{30} \times 500 = \cancel{666.66}\ 679.12.$$

$$\therefore \%\ \text{change} = \frac{T_1 - T_2}{T_1} \times 100\%$$

$$= \left(1 - \frac{T_2}{T_1}\right) \times 100\% = \left(1 - \frac{I_{a_2}^2}{I_{a_1}^2}\right) \times 100\%$$

$$= \left(1 - \frac{30^2}{40^2}\right) \times 100\%$$

$$= 43.75\%$$

$[\because T \propto I_{a_2}^2]$

DC machines

Q. What is a DC machines? Deduce the expression for emf generation in dc machines.

→ DC machines are rotating device and work on electromechanical energy conversion. Thus, they are even termed as "electromechanical energy conversion devices". DC machines are generator and motor.

Let,

ϕ = flux / pole in Weber

Z = total no. of conductors on armature

P = no. of poles.

A = no. of parallel paths in armature

N = Speed in rpm of armature

E = emf induced in armature conductors in volts.

Emf generated by armature = emf generated in one of the parallel path.

Also,

emf generated = flux cut per second in volts.

$\because$ Flux cut by one conductor in making one rev = ϕP.

$\therefore$ Emf generated in one conductor = $\frac{\phi P N}{60}$ volt.

Since Z no. of conductors are arranged in 'A' parallel paths, therefore no. of conductors per path = $\frac{Z}{A}$.

$\therefore$ Total emf generated = emf generated in one conductor no. of armature conductors in each circuit or parallel path,

$$E = \frac{\phi P N}{60} \times \frac{Z}{A}$$

or, $E = \frac{\phi Z p N}{60 A}$ volt.

If we consider angular velocity as ω rad/sec, then

$$N = \frac{\omega \times 60}{2\pi}$$

$$E = \frac{Z \phi \omega p}{2\pi A}$$

For a given DC machine, Z, P and A are constant, so defining armature constant K_a as $K_a = \frac{ZP}{2\pi A}$

The equation becomes,

$$E = K_a \phi \omega$$

This is the required emf equation of dc machine.

Types of dc motor:-

1. shunt motor or shunt wound.
2. series motor or series wound.
3. Compound motor or compound wound.

1. Shunt motor:-

This is the most common types of d.c. motor. The field winding is connected in parallel with the armature as shown in figure. with large no of Turns.

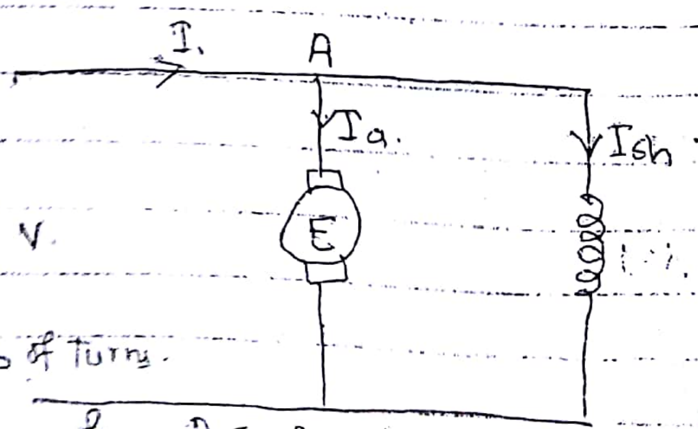


fig DC shunt motor

Characteristics:-

Its starting (or no load) torque is about 2.5 to 3 times greater than the full-load torque. By using shunt regulator, the variations of the speed of the motor can be well-acheived. It's runs practically at constant speed at almost all loads.

Uses:-

For driving pumps, drills, printing presses, etc.

2. Series motor:-

It possesses the field winding of few turns of heavy conductor, connected in series with the armature i.e. load current flows through the field and armature both.

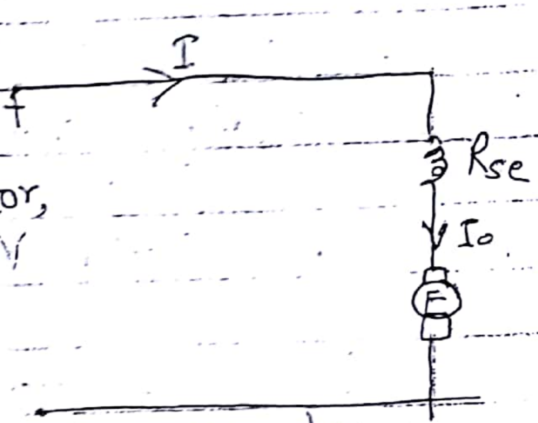


Fig. Series motor.

Characteristics:-

1. With increasing load, the speed decreases.
2. At no load, the speed of the motor is very high. So, it
3. should never be used without load.

Uses:-

For cranes, pumps, trains, trolley cars, etc. due to its very high starting torque.

3. Compound wound motor:-

It has a series as well as shunt windening. Depending on the types of field connections, a compound wound motor can be:

i) Cumulative-compound motor:-

It is one in which series field assists the field due to the shunt field windings.

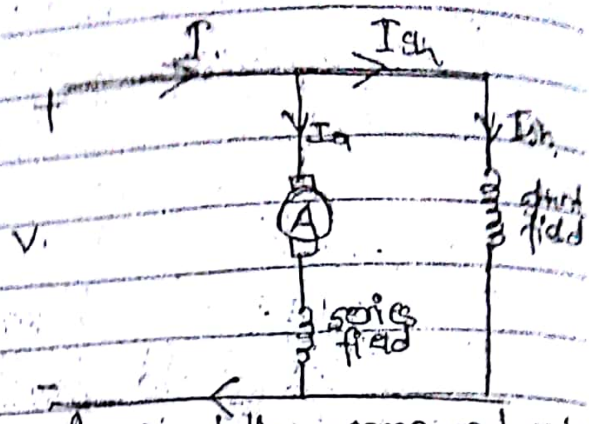


fig. Cumulative compound motor

Characteristics:

with the heavy starting loads, the torque increases. As the load increases, the speed decreases and vice-versa, similar to series motor.

uses:. machine tools, coal cutting machines, crusher, compressors, etc.

ii). Differential compound motor:-

It is one in which the field due to series winding opposes that due to shunt winding.

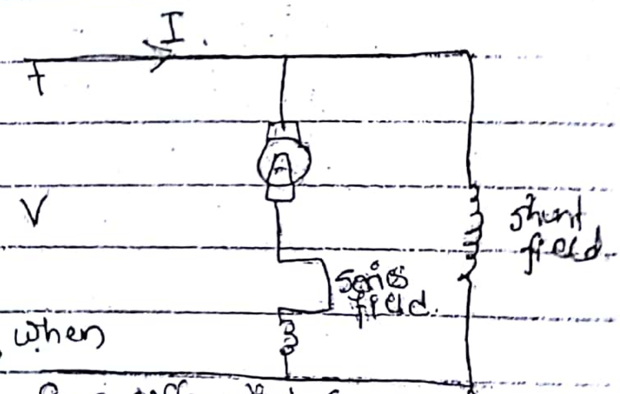


fig 2. Differential Compound motor

Characteristic:

Its speed remains constant. However, when such a motor is started, the series winding requires to be short-circuited; otherwise series winding would rise to its full-value before the shunt field does so.

Uses:

such motors are rarely used, since ordinary shunt motor serves the purpose of providing constant speed.

Speed Control of DC motors:

The speed of a d.c. motor is given by

$$N = \frac{V - I_a R_a}{K\phi} \quad \ldots\ldots (1)$$

Eqn. (1) shows that the speed is dependent upon the supply voltage V, the armature circuit resistance R_a, and the field flux ϕ, which is produced by the field current. In practice, the variation of these three factors is used for speed control. Thus, there are three general methods of speed control of dc motors:-

1. Armature resistance control:- (variation of resistance in the armature circuit).

In this method, a variable resistor R_e is put in the armature circuit. The field is directly proportion connected across the supply and therefore flux is not affected by variation of R_e as shown in fig. 1.

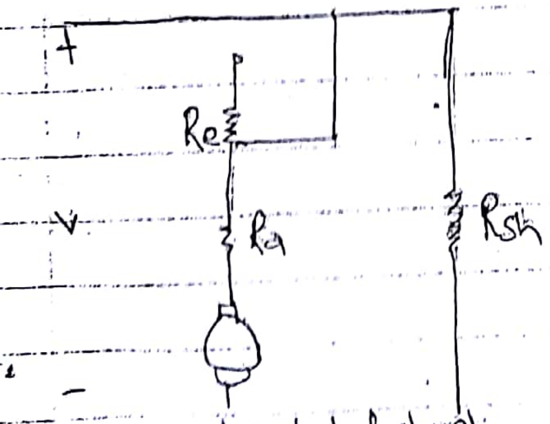


fig 1. speed control of shunt by armature resistance control

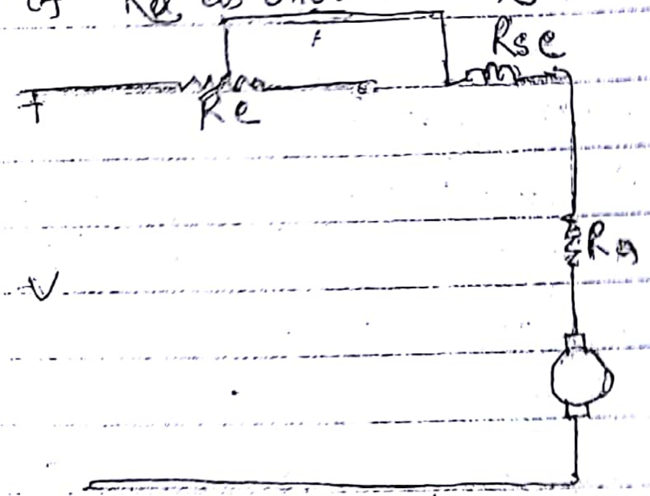
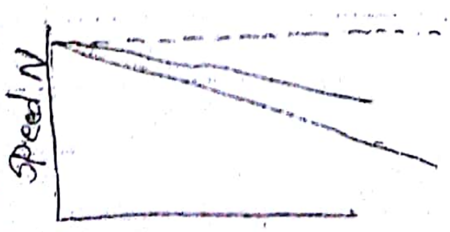


fig 2. series motor



shunt motor

series

fig.(2) shows the method of connection of external resistance Re in the armature circuit of d.c series motor.

In this case, the current and hence the flux are affected by the variation of armature circuit resistance.

This method suffers from the following drawbacks.

1. A large amount of power is wasted in external resistance Re
2. Control is limited to give speeds below normal and increase of the speed cannot be obtained by this method.
3. For a given value of Re, the speed reduction is not constant but varies with motor load. This method is only used for small motors.

b). Variation of field flux:-

Since the flux is produced by the field current, control of speed by this method is obtained by control of the field current. In the shunt motor, this is done by connecting a variable resistor Rc in series with the shunt field winding as shown in fig.(1). The resistor Rc is called shunt field regulator.

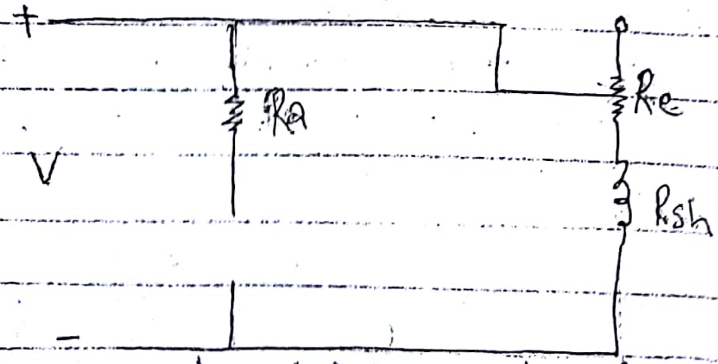


fig.(1) speed control by variation of field flux.

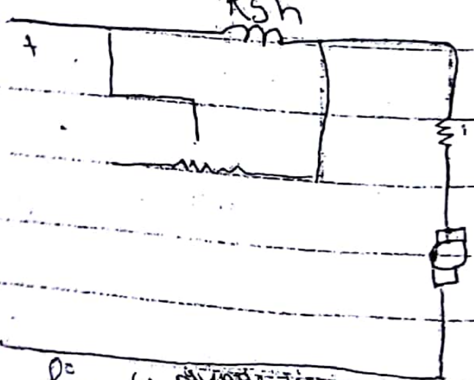


fig (2) parallel with series field of dc motor

The variation of field current in series is done by:

i) A variable resistance Rd is connected in parallel with series field winding as shown in fig. (2). The parallel resistor is called diverter.

ii) Using a tapped field control as shown below:-

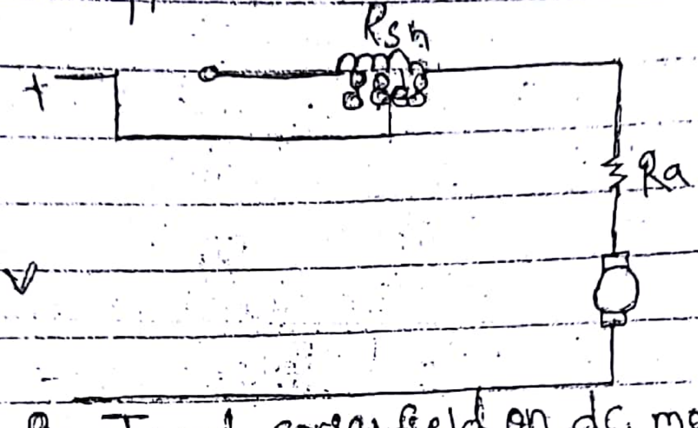


fig. Tapped series field on dc motor.

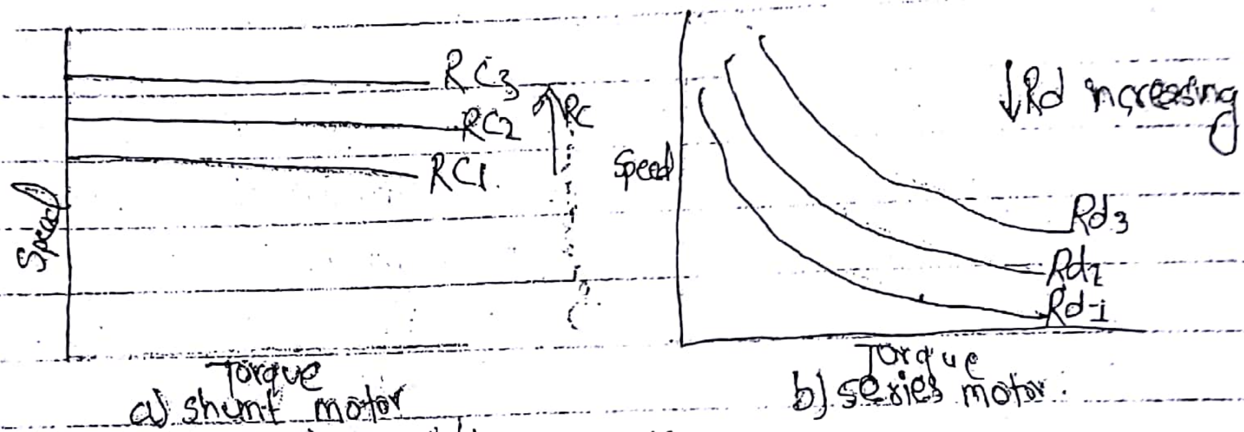


fig. 1. Typical speed/torque curve.

The advantages of field control are as follows:-

1. This method is easy and convenient.
2. Since shunt field current Ish is very small, the power loss in shunt field is small.

3. Variation of applied voltage:-

For separately excited motors, speed control is obtained by varying the applied voltage to the armature.

In the Ward-Leonard system, M is the main motor whose speed is to be controlled.

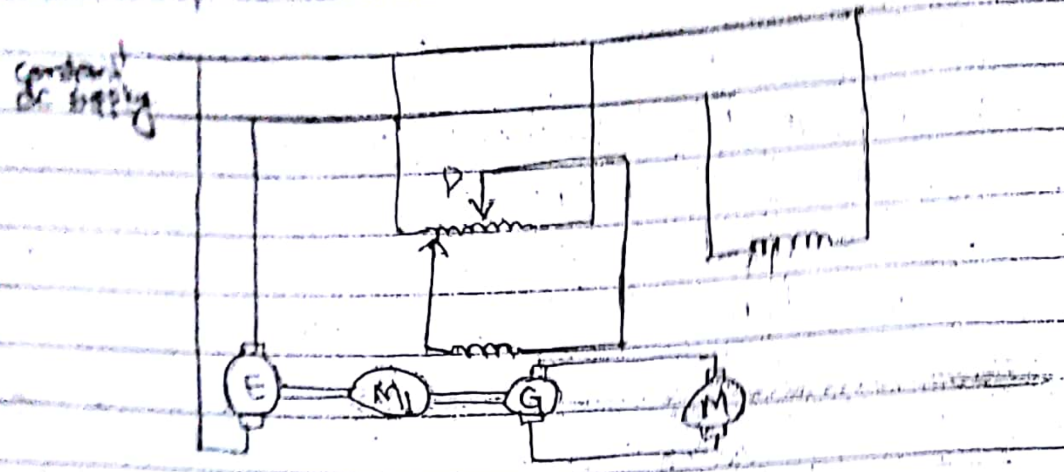


fig. Ward-Leonard system.

A constant speed auxilliary motor M_1 drives a dc generator G. The field of G is separately excited from an exciter E. The purpose of the exciter E is to supply a constant dc voltage to generator G and main motor field. The output of G can be varied over a wide range by means of potentiometer.

DisAdvantages:-

1 The use of extra machines makes the system costly.
2. The overall efficiency is low specially at light loads.

Disadvantages:-
1. The use of extra machines makes the system costly.
2. The overall efficiency is low specially

Advantages:

It offers smooth speed control over the whole range from zero to normal speed in either direction.

Characteristics of a shunt or separately excited DC motor:-

In both the cases of shunt and separately excited dc motors, the field is supplied from a constant voltage so that the field current is constant.

1) speed- armature current characteristics.

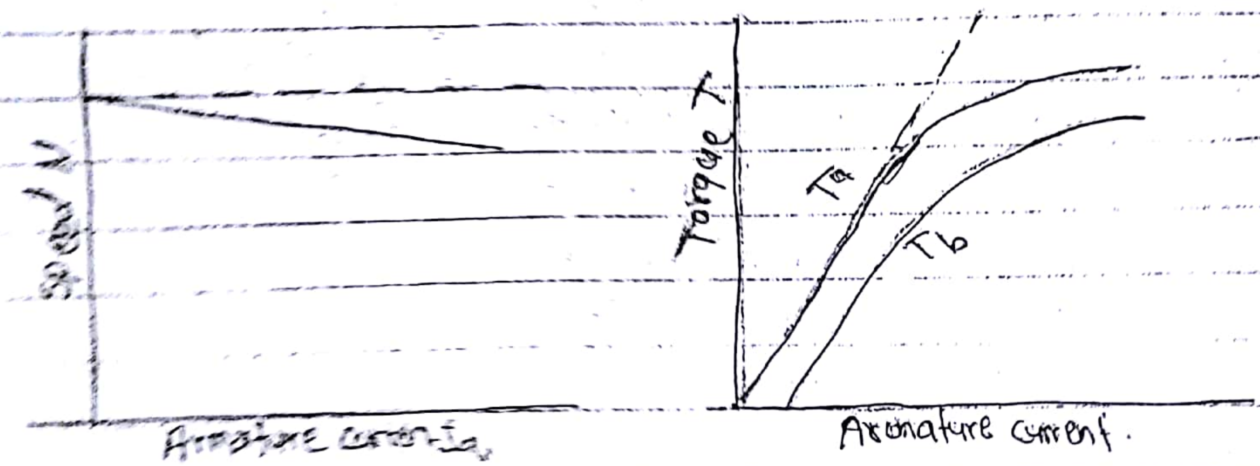


fig - 1). N – Ia   T – Ia.

The speed N of the motor decreases linearly with the increase in armature current as shown in fig (1)

Also,

$$T_g \propto \phi I_a$$

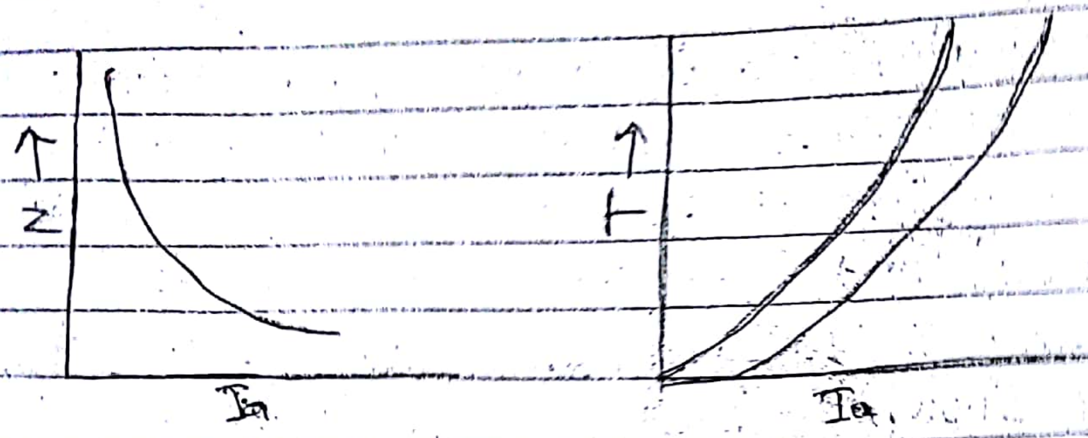
If the effect of armature reaction is neglected, ϕ is nearly constant and,

$$T_g \propto I_a. \quad \text{—}$$

Above eq^n. shows that graph between T_g and I_a is straight line passing through origin.

Characteristics of a DC series motor:

speed / armature current characteristic.



The motor speed N is given by

$$N \propto \frac{V - I_a(R_a + R_h)}{\phi}$$

At low values of I_a, the voltage drop $I_a(R_a + R_h)$ is negligibly small in comparision with V,

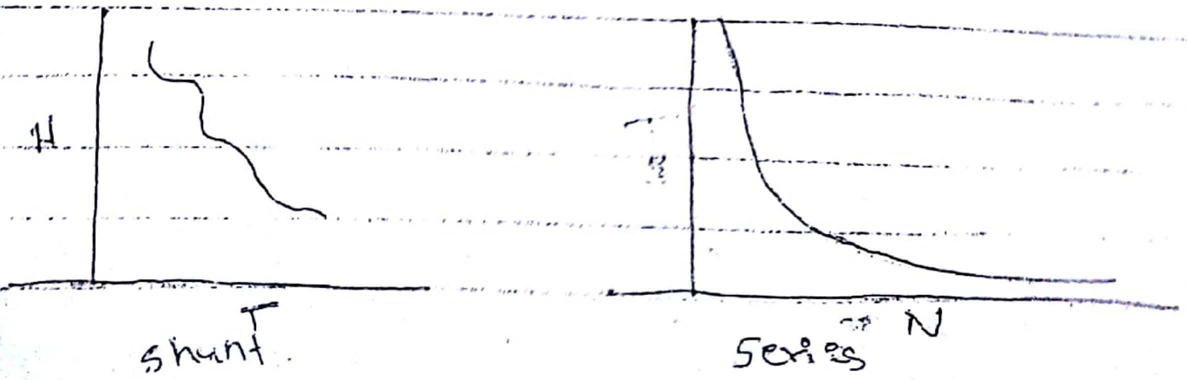
so $N \propto \frac{V}{\phi}$ or, $N \propto \frac{1}{\phi}$.

In a series motor, the flux ϕ is produced by armature current flowing in the field winding, so

$$\phi \propto I_a$$

or, $N \propto \frac{1}{I_a}$.

Thus speed is inversely proportional to armature current. The speed load characteristic is a rectangular hyperbola.



The characteristic shows that the dc series motor has a high torque at low speed and vice versa.

Characteristics of a Compound Motor:-

A compound motor has both shunt and series field winding, so its characteristics are intermediate between shunt and series motor. The cumulative compound motor is generally used in practise and characteristics is shown below:-

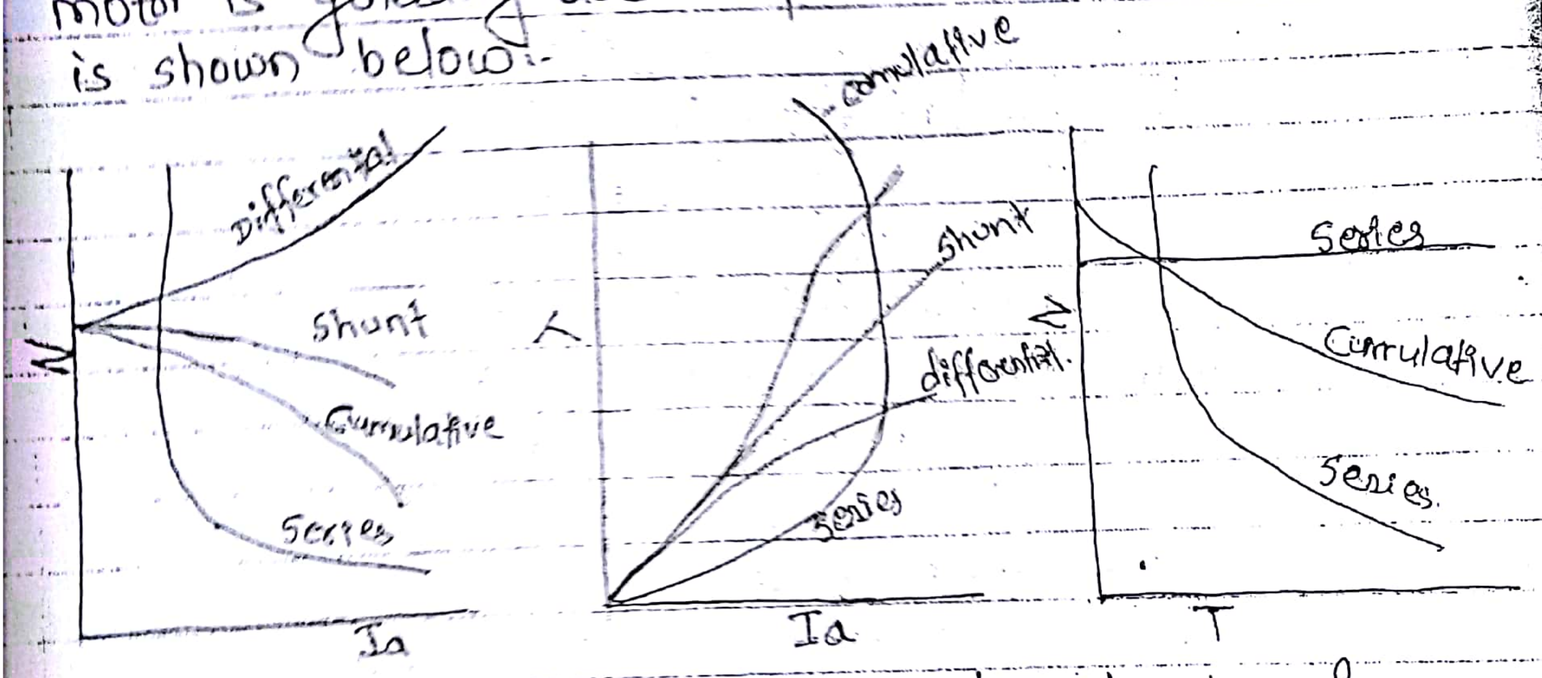


fig. (3) shows the speed-torque characteristics of compound motor. It is found that the compound motor has a high starting torque together with a safe no-load speed. These factors make it suitable for use with heavy intermittent loads such as lifts, hoists, etc.

Torque of a dc machine

When a dc machine is load either as a motor or as a generator, the rotor conductors carry current. These conductors lie in the magnetic field of the air gap. Thus, each conductor experiences a force & torque is produced around ~~when~~ the circumference of rotor and rotor starts rotating.

When the machine operates as a generator at constant speed, this torque is equal and opposite to that provided by the prime-mover. When the machine is operating as a motor, the torque is transferred to the shaft of rotor and drives the mechanical load.

The expression for the torque is same for generator and motor and it can be deduced as follows:-

The voltage equation of a dc motor is,

$$V = E - I_a R_a.$$

$$\text{or, } VI_a = EI_a - I_a^2 R_a \qquad [\text{multiply by } I_a]$$

But, VI_a = electrical power input to armature,

$I_a^2 R_a$ = Copper loss in the armature.

We know that,

input = output + losses

As the value of torque, the electromechanical power conversion takes place,

∴ Mechanical power developed by the armature,

$$P_m = \omega T = 2\pi n T.$$

Therefore,

$$P_m = EI_a = \omega T = 2\pi n T.$$

$$\text{But, } E = \frac{nP\phi Z}{A}$$

$$\therefore \frac{n P \phi Z}{A} I_a = 2\pi n T$$

$$\text{or, } T = \frac{PZ}{2\pi A} \phi I_a \quad \text{--- (1)}$$

Eq^n (1) is called the torque equation of dc motor. For a given dc machine, P, Z and A are constant, therefore $\left(\frac{PZ}{2\pi A}\right)$ is also constant.

$$\text{or, } T \propto K \phi I_a$$

$$\text{or, } T \propto I_a . \phi$$

Hence, the torque developed by a dc motor is directly proportional to the flux per pole and armature current.

Efficiency of a DC machine:-

Describe working principle, construction features and uses of induction motor.

→ <u>working principle:-</u>

Conversion of electrical power into mechanical power takes place in the rotating part of an electrical motor. In d.c. motors, the electric power is conducted directly to the armature through brushes and commutator. Hence, in this sense, a dc motor can be called as conduction motor. However, in ac motor, the rotor does not receive electrical power by conduction but by induction in exactly the same way as the secondary of a 2-winding transformer receives its power from the primary. This is why such motors are known as induction motor.

<u>Construction:-</u>

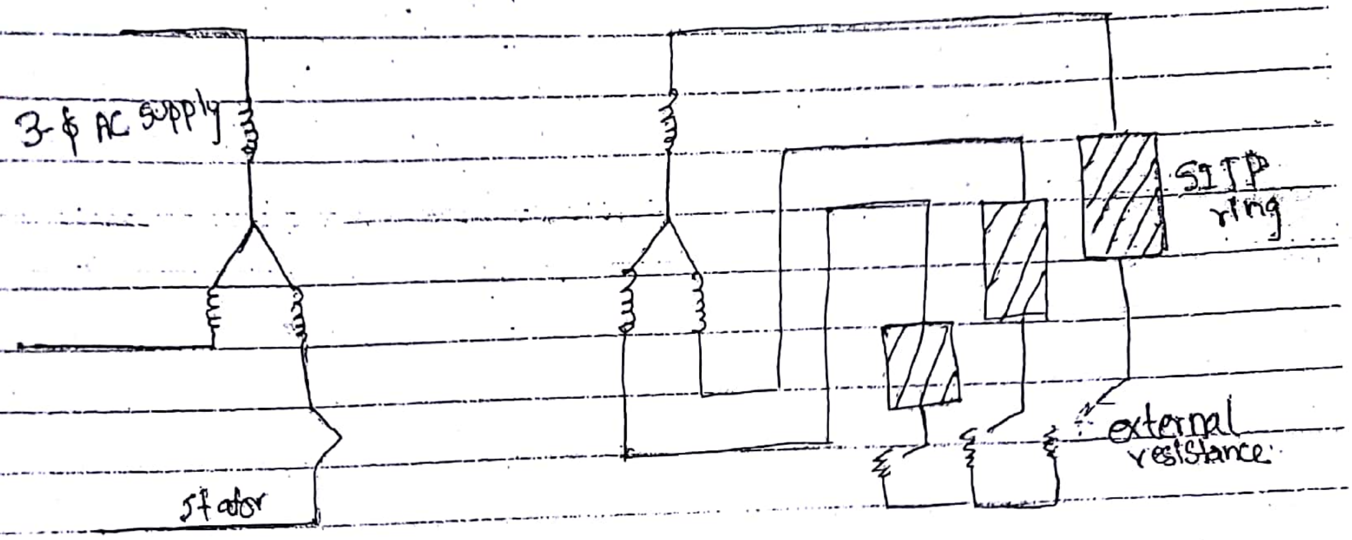
An induction motor consists of two main parts:-

1. <u>A stator:-</u>

It is made up of a number of stampings which are slotted to receive the winding. The stator carries a 3-ϕ winding and is fed from 3-ϕ supply. It is wound for a definite no. of poles, the exact no. of poles being determined by the requirements of speed. Greater the no. of poles, lesser the speed and vice-versa. When supplied with 3ϕ current, produces magnetic flux which ~~consists~~ is of constant magnitude but which revolves at synchronous speed. This revolving magnetic flux induces an emf in the rotor by mutual induction.

Induction motor

ii) a rotor:- It consists of a cylindrical laminated core with parallel slots for carrying the rotor conductors which ~~a~~ should be noted clearly are of copper, aluminium or alloys.



A stator is connected to 3-ϕ supply. The rotating-flux produced by the stator cuts the rotor bars and induces emf. It effects the flow of heavy current due to small resistance of the rotor field produced by the rotor will oppose the main field according to Lenz's law. This results in turning the ~~mo~~ rotor in direction of the rotating field tending to catch it up.

At starting, the rotor is at stand still but the field is travelling at synchronous speed with respect to the rotor. Therefore the emf induced in the rotor for it is maximum and also the current. The rotor runs at the speed ~~of~~ slightly less than the synchronous speed of motor. The no load speed of the motor is very near to synchronous speed. The synchronous speed depends upon the no. of poles and the frequency of supply voltage.

Advantages:-

1. It is very simple and extremely rugged, unbreakable.
2. Its cost is low and it is very reliable.
3. It has sufferiently high efficiency.
4. It requires minimum of maintainance.

Disadvantages:-

1. Its speed cannot be varied without sacrificing some of its efficiency.
2. Just like a d.c. shunt motor, its speed decreases with increase in load.
3. Its starting torque is somewhat inferior to that of d.c. shunt motor.

Uses:-

1. They are used in thermal power plants.
2. It can be used below rated speed.
3. Squirrel case induction motors with relatively low resistance, it is used for fans, most machinary tools, centrifugal pump, wood working tools,
4. Cage induction motor with relatively high resistance is used for crusher, compressor, etc.

Armature Reaction:-

The interaction between the fluxes produced by the field winding and by the current carrying armature windings is known as armature reaction. As a consequence of armature reaction, the air-gap filled is distorted and Magnetic Neutral Plane (MNP) is no longer coincident with the Geographical Neutral Plane (GNP). For maximum voltages at the terminals, the brushes have to be located at the MNP.

In a generator, the effect of armature reaction is to twist or distort the flux in the direction of rotation, whereas in the case of motors, it shifts the magnetic neutral plane backwards opposing the direction of rotation.

* Describe working principles, construction features and uses of Synchronous motors.

→ A synchronous motor converts 3 phase AC into mechanical energy. The speed of the synchronous motor is constant.

Principle:-

It operates on the fundamental principle of electromagnetic induction which states that: "when a rotating coil cuts the magnetic lines of force, an emf is induced in it." The direction of which at any instant is given by Fleming's right hand rule. The magnitude of induced emf at any instant is given by: $BLv \sin\theta$ volts.

Construction

Principle:-

~~Suppose~~ Synchronous motor is not self-starting. It needs a starting device. Now, if by some means the rotor is speed and up temporarily in the same direction as the rotating magnetic flux in the stator then, a relative motion between the stator and the rotor causes flux to reduce, and at or near zero relative speed, the motor ~~reduces~~ reaches the synchronous speed. At this stage, the attraction between the stator and rotor poles causes these to magnetically lock themselves. This inter locking of stator and rotor poles causes the motor to run only at synchronous speed.

Operation:-

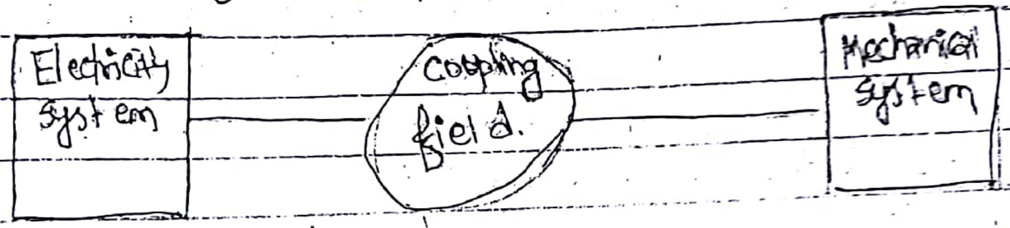
The operation of a synchronous motor is due to the interaction of the magnetic fields of the stator and the rotor. Synchronous motor is a doubly excited machine i.e. two electrical inputs are provided to it. Its stator which consists of a 3-ϕ winding is provided with a 3 ϕ supply and rotor is provided with a c supply. The 3-ϕ stator winding carrying 3-ϕ currents produces 3-ϕ rotating magnetic flux. The rotor ~~produces~~ locks in with the rotating magnetic field, the motor is said to be in synchronization. Once the motor is in operation, the speed of the motor is dependent only to the supply frequency when the motor load is increased beyond the breakdown load, motor falls out of synchronization and the field winding no longer follows the rotating magnetic field. Because the winding is smaller than that of an equivalent induction motor and can overheat on long operation and because large slip frequency voltage are induced in the rotor excitation winding synchronous motor protection devices sense this condition and interupt the power supply.

Uses:-

1. For high voltage service.
2. Power factor correction:- overexcited synchronous motors having leading p.f. are widely used for improving p.f. of those system which employ a large no. of induction motors and other devices having lagging p.f. such as welders and flourescent ~~lamp~~ lights, etc.
3. Constant speed application:-
   Because of their high efficiency and high speed, synchronous motor (above 600 rpm) are well ~~uses~~ suited for loads where constant speed is required such as centrifugal pump, paper mill, compressors, etc.
4. Voltage regulation:-

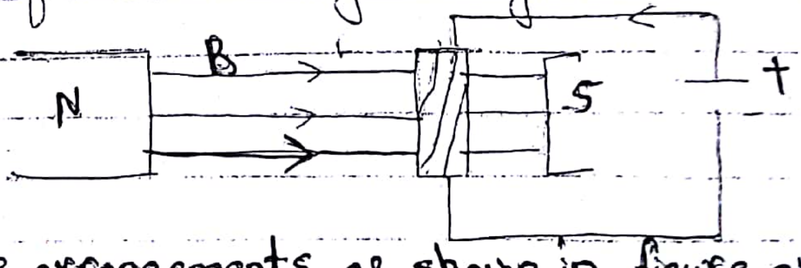
Explain how the electrical energy is converted to mechanical rotation in DC motor?

→ Electromechanical conversion involves the interchanging of energy between an electrical system and a mechanical system through the medium of a magnetic or electric field. The conversion process is essentially reversible except for a small amount which is lost as heat energy. When the conversion takes place from electrical to mechanical form, the device is called motor.



The word 'motor' is derived from latin word meaning 'mover'. When the electrical system is characterized by direct current the electromechanically conversion devices are called dc motors.

The motor action is derived from Ampere's law. According to this law, if a conductor of length 'l' carrying 'I' is placed in a magnetic field of flux density 'B', will experience a force 'F' given by $F = BIl$. The direction of this force is at right angles to the flux and current



simple arrangements as shown in figure are not practical to obtain motor actions. In real machines, suitable constructions are adopted to allow continous movement of conductors in magnetic field. Cylindrical geometry is adopted for construction of machines.

Explain in brief the basic requirements of measuring instruments:

→ The basic requirements of measuring instruments are as follows:-

1) Input device:- It receives the quantity under measurement and delivers a proportional electrical signal to the signal conditioning and processing unit. i.e. senses and converts the desired input to a more convenient and practicable form to be handled by measurement system.

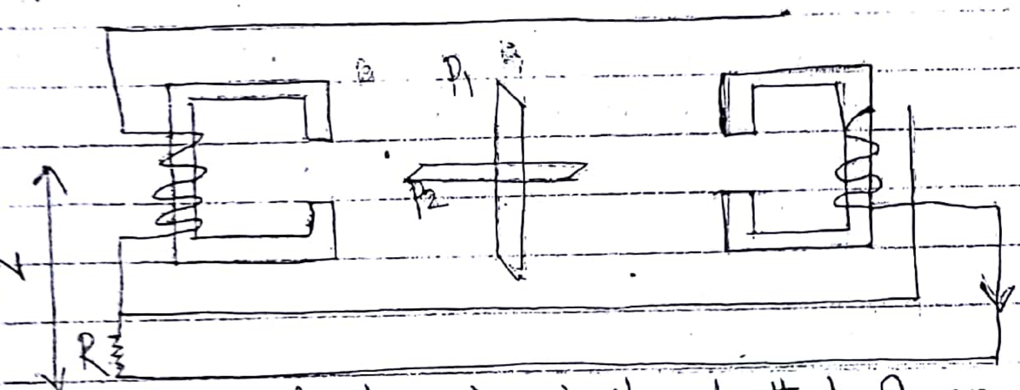
2. Signal conditioning and processing unit:-
This unit performs task like amplification, filtration, wave shaping, signal conversion, etc. It is important unit and plays very important role to modify a signal to a formal acceptable to the output device.

3. Output device:-
A device used to represent measured result or output. An output device may be a simple indicating meter, an oscilloscope, chart recorder or magnetic tape recorder or printer, etc. Output device may be of three types:-

a) Display type
b) Recording type
c) Storing type

4. Calibration element to provide a built in calibration facility.
5. External power supply is used to facilate the work of one or more of the elements like input device, ~~sing~~ signal conditioning and processing unit, output device, etc.
6. Feedback element is used to control the variation of physical quantity that is being measured.

Explain about induction type voltmeter.



Its construction is similar to that of an induction ammeter except for the difference that is ~~bind~~ winding is wind with a large number of turns of fine wire Since it is connected across the lines and carries very small current (5-10mA), the number of turns of its wire has to be large in order to produce an adequate amount of mmf ~~and emf~~. Split phase winding are obtained by ~~wi~~ connecting a high resistance R in series with the winding of one of magnet and an inductive coil in series with the winding of the other magnet as shown in figure above.

Explain the principle of operation of single phase induction motor.

→ Initially the rotor is stationary between the goals of the stator. As the alternating current is supplied to the stator winding, a sinusoidal pulsating magnetic field varying with time is produced which in turn produces pulsating current in the rotor. Since this pulsating current is incapable of producing a rotating torque in the stationary motor, so a single phase induction motor is not self starting. In order to obtain a rotating field, the stator is generally provided with two windings and four poles. The phase difference of 90° between two windings is obtained by splitting the phase.

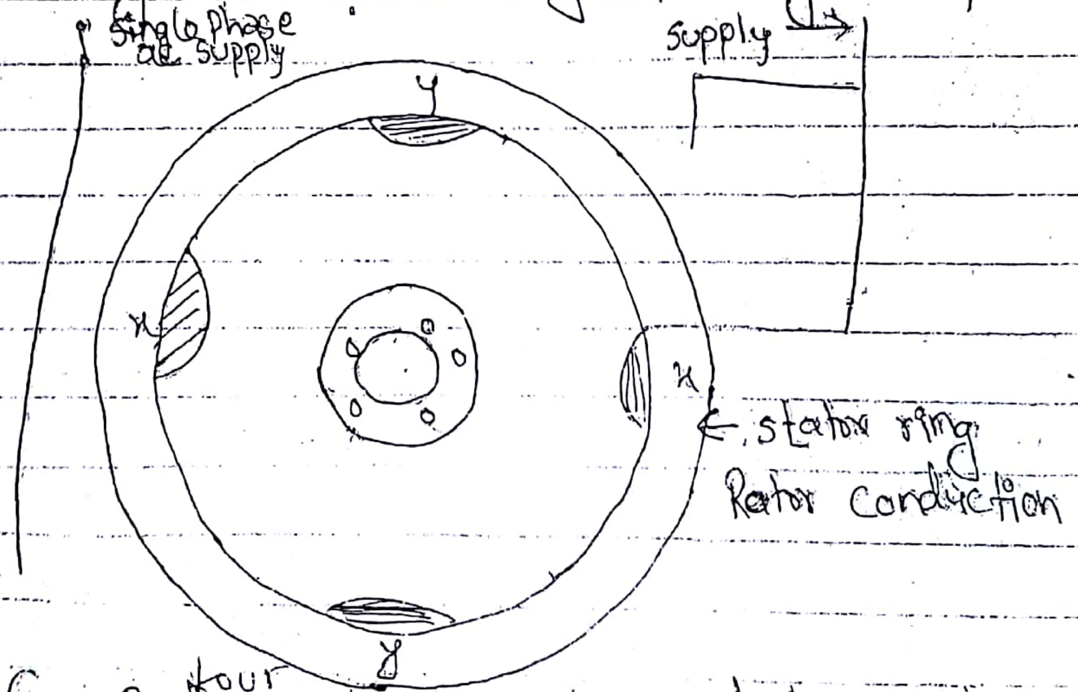


fig. A ~~four~~ four pole single phase induction motor.

The figure above shows the four pole single-phase induction motors. The combined effect of the field set up by two winding is more or less rotating field. Thereby providing a starting torque. After the starting the rotation of the field is carried out by currents set up in the main winding and the starting winding is generally switched out of the stator circuit. Such a motor is called split-phase induction motor.

Uses:- in fans, refrigerator, washing machines, hair dryers, etc.

Analogy of magnetic circuits with electrical circuits:-

Magnetic Circuits	Electrical circuit
1. flow is of flux (ϕ).	1) flow is of current (I)
2. mmf is the cause of current flow of flux.	2. emf is the cause of flow of current.
3. Resistance offered to the flow of flux is called reluctance (S).	3. Resistance offered to the flow of current is called resistance (R).
4. $S = \frac{1}{HA}$ where H is called permeability.	4. $R = \frac{1}{\sigma A}$ where σ = conductance.
5. Permeability (H) $= \frac{1}{S}$	5. Conductance (σ) $= \frac{1}{R}$

Power-factor:- It is defined as the ratio of resistance to impedance

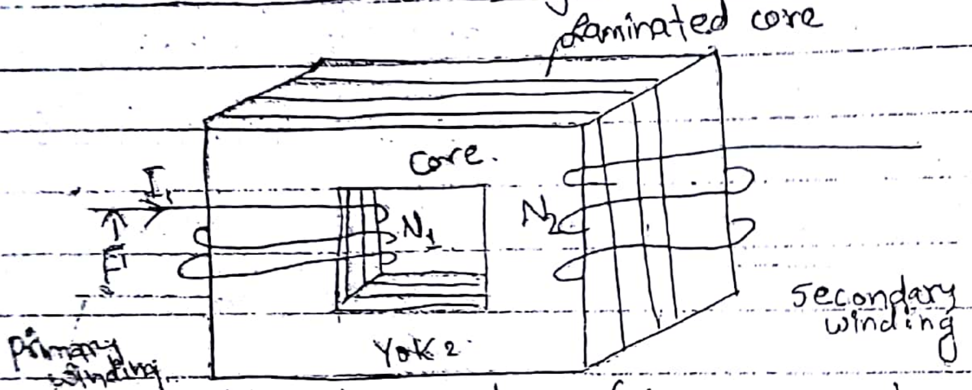
2) the ratio of true power to apparent power.

Transformer

What is transformer ?

A transformer is a static device which consists of two or more stationary electric circuits interlinked by a common magnetic circuit for the purpose of transfering electrical energy between them. This transfer takes place with no change in frequency.

Construction of single ϕ transformer



A single phase transformer consists of primary and secondary windings put on a magnetic core. Magnetic core is used to confine flux to a definite path. Transformer cores are made from thin sheets (called laminations) of high grade silicon steels. The laminations reduce eddy current loss and the silicon steel reduces hysteresis loss. The laminations are insulated from one another by heat resistant enamel insulator coating.

Properties of Ideal Transformer

An ideal transformer is an imaginary transformer which has the following properties.

1. Its primary and secondary winding resistance are negligible.

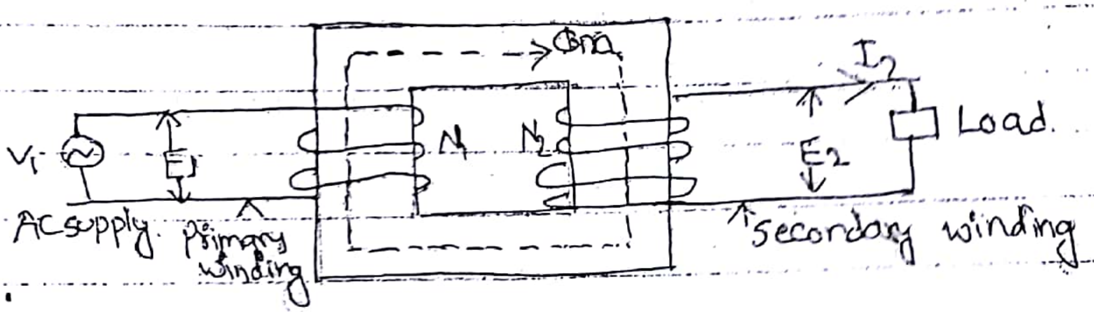
2. The core has infinite permeability (μ) so that negligible magnetic moment of force (mmf) is required to establish the flux in the core.

3. Its leakage flux and leakage inductances is zero. The entire flux is confined to the core and links both windings.

4. There are no losses due to resistance, hysteresis and eddy currents. Thus, the efficiency is 100%.

Imp. **Principle of transformer Operation.** (works on principle of mutual induction)

It transfers electrical energy from one circuit to another circuit without change in frequency.



The two coils windings are insulating from each other. When a source of alternating voltage V_1 is applied to coil N_1, an alternating current I_1 flows in it. This current produces an alternating flux Φm in magnetic circuit. This alternating flux links N_1 and induces in them an alternating voltage E_1 by self induction. If the transformer is ideal then all the flux produced by coil N_1 also links N_2 and induces in them a voltage E_2 by mutual induction. If secondary is connected to a load, a current I_2 flows through it. Thus, energy transfers.

(emf eqn:- already back)

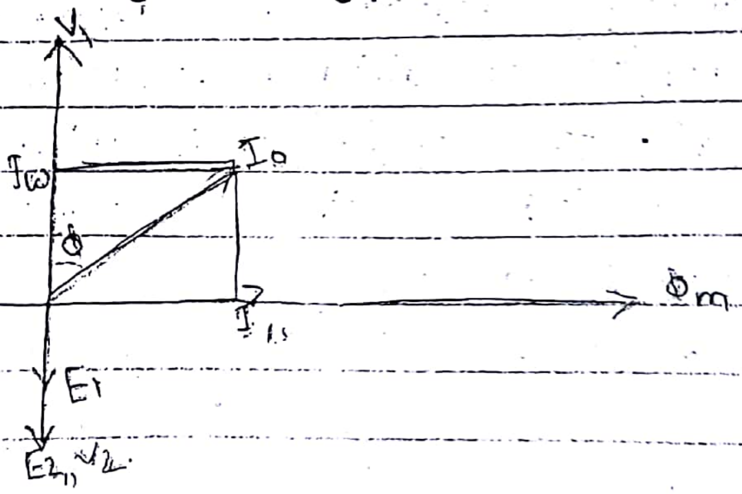
Transformer on no load

A transformer is said to be on no load when the secondary winding is open circuited. The secondary current is thus zero. When an alternating voltage is applied to the primary, a small current I_o flows in the primary. This is called no load current. The no load current has two components I_μ and I_ω.

I_μ is known as magnetizing component which magnetizes the core and sets up flux in the core, so is in phase with ϕm. The I_μ is also called reactive or wattless component.

$I_\omega \rightarrow$ known as active or wattful component and supplies hysteresis and eddy current loss. It is in phase with supplied voltage.

The phasor diagram is given as:-



$I_\omega = I_o \cos\phi$

$I_\mu = I_o \sin\phi$

$I_o = \sqrt{I_\omega^2 + I_\mu^2}$

$\cos\phi = \frac{I_\omega}{I_o}$

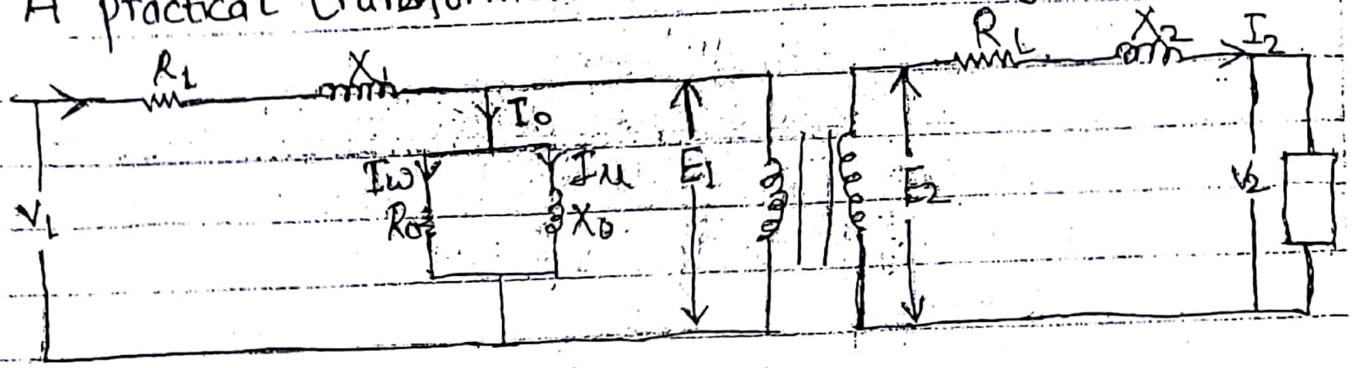
E_1 is equal and opposite to V_1. Since E_2 and E_1 are both induced by same flux, E_2 is in the same direction as E_1 but opposite to V_1.

core loss $= V_1 I_o \cos\phi_o = V_1 I_\omega$ (W) reactive = $V_1 I_0 \sin\phi_0 = V_1 I_\mu$ (VAR)

Reactive transformer has resistance and leakage reactance in both primary and secondary windings.

Equivalent circuit of a Transformer:-

A practical transformer is as shown in figure.



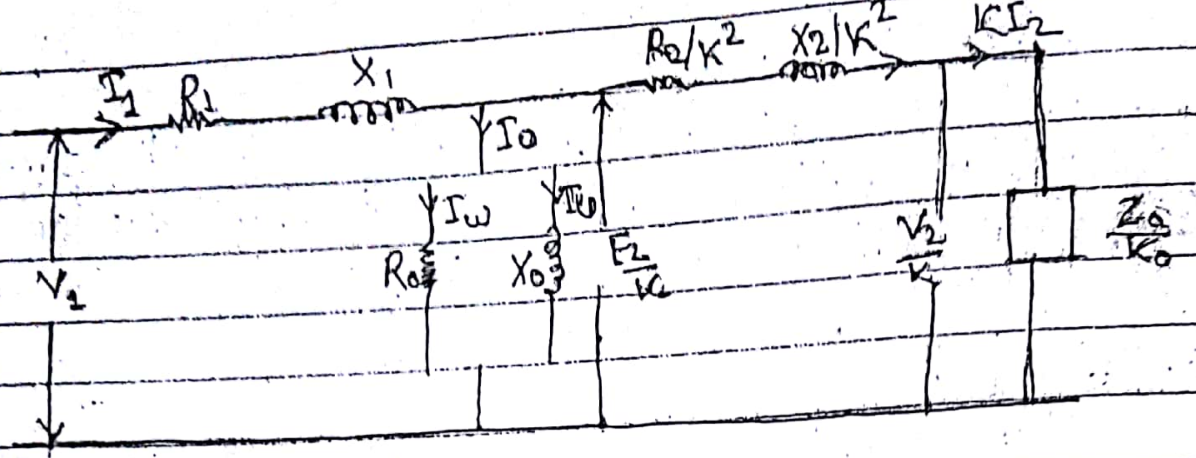
The equivalent circuit can be converted into simpler equivalent circuit by transferring all resistances, reactances, voltages, currents to either from primary to secondary or secondary to primary. If transfers from secondary to primary, it is called reffered to primary and if from primary to secondary, it is called reffered to secondary.

Lets convert from secondary to primary, i.e. referred to primary as

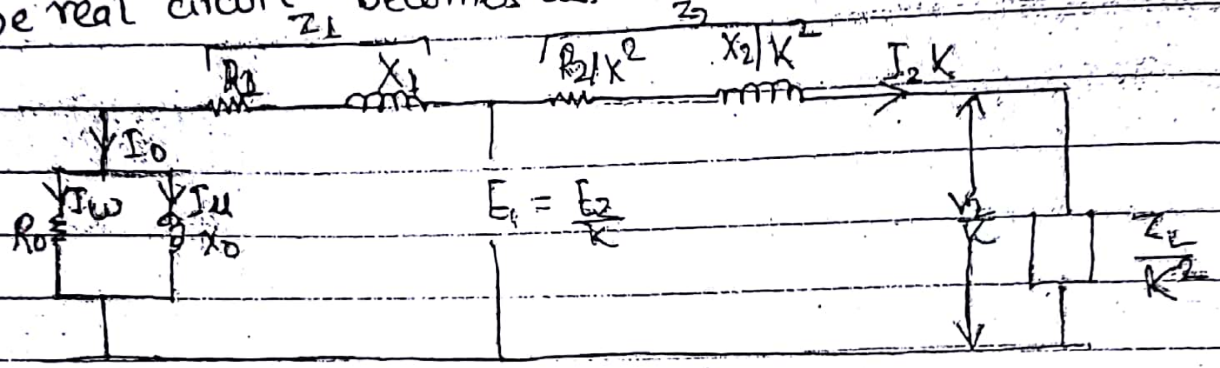
$$R_2' = \frac{R_2}{K^2}, \quad X_2' = \frac{X_2}{K^2}, \quad Z_L' = \frac{Z_L}{K^2}$$

$$V_2' = \frac{V_2}{K}, \quad E_2' = \frac{E_2}{K}, \quad I_2' = KI_2.$$

The circuit known as exact equivalent circuit referred to primary is shown below:



The real circuit becomes as:



Approximate equivalent circuit of the transformer referred to primary (calculation become easier).

Losses in transformer:-

a) Iron loss or Core loss:-

Iron loss occurs in the magnetic core of the transformer. This loss is the sum of hysteresis loss and eddy current loss.

b). Copper loss or I^2R loss:-

Copper loss is the I^2R loss which takes place in the primary and secondary winding because of the winding resistance.

Total copper loss = primary winding copper loss + secondary winding copper loss

$$P_c = I^2R_1 + I^2R_2.$$

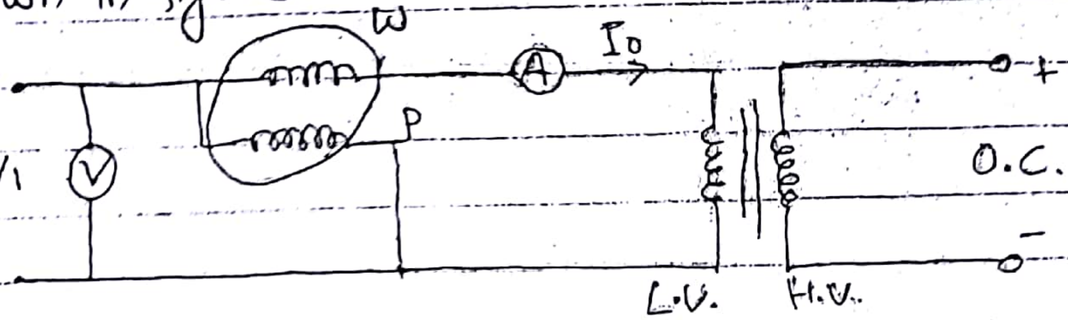
Transformer Test

Tests are performed to determine the circuit constants, efficiency and regulation without actually loading the transformer.

There are two tests:

1. Open circuit test:-

The high voltage side is left open. The circuit is shown in figure.



Thus the voltmeter reads the rated voltage V_1 of the primary. Since, the secondary is open circuited, a very small current I_0, called the no load current flows in the primary. The ammeter A, reads no load current I_0. The wattmeter reads the core loss (iron loss) only. There is no I^2R loss.

Ammeter reading = no load current I_0.

voltmeter reading = ~~no load~~ primary rated voltage V_1.

wattmeter reading = iron or core loss. P_i

From these measurement, the no load equivalent circuit can be determined.

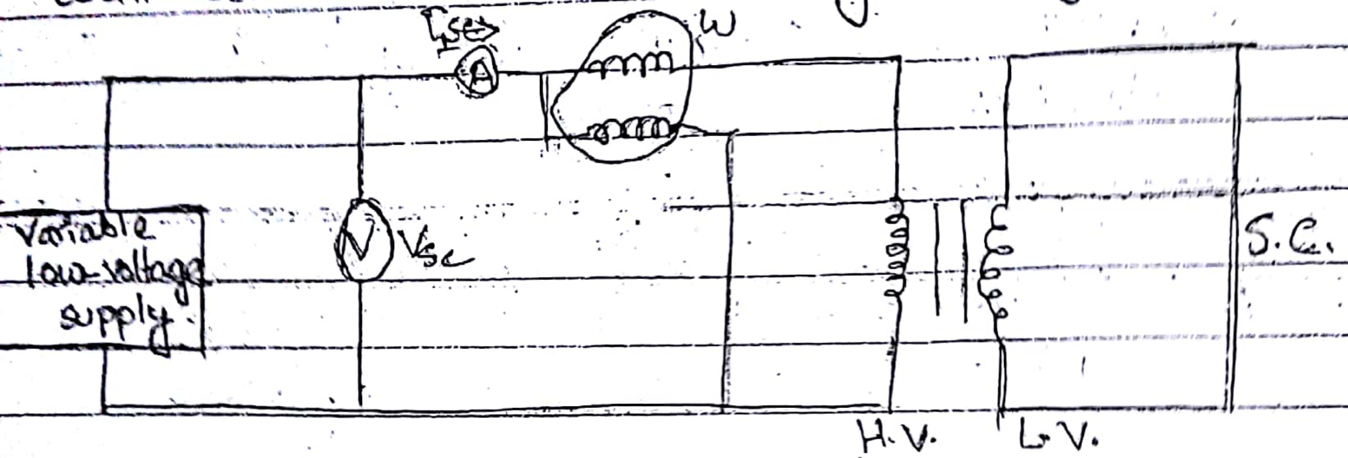
$$P_i = V_1 I_0 \cos \phi_0.$$

$$\text{no load p.f.,} \quad \cos\phi = \frac{P_i}{V_1 I_0}$$

$$I_w = I_0 \cos\phi_0, \quad I_\mu = I_0 \sin\phi_0, \quad R_0 = \frac{V_1}{I_w}, \quad X_0 = \frac{V_1}{I_\mu}$$

Short Circuit Test:-

In this test, usually the low voltage side is short circuited by a thick wire. An ammeter, a voltage and a wattmeter are connected on high-voltage side.



The ammeter reading I_{1sc} gives the full load primary current. The voltmeter reading V_{1sc} gives the value of the primary applied voltage when full-load currents are flowing in the primary and secondary. The wattmeter gives the full load copper loss. $V_2 = 0$, since short circuited.

$$V_{1sc} = I_{1sc} - I_C$$

$$P = V_{1sc} \cdot I_{1sc} \cos \phi_{sc}.$$

ammeter reading = I_{1sc} (full load primary current)

voltmeter reading = short circuit voltage V_{sc}.

Wattmeter reading = full load copper loss.

Now, equivalent resistance of the transformer referred to primary.

$$R_{e1} = \frac{P}{I_{1sc}^2}$$

Equivalent impedance,

$$Z_{e1} = \frac{V}{I}$$

Equivalent reactance referred to primary

$$X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2}$$

$$\cos\phi_{sc} = \frac{R_{e1}}{Z_{e1}}$$

with short circuit test performed on one side, the equivalent circuit constants referred to other side can be calculated as

$$Z_{e2} = Z_{e1}\left(\frac{N_2}{N_1}\right)^2$$

A test performed gave the following results:

OC test : 250V 1A 50W on HV side

SC test : 20 V 12A 100 W on LV side.

Find the equivalent transformer circuit referred to both sides.

→ solution:- (back page)

Q. What are main parts of transformer ?

1. An iron core :- It is either circular or rectangular in shape, and is laminated (to avoid eddy currents). The vertical portion is called limb; while top and bottom portions are yokes,

2. Two windings, one connected to the source (called primary winding) and other connected to load called secondary windings.

Efficiency of transformer:

The efficiency of a transformer, like that of any other piece of equipment, is the output power expressed as a percentage of the input power.

$$\text{Efficiency}, \eta = \frac{P_o}{P_i} \times 100\%$$

$$\text{or}, \eta = \frac{V_2 I_2 \cos\theta_2}{V_1 I_1 \cos\theta_1} \times 100\%$$

Because $P_i = P_o + \text{losses}$

$$\eta = \frac{P_o}{P_o + \text{losses}} \times 100\%$$

The power losses in a transformer consists of core losses due to hysteresis and eddy currents and copper losses due to the current flowing in the primary and secondary windings. As long as the supply frequency remains constant, the core losses tend to be a constant quantity. The copper loss have two components:

Primary winding copper loss $= I_1^2 R_1$

secondary winding copper loss $= I_2^2 R_2$

$$\eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + I_1^2 R_1 + I_2^2 R_2 + P_c} \times 100\%$$

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Differences between Practical transformer & Ideal Transformer

Practical Transformer	Ideal Transformer.
1. There are copper and eddy current losses.	1. There is no loss.
2. Efficiency is 93-97%.	2. Efficiency is 100%.
3. There is leakage of flux.	3. There is no leakage of flux.
4. Voltage regulation is never zero%..	4. Voltage regulation is 0%..
5. Its winding contains ohmic resistance also.	5. Its consists of purely inductive coils, wound on lossless core.
6. All constructed transformer are practical transformer.	6. It is impossible to construct an ideal transformer.

Describe working principle of single phase energy meter.

Principle:-

Induction-type energy meter (or watt-hour meter) is used for single phase AC measurement only, and is based on the fact that a torque is produced when a reaction between flux of an AC magnet and the eddy-currents induced by the flux.

Construction:-

It consists of:-

1) Two laminated electromagnets - One is excited by the load current, and is called the series magnet (or current coil), while the other is excited by current proportional to voltage called the shunt motor. The shunt magnet is in parallel with the load.

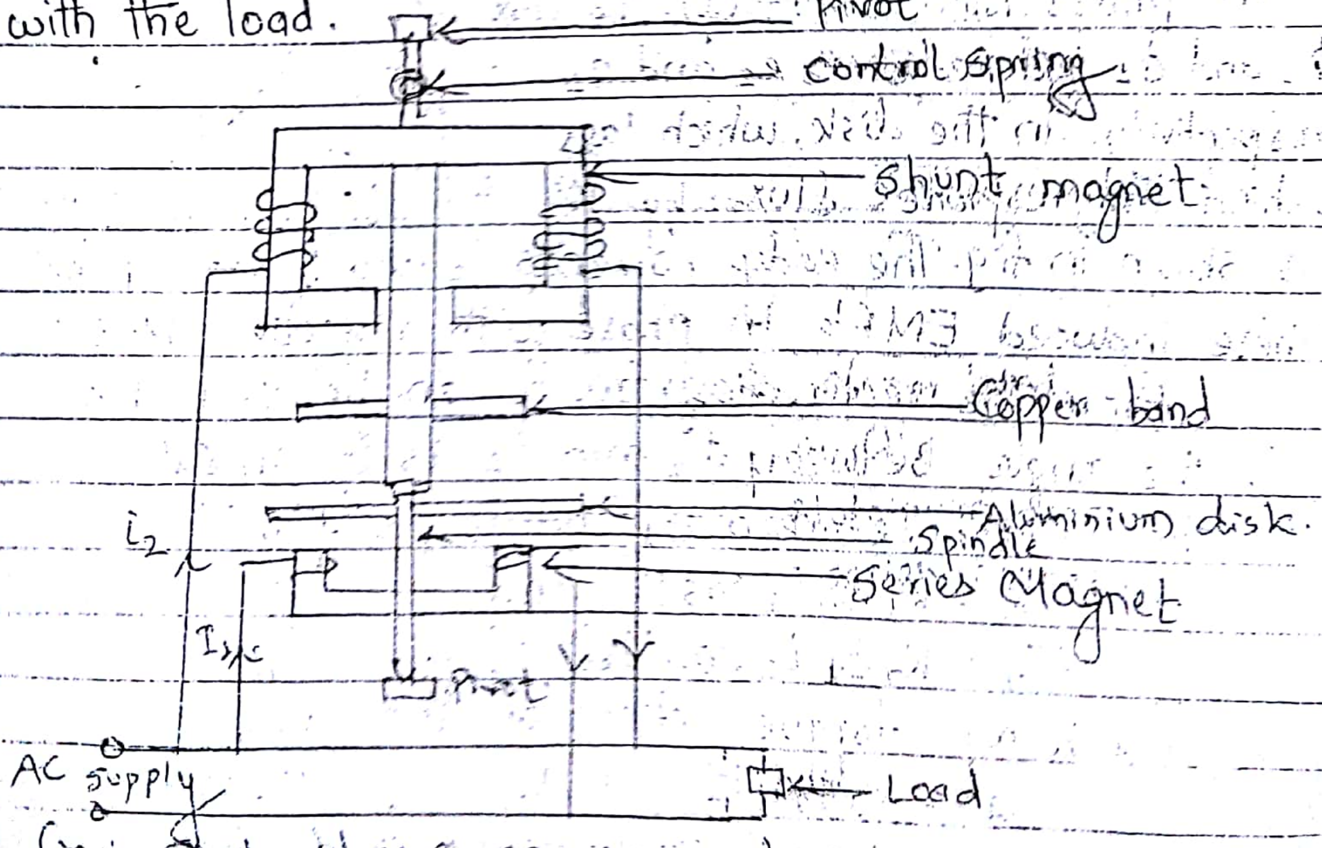


fig: Single phase energy meter.

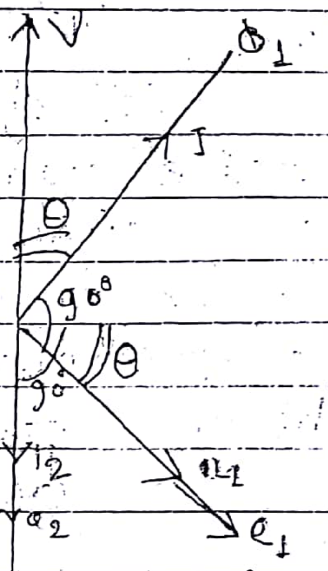
2. A thin aluminium disk is mounted between the two magnets so that it cuts the leakage fluxes of both the magnets, when current flows in the two magnet.

3. Lag adjuster:- In this instrument, it is absolutely essential that the shunt coil flux lags behind the voltage exactly by 90°.

4. Control:- The instrument is spring controlled. The springs are fixed to the spindle of the rotating aluminium disk having pointer.

Operation:-

The shunt magnet produces flux ϕ_2, which after correct adjustment by the copper bands, lags behind the applied voltage exactly by 90°. The series magnet produces flux ϕ_1, which is in phase with current (I). The flux ϕ_2 and ϕ_1 induce EMFs e_2 and e_1 respectively in the disk, which lags behind the respective fluxes by 90°



vector diagram of induction type single phase energy meter

as shown in fig. The eddy currents i_2 and i_1 set up by these induced EMF's in phase with respective EMFs.

From the vector diagram, it can be seen that,

i) the angle between ϕ_2 and i_1 is θ and

ii) the angle between ϕ_1 and i_2 is $(180\,\theta)$

$\therefore T_1 = K_1 \phi_2 i_1 \cos\theta$ and

$T_2 = K_2 \phi_1 i_2 \cos(180° - \theta)$

$\therefore$ Average torque acting on the disk,

$$T_d = K_1 \phi_2 i_1 \cos\theta - K_2 \phi_1 i_2 \cos(180 - \theta)$$

$$= K_1 \phi_2 i_1 \cos\theta + K_2 \phi_1 i_2 \cos\theta$$

But $i_2 \propto V$; $\phi_2 \propto V$; $i_1 \propto I$ and $\phi_1 \propto I$, ... (4)

$\therefore T \propto VI\cos\theta \propto$ power

i.e. the average torque acting on the aluminium disc is proportional to the power consumed by the circuit.

Advantages:-

1) It has a fairly long scale extending over 300° deflection
2) It has high working torque.
3) It is free from effects of stray field.
4) It is practically free from frequency errors.
5) It has good damping.

Disadvantages:-

1. It involves higher power consumption.
2. It is suitable only for ac measurements.
3. Error is introduced in it due to change in frequency and temperature.