CoCoCo:
Cooperation-Competition-Coordination

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Motivation: Technological Innovation for Climate Change

- Technological innovation must shift towards clean energy to tackle climate challenges.
- Key determinants of direction of technology:
  - Unpriced externalities
  - Varying markups
  - Heterogeneous ideologies
  - Social considerations like inequality
  - Coordination failures (underexplored)
- Existing literature addresses the first two extensively.

- *Proposal:* Extend the directed technological change framework (Acemoglu, 1998, 2002) to measure how coordination influences technological direction.

Cooperation, Competition, and Coordination

- **Cooperation:** Firms contribute to foundational knowledge (small ideas), benefitting all players.
- **Competition:** Firms race to achieve breakthrough innovations (big ideas) with winner-takes-all rewards.
- **Coordination:** Policies align cooperation and competition for targeted innovation progress.

Game Setup: Innovation Allocation Between Sectors

Players: Two firms (A and B).

Actions: Each firm allocates:

- Small idea effort (a^i_V, a^i_W) to contribute to knowledge stocks x_V and x_W in the *Efficiency* (V) and *Sustainability* (W) sectors.
- Big idea effort (c^i_V, c^i_W) to achieve breakthroughs in V and W.

Knowledge Stocks: Aggregate small ideas:

$$x_V = a^A_V + a^B_V, \quad x_W = a^A_W + a^B_W.$$

Big Idea Success: The probability of success in sector $j \in \{V, W\}$ is:

$$\Pr(S^i_j) = 1 - e^{-\lambda_j(x_j)\cdot c^i_j / C_j}, \quad \lambda_j(x_j) = \frac{x_j^\beta}{1 + x_j^\beta}, \quad C_j = c^A_j + c^B_j, \quad \beta > 1.$$

Payoff for Firm i:

$$\Pi^i = R_V \cdot \Pr(S^i_V) + R_W \cdot \Pr(S^i_W) - k_a \left((a^i_V)^2 + (a^i_W)^2\right) - k_c \left((c^i_V)^2 + (c^i_W)^2\right).$$

Proposition: Strategic Complementarity and Substitutability

1. **Strategic Complementarity:** If $\frac{\partial^2 \Pi}{\partial e_i \partial \bar{e}} = \Pi_{12}(e_i, \bar{e}) > 0$, the game exhibits strategic complementarity.
2. **Strategic Substitutability:** If $\frac{\partial^2 \Pi}{\partial e_i \partial \bar{e}} = \Pi_{12}(e_i, \bar{e}) < 0$, the game exhibits strategic substitutability.

Proposition: The model exhibits:

- *Strategic Complementarity* in small ideas effort (a^i_j) when the convexity of the arrival rate function exceeds the growth in marginal costs of effort, ensuring the marginal benefit of one firm's effort increases with the effort of others.
- *Strategic Substitutability* in big ideas effort (c^i_j) due to competition over $\frac{c^i_j}{C_j}$. As one firm increases its big idea effort, it reduces the marginal benefit of others' efforts by diluting their relative contribution to total success probability.

Characterization of Symmetric Nash Equilibrium (SNE)

In the SNE, each firm maximizes its own payoff, taking the other firm's effort as given:

$$\max_{a^i_j, c^i_j} \Pi^i = R_V \cdot \Pr(S^i_V) + R_W \cdot \Pr(S^i_W) - k_a \left((a^i_V)^2 + (a^i_W)^2\right) - k_c \left((c^i_V)^2 + (c^i_W)^2\right),$$

$$\text{where, } \Pr(S^i_j) = 1 - e^{-\lambda_j(x_j)\cdot \frac{c^i_j}{C_j}}, \quad \lambda_j(x_j) = \frac{x_j^\beta}{1 + x_j^\beta}, \quad x_j = a^i_j + a^{-i}_j, \quad C_j = c^i_j + c^{-i}_j.$$

Optimal efforts:

- **Small idea effort (a^i_j):** $a^i_j = \dfrac{R_j \cdot c^i_j \cdot e^{-\lambda_j(2a^i_j)\cdot \frac{c^i_j}{2c^i_j}} \cdot \frac{\beta\cdot(2a^i_j)^{\beta-1}}{(1+(2a^i_j)^\beta)^2}}{2k_a}$ **Big idea effort (c^i_j):** $c^i_j = \dfrac{R_j \cdot \lambda_j(x_j)\cdot e^{-\lambda_j(x_j)\cdot \frac{c^i_j}{C_j}} \cdot \frac{C_j - c^i_j}{C_j^2}}{2k_c}.$

Key Features:

- *Small Ideas:* Firms underinvest in small idea efforts due to the lack of coordination and free-riding on shared knowledge spillovers.
- *Big Ideas:* Firms overcompete in big idea efforts, as each firm's marginal returns depend on its relative contribution, leading to inefficiencies.

Characterization of Symmetric Cooperative Equilibrium (SCE)

In the SCE, firms jointly maximize total welfare:

$$W = \Pi^A + \Pi^B = 2 \cdot \left[R_V \cdot \Pr(S_V) + R_W \cdot \Pr(S_W) - k_a \left((a_V)^2 + (a_W)^2 \right) - k_c \left((c_V)^2 + (c_W)^2 \right) \right],$$

where:

$$\Pr(S_j) = 1 - e^{-\lambda_j(x_j) \cdot \frac{c_j}{C_j}}, \quad \lambda_j(x_j) = \frac{x_j^\beta}{1 + x_j^\beta}, \quad x_j = 2a_j, \quad C_j = 2c_j.$$

First-Order Conditions:

- *Small idea effort (a_j):*

$$a_j = \frac{R_j \cdot c_j \cdot e^{-\lambda_j(2a_j) \cdot \frac{c_j}{2c_j}} \cdot \frac{\beta \cdot (2a_j)^{\beta-1}}{(1+(2a_j)^\beta)^2}}{k_a}.$$

- *Big idea effort (c_j):*

$$c_j = \frac{R_j \cdot \lambda_j(x_j) \cdot e^{-\lambda_j(x_j) \cdot \frac{c_j}{C_j}} \cdot \frac{1}{C_j}}{2k_c}.$$

Key Features:

- *Small Ideas:* Higher effort due to internalization of positive spillovers in shared knowledge contributions.
- *Big Ideas:* Coordinated efforts reduce overcompetition, improving overall efficiency.

Equilibrium Technology Ratio

The equilibrium technology ratio given by the relative probabilities of successful innovation:

$$n^{\mathrm{EQ}} = \frac{\mathbb{E}[\text{Successes in } V]}{\mathbb{E}[\text{Successes in } W]} = \frac{\Pr(S_V^A) + \Pr(S_V^B)}{\Pr(S_W^A) + \Pr(S_W^B)} = \frac{2 \cdot \Pr(S_V)}{2 \cdot \Pr(S_W)} = \frac{\Pr(S_V)}{\Pr(S_W)}.$$

where, $\Pr(S_j) = 1 - e^{-\lambda_j(x_j)\cdot\frac{c_j}{C_j}}, \quad \lambda_j(x_j) = \frac{x_j^\beta}{1+x_j^\beta}, \quad x_j = 2a_j^*, \quad C_j = 2c_j^*.$

Final Expression for n^{EQ}: $n^{\mathrm{EQ}} = \frac{\Pr(S_V)}{\Pr(S_W)} = \frac{1-e^{-\frac{\left(\frac{R_V}{k_a^V}\right)^\beta}{2\cdot\left[1+\left(\frac{R_V}{k_a^V}\right)^\beta\right]}}}{1-e^{-\frac{\left(\frac{R_W}{k_a^W}\right)^\beta}{2\cdot\left[1+\left(\frac{R_W}{k_a^W}\right)^\beta\right]}}}.$

Comparative Statics: What Affects the Direction of Innovation?

1. Sectoral Returns (R_j):

- Higher R_j incentivizes small idea efforts (a_j), increasing spillovers (x_j) and arrival rates ($\lambda_j(x_j)$).
- *Effect:* If $R_V > R_W$, n^{EQ} shifts toward sector V.

2. Small Idea Costs (k_a^j):

- Lower k_a^j increases small idea efforts (a_j), boosting $\lambda_j(x_j)$.
- *Effect:* If $k_a^V < k_a^W$, n^{EQ} shifts toward sector V.

3. Strategic Complementarity (β):

- Higher $\beta > 1$ amplifies the role of small ideas via spillovers.
- *Effect:* As $\beta \to \infty$, the sector with the larger R_j / k_a^j dominates.

Note: The effects of competition across sectors cancels out in this model.

Appendix: Derivation of Symmetric Nash Efforts (SNE)

Maximizing Individual Payoff:

$$\Pi^i = R_V \cdot \Pr(S_V^i) + R_W \cdot \Pr(S_W^i) - k_a \left((a_V^i)^2 + (a_W^i)^2 \right) - k_c \left((c_V^i)^2 + (c_W^i)^2 \right),$$

$$\text{where } \Pr(S_j^i) = 1 - e^{-\lambda_j(x_j) \cdot \frac{c_j^i}{C_j}}, \quad \lambda_j(x_j) = \frac{x_j^\beta}{1 + x_j^\beta}, \quad x_j = 2a_j, \quad C_j = 2c_j.$$

First-Order Conditions:

- *Small Ideas Effort* (a_j^i): $\frac{\partial \Pi^i}{\partial a_j^i} = R_j \cdot c_j^i \cdot e^{-\lambda_j(x_j) \cdot \frac{c_j^i}{C_j}} \cdot \frac{\beta \cdot x_j^{\beta-1}}{(1+x_j^\beta)^2} - 2k_a a_j^i = 0,$

$$a_j^{\text{NE}} = \frac{R_j \cdot c_j^{\text{NE}} \cdot e^{-\lambda_j(2a_j^{\text{NE}}) \cdot \frac{c_j^{\text{NE}}}{2c_j^{\text{NE}}}} \cdot \frac{\beta \cdot (2a_j^{\text{NE}})^{\beta-1}}{(1+(2a_j^{\text{NE}})^\beta)^2}}{2k_a}.$$

- *Big Ideas Effort* (c_j^i): $\frac{\partial \Pi^i}{\partial c_j^i} = R_j \cdot \lambda_j(x_j) \cdot e^{-\lambda_j(x_j) \cdot \frac{c_j^i}{C_j}} \cdot \frac{C_j - c_j^i}{C_j^2} - 2k_c c_j^i = 0,$

$$c_j^{\text{NE}} = \frac{R_j \cdot \lambda_j(x_j) \cdot e^{-\lambda_j(x_j) \cdot \frac{1}{2}} \cdot \frac{1}{2}}{2k_c}.$$

Appendix: Derivation of Strategic Complementarity

Strategic Complementarity (Small Ideas):

1. **First Derivative (w.r.t. a_j^i):**

$$\frac{\partial \Pi^i}{\partial a_j^i} = R_j \cdot c_j^i \cdot e^{-\lambda_j(x_j)\cdot \frac{c_j^i}{C_j}} \cdot \frac{\beta \cdot x_j^{\beta-1}}{(1+x_j^{\beta})^2} - 2k_a a_j^i.$$

2. **Cross-Partial Derivative (w.r.t. a_j^i and a_j^{-i}):**

$$\frac{\partial^2 \Pi^i}{\partial a_j^i \partial a_j^{-i}} = R_j \cdot c_j^i \cdot e^{-\lambda_j(x_j)\cdot \frac{c_j^i}{C_j}} \cdot \left[-\frac{\beta \cdot c_j^i}{C_j} \cdot \frac{x_j^{\beta-1}}{(1+x_j^{\beta})^2} + \frac{\beta(\beta-1)\cdot x_j^{\beta-2}}{(1+x_j^{\beta})^2} - \frac{2\beta^2 \cdot x_j^{2\beta-1}}{(1+x_j^{\beta})^3} \right].$$

Sign: For $\beta > 1$, and small x_j, the positive terms dominate, establishing strategic complementarity.

Appendix: Derivation of Strategic Substitutability

1. First Derivative: Marginal returns to firm i's big ideas effort (c_j^i):

$$\frac{\partial \Pi^i}{\partial c_j^i} = R_j \cdot e^{-\lambda_j(x_j) \cdot \frac{c_j^i}{C_j}} \cdot \lambda_j(x_j) \cdot \frac{c_j^{-i}}{C_j^2} - 2k_c c_j^i.$$

2. Cross-Partial Derivative: Effect of other firm's effort (c_j^{-i}) on marginal returns to c_j^i:

$$\frac{\partial^2 \Pi^i}{\partial c_j^i \partial c_j^{-i}} = R_j \cdot \lambda_j(x_j) \cdot e^{-\lambda_j(x_j) \cdot \frac{c_j^i}{C_j}} \cdot \left[\frac{1}{C_j^2} - \frac{2c_j^{-i}}{C_j^3} - \frac{\lambda_j(x_j) \cdot (c_j^i)^2}{C_j^4} \right].$$

3. Parametric Condition for Strategic Substitutability: The game exhibits strategic substitutability if:

$$\frac{1}{C_j^2} < \frac{2c_j^{-i}}{C_j^3} + \frac{\lambda_j(x_j) \cdot (c_j^i)^2}{C_j^4}.$$

Sign The above inequality ensures that the negative terms dominate, leading to strategic substitutability.

- **Crowding Effect:** Increasing c_j^{-i} reduces the marginal benefit of c_j^i by increasing C_j, diluting c_j^i's impact on the success probability.
- **High Competition Sensitivity:** Substitutability is stronger when c_j^{-i} is large, $\lambda_j(x_j)$ is high, and total effort C_j is small, amplifying competition over success probability.