EMSR-b and DAVN Heuristics in Airline Revenue Management

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1 Introduction

Airline revenue management revolves around the challenge of determining which customer bookings to accept and which to reject, aiming to maximize overall revenue. A key problem faced by airlines is how to assign fares for the *products* they offer. Each *product* is characterized by the origin, the destination, the stopovers (if any), and the dates and times of travel.

Fare class is a term often used for charging the same fare on a given origin-destination itinerary. Airlines offer multiple fares even for the same origin-destination itinerary, each associated with different conditions and privileges. As tickets are sold over time, airlines must strategically adjust fare availability, closing lower fare classes and opening higher ones to optimize profitability.

Passengers in the economy cabin typically have access to several fare classes, even though all seats in that cabin are physically identical. Lower fare classes tend to sell out earlier due to higher demand, while higher fare classes may remain available longer, often offering additional benefits such as flexible cancellations or priority boarding. While customers who book early often choose lower fares, some may still opt for premium fares despite cheaper options being available. Airlines frequently adjust pricing based on factors such as remaining time before departure, demand patterns, and customer preferences to enhance revenue.

The key decision in revenue management is how many seats to allocate to each fare class to strike a balance between high occupancy and maximizing revenue. Allocating too many seats to lower fares may lead to full flights but lower profits, while reserving too many seats for higher fares risks unsold inventory, reducing potential revenue. Thus, airlines must carefully manage seat allocations to avoid these extremes.

Additionally, airline seats are a perishable resource—once a flight departs, any unsold seat represents lost revenue. To mitigate this, airlines employ overbooking strategies, selling more tickets than available seats to compensate for expected cancellations and no-shows. However, if more passengers show up than seats available, the airline incurs costs by compensating displaced travelers and arranging alternative flights. The complexity of revenue management arises from fluctuating demand, cancellation rates, and overbooking risks, all of which require airlines to make data-driven decisions on pricing and seat allocation.

The Expected Marginal Seat Revenue (EMSR-b) heuristic [1] is widely used to determine seat allocations among different fare classes dynamically. It extends Littlewood's rule [4] by incorporating multiple fare classes and probabilistic demand forecasts. The Displacement Adjusted Virtual Nesting (DAVN) heuristic [2] develops a model to generate the virtual revenue from each fare on a multi-leg airline network, taking into account how bookings on one leg impact availability across the network. This model can be combined with EMSR-b [5] (see also [3] for a detailed explanation) to optimize revenues across multiple connected flights; the combined model, often referred to as DAVN-EMSR-b, is powerful, as it can solve large-scale problems, and is hence widely used in the airline industry.

2 Definitions of Key Concepts

Before delving into the EMSR-b and DAVN heuristics, we define some key technical terms:

- **Fare Class** (i): A category of airline tickets distinguished by price and associated conditions (e.g., refundable vs. non-refundable).

- **Fare Price** (f_i): The price of fare class i to be paid by a customer.
- **Protection Level** (Q_i): The number of seats reserved for fare classes higher than and including i to ensure that they are available for potential high-revenue passengers. There is no protection level for the class with lowest fare.
- **Booking Limit** (B_i): The maximum number of seats that can be sold at a specific fare class i.
- **Demand** (d_i): The random number of bookings for fare class i by the customers during the booking period.
- **Leg** (ℓ): A leg refers to a single nonstop flight segment operated between two airports. For example, a direct flight from New York (JFK) to Los Angeles (LAX) is a leg.
- **Product:** A product consists of an itinerary (which may contain one or more legs) combined with a fare class. For example, a round-trip ticket from Boston (BOS) to San Francisco (SFO) via Chicago (ORD) in economy class is a product consisting of multiple legs.

3 EMSR-b Heuristic: Booking limits for a single leg

Littlewood's rule provides the foundation for airline revenue management by addressing the trade-off between selling a seat at a lower fare now versus reserving it for a potentially higher-paying customer in the future. The key idea underlying this rule is that an airline should accept a booking at a lower fare only if the expected revenue from selling it now is at least as great as the expected revenue from saving it for a later, higher-fare customer.

Mathematically, in a two-fare class system, Littlewood's rule states that a seat should be sold at fare f_1 if

$$f_1 \geq f_2 \cdot \Pr[d_2 > Q_2],$$

where $\Pr[d_2 > Q_2]$ represents the probability that demand d_2 for the higher fare f_2 exceeds the protection level Q_2. If this probability is high, the airline should protect more seats for high-fare customers. If it is low, the airline can safely sell more seats at the lower fare.

The EMSR-b heuristic generalizes Littlewood's rule to multiple fare classes on a single leg. Instead of considering only two fares, EMSR-b calculates a cumulative demand distribution for all fare classes above a given class and determines seat protection levels accordingly. The core idea remains the same: balance the trade-off between immediate revenue and potential higher future revenue.

By considering aggregated demand distributions and adjusting booking limits dynamically, EMSR-b enables airlines to make optimal seat allocation decisions that maximize total expected revenue on one leg.

The process consists of the following steps to compute the booking limits of the fare classes. Other data, such as airplane capacity, fare prices, demands, and demand distributions, are assumed to be part of the input.

Step 1: Sorting Fare Classes

Sort all fare classes in ascending order by fare price, such that

$$f_1 \leq f_2 \leq \cdots \leq f_n.$$

Step 2: Compute Aggregated Demand and Fares

For each fare class i, define the aggregated demand for all fare classes above and including i:

$$D_i = \sum_{j=i}^{n} d_j,$$

and the aggregate fares as

$$F_i = \frac{\sum_{j=i}^{n} f_j \cdot E[d_j]}{\sum_{j=i}^{n} E[d_j]}.$$

Step 3: Compute Protection Levels

The lowest fare class 1 has no protection level as no protection is needed for the lowest fare. For the remaining fare classes, determine the protection levels by solving Littlewood's equation for Q_{i+1}: For $i = 1, 2, \ldots, n-1$

$$f_i = F_{i+1} \cdot \Pr[D_{i+1} > Q_{i+1}].$$

A common assumption is that the arrivals are Poisson or normally distributed, which means the inverse cumulative distribution function of the distribution concerned can be used to solve for protection level Q_{i+1} in the Littlewood's equation, as shown below. For $i = 1, 2, \ldots, n-1$:

$$Q_{i+1} = \text{invcdf}\left(1 - \frac{f_i}{F_{i+1}}\right),$$

where the invcdf is of the random variable D_{i+1}. Note that, using the above, one needs to solve for the unknowns, Q_2, Q_3, $\ldots$ Q_n, and that Q_1 is not needed in this formulation.

Step 4: Compute Booking Limits

The booking limit for each fare class $i < n$ is given by:

$$B_i = C - Q_{i+1}.$$

In the above, if $B_i < 0$, B_i is forcibly set to zero. Also, for the highest fare class, $B_n = C$, where C is the capacity of the airplane.

The booking limit of a leg essentially signifies that once B_i seats have been sold in class i for this flight, no additional bookings are permitted for that class or any class with a lower fare. When overbooking is taken into account, C is often replaced with $C/(1 + cp)$ above, where cp represents the average cancellation probability across all fare classes. This modification temporarily increases the available capacity during the booking process to account for expected cancellations.

4 The DAVN Heuristic: Extending EMSR-b to Networks

While EMSR-b optimizes seat allocations for a single leg, the DAVN heuristic extends this concept to multi-leg itineraries. The DAVN heuristic applies revenue optimization techniques considering the demand for various products rather than just individual legs.

4.1 Linear Program Formulation

The DAVN heuristic solves the following linear program:

$$\max \sum_{j=1}^{n} f_j x_j$$

subject to:

$$\begin{aligned} 0 \le x_j \le E[d_j], &\quad \forall j = 1, 2, ..., n \\ \sum_{j \in A_\ell} x_j \le C_\ell, &\quad \forall \ell = 1, 2, ..., L \end{aligned} \qquad (\star)$$

where f_j is the fare price of product j, $E[d_j]$ is the expected demand for product j, C_ℓ is the capacity of leg ℓ, and A_ℓ is the set of products using leg ℓ.

Table 1: Input data for the 24 products, where the demand is assumed to have the Poisson distribution

Product (j_1, j_2)	Itinerary	$(E[d(j_1)], E[d(j_2)])$	(f_{j_1}, f_{j_2})
$(1, 13)$	A $\rightarrow$ X	$(58.8, 14.7)$	$(350, 500)$
$(2, 14)$	X $\rightarrow$ A	$(67.2, 16.8)$	$(375, 525)$
$(3, 15)$	A $\rightarrow$ Y	$(50.4, 12.6)$	$(400, 550)$
$(4, 16)$	Y $\rightarrow$ A	$(58.8, 14.7)$	$(430, 585)$
$(5, 17)$	A $\rightarrow$ Z	$(67.2, 16.8)$	$(450, 600)$
$(6, 18)$	Z $\rightarrow$ A	$(50.4, 12.6)$	$(500, 650)$
$(7, 19)$	X $\rightarrow$ Y via A	$(84, 21)$	$(600, 750)$
$(8, 20)$	Y $\rightarrow$ X via A	$(100.8, 25.2)$	$(610, 760)$
$(9, 21)$	X $\rightarrow$ Z via A	$(84, 21)$	$(620, 770)$
$(10, 22)$	Z $\rightarrow$ X via A	$(75.6, 18.9)$	$(630, 780)$
$(11, 23)$	Y $\rightarrow$ Z via A	$(84, 21)$	$(640, 790)$
$(12, 24)$	Z $\rightarrow$ Y via A	$(58.8, 14.7)$	$(650, 800)$

4.2 Computing Displacement Adjusted Revenue (DARE)

The displacement adjusted revenue (DARE) for product j on leg ℓ is computed as:

$$DARE_j^{\ell} = f_j - \sum_{i \neq \ell} \lambda_i,$$

where λ_ℓ is the dual price of constraint $(\star)$ for leg ℓ. These DARE values are then used as fares to apply EMSR-b on each leg separately, in order to determine the booking limits for each leg.

5 Illustrative Example

The use of EMSR-b is now illustrated using an example from [3]. Consider the following network with four cities: the hub, A, and three spoke cities, X, Y, and Z. This results in 6 legs: $l = 1$: A $\rightarrow$ X, $l = 2$: X $\rightarrow$ A, $l = 3$: A $\rightarrow$ Y, $l = 4$:Y $\rightarrow$ A, $l = 5$: A $\rightarrow$ Z, and $l = 6$: Z $\rightarrow$ A. Table 1 provides input data for the resulting 12 itineraries. For every itinerary, the airline offers two products (j_1 and j_2), where j_1 pays the higher fare with a lower cancellation probability and j_2 pays the lower fare with a higher cancellation probability. This implies there are $n = 24$ products. Each of the 6 legs have planes with a capacity of 100.

Note that the *DARE* values, which are obtained from solving the LP, can be represented in a matrix format in which the row is represented by the product and the column by the leg. Table 2 shows this *DARE* matrix, where a negative one (-1) implies the associated product does *not* use the leg concerned. Thus, from inspecting Table 2, it is clear that on the first leg, the virtual revenues for products numbered 1, 8, 10, 13, 20, and 22, are respectively 350, 215, 350, 500, 365, and 500. This set of values is now employed in the EMSR-b calculations for the first leg. In a similar manner, EMSR-b calculations are performed *separately* for each leg.

The following average cancellation probabilities were used for the 6 legs from the first to the sixth: 0.225, 0.2, 0.1, 0.22, 0.15, 0.21, and 0.14, respectively. Each leg is assumed to have a capacity of 100. The 24 products have the following expected demand from the first product to the 24th: 58.8, 67.2, 50.4, 58.8, 67.2, 50.4, 84, 100.8, 84, 75.6, 84, 58.8, 14.7, 16.8, 12.6, 14.7, 16.8, 12.6, 21, 25.2, 21, 18.9, 21, and 14.7, respectively.

The EMSR-b calculations can be explained as follows. Consider for instance, the first leg, where the average cancellation probability is 0.225; the fares and the mean demand are 215 and 100.8 (product 8), 350 and 58.8 (product 1), 350 and 75.6 (product 10), 365 and 25.2 (product 20), 500 and 14.7 (product 24), 500 and 18.9 (product 22), respectively. Then, the booking limits computed from EMSR-b are: 0, 10, 77, 100, 130 and 130, respectively. Table 3 shows the booking limits and fares for *all the six legs.*

Table 2: Table showing the $DARE$ values in matrix format where the column denotes the leg and the row denotes the product

350	-1	-1	-1	-1	-1
-1	375	-1	-1	-1	-1
-1	-1	400	-1	-1	-1
-1	-1	-1	430	-1	-1
-1	-1	-1	-1	450	-1
-1	-1	-1	-1	-1	500
-1	230	225	-1	-1	-1
215	-1	-1	260	-1	-1
-1	225	-1	-1	245	-1
350	-1	-1	-1	-1	280
-1	-1	-1	245	245	-1
-1	-1	370	-1	-1	280
500	-1	-1	-1	-1	-1
-1	525	-1	-1	-1	-1
-1	-1	550	-1	-1	-1
-1	-1	-1	585	-1	-1
-1	-1	-1	-1	600	-1
-1	-1	-1	-1	-1	650
-1	380	375	-1	-1	-1
365	-1	-1	410	-1	-1
-1	375	-1	-1	395	-1
500	-1	-1	-1	-1	430
-1	-1	-1	395	395	-1
-1	-1	520	-1	-1	430

Table 3: Table showing the fares and booking limits on each of the legs

Leg	Fare	Booking Limit
1	215	0
1	350	10
1	350	77
1	365	100
1	500	130
1	500	130
2	225	0
2	230	1
2	375	76
2	375	93
2	380	111
2	525	125
3	225	0
3	370	24
3	375	43
3	400	88
3	520	105
3	550	112
4	245	0
4	260	12
4	395	42
4	410	66
4	430	117
4	585	129
5	245	0
5	245	0
5	395	24
5	395	43
5	450	104
5	600	118
6	280	0
6	280	32
6	430	58
6	430	71
6	500	117
6	650	127

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