Answer of 1

Solution of 1(A):

```
Algorithm: mergeSortPlus(S)
Input: sequence S with n integers
Output: sequence S sorted

return mergeSortPlus(S,0, S.size() - 1);


Algorithm: mergeSortPlus(S, lower, upper)
Input: sequence S with n integers with upper and lower bounds
Output: sequence S sorted

Size ← upper – lower + 1
Mid ← (upper + lower) / 2

If Size <= 20 then
    return insertionSort(S, lower, upper)
else if Size() > 1 then
    (S1 , S2 ) ← partition(S, lower, mid,  upper)
    mergeSortPlus (S1, lower, mid )
    mergeSortPlus (S2 , mid + 1, upper)
    S ← merge(S1 , S2 )
return S

Algorithm: InsertionSort(arr, lower, upper)
Input: arr is an array of integers with lower and upper bounds to sort
Output: a sorted array

n ← arr.length
temp ← 0
j ← 0
for i ← lower + 1 to upper do
    temp ← arr[i]
    j ← i

    while j > 0 && temp < arr[j - 1]
        arr[j] ← arr[j - 1]
        j ← j – 1
    arr[j] ← temp
```

return arr

Solution of 1 (B):

```java
public class MergeSortPlus extends Sorter {

    //public sorter
    public int[] sort(int[] input){

       mergeSortPlus(input, 0, input.length - 1);

       return input;
    }

    void insertionSort(int[] arr, int lower, int upper) {
       int temp = 0;
       int j    = 0;

       for (int i = lower + 1; i <= upper; i++) {
          temp = arr[i];
          j    = i;

          while (j > lower && temp < arr[j - 1]) {
             arr[j] = arr[j - 1];
             j--;
          }

          arr[j] = temp;
       }
    }

    void mergeSortPlus(int[] arr, int lower, int upper) {
       if (upper - lower + 1 <= 20) {
          insertionSort(arr, lower, upper);
       } else {
          int mid = lower + (upper - lower) / 2;

          mergeSortPlus(arr, lower, mid);
          mergeSortPlus(arr, mid + 1, upper);

          merge(arr, lower, mid, upper);
       }
    }

    void merge(int[] arr, int low, int mid, int high) {
       int n1 = mid - low + 1;
       int n2 = high - mid;

       int[] arr1 = new int[n1];
       int[] arr2 = new int[n2];

       System.arraycopy(arr, low, arr1, 0, n1);
       System.arraycopy(arr, mid + 1, arr2, 0, n2);

       int left = 0, right = 0;
       int index = low;
```

```
        while (left < n1 && right < n2) {
            if (arr1[left] < arr2[right]) {
                arr[index++] = arr1[left++];
            } else {
                arr[index++] = arr2[right++];
            }
        }

        while (left < n1) {
            arr[index++] = arr1[left++];
        }

        while (right < n2) {
            arr[index++] = arr2[right++];
        }
    }

}
```

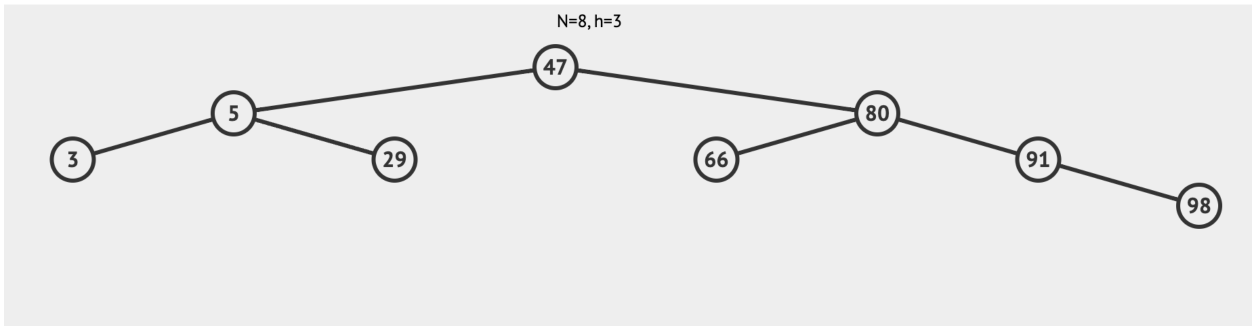
Solution of 1 (C):

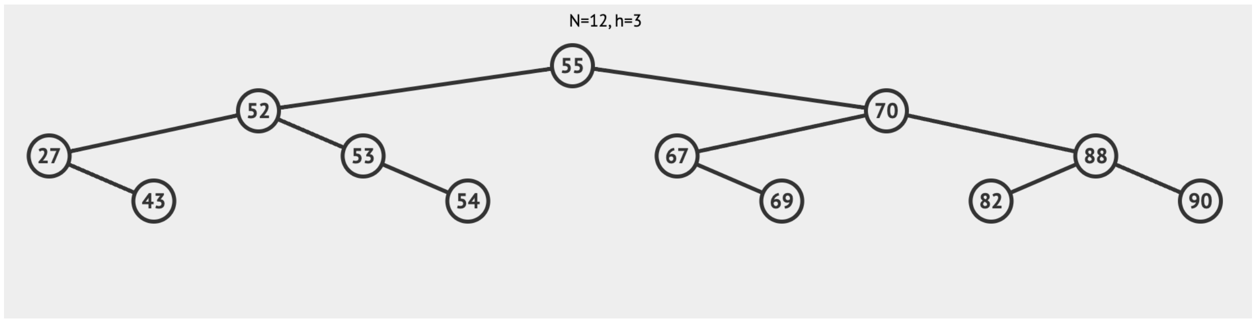
The MergeSortPlus seems to run faster than the MergeSort due to the enhanced algorithm benefit of using InsertionSort. I compared it on large datasets of 100,000, 200,000, 300,000 and 500,000 elements comparing the time it took for each algorithm to finish. Each time, MergeSortPlus was faster.

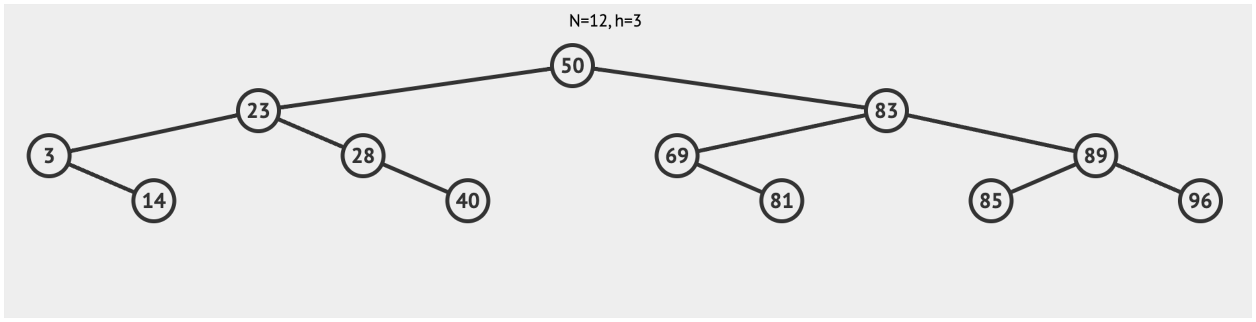
The results are conclusive and in favor of MergeSortPlus being the faster algorithm.

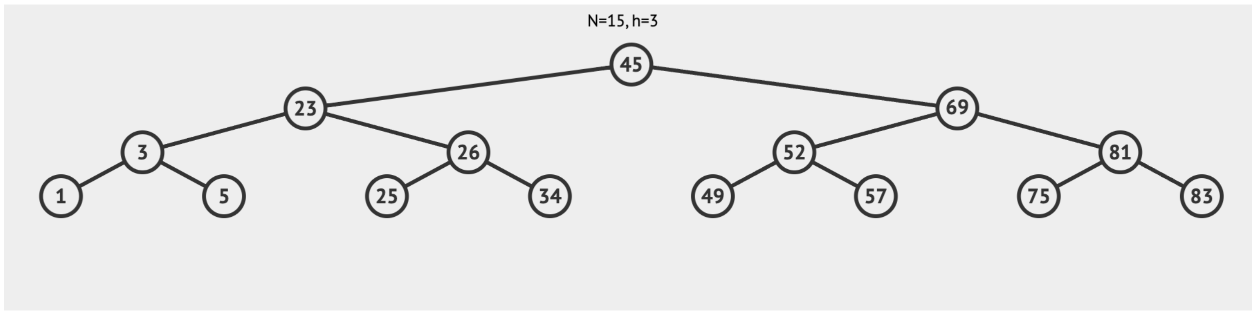
Answer of 2

Solution of 2 (A):









Solution of 2 (B):

Statement: Every binary tree of height 3 has at most 2^3=8 leaves.
Verdict: It Is true, for a binary tree each node can have 0, 1 or 2 nodes.
Hence, at height 0, we can have at most one element, the root node.
At Height 1, we can only have at most 2^1 = 2 leaf nodes.
At Height 2, we can only have at most 2^2 = 4 leaf nodes.

We can conclude, at height 3 we will have at most 2^3 = 8 leaf nodes.

Solution of 2 (C):

Going on with this pattern, at height n, we will have at most 2^n leaf nodes.

Answer of 3

```
List<Set<Integer>> powerSet(List<Integer> X){
    List<Set<Integer>> P = new ArrayList<>();
    Set<Integer> S = new HashSet<>();

    P.add(S);

    while (!X.isEmpty()){
        Integer f = X.remove(0);

        List<Set<Integer>> newSubsets = new ArrayList<>();

        for (var x : P) {
            Set<Integer> newSubset = new HashSet<>(x);
            newSubset.add(f);
            newSubsets.add(newSubset);
        }

        P.addAll(newSubsets);
    }

    return P;
}
```

Answer of 4

```
import java.util.ArrayList;
import java.util.HashSet;
import java.util.List;
import java.util.Set;

public class Problem04Solution {
    public static void main(String[] args) {
        Problem04Solution solution = new Problem04Solution();

        List<List<Integer>> inputs = new ArrayList<>(List.of(
                        new ArrayList<>(List.of(1, 3, 9, 4, 8, 5)),
                        new ArrayList<>(List.of(1, 3, 9)),
                        new ArrayList<>(List.of(1, 3, 9, 4, 8, 5))
                       ));

        int[] ks = new int[]{2, 5, 0};

        for (int i = 0; i < inputs.size(); i++){
            System.out.printf("Array: %s%n", inputs.get(i));
            System.out.printf("Sum of %d exists in subset: %s %n%n",
                              ks[i],
                              solution.subsetSumExists(inputs.get(i),
ks[i]));
        }


    }

    Set<Integer> subsetSumExists(List<Integer> X, int k){
        List<Set<Integer>> P = new ArrayList<>();
        Set<Integer> S = new HashSet<>();

        if (k == 0)
            return S;

        P.add(S);

        Integer sum = 0;

        while (!X.isEmpty()){
            Integer f = X.remove(0);

            List<Set<Integer>> newSubsets = new ArrayList<>();

            sum = 0;

            for (var x : P) {
                Set<Integer> newSubset = new HashSet<>(x);
                newSubset.add(f);

                sum = newSubset.stream().mapToInt(Integer::valueOf).sum();

                if (sum == k){
                    return newSubset;
                }

                newSubsets.add(newSubset);
            }

            P.addAll(newSubsets);
        }

        return null;
    }
}
```