Answer of 1

TSP is NP-Complete if:

i. TSP is in NP
ii. TSP is NP-hard

1. TSP is in NP:

- TSP is in NP because, given a proposed tour and its cost, we can verify in polynomial time if it visits each city exactly once and the total cost is $\leq D$.

2. Reduction from Hamiltonian Cycle (HC) to TSP:

- **Hamiltonian Cycle (HC)**: Given a graph G, determine if there is a cycle that visits each vertex exactly once.
- **Reduction**: Given a graph G with n vertices, create an equivalent TSP instance:
    - Each vertex is a city.
    - If (u,v) is an edge in G, set distance $d(u,v)=1$; otherwise, set $d(u,v)=n+1$.
    - Set the target distance $D= n$.
- A Hamiltonian Cycle in G corresponds to a TSP tour of length n, and vice versa.

3. Conclusion:

- TSP is in NP, and HC reduces to TSP in polynomial time, so TSP is NP-complete.

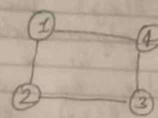
Answer of 2

a. False.
b. False. Reducibility is not symmetric.
c. True. If NP-complete can be solved using polynomial time, then all NP-complete problems can be solved in polynomial time. Thus proving, P= NP.
d. False. We can only conclude B is NP-hard. B may not be in NP.

Answer of 3

Answer of 3

$G = \{(V, E)$
$V = \{1, 2, 3, 4\}$
$E = \{(1,2), (2,3), (3,4), (4,1)\}$



Smallest Vertex cover is $\{1,3\}$ and $\{2,4\}$ with size $s = 2$.

Vertex Cover Approx Algorithm.

$E \leftarrow \{(1,2), (2,3)\ (3,4), (1,4)\}$
$C = \{\}$

#1
E is not empty.
$e = (1,2)$
Add 1 and 2 to C.
$C = \{1,2\}$

$E = \{(3,4)\}$

#2 E is not empty.

$e = (3,4)$
Add 3 and 4 to C.
$C = \{1,2,3,4\}$
$E = \phi$

#3 Edge is empty

$C = \{1, 2, 3, 4\}$ is the vertex cover of size 4
$\approx 2 * 2$ or $2 * s$.

Answer of 4

Answer of 4

Given a positive integer k, and a graph G, is there a vertex cover for G having size $\leq k$? Show that this decision problem belongs to NP.

Input: Graph $G = (V, E)$, Vertex cover V_c where $V_c \in V$ and required size k.

Verification:

1. Check the elements in V_c are unique and less than or equal to k.
2. For each edge $e \in E$, validate at least one vertex ~~belong~~ v from edge e is in vertex cover V_c.

(1.) takes $O(n)$.
(2) $O(m) = O(n^2)$ as there can be total $\frac{n(n-1)}{2}$ edges max.

This takes polynomial time. Hence the Vertex Cover decision problem belongs to NP.