First Exercise

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https://github.com/adhikarijenish/Computational_Physics.git

October 25, 2024

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1 Just do one big experiment:

In this exercise I have done one big experiment to estimate π unsing 10000 pairs of random numbers. In fig. 1 you can see the histogram of radius of random pairs and its squared. If we compair true value of π and mean indicator variable fig. section 1, they almost overlapp. The experimental value is found to be $\pi_{\text{exp}} = 3.15 \pm 1.64$.
In fig. 1 left we can see radius around 0.8, we can explain this due to eq. (1).

$$\begin{aligned} \sqrt{x} &> x \\ x^2 - x &< 0 \\ x(x-1) &< 0 \\ x &< 1 \end{aligned} \tag{1}$$

2 Split into 100 experiments

If you have same number of random numbers but in a different way, such as Pairs $P = 100$ and $X = 100$. We find an estimator for π for each experiment. If one plots the histogram of such estimator, one finds fig fig. 3. We can estimate the the observable as the mean of all estimators and found the value to be $\pi_{\text{exp}} = 3.1476 \pm 0.17344$.

3 A Zillion Little Experiments

If we compute many experiment insted of taking random numbers many times, it doesn't affect the outcome. We can still interprete many experiment as drawing random numbers many time. The value of estimator if found to be $\pi_{\text{exp}} = 3.14 \pm 1.64$, the reasult is almost same as in case of $P = 10000$.

4 Stop and think

The estiamtes of the previous parts as compatible with the know value of $\pi \approx 3.14159265358$. I would say no, the standard deviation of a single experiment makes no sense as an uncertainty, because in that case we generate a pair of random numbers for our computation, which has to be random. Thus has its intinsic random distribution.
The reorganization of random numbers matters in many way;

- Taking just few random numbers generated by computer, has certain correlation. Only on many draw of such number it is random.
- Some experiments demand lot of computational resources. So, in such case conducting more experiment is not feasible.

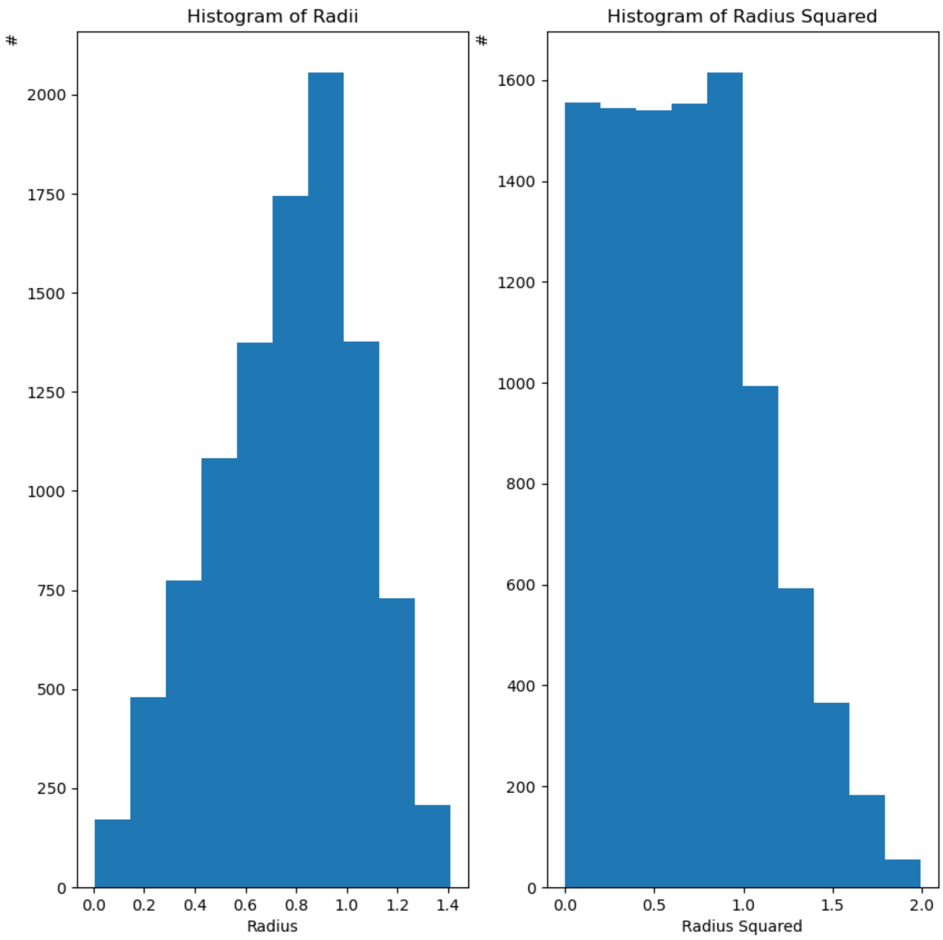


Figure 1: One Big Experiment

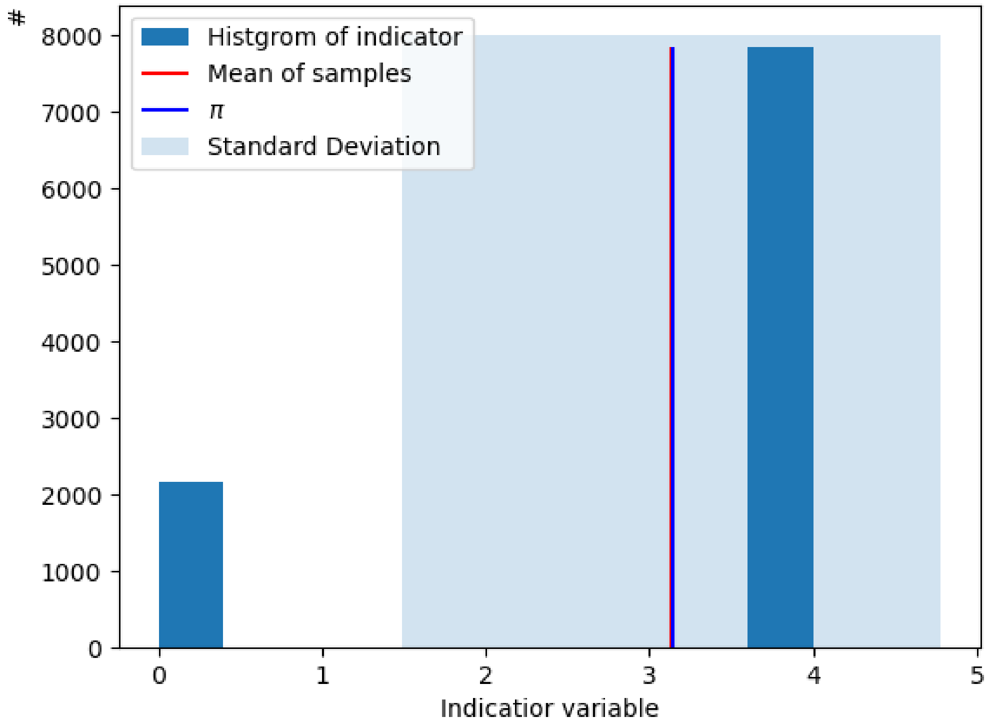


Figure 2: Histogram of indicator variable $4[x^2 + y^2 \leq 1]$

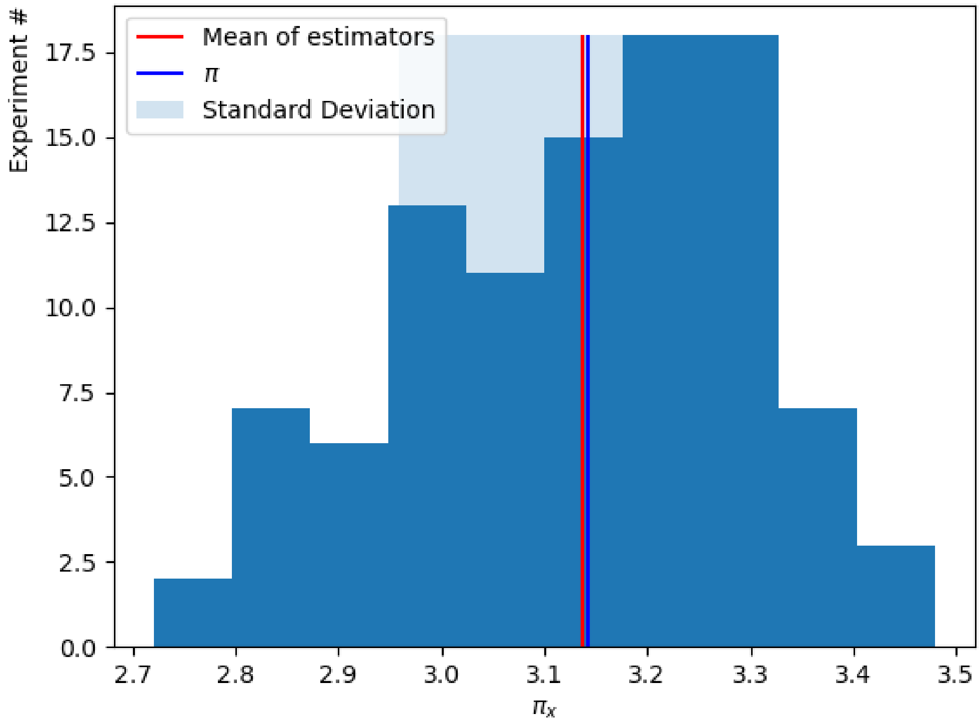


Figure 3: Split into many experiments

5 More Experiments vs. Longer Experiments

Here we'd like to investigate how the uncertainty behaves as the function of experiments and the length of each experiment. All possible combination of $P \in [10, 100, 1000, 1000]$ and $X \in [10, 100, 1000, 1000]$ can be found in table 1. The histogram of all possible estimator can be found in fig. 4 with their uncertainties. The horizontal line is a true value of π.

In fig. 4 we can see already $XP \approx 10^3$ the π_{exp} is close to π. We can also find the optiomal value to be $P = 1000$ and $X = 100$. The final estimate of $\pi_{\text{exp}} = 3.1476 \pm 0.02722$. As we can see in fig. 4 there is huge uncertainties $XP \leq 10^5$. Thus greater influance in the reasult. The rest two plots are not great to analyse. If intrested see fig. 5a and fig. 5b.

Table 1: Combination of P ans X

P	X	XP	π_x	$\Delta\pi_x$
10	10	100	3.24	0.54813
10	100	1000	3.18	0.53447
10	1000	10000	3.1244	0.50642
10	10000	100000	3.1356	0.52595
100	10	1000	3.148	0.18955
100	100	10000	3.1388	0.17893
100	1000	100000	3.1404	0.16582
100	10000	1000000	3.1416	0.16569
1000	10	10000	3.1336	0.075594
1000	100	100000	3.1386	0.052451
1000	1000	1000000	3.139	0.051337
1000	10000	10000000	3.1425	0.052117
10000	10	100000	3.1332	0.014394
10000	100	1000000	3.1437	0.015649
10000	1000	10000000	3.1411	0.016698
10000	10000	100000000	3.1417	0.016515

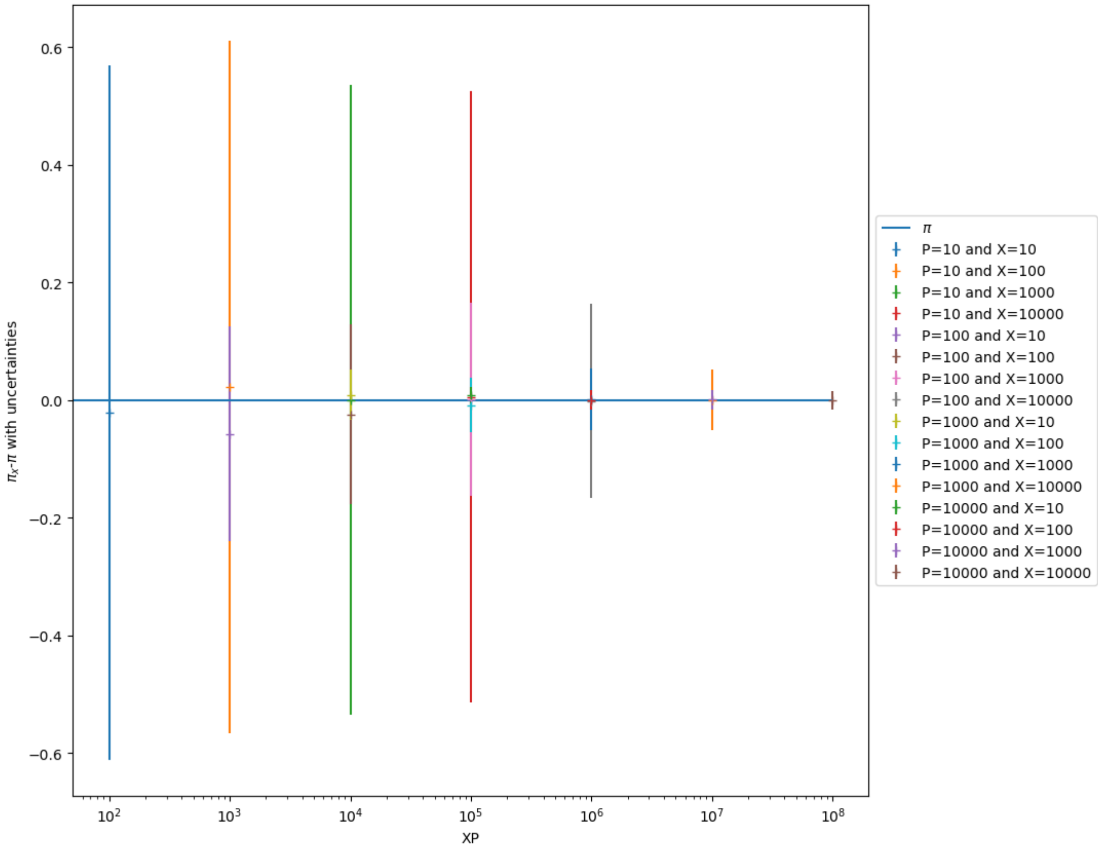
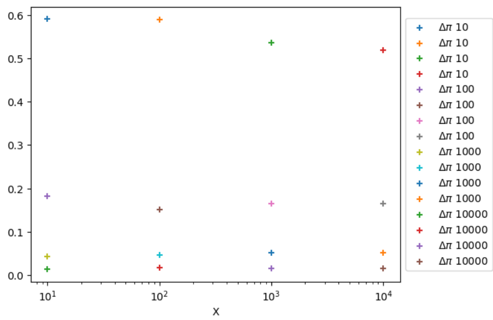
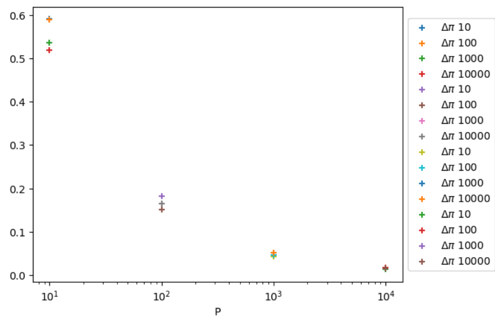


Figure 4: Best XP



(a) Error as function of X



(b) Error as function of P

Figure 5: Error of combination of P and X