CHP = 2

Signal Measurements

In measuring a quantity, we always compare it with some established reference standard. Such standard is called a unit of a quantity. Measurement implies comparison with standard value. To say that a robe is 30 m long is to say that is 30 times as long as an object whose length has been defined to be 1 meter.

Fundamental units:

Basic units of mass (M), length (L), time (T) are known as fundamental units. These are also known as primary fundamental unit.

Derived units:

All other units which can be expressed in terms of fundamental units with the help of physical equations are called derived units. For example; there is no unit for speed among the base unit, however a suitable unit can be derived from the equation.

$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{m}{s} = m/s.$$

This suggest that the unit of speed is m/s.

// STANDARDS :

1/ International Standard (International bureau of measurement)
2/ Primary Standard (National standard)
3/ Secondary Standard (Laboratories Standard)
4/ Working Standard (General Standard).

A Standard is a physical representation of a unit of measurement. The term standard is applied to up piece of equipment having a known measure of physical quantity. They are used for obtaining the value of physical properties of other equipment by comparison measure.

1/ International Standard :

They are defined on the basis of international agreement. They are check and evaluated regularly against absolute measurement in terms of fundamental units. They are maintained at the international bureau of weight and measurement. They are not available to the ordinary user of measuring instrument for the purpose of comparison or calibration.

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2) Primary Standard:
The standard are maintained by national laboratories.
They are not available for the use outside the national laboratories. One of the main function of primary standard is the verification and calibration of secondary standard.

3) Secondary standard:
They are basis reference standard used in industrial measurement, laboratories. These standards are checked locally against the reference standard. These are normally sent periodically to the national standard laboratories for calibration and comparison against primary standard. They are sent back to the industry by national laboratory. With a certification as regard their measured values in terms of primary standard.

4) Working Standard:
They are used to verify and calibrate general laboratories instruments for their accuracy and performance. These are used for general purpose measurement. They are calibrated and compare by secondary standard.

Measurement Instrument

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Mechanical Instrument :
They are reliable for and stable condition. They have many parts that are rigid, heavy and bulky, consequently has large mass. During dynamic conditions, they are unable to respond rapidly.

Electrical Instrument :
Electrical system normally depends upon mechanical measurement, movement as indicating device. They are too slow for present day where there is requirement of fast measurement.

Electronic Instrument :
These instrument requires use of semi-conductor devices. In electronic devices or instrument only movement involved is that of electrons, the response time is extremely small. Very weak signals can be detected by using pre-amplifiers and amplifiers. Power amplification provided by the electronic amplifiers which result in higher sensitivity.

During electronic system non-electrical variable are connected into electrical variable using transducer, power consumption is very low.

Similarly, instrument can be classified into major categories depending upon the way of representing result of measurement.

@ Deflection Type :

The measured quantity produces some physical effect which deflects or produces of mechanical displacement of the moving system of instrument. And opposing effect is built in the instrument which tries to oppose the deflection of moving system. The opposing system is so designed that its magnitude increases with increase in deflection of moving system caused by the quantity of measurement. The deflection of instrument produces up basis of determining the quantity to be measured. Opposing force is equal to deflection effect produced by the measured quantity or measuring.

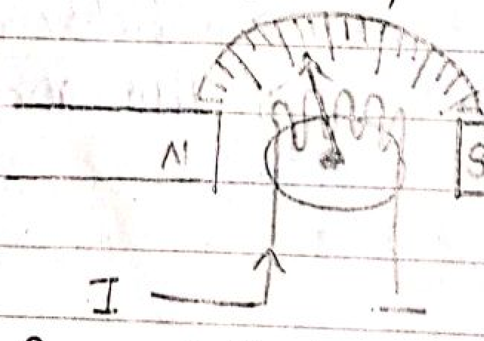

fig: Deflection type Measuring instrument.

⑥ Null type :
A null type instrument attempts to maintain the deflection at zero by suitable application of an effect opposing that generated by the measured quantity. A detector that detects the null condition that is a device which indicates zero deflection, (balanced condition) when the effect produced by measured quantity is equal to the effect produced by opposing quantity. The accuracy is higher than deflection type and they are highly sensitive.
For ex: Wheat stone bridge or meter bridge.

characteristics of Instrument (Performance parameter)

Static characteristics

→ Accuracy
→ Sensitivity
→ Reproducibility and Repeatability
→ Drift
→ Static error
→ Dead zone.

Dynamic Characteristics

→ Speed of response
→ Measuring Lag
→ Fidelity
→ Dynamic error.

When an input is applied to a system or instrument, the response doesnot take its maximum value or constant value immediately, there will be some delay in a time, known as transient period, and the behaviour shown by the system or instrument during this period is known as dynamic characteristics.

After the transient period, the response takes its constant value. This time period is known as steady state period, and the behaviour shown by the system during this period is known as static characteristics.

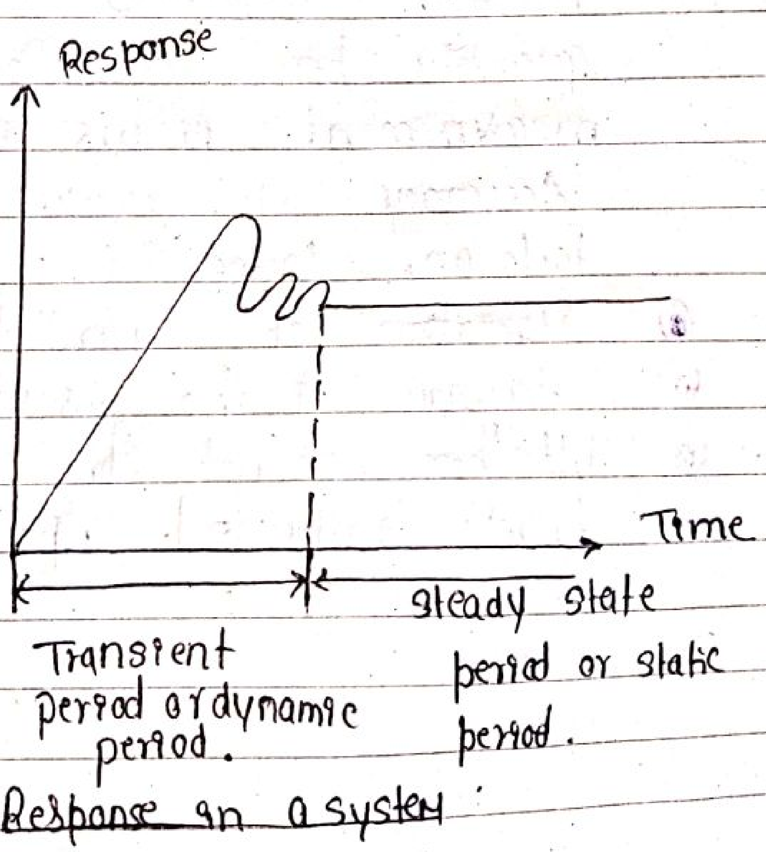

Fig: Response in a system

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4.1 Static characteristics :

Static characteristics are set of criteria that gives of meaningful description of quality of measurement where the quantity to be measured are constant or change should with time.

(a) Accuracy :

It is the degree of correctness with which a measuring means gives the truth value with reference to accepted engineering standard. It is the closeness with which an instrument reading approaches the true value of the quantity being measured. The accuracy of measurement means conformity to truth.

Accuracy of measured signals depends upon following factors;

(i) Variation of signal to be measured.
(ii) Accuracy of the observer.
(iii) Whether or not the quantity is being truely impressed upon the measurement.

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(b) Sensitivity :

It is the ratio of magnitude of output signal to the magnitude of input signal or it is the ratio of response to the quantity be measured.

The ratio of magnitude of measured quantity to the magnitude of response is defined as deflection factor or inverse sensitivity.

$$\text{Sensitivity} = \frac{\text{Magnitude of output Signals}}{\text{magnitude of input signals}}$$

or, $$\text{Sensitivity} = \frac{\text{Response}}{\text{Quantity being measured}}$$

$$\text{Deflection Factor} = \frac{1}{\text{Sensitivity}}$$

The sensitivity of an element should be high.

Que A wheat stone bridge required a change of 7Ω in unknown arm of the bridge to produce a change in deflection of 3 mm of Galvanometer, determine the sensitivity and deflection factor ?

Here,

magnitude of input signal = 7Ω

magnitude of output signal = 3 mm.

Then,

$$\text{Sensitivity} = \frac{\text{Magnitude of output signal}}{\text{magnitude of input signal}}$$

$$= \frac{3mm}{7\Omega}$$

$\therefore$ Sensitivity = 0.43 mm/Ω

$\therefore$ Deflection Factor $= \frac{1}{\text{sensitivity}}$

$$= \frac{1}{0.43}$$

$= 2.33\ \Omega/mm$

c.) **Reproducibility and Repeatability :**

Reproducibility relates to the closeness of output reading for the same input when their changes in the method of measurement, observer, measuring instrument, location, conditions of use and time of measurement. It may be specified in terms of scale reading over a given period of time.

Repeatability relates to the closeness of output reading when the same input is applied repeadedly over a short period of time with the same measurement condition, same instrument, same location and same condition of use maintained throughout the observation by the same observer.

d.) **Drift**

It is an undesired gradual departure of instrument output over a period of time i.e, unrelated to change in input operating condition or load. An instrumet is said to have no drift if it reproduces same change at different time for same variation in measured variable.

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The drift may be cause by following factors;
- Mechanical vibrations
- Electromagentic field
- Thermal emf.
- Temperature change etc.

• Static error :
Study error is defined as the difference between measured value and true value of the quantity.

i.e $\delta A = X_m - X_t$

Where,

$\delta A \rightarrow$ static error
$X_m \rightarrow$ measure value
$X_t \rightarrow$ true value.

'δA' is also called absolute study error of the quantity 'X' i.e $\delta A = \varepsilon_0$.

$\varepsilon_0 \rightarrow$ absolute static error.

The relative static error (ε_r) is the ratio of absolute static error to the true value of quantity under measurement.

$$\varepsilon_r = \frac{\varepsilon_0}{X_t} = \frac{\delta A}{X_t}$$

$$\Rightarrow \delta A = X_t \varepsilon_r$$

$$\because \delta A = X_m - X_t$$

or, $X_t \varepsilon_r + X_t = X_m$

or, $X_t (1 + \varepsilon_r) = X_m$

$$\therefore X_t = \frac{X_m}{1 + \varepsilon_r}$$

Static Correctness $= -\delta A$

$= X_t - A_m$

Que A Meter reads 127.50V and the true value of the voltage is 127.43V, determine static error and static correction of this instrument?

Soln :

Measured value $(A_m) = 127.50V$

true value $(A_t) = 127.43V$

Then,

Static error $(\delta A) = A_m - A_t$

$= 127.50 - 127.43$

$= 0.07$

$\therefore$ Static Correction $= -\delta A$

$= -0.07$

5) Dead zone :
It is defined as the largest change of the input quantity for which there is no change of the instrument. For example; the input to the instrument may not be sufficient to overcome the friction and will not move. It will only move when the input is such that it produces a driving force which can overcome friction force.

2) Dynamic characteristics :

Dynamic characteristics are criteria based upon the dynamic relation many measurements are concerned with rapidly varying quantities and therefore for such a case we must test the dynamic relationship which exist between output and input. This is normally turned with the help of differential equation.

@ Speed of Response :
→ It is defined as the rapidity with which a measurement system respond to change in the measurend quantity. The

behaviour of the system when input vary with time (i.e, input are dynamic in nature) and so does the output. Such a response in measurement system is found occur in industrial, aerospace, biological application.

(b) Measuring Lag :

It is the retardation or delay in the response of measurement system to change in the measured quantity. The lag is caused by conditions such as capacitance, inertia or resistance. They are of two types.

- Retardation type lag.
- Time delay type lag.

(c) Fidelity :

It is defined as the degree to which a measurement system indicates change in measured quantity without any dynamic error. It refers to the ability of the system to reproduces the output in the same form as the input.

(d) Dynamic Error:
It is the difference between the true value of the quantity changing with the time and the value indicated by the measurement system. It is also called measurement error.

Errors are inherent in the act of measurement itself since the perfect accuracy is not attaindable, a ~~desp~~ description of each measurement should include an attempt to evaluate the magnitude and source of its error. The study of error is the first step in finding way to reduce them. Errors may arise from different source and are usually classified as below.

Statistical Analysis

A statistical analysis of measurement data is common practice because it allows an analytical determination of the uncertainty of the final test result. To make statistical methods and interpretations meaningful, a large number of measurements is usually required.

1) Arithmetic Mean
2) Deviation From Mean
3) Average Deviation
4) Standard Deviation

1) Arithmetic Mean.

The based approximation is made when the number of reading of the same quantity is very large. Theoritically, large no. of reading would give the best result. The arithmetic Mean is given by,

$$\overline{X} = \frac{X_1 + X_2 + X_3 + ---- + X_n}{n} = \frac{\sum_{i=1}^{n} X_i}{n}$$

Where,

$\overline{X}$ = Arithmetic mean

$n \rightarrow$ total no. of reading/observation.

$X_1, X_2, X_3 ---, X_n$ are the reading taken.

2) Deviation from Mean

Deviation is the departure of given reading from the arithmetic mean of the group of readings. If the deviation of first X_1 is called d_1, X_2 is called d_2 and so on. Then;

$$d_1 = X_1 - \overline{X}$$
$$d_2 = X_2 - \overline{X}$$
$$\vdots$$
$$d_n = X_n - \overline{X}$$

Where,

$\overline{X} \rightarrow$ Arithmetic mean.

Note that the deviation from mean may have a positive or negative value and that algebric sum of all deviation is zero.

(3) Average Deviation :

The average deviation is an indication of precision of the instrument used in the measurement. Highly precise instrument will give a low average deviation. The average deviation is defined as the ratio of sum of absolute value of deviation to the total number of readings.

$$D = \frac{|d_1| + |d_2| + |d_3| + --- + |d_n|}{N}$$

(4) Standard Deviation:

Standard deviation of a finite number of data is the square root of sum of all the individual deviations square divided by (N-1).

$$S.D\ (\sigma) = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + ---- d_n^2}{N-1}}$$

$$\text{variance} = (\sigma)^2$$

\# A set of independent current Measurement was taken by six observers and recorded as 12.8 mA, 12.2 mA, 12.5 mA, 13.1 mA, 12.9 mA and 12.4 mA. Calculate,

i) arithmetic mean
ii) deviation from mean.
iii) average deviation
iv) standard deviation
v) Varience
vi) probable error (0.674 σ)

Here,
Given data : 12.8 , 12.2 , 12.5, 13.1, 12.9,
12.4

$N = 6$

(i) $$A.M\,(\overline{X}) = \frac{12.8 + 12.2 + 12.5 + 13.1 + 12.9 + 12.4}{6}$$

$$(\overline{X}) = 12.65 \text{ mA}$$

(ii) For standard deviation,

$$d_1 = X_1 - \overline{X} = 12.8 - 12.65 = 0.15$$
$$d_2 = X_2 - \overline{X} = 12.2 - 12.65 = -0.45$$
$$d_3 = X_3 - \overline{X} = 12.5 - 12.65 = -0.15$$
$$d_4 = X_4 - \overline{X} = 13.1 - 12.65 = 0.45$$
$$d_5 = X_5 - \overline{X} = 12.9 - 12.65 = 0.25$$
$$d_6 = X_6 - \overline{X} = 12.4 - 12.65 = -0.25$$

(iii) $\therefore$ Average deviation (D) $= \dfrac{|d_1| + |d_2| + |d_3| + |d_4| + |d_5| + |d_6|}{N}$

$$= \frac{0.15 + 0.45 + 0.15 + 0.45 + 0.25 + 0.25}{6}$$

$$= \frac{1.7}{6} = 0.2833.$$

(iv) Standard deviation $(\sigma) = \sqrt{\dfrac{d_1^2+d_2^2+d_3^2+d_4^2+d_5^2+d_6^2}{N-1}}$

$$= \sqrt{\dfrac{(0.15)^2+(0.45)^2+(0.15)^2+(0.45)^2+(0.25)^2+(0.2}{6-1}}$$

$$= 0.3391$$

$\therefore$ variance $= \sigma^2$

$$= (0.3391)^2$$

$$= 0.115$$

(v) probable error $= 0.674\,\sigma$

$$= 0.674 \times 0.3391$$

$$= 0.2286$$

Probable Error ;

The best value of normal distribution is obtained by taking arithmetic mean of the various value of the given reading. If 'σ' be the standard deviation then probable error is 0.67σ. The probable error has been used in experimental work in the past.

Limiting Error :

In most instrument the accuracy is guaranteed to be within a certain percentage of full scale reading. The limit of thin deviation from the specified value are defined as limiting error. If 'Aa' be the actual quantity, 'As' be the nominal quantity. Then $\pm \delta A$ is the limiting error.

i.e,

$$A_a = A_s \pm \delta_A$$

Relative limiting error, $\varepsilon_r = \frac{A_a - A_s}{A_s} = \frac{\pm \delta_A}{A_s}$

$$\Rightarrow \pm \delta_A = \varepsilon_r A_s.$$

Qu A 0-150V voltmeter has a guaranteed accuracy of 1% of full deflection. The voltage measured by this instrument is 75 V. Calculate the limiting error in percentage?

An Here, nominal value = 150 V

Relative % error (ε_r) = 1 %

ε_r = 0.01

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Since, $\varepsilon_r = \frac{\delta A}{A_s}$

$\therefore \delta A = \varepsilon_r \times A_s = 0.01 \times 150 = 1.5V.$

Now,

$\delta A = 1.5V$

$A_s = 75V$

$\varepsilon_r = ?$

Then

$\varepsilon_r = \frac{\delta A}{A_s} = \frac{1.5V}{75V} = 0.02$

$\varepsilon_r = 2\%$

$\therefore$ Actual value $(A_a) = A_s \pm \delta A$

$= 75 \pm 0.02$

Qu. The resistance of an unknown resister is determine by wheat-stone bridge meter. The solution of unknown resistor is stated as $R_x = \frac{R_1 R_2}{R_3}$ where,

$R_1 = 500 \pm 1\%$

$R_2 = 615 \pm 1\%$

$R_3 = 100 \pm 0.5\%$

Calculate;

(i) Nominal value of R_x

(ii) Limiting error in ohm (Ω) of R_x and percentage error.

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Here we have given;

$R_1 = 500 \pm 1\%$
$R_2 = 615 \pm 1\%$
$R_3 = 100 \pm 0.5\%$

Then,

Nominal value of $R_x = \frac{R_1 R_2}{R_3} = \frac{500 \times 615}{100}$

$= 3075\,\Omega$

Now, for limiting error

$$\frac{\delta R_x}{R} = \varepsilon_r = \pm\left(\frac{\delta R_1}{R_1} + \frac{\delta R_2}{R_2} + \frac{\delta R_3}{R_3}\right) \quad \text{(i)}$$

Where,

$R_1 = 500 \pm 1\%$, $\delta R_1 = 1\%$ of 500
$= R_n \pm \delta R_1$ $\qquad = 5\,\Omega$

$\delta R_2 = 1\%$ of $650 = 6.5\,\Omega$
$\delta R_3 = 0.5\%$ of $100 = 0.5\,\Omega$

Then from (i)

$$\varepsilon_r = \pm\left(\frac{5}{500} + \frac{6.5}{650} + \frac{0.5}{100}\right)$$

$\varepsilon_r = 0.025$

$\varepsilon_r = 2.5\%$

Again,

$R_x = R_x \pm \delta R_x$

$\therefore \frac{\delta R_x}{R_x} = \varepsilon_r$

or, $\delta R_x = \varepsilon_r \times R_x$

$= 0.025 \times 3075$

$= 76.875$

$\therefore R_x = (3075 \pm 76.875)$

Measurement of Resistance, Inductance and Capacitance

Resistance, inductance and capacitance can be measured with the help of bridge circuit. A bridge circuit is a special class of instrument consisting of four arms, where three arms have the known values of parameter and one arm has unknown value. The value of unknown arm is found in terms of known values of the other three parameters. For the measurement of resistance, DC bridge is used, whereas inductance and capacitance can be measured with the help of AC bridge.

Wheat Stone Bridge: (Measurement of Resistance)

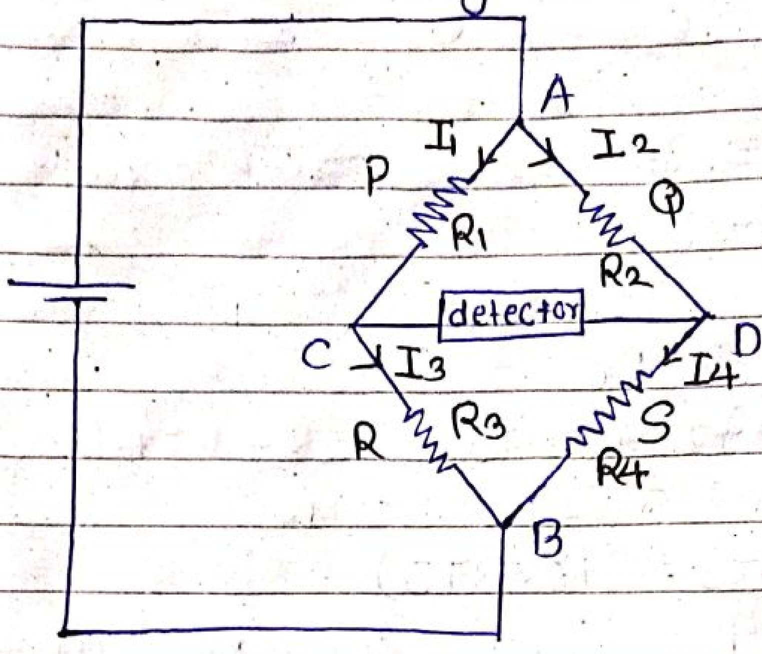

Fig: Wheat Stone Bridge:

A bridge is the name used to denote a special class of measuring circuit. They are most often used for making measurement of resistance, capacitance and inductance. Bridge are used for resistance measurement when the very accurant determination of a particular resistance is required. The most well known and widely used resistance bridge is wheat-stone bridge, it is used for accurately measuring resistance value from mΩ to MΩ range. The value of resistance obtain from the bridge are far more accurate than the value obtained from ohm meter.

The bridge has four arms together with a source of emf and a null detector usually as a galvanometer or other sensitive meter. The current through the galvanometer depends upon the potential difference between point C and D. The bridge is said to be balanced when the potential difference across the galvanometer is zero. So, that there is no current through the galvanometer.

At balanced condition.

$$I_1R_1 = I_2R_2 \text{ --- (i)}$$

and,

$$I_1 = I_3 = \frac{E}{R_1+R_3} \text{ --- (ii)}$$

$$I_2 = I_4 = \frac{E}{R_2+R_4} \text{ --- (iii)}$$

Substituting the value in (i), we get

$$\frac{E \times R_1}{R_1+R_3} = \frac{E \times R_2}{R_2+R_4}$$

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or, $R_1(R_2 + R_4) = R_2(R_1 + R_3)$

or, $R_1R_2 + R_1R_4 = R_2R_1 + R_2R_3$

or, $R_1R_4 = R_2R_3$

$R_4 = \frac{R_2R_3}{R_1}$

$\therefore R_4 = \frac{R_2}{R_L} \times R_3$

In practical bridge, the ratio of $\frac{R_2}{R_1}$ is controlled by a variable switch, R_3 is continuously adjustable variable resistor. A number of detector including galvanometer or even headphone are available to determine a null or balanced condition.

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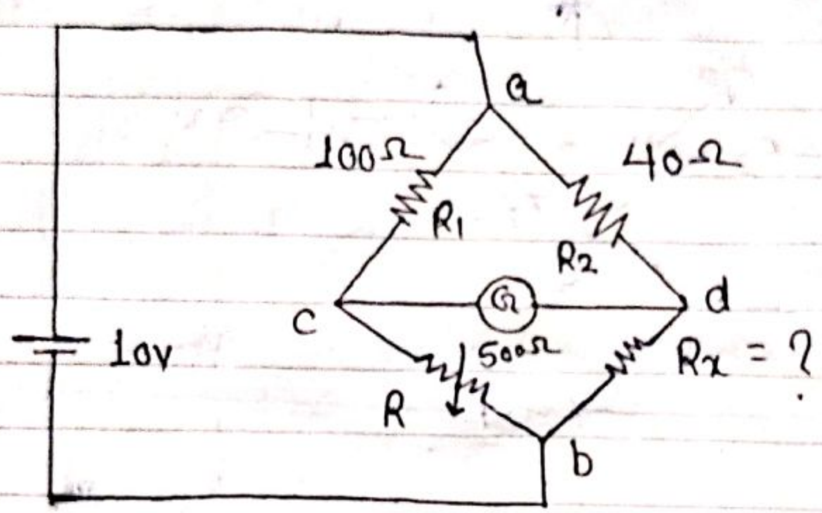
Que Given a wheat-stone bridge with $R_1 = 100\Omega$, $R_2 = 40\Omega$, $R_3 = 500\Omega$, $V_0 = E = 10V$, $R_G = 600\Omega$, Find the value of R_x when the bridge is in balanced condition ?



At balanced condition,

$$R_x = \frac{R_2}{R_1} \times R_3$$

$$= \frac{40}{100} \times 500$$

$$= 200\,\Omega$$

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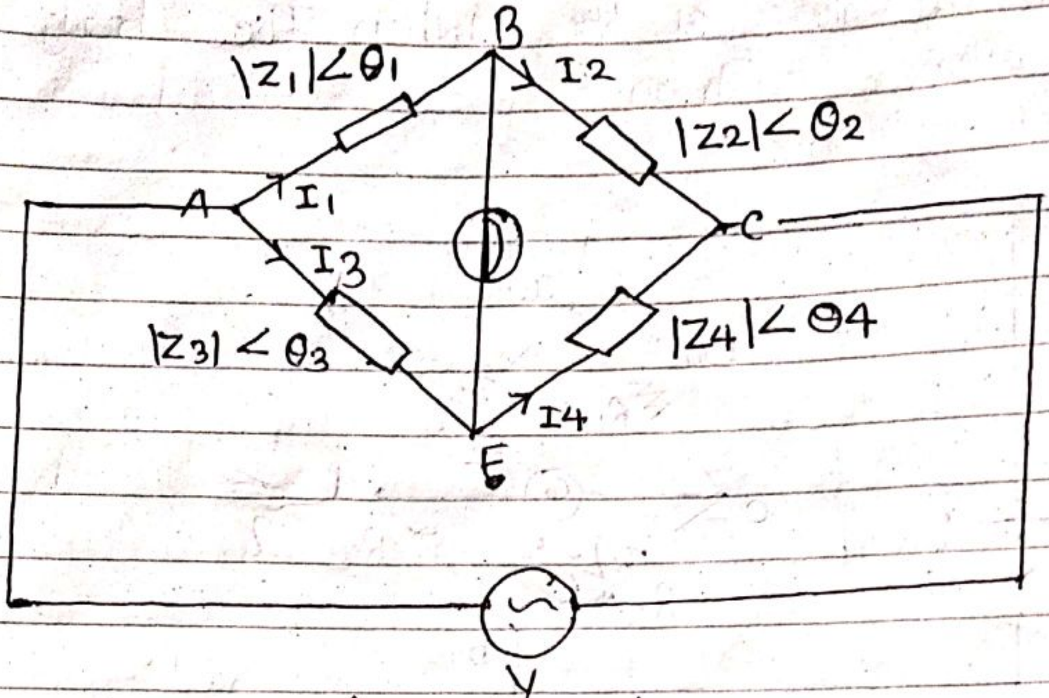
Balanced Condition for AC Bridge.



When the bridge is balanced,

$V_{AB} = V_{AF}$

or, $I_1|Z_1|\angle\theta_1 = I_3|Z_3|\angle\theta_3$ —— (1)

As no current flows through the detector,

$$I_1 = I_2 = \frac{V}{|Z_1|\angle\theta_1 + |Z_2|\angle\theta_2} \quad \text{—— (2)}$$

$$I_3 = I_4 = \frac{V}{|Z_3|\angle\theta_3 + |Z_4|\angle\theta_4} \quad \text{—— (3)}$$

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From equation (1), (2) and (3)

$$\frac{V}{|Z_1|\angle\theta_1 + |Z_2|\angle\theta_2} |Z_1|\angle\theta_1 = \frac{V}{|Z_3|\angle\theta_3 + |Z_4|\angle\theta_4} |Z_3|\angle\theta_3$$

or, $|Z_1||Z_3|\angle(\theta_1+\theta_3) + |Z_1||Z_4|\angle(\theta_1+\theta_4) = |Z_1||Z_3|\angle(\theta_1+\theta_3) +$

$|Z_2||Z_3|\angle(\theta_2+\theta_3)$

or, $|Z_1||Z_4|\angle(\theta_1+\theta_4) = |Z_2||Z_3|\angle(\theta_2+\theta_3)$ — (4)

eqn (4) gives the necessary condition for AC bridge to be balanced.
Thus, there are two conditions;

(i) The product of magnitude of any two opposite arms impedance must be equal to that of other two i.e $|Z_1||Z_4| = |Z_2||Z_3|$.

(ii) The sum of angle of any two opposite arms impedance must be equal to that of other two i.e $(\theta_1+\theta_4) = (\theta_2+\theta_3)$.

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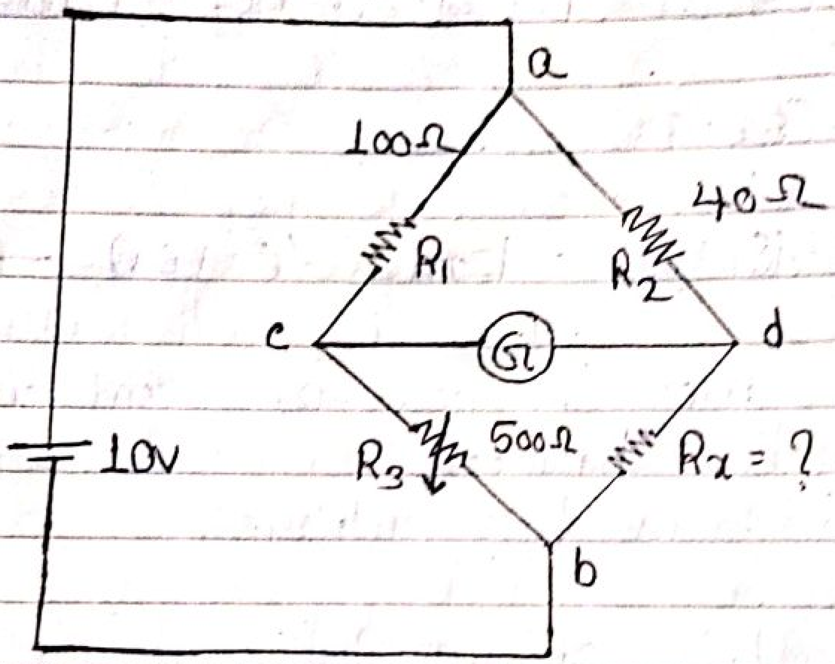
Given a Wheatstone bridge with $R_1 = 100\Omega$, $R_2 = 40\Omega$, $R_3 = 500\Omega$, $V_0 = E = 10V$, $R_G = 600\Omega$, Find the value of 'R_x', When the bridge is in balanced condition?



At balanced condition,

$$R_x = \frac{40}{100} \times 500$$

$$= 200\Omega$$

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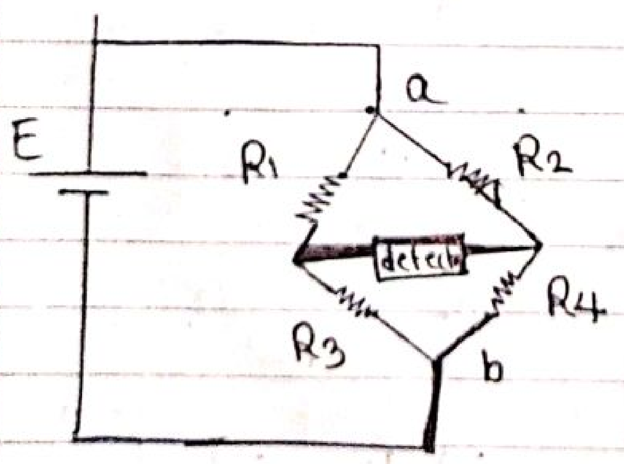
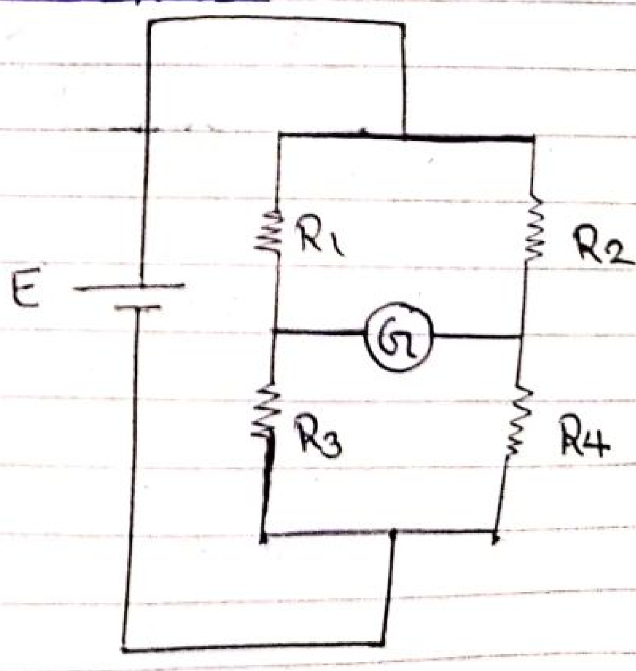
Causes of error of wheat-stone bridge or (Measurement error)

1) Insufficient sensitivity of null detector.
2) Change in resistance of bridge arm due to heating effect of current through the resistor.
3) Thermal emf in the bridge circuit or galvanometer circuit can also cause problem when low value resistor are being measured.
4) Error due to resistance of lead and contact exterior to the actual bridge circuit play a role in the measurement of very low resistance value.

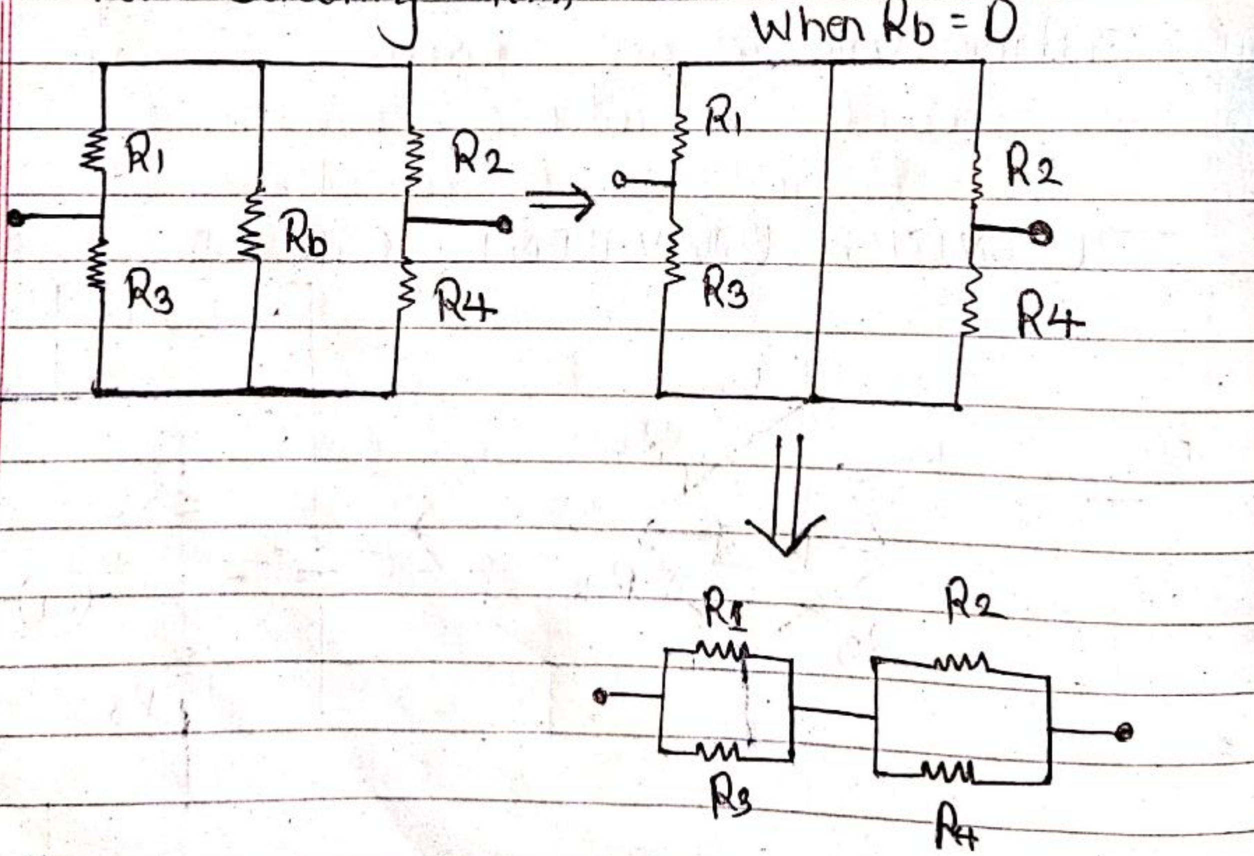
⇒ THEVENIN'S EQUIVALENT CIRCUIT:





To determine whether or not the galvanometer has required sensitivity to detect an unbalanced condition, it is necessary to calculate galvanometer current I_g. Different galvanometer not only may require different current sensitivity but they also have different internal resistance. Since, we are interested in the current through the galvanometer, the thevenin's equivalent circuit is determined by looking into the galvanometer terminal.

For detecting R_{th},

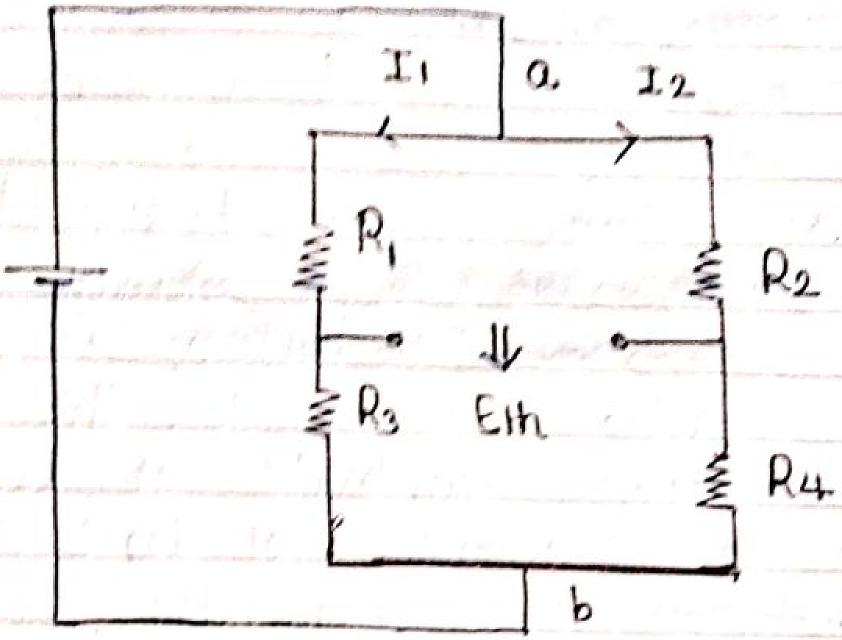


$R_h \rightarrow$ Internal resistance

$\therefore$ Equivalent Resistance

$$R_{th} = (R_1 \| R_3) + (R_2 \| R_4)$$

$$R_{th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$



$$\therefore \quad E_{th} = E_{ac} - E_{ad}$$

$$= I_1 R_1 - I_2 R_2$$

$$= \left(\frac{E}{R_1 + R_2}\right) R_1 - \left(\frac{E}{R_2 + R_4}\right) R_2$$

$$= E \left(\frac{R_1}{R_1 + R_3} - \frac{R}{R_2 + R_4}\right)$$

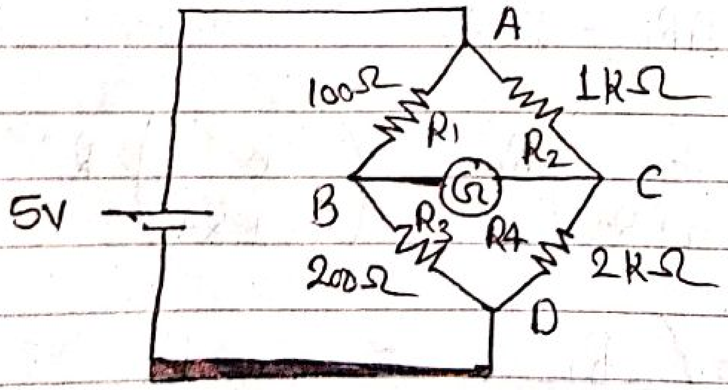
Now galvanometer current is given by

$$\boxed{I_g = \frac{E_{th}}{R_{th} + R_g}}$$

Where, $R_g \rightarrow$ Internal resistance of galvanometer.

\# A circuit shows a wheat-stone bridge with the value of bridge element at balanced condition. The battery is 5V, internal resistance is neglible. The galvanometer has a current sensitivity of 10mm/μA and internal resistance of 100Ω. Calculate the deflection of galvanometer caused by $+5\Omega$ unbalanced in the arm 'CD'.

Soln:



Here, Given

$R_1 = 100\Omega$, $R_2 = 1k\Omega$, $R_3 = 200\Omega$ and $R_4 = 2000+5$
$= 2005\Omega$

Sensitivity = $10mm/\mu A$.

Equivalent resistance,

$$R_{th} = (R_1 \| R_3) + (R_2 + R_4)$$

$$= \frac{R_1 R_2}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

$$= \frac{100 \times 200}{100 + 200} + \frac{1000 \times 2005}{1000 + 2005}$$

$$= \frac{20000}{300} + \frac{2005000}{3005}$$

$$\therefore R_{th} = 733.89\Omega$$

For E_{th},

$$E_{th} = E\left(\frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R_4}\right)$$

$$= 5\left(\frac{100}{100 + 200} - \frac{1000}{1000 + 2005}\right)$$

$$= 2.77$$

Then galvanometer current,

$$I_g = \frac{E_{th}}{R_{th} + R_g}$$

$$= \frac{2.77 \times 10^{-3}}{733.89 + 100}$$

$$= 3.325 \times 10^{-6} \text{ A}$$

$$= 3.32 \mu A$$

Deflection $= I_g \times$ sensitivity

$$= 3.32 \mu A \times 10 \text{ mm}/\mu A$$

$$= 33.2 \text{ mm}$$

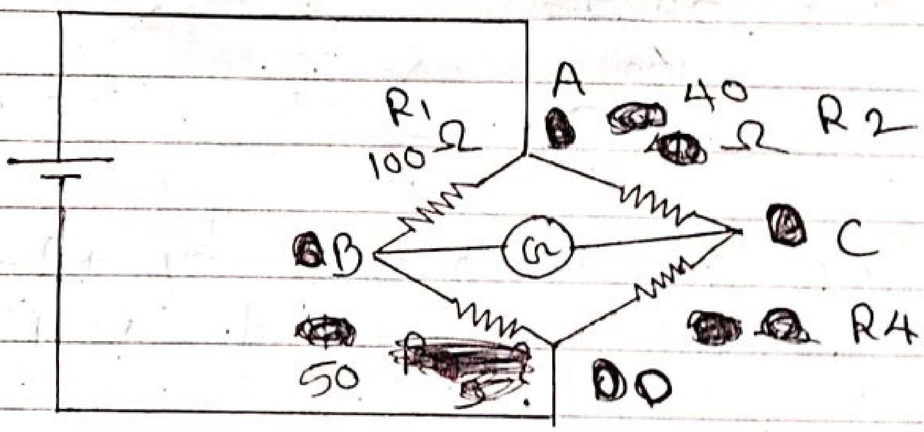
Qu. Given a Wheat-stone bridge with $R_1 = 100\Omega$, $R_2 = 40\Omega$, $R_3 = 50\Omega$, $V_0 = 10V$, $R_m = R_g = 600\Omega$

(i) Find the value of R_x at balanced condition.

(ii) If the R_x is changed by +2 from its value at balanced, find I_g that will flow through galvanometer.

(iii) Find the deflection of galvanometer, if sensitivity is 10 mm/μA.

Soln;



given,

Resistance of arm (AB) $(R_1) = 100\Omega$

Resistance of arm (AC) $(R_2) = 40\Omega$

Resistance of arm (BD) $(R_3) = 50\Omega$

Resistance of arm (CD) $(R_4) = R_x = ?$

Supplied voltage $(V_0) = 10$ V.

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Sensitivity of galvanometer = 10 mm/μA.
Internal resistance (R_g) = 800 Ω.

(i) Value of $R_4 = R_x$

Now, when bridge is in balanced condition then,

$$R_4 = \frac{R_2 \times R_3}{R_1}$$

$$= \frac{50 \times 40}{100}$$

$$= 20\ \Omega \quad \#$$

(ii) Since, R_x is changed by +2 Ω from its value at balanced i.e $R_x = 20\ \Omega$.

$\therefore R_x' = 20\ \Omega + 2 = 22\ \Omega$.

Then, equivalent resistance (R_{th})
$= (R_1 \parallel R_3) + (R_2 \parallel R_x')$

$= (100 \parallel 50) + (40 \parallel 22)$

$\therefore R_{th} = 47.528\ \Omega$

Also, Thevenin Voltage (V_{Th}) = $E_{th} = E_{AB} - E_{AC}$

$$= I_1R_1 - I_2R_2$$

$$= \left(\frac{E}{R_1+R_3}\right)R_1 - \left(\frac{E}{R_2+R_x'}\right) \times R_2$$

$$= E\left[\frac{R_1}{R_1+R_3} - \frac{R_2}{R_2+R_x'}\right]$$

$$= 10V\left[\frac{100V}{100+50} - \frac{40\Omega}{40+22}\right]$$

$$= 10V \times 0.02184$$

$$= 0.2184\ V.$$

So, galvanometer current (I_g) = $\dfrac{E_{th}}{R_{th}+R_g}$

$$= \frac{0.2184\,V}{47.526\,\Omega + 600\,\Omega}$$

$I_g = 337.3\,\mu A$ #

(iii) Deflection of galvanometer is

$= I_g \times$ Sensitivity of galvanometer.

$= 337.3\ \mu A \times 10\ mm/\mu A$.

$= 3373$ mm #

AC BRIDGE :

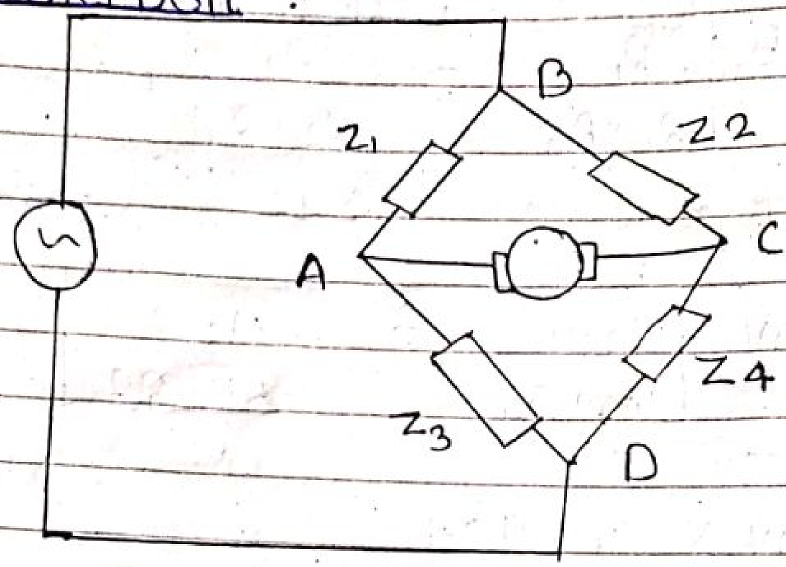

fig: AC-Bridge

The ac bridge is a natural outgrowth of dc bridge and in its basic form it consist of four bridge arm, a source of excitation and a null detector. The power source supply ac voltage to the bridge at the desired frequency.

For measurement at low frequency the power line may serve as source of excitation at higher frequencies and oscillator generally supply the excitation voltage. The null deflection must response to ac unbalanced current and in its form consist of a pair of headphone or any means.

The four bridge arm Z_1, Z_2, Z_3 and Z_4 are indicated as shown in figure. The balanced condition in this ac bridge is reached when the ~~dece~~ detector response is zero. The condition for bridge balance requires that the potential difference from 'A' to 'C' is zero. This will be the case when the voltage drop from 'B' to 'A' is equal to the voltage drop from 'B' to 'C' in both magnitude and phase.

In balanced condition,

$E_{BA} = E_{BC}$ & $I_1 = I_2$

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$\Rightarrow$ $I_1 Z_1 = I_2 Z_2$

or, $\frac{E}{Z_1+Z_3} \times Z_1 = \frac{E}{Z_2+Z_4} \times Z_2$

or, $Z_1(Z_2+Z_4) = Z_2(Z_1+Z_3)$

or, $\cancel{Z_1Z_2} + Z_1Z_4 = \cancel{Z_1Z_2} + Z_2Z_3$

or, $\boxed{Z_1Z_4 = Z_2Z_3}$ → Impedence form.

$\boxed{Z_4 = \frac{Z_2Z_3}{Z_1}}$

$\boxed{Y_1Y_4 = Y_2Y_3}$ → Admittance form.

From above equation, it states that the product of impedance of one pair of opposite arm with the impedance expressed in complex notation.

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If impedances are expressed in terms of magnitude and phase angle then balanced equation can be written as;

$$Z_1 Z_4 = Z_2 Z_3$$

or, $Z_1 \angle \theta_1 \; Z_4 \angle \theta_4 = Z_2 \angle \theta_2 \; Z_3 \angle \theta_3$

or, $Z_1 Z_4 \angle (\theta_1 + \theta_4) = Z_2 Z_3 \angle (\theta_2 + \theta_3)$

$\Rightarrow Z_1 Z_4 = Z_2 Z_3$ — (A)

and $\angle (\theta_1 + \theta_4) = \angle (\theta_2 + \theta_3)$. — (B)

The product of magnitude of apposite arm must be equal from equation (A) and the sum of phase angle of the opposite arm must be equal from equation (B).

The impedance of the AC bridge are given as follow $Z_1 = 100\Omega\angle 80$, $Z_2 = 250\Omega\angle 0$, $Z_3 = 400\Omega\angle 30'$ and $Z_4 = ?$, $\theta_4 = ?$

Here,

We have given magnitude and phase angle of arms of AC-bridge as follows.

$Z_1 = 100\Omega$ $\phi_1 = 80°$

$Z_2 = 250\Omega$ $\phi_2 = 0'$

$Z_3 = 400\Omega$ $\phi_3 = 30'$

$Z_4 = \text{unknown} = ?$ $\phi_4 = ?$

Then, the first condition for bridge balance gives that,

$Z_1 Z_4 = Z_2 Z_3$ and on solving we get Z_4.

i.e

$$Z_4 = \frac{Z_2 Z_3}{Z_1}$$

$$= \frac{250 \times 400}{100}$$

$$= 1000\Omega$$

$$\therefore Z_4 = 1\,k\Omega$$

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Again,

To find phase angle ϕ_4, second condition for bridge balace gives that,

$$\angle\phi_1 + \angle\phi_4 = \angle\phi_2 + \angle\phi_3$$

$$\text{or, } 80 + \phi_4 = 0^\circ + 30^\circ$$

$$\phi_4 = 30^\circ - 80^\circ = -50^\circ.$$

Hence, the unknown impedance Z4 can be written,

$$Z_4 = 1k \angle -50$$

indicating that we are dealing with capacitive element, possibly in series.

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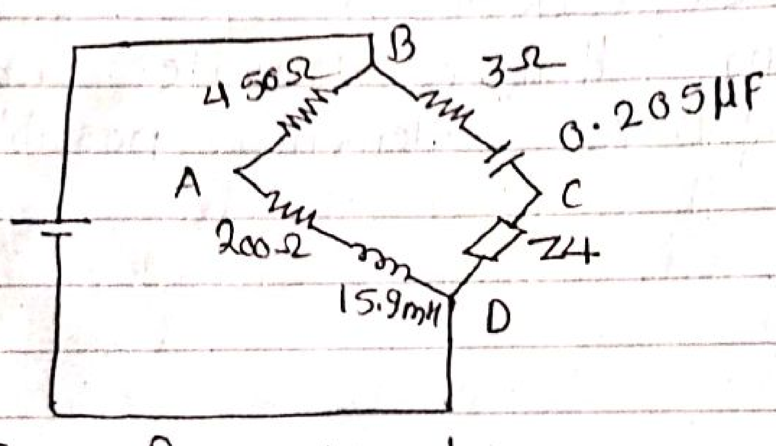
(#) The AC-bridge is in balanced with the following constant arm AB, R = 450 Ω, BC = R, 3Ω in series with C = 0.205 μF, CD = ? DA, R = 200Ω series with L = 15.9mH. The oscillator frequency is 1kHz. Find the constant of CD?

Solution;

We have given, the parameter of Constant arm AB, BC, DA and need to find Constant of CD.



From fig we get,

$Z_1 = R = 450\,\Omega$

$Z_2 = R - jX_C = 3\Omega - \frac{j}{\omega C}$ —— (i)

Since, frequency of oscillator (F) = 1 kHz = 1000 Ω

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 1000 \times 0.205 \times 10^{-6}}$$

$$= 776.36\ \Omega$$

and $X_L = 2\pi f L = 2\pi \times 1000 \times 15.9 \times 10^{-3}$
$= 99.90\ \Omega$

$\therefore Z_2 = (3 - j776.36)\ \Omega$

$Z_3 = (200 + j99.90)\ \Omega$

$Z_4 = ?$

Now,
$Z_1 Z_4 = Z_2 Z_3$

$450\, Z_4 = (3 - j776.36)(200 + j99.90)$

$\therefore Z_4 = 385.70 \angle -63.23$

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Measurement of Inductance :

Inductance can be measured by using one of these methods whichever appropriate as suggested by the quality factor.

(i) Maxwell's Bridge ($1 < Q < 10$)
(ii) Hay's Bridge ($Q > 10$)

$$Q\text{-factor} = \tan\theta = \frac{X_L}{R} = \frac{\omega L}{R}$$

Maxwell's bridge is used for measurement of inductance of the coil having moderate quality factor ($1 < Q < 10$). It is neither suitable for measurement of inductance of the coil having high quality factor nor for low quality factor whereas Hay's bridge is used for the measurement of inductance of coil having high quality factor.

Maxwell's Bridge ($1 < Q < 10$)

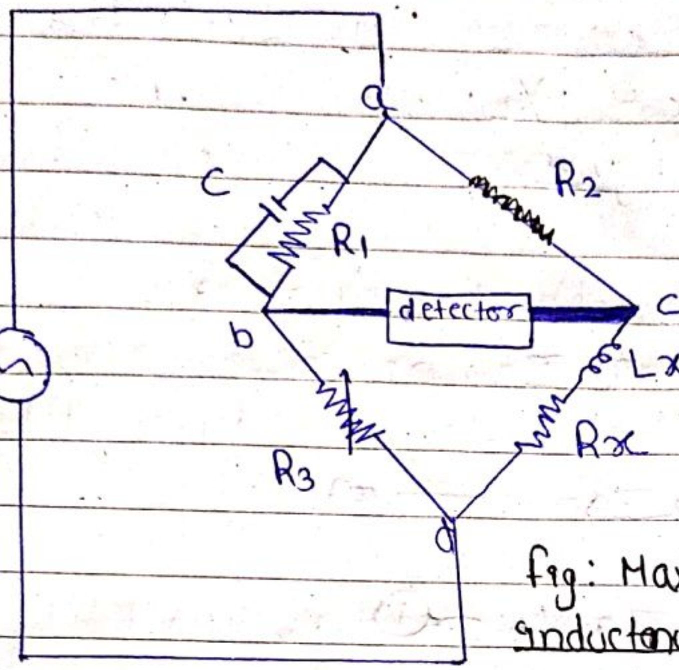

fig: Maxwell's bridge for inductance measurement

Maxwell's bridge measures an unknown inductance in terms of known capacitance. One of the ratio arm has a resistance and capacitance in parallel and the corresponding opposite arm has a combination of resistance and inductance in series as shown in figure ;

Z_1, Z_2, Z_3 and Z_4 or Z_x are the respective impedances at arm AB, AD, BC and CD respectively.

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$Z_2 = R_2$
$Z_3 = R_3$
$Z_x = R_x + jX_L$
$= R_x + j\omega L_x$

At balanced condition,
$Z_1 Z_x = Z_2 Z_3$

or, $Z_x = \frac{Z_2 Z_3}{Z_1}$ — (i)
$= Z_2 Z_3 Y_1$ — (i)

Now,
$Z_1 = R_1 \parallel (-jX_{C_1})$

$\Rightarrow \frac{-jR_1 X_{C_1}}{R_1 - jX_{C_1}}$

$Y_1 = \frac{1}{Z_1} = \frac{R_1 - jX_{C_1}}{-jR_1 X_{C_1}}$

$= \frac{R_1 - j\frac{1}{\omega C_1}}{-jR_1 X_{C_1}}$

$= \frac{(R_1 C_1 \omega - j)/\omega C_1}{\frac{-jR_1}{\omega C_1}}$

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$$= \frac{R_1C_1\omega - j}{-jR_1}$$

$$= \frac{R_1\omega C_1}{-R_1 j} - \frac{j}{-jR_1}$$

$$= \frac{1}{R_1} - \frac{\omega C_1}{j}$$

$$\Rightarrow \boxed{Y_1 = \frac{1}{R_1} + j\omega C_1}$$

$$Z_2 = R_2$$
$$Z_3 = R_3$$
$$\therefore Z_x = R_x + j\omega L_x$$

Substituting z_1, z_2, z_3 in eqn (9),

$$R_x + j\omega L_x = R_3R_2\left(\frac{1}{R_1} + j\omega C_1\right)$$

$$= \frac{R_2R_3}{R_1} + R_2R_3 j\omega C_1$$

Now, Comparing real and imaginary part we get,

$$\boxed{R_x = \frac{R_2 R_3}{R_1}}$$

&

$$\boxed{L_x = C_1 R_2 R_3}$$

Where resistance is expressed in ohm (Ω) and capacitance in farad (F) and inductance in Henry (H).

→ Maxwell's bridge is not used for measurement of inductance of the coil having high Q-factor.
This bridge is also not used for measurement of inductance of the coil having low Q-factor because of the convergence problem. i.e it become difficult to obtain balanced condition as the balanced condition goes on shifting.

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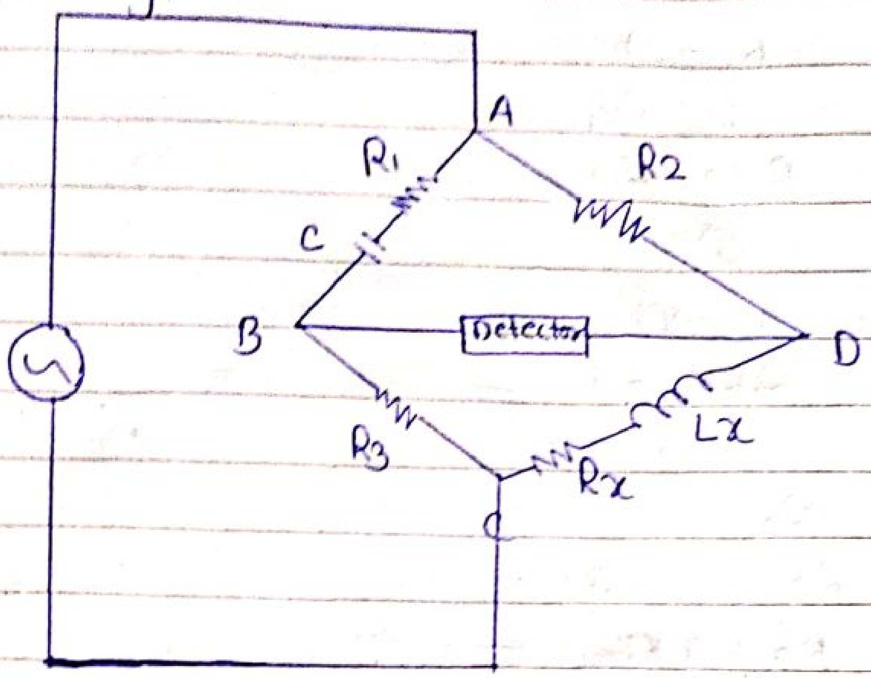
Hay Bridge



(fig: Hay - Bridge)

The Hay's bridge is used for the measurement of inductance of the coil having high Q-factor.
The Hay's bridge consist of a resistor R_1 in series with capacitor C_1 in arm AB. a resistor R_x & inductor L_x in series in arm CD.

Here, $Z_1 = R_1 - jX_c$

$$= R_1 - \frac{j}{\omega C_1}$$

$$= \frac{R_1\omega C_1 - j}{\omega C_1}$$

$$\Rightarrow Y_1 = \frac{\omega C_1}{R_1\omega C_1 - j}$$

$Z_2 = R_2$

$Z_3 = R_3$

$Z_4 = R_x + jX_L$ — (9)

$$= R_x + j\omega L_x$$

We have at balanced condition,

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_4 = \frac{Z_2 Z_3}{Z_1}$$

$$R_x + jX_L = R_2R_3 \left(\frac{1}{\frac{R_1\omega C_1 - j}{\omega C_1}} \right)$$

or, $R_x + j\omega L_x = R_2R_3 \left(\frac{\omega C_1}{R_1\omega C_1 - j} \right)$

$$= \frac{R_2R_3\omega C_1}{(R_1\omega C_1 - j)} \times \frac{(R_1\omega C_1 + j)}{(R_1\omega C_1 + j)}$$

$$= \frac{R_1 R_2 R_3 \omega^2 C_1^2 + j R_2 R_3 \omega C_1}{(R_1 C_1 \omega)^2 + 1}$$

Comparing both sides,

$$\boxed{R_x = \frac{R_1 R_2 R_3 \omega^2 C_1^2}{(R_1 C_1 \omega)^2 + 1}}$$

&

$$\boxed{L_x = \frac{R_2 R_3 C_1}{(R_1 C_1 \omega)^2 + 1}}$$

Note :

Second balanced condition;

$\theta x = \theta_1$, $\quad Q = \tan\theta$

$$Q = \frac{1}{\omega R_1 C_1}$$

($1/Q^2$ is neglible)

$L_x = R_2 R_3 C_1$

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Measurement of Capacitance using Schering Bridge:

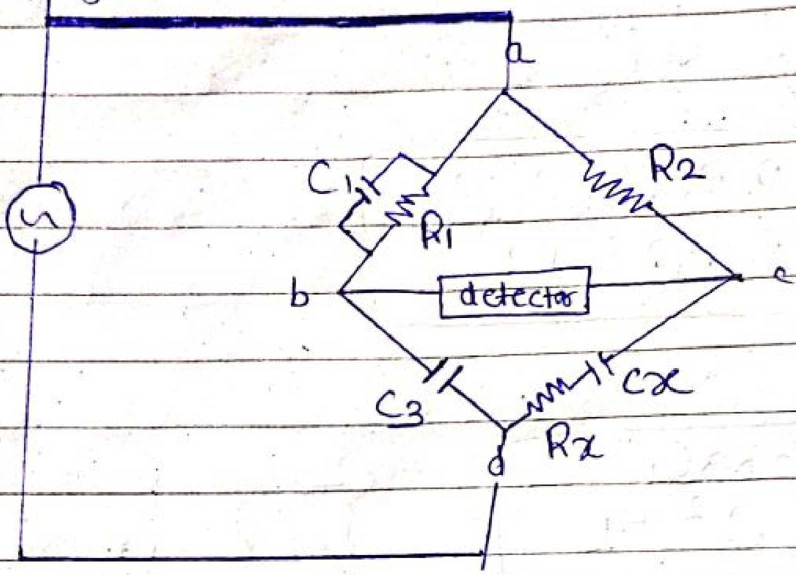

Fig: Schering's bridge for the measurement of capacitance.

Schering Bridge is used for the measurement of capacitance. Arm AB contains parallel combination of resistor and capacitor and the corresponding opposite arm contains series combination of resistor capacitor.

The Standard arm BC contains only a capacitor and a resistor in its corresponding opposite arm.

At balanced conditions ;

$$Z_1 Z_x = Z_2 Z_3$$

$$Z_x = Z_2 Z_3 Y_1 \quad \text{——} (9)$$

$$Z_1 = R_1 // -jX_c$$

$$= \frac{-jX_c R_1}{R_1 - jX_c}$$

$$Y_1 = \frac{1}{R_1} + j\omega C_1$$

$$Z_2 = R_2$$

$$Z_3 = -jX_{c_3} = -\frac{j}{\omega C_3}$$

$$Z_x = R_x - jX_{cx}$$

$$= R_x - \frac{j}{\omega C_x}$$

Substuting Y_1, Z_2, Z_3, Z_x in (9) we get,

$$R_x - \frac{j}{\omega C_x} = -\frac{R_2 j}{\omega C_3}\left(\frac{1}{R_1} + j\omega C_1\right)$$

$$= \frac{-R_2 j}{R_1 \omega C_3} + \frac{R_2 C_1}{C_3}$$

Comparing real and imaginary parts,

$$\boxed{R_x = \frac{R_2 C_1}{C_3}}$$

and, $\boxed{C_x = \frac{R_1}{R_2} C_3}$

This is the required expression for measurement of capacitance using Schering brigde.

+ WEIN BRIDGE FOR The measurement of frequency.

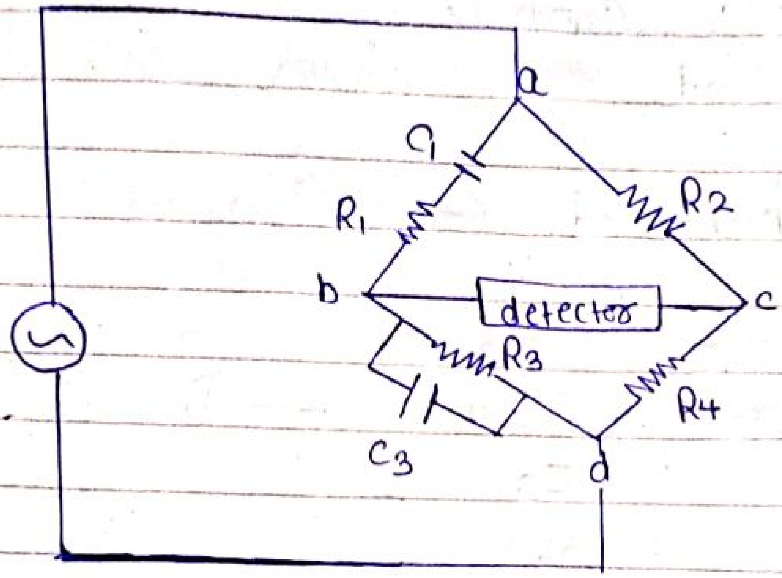

fig: Wein bridge for the measurement of frequency.

→ The WEIN bridge is used for the measurement of frequency but also for determination of various circuit Components.

The wein Bridge also finds application in audio and high frequency ascillator at the frequency determining element.

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The wein-bridge has a series R-C combination in arm 'ab' and parallel combination of R-C in arm 'bd' as shown in figure.

At balanced condition,

$$Z_1 Z_4 = Z_2 Z_3$$
$$Z_2 = \frac{Z_1 Z_4}{Z_3} \quad \text{— (9)}$$

$$Z_2 = R_2$$

$$Z_1 = R_1 - jX_{C_1}$$
$$= R_1 - \frac{j}{\omega C_1}$$

$$Z_3 = R_3 // -jX_{C_3}$$

$$= \frac{-R_3 j X_{C_3}}{R_3 - jX_{C_3}}$$

$$= \frac{-R_3 j \frac{1}{\omega C_3}}{\frac{R_3 \omega C_3 - j1}{\omega C_3}}$$

$$= \frac{-R_3 j}{R_3 C_3 \omega - j}$$

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$$Y_3 = \frac{\omega R_3 C_3 - j}{-jR_3} \quad ; \quad Z_4 = R_4$$

$$= \frac{\omega C_3}{-j} + \frac{1}{R_3}$$

$$= \frac{1}{R_3} + j\omega C_3$$

from ⑨,

$$Z_2 = \frac{R_4 \left(R_1 - \frac{j}{\omega C_1} \right)}{} \cdot \left(\frac{1}{R_3} + j\omega C_3 \right)$$

or, $$R_2 = \left(R_4 R_1 - \frac{R_4 j}{\omega C_1} \right) \left(\frac{1}{R_3} + j\omega C_3 \right)$$

$$R_2 = \frac{R_1 R_4}{R_3} + R_1 R_4 j\omega C_3 - \frac{R_4 j}{R_3 \omega C_1} + \frac{R_4 C_3}{C_1}$$

Here, Real part is,

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1}$$

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and imaginary parts,

$$R_1 R_4 C_3 \omega - \frac{R_4}{R_3 \omega C_1} = 0$$

or, $R_1 R_4 R_3 C_1 C_3 \omega^2 - R_4 = 0$

or, $\omega^2 = \frac{1}{R_1 R_3 C_1 C_3}$

or, $\omega = \sqrt{\frac{1}{R_1 R_3 C_1 C_3}}$

or, $2\pi f = \sqrt{\frac{1}{R_1 R_3 C_1 C_3}}$

$$\boxed{\therefore f = \frac{1}{2\pi}\sqrt{\frac{1}{R_1 R_3 C_1 C_3}}}$$

This is the required expressions for frequency.