Pre-University Examination Questions paper collection

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Lumbini Engineering College (Internal Exam)

Level: Bachelor
Program: Com/EnE

Course: Engineering Mathematics IV

Year: 2025
F.M: 100
P.M: 45
Time: 3 hrs.

Attempt all the questions:

1.a) Define analytic function. State and prove the necessary condition for f(z) to be analytic. Also check whether cosz is analytic or not? (8)

OR

Define Harmonic function. If a function f(z) is analytic then show that $U_x = V_y$ and $U_y = -V_x$.

b) Write Define conformal mapping. Name the type of conformal mapping. Translate the rectangular region ABCD in Z Plane bounded by x=1, x=3, y=0, and y=3 under the transformation w= z+ (2+i). Show with figure. ….(7)

2.a) State Cauchy Integral formula for derivative. Evaluate $\oint_c \frac{e^z}{(z+1)^3(z-2)}dz$, where c:|z-1|=3 (8)

b) Find the Laurent Series expansion of $\frac{z^2-1}{z^2+5z+6}$ in the region i) |z|<2 ii) 2<|z|<3 (7)

3.a) State and prove that second shifting theorem of Z-transform. Obtain z-transform of $(1-e^{-a})$, a>0. (7)

b) Solve the difference equation using z-transform.
$y_{n+2}-3y_{n+1}+2y_n=4^n$, $y_0=0$, $y_1=1$ (8)

4.a) Find the Fourier cosine transform of $f(x)=e^{-x}$, x>0 and hence show that $\int_0^\infty \frac{1}{(1+x^2)^2}dx = \frac{\pi}{4}$ (7)

b) What are the Fourier sine and cosine integrals of a function f(x)?
Show that $\int_0^\infty \frac{\cos\frac{\pi}{2}w \cos xw}{1-w^2}dw = \begin{cases} \frac{\pi}{2}\cos x; & |x| < \frac{\pi}{2} \\ 0 & |x| > \frac{\pi}{2} \end{cases}$

5.a) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ with given boundary conditions u(0,t)=0, u(L,t)=0 and initial deflection u(x,0)=f(x), initial velocity $\frac{\partial u}{\partial t}$=g(x). (8)

b) A rod of length l has it's ends A and B maintained at 0^0C and 100^0C respectively, until strady state condition prevails. If the changes consist of raising the temperature A to 20^0C and reducing that of B to 80^0c. Find the temperature distribution in the rod at time t. (7)

6.a) Change the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$ into cylindrical polar co-ordinate form. (8)

OR

Derive the solution of two dimensioned wave equation under the condition when a circular membrane of radius R is vibrating.

b) Show that U= e^{2x}(x cosy − y siny) is a harmonic function. Find an analytic function for U. If U is the real part. (7)

7. Solve the following: Any Two
(2x5=10)

i) Find the solution of the differential equation. y^2u_x-x^2u_y =0 by separating the variables method.

ii) Find the image of infinite strip $\frac{1}{4}$ < z< $\frac{1}{2}$ under the transformation w= $\frac{1}{z}$

iii) Find the z transform of sin($\frac{n\pi}{2}$) and cos($\frac{n\pi}{2}$).

The End

$z^2-2z-z+2$.
$z(z-2)-1(z-$

GANDAKI COLLEGE OF ENGINEERING AND SCIENCE

Level: Bachelor | Semester: Spring | Year : 2025

Programme: BE CE IV | Full Marks: 100

Course: Applied Mathematics | Pass Marks: 45

Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define harmonic function. Show that the function $u = y^3 - 3x^2y$ is harmonic then find harmonic conjugate and corresponding analytic function. 7

   b) State Cauchy- Residue theorem, then evaluate the following integrals. 8

   $\int_c \frac{Z-23}{Z^2-4z-5} \, .dz \quad C: \lfloor Z-2 \rfloor = 4$

2. a) Define bilinear transformation. Find the bilinear transformation which maps $Z_1 = 0$, $Z_2 = 1$, $Z_3 = \infty$ into $W_1 = i$, $W_2 = -1$, $W_3 = -i$. 7

   b) Define analytic function. If $f(z) = u(x,y) + iv(x,y)$ is analytic in Domain D, then the partial derivatives u_x, V_x, u_y, V_y exists and satisfy $u_x = V_y$ and $u_y = -V_x$. 8

3. a) State and prove first shifting theorem of z- transform. Using it evaluate the z- transform of a^n cosbt and a^n sinbt. 7

   b) Solve the difference equation by using z-transform: $y_{n+2} - 4y_{n+1} + 4y_n = 2^n$ with $y_0=0$, $y_1=1$ 8

   OR

   State and prove Initial and final value theorem.

4. a) Derive fourier integral from fourier series. 7

   OR

   Find the fourier integral in complex form.

b) Define fourier cosine and sine transform. Find the fourier sine transform of e^{-x} (x> 0) and hence show that. 8

$$\int_0^\infty \frac{x\sin mx}{x^2+1} \cdot dx = \frac{\pi}{2} \cdot e^{-m}$$

5. a) Derive one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ with necessary assumptions. 7

b) Find u(x, t) from one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, with boundary condition $u(0, t) = 0 = u(L, t)$, initial deflection f(x) and initial velocity $\frac{\partial u}{\partial t}\Big|_{t=0} = g(x)$. 8

6. a) The ends A and B of a rod 20 cm long temperature at 30^0 c and 80^0 c until steady state prevails. If the change consists of raising the temperature of A to 40^0 c and reducing that of B to 60^0. Find the temperature distribution in the bar at time t. 7

b) Express the Laplacian. 8

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \text{ in polar co-ordinates.}$$

7. Write short notes on (Any Two) 2×5

a) Solve the partial differential equation $u_{xy} - u = 0$.

b) Check the analyticity of the function f(z) = *arg* z

c) Verify the given function.

$u = \sin 9t. \sin\frac{x}{4}$, satisfy one dimensional wave equation.

COSMOS COLLEGE
OF MANAGEMENT AND TECHNOLOGY

Term Test II

Date: 2082/04/04			
Level	BE	Full Marks	70
Programme	BEIT, BCE	Time	
Semester	IV	2 hrs	

Subject: - Applied Mathematics

- ✓ *Candidates are required to give their answers in their own words as far as practicable.*
- ✓ *Attempt 70 marks questions.*
- ✓ *The figures in the margin indicate Full Marks.*
- ✓ *Assume suitable data if necessary.*

1 a) Define analytic function. Show that the function $u(x, y) = 3x^2y + x^2 - y^3 - y^2$ is a harmonic function. Also fond the harmonic conjugate of u. [8]

b) State Cauchy's Residue theorem and use it to evaluate $\oint Tan\, z\, dz$, where c: $|z| = 2$ [7]

2 a) Evaluate $\oint_C \frac{\cot z}{\left(z-\frac{\pi}{2}\right)^2} dz$, where C is the ellipse $4x^2 + 9y^2 = 36$. [7]

b) State Taylor series. Expand the function $f(z) = \frac{1}{z - z^3}$ in the region [8]

(i) $1 < |z+1| < 2$. (ii) $|z+1| > 2$

OR

Show that bilinear transformation $w = \frac{5-4z}{4z-2}$ maps the circle $|z| = 1$ in Z plane onto the circle $u^2 + v^2 + u - \frac{3}{4} = 0$ in w plane

3 a) Show that $Z[nf(t)] = -z\frac{d}{dz}[F(z),]$ where $F(z) = Z[f(t)]$.

Find $Z^{-1}\left[\frac{z}{(z+1)^2(z-1)}\right]$ [7]

b) Solve $y_{n+2} - 3y_{n+1} + 2y_n = 0$ where $y(0) = 0$, $y_1(0) = 1$, by using z-transform. [8]

4 a) State the first shifting theorem for Z transformation and hence find $Z[e^{-at}t]$. [7]

b) Find Fourier cosine transform of $f(x) = e^{-mx}$ for $m > 0$. [8]

Then prove that $\int_0^\infty \frac{\cos kx}{1+x^2} dx = \frac{\pi}{2} e^{-k}$

OR

Define convolution of the two functions. If f(x) and g(x) are piecewise continuous, bounded absolutely integrable on the x- axis. Prove that $F(f*g) = \sqrt{2\pi}\, F(f).F(g)$

5 a) Show that $\int_0^\infty \frac{\cos wx + w \sin wx}{1+w^2} dx = \begin{cases} 0 \text{ if } x < 0 \\ \frac{\pi}{2} \text{ if } x = 0 \\ \pi e^{-x} \text{ if } x > 0 \end{cases}$ [7]

b) Find the solution of one -dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, under boundary and initial condition. [8]

6 a) Solve: $u_{xx}+u_{yy}=0$ by using separation of variables method [7]

OR

Find the solution of one -dimensional wave equation with initial deflection $\frac{1}{2}\sin 3x + \sin x$ and initial velocity is zero.

b) Show that the Laplacian in u in polar coordinate is $\nabla^2 u = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta}$. [8]

7 a) Find Maclaurin expansion of the function $f(z) = \frac{z+2}{1-z^2}$. [2.5]

b) Show that $\oint_C \frac{dz}{z} = 2\pi i$, where C is the unit circle, counter-clockwise. [2.5]

c) Solve the differential equation: [2.5]
$u_{xy} - u = 0.$

d) Find z–transform of na^n [2.5]

POKHARA UNIVERSITY

Level: Bachelor
Program: BE
Course: Applied Mathematics

Semester: Spring

Year: 2025
Full Marks: 100
Pass Marks: 45
Time: 3 hrs.

Candidates are required to answer in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Define harmonic function. Determine the value of 'a' , when u = cosax .cosh2y ia harmonic and also, find its harmonic conjugate . 8

   b) State Cauchy's integral formula and using it integrate : $\oint_c \frac{z^2}{(z^4-1)}dz$,where c is the circle $|z+i|=1$ in counter clockwise 7

2. a) Find the image of triangular region of the z-plane bounded by the lines $x=0, y=0$ and $x+y=1$ under the transformation of $w = z\,e^{i\pi/4}$ and show the sketch in the diagram. 8

   b) Define singularities of a function $f(z)$. Find the residue of a function $f(z) = \frac{z^3}{(z-3)(z-2)(z-1)^4}$ at z = 1.

   OR

   Define Laurent series. find laurent series of the function $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the region given by i) $0 < |z+1| < 1$ ii) when $1 < |z+1| < 3$ 7

3. a) State and prove second shifting theorem of z-transform. Find z-transform of e^{-iat} and hence find Z(cos at) 8

   b) Use z-transform to solve the difference equation : $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, where $y_0 = 0$ and $y_1 = 0$ 7

4. a) Using fourier cosine integral, show that 8

$$\int_0^\infty \frac{\sin\omega\cos\omega\, d\omega}{\omega} = \begin{cases} \pi/2 & if\ 0 \le x < 1 \\ \pi/4 & if\ x = 1 \\ 0 & if\ x > 1 \end{cases}$$

   b) Find fourier sine transform of $f(x) = e^{-x}$ for $x > 0$ and then show that $\int_0^\infty \frac{x \sin mx}{1+x^2}dx = \frac{\pi}{2}e^{-m}$ for $m > 0$. 7

5. a) A tightly stretched string of length L, fixed at its ends, is initially in a position given by $u(x,0) = u_0 \sin^3\left(\frac{x\pi}{L}\right)$. If it is released from the rest from this position, find the displacement at any point x at time t. 8

   b) Find the temperature in a laterally insulated bar of length L=10cm whose ends are kept at zero temperature, assuming that the initial temperature is

$$f(x) = \begin{cases} x & if\ 0 < x < L/2 \\ L-x & if\ L/2 < x < L \end{cases}$$

7

OR

State one dimensional wave equation and Derive it.

6. a) Find the solution of differential equation $xu_x + 2yu = 0$ using separation of variables. 7

b) Express the Laplacian $\nabla^2 u = \frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2}$ into polar coordinates. 8

7. **Attempt any two of the questions** 2 * 5

a) Check the analyticity of $y = \text{Re}\,(z)^3$

b) Find the poles of the function $f(z) = \frac{\sinh z}{(z-i\pi)}$

c) Find the z-transform of $Z(n.a^n)$

7
7.

Universal Engineering & Science College

Affiliated to Pokhara University

Chakupat, Lalitpur

Level: Bachelor
Programme : BE Computer
Course:Applied Mathematics

Semester: IV
Time: 3 hours

Year: 2025
Full Marks: 100
Pass Marks: 45

Pre-Board Examination - 2082 (Spring 2025)

Candidates are requested to give their answers in their own words as far as practicable. Figure in the margin indicates full marks.

Attempt **All** the questions:

1. a. Define the Laplace equation and harmonic function. Is $v = (x^2-y^2)^2$ harmonic? If yes find its harmonic conjugate. 8

   b. State Cauchy Integral formula for derivative. Evaluate $\oint_c \frac{z^6}{(2z-1)^6}$ dz, where c is the unit circle $|z|=1$, counter clockwise 7

2. a. State Laurent's theorem. Find Laurent's series for 7

$$f(z) = \frac{1}{(z-z^3)} \text{ in the region } 01<|z+1|<2.$$

   b. Define singularity, zeros, and poles of a function. Evaluate $\oint_c f(z)dz$ where $f(z) = \frac{e^{2z}}{(z+1)^3}$ where c is the ellipse $4x^2+9y^2=16$. 8

3. a. Define Z - transform. State and prove the Second shifting theorem of Z- transform. Evaluate $Z(t^2e^{-bt})$ 7

OR

Find $Z^{-1}\frac{z^2+1}{z^2-2z+2}$.

   b. Solve $U_{n+2} - 2\cos\alpha\, U_{n+1} + U_n = 0$ where, by using z-transform 8

4. a. Find the Fourier integral of the function 7

$$f(x)=\begin{cases}\pi/2 & \text{if } 0\le x<1\\ \pi/4 & \text{if } x=1\\ 0 & \text{if } x>1\end{cases}$$

b. Define the convolution of the two functions. State and prove the convolution theorem on Fourier transform. 8

5. a. Solve one-dimensional wave equation with initial deflection is 0.01sin3x and initial velocity is zero and $L=\pi$, $c^2=1$ 7

b. Find the solution of one-dimensional heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$, having zero temperature in endpoints and initial temperature 8

6. a. Derive two-dimensional heat equation with required assumption. 7

b. Find the deflection u(x,y,t) of a square membrane with a = b=1 with c=1if the initial velocity is zero and the initial deflection is $0.1\sin 3\pi x\ \sin 4\pi y$ 8

7. Attempt all.

a. Express f(z) = sinz in the form u + iv 2

b. Find z–transform of na^n 2

c. solve $u_{xx}-u_{yy}=0$ 2

d. Find the unit tangent vector to the curve $\vec{r}(t)=2\cos t\,\vec{i}+\sin t\,\vec{j}$ at $(\sqrt{2},\sqrt{2},0)$. 2

e. Sketch the paraboloid $z=x^2+y^2$. 2

g Hazards

terrupts

POKHARA ENGINEERING COLLEGE
Final Assessment

Level: Bachelor (4th Sem)
Programme: BE (Computer/IT)
Course: Applied Mathematics

Year :2025
Full Marks: 100
Pass Marks: 45
Time :3:00 hrs

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Define harmonic function.Show that the function$u(x,y) = 3x^2y + x^2 - y^3 - y^2$ is harmonic function.Find the function$v(x,y)$ such that$u + iv$is analytic function. -7

   b) State and prove cauchy integral formula.Integrate$\oint_c \frac{z^3}{2z-i}dz$ counterclockwise around the unit circle. -8

2. a) Find the Laurent series for$f(z) = \frac{z^2}{z^2+5z+6}$ in the region $2 < |z| < 3$ -7

   b). State Cauchy residue theorem. Integrate$f(z) = \frac{z+1}{z^4-2z^3}$ around $c: |z| = \frac{1}{2}$ using cauchy residue theorem. -8

   OR

   Define a Bilinear map.Find the bilinear transformation which maps$z_1 = 0, z_2 = 1, z_3 = \infty$ into $w_1 = i, w_1 = -1, w_3 = -i$

3. a) Define Fourier integral of $f(x)$.Choosing a suitable function, show that -7

$$\int_0^\infty \left[\frac{\cos x\omega + \omega \sin x\omega}{1+\omega^2}\right] d\omega = \begin{cases} 0 & if\ x < 0 \\ \frac{\pi}{2} & if\ x = 0 \\ \pi e^{-x} & if\ x > 0 \end{cases}$$

   b) Find Fourier sine and cosine transform of$f(x) = 2e^{-5x} + 5e^{-2x}$ -8

   OR

   Find the Fourier sine transform ofe^{-x} for $x > 0$ and then by using parseval's identity show that:

$$\int_0^\infty \frac{x^2}{(1+x^2)^2}dx = \frac{\pi}{4}$$

4. a) State and prove first shifting theorem on Z –transform.Find the Z –transform of $e^{\frac{in\pi}{2}}$ and then find $Z\left(\cos\frac{n\pi}{2}\right)$ and $Z\left(\sin\frac{n\pi}{2}\right)$ -7

b) Use Z –transform to solve the difference equation $y_{n+2} - 3y_{n+1} + 2y_n = 4^n, y_0 = 0, y_1 = 1.$ -8

5. a) Derive onedimensional heat equation with necessary assumptions. -7

b) Change the Laplacian $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar form $r^2 u_{rr} + r u_r + u_{\theta\theta} = 0.$ -8

6. a) Find the solution of one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ with initial velocity $g(x)$, initial deflection $f(x)$ and boundary conditions $u(0, L) = 0$ and $u(L; t) = 0.$ -7

b) Find the temperature distribution in a laterally insulated thin copper bar ($c^2 = 1.158\, cm^2/sec$),100cm long and of constant cross section whose endpoints at $x = 0$ and $x = 100$ are kept at $0°C$ and whose initial temperature is -8

i. $f(x) = \sin(0.01)\pi x$

ii. $f(x) = \sin^3(0.01)\pi x$

7. **Write Short notes on (Any two) $2\times 5 = 10$**

a) Show that $f(z) = z^3$ is analytic for all z.

b) Determine the location and order of zeros of $f(z) = \tan \pi z$

c) Verify the function $u = \sin 9t \sin\frac{x}{4}$ to satisfy one dimensional wave equation.

NEPAL COLLEGE OF INFORMATION TECHNOLOGY
Assessment Spring 2025

Level: Bachelor | Semester – Spring | Year: 2025
Programme: BE_COM(M & D)_IT(DAY) | Full Marks : 100
Course: Applied Mathematics | Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a. State and prove cauchy-Reimann euation as necessary conditions for function to be analytic. 5
   b. Define Harmonic function.show that u=cosx coshy is harmonicand find corresponding analytic function. 5
   c. State and prove cauchy –integral formuia 5
2. a. Solve: $\oint_C \frac{1}{z^2-1}\,dz$,the counter clockwise around the circle |z-1|=1.(chapter-1) 5
   b. Define Laurent series.Find the Laurent expansion for f(z)=$\frac{1}{z-z^3}$ *in the region given by* $1 < |z+1| < 2$ 5
   c. Define Cauchy –Residue theorem. By using Cauchy residue theorem evalute the integration $\oint_c \frac{z-23}{z^2-4z-5}dz,$ *where Cis the circle* $|z| = 6$ 5
3. a. Define bilinear transformation as well as fixed point.Find the bilinear transformation which makes the pointz=1.i.-i onto w=i,0,-1.*Also find the image of unit circle* $z = 1$ 5
   b. State and prove first shifting theorem of Z transform.Using it o find the z_transform of $Z(e^{-at}coswt)$ 5
   c. State and prove Initial and final value problem of Z-transform.Find the z-transform of $Z(a^n sinbt)$. 5
4. a. Prove that $Z(y_{n+K})=z^k(\bar{y} - y_0 - \frac{y_1}{z} - \cdots - \frac{y_{k-1}}{z^{k-1}})$ 5
   b. Solve the difference equation: 5
   $y_{n+2} + 6y_{n+1} + 9y_n = 2^n \quad with\ y_0 = y_1 = 0$
5. a. Using Fourier integral representation ,show that 5

$$\int_0^\infty \frac{cosxw + wsinxw}{1+w^2}dw = 0 \ if\ x < 0$$
$$= \frac{\pi}{2}\ if\ x = 0$$
$$= \pi e^{-x}\ if\ x > 0$$

   b. Find the Fourier sine transform of 5
   f(x)=e^{-x} *for* $x > 0$. *Then prove that* $\int_0^\infty \frac{sinmx}{1+x^2}dx = \frac{\pi}{2}e^{-m}$ *for* $m > 0$
6. a. Solve the following equation: 5

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0 \qquad by\ the\ seperation\ of\ variables.$$

b. A tightly stretche string with fixed end points x=0 and x=L is initially in a position given by u(x,0)=$sinx^3\left(\frac{\pi x}{L}\right)$.If it is realesed from rest from this position. Find the deflection u(x,t). 5

OR

Derive one dimensional Heat equation.

c. Determine the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial 2_u}{\partial x^2}$ Subject to the boundary condition u(0,t)=u(L,t)=0 and initial condition is u(x,0)=L ,being the length of the bar 5

OR

Derive one dimensional Wave equation with required assumptions.

7 a.. When two dimensional Heat equation $\frac{\partial u}{\partial t} = c^2(\frac{\partial 2_u}{\partial x^2} + \frac{\partial 2_u}{\partial y^2})$ become Laplace equation.By the concepr of solution of laplace equation with rectangular boundaries,solve the problem: 1+7

A rectangular plate with insulated surface is 8 cm wide so long compared to its width that it may be considered infinite in length without introducing an appreciable.if the temperature along the short edge y=o is given by

u(x,0)=100 sin$\frac{\pi x}{8}$, $0 < x < 8$

while two long edges x=0 and x=8 as well as the other short edge are kept at o^{0}c.

Show that steady state temperature at any point of the plate is given by

u(x,y)=100$e^{-\pi y/8}$ $sin\frac{\pi x}{8}$

OR

Derive two dimensional heat equation with required assumptions.

b. Derive polar form of Laplace equation. 7

OR

Aplate was insulated surface has the shape of quadrant of a circle of radius 10 cm.The bounding raddi $\theta = 0$ $and \theta = \frac{\pi}{2}$ are kept at 0^{0}c and temperature along the circular quadrant is kept at 100($\pi\theta - 2\theta^{2)0}c$ for 0≤ $\theta \le \frac{\pi}{2}$

$e^{\alpha x}(C_1 \sin \beta x + C_2 \cos \beta x)$

$C_3 e^{-c^2 p^2 t}$

NEPAL ENGINEERING COLLEGE

Changunarayan, Bhaktapur

(Assessment Spring Semester 2025)

Level: Bachelor | Full Marks: 100

Programme: BE | Pass Marks: 45

Course: .Applied Mathematics | Time: 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Show that $e^x(x\cos y - y\sin y)$ is a harmonic function. Find the analytic function for which $e^x(x\cos y - y\sin y)$ is imaginary part. 8

   b) State Cauchy integral formula. Evaluate $\oint_C \frac{z+1}{z^3-2z^2}dz$, where C is the $|z| = 1$,counterclockwise. 7

2. a) Find the bilinear transformation which maps the points $z = 0, -1, i$ onto $w = i, 0, \infty$. Also find the image of the unit circle |z|=1. 8

   b) Find the Laurent series for $f(z) = \frac{7z-2}{z^3-z^2-2z}$ in the region given by (i) $0 < |z+1| < 1$ (ii) $1 < |z+1| < 3$ 7

3. a) State Cauchy Residue Theorem. By applying Cauchy Residue Theorem, evaluate $\oint_C \frac{4-3z}{z(z-1)(z-2)}dz$ where $C: |z| = \frac{3}{2}$. 8

   b) State and prove first shifting theorem of Z transform. Obtain $Z^{-1}\left(\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}\right)$. 7

4. a) Using Z transform and inverse Z transform solve the equation $y_{n+2} - 2y_{n+1} + y_n = 2^n; y_0 = 2, y_1 = 1$ 7

   b) Show that $\int_0^\infty \frac{\sin \pi w \sin xw}{1-w^2}\,dw = \begin{cases} \frac{\pi}{2}\sin x & if\ 0 \le x \le \pi \\ 0 & if\ x > \pi \end{cases}$ 8

5.

a) Find the Fourier transform of $e^{-|x|}$.

Hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2} dx$. 8

OR

Find the fourier cosine transform of $f(x) = e^{-mx}$ for $m > 0$, and then show that $\int_0^\infty \frac{coskx}{1+x^2} dx = \frac{\pi}{2} e^{-k}$

b) A tightly stretched string with fixed end points $x = 0, and\ x = l$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points a velocity $kx(l-x)$, find the displacement of the string at any distance x from one end at any time t. 7

6.

a) Derive the one dimensional heat equation with necessary assumption 7

b) Solve; $r^2 \frac{\partial^2 u}{\partial r^2} + r \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0.$ 8

OR

A square plate is bounded by the lines $x = 0, y = 0, \quad x = 20, y = 20,$ its faces are insulated. The temperature along the upper horizontal edge is given by $U(x, 20) = x(20-x)$ when $0 < x < 20$ which other three edges are kept at 0^0 C. Find the steady state temperature in the plate.

7. Solve any two (2×5= 10)

a) Find the fixed point(s) and nature of the fixed point(s) of $f(z) = \frac{2-2z}{z+1}$.

b) Prove that $Z[nf(t)] = -z \frac{d}{dz} F(z)$,where F(z) = Z[f(t)] .

c) Solve $xU_{xy} + 2yU = 0$, using variable separation.

THE END

POKHARA ENGINEERING COLLEGE
Final Assessment

Level: Bachelor (4^{th} Sem) | Year :2025
Programme: BE (Computer/IT) | Full Marks: 100
Course: Applied Mathematics | Pass Marks: 45
Time :3:00 hrs

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define harmonic function.Show that the function $u(x,y) = 3x^2y + x^2 - y^3 - y^2$ is harmonic function.Find the function $v(x,y)$ such that $u + iv$ is analytic function. -7

b) State and prove cauchy integral formula.Integrate $\oint_c \frac{z^3}{2z-i}dz$ counterclockwise around the unit circle. -8

2. a) Find the Laurent series for $f(z) = \frac{z^2}{z^2+5z+6}$ in the region $2 < |z| < 3$ -7

b) State Cauchy residue theorem. Integrate $f(z) = \frac{z+1}{z^4-2z^3}$ around $c: |z| = \frac{1}{2}$ using cauchy residue theorem. -8

<u>**OR**</u>

Define a Bilinear map.Find the bilinear transformation which maps $z_1 = 0, z_2 = 1, z_3 = \infty$ into $w_1 = i, w_1 = -1, w_3 = -i$

a) Define Fourier integral of $f(x)$.Choosing a suitable function, show that -7

$$\int_0^\infty \left[\frac{\cos x\omega + \omega \sin x\omega}{1+\omega^2}\right] d\omega = \begin{cases} 0 & if\ x < 0 \\ \frac{\pi}{2} & if\ x = 0 \\ \pi e^{-x} & if\ x > 0 \end{cases}$$

b) Find Fourier sine and cosine transform of $f(x) = 2e^{-5x} + 5e^{-2x}$ -8

<u>**OR**</u>

Find the Fourier sine transform of e^{-x} for $x > 0$ and then by using parseval's identity show that:

$$\int_0^\infty \frac{x^2}{(1+x^2)^2}dx = \frac{\pi}{4}$$

4. a) State and prove first shifting theorem on Z −transform.Find the Z −transform of $e^{\frac{in\pi}{2}}$ and then find $Z\left(\cos\frac{n\pi}{2}\right)$ and $Z\left(\sin\frac{n\pi}{2}\right)$ -7

b) Use Z −transform to solve the difference equation $y_{n+2} - 3y_{n+1} + 2y_n = 4^n, y_0 = 0, y_1 = 1.$ -8

5. a) Derive onedimensional heat equation with necessary assumptions. -7

b) Change the Laplacian $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar form $r^2 u_{rr} + ru_r + u_{\theta\theta} = 0.$ -8

6. a) Find the solution of one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ with initial velocity $g(x)$, initial deflection $f(x)$ and boundary conditions $u(0,L) = 0$ and $u(L,t) = 0.$ -7

b) Find the temperature distribution in a laterally insulated thin copper bar $(c^2 = 1.158\ cm^2/sec)$, $100cm$ long and of constant cross section whose endpoints at $x = 0$ and $x = 100$ are kept at $0°C$ and whose initial temperature is -8

i. $f(x) = \sin(0.01)\pi x$

ii. $f(x) = \sin^3(0.01)\pi x$

7. **Write Short notes on (Any two) $2\times 5 = 10$**

a) Show that $f(z) = z^3$ is analytic for all z.

b) Determine the location and order of zeros of $f(z) = \tan \pi z$

c) Verify the function $u = \sin 9t \sin\frac{x}{4}$ to satisfy one dimensional wave equation.

Pokhara University

School of Engineering

Final Internal assessment

Program: BE — Full Marks : 100

Course: Applied Mathematics — Pass marks: 45

Attempt all the questions

1. a. Define harmonic function. Show that v = e^x (xcosy − y siny) is a harmonic function. Find harmonic conjugate u of v 8

   b. State and prove Cauchy–integral formula and hence evaluate $\oint_C \frac{2z^2+4z}{z-2}dz;\ c:|z|=1$ 7

2. a. Define conformal mapping with examples. Find the bilinear transformation that maps $i, 1, 2+i$ into $4i, 3-i, \infty$. 7

   b. Define singularity, zeros and pole of a function. State Cauchy Residue theorem. Evaluate $\oint_C \left(\frac{z^2 \sin z}{4z^2-1}\right)dz$ where c : | z | = 2, counter-clockwise 8

   Or

   Find the Laurentz expansion for $f(z)=\frac{1}{z^2+4z+3}$ in the region (i) $1<|z|<3$ (ii) $0<|z+1|<2$ (iii) $|z|>1$

3. a. Using the Fourier integral show that $\int_0^\infty \frac{\cos xw}{1+w^2}dw=\frac{\pi}{2}e^{-x}$ for x > 0

   b. Define Fourier transform. Define Fourier sine & cosine transforms. If f(x) in continuous piecewise in each finite interval and absolutely integrable on x-axis, $f(x)\to 0$ as $x\to\infty$ then 8

   (i) $\hat{f}_c\{f'(x)\} = w\ \hat{f}_s\{f(x)\} - \sqrt{\frac{2}{\pi}}f(0)$

   (ii) $F_c\{f'(x)\} = w\ F_s\{f(x)\} - \sqrt{\frac{2}{\pi}}f(0)$

4. a. Define Z - transform. State and prove first shifting theorem of Z-transform. Using it evaluate $Z(te^{2t}), Z(ne^{-2t})$ Z(e^{-at}), Z(te^{-at}) and Z(e^{-at}/n!). 8

   b. Solve the differential equation $y_{k+2}+2y_{k+1}+y_k=k$ where $y_0=0,\ y_1=0$ using z – transform. 7

5. a. Solve the partial differential equation $x^2U_y+y^3U_x=0$. 7

   b. Derive one dimensional wave equation and solve it 8

6. a. Find the temperature in a laterally insulated bar of length L whose ends are kept at a zero temperature, assuming that the initial temperature is 8

   $$f(x)=\begin{cases} x & \text{if } 0<x<\frac{L}{2} \\ L-x & \text{if } \frac{L}{2}<x<L \end{cases}$$

   b. Solve the equation of circular membrane completely 7

7. Attempt all questions 2.5x4=10

   a. If f(z)= $z+\frac{1}{z}$, check analyticity of f(z)

   b. Expand $\frac{1}{z}$ at $z=2$ as Taylor's series.

   c. Find the inverse Z – transform of $\frac{z}{z^2-2z+2}$.

   d. Verify that $U=e^{-3t}\cos 5x$ satisfy one dimensional heat equation.

UNITED TECHNICAL COLLEGE

Level: Bachelor | Semester: Spring | Year : 2025

Programme: B.E.Computer | Full Marks: 50

Course: Applied Mathematics | Pass Marks: 23

Time : 1.5hrs.

Attempt only 50 marks questions.

1 a. show that the necessary condition for analytic of $f(z) = u + iv$ is $u_x = v_y$ and $u_y = -v_x$ 8

b. Define Laplace equation. If $u = 3x^2y+x^2-y^3-y^2$ show that there exist a function $v(x, y)$ such that $f(z) = u +iv$ is analytic . 7

2 a. State and prove Cauchy's Integral formula. 8

Integrate $\oint_C \frac{dz}{z^2+4}$ where C: $4x^2+ (y -2)^2 = 4$

OR

Find the Laurent's series for $f(z) = \frac{1}{z-z^3}$ in $1 < |z + 1| < 2$

b. Define conformal mapping. Determine the region of Transformation $w =3z$ in the w-plane where the region in z-plane enclosed by the lines 7

$y= 1$, $y = 2$, $x = 1$, $x = 2$.

3 a. Solve: $Y_{n+2}+ 6Y_{n+1}+9Y_n=2^n$ When $Y_0 = Y_1 = 0$. b) Find $Z^{-1}\left(\frac{z^2+1}{z^2-2z+2}\right)$ 8 7

4 a. if $f(n)= 0$ for $n<0$ such that $Z(f(t)) =F(z)$ then $Z(f(t- kT)) = z^{-k} F(z)$ for $n>0$,$k>0$ 8

b. Derive two dimensional wave equation with necessary assumptions. 7

OR

A uniform rod of length 100cm has its ends maintained at a temp. 0° and initial temp. of rod is $f(x) = \sin^3(0.01)\pi x$.

5 a. *Find the deflection* $u(x ,y ,t)$ of the square membrane with $a = b = c = 1$. *If* initial velocity is zero and initial deflection is $k \sin\pi x \sin\pi y$. 8

OR

Find the solution of one –dimensional wave equation by D-Alembert's Method.

b. Solve : $U_{xx} + U_{yy} = 0$ separating variables.. 7

6 a. State and Prove Convolution Theorem for Fourier transform 8

b. find the Fourier cosine integral of $f(x) = e^{-x} + e^{-2x} \quad x > 0$. 7

7 a. Find the Z- transform of $n2^n$ and $e^{-at} - e^{-bt}$ 8

b. Find the Fourier sine and cosine transforms of $f(x) = e^{-mx}, m > 0$ 7

National Academy of Science and Technology

(Affiliated to Pokhara University)

Dhangadhi, Kailali

Pre University Examinations

Level: Bachelor Semester: IV_Spring Year : 2025

Program: B.E. Computer F.M. : 100

Course: **Applied Mathematics** P.M. : 45

Time : 3 hrs.

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt all the questions.

1.a) Show that the function u= Sinx Coshy is harmonic and find its harmonic Conjugate **[8]**

b) Evaluate: f(z)=$\frac{z^4}{(z+1)(z-i)^2}$. where c is $9x^2 + 4y^2 = 36$ **[7]**

2.a) Find the Taylor's and Laurent's series which represents the function $f(z)=\frac{z^2-1}{(z+2)(z+3)}$ in the regions:

i) $|z|<2$ ii) $2<|z|<3$ iii) $|z|>3$. **[7]**

b) State Cauchy's residue theorem. Using residue theorem, evaluate the integral. Calculate the residue of the function f(z) at each of its singular points, where **[8]**

$$f(z)=\frac{z+2}{(z-2)^2(z^2+1)}$$

3.a) Define Z-transform of a function f(n). Find the Z-transform of $e^{in\pi/2}$ and hence find $Z[\cos(\frac{n\pi}{2})]$ *and* $Z[\sin(\frac{n\pi}{2})]$ **[8]**

b)Using Z-transform, solve the difference equation **[7]**

$y_{n+2} - 4y_{n+1} + 4y_n = 2^n$, $y_0 = 0$, $y_1 = 1$.

4.a) Show that: $\int_0^\infty \frac{(1-Cos\pi w)}{w} sinxw\, dw = \begin{cases} \frac{\pi}{2} & if\ 0<x<\pi \\ 0 & if\ x>\pi \end{cases}$ **[7]**

b) Find the Fourier sine transform of the function

$$f(x)=\begin{cases} x^2 & for\ 0<x<1 \\ 0 & for\ x>1 \end{cases}$$ [8]

OR

Find the Fourier cosine transform of $f(x)=e^{-mx}$ for m>0 and then

Show that $\int_0^{\infty} \frac{coskx}{1+x^2}dx = \frac{\pi}{2}e^{-k}$.

5.a) A tightly stretched string of length 100 cm is drawn a side at its midpoint perpendicular to the equilibrium position so that its initial position is given by

$$f(x)=\begin{cases} x\ if & 0<x<50 \\ 100-x\ if & 50<x<100 \end{cases}$$ [7]

b) Derive one dimensional heat equation $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ [8]

6.a) Express the Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar co-ordinates. *[8]*

b) Find the solution of one dimensional wave equation with the appropriate initial and boundary conditions. [7]

OR

Solve $u_{xx}+u_{yy}=0$ using separation of variables method.

7. *Solve:* [4x2.5 = 10]

i) Express f(z) = coshz in the form f(z)=u+iv.

ii) Show that the function $u = CosxCoshy$ is a solution of two dimensional Laplace equation.

iii) Show that Fourier cosine transforms satisfy linearity property.

iv) Find $Z(e^n)$

$\frac{z\{1-9\}}{2}$

Pokhara University

Everest Engineering College

Final Internal Assessment

Spring- 2025

Level: Bachelor

Program: BE CMP(4th Semester)

Faculty: Science & Technology

Subject: Applied Mathematics

F.M. 100

P.M. 45

Time: 3hrs

Attempt all questions.

1.a) Define harmonic function. Is $v = \arg z$ is harmonic? If yes, find its harmonic conjugate. [8]

b) Define Mobius transformation. Find the bilinear transformation which maps the points $z = 0, 1, \infty$ in to the points $w = -5, -1, 3$ and hence find the fixed points. [7]

2. a) State Cauchy's Integral Formula for derivatives. Evaluate

$\oint_C \frac{z^6}{(2z-1)^6} dz$, where $C: |z| = 1$. [7]

b) Expand in Laurent's series of the function $\frac{z^2-1}{z^2+5z+6}$ in the region

i) $|z| < 2$ ii) $|z| > 3$ iii) $2 < |z| < 3$ [8]

3. a) Show that $\int_0^\infty \frac{\sin \pi\omega \sin \omega x}{1-\omega^2} d\omega = \begin{cases} \frac{\pi}{2} \sin x & \text{for } 0 \le x \le \pi \\ 0 & \text{for } x > \pi \end{cases}$ [7]

b) Define Fourier transform. Find the Fourier transform of $e^{-\frac{x^2}{2}}$ [8]

OR

Find the Fourier sine transform of $\frac{e^{-ax}}{x}$, $x > 0$ and hence show that

$\int_0^\infty \tan^{-1}\frac{x}{a} \sin x \, dx = \frac{\pi}{2} e^{-a}$.

4. a) Find the Z-transform of e^{-iat} and hence find the Z-transform of $\cos at$ and $\sin at$ [7]

b) State and Prove that Initial Value Theorem and Final Value Theorem for Z-transform. [8]

5. a) Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ where $y_0 = 0 = y_1$ by Z – transform method. [7]

b) A tightly stretched string with fixed end points $x = 0$ and $x = L$ is initially in a position given by $u(x,0) = u_0 \sin^3\left(\frac{\pi x}{L}\right)$. If it is released from rest from this position. Find the displacement. [8]

6. a) Find the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ with boundary condition $u(0,t) = 0 = u(L,t)$ and initial temperature $f(x)$. [7]

OR

Find the temperature in a laterally insulated bar of length L whose ends are kept at a zero temperature, assuming that the initially temperature is $f(x) = \begin{cases} x & \text{if } 0 < x < \frac{L}{2} \\ L - x & \text{if } \frac{L}{2} < x < L \end{cases}$

b) Change the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial^2 y} = 0$ into polar co-ordinates. [8]

7. Attempt all: [10]

a) Find the Fourier cosine transform of $e^{-mx}, m > 0$

b) Solve $u_x + u_y = 0$ by using separation of variables.

c) Verify that $u = x^2 + t^2$ is a solution of one-dimensional wave equation.

d) Check that the function $z\bar{z}$ is analytic.

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